Renormalization Group and Gradient Flow

Non-perturbative determination of the β function and anomalous dimensions

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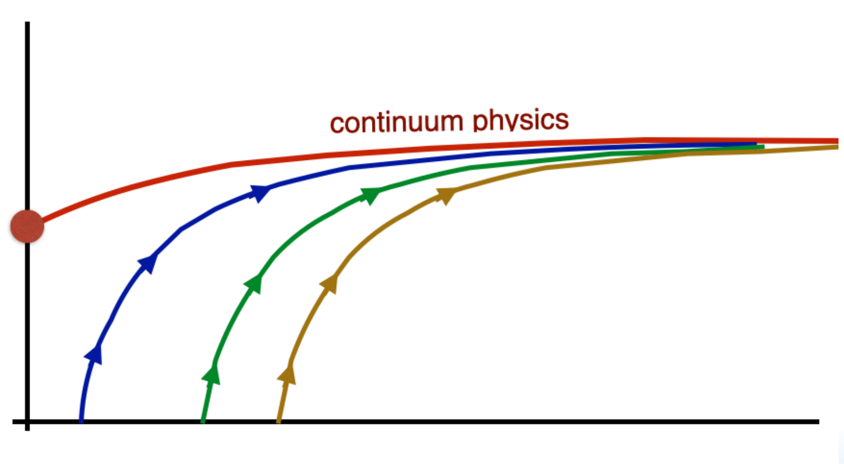
BNL-HET & RBC Workshop
DWQ@25

Dec 13 2021



- Numerical results presented here use Mobius DWFs
- Configurations were generated by the Grid code

The gradient flow transformation

GF is a powerful tool in analyzing and interpreting lattice data. See eg. the talks

- M. Luscher : The Chiral Anomaly & the Gradient Flow
- R. Harlander : The Small GF-time Expansion of the Effective Electro-weak Hamiltonian

- GF is a UV regularization scheme
  - momentum space (perturbative) : flow time $\sqrt{8t} \propto \mu^{-1}$

- GF is not an RG transformation
  - GF is a smearing transformation that naturally defines "block spins" with $\sqrt{8t/a^2} \propto b$
  - but GF does not include coarse graining (dilatation)

- GF can be used to define a position space RG transformation if the coarse graining is incorporated when taking expectation values
  - allows the formal use of Wilsonian RG language
  - applicable in the vicinity of non-perturbative fixed points

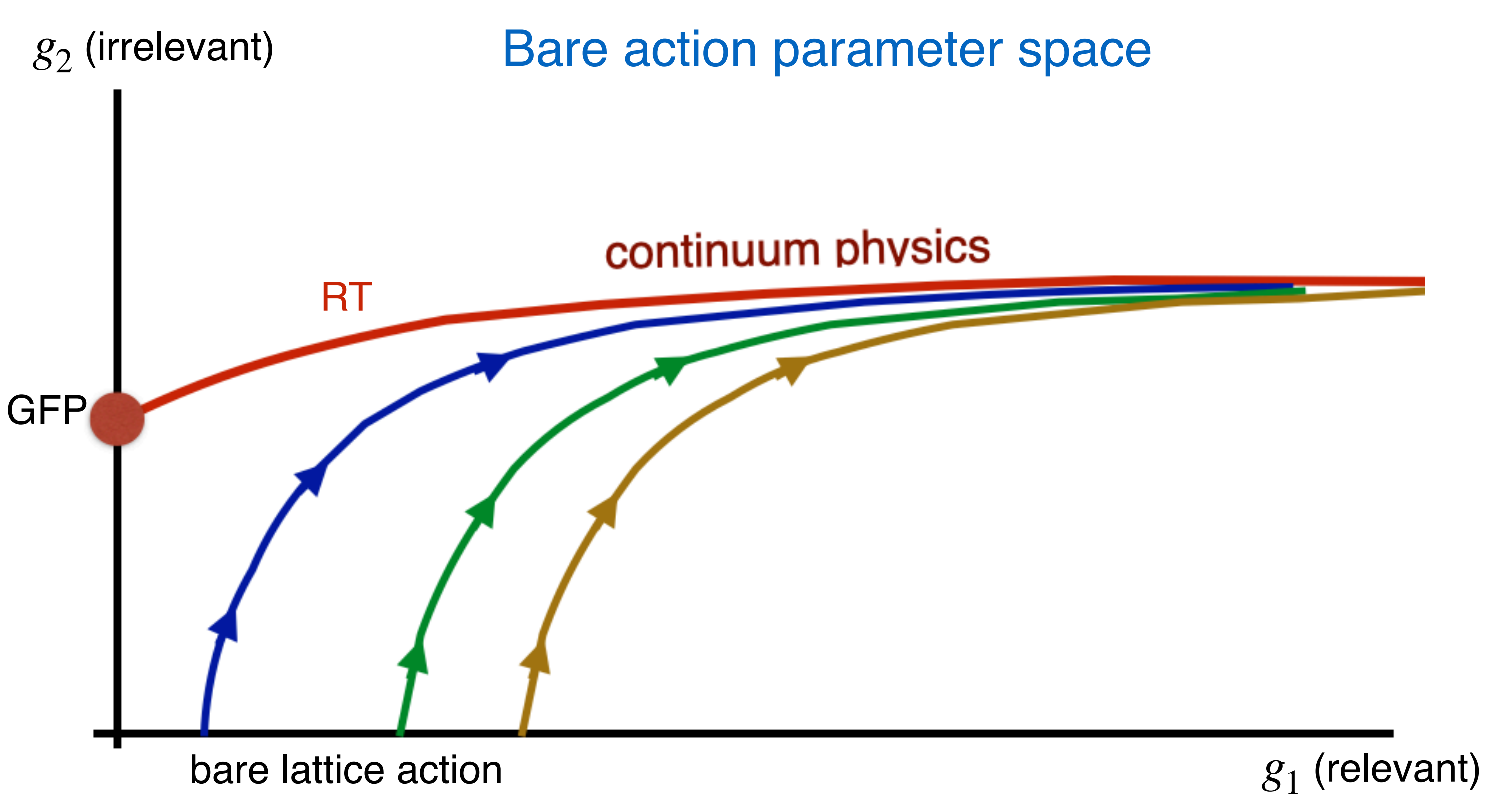
A. Carosso, AH, E. Neil, PRL 121,201601 (2018)

Sonoda, H., Suzuki, H. PTEP,023B05 (2021)

Gradient flow vs RG

In Wilsonian RG language:

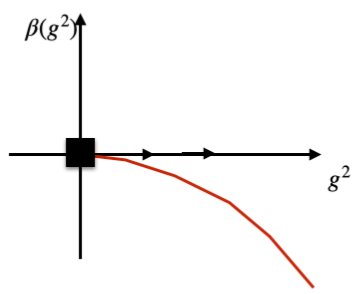
✦ RG transformation: determines the location of FP and its renormalized trajectory (RT)
  - RT describes continuum physics
✦ Lattice action: starting point of RG flow
  - at large flow time RG flows from different bare couplings overlap and describe continuum physics along the RT
✦ Operator: has given quantum number; reduces to scaling operator on the RT



gauge-fermion system, $m_f = 0$

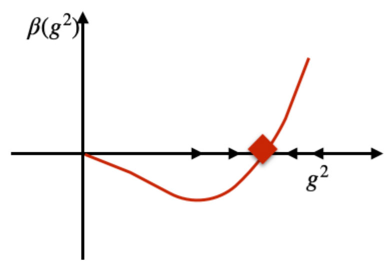
The phase diagram as the function of N_f

- $N_f = 0$ YM is confining (zero temperature)

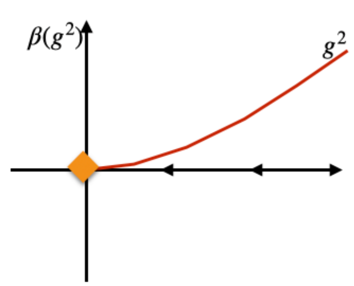


- QCD with 2 light flavors is chirally broken, confining (zero temperature)

- As N_f increases a phase transition to a conformal phase occurs
  ($N_f^* \approx 8 - 10$ for SU(3) color, fundamental fermions)
  - the conformal phase is characterized by an infrared fixed point
  - there might be a UV fixed point with a new relevant operator (4-fermion?)



- At $N_f = 16.5$ (SU(3) color) asymptotic freedom is lost
  - the system could be "trivial"
  - or a UV-safe fixed point might emerge (again, new relevant operator)

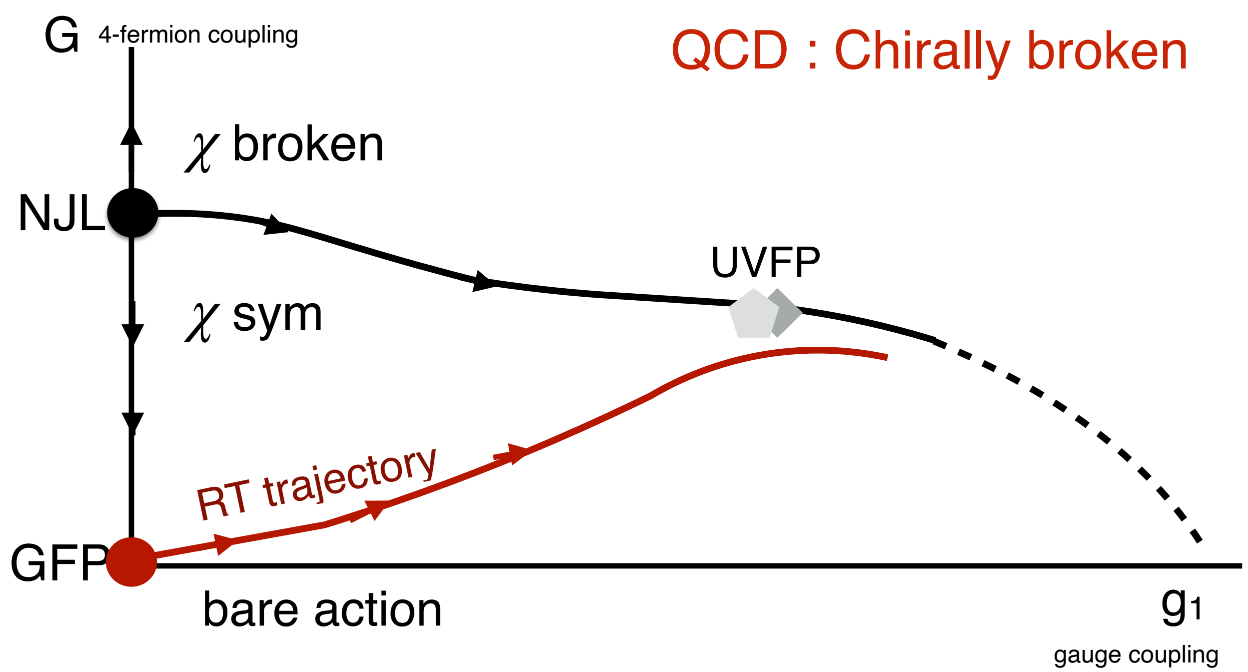


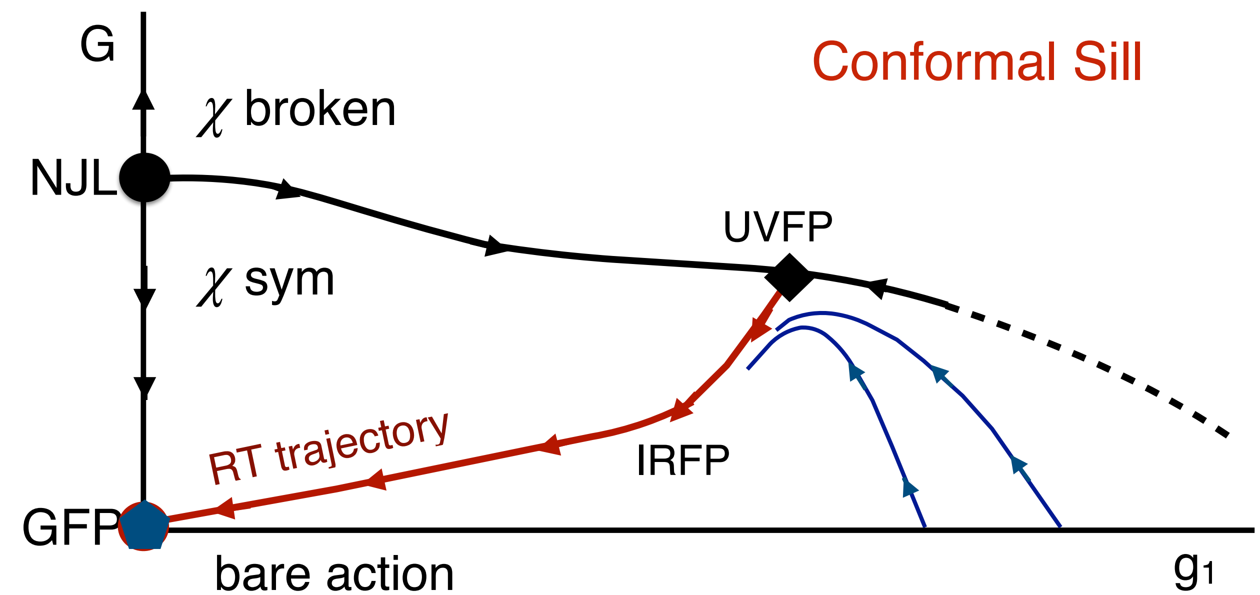
Gauge-fermion systems with 4-fermion interaction

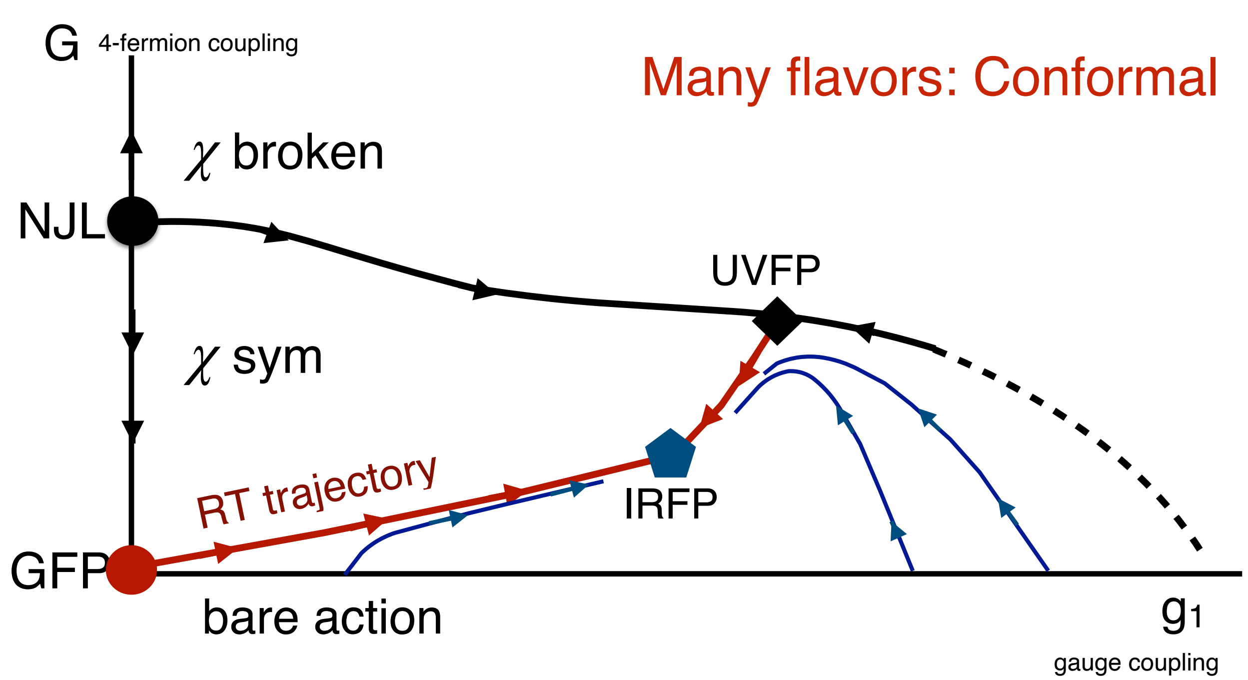
Conjectured phase diagram in the extended parameter space

AH, O. Witzel, LAT'19. (2019) 094

Kaplan et al, *Phys.Rev.D* 80 (2009) 125005
Gorbenko et al, *JHEP* 10 (2018) 108







Gauge-fermion systems with 4-fermion interaction

Exploring the properties of the extended phase diagram is very interesting.
And also hard.

For now I stay with gauge-fermion systems and explore the RT up to a possible IRFP
RT is a 1-dimensional line / 1-parameter function:

- RG $\beta(g^2)$ function
- anomalous dimension $\gamma_{\mathcal{O}}(g^2)$

The continuous β function from gradient flow

Need an operator that maps the flow along the RT.
It has to be dimensionless (no canonical or anomalous dimension)

GF gauge coupling works : $g_{GF}^2 = \mathcal{N} t^2 \langle E \rangle$

Beta function : $$\beta(g_{GF}) = -t \frac{dg_{GF}^2}{dt}$$

Lattice details:

The RG picture is valid

- in infinite volume
- in $am_f = 0$ chiral limit

The continuum limit :

$t/a^2 \to \infty$ while keeping g_{GF}^2 fixed

- the flows approach the RT
- the correct scaling operator is projected out
- Remaining cut-off effects are removed by $a^2/t \to 0$
-

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3
Fodor et al,EPJWeb Conf. 175, 08027 (2018)
Peterson et al, Lat2021, 2109.09720

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- Step scaling function requires that the volume is the only scale; valid only in the deconfined regime

- Continuous β function requires infinite volume extrapolation but it is correct even in the confining regime

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3
Fodor et al,EPJWeb Conf. 175, 08027 (2018)
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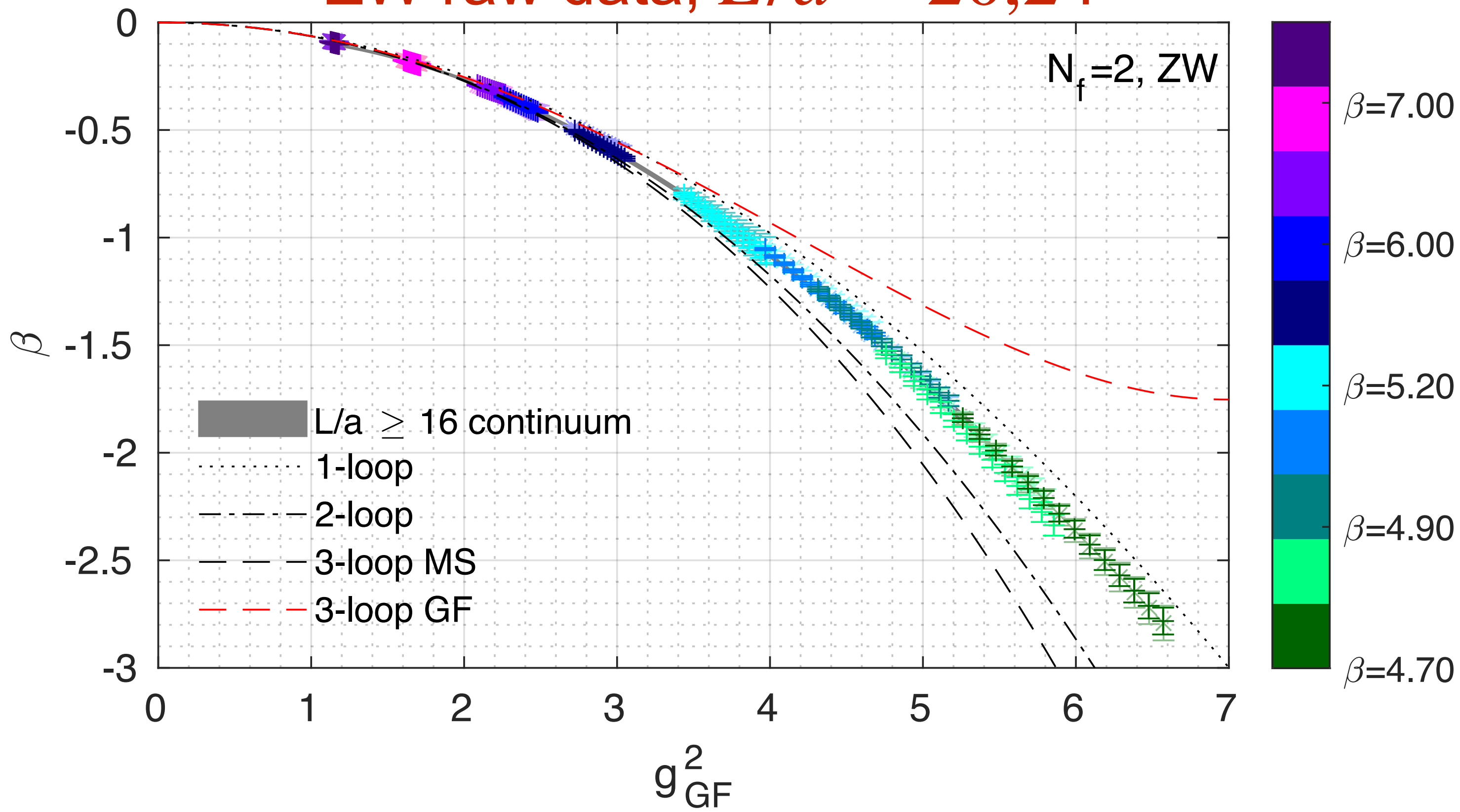
The continuous β function, $N_f = 2$

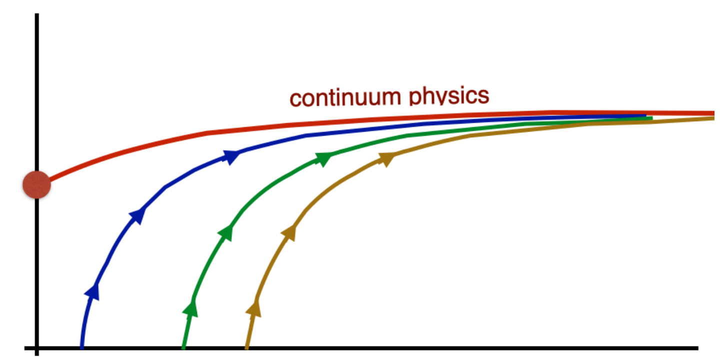
AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

We use

- Symanzik gauge action, Mobius domain wall fermions (Grid)
- Zeuthen (Z), Wilson(W) and Symanzik(S) flows
- Wilson plaquette(W), clover(C) and Symanzik(S) operators

ZW raw data, $L/a = 20{,}24$





colored points show raw data:
fixed bare coupling, changing flow time

✦ Minimal finite volume effects ($\propto t^2/L^4$)

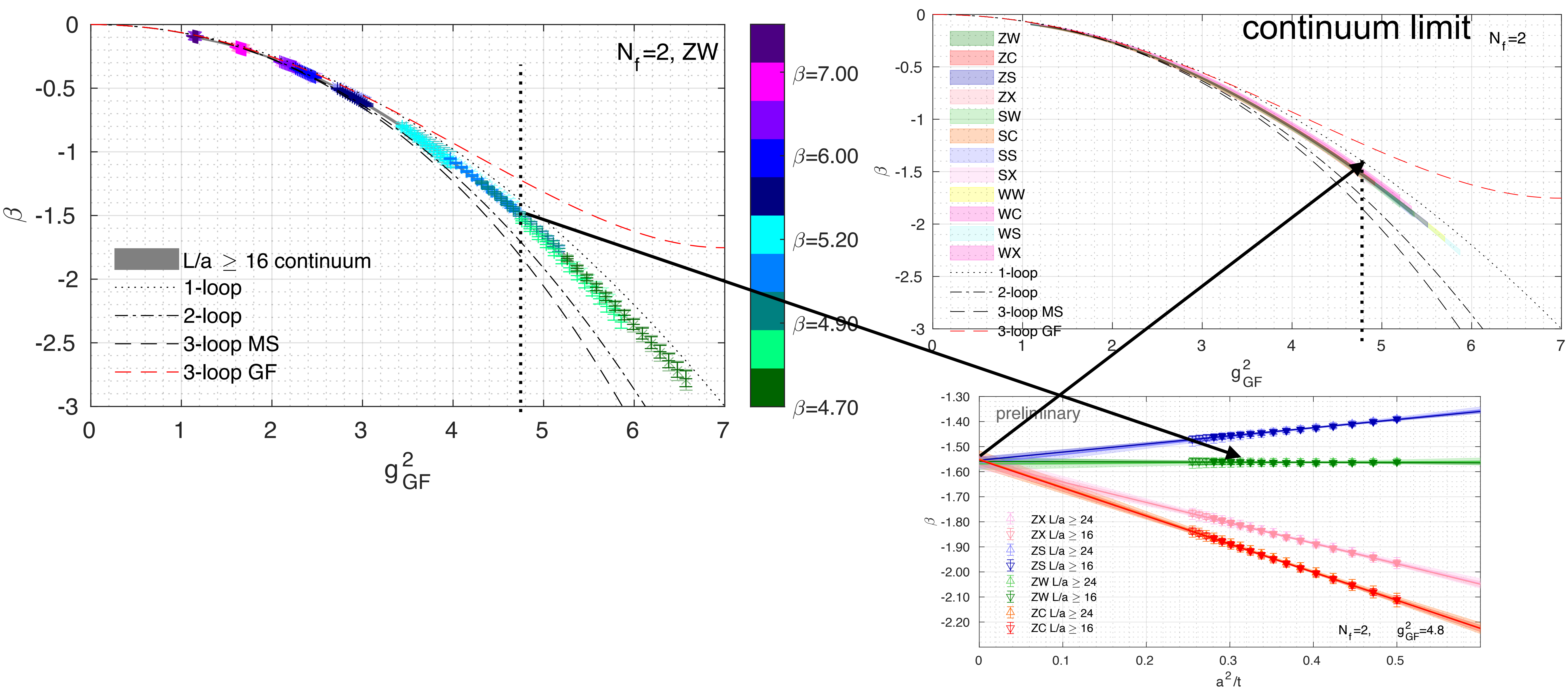
✦Different bare couplings overlap, form a unique curve, indicating that the
- RG flow reached the RT
- cutoff effects are small

The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

Analysis steps for continuum limit:

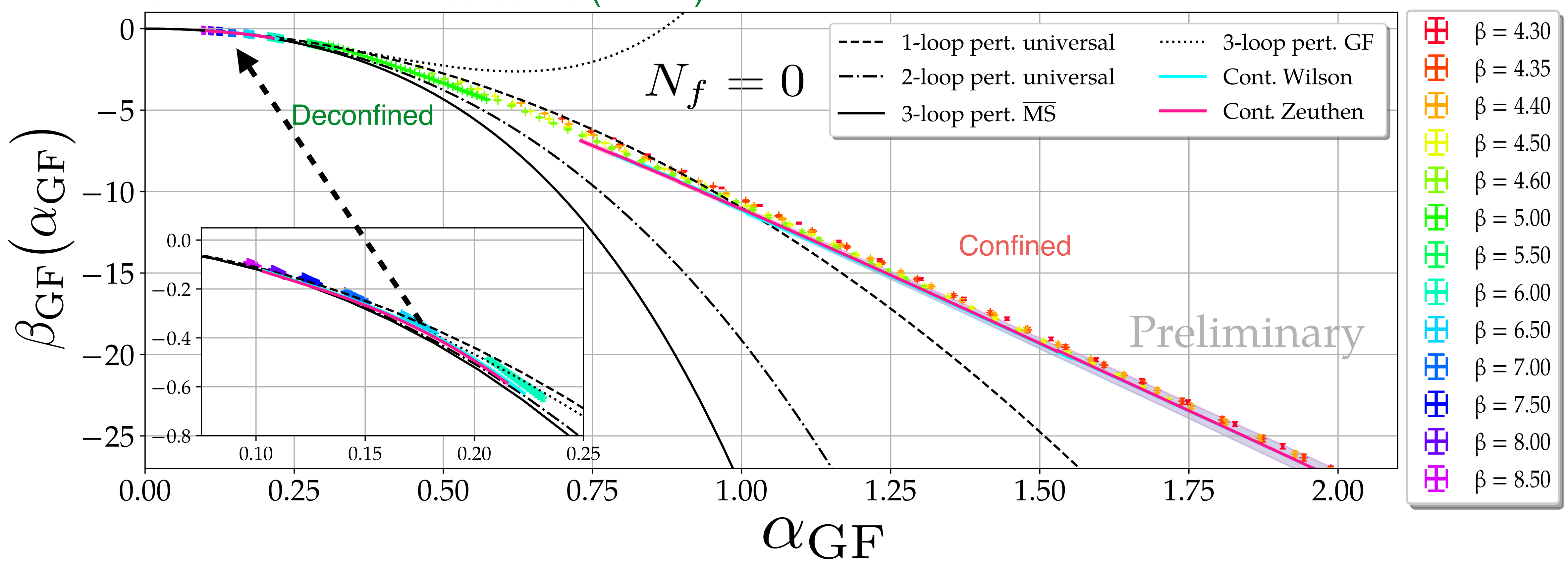
1. Infinite volume extrapolation ($1/L^4$ in the chirally symmetric regime)
2. Infinite flow time extrapolation (a^2/t) at fixed g^2_{GF}



The continuous β function, $N_f = 0$

Check the confining regime

C. Peterson et al 2109.09720 (Lat'21)



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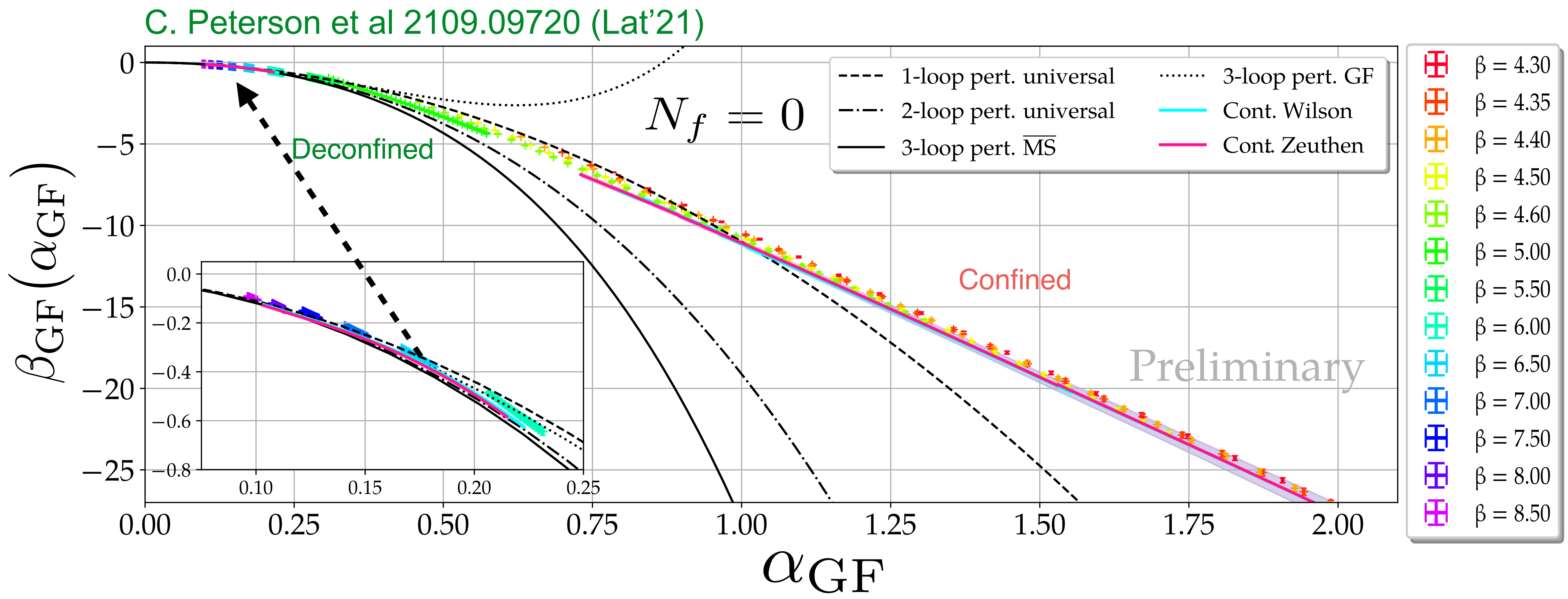


✦In the confining regime $\beta(g^2) \sim \alpha g^2$: non-perturbative; Topology?

Nakamura, Schierholz 2106.11369

$N_T = N_I + N_A$ instantons → vacuum energy density: $\langle E \rangle = \langle E \rangle_{N_T=0} + N_T * S_I / V$

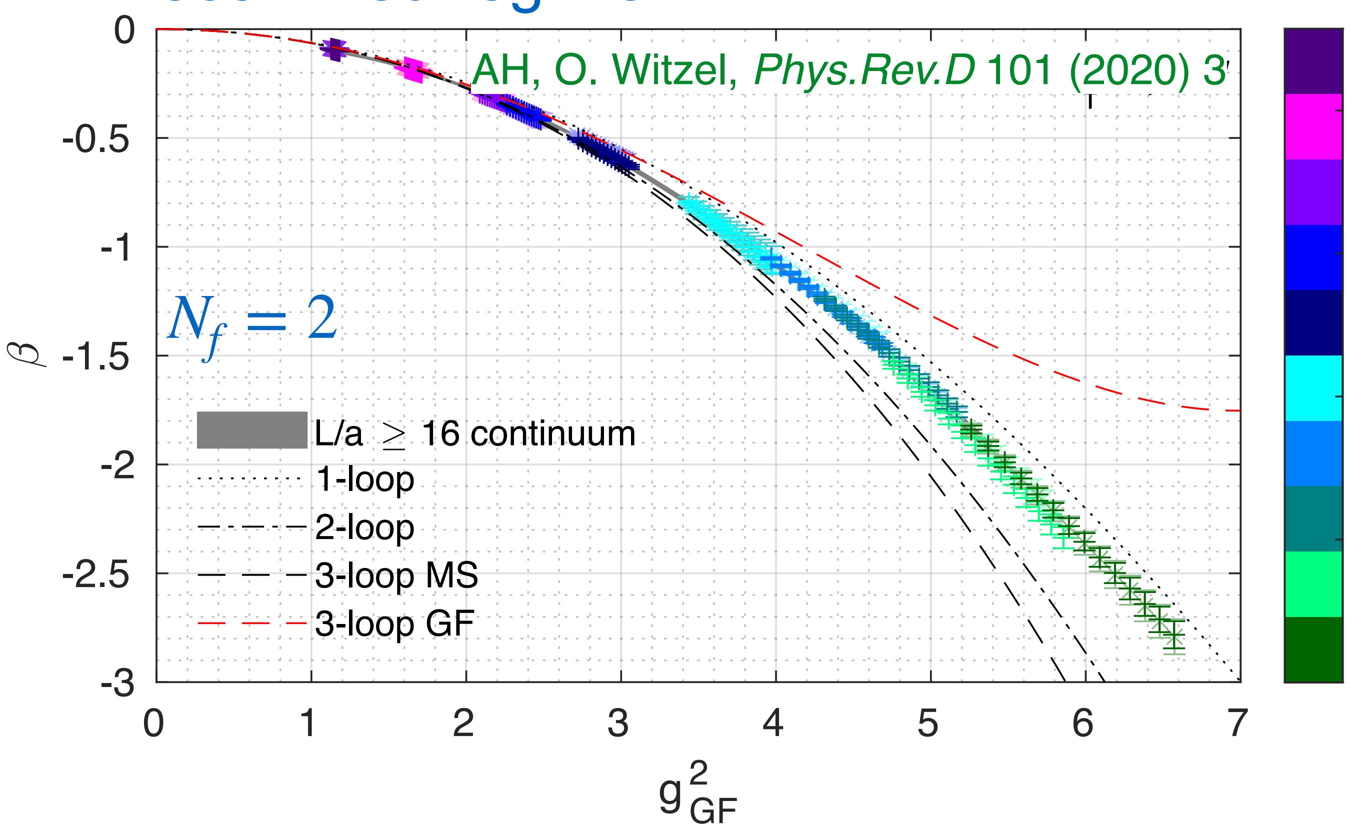
GF coupling: $g^2_{GF} = g^2_{GF,0} + ct^2 N_T / V$

If $lim_{V\to\infty} N_T/V =$ finite, topology dominates as $t \to \infty$, and $\beta(g^2) \propto g^2$

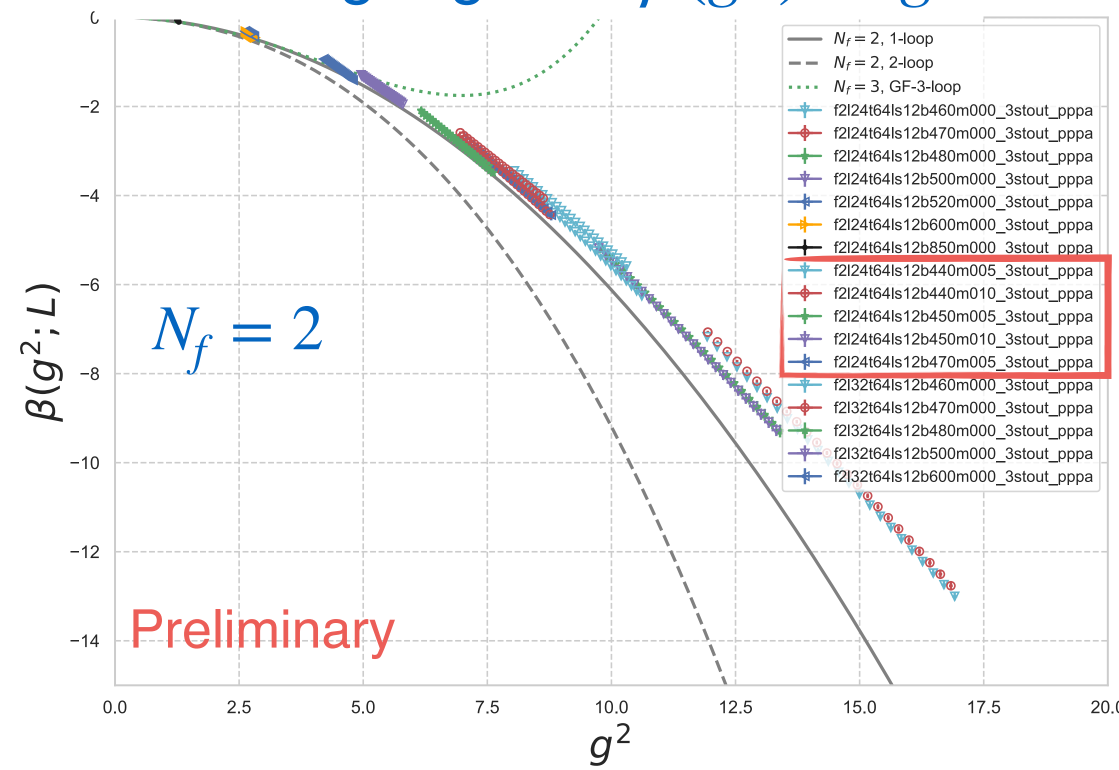
(Our flow time and volumes are not that large, other effects must contribute)

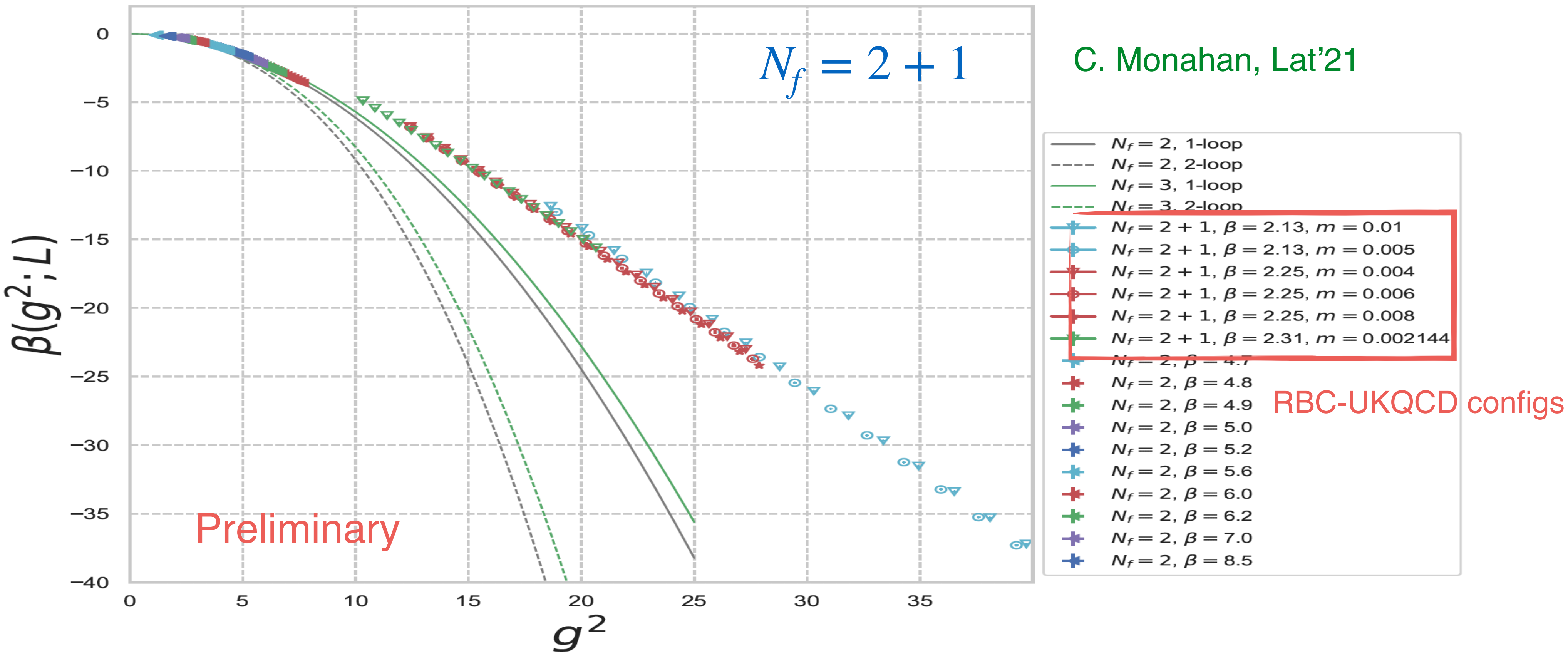
The continuous β function, $N_f = 2,\ 2+1$ DWF, $m_f = 0$

Deconfined regime



Confining regime : $\beta(g^2) \propto g^2$





C. Monahan, Lat'21

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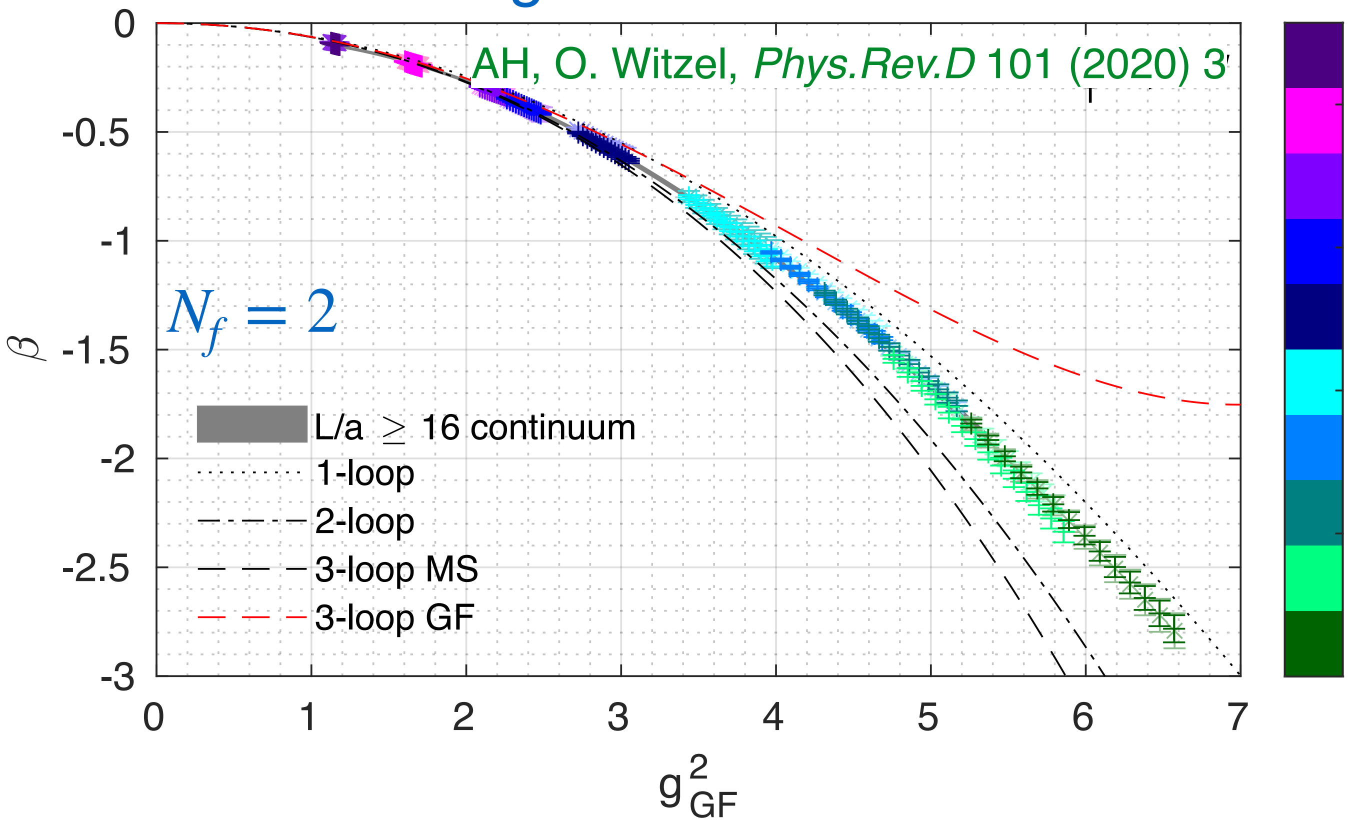
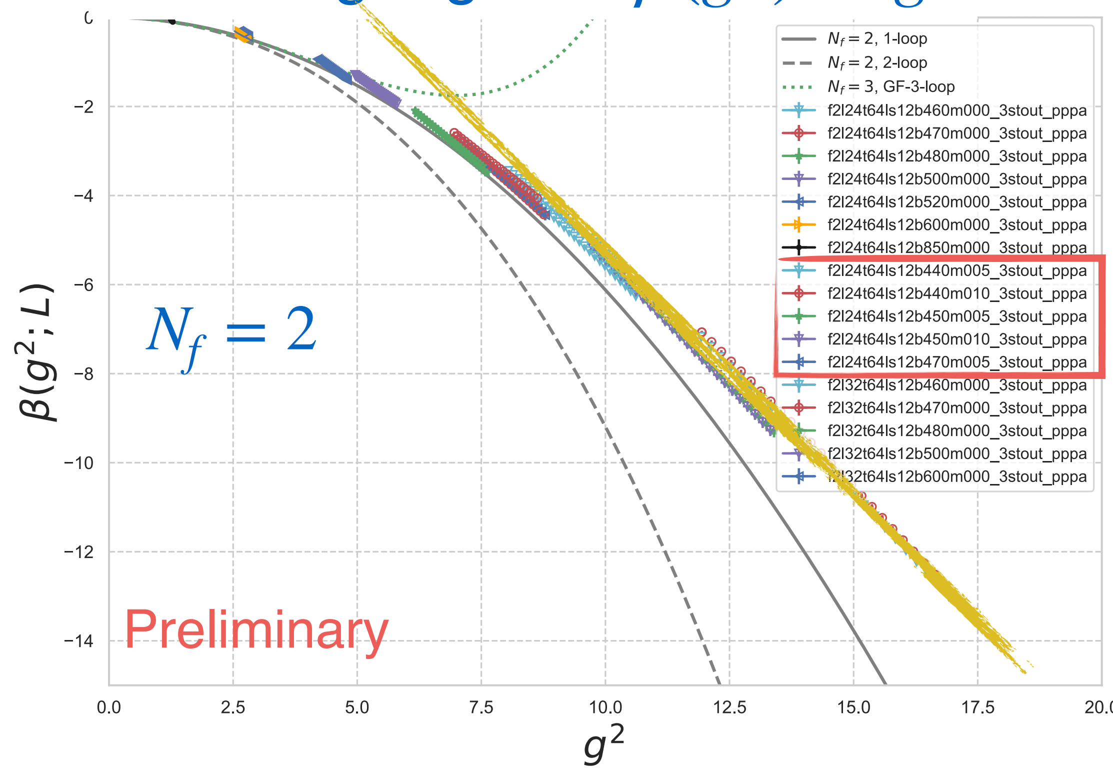
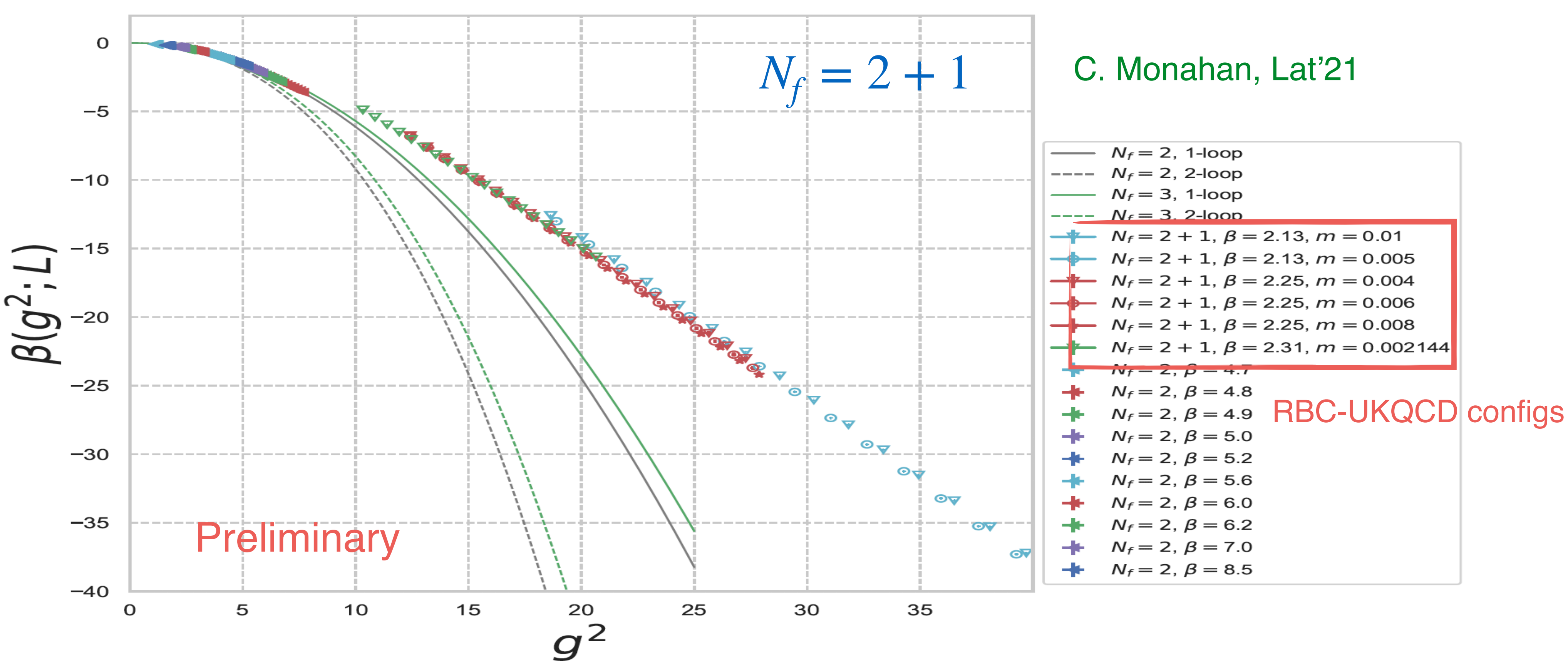
Deconfined regime



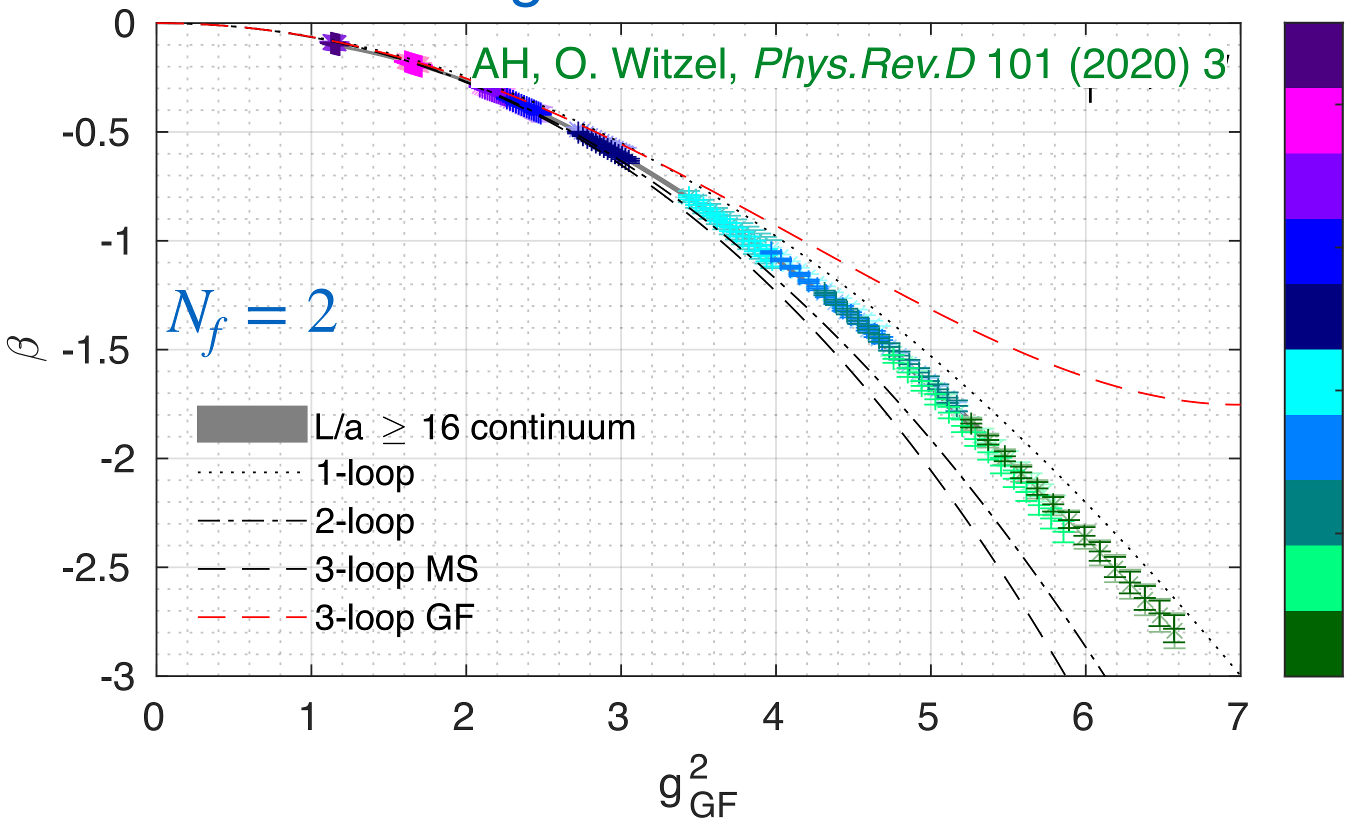
Confining regime : $\beta(g^2) \propto g^2$



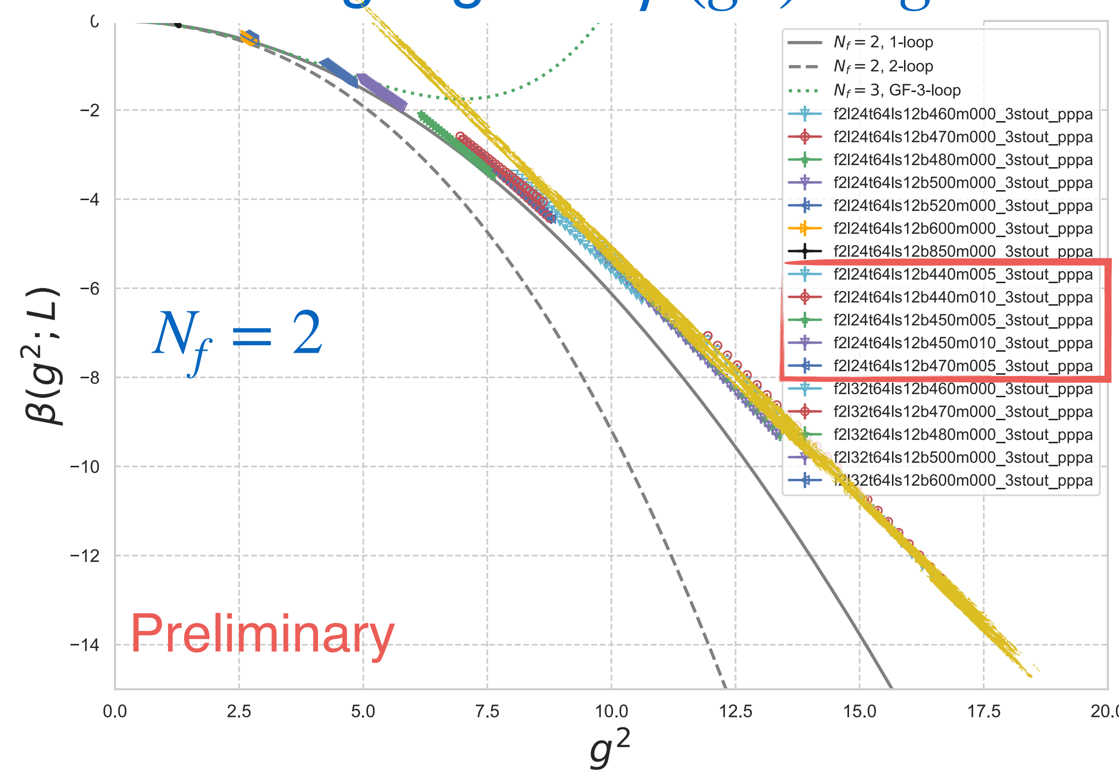


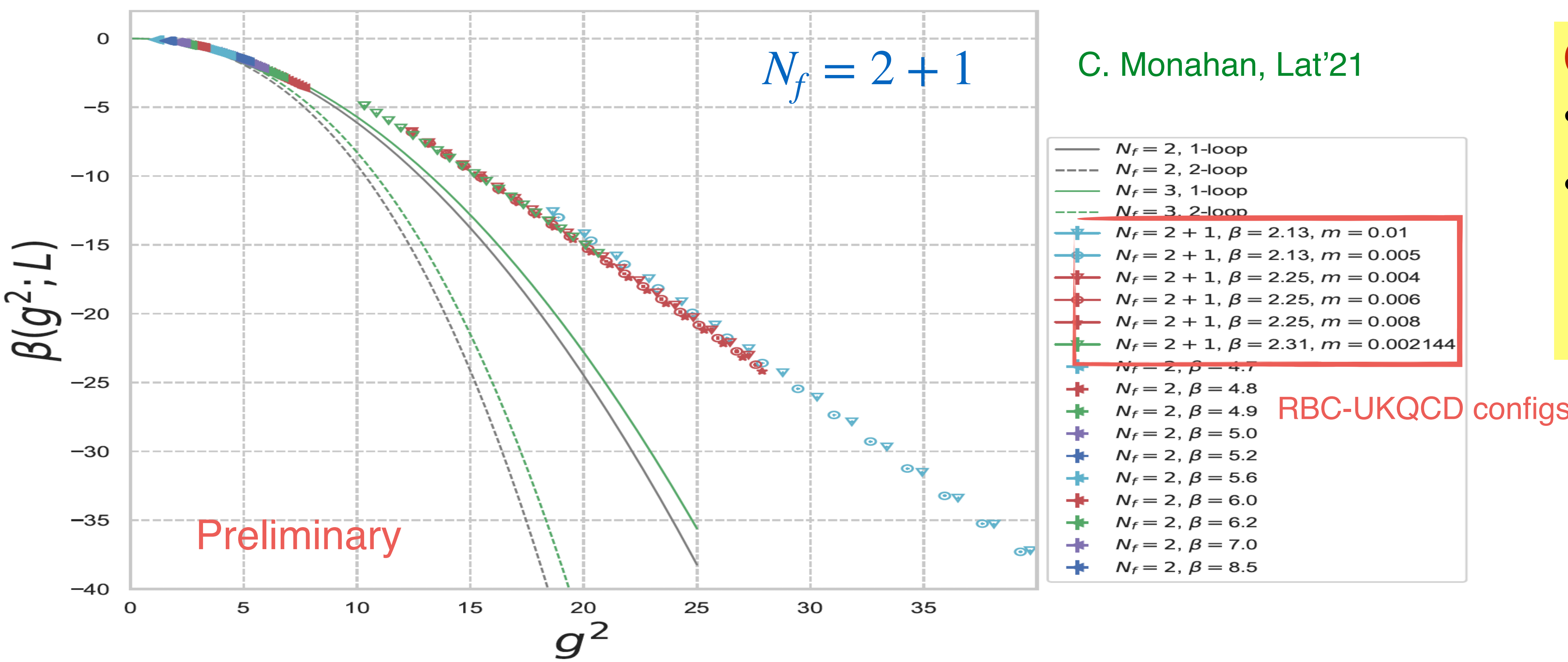
The continuous β function, $N_f = 2,\ 2+1$ DWF,$m_f = 0$

Deconfined regime



Confining regime : $\beta(g^2) \propto g^2$





C. Monahan, Lat'21

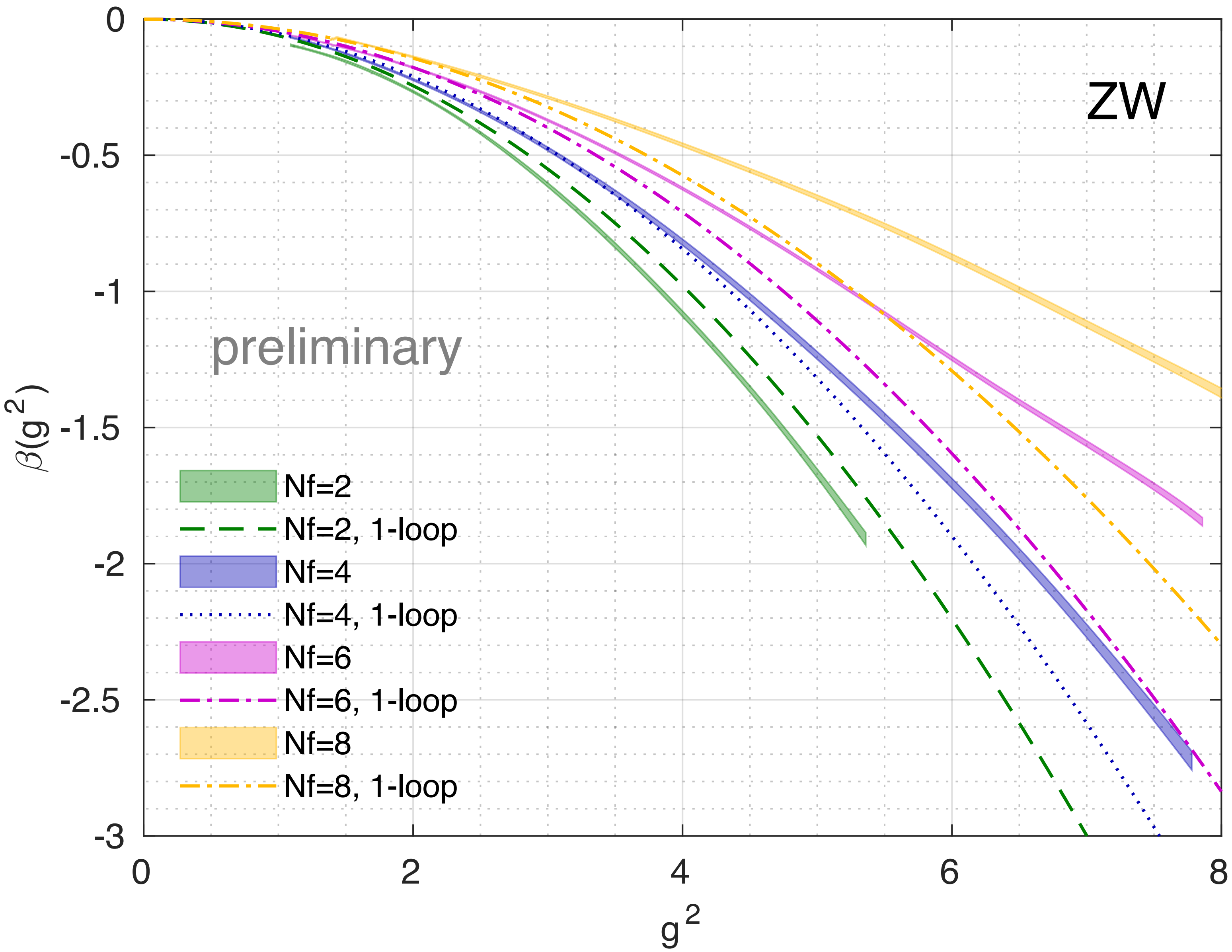
Confining regime :
- mild mass dependence
- $\beta(g^2) \propto g^2$ for $N_f = 0, 2, 2+1$

Why?

The continuous β function, $N_f = 0 - 8$

Continuum limit in the deconfined regime
In all cases:

- The β functions runs slower than 1-loop PT
- minimal cutoff effects if Zeuthen flow+Wilson op (or tree level improved coupling)



Anomalous dimension and $Z_{\mathcal{O}}$ renormalization factors

- A new method to determine RG $Z_{\mathcal{O}}$ factors for local composite operators
with C. Monahan, M. Rizik, A. Shindler and O. Witzel

- Combine the RG beta function and the anomalous dimension (A. Carosso, AH, E. Neil, PRL 121,201601 (2018)) to predict the running anomalous dimension

Anomalous dimension and $Z_{\mathcal{O}}$ renormalization factors

Consider a GF two-point function

$$G_{\mathcal{O}}(x_4, t) = \int d^3x \langle \mathcal{O}(\mathbf{x}, x_4; t)\mathcal{O}(\mathbf{x},0; t=0)\rangle,$$

If $\mathcal{O}$ is a scaling operator, an RG transformation with scale change $b \propto \sqrt{8t/a^2}$ predicts

$$G_{\mathcal{O}}(g_i, x_4) = b^{-\Delta_{\mathcal{O}}} G_{\mathcal{O}}(g_i^{(b)}, x_4/b), \qquad x_4 \gg b$$

where the scaling dimension is

- $\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$ for a non-linear RG
- $\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} + \eta$ for a linear RG (mesons)
  ($\eta/2$ is the anomalous dimension of the fermion)

For the vector current $\gamma_V = 0$. The ratio

$$\mathcal{R}_{\mathcal{O}}(x_4; t) = \frac{G_{\mathcal{O}}(x_4; t)}{G_V(x_4; t)}$$

is independent of η

Anomalous dimension and $Z_{\mathcal{O}}$ renormalization factors

The logarithmic derivative

$$2t\frac{d\mathcal{R}(x_4,t)}{dt} = 2t\frac{d}{dt}\frac{G_{\mathcal{O}}(x_4;t)}{G_V(x_4;t)} = \gamma_{\mathcal{O}}(g_r^2)$$

predicts the running anomalous dimension $\gamma_{\mathcal{O}}(g_r^2)$ as the function of $g_r^2 = g_{GF}^2(t)$.

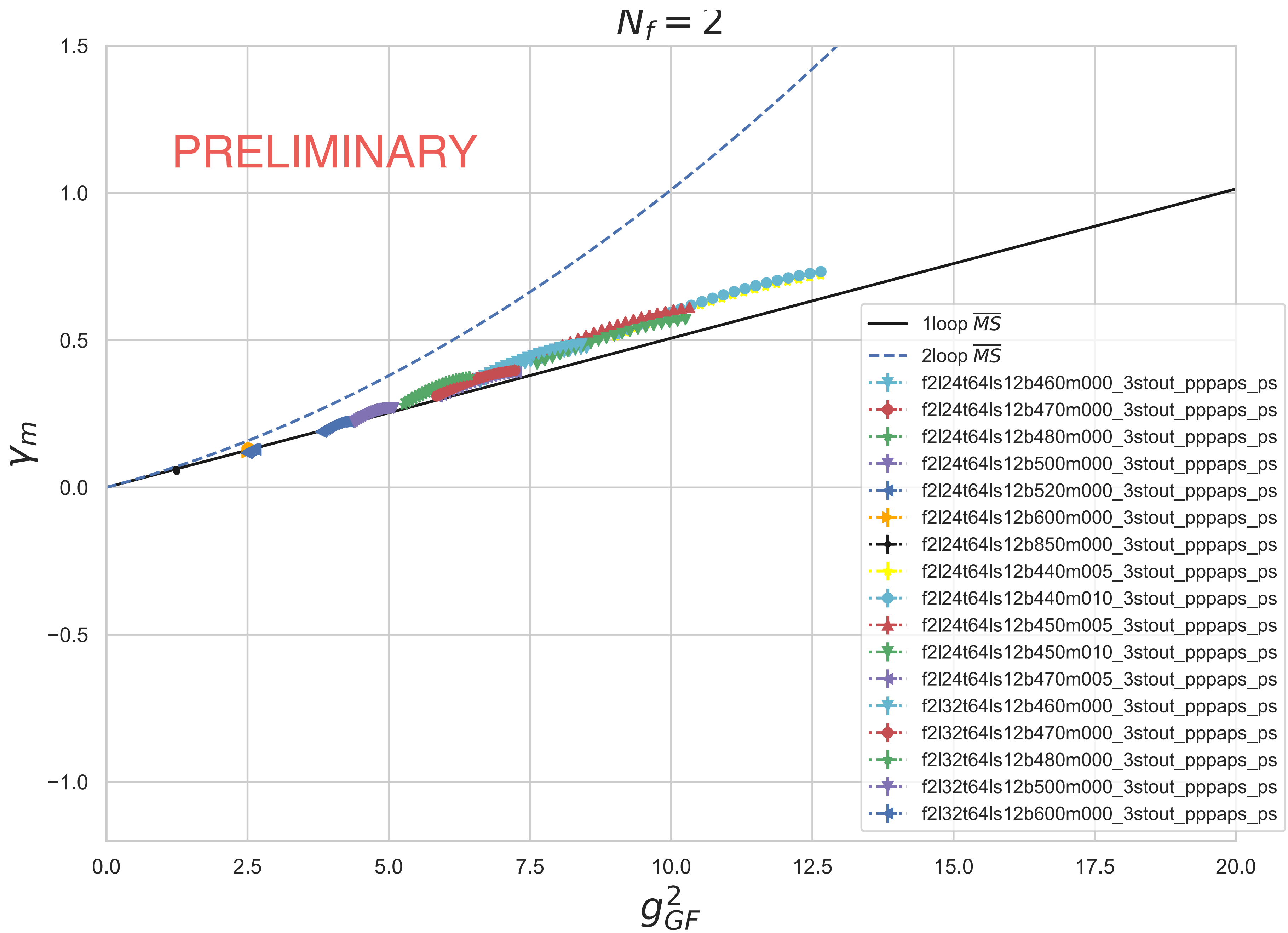
Connect the UV and IR as

$$\exp\int_{g_{IR}}^{g_{UV}} dg' \frac{\gamma_{\mathcal{O}}(g'^2)}{\beta(g'^2)} = \frac{Z_{\mathcal{O}}^{UV}}{Z_{\mathcal{O}}^{IR}}$$

A perturbative calculation connects the lattice GF $Z_{\mathcal{O}}^{UV}$ and $Z_{\mathcal{O}}^{\overline{MS}}$

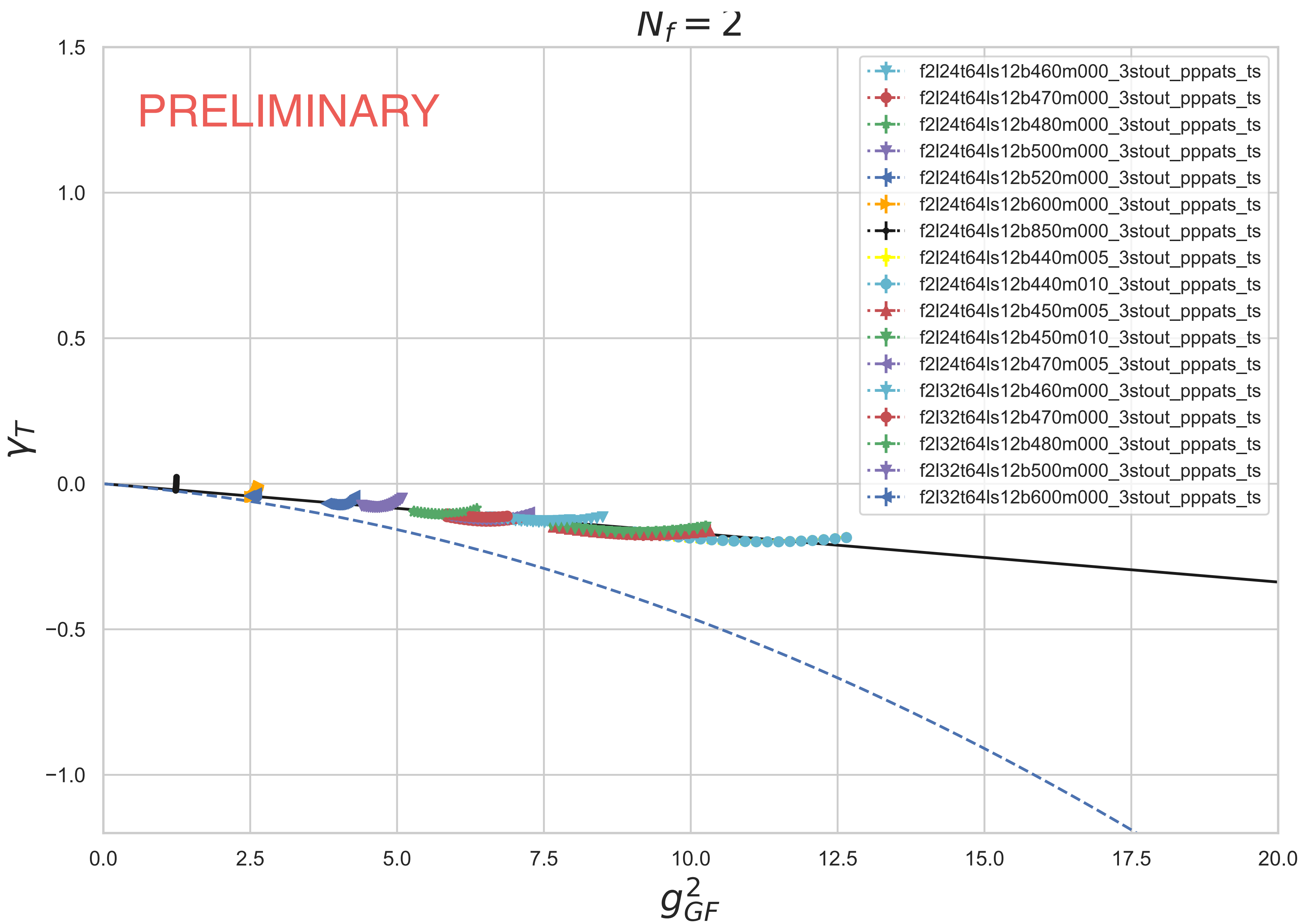
(talks by C. Monahan and M. Rizik at Lat21)

Anomalous dimension for $N_f = 2$ — scalar



- Small finite volume effects
- Weak mass dependence
- Overlap indicates small cutoff effects
- γ_m very close to 1-loop perturbative prediction

Anomalous dimension for $N_f = 2$ —tensor



Very similar to scalar; baryon is possible but much noisy

Summary and Outlook

The equivalence of Wilsonian RG and gradient flow allows a theoretically solid description of the strong coupling regime of lattice models

✦ This is particularly important in near-conformal / conformal systems where new fixed points, new relevant operators appear

The continuous β function:

✦In QCD-like chirally broken systems it is well controlled with minimal cutoff effects with improved action/flow/operator in QCD-like chirally broken systems

Anomalous dimension $\gamma(g^2)$ can be calculated for any operator

✦predict the universal γ at the of conformal systems

✦predict RG factors in QCD-like systems

Unexpected observations:

✦ In the confining regime (large g^2) $\beta(g^2) \sim \alpha_0 + \alpha_1 g^2$ - topology?

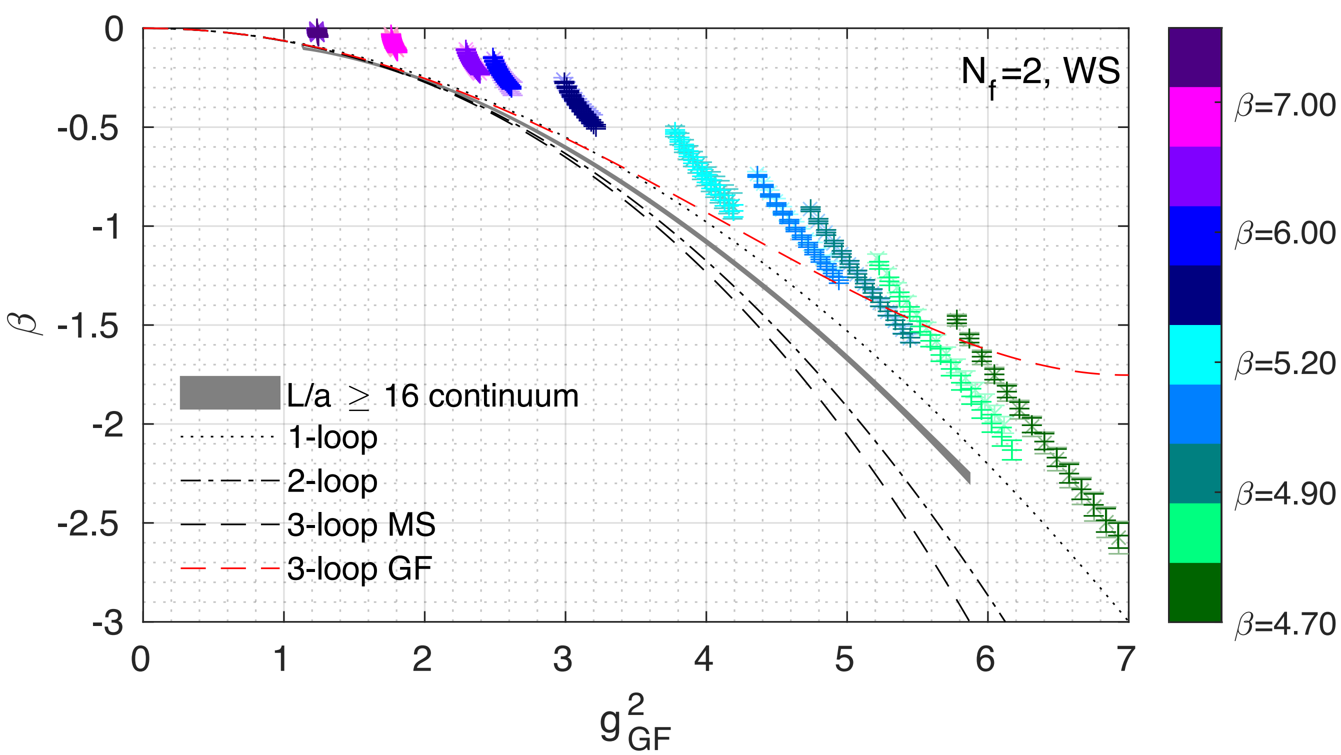
✦ Both the β function and anomalous dimensions lie close to the 1-loop perturbative prediction

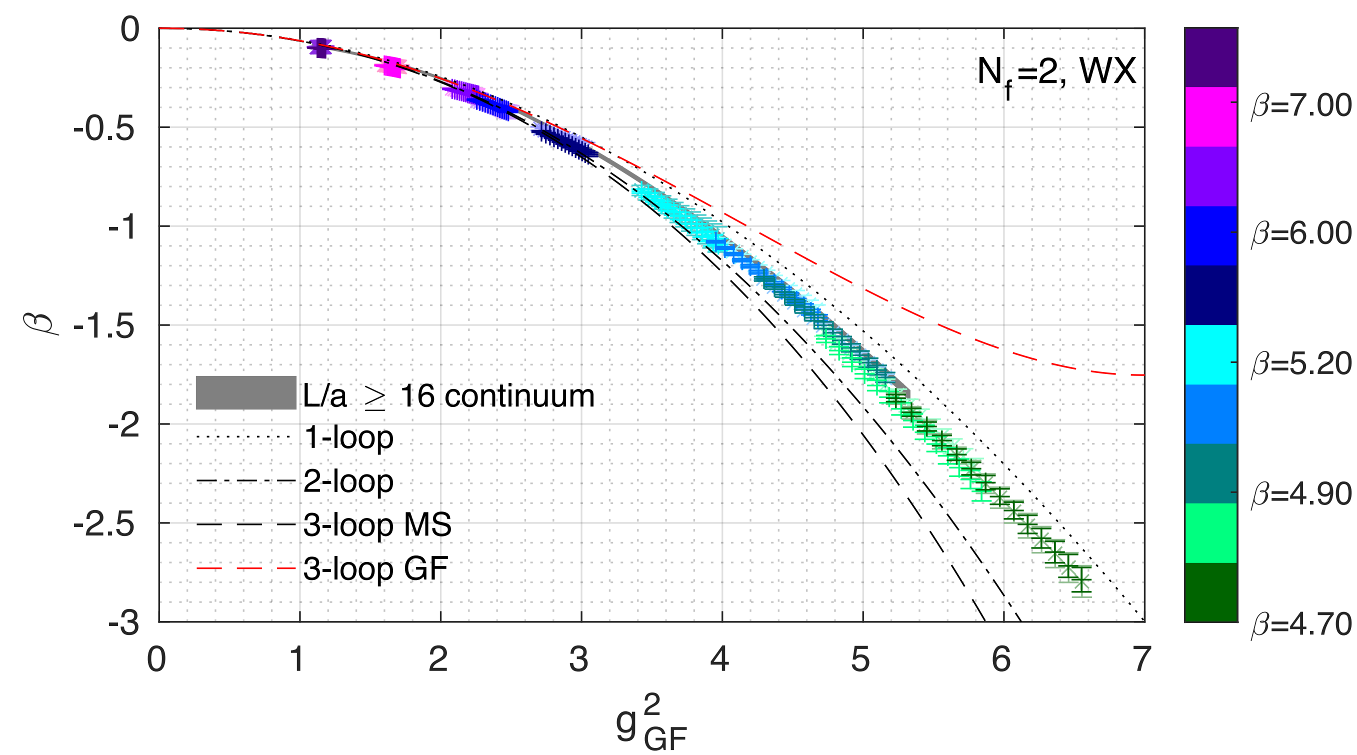
EXTRA SLIDES

The continuous β function, $N_f = 2$

AH, O. Witzel, *Phys.Rev.D* 101 (2020) 3

- Other flow/operator combos predict the same continuum limit, sometimes with larger cut-off effects (WS)
- It is possible to "optimize" the operator for different flows and find "scaling operators" (e.g. $X = 0.25W + 0.75C$ is optimal for Wilson flow) (WX)





GF β function is closest to 1-loop; consistent with GF 3-loop up to $g^2_{GF} \approx 2.5$

Harlander et al, *JHEP* 06 (2016) 161