High order QCD evolution in the Regge limit

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Based on: 1501.03754
+1604.07417 (w/ Matti Herranen)
[NNLO BK equation in N=4 SYM]

RBRC Virtual Workshop: Small-x Physics in the EIC Era, Dec. 15th 2021

Multi-loop corrections at small-x:

why?

1. physics case: *not this talk*

2. feasibility: *how hard?*

I hope to convince you that small-x can fit into a more or less standard automation framework.

Plan:
- Evolution equations
- 'spacelike-timelike' correspondence
- highlights: why helpful

small-x evolution equations

Nonlinear $\longrightarrow$ Linear

colors

$\downarrow$

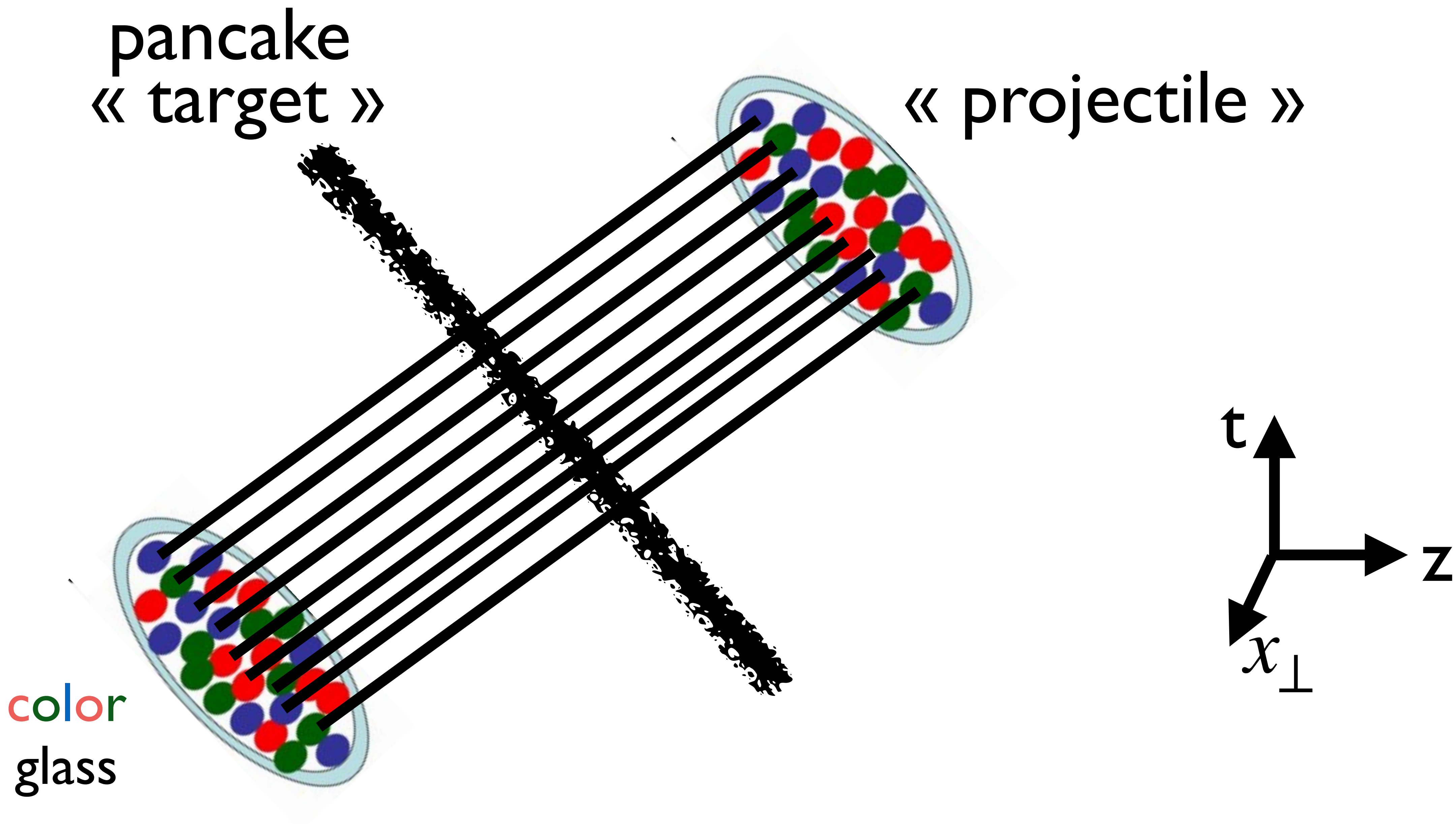
planar (large N_c)

B-JIMWLK	BFKL (/BKP)
BK	planar BFKL

DGLAP

which one to aim for?

tracking color



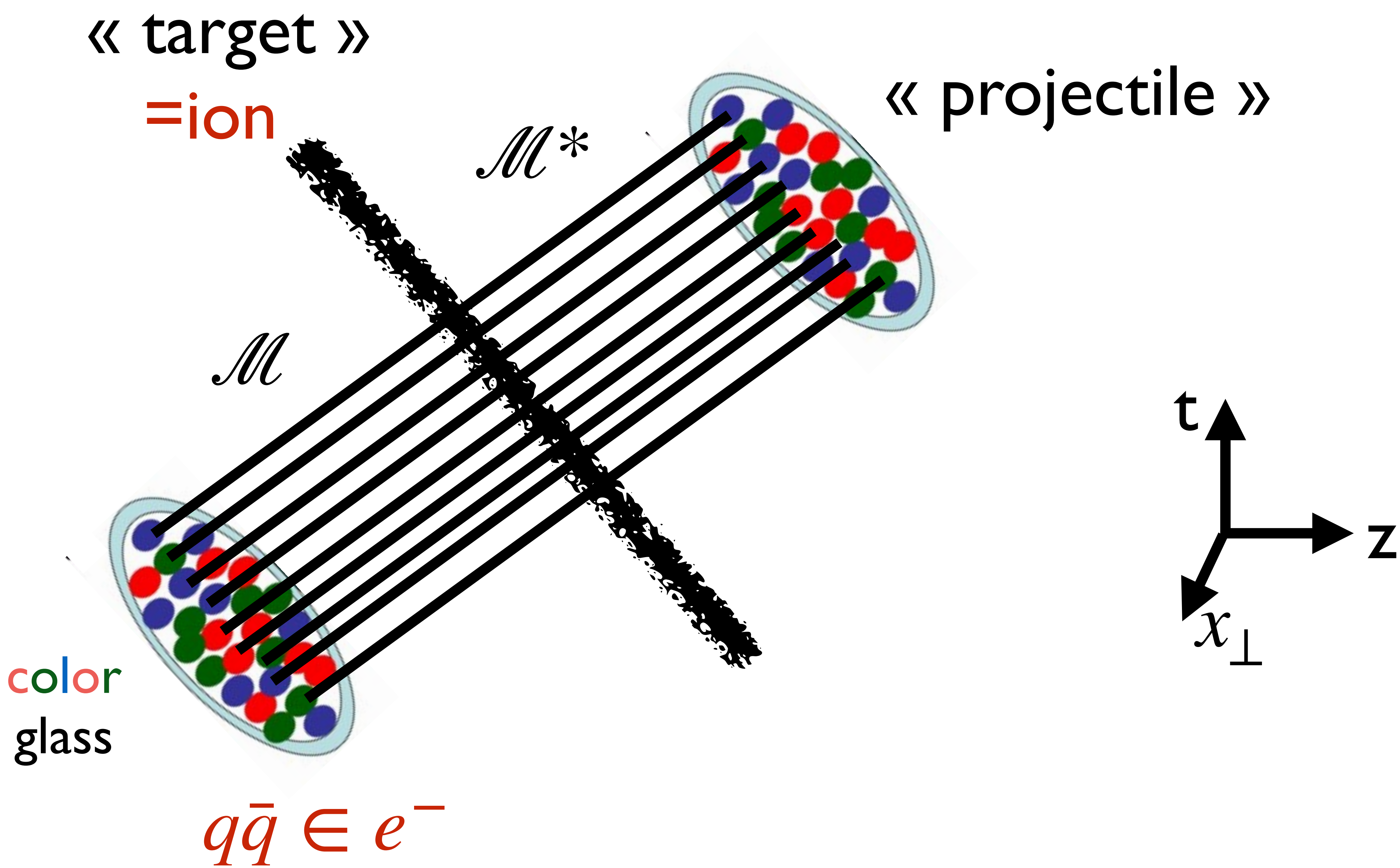
amplitude = product of color rotation for each charge

$$\mathscr{M} \propto \langle U(\mathbf{x}_1) \cdots U(\mathbf{x}_n) \rangle_{\text{target}}$$

Comments

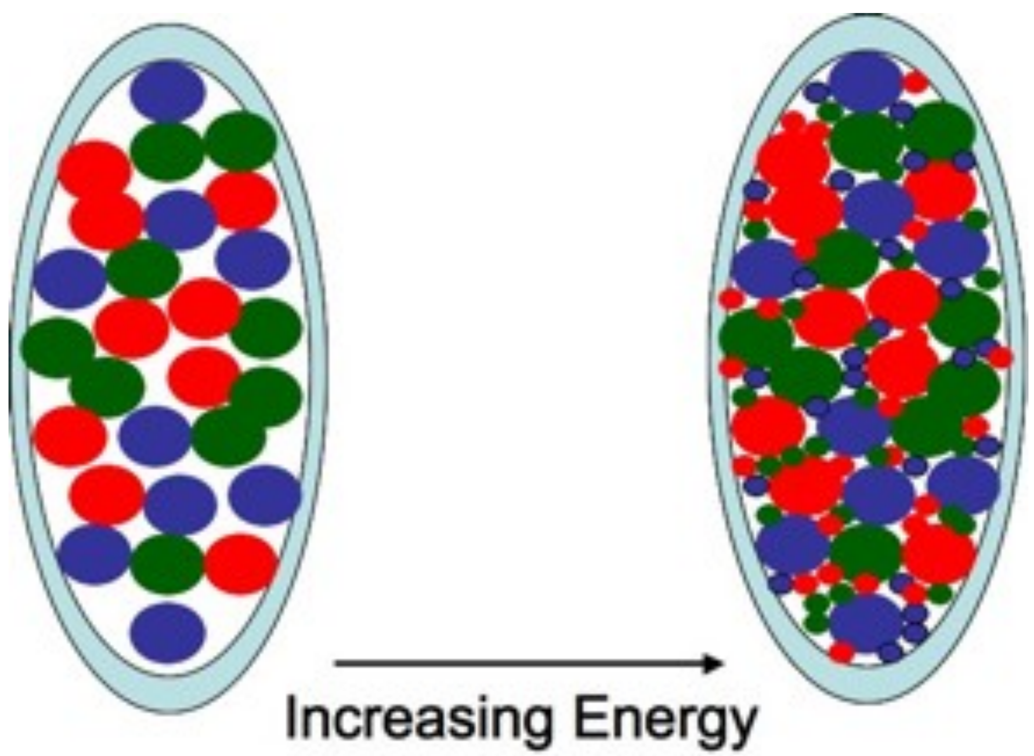
a well-defined problem for precision calculation:

- Target can be strong: U=O(1)
- Projectile weak: $n \ll \alpha_s^{-1}$ (EIC!)
- inclusive cross-sections $(\mathrm{H_1 H_2} \rightarrow \mathrm{jet + X})$ vs $(pX -> pX)$: mathematically *similar* [identical?]

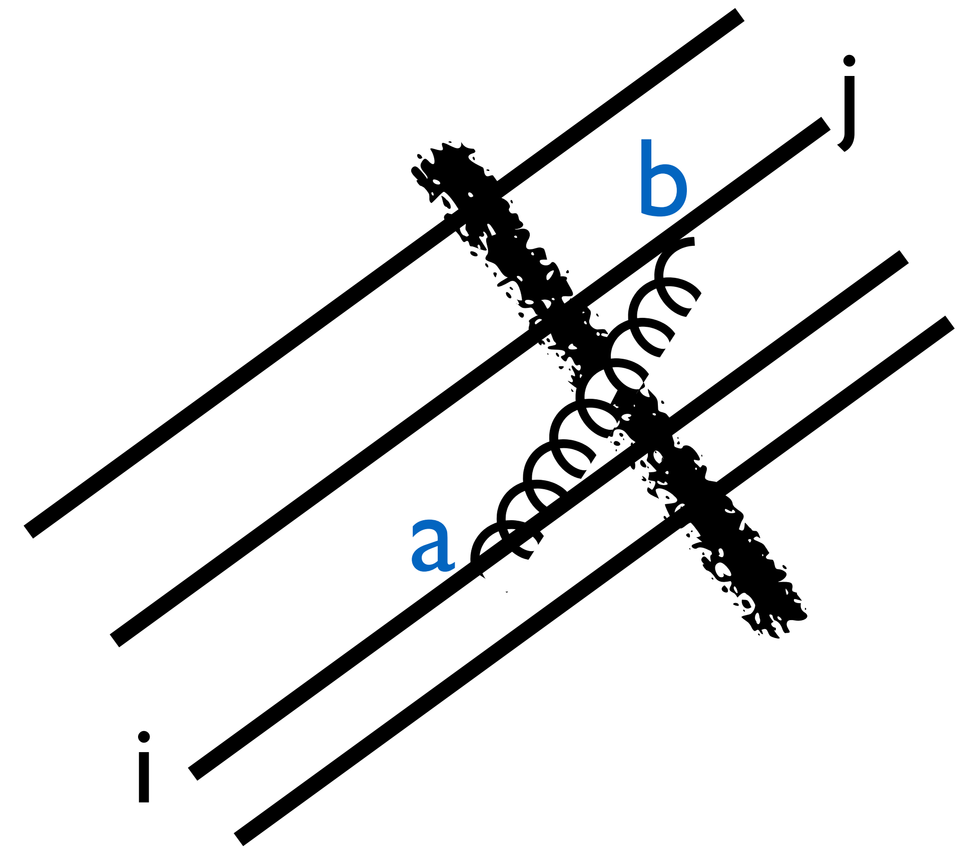


« target »
=ion
$\mathscr{M}^*$
« projectile »
$\mathscr{M}$
t
z
$x_\perp$
color
glass
$q\bar{q} \in e^-$

rapidity evolution: B-JIMWLK



[cartoon from McLarren '09]

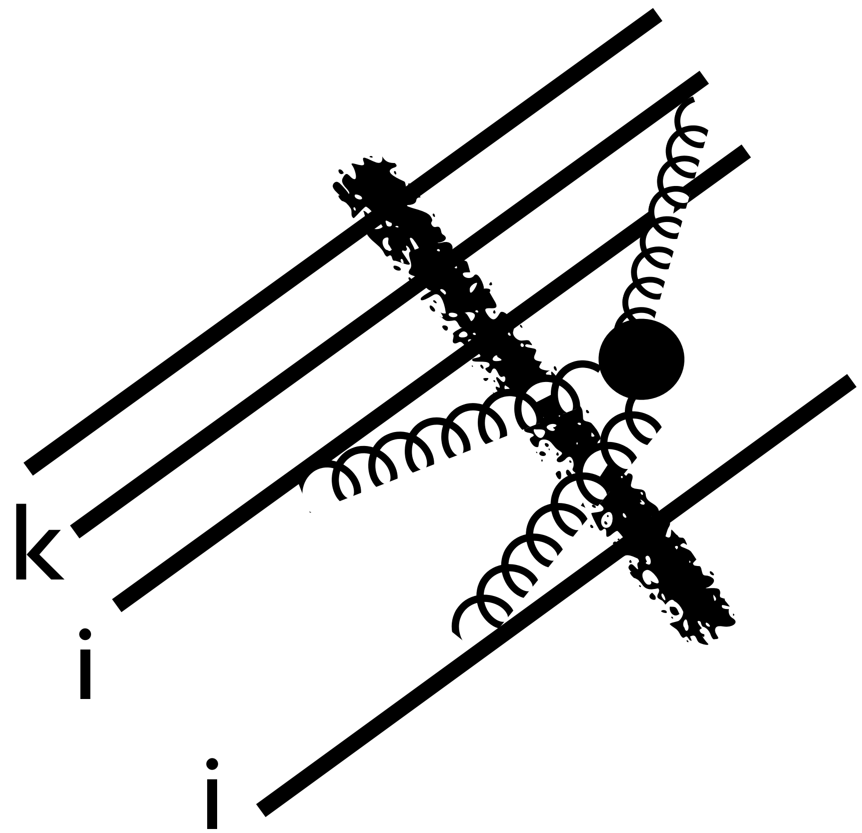


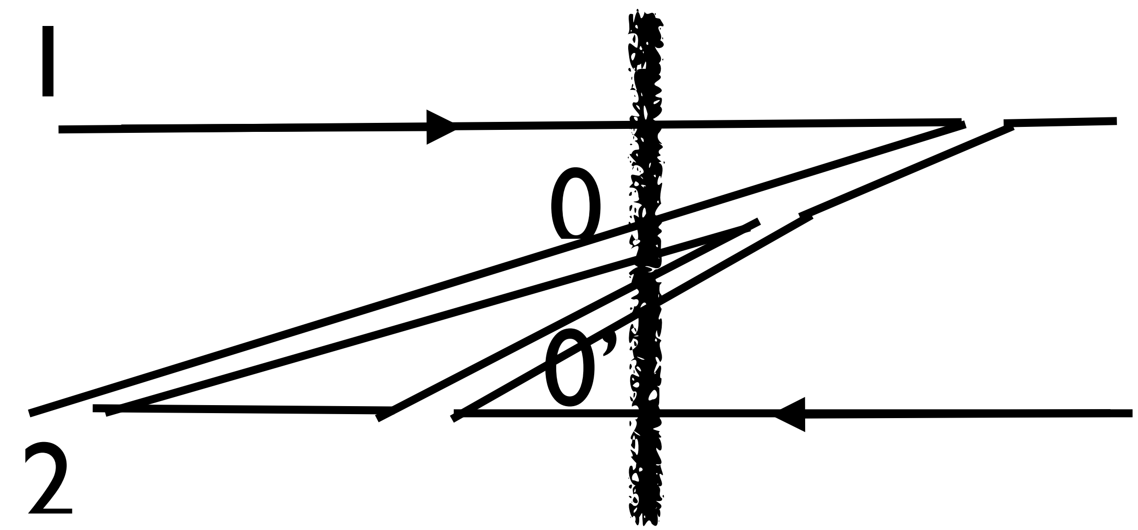
$$\frac{d}{d\eta}U_1\cdots U_n = \frac{\alpha_s}{\pi}\int \frac{d^2 z_0 z_{ij}^2}{z_{0i}^2 z_{0j}^2}\Big(U_1\cdots U_n T_i^a T_j^b {\color{blue}U_0^{ab}} - C_F U_1\cdots U_n\Big)$$

$${\color{red}+O(\alpha_s^2)}$$

SU(3)$_c$ generators

Color vs planar





n->n+2 Wilson lines

$$\frac{d}{d\eta}U_1\cdots U_n \supset \sum_{i,j,k} U_1\cdots U_n T_i^a T_j^b T_k^c U_0^{aa'} U_{0'}^{bb'} f^{abc}(\cdots)$$

NLO: [Kovner,Mulian&Lublinski '14, Balitsky&Chirilli '14, SCH '15]

dipole → 3 dipoles

$$\frac{d}{d\eta}U_{12} \supset U_{10}U_{00'}U_{0'2}$$

NLO: [Balitsky-Chirilli '07]

Linearization is easy

Weak target: expand Wilson lines around identity:

$$U_{ij} \to 1 + \epsilon \mathscr{U}_{ij}$$

(planar, nonlinear) BK: $$\frac{d}{d\eta} U_{12} = \frac{\lambda}{8\pi^2} \int \frac{d^2 z_0}{\pi} \frac{z_{12}^2}{z_{10}^2 z_{02}^2} \left(U_{10} U_{02} - U_{12} \right)$$

planar BFKL: $$\frac{d}{d\eta} \mathscr{U}_{12} = \frac{\lambda}{8\pi^2} \int \frac{d^2 z_0}{\pi} \frac{z_{12}^2}{z_{01}^2 z_{02}^2} \left(\mathscr{U}_{10} + \mathscr{U}_{02} - \mathscr{U}_{12} \right)$$

(non-planar: $U(\mathbf{x}) = e^{igT^a W^a(\mathbf{x})}$, $W^a(\mathbf{x})$ = reggeized gluon)

small-x evolution equations

Nonlinear $\xrightarrow{\text{easy}}$ Linear

	Nonlinear	Linear	
colors	~~B-JIMWLK~~	~~BFKL (/BKP)~~	hard
planar (large N_c)	BK	planar BFKL	

=our focus in 2016 (NNLO in N=4 SYM)

Our motivations then:

1. BFKL convergence is slow:
   how to resum large effects

[~'98]
[Salam;
Ball,Forte ~'00,…
Iancu,Mueller et al '14]

2. Multi-loops are standard in many QCD contexts

3. Purely theoretical:

-partonic amplitudes in Regge limit:
 unique insight into scattering at high loops

-generally interesting limit (pomeron→graviton in AdS CFT,…)

-new qualitative features @NNLO(non-planar pomeron loop…)

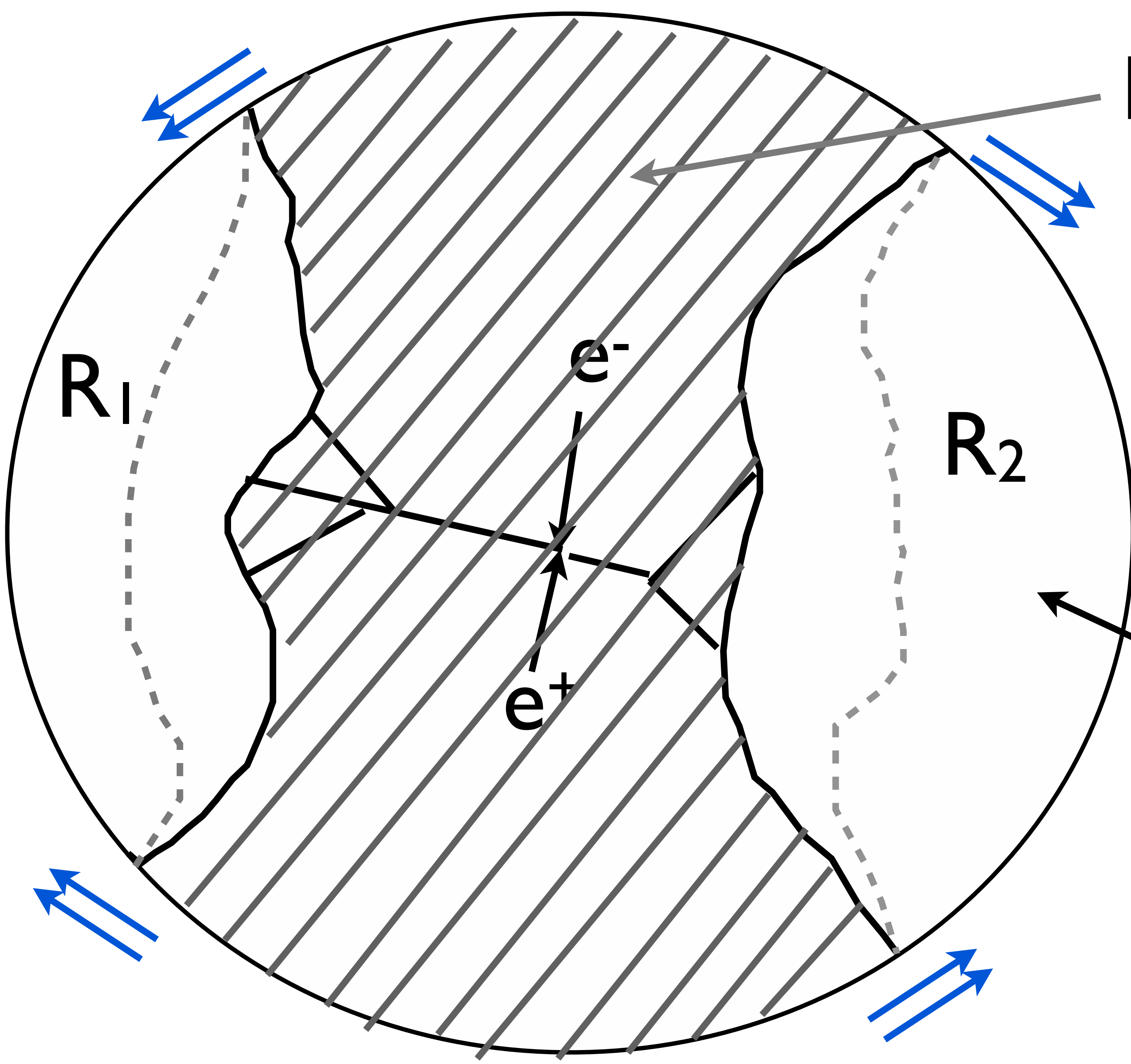
Tool: A surprising equivalence

Rapidity evolution (small x amplitude)	$\Leftrightarrow$	Soft evolution (small E cross-section)
Transparent		Allowed region
Opaque		Vetoed region
Rapidity Y		Soft veto
dipoles saturate		veto region grows

[Weigert ’03; Hatta ’08-…, Hofman& Maldacena ‘08]

Non-global logs

Q: Cross-section for $e^+e^- \to X$,
less than E_0 energy outside some region R



Radiation vetoed 'opaque'

'transparent' potato-shaped regions

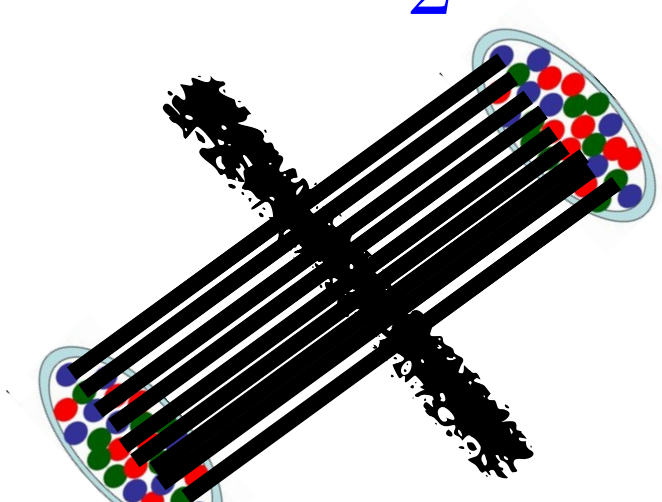
- Quantitative equivalence:

BK: $$\frac{d}{d\eta}U_{12} = \frac{\lambda}{8\pi^2}\int \frac{d^2 z_0}{\pi}\frac{z_{12}^2}{z_{10}^2 z_{02}^2}(U_{10}U_{02} - U_{12})$$ Rapidity evolution

BMS: $$E\frac{d}{dE}U_{12} = \frac{\lambda}{8\pi^2}\int \frac{d^2 \Omega_0}{4\pi}\frac{\alpha_{12}}{\alpha_{10}\alpha_{02}}(U_{10}U_{02} - U_{12})$$ Soft evolution

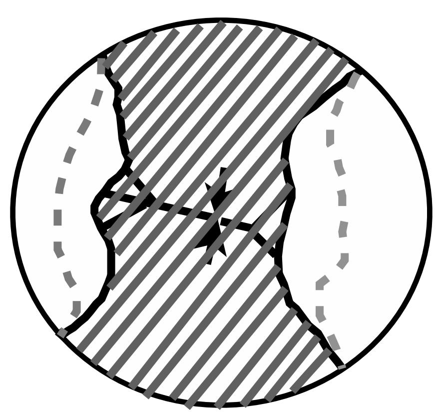
- Conformal (stereographic) symmetry of pQCD:

$$\alpha_{ij} \equiv \frac{1-\cos\theta_{ij}}{2} \to z_{ij}^2 \equiv (z_i - z_j)^2, \qquad \frac{d\Omega}{4\pi} \to \frac{d^2 z}{\pi}$$



$$\longleftrightarrow$$

$$x^+ \leftrightarrow 1/x^+$$

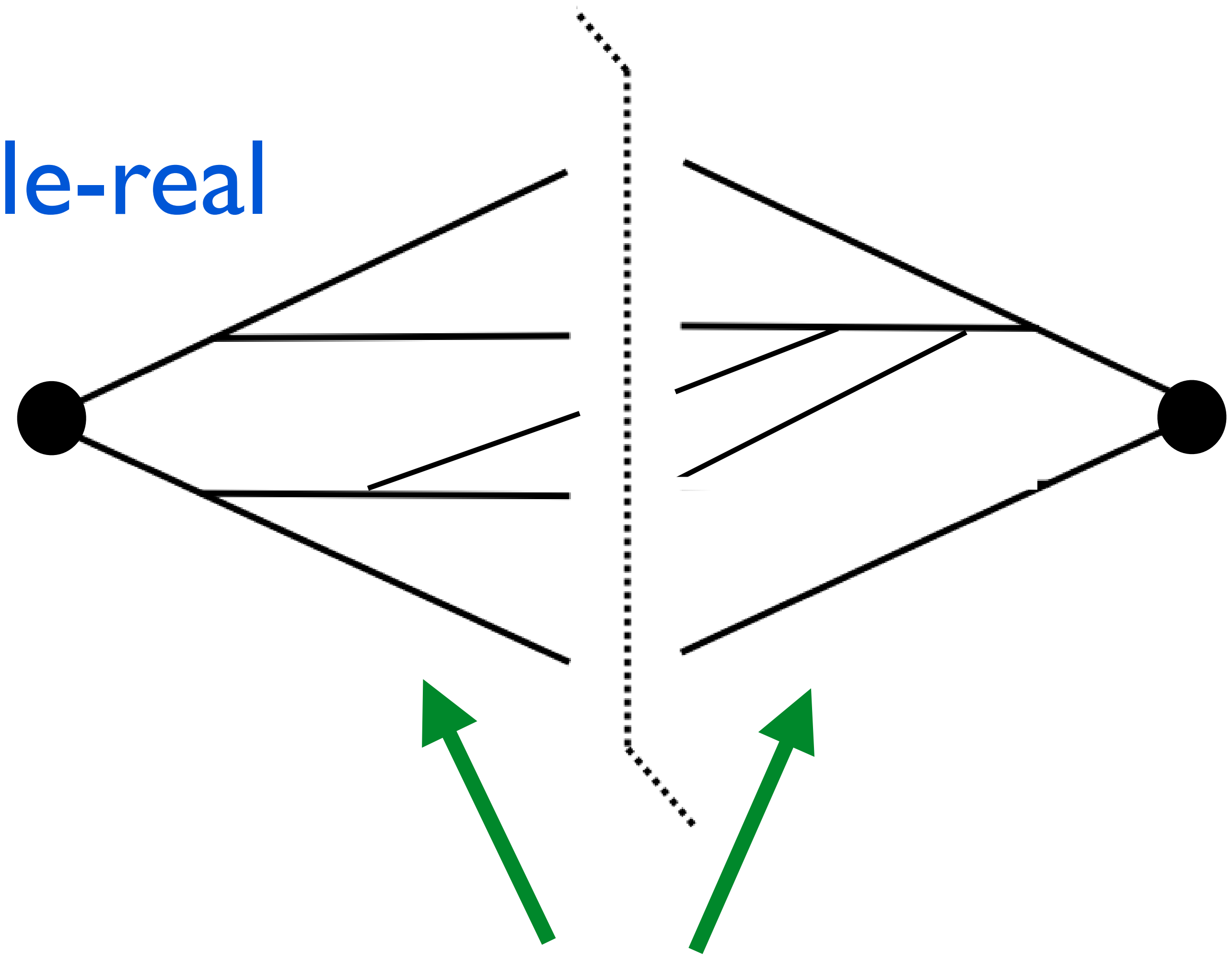


[Weigert '03; Hatta '08-…, Hofman& Maldacena '08]

why is it useful?

break 3-loop calculation into physical building blocks!!

ex: triple-real

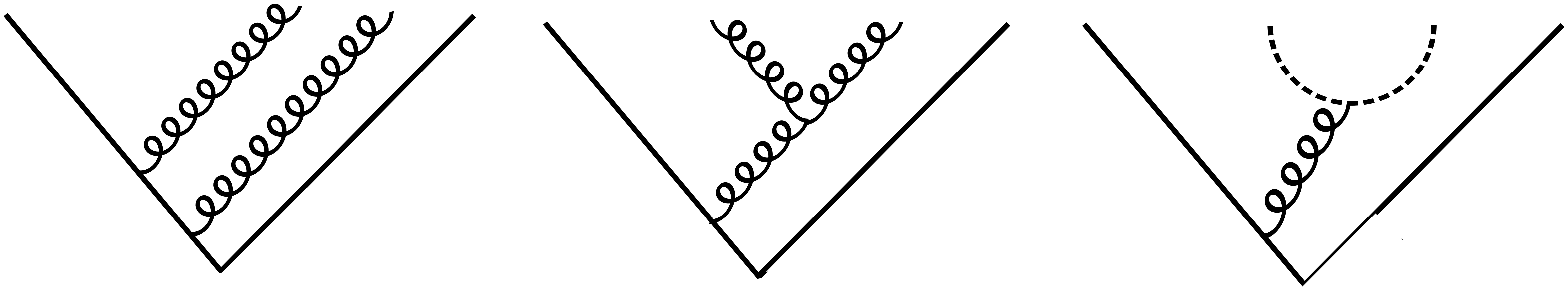


on-shell trees

(2 hard partons
+ 3 soft gluons)

According to [Mueller 1804.07249]:
correspondence diagram by diagram! $\rightarrow$ running couplings

let's describe NLO in detail:



[Catani&Grazzini '99]

Square of tree-level soft current relatively simple:

$$|\mathcal{S}|^2 = \frac{s_{12}}{s_{10}s_{00'}s_{0'2}}\left[1 + \frac{s_{12}s_{00'} + s_{10}s_{0'2} - s_{10'}s_{20}}{2(s_{10}+s_{10'})(s_{02}+s_{0'2})}\right]$$

N=4SYM

$$+ (n_F - 4)\frac{s_{12}}{s_{00'}(s_{10}+s_{10'})(s_{20}+s_{20'})}$$

$$+ (2 + n_s - 2n_F)\frac{(s_{10}s_{20'} - s_{10'}s_{20'})^2}{2s_{00'}^2(s_{10}+s_{10'})^2(s_{20}+s_{20'})^2}$$

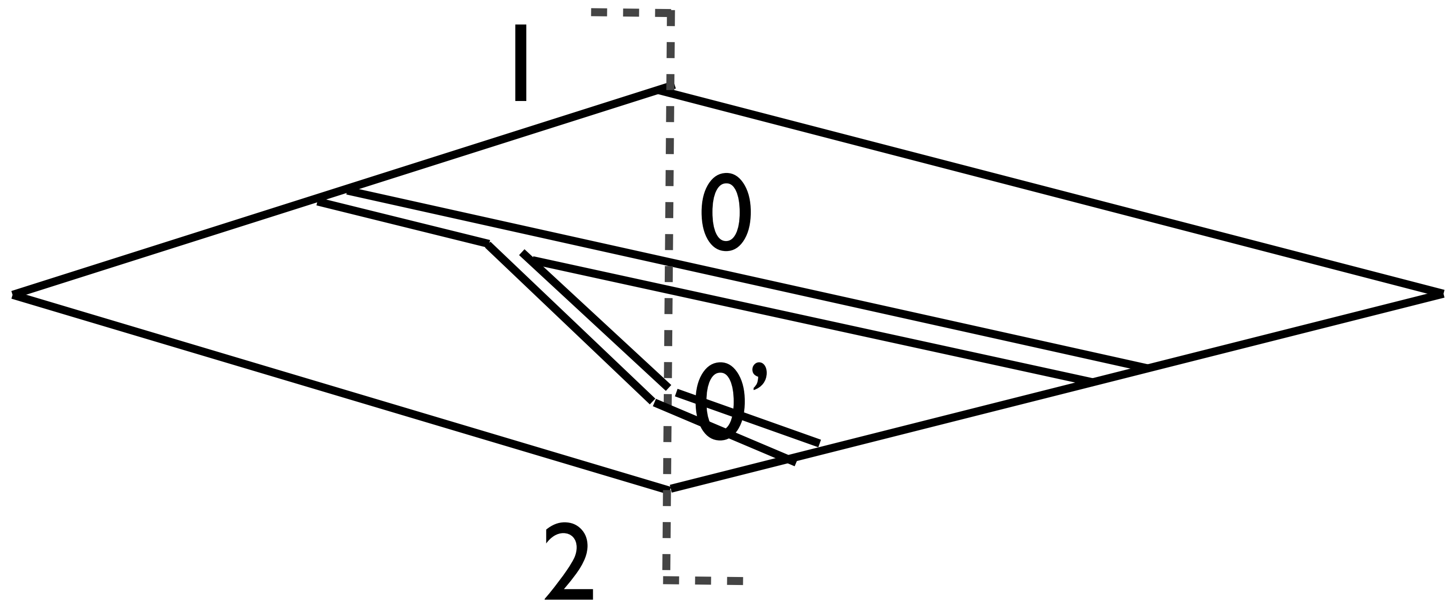
general gauge thy

[SCH, '15]

- Crucial step: subtract subdivergences
- Two soft gluons $\neq$ [one soft]^2

$$|\mathcal{S}|^2 = \frac{s_{12}}{s_{10}s_{00'}s_{0'2}}\left[1 + \frac{s_{12}s_{00'} + s_{10}s_{0'2} - s_{10'}s_{20}}{2(s_{10}+s_{10'})(s_{02}+s_{0'2})}\right]$$

- Amplitude depends on ratio of soft energies
- NLO BK ~ the integral over that ratio

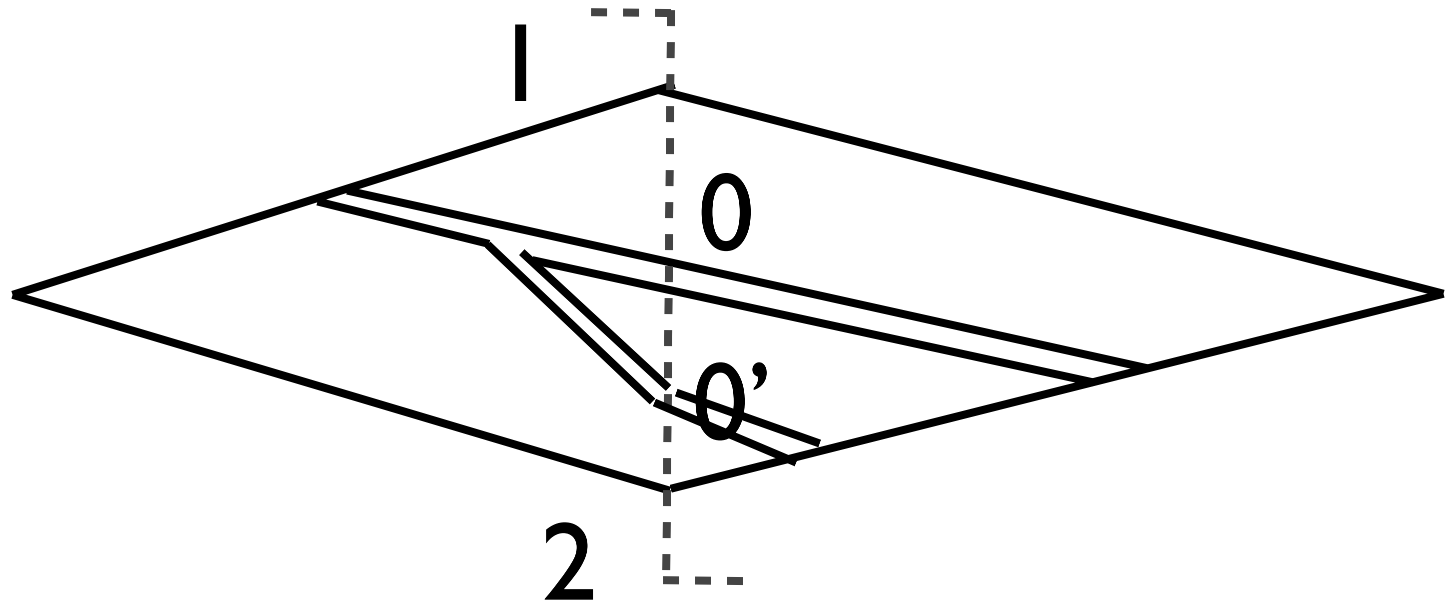


0. Pull out angular/transverse integrals:

$$E\frac{d}{dE}U_{12} \supset \int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K_{[1\,00'\,2]} U_{10}U_{00'}U_{0'2}$$

1. Integrate over relative energies:

$$K_{[1\,00'\,2]} = \int_0^\infty \tau d\tau \left[\left|\mathcal{S}(\tau\beta_0, \beta_{0'})\right|^2 \qquad \right]$$



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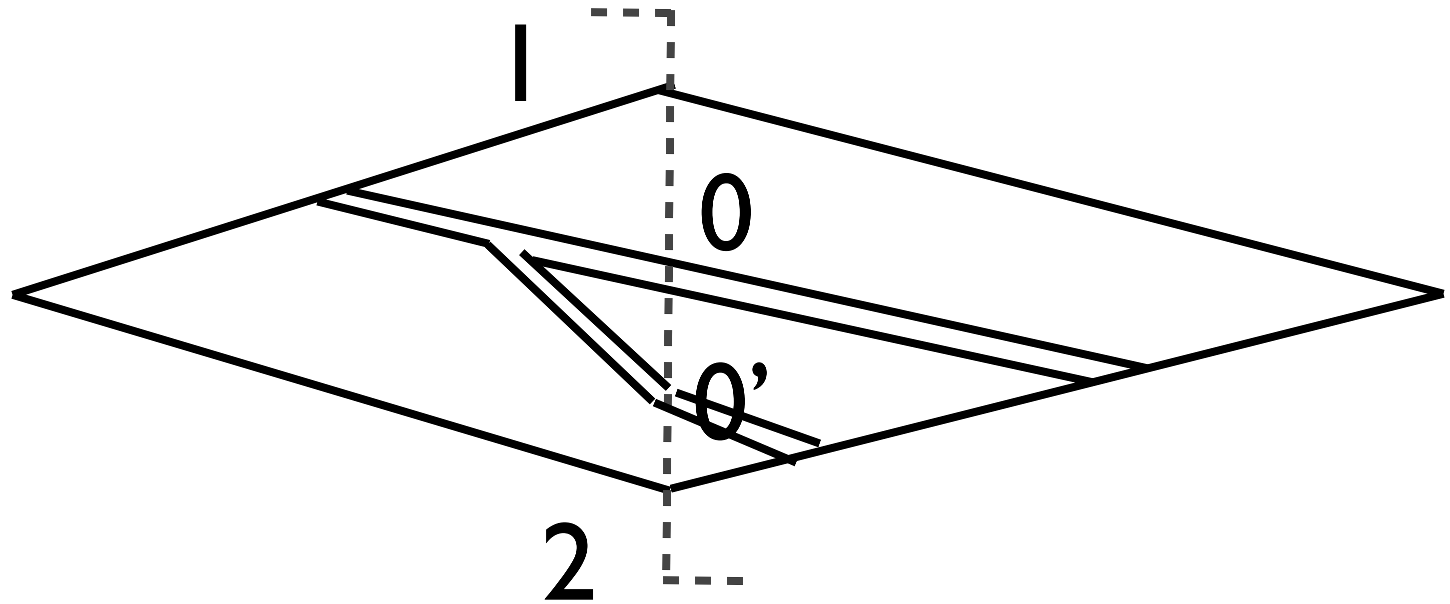
1. Integrate over relative energies:

$$K_{[1\,00'\,2]} = \int_0^\infty \tau d\tau \left[\left|\mathcal{S}(\tau\beta_0,\beta_{0'})\right|^2 - \Big|_{\tau\to 0}\theta(\tau<1) - \Big|_{\tau\to\infty}\theta(\tau>1)\right]$$

Subtract iterations of LO



0. Pull out angular/transverse integrals:

$$E\frac{d}{dE}U_{12} \supset \int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K_{[1\,00'\,2]} U_{10}U_{00'}U_{0'2}$$

1. Integrate over relative energies:

$$K_{[1\,00'\,2]} = \int_0^\infty \tau d\tau \left[\begin{array}{l} \left|\mathcal{S}(\tau\beta_0,\beta_{0'})\right|^2 \\ -\Big|_{\tau\to 0}\theta(Q^2_{[1\tau 00']}<Q^2_{[10'2]}) \\ -\Big|_{\tau\to\infty}\theta(Q^2_{[00'2]}<Q^2_{1\tau 02]}) \end{array} \right]$$

Best: order w/Lorentz-invariant trans. mom $Q^2_{[i0j]} \equiv \dfrac{s_{i0}s_{0j}}{s_{ij}}$

- That's basically it! NLO (planar) evolution:

$$K^{(2)}U_{12} = \int_{\beta_0,\beta_{0'}} \frac{\alpha_{12}}{\alpha_{10}\alpha_{00'}\alpha_{0'2}} K^{(2)}_{[1\,00'\,2]} \left(U_{10}U_{02}+U_{10'}U_{0'2}-2U_{10}U_{00'}U_{0'2}\right) + \gamma_K^{(2)} K^{(1)} U_{12}$$

$$K^{(2)}_{[1\,00'\,2]} = 2\log\frac{\alpha_{12}\alpha_{00'}}{\alpha_{10'}\alpha_{02}} + \left(1+\frac{\alpha_{12}\alpha_{00'}}{\alpha_{10}\alpha_{0'2}-\alpha_{10'}\alpha_{02}}\right)\log\frac{\alpha_{10}\alpha_{0'2}}{\alpha_{10'}\alpha_{02}}$$

✔

- Precisely Balitsky&Chirilli's (N=4) result!!!

[Balistky&Chirilli '07,'08]

- Eigenvalues match 'Pomeron trajectory'

[Fadin&Lipatov(&Kotikov) '98;
Ciafaloni&Gamici '98]

simple hard-earned lessons

- use covariant cutoffs:

$$Q^2_{[102]} = \frac{p_1 \cdot p_0 \; p_0 \cdot p_2}{p_1 \cdot p_2} \quad \begin{matrix} < \mu^2 = \text{soft} \\ > \mu^2 = \text{hard} \end{matrix} \qquad Q^2_{[100'2]} = \left(\frac{p_1 \cdot p_0 \; p_0 \cdot p_{0'} p_{0'} \cdot p_2}{p_1 \cdot p_2} \right)^{1/2}$$

(for BK: land automatically on 'conformal dipoles')

- exploit real-virtual cancellations

$$\big(U_{10} U_{02} + \underbrace{U_{10'} U_{0'2}}_{\text{single-real}} - \underbrace{2 U_{10} U_{00'} U_{0'2}}_{\text{double-real}} \big)$$

- fun combinatorics: subtractions @ 3-loops

$$F^{\rm sub}_{[1\,0\,2]} \equiv F_{[1\,0\,2]} = 1, \tag{4.20a}$$

$$F^{\rm sub}_{[1\,00'\,2]} \equiv F_{[1\,00'\,2]} - [1\,0\,0'][1\,0'\,2] - [0\,0'\,2][1\,0\,2], \tag{4.20b}$$

$$\begin{aligned} F^{\rm sub}_{[1\,00'0''\,2]} \equiv\ & F_{[1\,00'0''\,2]} - [1\,0\,0'][1\,0'0''\,2] - [0\,0'\,0''][1\,00''\,2] - [0'\,0''\,2][1\,00'\,2] \\ & -[1\,00'\,0''][1\,0''\,2] - [0\,0'0''\,2][1\,0\,2] \\ & -[1\,0\,0'][1\,0'\,0''][1\,0''\,2] - [0'\,0''\,2][0\,0'\,2][1\,0\,2] - [0\,0'\,0''][1\,0\,0''][1\,0''\,2] \\ & -[0\,0'\,0''][0\,0''\,2][1\,0\,2] - [1\,0\,0'][0'\,0''\,2][1\,0'\,2] - [0'\,0''\,2][1\,0\,0'][1\,0'\,2]. \end{aligned} \tag{4.20c}$$

energy step functions

- Cleanly removes iterations of lower-loop evolution ✓
- left with convergent energy integrals!

NNLO

[Herranen+SCH, '16]

- Triple real at tree-level

  ⇒ extract from known 4-particle integrand ✓

- Double real at one-loop

  ⇒ extract from known one-loop 6-point ✓

  [~'94]

- Single real at two-loops

  ⇒ not needed: contribution really just $\gamma_K^{(3)}$ ✓

- Fully virtual IR divergences at three-loops

  ⇒ not needed: KLN fixes from rest ✓

Schematic result:

explicit transverse functions

$$K^{(3)}U_{12} = \frac{11\pi^4}{45}K^{(1)}U_{12} + \int_{\beta_0,\beta_{0'}} \frac{\alpha_{12}}{\alpha_{10}\alpha_{00'}\alpha_{0'2}} K^{(3)}_{[1\,00'\,2]}\left(U_{10}U_{02}+U_{10'}U_{0'2}-2U_{10}U_{00'}U_{0'2}\right)$$
$$+\int_{\beta_0,\beta_{0'},\beta_{0''}} \frac{\alpha_{12}}{\alpha_{10}\alpha_{00'}\alpha_{0'0''}\alpha_{0''2}} \Big[K^{(3)}_{[1\,00'0''\,2]}\left(2U_{10'}U_{0'2}-2U_{10}U_{00'}U_{0'0''}U_{0''2}\right)$$
$$-(1+P)\left(K^{(3)c.t.}_{[1\,00'0''\,2]}\left(2U_{10'}U_{0'2}-2U_{10}U_{00'}U_{0'2}\right)\right)\Big], \quad (4.34c)$$

(Planar) QCD: expect different functions, similar structure

the supersymmetric result could be independently tested

Tests

- Collinear limit $\nu \to \pm i$ controlled by small-x limit of DGLAP

[Jaroscewicz '83;
Ball, Falgari, Forte, Marzani... 07]
[Velizhanin '15]

$$\omega^{(3)} \to +g^6\Big(\frac{1024}{\gamma^5} - \frac{512}{\gamma^3}\zeta_2 + \frac{576}{\gamma^2}\zeta_3 - \frac{464}{\gamma}\zeta_4 + 840\zeta_5 + 64\zeta_2\zeta_3 + \gamma\Big(-40{\zeta_3}^2 - 373\zeta_6\Big) + \gamma^2\left(-8\zeta_2\zeta_5 - 86\zeta_3\zeta_4 + \frac{1001}{4}\zeta_7\right)\Big)\,. \tag{21}$$

✔

- Analytic expression for m=0 conjectured using Integrability of planar N=4

$$\begin{aligned}\frac{F^{(3)}_{0,\nu}}{32} = {} & -S_5 + 2S_{-4,1} - S_{-3,2} + 2S_{-2,3} - S_{2,-3} - 2S_{3,-2} + 4S_{-3,1,1} + 4S_{1,-3,1} + 2S_{1,-2,2} \\ & +2S_{1,2,-2} + 2S_{2,1,-2} - 8S_{1,-2,1,1} + \zeta_2\big(S_1S_2 - 3S_{-3} + 2S_{-2,1} - 4S_{1,-2}\big) - \tfrac{49}{2}\zeta_4 S_1 \\ & +7\zeta_3\big(2S_{1,-1} + 2(S_1 - S_{-1})\log 2 - S_{-2} - \log^2 2\big) + (8\zeta_{-3,1} - 17\zeta_4)\big(S_{-1} - S_1 + \log 2\big) \\ & -\tfrac{1}{2}\zeta_3 S_2 + 4\zeta_5 - 6\zeta_2\zeta_3 + 8\zeta_{-3,1,1}\,. \end{aligned} \tag{C.3}$$

✔

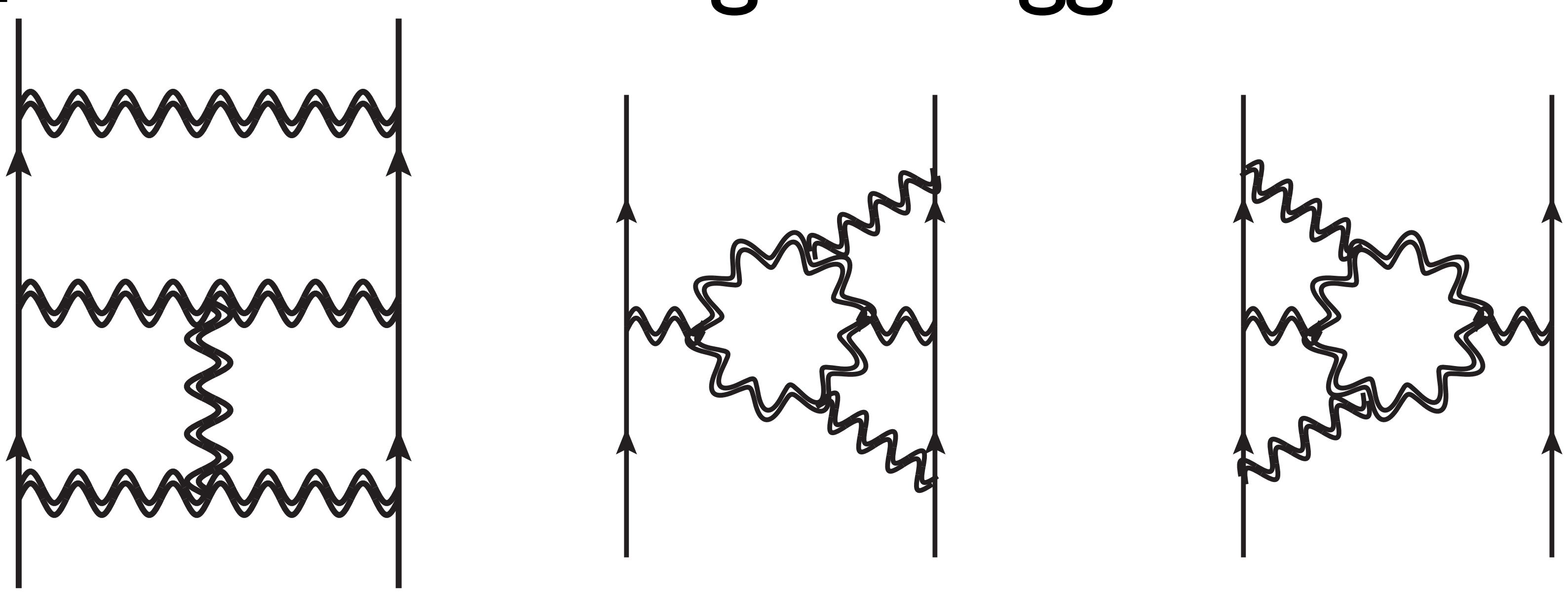
[new results for m>0]

[Gromov, Levkovich-Maslyuk & Sizov, '15]

- Lots of Regge know-how in perturbative scattering amplitude community

  [Bartels, Chachamis, Del Duca, Dixon, Drummond, Duhr, Dulat, Gardi, Henn, Magnea, Mistlberger, Sabio-Vera, Vernazza, … + many others, even in this room!]

- ex: parton scattering in Regge limit at high loops:

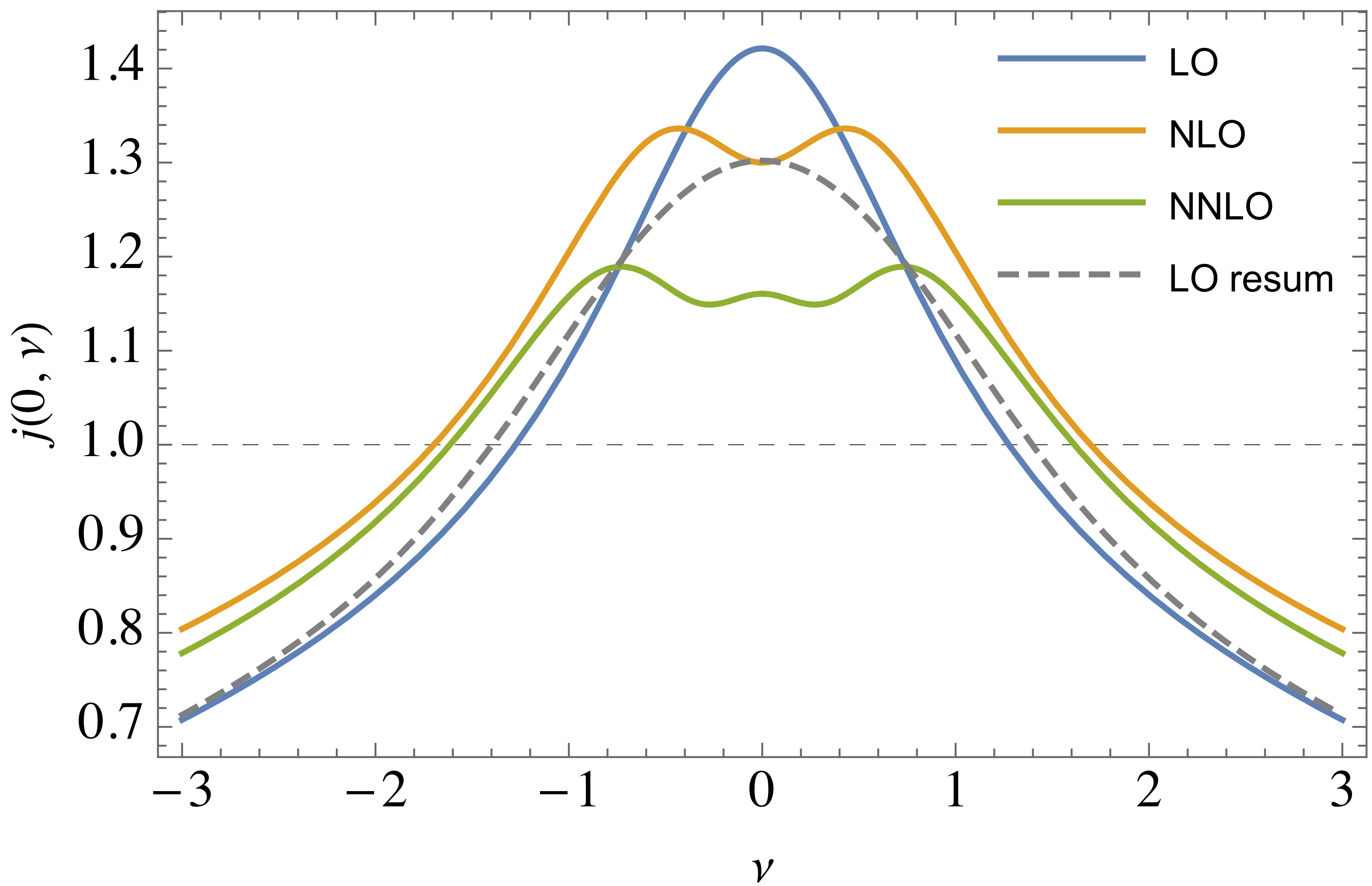


[SCH, Gardi, Reichel, Vernazza '17-'20…]

Conclusions

spacelike-timelike correspondence:
small-x pQCD $\simeq$ solved/automated cross-section calculations

Lots of computable objects:
- -Evolution of color charges
- -of TMDs?
- -N(N)LO impact factors: $e^- \simeq$ virtual $q\bar{q} + \ldots$
- -jets, $\ldots$
- -resummation? how low Q^2 can pQCD handle?



- Pomeron trajectory = linearized eigenvalue

$$U_{ij} = 1 - \frac{1}{N_c}\mathcal{U}_{ij}$$

for eigenfunction: $\mathcal{U}_{m,\nu} = |z_i - z_j|^{i\nu} e^{im\arg(z_i - z_j)}$

$$\frac{d}{d\eta}\mathcal{U}_{m,\nu} = \left[j(m,\nu) - 1\right]\mathcal{U}_{m,\nu} \qquad (\Delta = 2 + i\nu)$$

[see Brower,Polchinski,Strassler&Tan]

[more on DGLAP vs BFKL: use *dimensions* instead of γ]

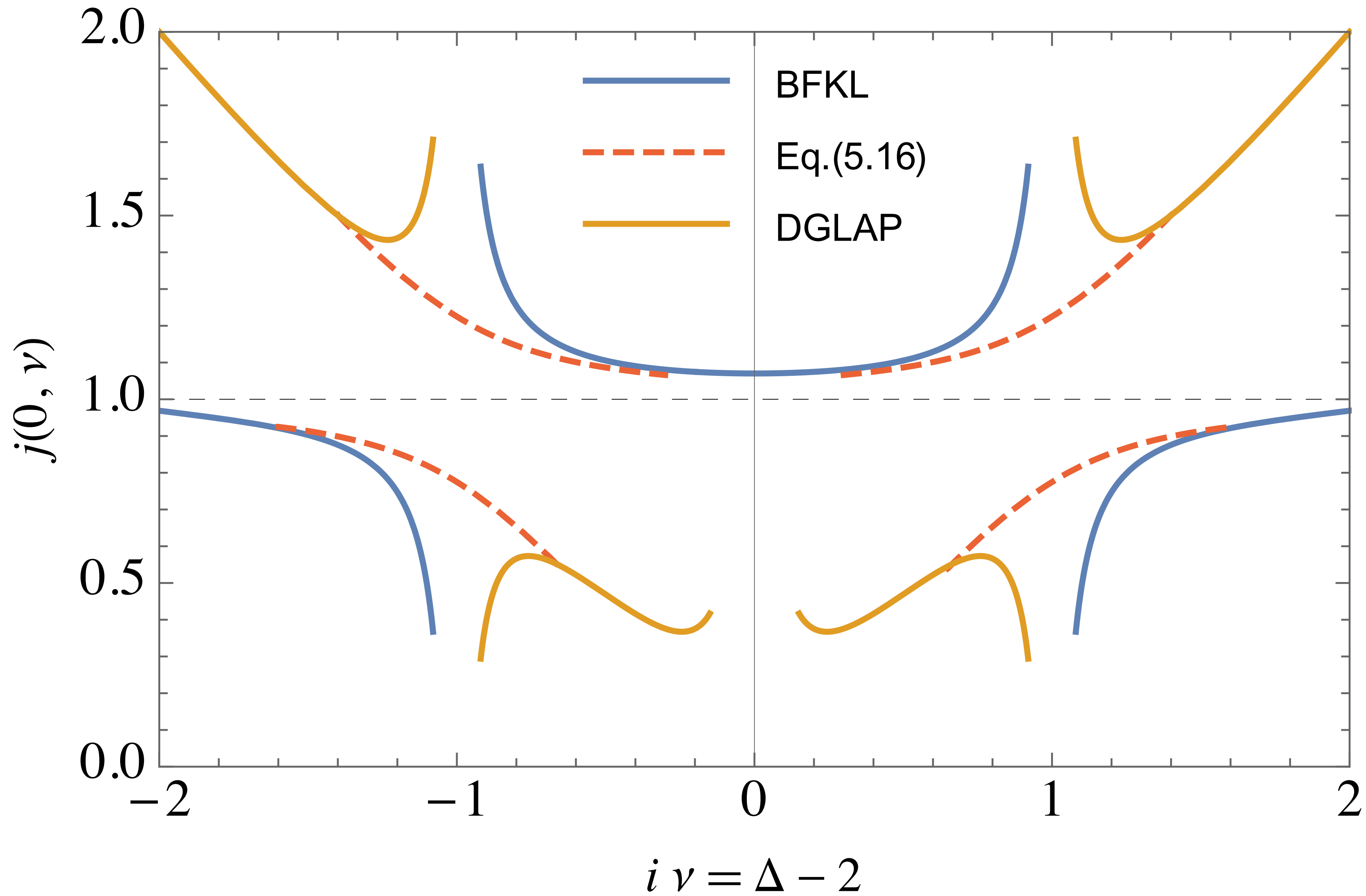


Figure 6. Level repulsion between the Pomeron and DGLAP trajectories for $m = 0$ as a function of scaling dimension, illustrating the $\nu = \pm i$ singularities. (LO expressions plotted with $\lambda = g^2_{\rm YM} N_c = 1$.)

$$j \approx 1 + \frac{\Delta - 3 \pm \sqrt{(\Delta-3)^2 + 32 g^2}}{2}, \qquad \Delta = 2 + i\nu. \tag{5.16}$$

[for polarized PDFs: level crossing is at ν=0] Bartels, Ermolaev&Ryskin '96]

[cf Sievert & Kovchegov's talks]

A slide from Ian Balitsky's talk (@Edinburgh '18?):

NLO evolution of composite "conformal" dipoles in QCD

I. B. and G. Chirilli

$$a\frac{d}{da}[\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{comp}}_a = \frac{\alpha_s}{2\pi^2}\int d^2z_3\,\Big([\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}]^{\mathrm{comp}}_a$$

$$\times\frac{z_{12}^2}{z_{13}^2z_{23}^2}\Big[1+\frac{\alpha_sN_c}{4\pi}\Big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2-z_{23}^2}{z_{13}^2z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2} + \frac{67}{9}-\frac{\pi^2}{3}\Big)\Big]$$

=O(eps)term in LO BK

$$+\frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\Big\{\Big[-2+\frac{z_{23}{}^2z_{23}^2+z_{24}^2z_{13}^2-4z_{12}^2z_{34}^2}{2(z_{23}{}^2z_{23}^2-z_{24}^2z_{13}^2)}\ln\frac{z_{23}{}^2z_{23}^2}{z_{24}^2z_{13}^2}\Big]$$

$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_3}U_{z_4}U^\dagger_{z_2}U_{z_3}U^\dagger_{z_4}\} - (z_4\to z_3)]$$

QCD NGL'

$$+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2}\Big[2\ln\frac{z_{12}^2z_{34}^2}{z_{23}{}^2z_{23}^2}+\Big(1+\frac{z_{12}^2z_{34}^2}{z_{13}^2z_{24}^2-z_{23}{}^2z_{23}^2}\Big)\ln\frac{z_{13}^2z_{24}^2}{z_{23}{}^2z_{23}^2}\Big]$$

$$\times[\mathrm{tr}\{U_{z_1}U^\dagger_{z_3}\}\mathrm{tr}\{U_{z_3}U^\dagger_{z_4}\}\mathrm{tr}\{U_{z_4}U^\dagger_{z_2}\} - \mathrm{tr}\{U_{z_1}U^\dagger_{z_4}U_{z_3}U^\dagger_{z_2}U_{z_4}U^\dagger_{z_3}\} - (z_4\to z_3)]\Big\}$$

N=4

$$b=\frac{11}{3}N_c-\frac{2}{3}n_f$$

$K_{\mathrm{NLO\ BK}}$ = Running coupling part + Conformal "non-analytic" (in j) part + Conformal analytic ($\mathcal{N}=4$) part

Linearized $K_{\mathrm{NLO\ BK}}$ reproduces the known result for the forward NLO BFKL kernel.

Wait. QCD is not conformal!

- QCD non-global logs in the same way
- Regge and Soft kernels don't quite agree:

$$K_{Regge} - K_{Soft} = (11C_A - 4n_F T_F - n_S T_S) \int \left(\frac{z_{ij}^2}{z_{0i}^2 z_{0j}^2} \log(\mu^2 z_{ij}^2) + \frac{z_{0j}^2 - z_{0i}^2}{z_{0i}^2 z_{0j}^2} \log \frac{z_{0i}^2}{z_{0j}^2} \right)$$

- diff prop to β = conformal breaking, as expected!

$\Rightarrow$ difference computable from matter loops!

Rapidity vs Soft divergences

- Work in d=4-2ε dimensions:

K_{Soft} does not depend on ε

$K_{Regge}(\epsilon)$ does

- In the conformal dimension, they are equal!

$$K_{Regge}(2\epsilon = -\beta(\alpha_s)) = K_{soft}$$

- Given the ε-dependence at lower loops, they are equivalent to each other!!!

[Vladimirov '16]

- Full non-planar NLO result also available (N=4&QCD)

$$K^{(2)} = \int_{i,j,k}\int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K^{(2)\ell}_{ijk;00'}\, i f^{abc}\left(L^a_{i;0}L^b_{j;0'}R^c_k - R^a_{i;0}R^b_{j;0'}L^c_k\right)$$
$$+\int_{i,j}\int \frac{d^2\Omega_0}{4\pi}\frac{d^2\Omega_{0'}}{4\pi} K^{(2)N=4,\ell}_{ij;00'}\left(f^{abc}f^{a'b'c'}U_0^{bb'}U_{0'}^{cc'} - \frac{C_A}{2}(U_0^{aa'}+U_{0'}^{aa'})\right)(L_i^a R_j^{a'} + R_i^{a'} L_j^a)$$
$$+\int_{i,j}\int \frac{d^2\Omega_0}{4\pi}\frac{\alpha_{ij}}{\alpha_{0i}\alpha_{0j}}\gamma_K^{(2)}\left(R^a_{i;0}L^a_j + L^a_{i;0}R^a_j\right) + K^{(2)N\neq 4}. \tag{3.32}$$

known

[SCH '15]

$$L^a_{i;0} \equiv (L_i^{a'}U_0^{a'a} - R_i^a)$$

$$\begin{aligned}
\alpha_{0i}\alpha_{0'j}K^{(2)\ell}_{ijk;00'} &= \frac{\alpha_{ij}}{\alpha_{00'}}\log\frac{\alpha_{0'i}\alpha_{0'j}\alpha_{0k}^2}{\alpha_{0i}\alpha_{0j}\alpha_{0'k}^2} + \frac{\alpha_{ik}\alpha_{jk}}{\alpha_{0k}\alpha_{0'k}}\log\frac{\alpha_{ik}\alpha_{0'j}\alpha_{0k}}{\alpha_{jk}\alpha_{0i}\alpha_{0'k}} + \frac{\alpha_{0'i}\alpha_{jk}}{\alpha_{00'}\alpha_{0'k}}\log\frac{\alpha_{jk}\alpha_{0i}\alpha_{00'}\alpha_{0'k}}{\alpha_{0k}^2\alpha_{0'i}\alpha_{0'j}} \\
&\quad -\frac{\alpha_{ik}\alpha_{0j}}{\alpha_{0k}\alpha_{00'}}\log\frac{\alpha_{ik}\alpha_{0'j}\alpha_{00'}\alpha_{0k}}{\alpha_{0'k}^2\alpha_{0i}\alpha_{0j}} + \frac{\alpha_{ik}\alpha_{0'j}}{\alpha_{0'k}\alpha_{00'}}\log\frac{\alpha_{ik}\alpha_{00'}}{\alpha_{0k}\alpha_{0'i}} - \frac{\alpha_{0i}\alpha_{jk}}{\alpha_{0k}\alpha_{00'}}\log\frac{\alpha_{jk}\alpha_{00'}}{\alpha_{0'k}\alpha_{0j}} \\
K^{(2)N=4,\ell}_{ij;00'} &= \frac{\alpha_{ij}}{\alpha_{0i}\alpha_{00'}\alpha_{0'j}}\left(2\log\frac{\alpha_{ij}\alpha_{00'}}{\alpha_{0'i}\alpha_{0j}} + \left[1+\frac{\alpha_{ij}\alpha_{00'}}{\alpha_{0i}\alpha_{0'j}-\alpha_{0'i}\alpha_{0j}}\right]\log\frac{\alpha_{0i}\alpha_{0'j}}{\alpha_{0'i}\alpha_{0j}}\right).
\end{aligned} \tag{3.33}$$

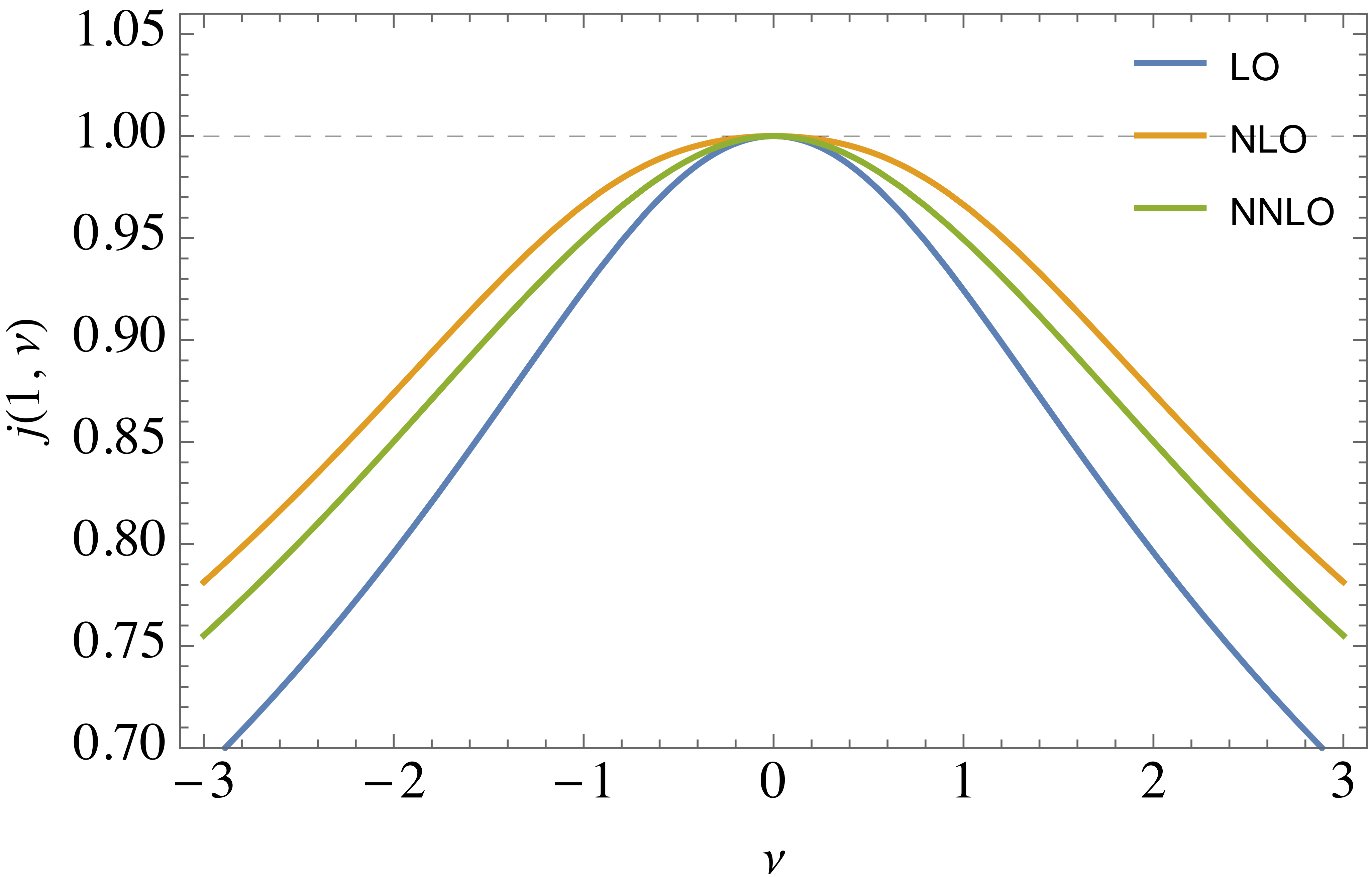
Precisely the same as NLO B-JIMWLK result ✔

[Kovner,Mulian&Lublinski '14,
Balitsky&Chirilli '14]

matter loop contributions to NGLs:

$$
\begin{aligned}
K^{(2)N\neq 4} = & \int_{i,j}\int \frac{d\Omega_0}{4\pi}\frac{d\Omega_{0'}}{4\pi}\frac{1}{\alpha_{00'}}\left[\frac{\alpha_{ij}\log\frac{\alpha_{0i}\alpha_{0'j}}{\alpha_{0'i}\alpha_{0j}}}{\alpha_{0i}\alpha_{0'j}-\alpha_{0'i}\alpha_{0j}}\right](L_i^a R_j^{a'}+R_i^{a'}L_j^a) \\
& \times\left\{2n_F\mathrm{Tr}_R\big[T^aU_0T^{a'}U_{0'}^\dagger\big]-4f^{abc}f^{a'b'c'}U_0^{bb'}U_{0'}^{cc'}-(n_FT_R-2C_A)(U_0^{aa'}+U_{0'}^{aa'})\right\} \\
& +\int_{i,j}\int \frac{d\Omega_0}{4\pi}\frac{d\Omega_{0'}}{4\pi}\frac{1}{2\alpha_{00'}^2}\left[\frac{\alpha_{0i}\alpha_{0'j}+\alpha_{0'i}\alpha_{0j}}{\alpha_{0i}\alpha_{0'j}-\alpha_{0'i}\alpha_{0j}}\log\frac{\alpha_{0i}\alpha_{0'j}}{\alpha_{0'i}\alpha_{0j}}-2\right](L_i^aR_j^{a'}+R_i^{a'}L_j^a) \\
& \times\left\{\begin{matrix} 2(n_S-2n_F)\mathrm{Tr}_R\big[T^aU_0T^{a'}U_{0'}^\dagger\big]+2f^{abc}f^{a'b'c'}U_0^{bb'}U_{0'}^{cc'} \\ -((n_S-2n_F)T_R+C_A)(U_0^{aa'}+U_{0'}^{aa'}) \end{matrix}\right\} \\
& +\int_{i,j}2\pi i b_0\log(\alpha_{ij})\big(L_i^aL_j^a-R_i^aR_j^a\big). \qquad (3.34)
\end{aligned}
$$

m=1 (leading Odderon trajectory)



note: Odderon intercept=1 to all orders in λ.
Agrees with strong coupling!

[Tan et al '14]

On the Odderon intercept

- m=1,v=0 is a very special wavefunction:

$$\mathcal{U}_{12} = 1 - \frac{1}{N_c}(z_1 - z_2)$$

- Strings of dipoles in planar limit telescope:

$$\mathcal{U}_{10}\mathcal{U}_{02} = 1 - \frac{1}{N_c}((z_1 - \cancel{z_0}) + (\cancel{z_0} - z_2)) + O(1/N_c^2)$$

$$= 1 - \frac{1}{N_c}(z_1 - z_2) = \mathcal{U}_{12}$$

$$\mathcal{U}_{10}\mathcal{U}_{00'2}\mathcal{U}_{0'2} = \mathcal{U}_{12}$$

...

- Cancel in evolution. Thm: Odderon intercept vanishes to all order in λ in planar limit