Funded by
Deutsche Forschungsgemeinschaft
German Research Foundation

The effective electroweak Hamiltonian in the gradient-flow formalism

Robert Harlander

RWTH Aachen University

based on work with Fabian Lange and contributions from others

Institute for Theoretical Particle Physics and Cosmology

RWTH AACHEN UNIVERSITY

The gradient flow

flowed gauge field:

$$\boxed{\begin{aligned}\frac{\partial}{\partial t}B_\mu(t,x) &= \mathscr{D}_\nu G_{\nu\mu}(t,x)\\ B_\mu(t=0,x) &= A_\mu(x)\end{aligned}}$$

flowed quark field:

$$\boxed{\begin{aligned}\frac{\partial}{\partial t}\chi(t,x) &= \mathscr{D}^2\chi(t,x)\\ \chi(t=0,x) &= \psi(x)\end{aligned}}$$

Lüscher '10, '13

The gradient flow

flowed gauge field:

$$\frac{\partial}{\partial t} B_\mu(t,x) = \mathscr{D}_\nu G_{\nu\mu}(t,x)$$
$$B_\mu(t=0,x) = A_\mu(x)$$

flowed quark field:

$$\frac{\partial}{\partial t} \chi(t,x) = \mathscr{D}^2 \chi(t,x)$$
$$\chi(t=0,x) = \psi(x)$$

Lüscher ’10, ‘13

The gradient flow

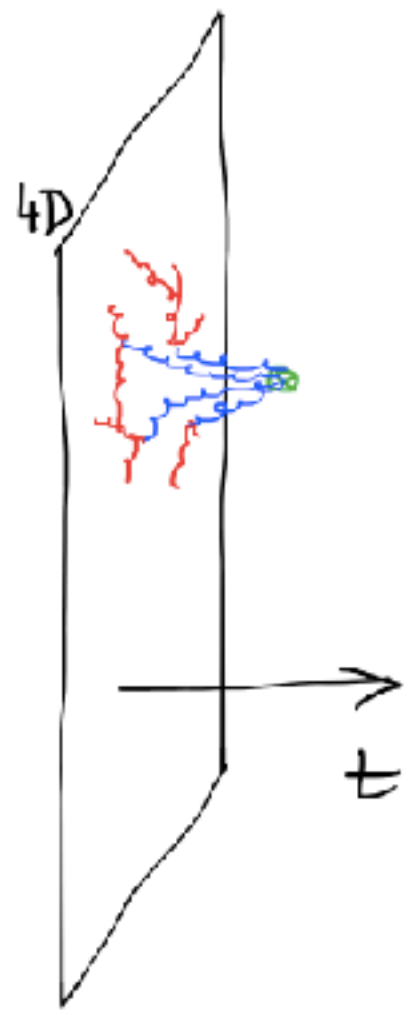
flowed gauge field:

$$\frac{\partial}{\partial t} B_\mu(t,x) = \mathscr{D}_\nu G_{\nu\mu}(t,x)$$
$$B_\mu(t=0,x) = A_\mu(x)$$

flowed quark field:

$$\frac{\partial}{\partial t} \chi(t,x) = \mathscr{D}^2 \chi(t,x)$$
$$\chi(t=0,x) = \psi(x)$$

Lüscher ’10, ‘13



$$B_\mu(t,p) \sim e^{-tp^2} A_\mu(p)$$

Perturbative approach

Use 5-dimensional QFT formulation: Lüscher, Weisz ‘11

$$\mathscr{L} = \mathscr{L}_{\text{QCD}} + \mathscr{L}_B \qquad \mathscr{L}_B \sim \int_0^\infty dt\, L_\mu \left(\partial_t B_\mu - \mathscr{D}_\nu G_{\nu\mu}\right)$$

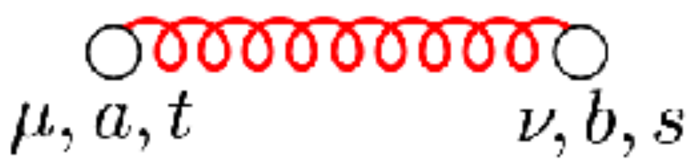
Lagrange multiplier

Perturbative approach

Use 5-dimensional QFT formulation: Lüscher, Weisz '11

$$\mathscr{L} = \mathscr{L}_{\rm QCD} + \mathscr{L}_B \qquad \mathscr{L}_B \sim \int_0^\infty dt\, L_\mu \left(\partial_t B_\mu - \mathscr{D}_\nu G_{\nu\mu}\right)$$

Lagrange multiplier



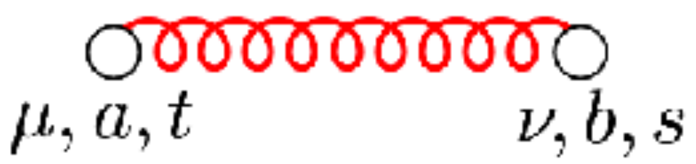
$$\frac{\delta^{ab}}{p^2}\left(\delta_{\mu\nu} - \xi\frac{p_\mu p_\nu}{p^2}\right) e^{-(t+s)p^2}$$

Perturbative approach

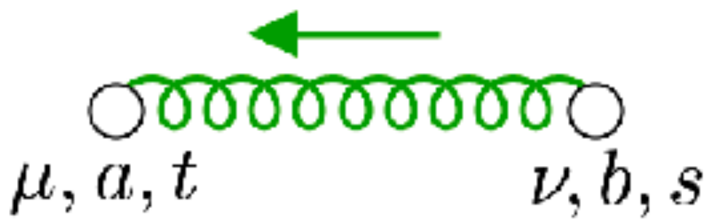
Use 5-dimensional QFT formulation: Lüscher, Weisz '11

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_B \qquad \mathcal{L}_B \sim \int_0^\infty dt\, L_\mu \left(\partial_t B_\mu - \mathcal{D}_\nu G_{\nu\mu} \right)$$

Lagrange multiplier



$$\frac{\delta^{ab}}{p^2} \left(\delta_{\mu\nu} - \xi \frac{p_\mu p_\nu}{p^2} \right) e^{-(t+s)p^2}$$



$$\delta_{ab}\delta_{\mu\nu}\, \theta(t-s)\, e^{-(t-s)p^2}$$

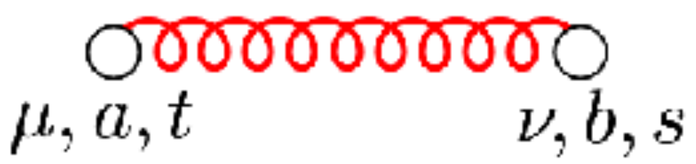
"gluon flow line"

Perturbative approach

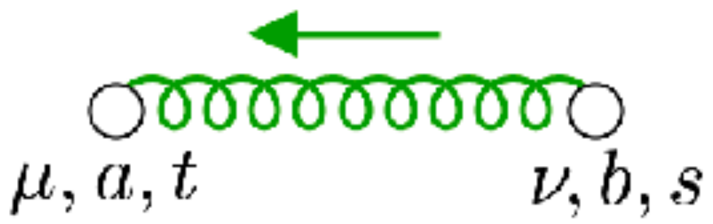
Use 5-dimensional QFT formulation: Lüscher, Weisz '11

$$\mathscr{L} = \mathscr{L}_{\rm QCD} + \mathscr{L}_B \qquad \mathscr{L}_B \sim \int_0^\infty dt\, L_\mu \left(\partial_t B_\mu - \mathscr{D}_\nu G_{\nu\mu}\right)$$

Lagrange multiplier

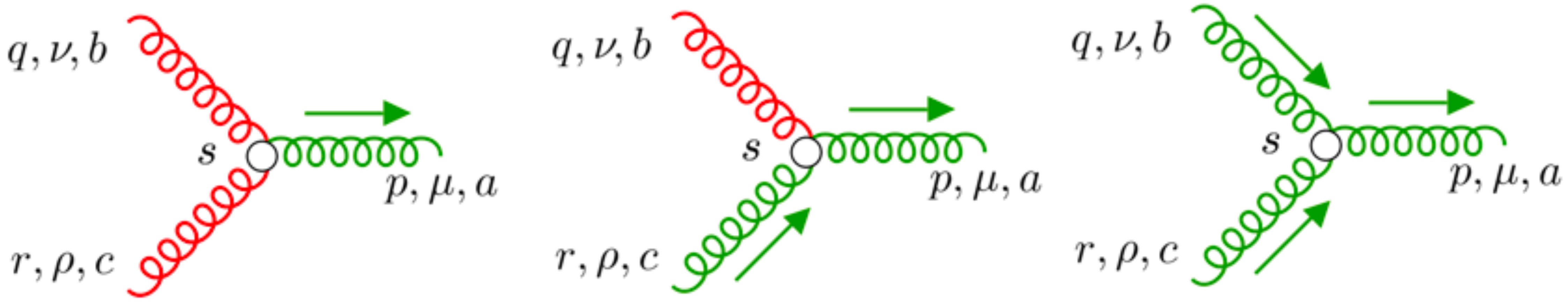


$$\frac{\delta^{ab}}{p^2}\left(\delta_{\mu\nu} - \xi \frac{p_\mu p_\nu}{p^2}\right) e^{-(t+s)p^2}$$



$$\delta_{ab}\delta_{\mu\nu}\,\theta(t-s)\, e^{-(t-s)p^2}$$

"gluon flow line"



$$-\mathrm{i}g f^{abc} \int_0^\infty \mathrm{d}s\, \big(\delta_{\nu\rho}(r-q)_\mu + 2\delta_{\mu\nu} q_\rho - 2\delta_{\mu\rho} r_\nu + (\kappa - 1)(\delta_{\mu\rho} q_\nu - \delta_{\mu\nu} r_\rho)\big)$$

The first application

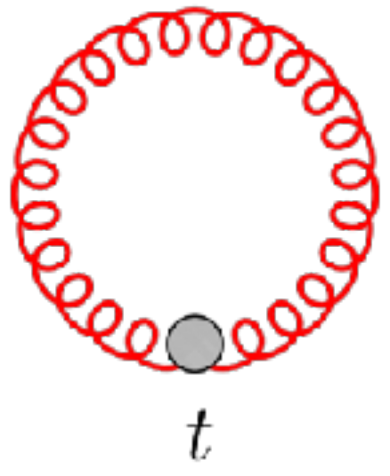
The first application: Lüscher '10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle \, :$$

The first application

The first application: Lüscher '10

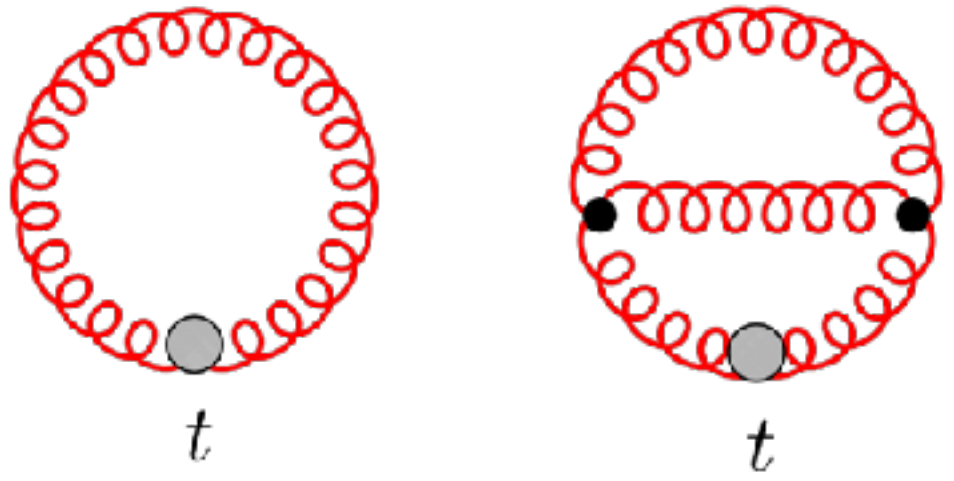
$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle :$$



The first application

The first application: Lüscher '10

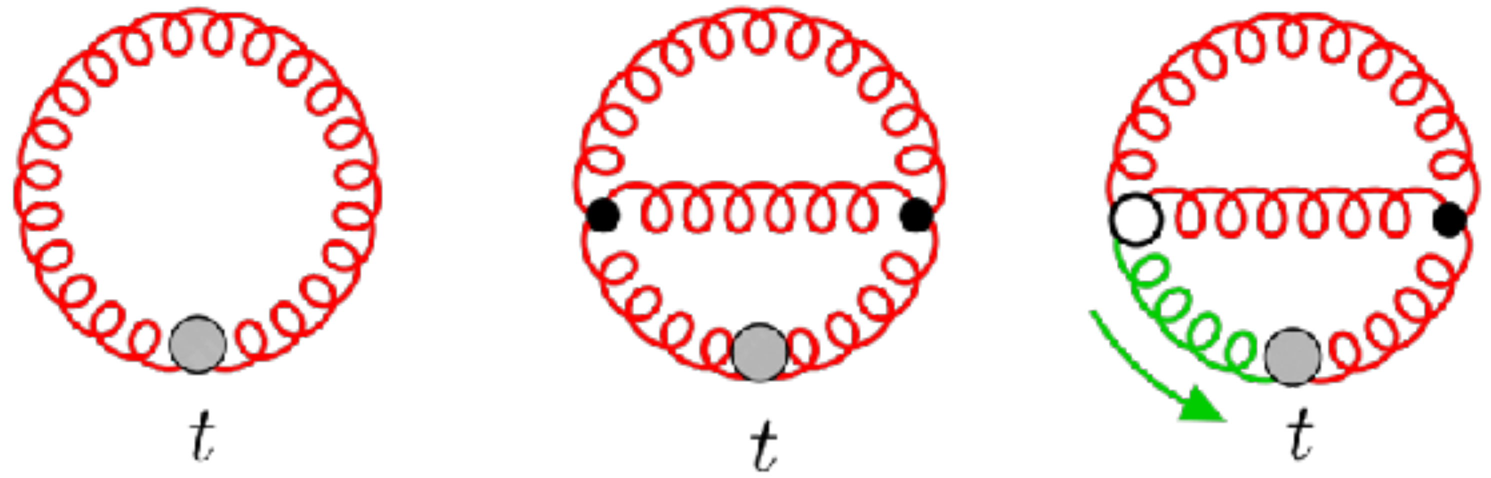
$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle \; :$$



The first application

The first application: Lüscher ‘10

$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle \; :$$

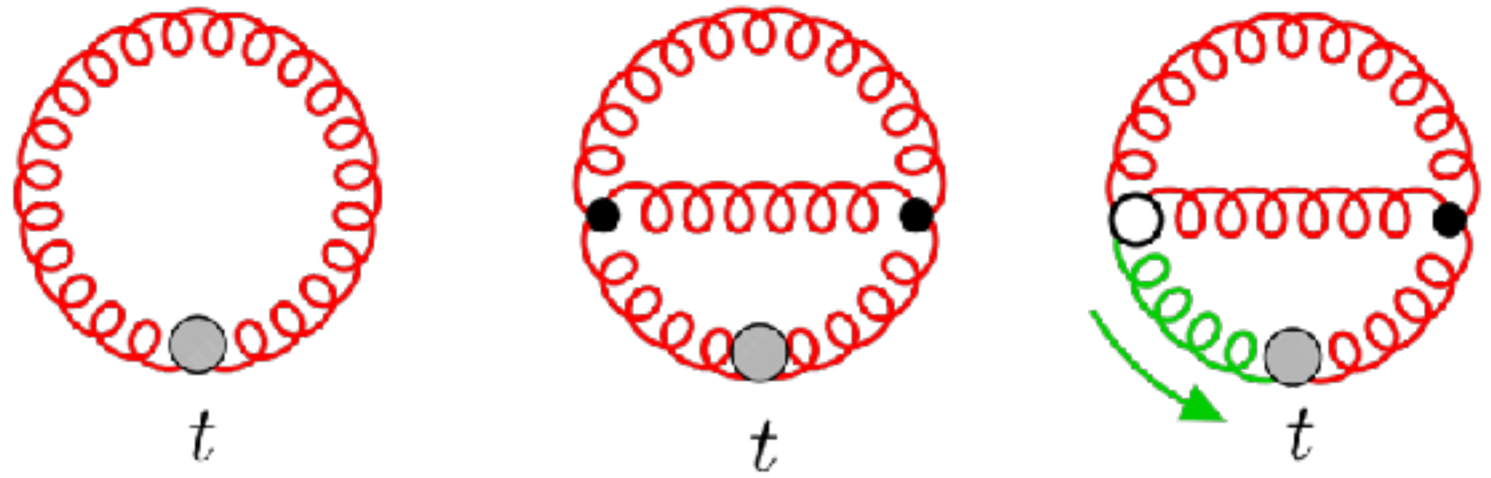


The first application

The first application: Lüscher ‘10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle :$$

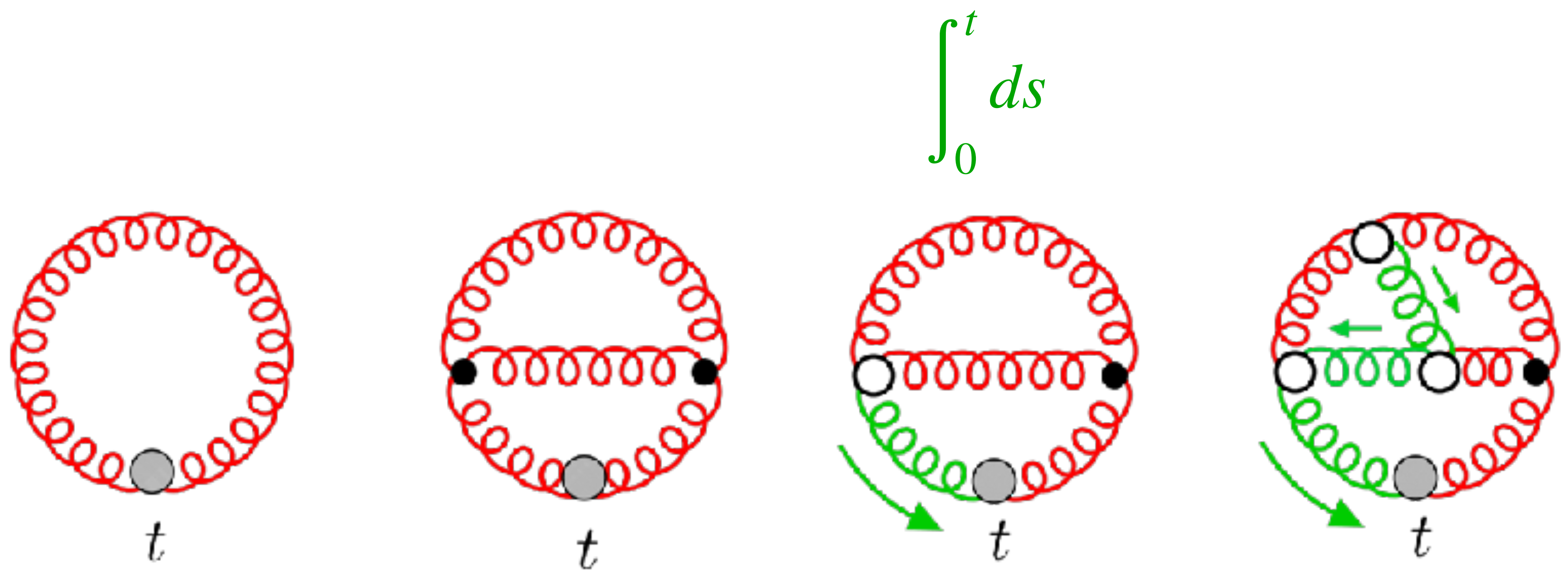
$$\int_0^t ds$$



The first application

The first application: Lüscher '10

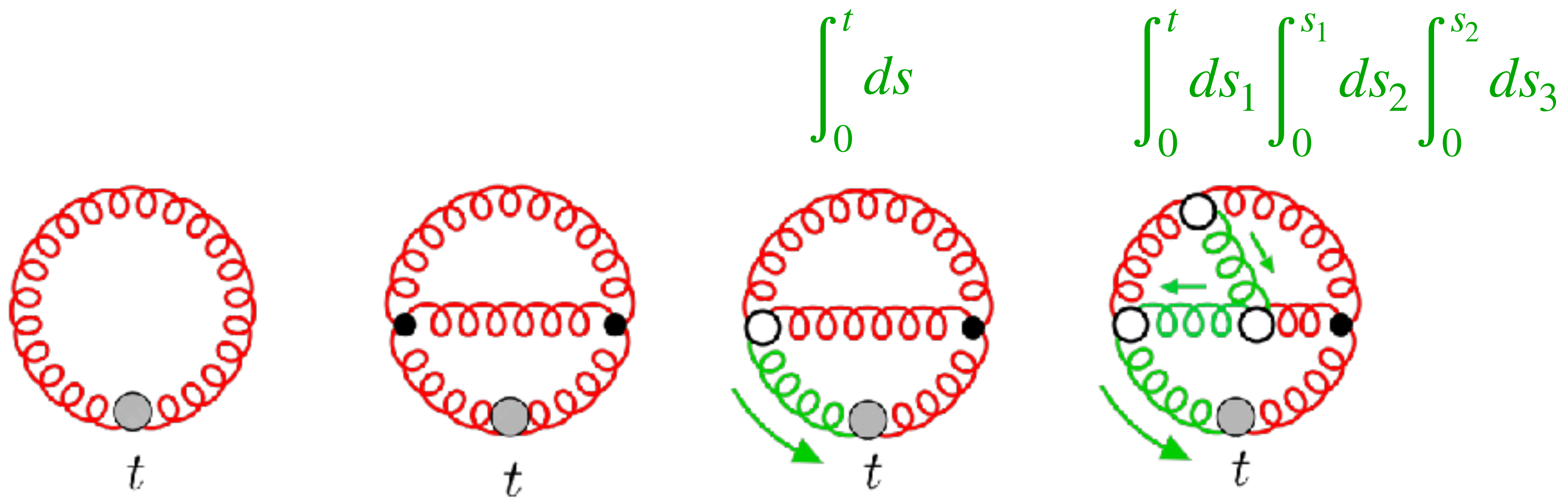
$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle$:



The first application

The first application: Lüscher '10

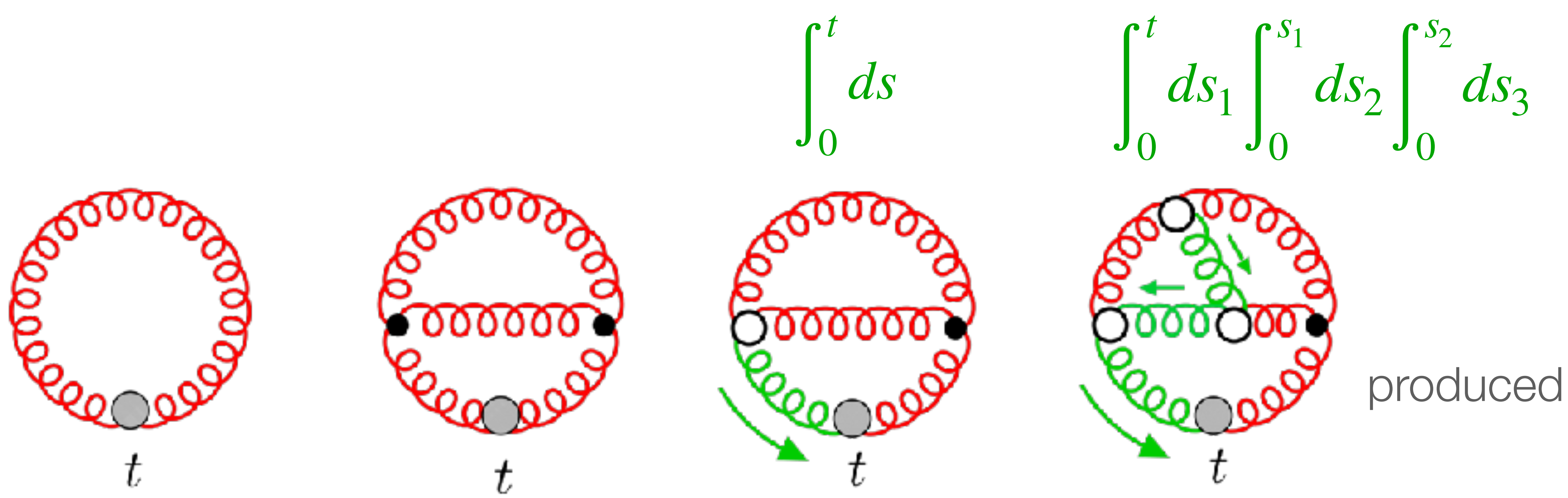
$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle :$$



The first application

The first application: Lüscher '10

$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle :$$



produced with FeynGame

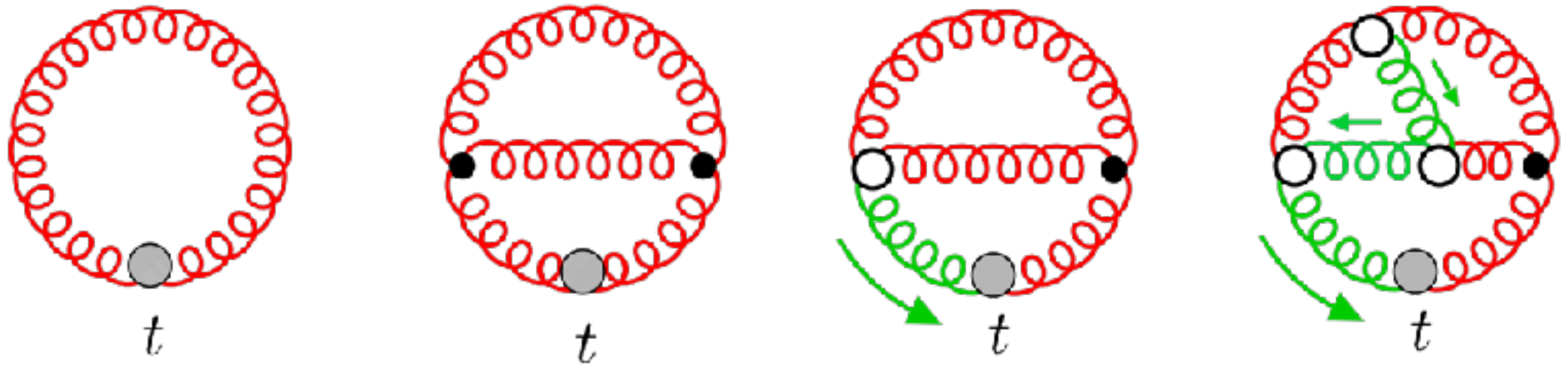
The first application

The first application: Lüscher '10

$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle :$$

$$t^2 \langle G_{\mu\nu}^2(t) \rangle = \frac{3\alpha_s}{4\pi} \left[1 + k_1\, \alpha_s + k_2\, \alpha_s^2 + \ldots \right]$$

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$



produced with FeynGame

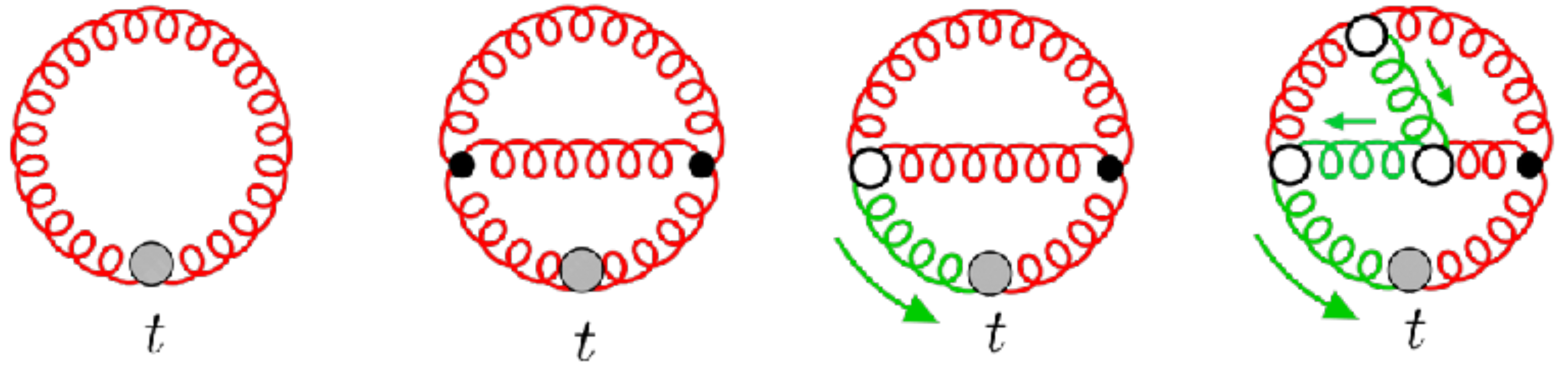
The first application

The first application: Lüscher ‘10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle \; :$$

$$t^2\langle G_{\mu\nu}^2(t)\rangle = \frac{3\alpha_s}{4\pi}\left[1 + k_1\,\alpha_s + k_2\,\alpha_s^2 + \ldots\right]$$
$$= \frac{3\alpha_s}{4\pi}\left[1 + 1.8\,\alpha_s + 5.4\,\alpha_s^4 + \ldots\right]$$

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$



produced with FeynGame

The first application

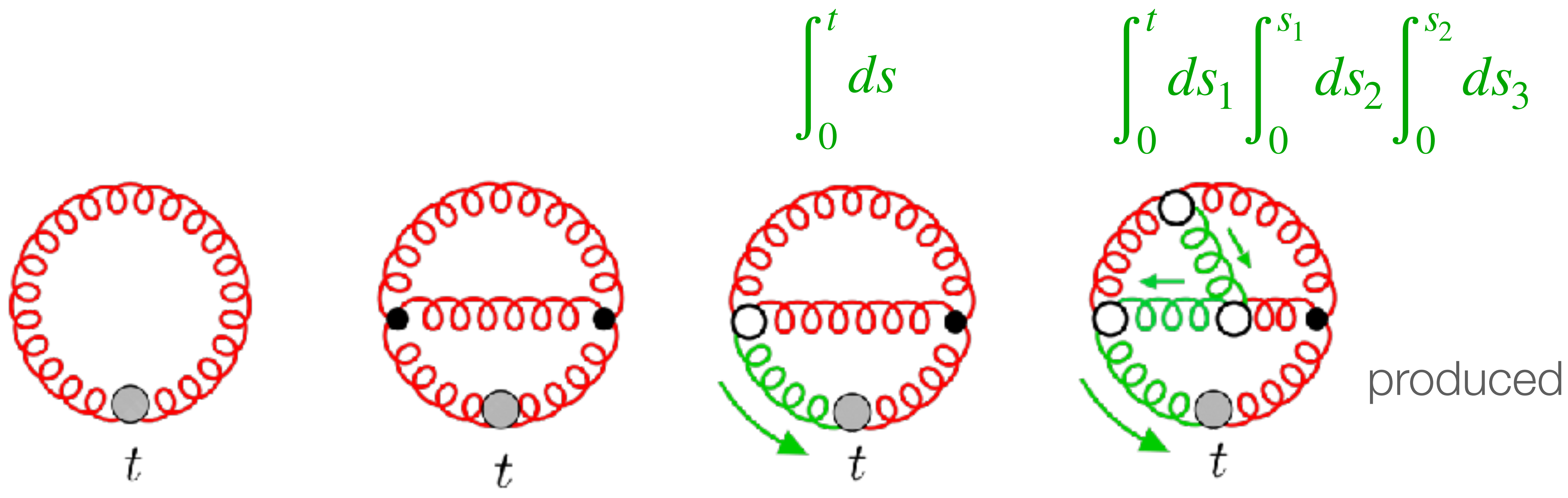
The first application: Lüscher '10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle \; :$$

$$t^2\langle G^2_{\mu\nu}(t)\rangle = \frac{3\alpha_s}{4\pi}\left[1 + k_1\,\alpha_s + k_2\,\alpha_s^2 + \ldots\right]$$

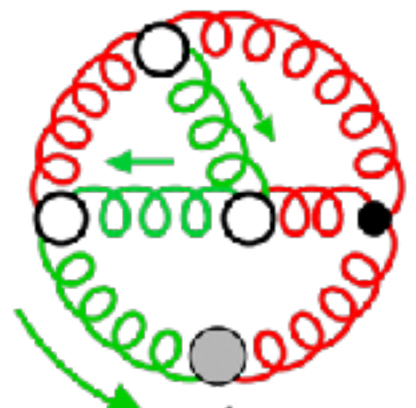
$$= \frac{3\alpha_s}{4\pi}\left[1 + 1.8\,\alpha_s + 5.4\,\alpha_s^4 + \ldots\right]$$

Lüscher '10



produced with FeynGame

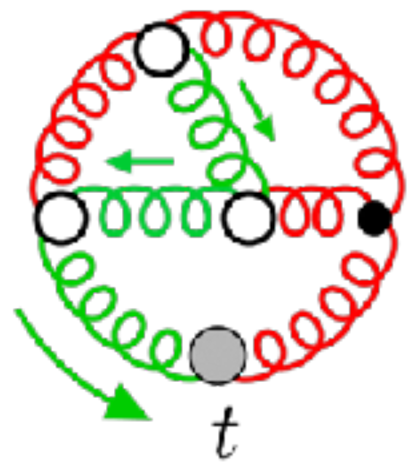
Three-loop calculation



Three-loop calculation

The usual problems:

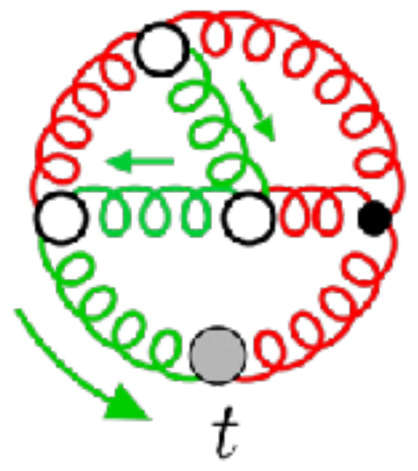
- many diagrams (NLO: 20; NNLO: 3651)
- many integrals
- complicated integrals



Three-loop calculation

The usual problems:

- many diagrams (NLO: 20; NNLO: 3651)
- many integrals
- complicated integrals

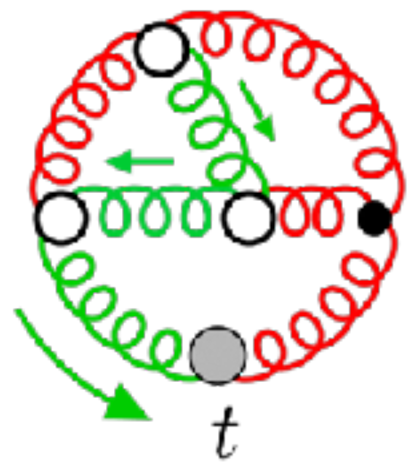


$$I(t,\mathbf{n},\mathbf{a},D) = \left(\prod_{f=1}^{N}\int_0^{t_f^{\rm up}} \mathrm{d}t_f\right)\int_{p_1,p_2,p_3} \frac{\exp[\sum_{k,i,j} a_{kij} t_k p_i p_j]}{p_1^{2n_1} p_2^{2n_2} p_3^{2n_3} p_4^{2n_4} p_5^{2n_5} p_6^{2n_6}}$$

Three-loop calculation

The usual problems:

- many diagrams (NLO: 20; NNLO: 3651)
- many integrals
- complicated integrals



$$I(t,\mathbf{n},\mathbf{a},D) = \left(\prod_{f=1}^{N}\int_0^{t_f^{\rm up}} \mathrm{d}t_f\right)\int_{p_1,p_2,p_3} \frac{\exp[\sum_{k,i,j} a_{kij} t_k p_i p_j]}{p_1^{2n_1} p_2^{2n_2} p_3^{2n_3} p_4^{2n_4} p_5^{2n_5} p_6^{2n_6}}$$

The usual solutions:

- automatic diagram generation
- reduce to master integrals
- evaluate master integrals

Artz, RH, Lange, Neumann, Prausa ’19

The perturbative toolbox

[For details, see: Artz, RH, Lange, Neumann, Prausa '19]

Diagram generation: qgraf Nogueira '93-...

Diagram analyzation: q2e/exp RH, Seidensticker, Steinhauser '98

Algebraic manipulations: FORM Vermaseren '89-...

Reduction to masters: Kira $\otimes$ FireFly

Chetyrkin, Tkachov '81
Laporta '00

Usovitsch, Uwer, Maierhöfer '17-... $\otimes$ Klappert, Klein, Lange '21

Sector Decomposition:

Binoth, Heinrich '00

$$\int d^Dk \int d^Dp \int_0^t ds \frac{e^{-tp^2-s(k-p)^2}}{k^2p^2(k-p^2)} = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C + \ldots$$

The first application

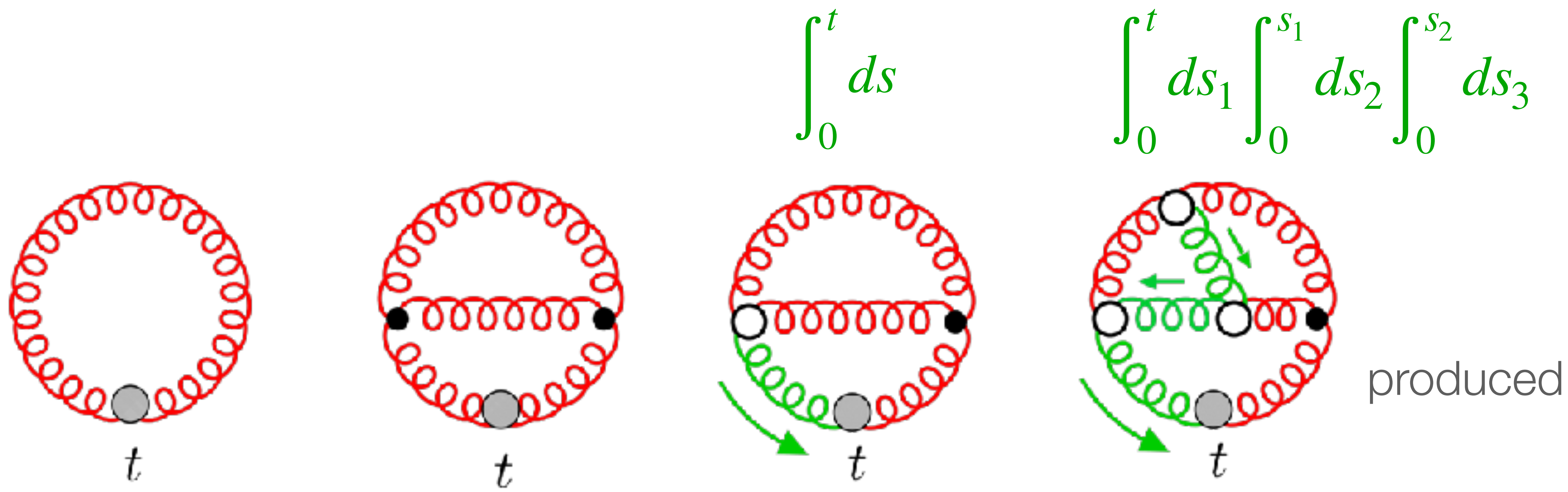
The first application: Lüscher '10

$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle \; :$$

$$t^2 \langle G_{\mu\nu}^2(t) \rangle = \frac{3\alpha_s}{4\pi} \left[1 + k_1\, \alpha_s + k_2\, \alpha_s^2 + \ldots \right]$$

$$= \frac{3\alpha_s}{4\pi} \left[1 + 1.8\, \alpha_s + 5.4\, \alpha_s^4 + \ldots \right]$$

Lüscher '10



produced with FeynGame

The first application

The first application: Lüscher ‘10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle \, :$$

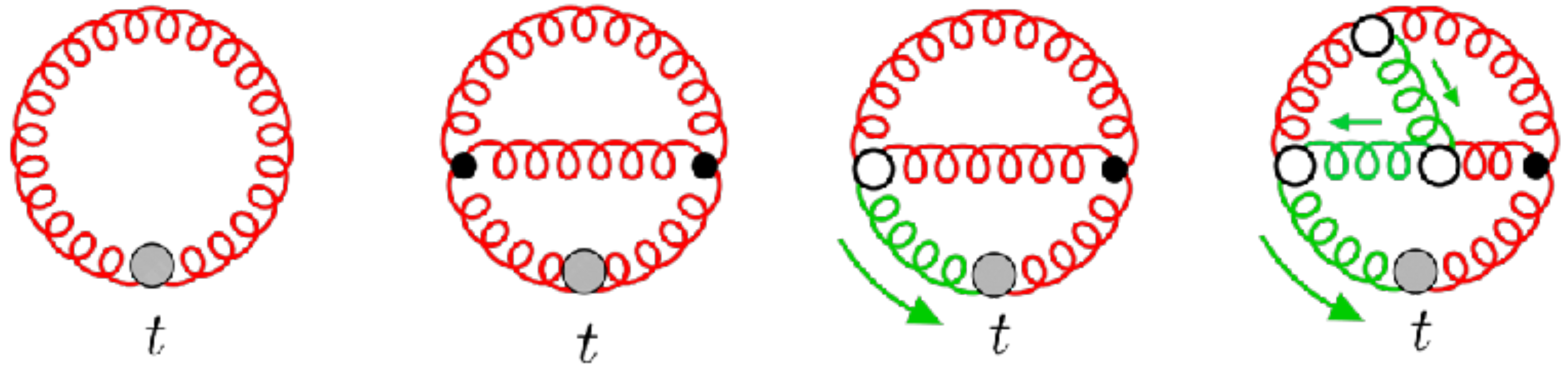
$$t^2\langle G_{\mu\nu}^2(t)\rangle = \frac{3\alpha_s}{4\pi}\left[1 + k_1\,\alpha_s + k_2\,\alpha_s^2 + \ldots\right]$$

$$= \frac{3\alpha_s}{4\pi}\left[1 + 1.8\,\alpha_s + 5.4\,\alpha_s^4 + \ldots\right]$$

Lüscher ‘10

RH, Neumann ‘16

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$

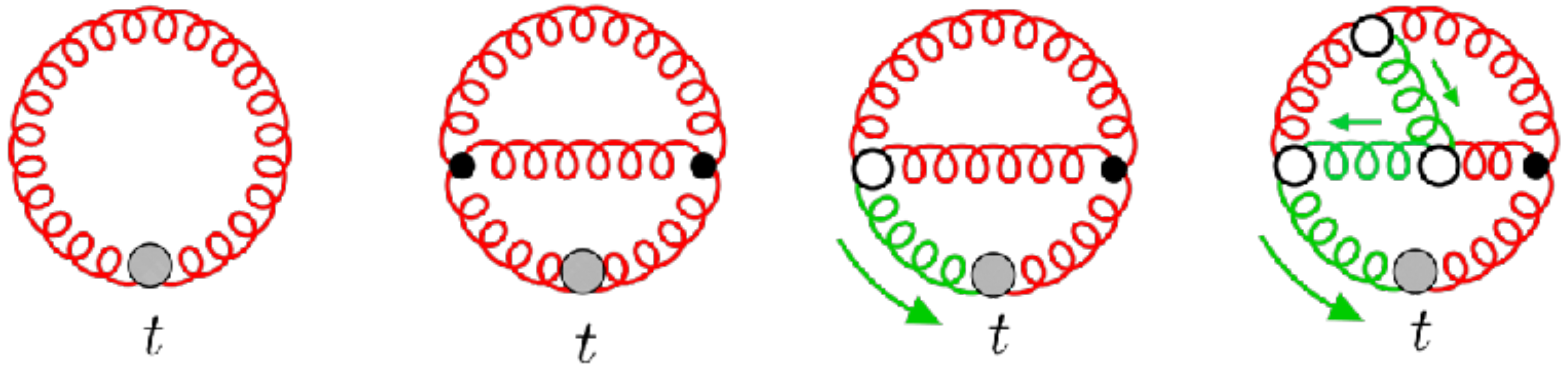


produced with FeynGame

The first application

The first application: Lüscher ‘10

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$



produced with FeynGame

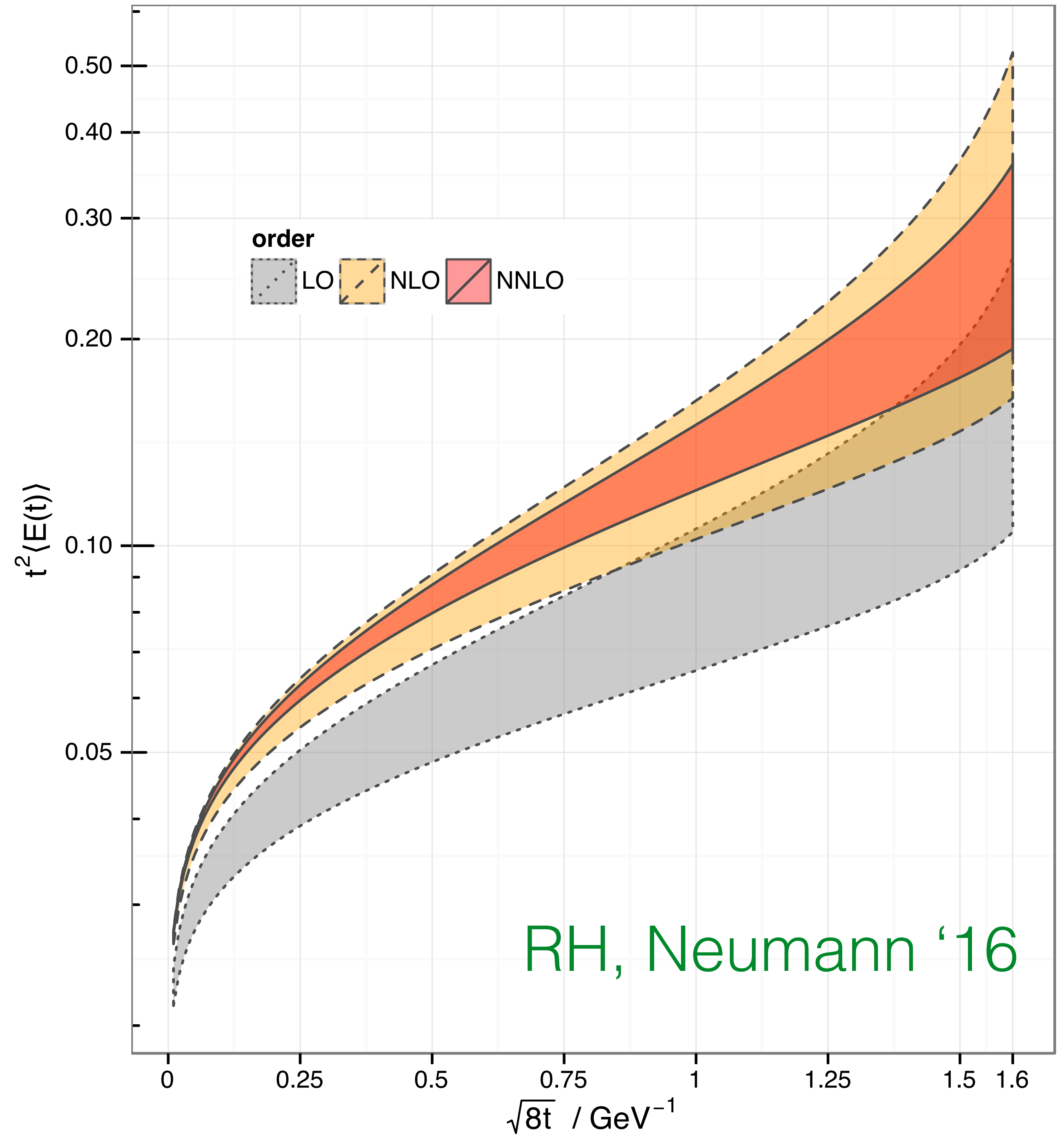
$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t)\rangle :$$

$$t^2\langle G^2_{\mu\nu}(t)\rangle = \frac{3\alpha_s}{4\pi}\left[1 + k_1\,\alpha_s + k_2\,\alpha_s^2 + \ldots\right]$$

$$= \frac{3\alpha_s}{4\pi}\left[1 + 1.8\,\alpha_s + 5.4\,\alpha_s^4 + \ldots\right]$$

Lüscher ‘10

RH, Neumann ‘16



The first application

The first application: Lüscher '10

$$\langle G_{\mu\nu}(t) G^{\mu\nu}(t) \rangle :$$

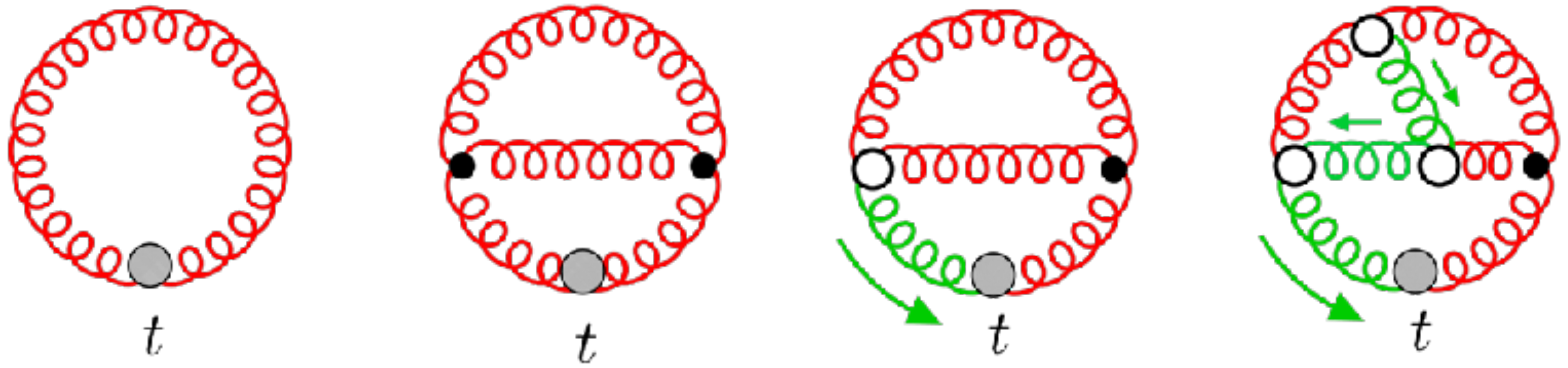
$$t^2 \langle G_{\mu\nu}^2(t) \rangle = \frac{3\alpha_s}{4\pi} \left[1 + k_1\, \alpha_s + k_2\, \alpha_s^2 + \ldots \right]$$

$$= \frac{3\alpha_s}{4\pi} \left[1 + 1.8\, \alpha_s + 5.4\, \alpha_s^4 + \ldots \right] \equiv \frac{3\alpha_s^{\mathrm{GF}}(t)}{4\pi}$$

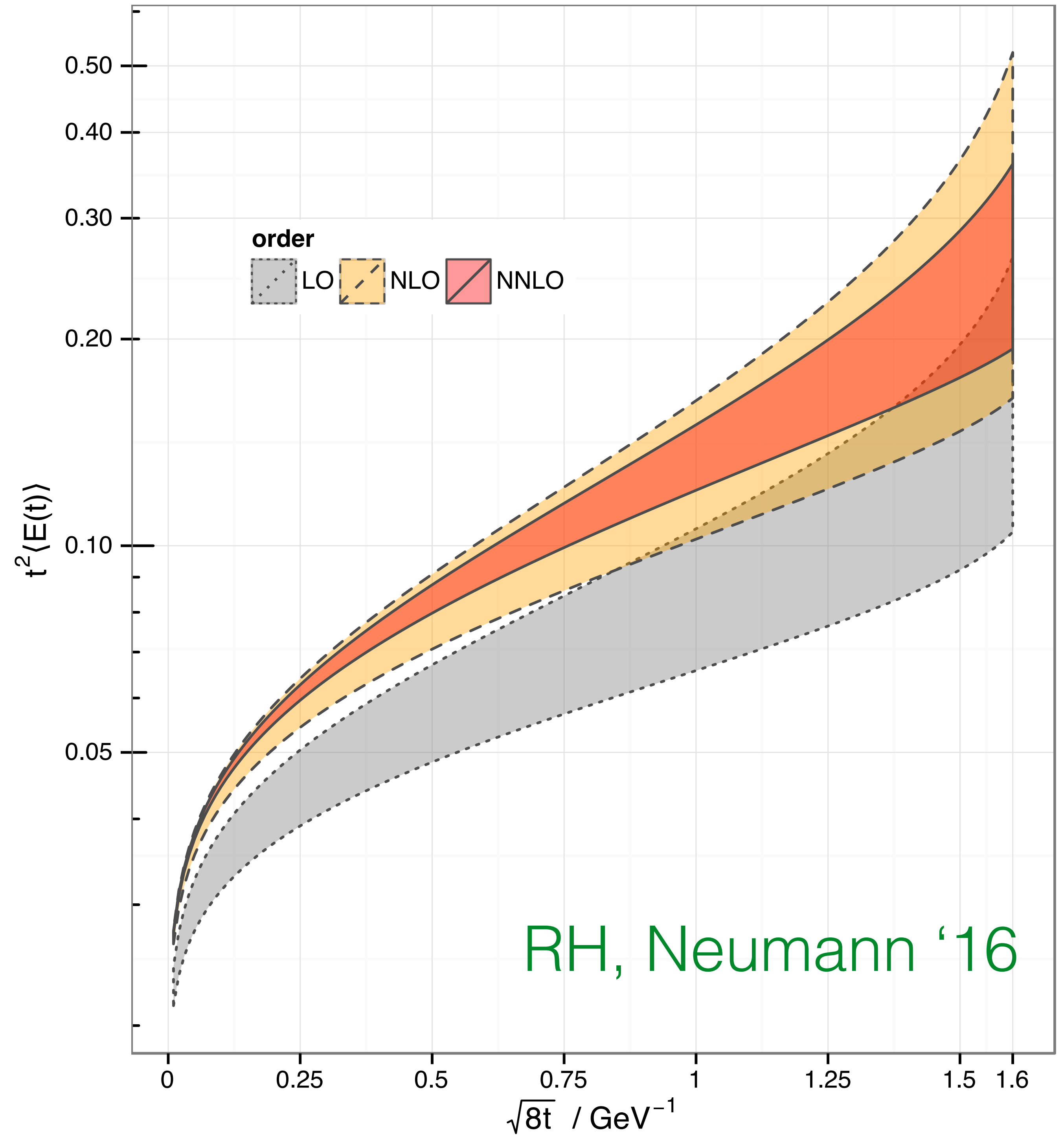
Lüscher '10

RH, Neumann '16

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$



produced with FeynGame

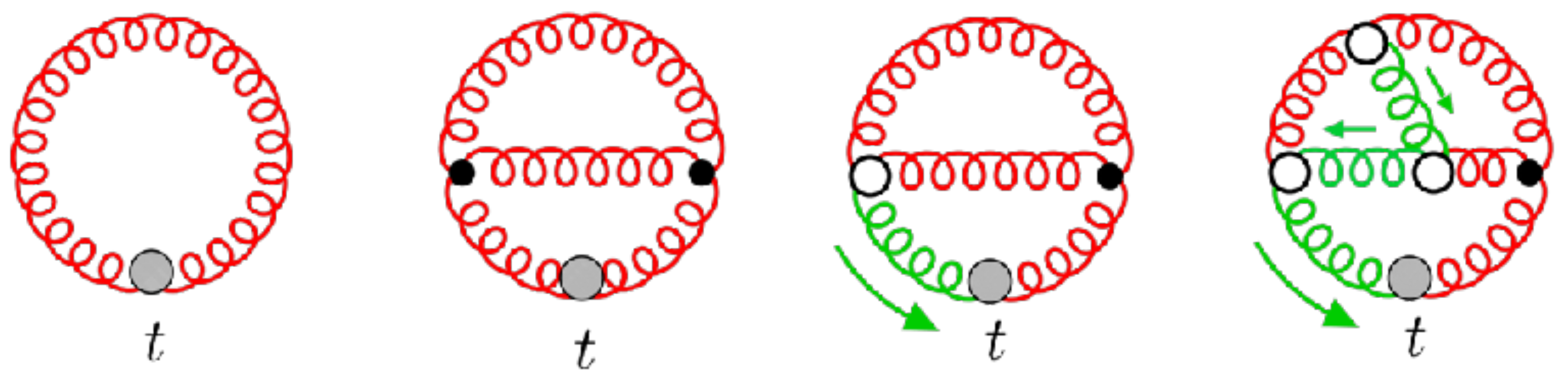


The first application

The first application: Lüscher '10

$$\langle G_{\mu\nu}(t)G^{\mu\nu}(t)\rangle :$$

$$\int_0^t ds \qquad \int_0^t ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3$$



produced with FeynGame

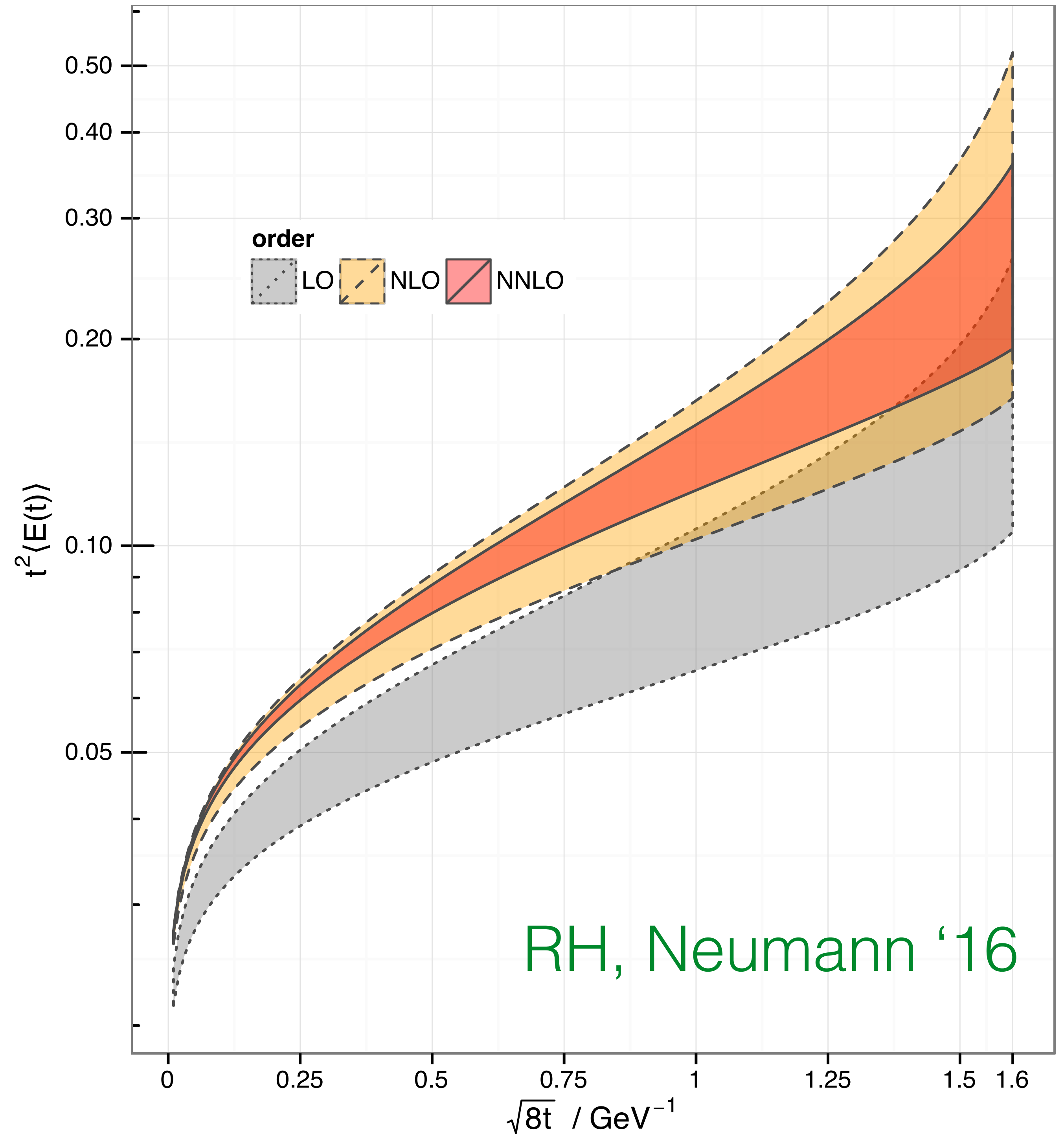
$$t^2\langle G_{\mu\nu}^2(t)\rangle = \frac{3\alpha_s}{4\pi}\left[1 + k_1\,\alpha_s + k_2\,\alpha_s^2 + \ldots\right]$$

$$= \frac{3\alpha_s}{4\pi}\left[1 + 1.8\,\alpha_s + 5.4\,\alpha_s^4 + \ldots\right] \equiv \frac{3\alpha_s^{\rm GF}(t)}{4\pi}$$

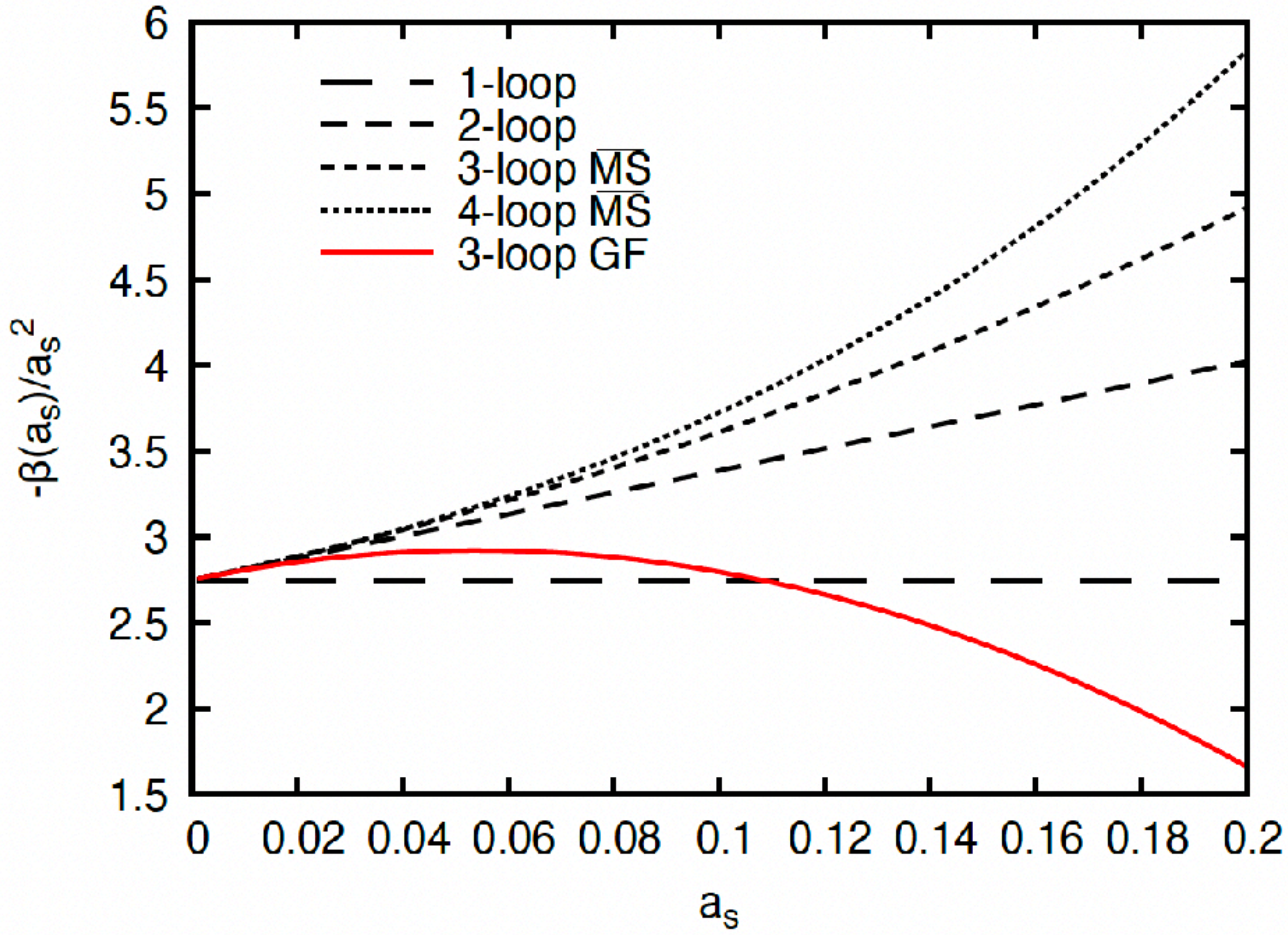
Lüscher '10

RH, Neumann '16

$$\mu^2\frac{d}{d\mu^2}\alpha_s^{\rm GF}(\mu) = \beta^{\overline{\rm GF}}(\alpha_s^{\rm GF})$$



GF beta function



$$\mu^2 \frac{d}{d\mu^2}\alpha_s^{\mathrm{GF}}(\mu) = \beta^{\overline{\mathrm{GF}}}(\alpha_s^{\mathrm{GF}})$$

see also

Dalla Brida, Lüscher '17

Dalla Brida, Ramos '19

→ A. Hasenfratz' talk

A common problem

Observable: $$R = \sum_n C_n \langle \mathcal{O}_n \rangle$$

perturbation theory

lattice

match renormalization schemes?

A common problem

Observable: $$R = \sum_n C_n \langle \mathcal{O}_n \rangle$$

perturbation theory

lattice

match renormalization schemes?

Instead: $$R = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

perturbation theory

lattice

gradient flow renormalization

Small flow-time expansion

Observable: $$R = \sum_n C_n \langle \mathcal{O}_n \rangle = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

small flow-time expansion:
Lüscher, Weisz '11

$$\tilde{\mathcal{O}}_n(t) \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t)\, \mathcal{O}_m$$

$$\tilde{C}_n(t) \overset{t\to 0}{\to} \sum_m C_m\, \zeta^{-1}_{mn}(t)$$

$\Rightarrow$ need $\zeta_{nm}(t)$ for small t $\Rightarrow$ perturbation theory

Determining $\zeta(t)$

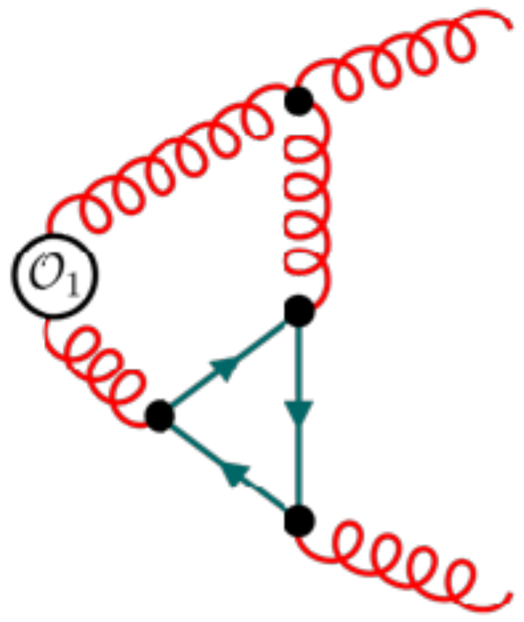
Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \, \langle \mathcal{O}_m \rangle$$

Determining $\zeta(t)$

Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

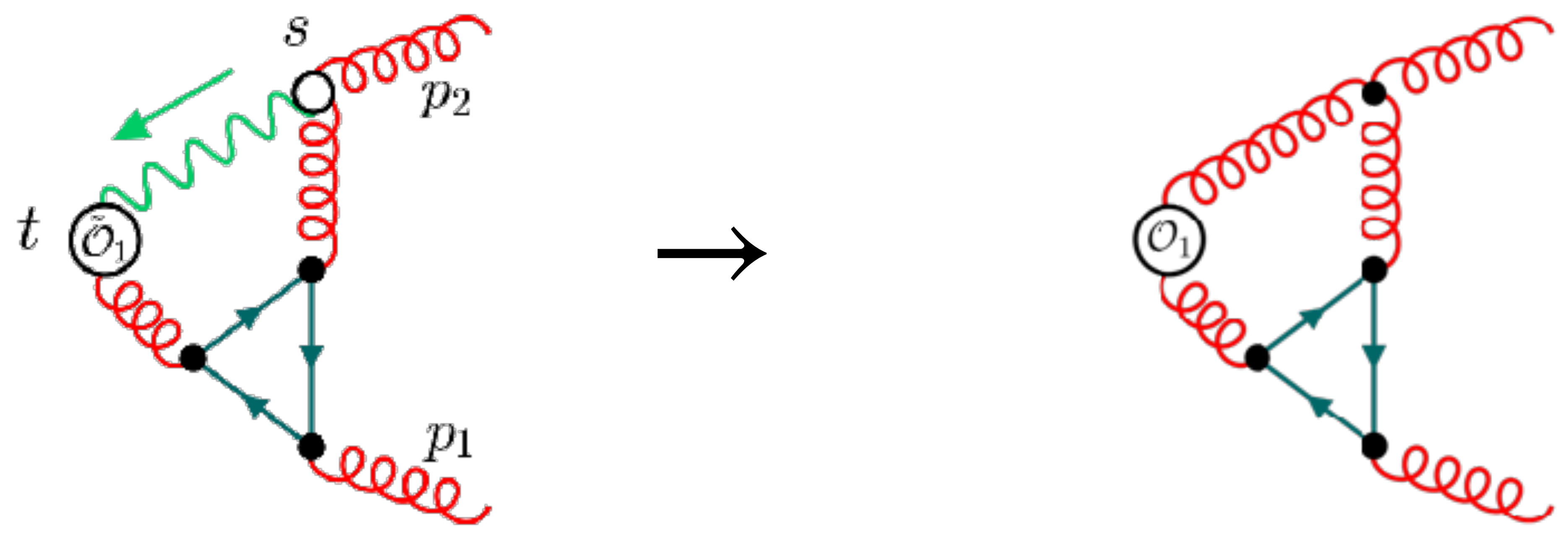
$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \, \langle \mathcal{O}_m \rangle$$



Determining $\zeta(t)$

Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

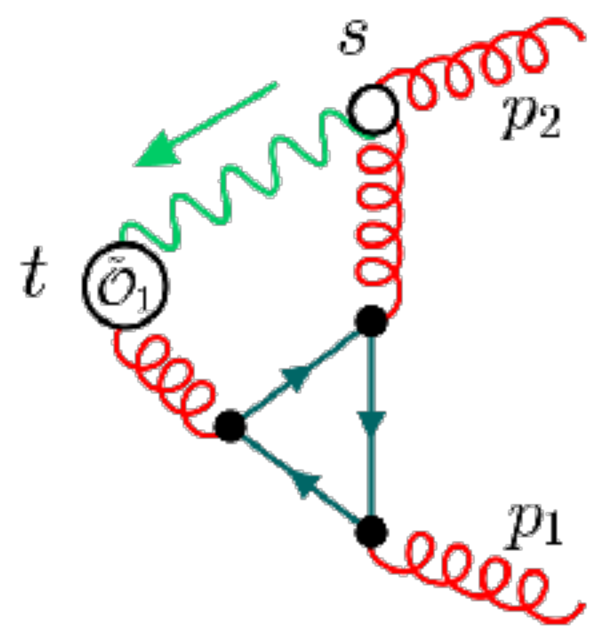
$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \, \langle \mathcal{O}_m \rangle$$



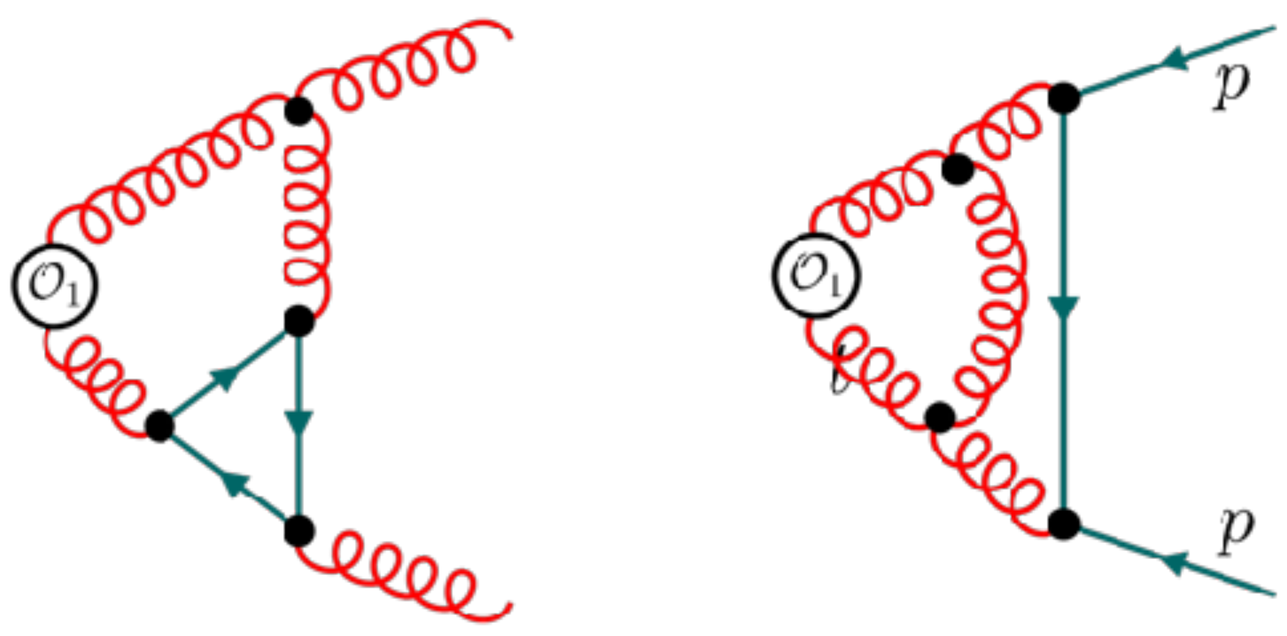
Determining $\zeta(t)$

Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \, \langle \mathcal{O}_m \rangle$$



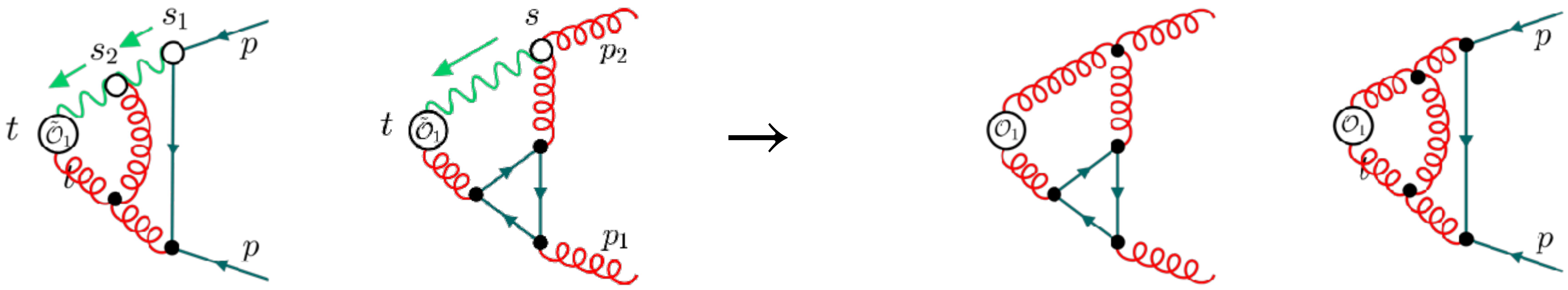
$\to$



Determining $\zeta(t)$

Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

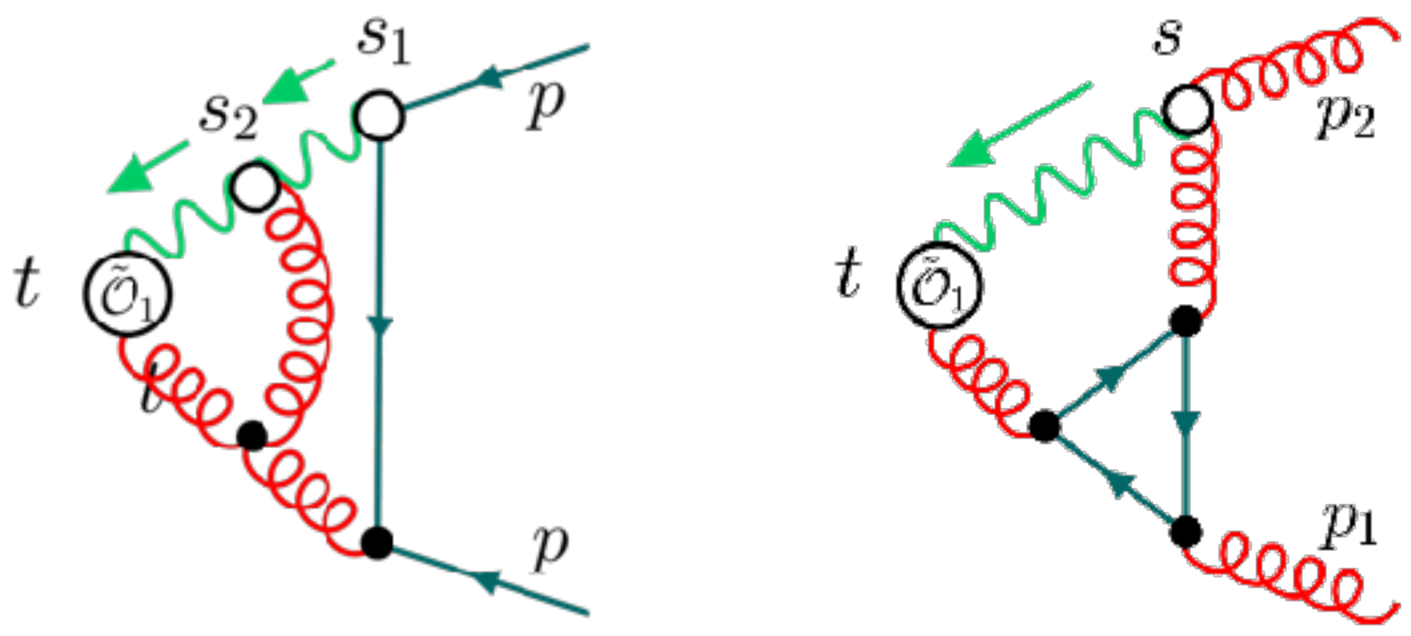
$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \, \langle \mathcal{O}_m \rangle$$



Determining $\zeta(t)$

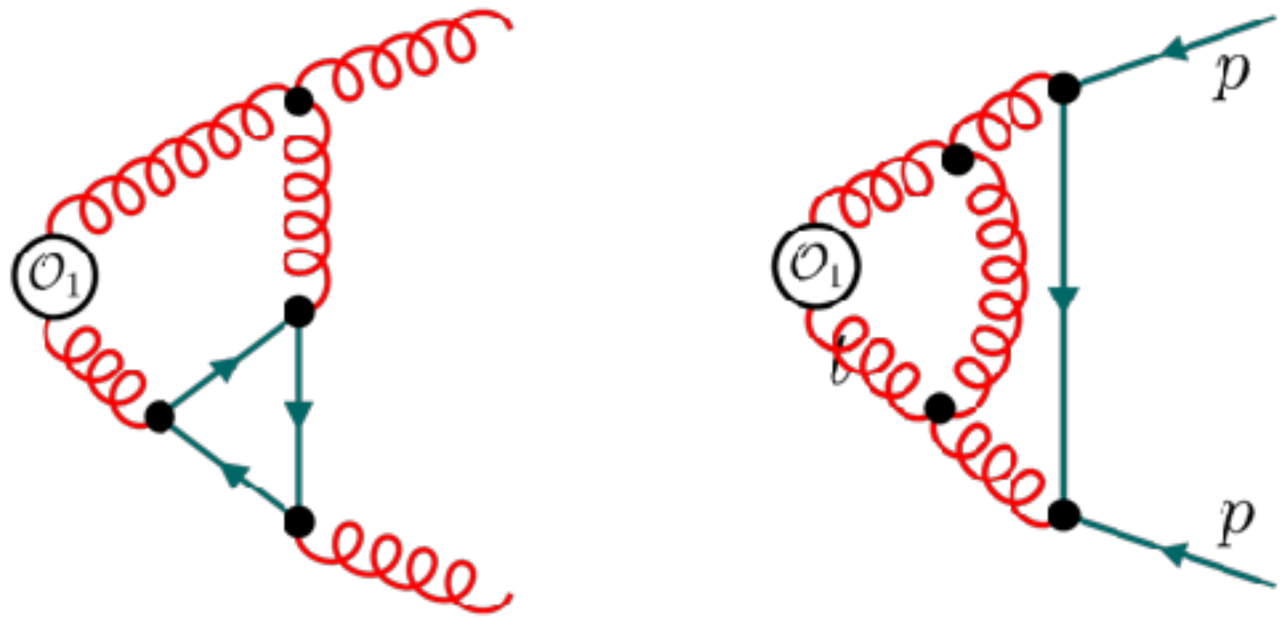
Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t) \langle \mathcal{O}_m \rangle$$



$p = m = 0$

$\rightarrow$

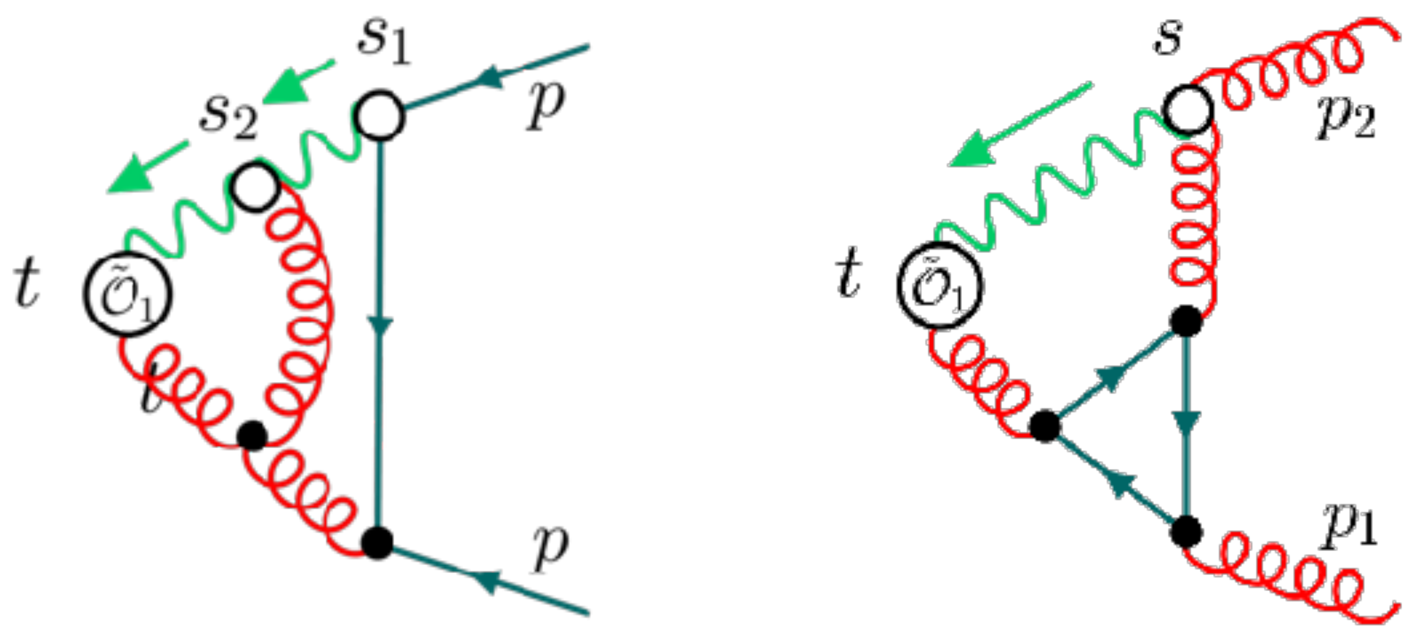


$p = m = 0$

Determining $\zeta(t)$

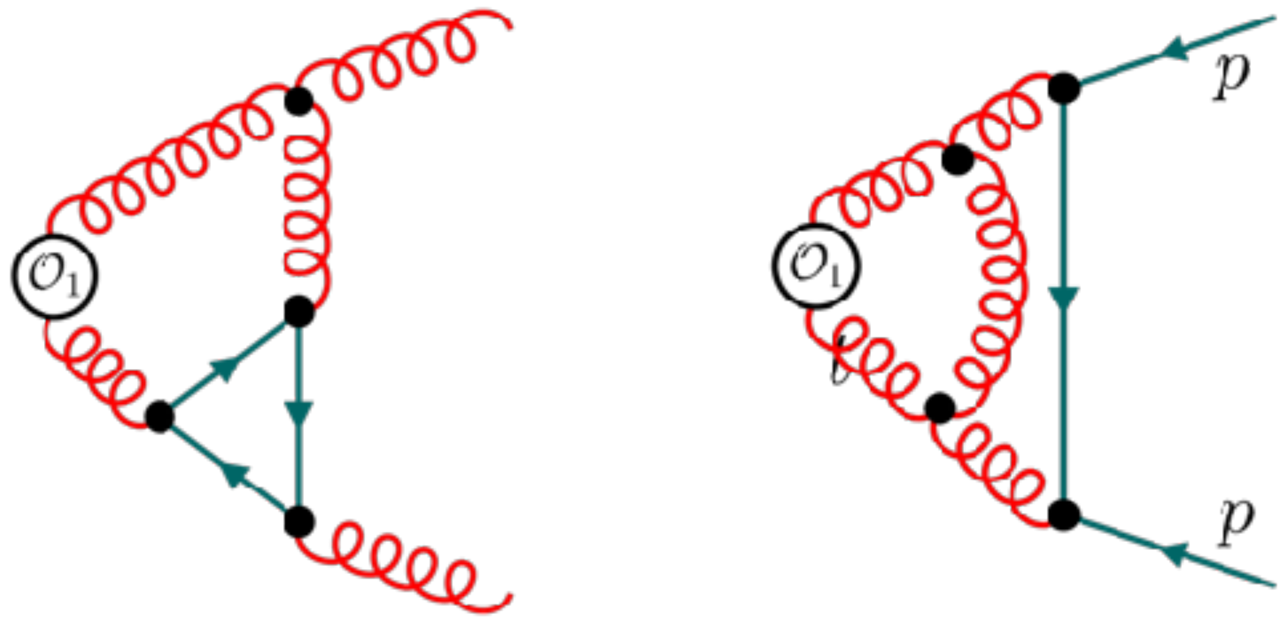
Matching: calculate a set of suitable Green's functions and solve for $\zeta_{nm}(t)$

$$\langle \tilde{\mathcal{O}}_n(t) \rangle \overset{t\to 0}{\to} \sum_m \zeta_{nm}(t)\, \langle \mathcal{O}_m \rangle$$



$p = m = 0$

$\rightarrow$



$p = m = 0$

only tree-level diagrams survive on r.h.s.

Gorishnii, Larin, Tkachov '83

Ex. 1: QCD energy-momentum tensor

Suzuki, Makino '13, '14

$$T_{\mu\nu} = \sum_n C_n \mathcal{O}_{n,\mu\nu}$$

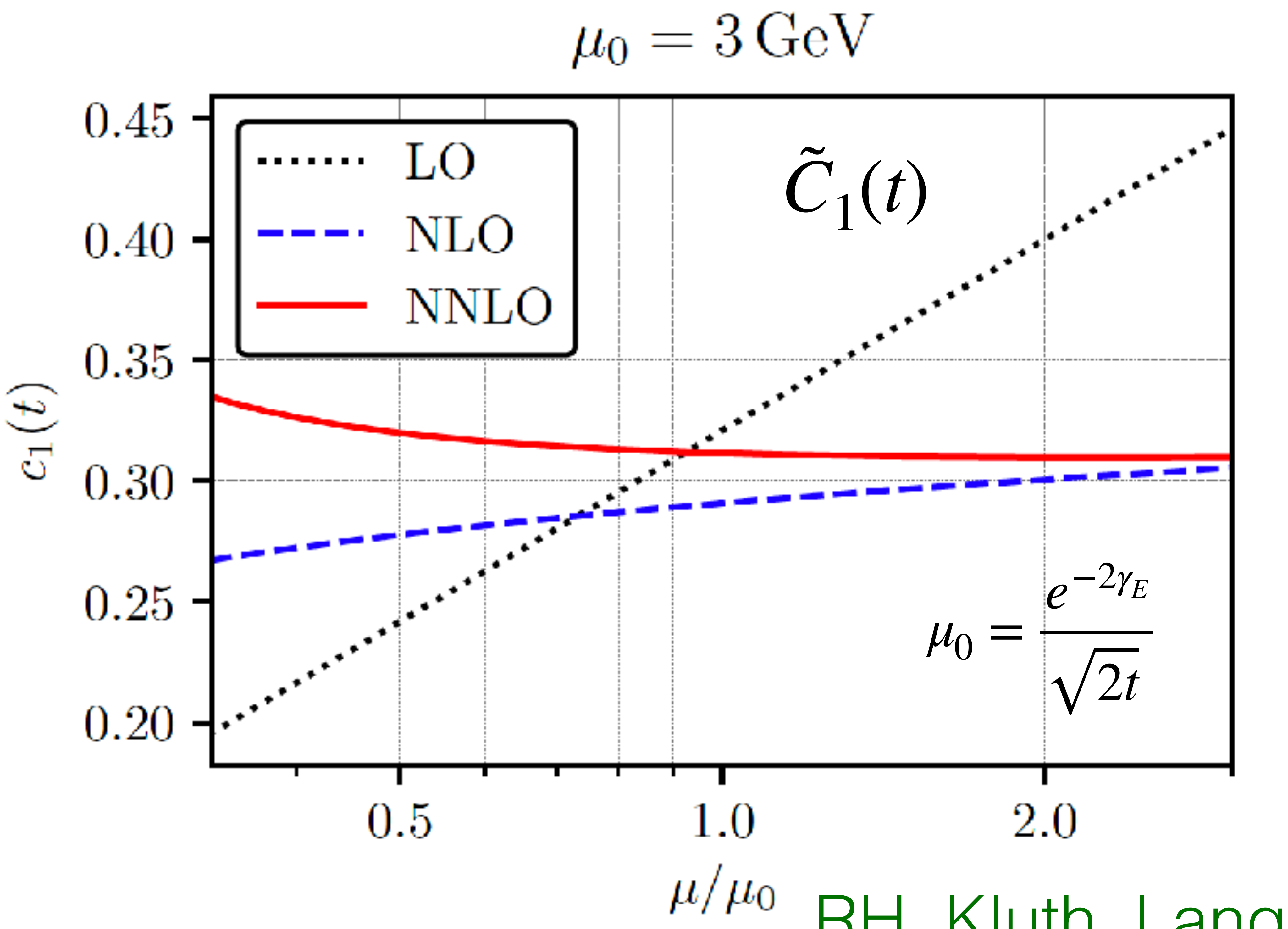
$$T_{\mu\nu} = \sum_n \tilde{C}_n(t) \tilde{\mathcal{O}}_{n,\mu\nu}(t)$$

$$\mathcal{O}_{1,\mu\nu} = \frac{1}{g_0^2} F^a_{\mu\rho} F^a_{\nu\rho} \qquad C_1 \equiv 1$$

$$\mathcal{O}_{2,\mu\nu} = \frac{\delta_{\mu\nu}}{g_0^2} F^a_{\rho\sigma} F^a_{\rho\sigma} \qquad C_2 \equiv -\frac{1}{4}$$

$$\mathcal{O}_{3,\mu\nu} = \bar{\psi} \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi \qquad C_3 \equiv \frac{1}{4}$$

$$\mathcal{O}_{4,\mu\nu} = \delta_{\mu\nu} \bar{\psi} \overleftrightarrow{\not{D}} \psi \qquad C_4 \equiv 0$$

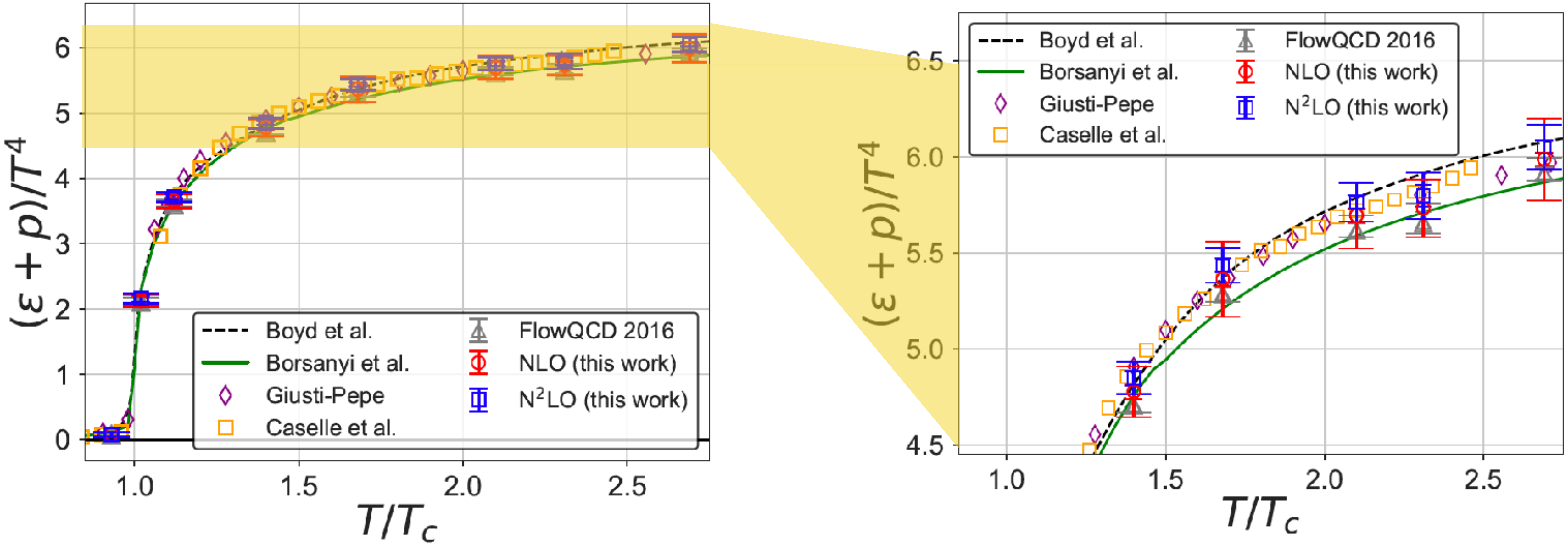


RH, Kluth, Lange '18

application: see WHOT collaboration

Application

Entropy density: $$\varepsilon + p = -\frac{4}{3}\left\langle T_{00}(x) - \frac{1}{4}T_{\mu\mu}(x)\right\rangle$$

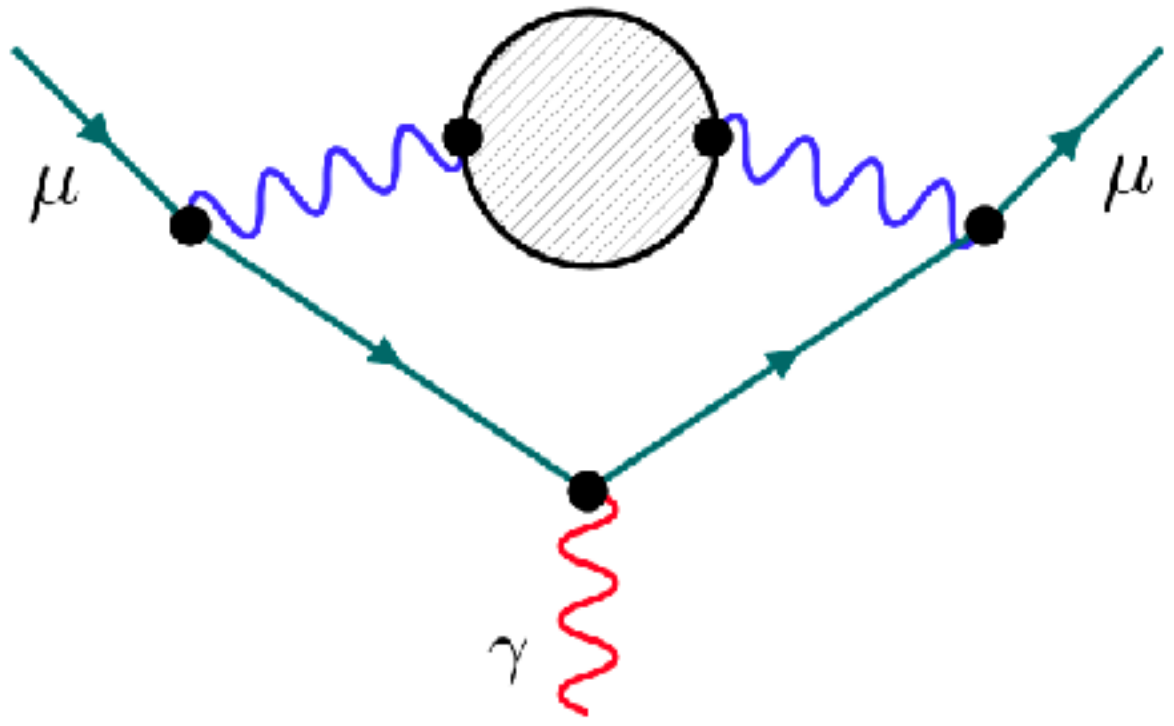


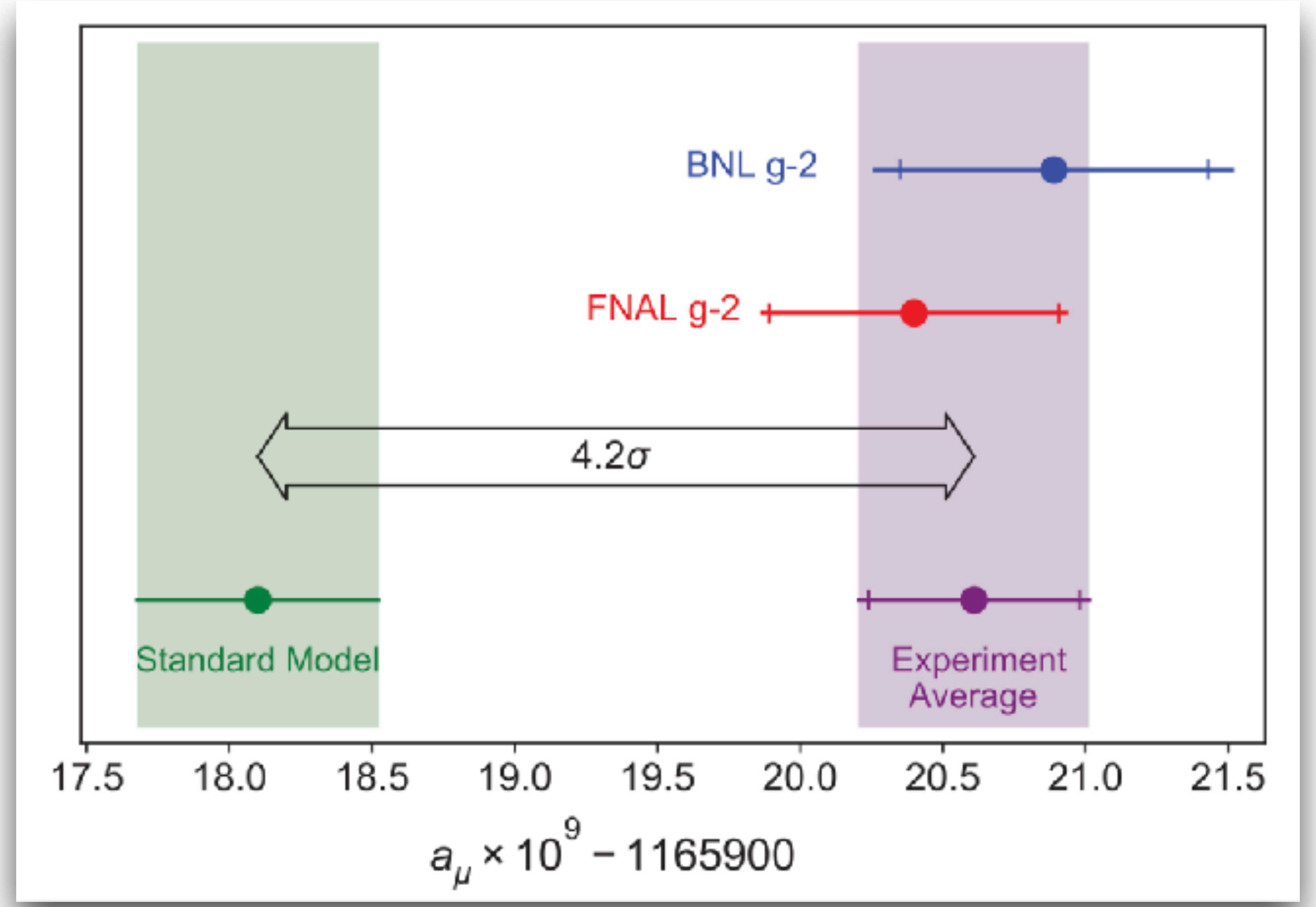
Iritani, Kitazawa, Suzuki, Takaura 2019

Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx} \langle T j(x) j(0) \rangle$$

contribution to $(g-2)_\mu$

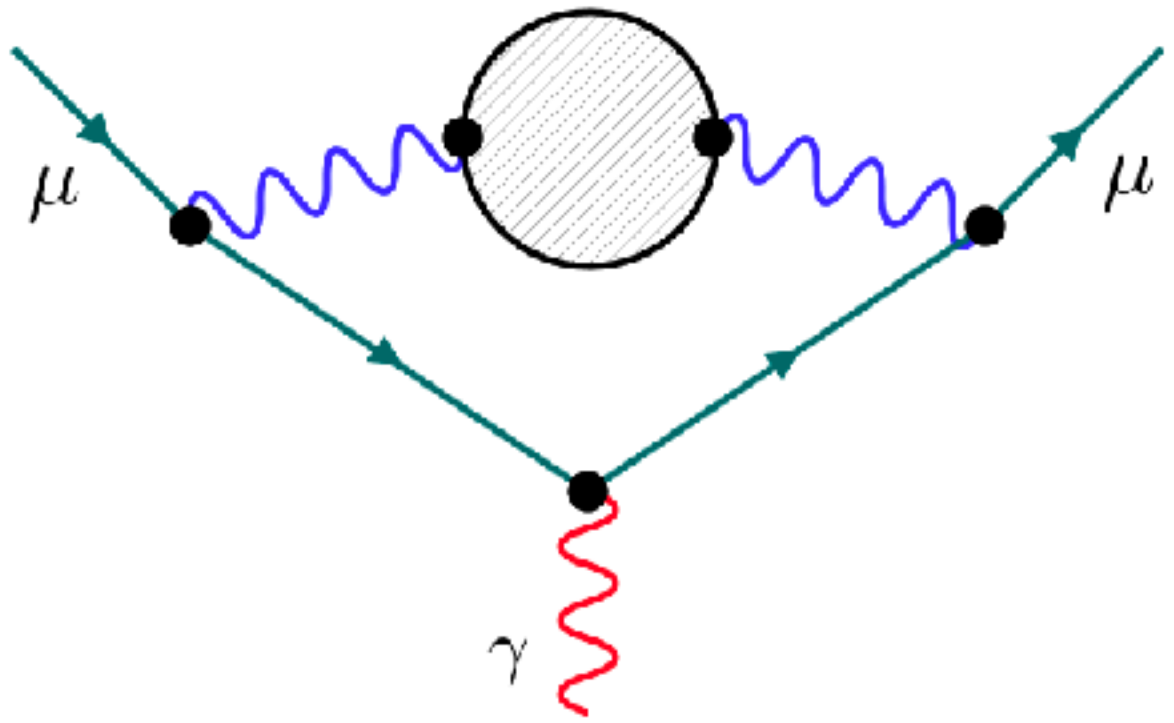




Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx} \langle T\, j(x)\, j(0) \rangle$$

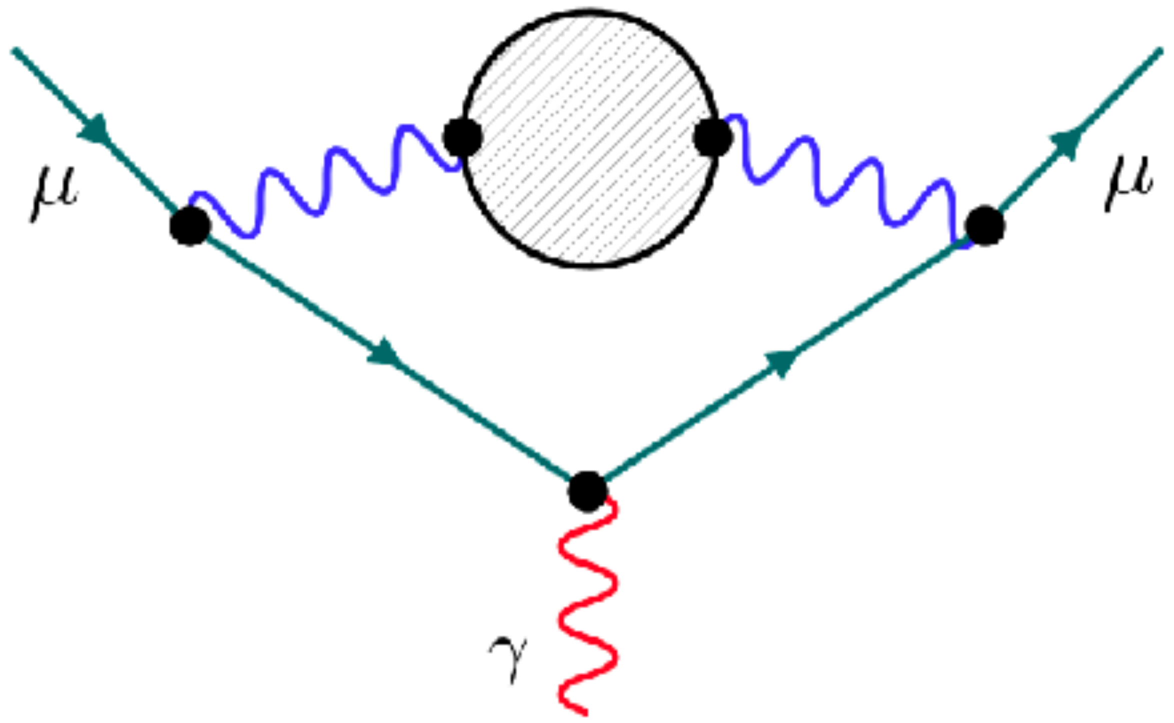
contribution to $(g-2)_\mu$



Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx}\langle T\, j(x)\, j(0)\rangle \to \sum_n C_n(Q)\langle \mathcal{O}_n\rangle$$

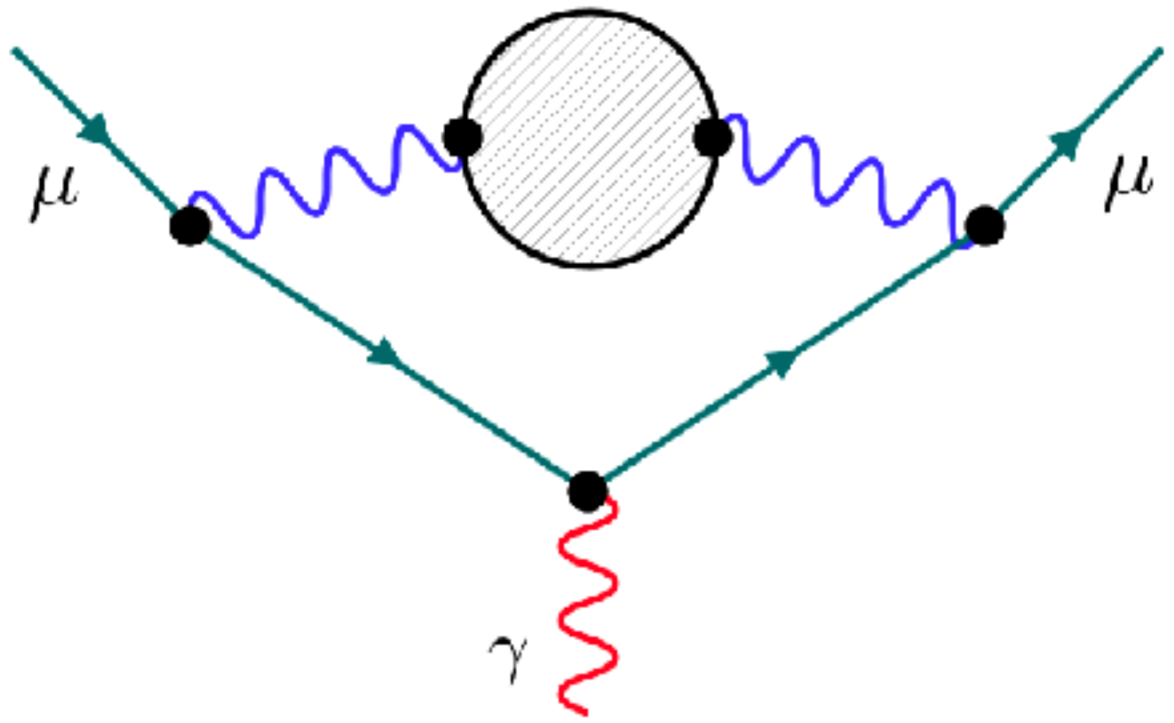
contribution to $(g-2)_\mu$



Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx} \langle T j(x) j(0) \rangle \to \sum_n C_n(Q) \langle \mathcal{O}_n \rangle$$

contribution to $(g-2)_\mu$

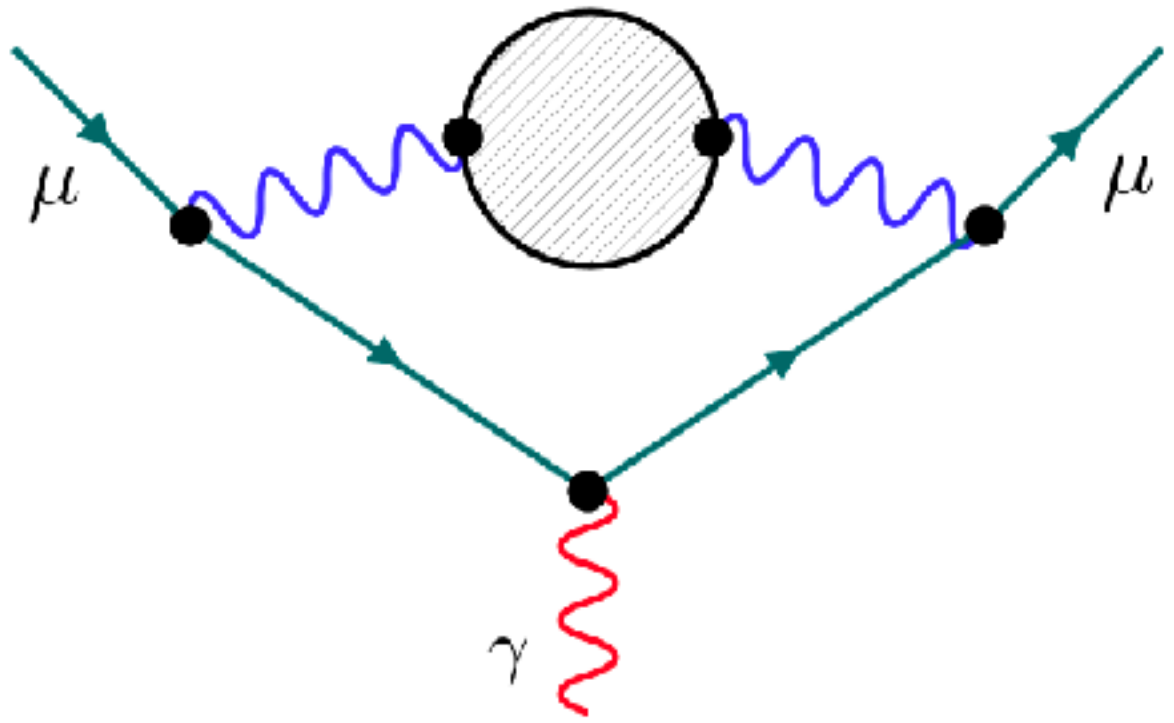


Known to
high orders in
perturbation theory

Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx} \langle T j(x) j(0) \rangle \to \sum_n C_n(Q) \langle \mathcal{O}_n \rangle \; = \sum_n \tilde{C}_n(Q, t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

contribution to $(g-2)_\mu$

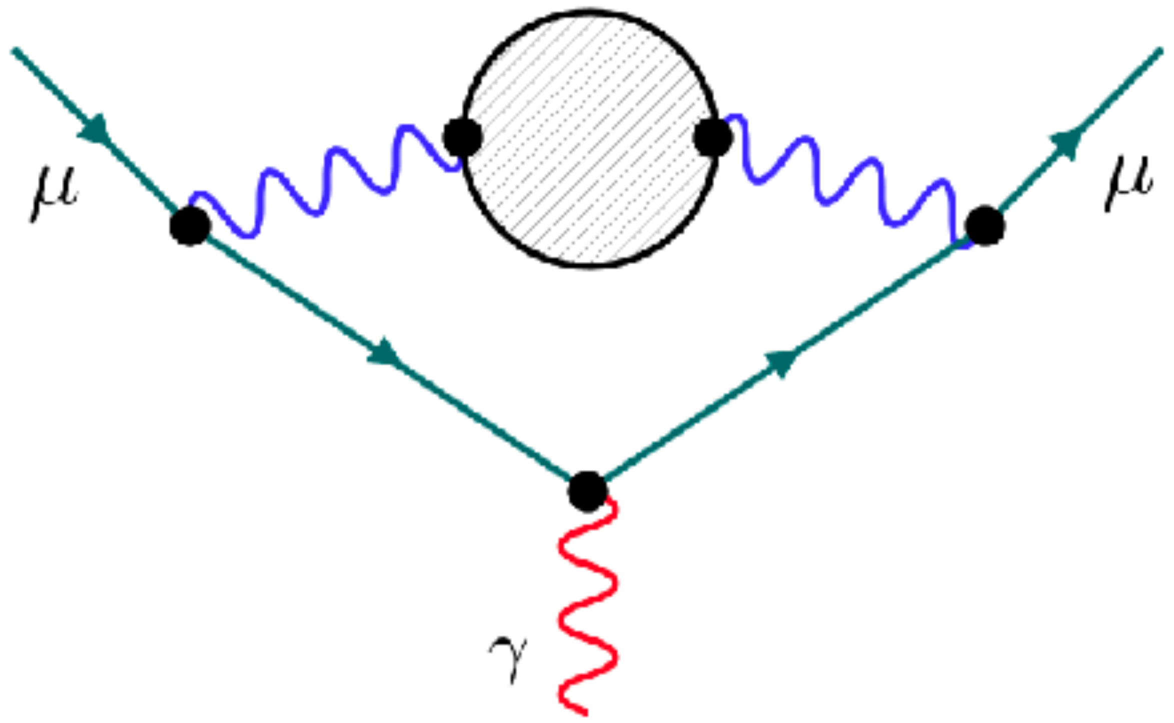


Known to
high orders in
perturbation theory

Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx}\langle T j(x) j(0)\rangle \to \sum_n C_n(Q)\langle \mathcal{O}_n\rangle = \sum_n \tilde{C}_n(Q,t)\langle \tilde{\mathcal{O}}_n(t)\rangle$$

contribution to $(g-2)_\mu$



Known to
high orders in
perturbation theory

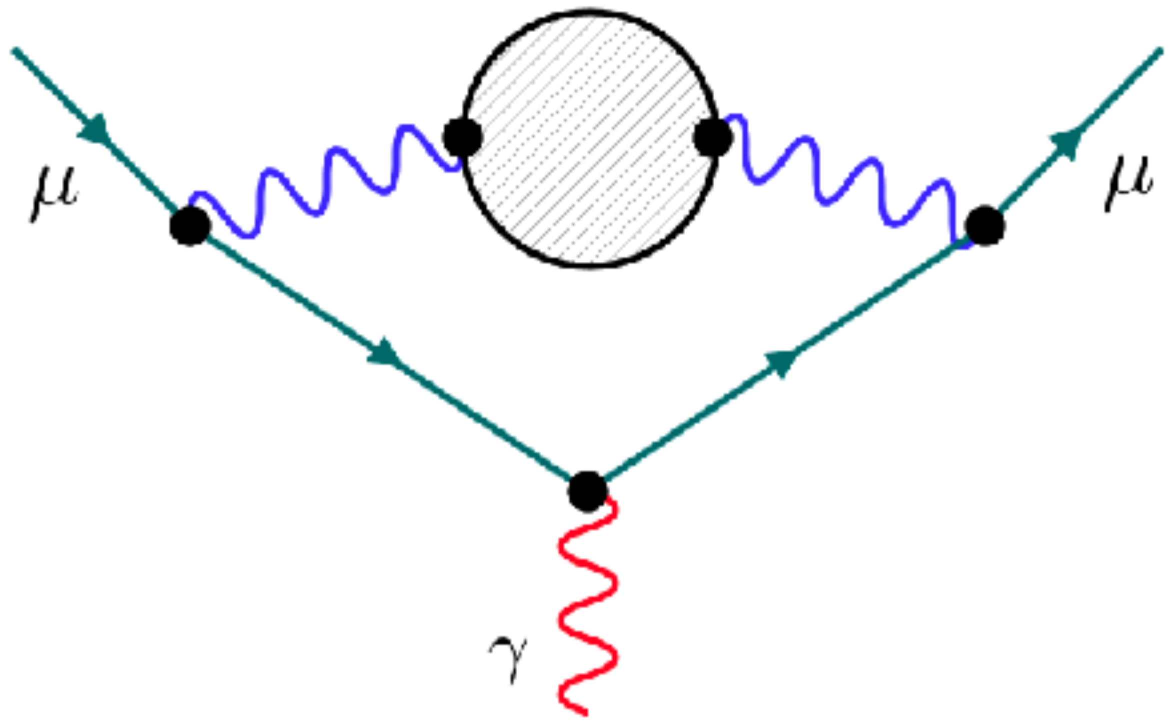
$\mathcal{O}_1 = 1 \qquad \mathcal{O}_2 = m^2 \qquad \mathcal{O}_3 = m^4$

$\mathcal{O}_4 = F^a_{\mu\nu}F^a_{\mu\nu} \qquad \mathcal{O}_5 = m\bar{\psi}\psi$

Ex. 2: Hadronic vacuum polarization

$$\int d^4x\, e^{iQx}\langle T j(x) j(0)\rangle \to \sum_n C_n(Q)\langle \mathcal{O}_n\rangle \;=\; \sum_n \tilde{C}_n(Q,t)\langle \tilde{\mathcal{O}}_n(t)\rangle$$

contribution to $(g-2)_\mu$



Known to high orders in perturbation theory

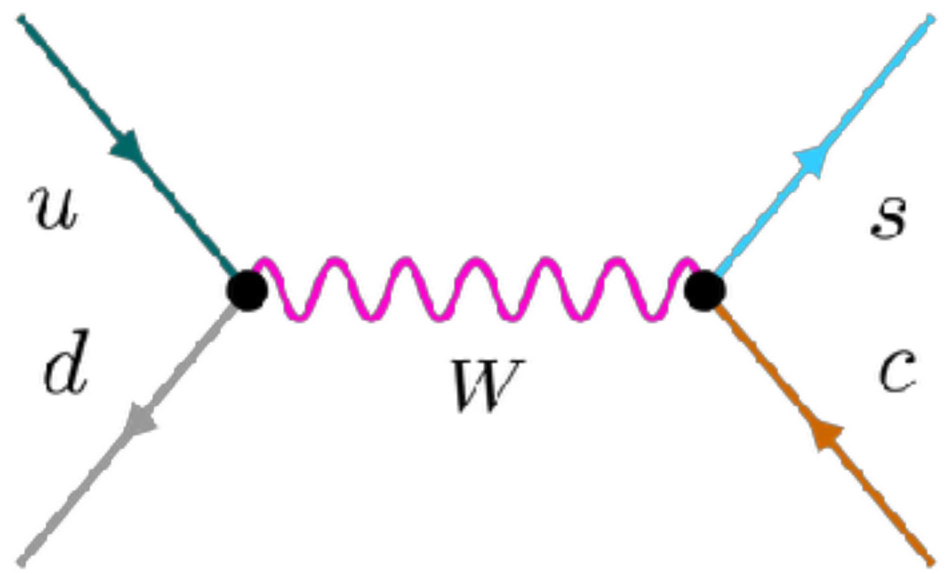
$\mathcal{O}_1 = 1 \qquad \mathcal{O}_2 = m^2 \qquad \mathcal{O}_3 = m^4$

$\mathcal{O}_4 = F^a_{\mu\nu}F^a_{\mu\nu} \qquad \mathcal{O}_5 = m\bar{\psi}\psi$

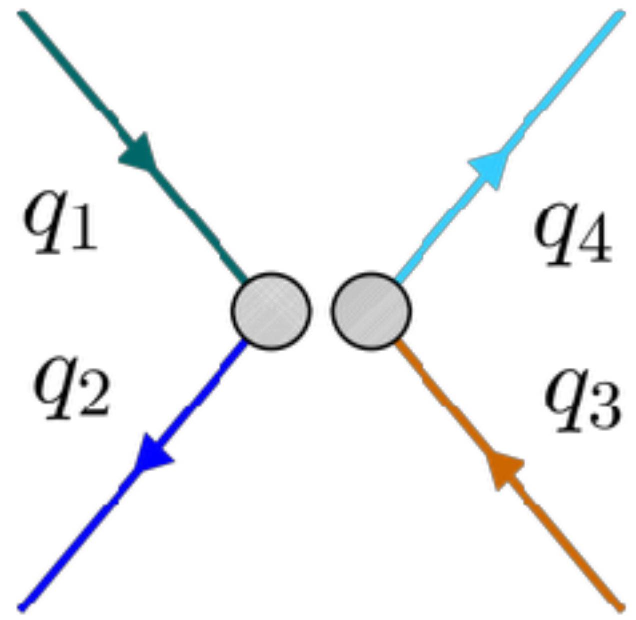
$\tilde{C}_n(Q,t)$ to NNLO

RH, Lange, Neumann '20

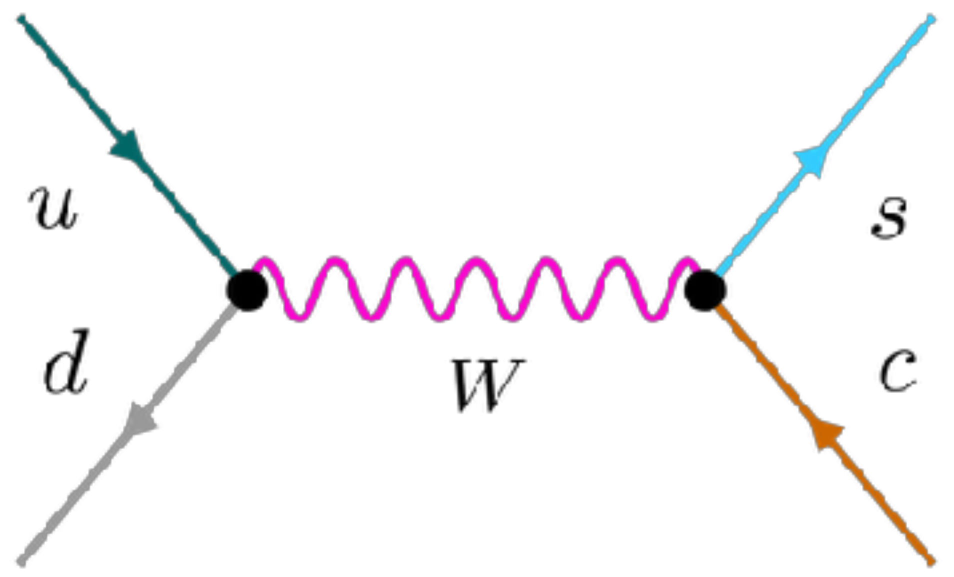
Electroweak Hamiltonian



$$M_W \to \infty$$

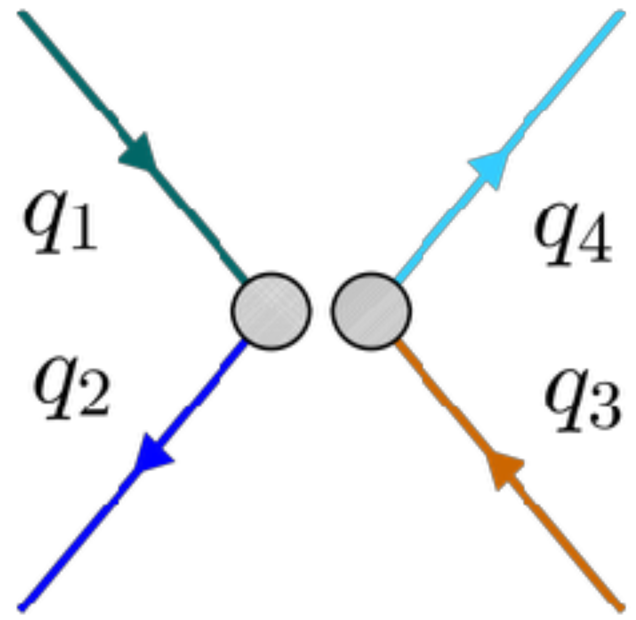


Electroweak Hamiltonian



$M_W \to \infty$

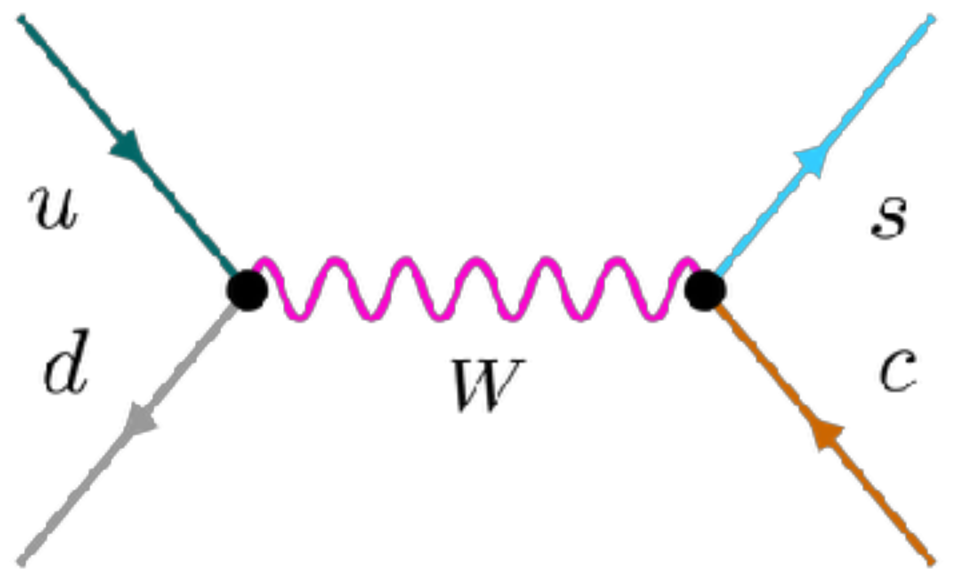
$\longrightarrow$



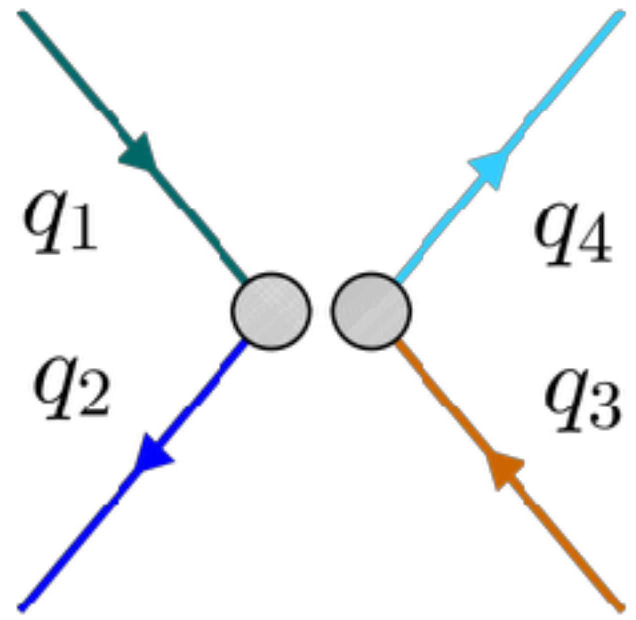
$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

Electroweak Hamiltonian



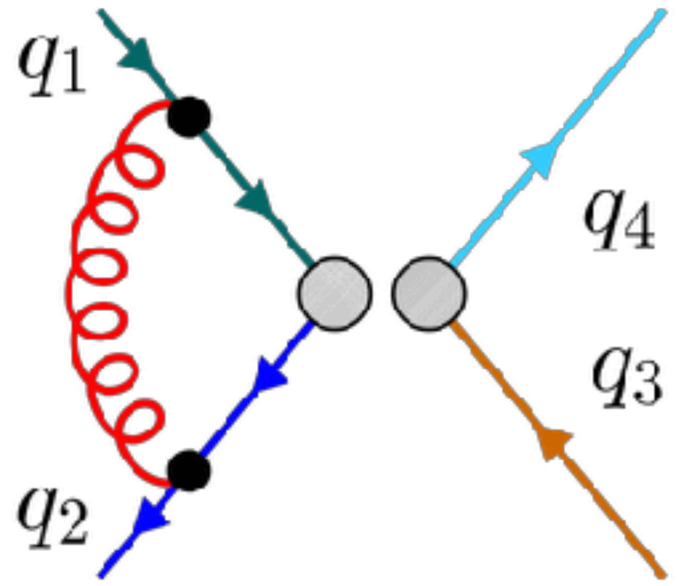
$$\xrightarrow{M_W \to \infty}$$



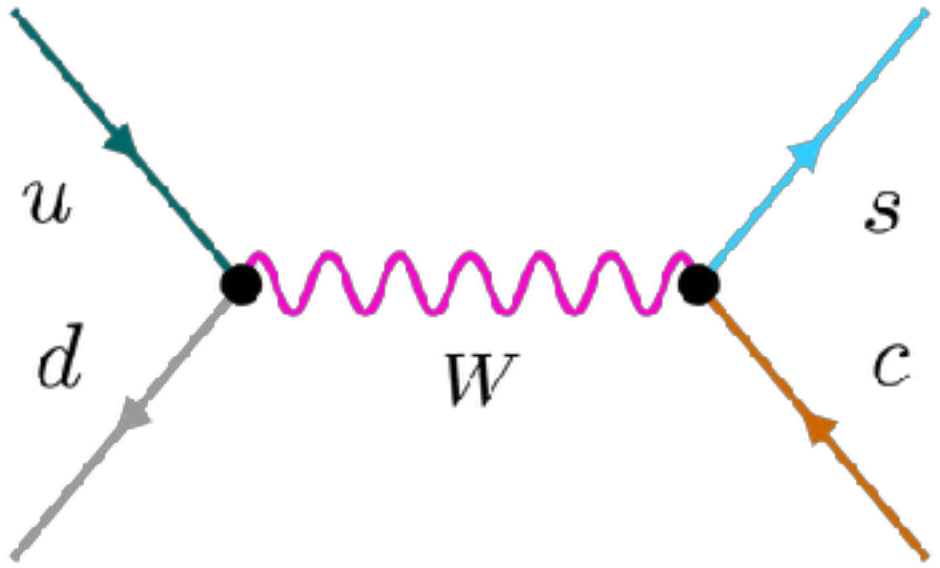
$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

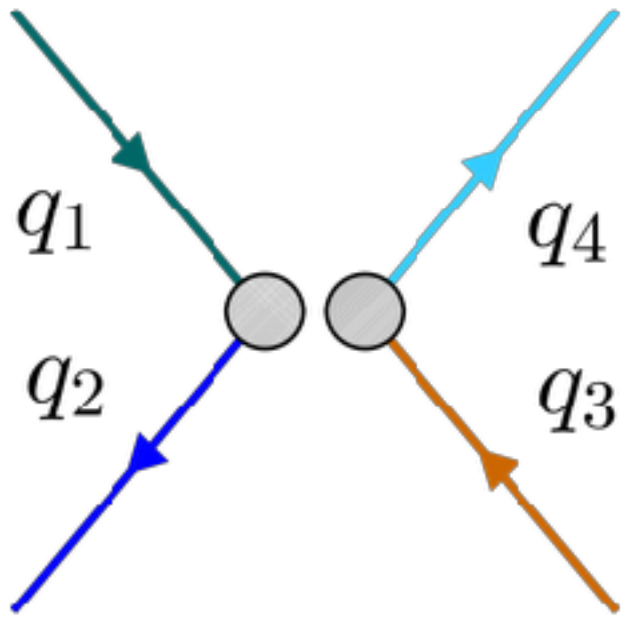
In $D = 4 - 2\epsilon$



Electroweak Hamiltonian



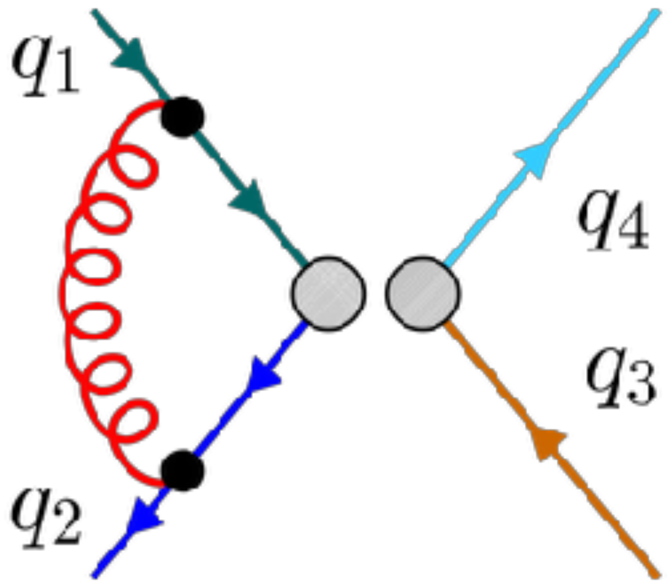
$$M_W \to \infty$$
$$\longrightarrow$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

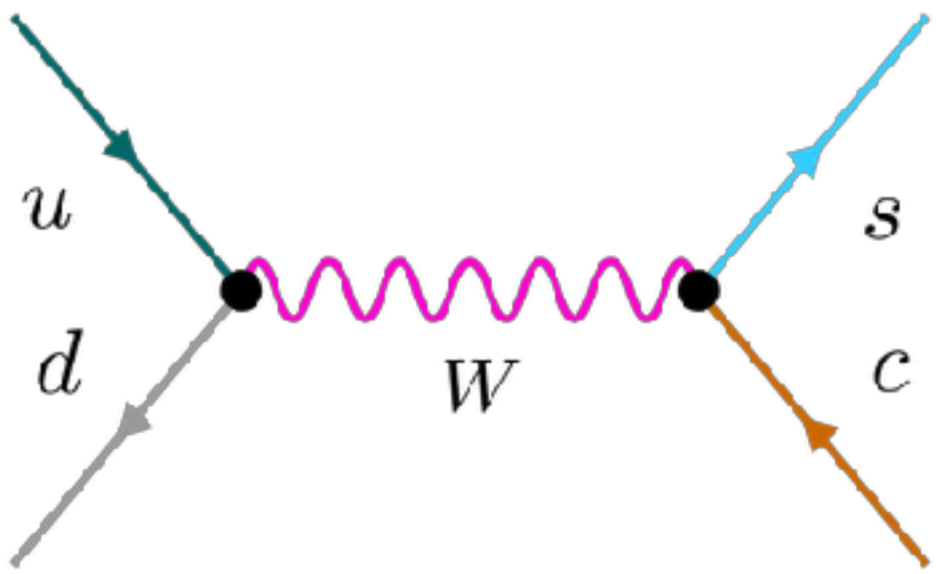
+ penguin operators

In $D = 4 - 2\epsilon$

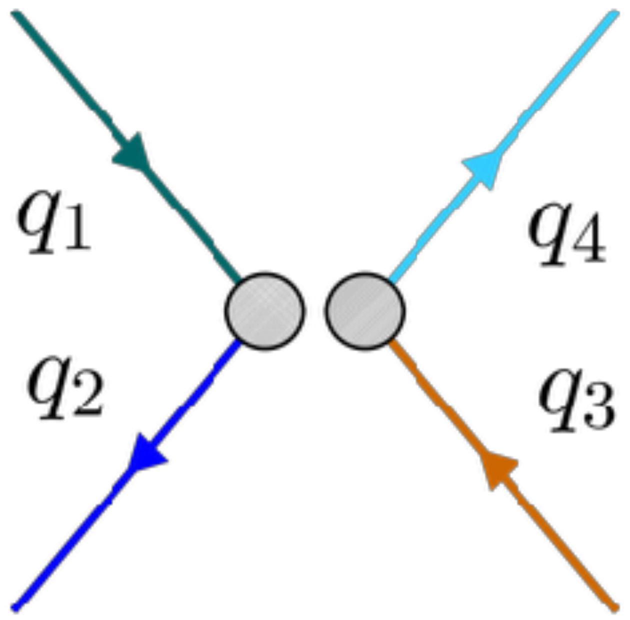


UV divergences $\sim \mathcal{O}_1 \,, \, \mathcal{O}_2$

Electroweak Hamiltonian



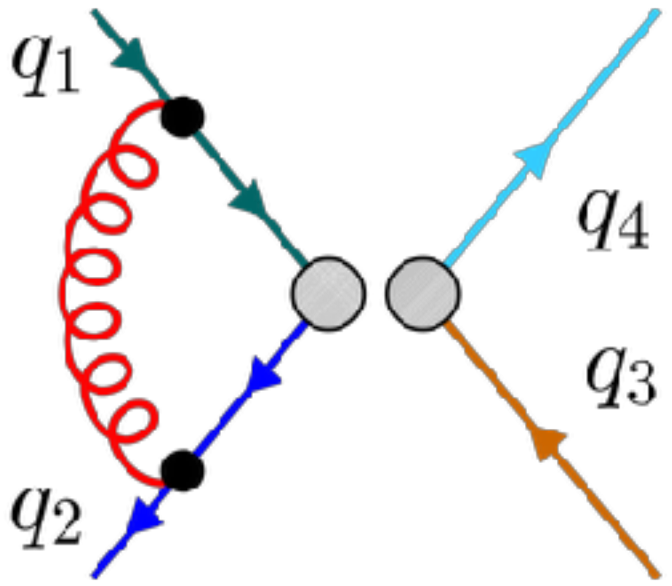
$$M_W \to \infty$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

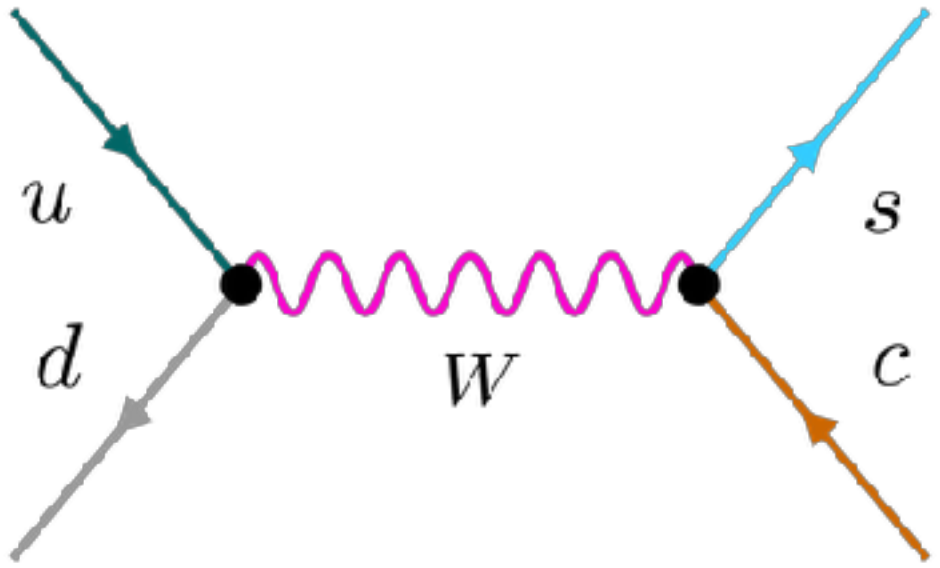
In $D = 4 - 2\epsilon$



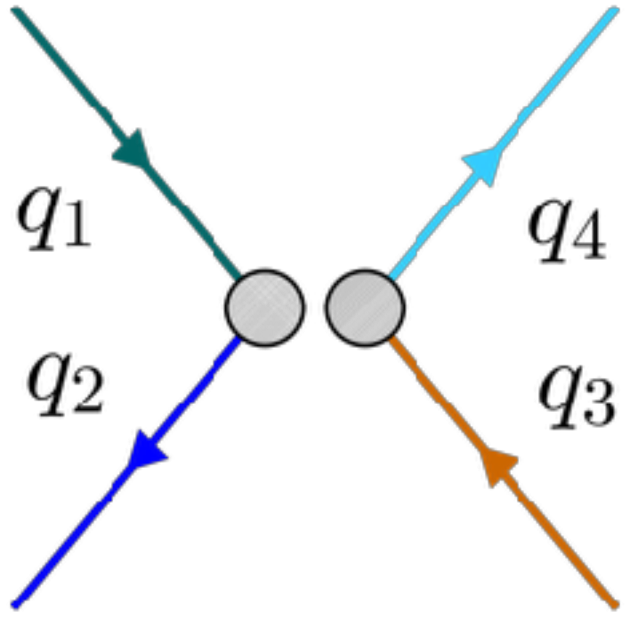
UV divergences $\sim \mathcal{O}_1 \,, \, \mathcal{O}_2$

and $\sim \mathcal{E}_1^{(1)}, \, \mathcal{E}_2^{(1)}$

Electroweak Hamiltonian



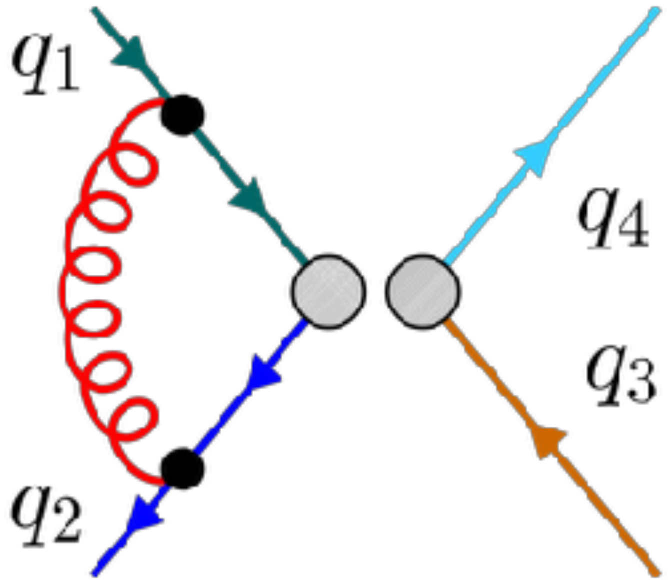
$$\xrightarrow{M_W \to \infty}$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

In $D = 4 - 2\epsilon$

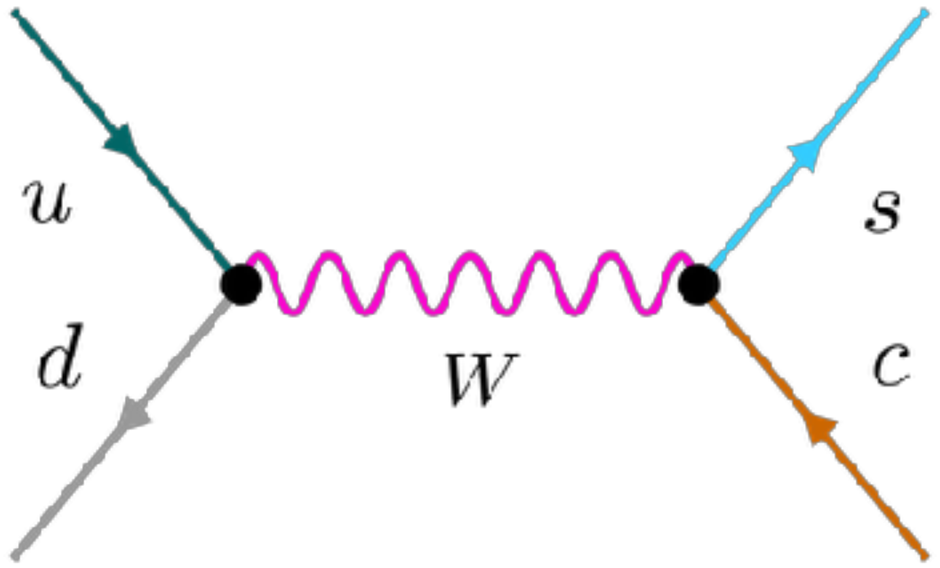


UV divergences $\sim \mathcal{O}_1\,,\ \mathcal{O}_2$

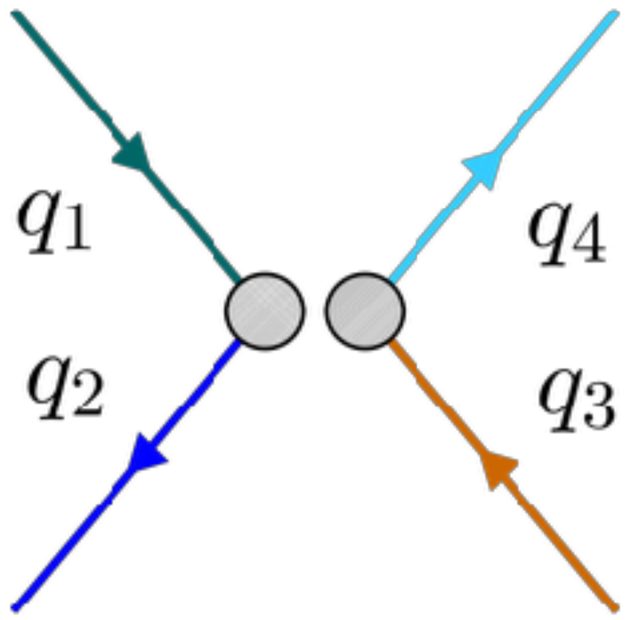
and $\sim \mathcal{E}_1^{(1)},\ \mathcal{E}_2^{(1)}$

$$\mathcal{E}_1^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L T q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma T q_4) - 16\mathcal{O}_1$$
$$\mathcal{E}_2^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma q_4) - 16\mathcal{O}_2$$

Electroweak Hamiltonian



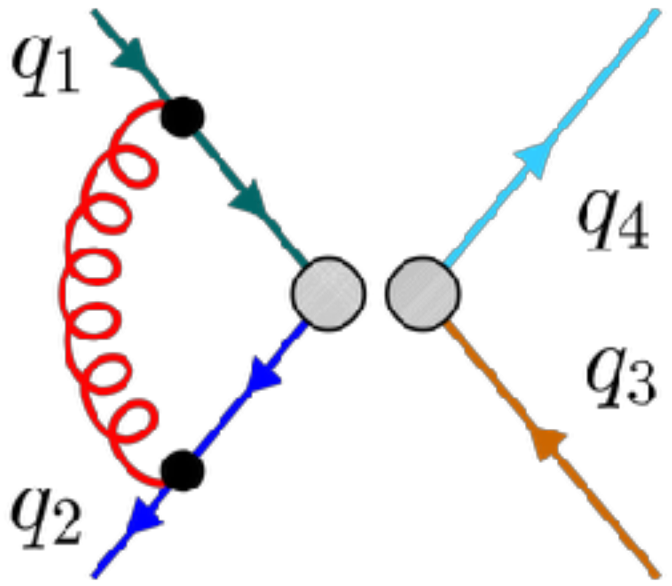
$$M_W \to \infty \longrightarrow$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

In $D = 4 - 2\epsilon$



UV divergences $\sim \mathcal{O}_1\,,\ \mathcal{O}_2$

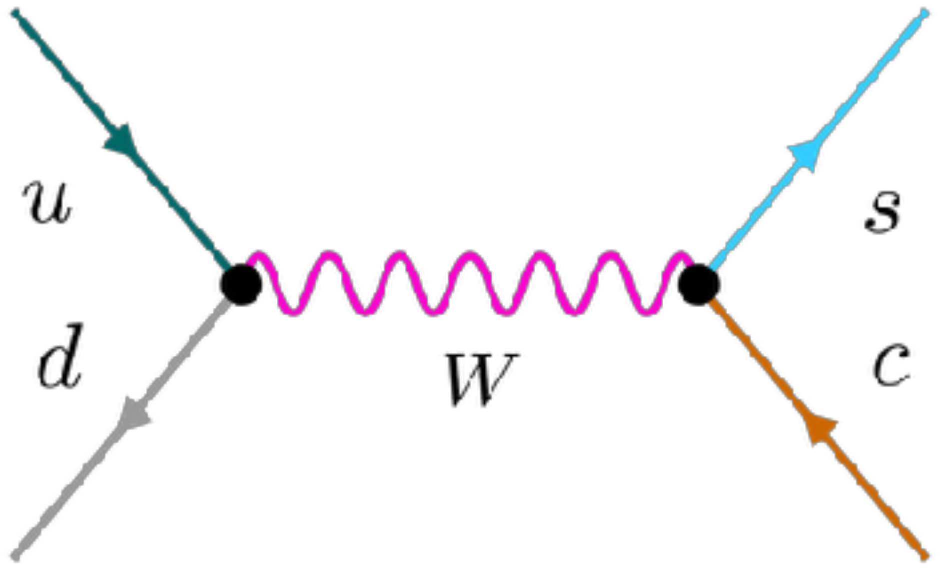
and $\sim \mathcal{E}_1^{(1)},\ \mathcal{E}_2^{(1)}$

$$\mathcal{E}_1^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L T q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma T q_4) - 16\mathcal{O}_1$$
$$\mathcal{E}_2^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma q_4) - 16\mathcal{O}_2$$

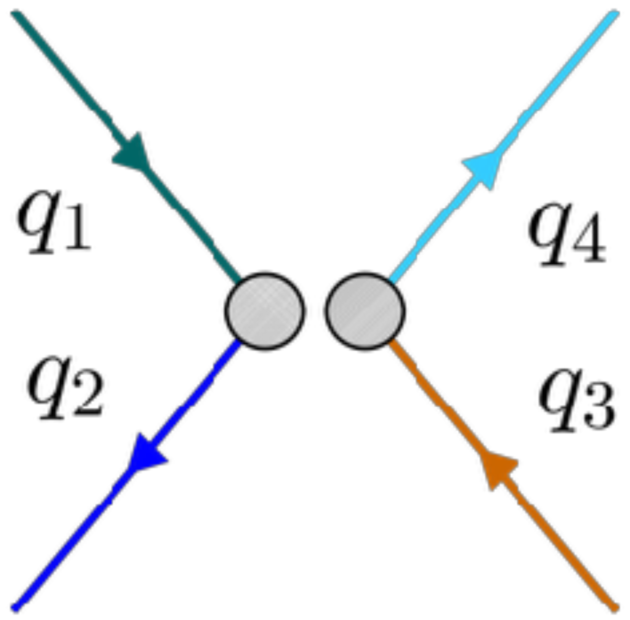
In $D = 4$: $\mathcal{E}_i^{(n)} = 0$ evanescent operators

Chetyrkin, Misiak, Münz ‘98

Electroweak Hamiltonian



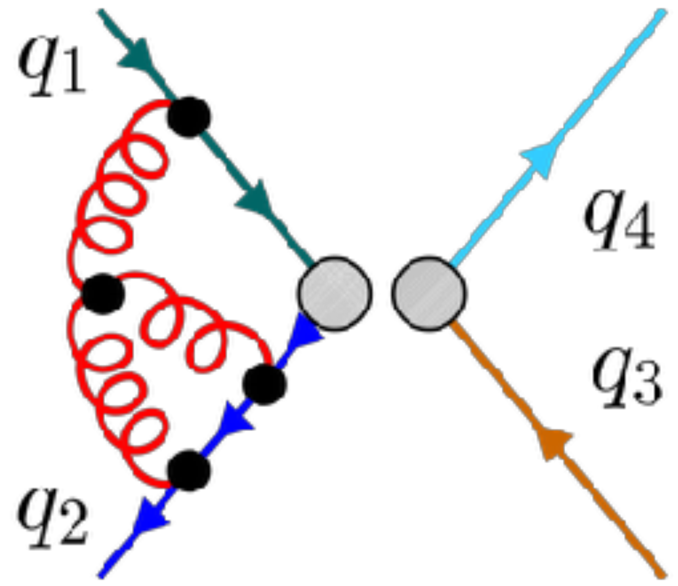
$$M_W \to \infty \longrightarrow$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma^\mu_L T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma^\mu_L q_4)$$

+ penguin operators

In $D = 4 - 2\epsilon$



UV divergences $\sim \mathcal{O}_1\,,\ \mathcal{O}_2$

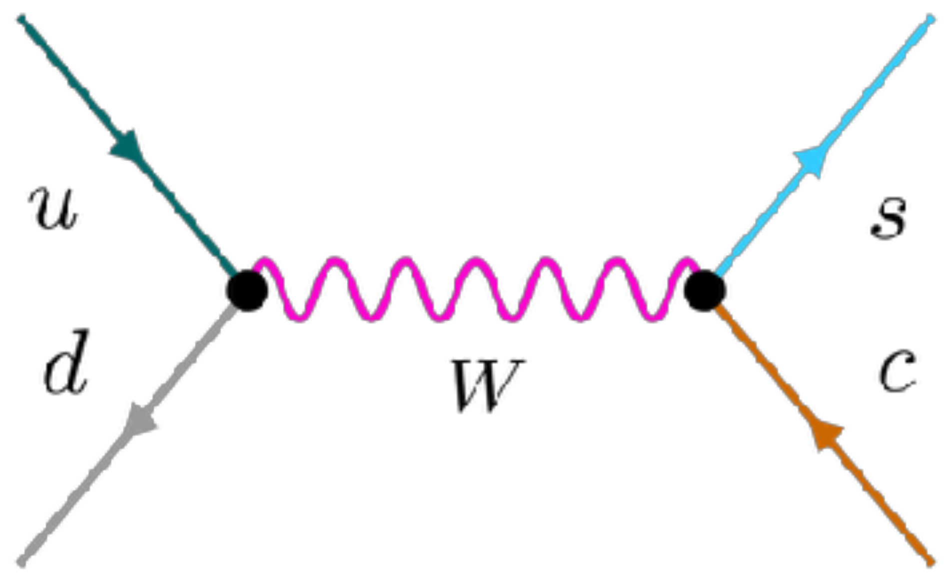
and $\sim \mathcal{E}_1^{(1)},\ \mathcal{E}_2^{(1)}$

$$\mathcal{E}_1^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L T q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma^\sigma_L T q_4) - 16\mathcal{O}_1$$
$$\mathcal{E}_2^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma^\sigma_L q_4) - 16\mathcal{O}_2$$

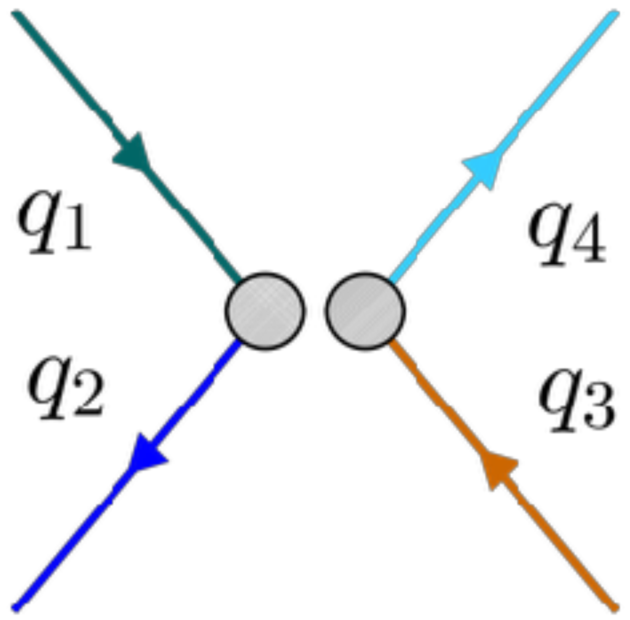
In $D = 4$: $\mathcal{E}_i^{(n)} = 0$ evanescent operators

Chetyrkin, Misiak, Münz '98

Electroweak Hamiltonian



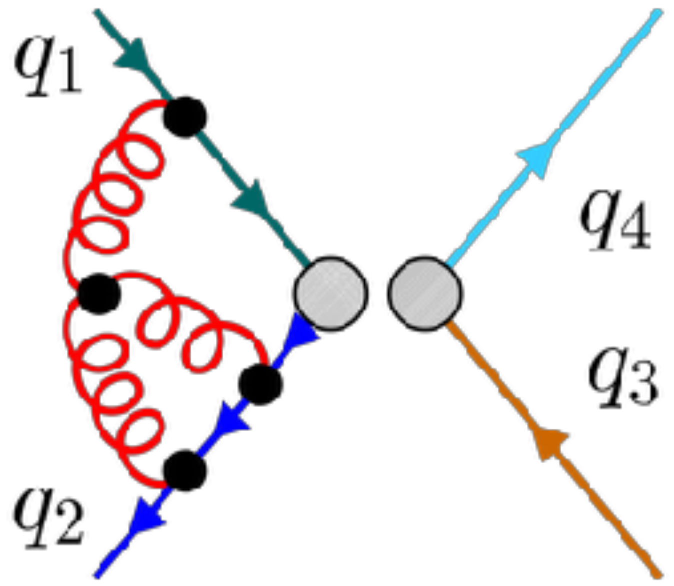
$$M_W \to \infty \longrightarrow$$



$$\mathcal{O}_1 = (\bar{q}_1 \gamma_\mu^L T q_2)(\bar{q}_3 \gamma_L^\mu T q_4)$$
$$\mathcal{O}_2 = (\bar{q}_1 \gamma_\mu^L q_2)(\bar{q}_3 \gamma_L^\mu q_4)$$

+ penguin operators

In $D = 4 - 2\epsilon$



UV divergences $\sim \mathcal{O}_1\,,\ \mathcal{O}_2$

and $\sim \mathcal{E}_1^{(1)},\ \mathcal{E}_2^{(1)}$

and $\sim \mathcal{E}_1^{(2)},\ \mathcal{E}_2^{(2)}$

$$\mathcal{E}_1^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L T q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma T q_4) - 16\mathcal{O}_1$$
$$\mathcal{E}_2^{(1)} = (\bar{q}_1 \gamma_\mu \gamma_\rho \gamma_\sigma^L q_2)(\bar{q}_3 \gamma^\mu \gamma^\rho \gamma_L^\sigma q_4) - 16\mathcal{O}_2$$

$$\mathcal{E}^{(2)} = (\bar{q}_1 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_L^\tau q_2)\cdots$$

In $D = 4$: $\mathcal{E}_i^{(n)} = 0$ evanescent operators

Chetyrkin, Misiak, Münz ‘98

Renormalization

$$\mathcal{O}_R \equiv \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{E}_1^{(1)} \\ \mathcal{E}_2^{(1)} \\ \mathcal{E}_1^{(2)} \\ \mathcal{E}_2^{(2)} \end{pmatrix}_R = Z \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{E}_1^{(1)} \\ \mathcal{E}_2^{(1)} \\ \mathcal{E}_1^{(2)} \\ \mathcal{E}_2^{(2)} \end{pmatrix} \equiv \mathcal{O}$$

$\overline{\mathrm{MS}}$, except: $\langle \mathcal{E}_{i,R}^{(n)} \rangle \stackrel{!}{=} 0$

Z is known through NNLO!

Gorbahn, Haisch '05

Gradient flow: $\tilde{\mathcal{O}}(t) = \zeta^B(t)\mathcal{O} = \zeta^B(t)Z^{-1}\mathcal{O}_R \equiv \zeta(t)\mathcal{O}_R$

$\Rightarrow$ cancellation of UV-poles in $\zeta^B(t)Z^{-1}$ is highly non-trivial check

also: $\tilde{\mathcal{E}}_i(t) = 0 \cdot \mathcal{O}_{j,R} + \zeta_{ij}(t)\mathcal{E}_j$ so that $\langle \tilde{\mathcal{E}}_i(t) \rangle = 0$

Results

$$\zeta_{11}(t) = 1 + a_s\left[-\frac{1}{6} - \frac{8}{3}\ln 2 - 2\ln 3 - \frac{1}{2}L_{\mu t}\right] + a_s^2\left[-\frac{10669}{576} + \frac{1}{2}c_{\chi,\mathrm{A}} + \frac{2}{9}c_{\chi,\mathrm{F}}\right.$$
$$- \frac{181}{96}\zeta_2 - \frac{235}{8}\ln 2 + \frac{1537}{48}\ln 3 + \frac{14}{3}\ln^2 2 + \frac{8}{3}\ln 2\ln 3 + \ln^2 3$$
$$+ \frac{829}{48}\mathrm{Li}_2(1/4) + n_\mathrm{f}\left(\frac{305}{432} + \frac{1}{12}c_{\chi,\mathrm{R}} + \frac{5}{24}\zeta_2\right) + L_{\mu t}\Big(-\frac{61}{96} - 6\ln 2$$
$$\left.- \frac{9}{2}\ln 3 + n_\mathrm{f}\,(\frac{1}{12} + \frac{4}{9}\ln 2 + \frac{1}{3}\ln 3)\Big) + L_{\mu t}^2\Big(-\frac{5}{16} + \frac{1}{24}n_\mathrm{f}\Big)\right],$$

$$\zeta_{12}(t) = a_s\left(\frac{5}{6} + \frac{1}{3}L_{\mu t}\right) + a_s^2\Big(\frac{773}{432} + \frac{67}{432}\zeta_2 + \frac{163}{108}\ln 2 - \frac{115}{24}\ln 3 + \frac{3}{8}\mathrm{Li}_2(1/4)$$
$$+ n_\mathrm{f}\,(-\frac{145}{1296} - \frac{1}{36}\zeta_2) + L_{\mu t}\,(\frac{205}{144} - \frac{8}{9}\ln 2 - \frac{2}{3}\ln 3 - \frac{5}{54}n_\mathrm{f}) + L_{\mu t}^2\,(\frac{3}{8}$$
$$- \frac{1}{36}n_\mathrm{f})\Big),$$

$$\zeta_{21}(t) = a_s\left(\frac{15}{4} + \frac{3}{2}L_{\mu t}\right) + a_s^2\Big(\frac{1043}{96} + \frac{67}{96}\zeta_2 + \frac{163}{24}\ln 2 - \frac{345}{16}\ln 3$$
$$+ \frac{27}{16}\mathrm{Li}_2(1/4) + n_\mathrm{f}\,(-\frac{145}{288} - \frac{1}{8}\zeta_2) + L_{\mu t}\,(\frac{241}{32} - 4\ln 2 - 3\ln 3 - \frac{5}{12}n_\mathrm{f})$$

RH, Lange (in prep)

Results

$$\zeta_{11}(t) = 1 + a_s\left[-\frac{1}{6} - \frac{8}{3}\ln 2 - 2\ln 3 - \frac{1}{2}L_{\mu t}\right] + a_s^2\left[-\frac{10669}{576} + \frac{1}{2}c_{\chi,\mathrm{A}} + \frac{2}{9}c_{\chi,\mathrm{F}}\right.$$
$$- \frac{181}{96}\zeta_2 - \frac{235}{8}\ln 2 + \frac{1537}{48}\ln 3 + \frac{14}{3}\ln^2 2 + \frac{8}{3}\ln 2\ln 3 + \ln^2 3$$
$$+ \frac{829}{48}\mathrm{Li}_2(1/4) + n_\mathrm{f}\left(\frac{305}{432} + \frac{1}{12}c_{\chi,\mathrm{R}} + \frac{5}{24}\zeta_2\right) + L_{\mu t}\left(-\frac{61}{96} - 6\ln 2\right.$$
$$\left.\left. - \frac{9}{2}\ln 3 + n_\mathrm{f}\left(\frac{1}{12} + \frac{4}{9}\ln 2 + \frac{1}{3}\ln 3\right)\right) + L_{\mu t}^2\left(-\frac{5}{16} + \frac{1}{24}n_\mathrm{f}\right)\right],$$

$$\zeta_{12}(t) = a_s\left(\frac{5}{6} + \frac{1}{3}L_{\mu t}\right) + a_s^2\left(\frac{773}{432} + \frac{67}{432}\zeta_2 + \frac{163}{108}\ln 2 - \frac{115}{24}\ln 3 + \frac{3}{8}\mathrm{Li}_2(1/4)\right.$$
$$+ n_\mathrm{f}\left(-\frac{145}{1296} - \frac{1}{36}\zeta_2\right) + L_{\mu t}\left(\frac{205}{144} - \frac{8}{9}\ln 2 - \frac{2}{3}\ln 3 - \frac{5}{54}n_\mathrm{f}\right) + L_{\mu t}^2\left(\frac{3}{8}\right.$$
$$\left.\left. - \frac{1}{36}n_\mathrm{f}\right)\right),$$

$$\zeta_{21}(t) = a_s\left(\frac{15}{4} + \frac{3}{2}L_{\mu t}\right) + a_s^2\left(\frac{1043}{96} + \frac{67}{96}\zeta_2 + \frac{163}{24}\ln 2 - \frac{345}{16}\ln 3\right.$$
$$+ \frac{27}{16}\mathrm{Li}_2(1/4) + n_\mathrm{f}\left(-\frac{145}{288} - \frac{1}{8}\zeta_2\right) + L_{\mu t}\left(\frac{241}{32} - 4\ln 2 - 3\ln 3 - \frac{5}{12}n_\mathrm{f}\right)$$

NLO in DRED:

Suzuki et al. '21

RH, Lange (in prep)

Flow-time evolution

$t \to 0:$ $$\tilde{\mathcal{O}}_n(t) = \sum_m \zeta_{nm}(t)\,\mathcal{O}_m$$

matrix notation: $$\tilde{\mathcal{O}}(t) = \zeta(t)\,\mathcal{O}$$

$$t\frac{\partial}{\partial t}\tilde{\mathcal{O}}(t) = \left(t\frac{\partial}{\partial t}\zeta(t)\right)\mathcal{O} = \left(t\frac{\partial}{\partial t}\zeta(t)\right)\zeta^{-1}(t)\,\tilde{\mathcal{O}}(t)$$

$$t\frac{\partial}{\partial t}\tilde{\mathcal{O}}(t) = \tilde{\gamma}(t)\tilde{\mathcal{O}}(t) \qquad \text{with} \qquad \tilde{\gamma}(t) = \left(t\frac{\partial}{\partial t}\zeta(t)\right)\zeta^{-1}(t)$$

RH, Lange, Neumann '20

Conclusions and Outlook

$$R = \sum_n C_n \langle \mathcal{O}_n \rangle \; = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

unique possibilities for lattice/p.t. synergies

Conclusions and Outlook

$$R = \sum_n C_n \langle \mathcal{O}_n \rangle = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

unique possibilities for lattice/p.t. synergies

Applications so far:

- energy-momentum tensor (NNLO) — RH, Kluth, Lange '18
- hadronic vacuum polarization (NNLO) — RH, Lange, Neumann '20
- coming soon: flavor physics (NNLO)

Conclusions and Outlook

$$R = \sum_n C_n \langle \mathcal{O}_n \rangle = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

unique possibilities for lattice/p.t. synergies

Applications so far:

- energy-momentum tensor (NNLO) — RH, Kluth, Lange '18
- hadronic vacuum polarization (NNLO) — RH, Lange, Neumann '20
- coming soon: flavor physics (NNLO)

See also:

- ~~CP~~ operators (NLO) — SymLat '20
- Bag parameter (NLO) — A. Suzuki, Taniguchi, H. Suzuki, Kanada '20

Conclusions and Outlook

$$R = \sum_n C_n \langle \mathcal{O}_n \rangle = \sum_n \tilde{C}_n(t) \langle \tilde{\mathcal{O}}_n(t) \rangle$$

unique possibilities for lattice/p.t. synergies

Applications so far:

- energy-momentum tensor (NNLO) — RH, Kluth, Lange ‘18
- hadronic vacuum polarization (NNLO) — RH, Lange, Neumann ‘20
- coming soon: flavor physics (NNLO)

See also:

- $\cancel{CP}$ operators (NLO) — SymLat ‘20
- Bag parameter (NLO) — A. Suzuki, Taniguchi, H. Suzuki, Kanada ‘20

3-loop gradient flow coupling / beta-function — RH, Neumann, Lange ’16, ‘20