Interplay of lattice fermion and Pauli-Villars fields

Yigal Shamir

Tel Aviv University

DWQ@25

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with emphasis on the supporting actor: PV fields

Outline

1. Domain-wall fermions and their associated PV fields

   Axial symmetry

   Effective 4-dim operator and Ginsparg-Wilson relation

2. Imperfect chiral symmetry

   Super-critical Wilson kernel and residual mass

   localization, mobility edge

   Dislocation suppressing determinant ratio

3. Taming lattice artifacts with PV fields

   PV fields anti-screen! (And then decouple.)

1. The domain-wall / boundary mode

David B. Kaplan '92

YS '93

$$D_5 = \begin{pmatrix} D_W - 1 & P_R & 0 & \dots \\ P_L & D_W - 1 & P_R & \dots \\ 0 & P_L & D_W - 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Wilson kernel $D_W = \not{D} - W + M$ $(M = -m_0)$
- Free DW operator at $p_\mu = 0$:

$$\Rightarrow \quad P_R \begin{pmatrix} M-1 & 1 & 0 & \dots \\ 0 & M-1 & 1 & \dots \\ 0 & 0 & M-1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + P_L \begin{pmatrix} M-1 & 0 & 0 & \dots \\ 1 & M-1 & 0 & \dots \\ 0 & 1 & M-1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RH zero mode: $(1-M)^s$ provided DW height $0 < M < 2$
- LH zero mode on other boundary
- DWQ mass m: link connecting $s = 1$ and $s = N$
- Quantum wave function controlled by edge of spectrum of Wilson kernel.

Integrating out 5-d bulk mode

- Integrating out 5-d fermions away from boundaries (gauge field is 4-dimensional)

$$S_{\text{ind}} \;=\; N\Big(c\,F^2 + \cdots\Big) + O(1)$$

- Integrating out N_W Wilson fermions — DeGrand, Hasenfratz '93

  hopping parameter expansion: $\kappa = 1/(2d + 2m_0)$

$$S_{\text{ind}} \;=\; N_W \sum_{\ell} \frac{\kappa^{\ell}}{\ell} \sum_{\mathcal{C}_\ell} \mathrm{Tr}\left(\tfrac{1}{2}(1 \pm \gamma_\mu)U_\mu \cdots + \text{h.c.}\right)_{\mathcal{C}_\ell}$$

- Can't send $\kappa \to 0$ since domain-wall height $M = -m_0$ is bounded.

$\Rightarrow$ Use Pauli-Villars fields to subtract out $S_{\text{ind}} \propto N$.

$$Z_{DW} \;=\; \frac{\mathrm{Det}(DWF)}{\mathrm{Det}(PV)} \;=\; \frac{\mathrm{Det}\big(D_5(m)\big)}{\mathrm{Det}\big(D_5(m=1)\big)}$$

Axial symmetry

Furman, YS '94

- 5-dim vector current is conserved: $\Delta_\mu j_\mu(x,s) + \tilde{\Delta}_5 j_5(x,s) = 0$.

$\Rightarrow$ 4-dim conserved vector current: $\Delta_\mu V^a_\mu(x) = 0$

- Axial symmetry: rotate $\psi(x,s)$ with opposite chiral phases for $1 \le s \le N/2$ and for $N/2 < s \le N$.

$\Rightarrow$ 4-dim partially conserved axial current: $\Delta_\mu A^a_\mu = 2mJ^a_5 + 2J^a_{5q}$

where $J^a_5(x) = j^a_5(x, s=N)$ made of fields at boundaries

$J^a_{5q}(x) = j^a_5(x, s=N/2)$ made of fields mid-way in 5th direction

- Effective quark field: $q_R(x) = P_R\,\psi(x, s=1)$

$$q_L(x) = P_L\,\psi(x, s=N)$$

- Restore chiral symmetry ($m=0$):

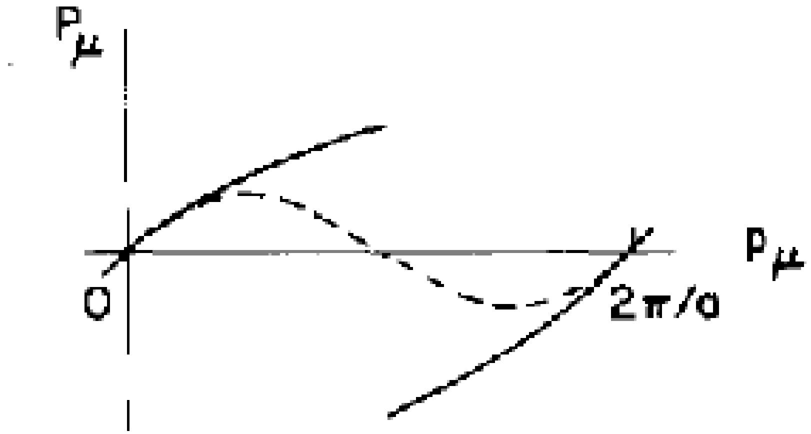
$$\{\langle q(x)\bar{q}(y)\rangle\,, \gamma_5\} \to 0 \qquad \text{for} \quad N \to \infty$$

exponentially fast (only) in perturbation theory

Effective 4-d operator $(m = 0, \quad N \to \infty)$

- Effective boundary operator $D_4 = \langle q(x)\bar{q}(y)\rangle^{-1}$ anti-commutes with γ_5

$\Rightarrow$ must have poles! (correspond to zeros of the propagator)



Karsten, Smit '81

Nielsen, Ninomiya '81

$D_4 = \sum_\mu \gamma_\mu P_\mu(p)$. For example, P_1 as a function of $(p_1, 0, 0, 0)$

Plot from Karsten & Smit

Effective 4-d operator $(m = 0, \quad N \to \infty)$

- Try again — Kikukawa, Noguchi '99

  use tight relation of DWF and PV fields

$$\frac{\mathrm{Det}\big(D_5(m=0)\big)}{\mathrm{Det}\big(D_5(m=1)\big)} = \frac{\mathrm{Det}(D_4)}{\mathrm{Det}(D_4+1)} = \int dq d\bar{q} \exp\left(\bar{q}\, \underbrace{\big(D_4+1\big)^{-1}}_{PV} D_4\, q\right)$$

- Define $D_4^{\mathrm{eff}} = \big(D_4+1\big)^{-1} D_4$ (Note: D_4^{eff} *not* ultra-local)

$$\{\big(D_4^{\mathrm{eff}}\big)^{-1}, \gamma_5\} = \{D_4^{-1}+1, \gamma_5\} = 2\gamma_5\, \delta^4(x-y)$$

$\Rightarrow$ Ginsparg-Wilson relation

$$\{D_4^{\mathrm{eff}}, \gamma_5\} = 2\, D_4^{\mathrm{eff}}\, \gamma_5\, D_4^{\mathrm{eff}}$$

Neuberger '97

$\Rightarrow$ Modified chiral symmetry — Lüscher '98

2. Imperfect chiral symmetry

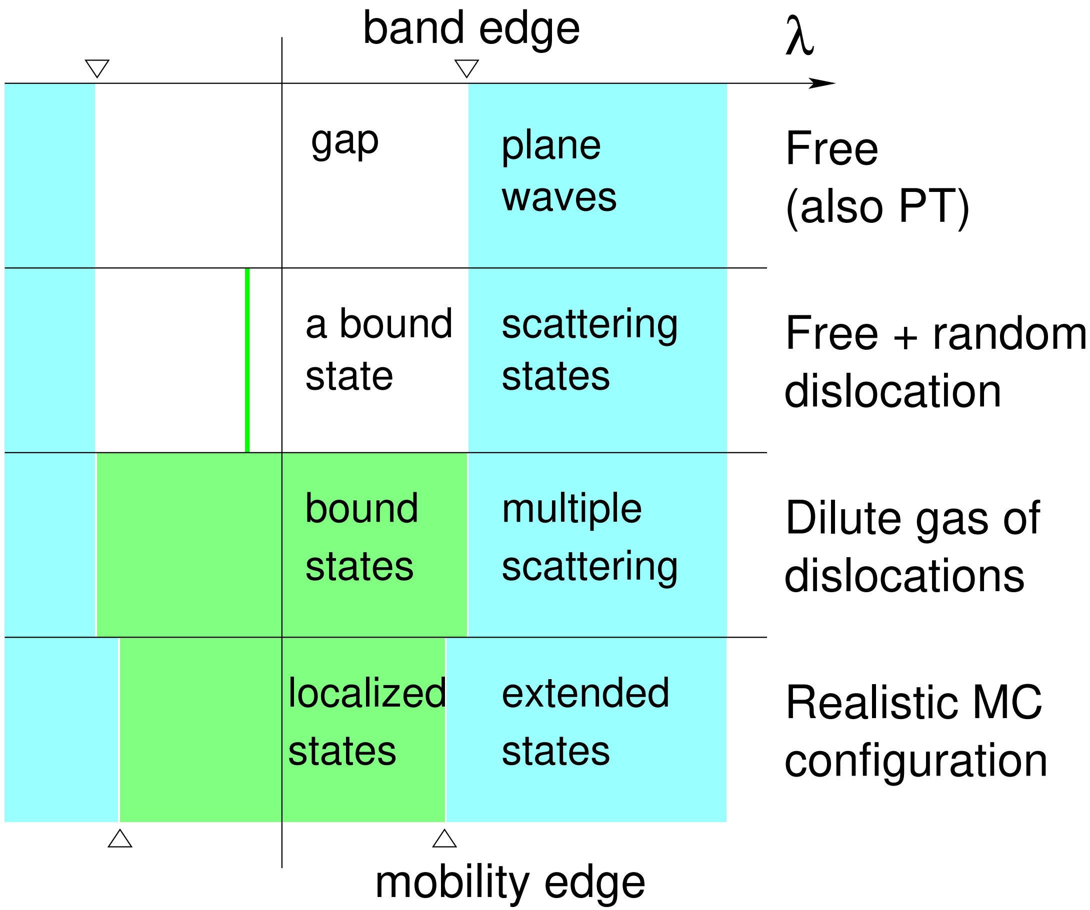
Golterman, YS, Svetitsky, '03 – '05

- Kernel $H_W = \gamma_5 D_W$ is gapped in the free theory (and perturbation theory)

$\Rightarrow$ zero mode falls off exponentially away from boundary

- Non-perturbativley:
  super-critical
  no gap!

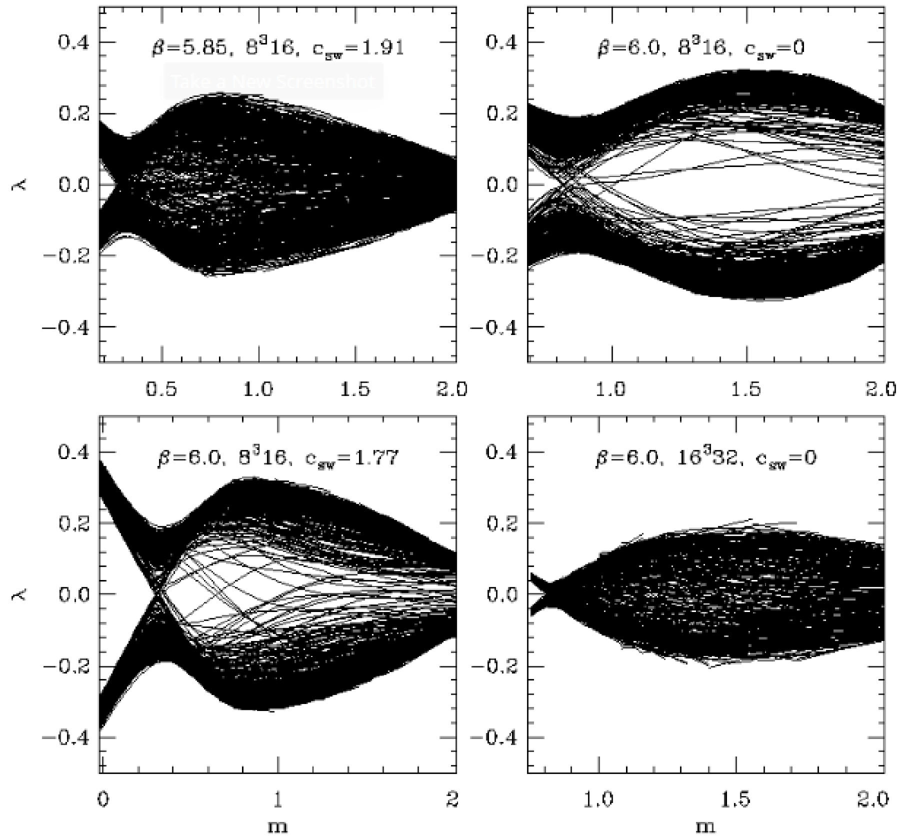
- In particular
  H_W has a zero mode
  when changing the
  topological charge
  Narayanan, Neuberger '93



In real life

- Spectrum of H_W vs. DW height:

Edwards, Heller, Narayanan '98



Residual mass and DWQ wave function

$$\Delta_\mu A^a_\mu = 2mJ^a_5 + 2J^a_{5q}$$

- The residual mass: $J^a_{5q} = m_{\rm res}\, J^a_5 + O(a^2)$

CP-PACS hep-lat/0007014

RBC hep-lat/0007038

$\Rightarrow$ $m_{\rm res} \;\sim\; \langle J^a_{5q}\, J^a_5 \rangle \,/\, \langle J^a_5\, J^a_5 \rangle$

- In terms of 5th direction DW transfer matrix $T \;\sim\; \exp(-H_W)$

$\Rightarrow$ $m_{\rm res} \;\sim\; {\rm Tr}\,{:}T{:}^N \;\sim\; \int_0^1 d\lambda\, \rho(\lambda)\, \lambda^N$

YS hep-lat/0003024

RBC/UKQCD 0705.2340

$\Rightarrow$ Dominated by eigenvalues of ${:}T{:}$ near 1

$\Rightarrow$ Possibilities (1st: gap; 2nd, 3rd: no gap)

Exponential $\rho(\lambda) \sim (\lambda_0 - \lambda)^\delta, \;\; \lambda_0 < 1 \;\; \Rightarrow \;\; m_{\rm res} \;\propto\; \lambda_0^N / N^{1+\delta}$

Power law $\rho(\lambda) \sim (1-\lambda)^\delta \;\; \Rightarrow \;\; m_{\rm res} \;\propto\; 1/N^{1+\delta}$

Minimal suppression $\rho(1) \neq 0 \;\; \Rightarrow \;\; m_{\rm res} \;\propto\; 1/N$

Dislocation Suppressing Determinant Ratio

Vranas hep-lat/0606014
JLQCD hep-lat/0607020
Renfrew *et al.* 0902.2587

- Add a function of H_W to the Boltzmann weight to suppress kernel's near-zero spectral density

$$\text{DSDR} = \frac{\mathrm{Det}(H_W^2 + \epsilon_1)}{\mathrm{Det}(H_W^2 + \epsilon_2)} \qquad \epsilon_1 \ll \epsilon_2 < 1/N$$

- Note PV-type denominator, to restrict influence to kernel's low modes

⇒ Suppress $\rho(\lambda)$ by $\big((1-\lambda)^2 + \epsilon_1\big)/\epsilon_2$ for near-unity ev's of $:T$:

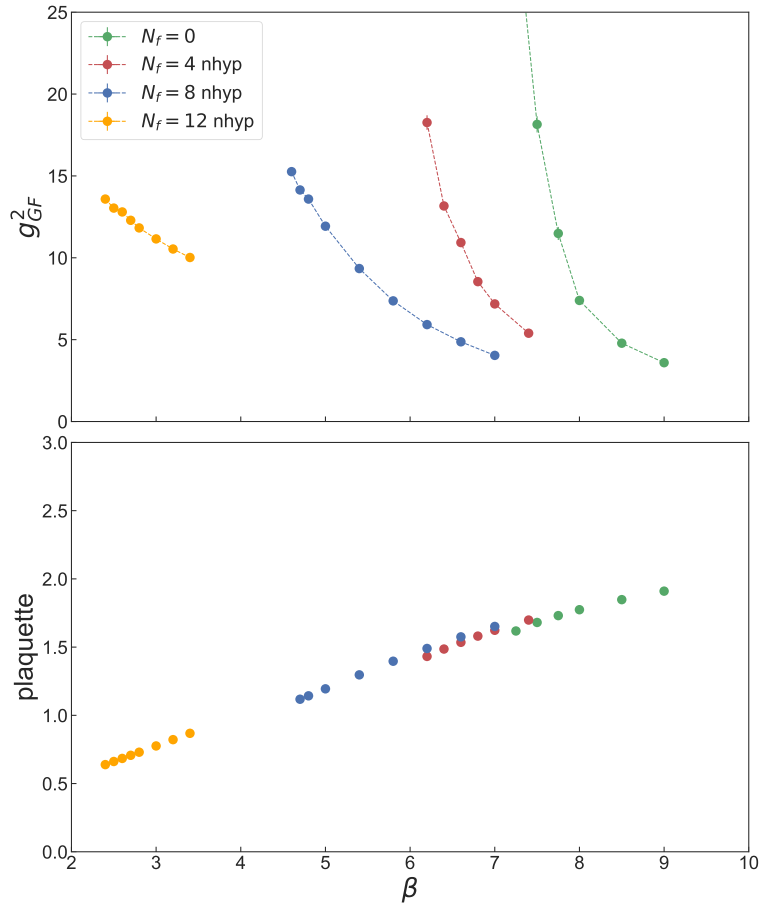
⇒ Improve $m_{\rm res}$ from $\dfrac{c}{N}$ to $\dfrac{c}{\epsilon_2}\dfrac{1}{N^3} + \dfrac{\epsilon_1}{\epsilon_2}\dfrac{c}{N}$

- But watch for reduced topological activity

3. Taming lattice artifacts with PV fields

Hasenfratz, YS, Svetitsky 2109.02790

- Q: What makes discretization effects of DWF so small?
- Possible answer:
  The PV fields remove (most of) the fermion discretization effects at the lattice scale.
- Shown: Plaquette and GF coupling for SU(3) gauge theory with various N_f using staggered fermions
- Small plaquette values mean large short-distance effects



Try adding (massive) PV fields

- PV mass is fixed at $O(1)$ value in lattice units

$\Rightarrow$ decouple in the continuum limit.

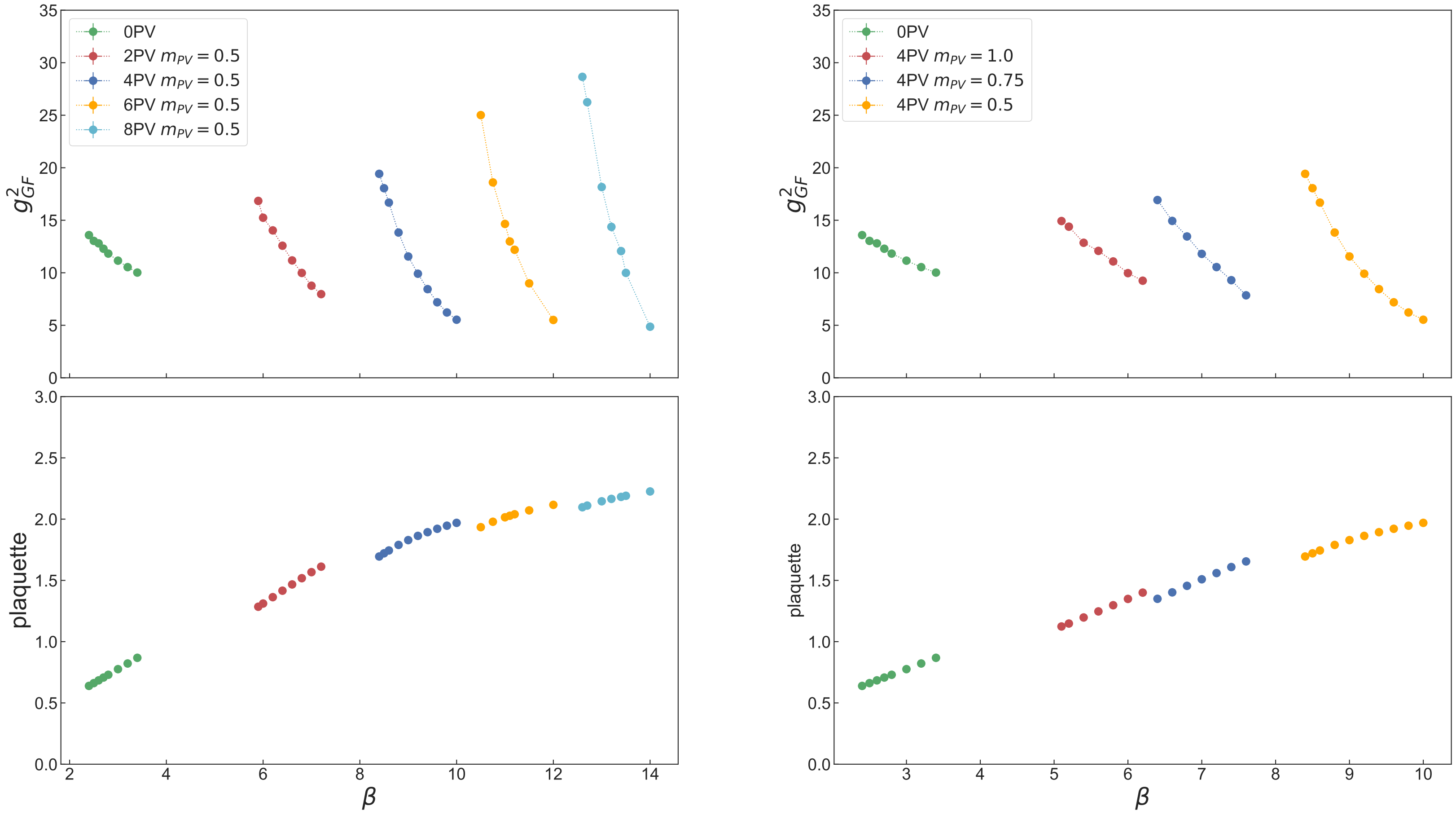
- True for *any number* of PV fields!

- PV fields *anti-screen*!
  Generate large renormalized coupling from weak bare coupling

- Laboratory: $N_f = 12$ staggered theory
  Exhibits bulk transition into S_4 phase
  Watch GF coupling and plaquette before encountering S_4 phase.
  Volume: mostly 8^4, some 12^4.

- Use same-type PV fields: here, staggered.

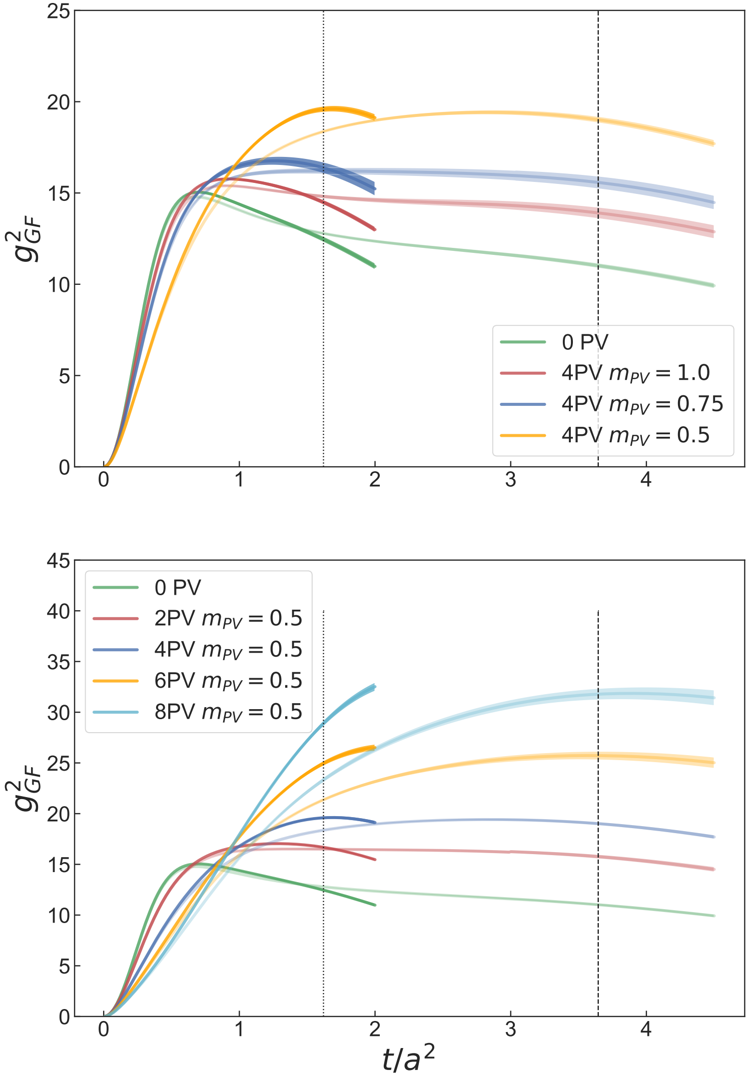
Results



$\Rightarrow$ Same GF coupling at much larger plaquette values (and β)

Gradient-flow coupling

- Short curves: 8^4, long curves: 12^4
  Vertical lines: where we measure g^2_{GW}
- Flow rises sharply till PVs decouple.
  Then flattens:
  $N_f = 12$ is (nearly) conformal.
- If $N_f = 12$ has IRFP at $g^2_* \sim 6$
  these g^2_{GW} values
  are on other side of IRFP.



Might be good for

- Many-fermion systems
  such as (near) conformal systems:
  can reach large renorml. coupling
  even if bare coupling is weak!
- Large lattice spacing
  for large physical volume
  expect small scaling violations
- Now trying with Wilson fermions
- ...Any use for DWF?

Thank you