STAGGERING TOWARDS THE CONTINUUM: IS a SMALL ENOUGH FOR a_μ^{HVP} ?

WORK IN PROGRESS

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DWQ@25 (on line)

INTRODUCTION

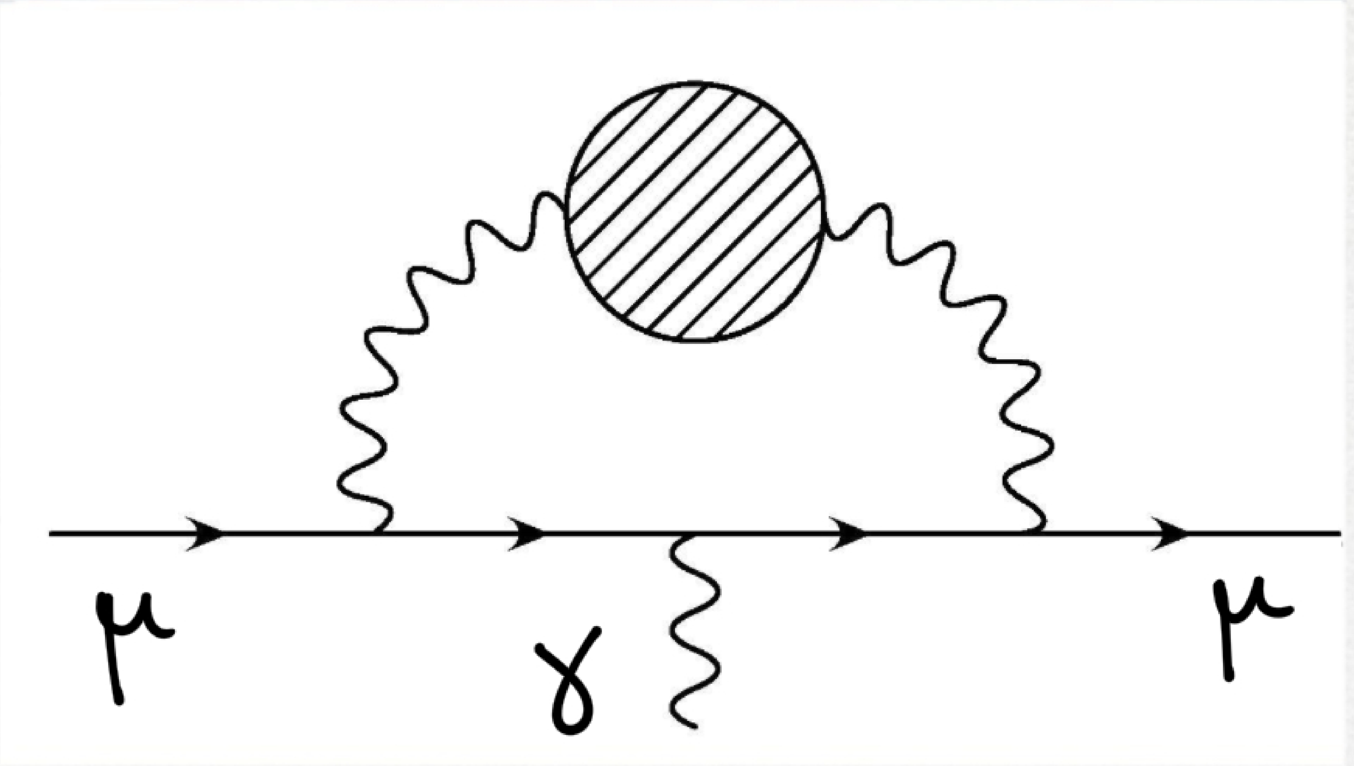
- a_μ is a low-energy effect: $\underline{\text{EFT}\,(\pi,\gamma,\mu)}$ — Aubin et al. PRD102 ('20) 9, 094511

$$\mathcal{L}_{EFT} = \bar\mu\, i\not{D}\mu - \frac{f_\pi^2}{4}\mathrm{Tr}\left[D_\alpha U D^\alpha U^\dagger\right] + \text{Higher-dim. Op's (LEC's)}$$

$$a_\mu^{HVP} \sim \alpha^2 \int_0^\infty dq^2\, f(q^2)\,\hat\Pi(q^2)\;;\quad f(q^2) \underset{q^2\to\infty}{\sim} \frac{m_\mu^4}{q^6}$$

Lautrup et al. Phys. Rep. ('72)
Blum PRL ('03)

$$\hat\Pi(q^2) \underset{q^2\to\infty}{\sim} (q^2)^{k-1} \qquad (N^kLO)$$



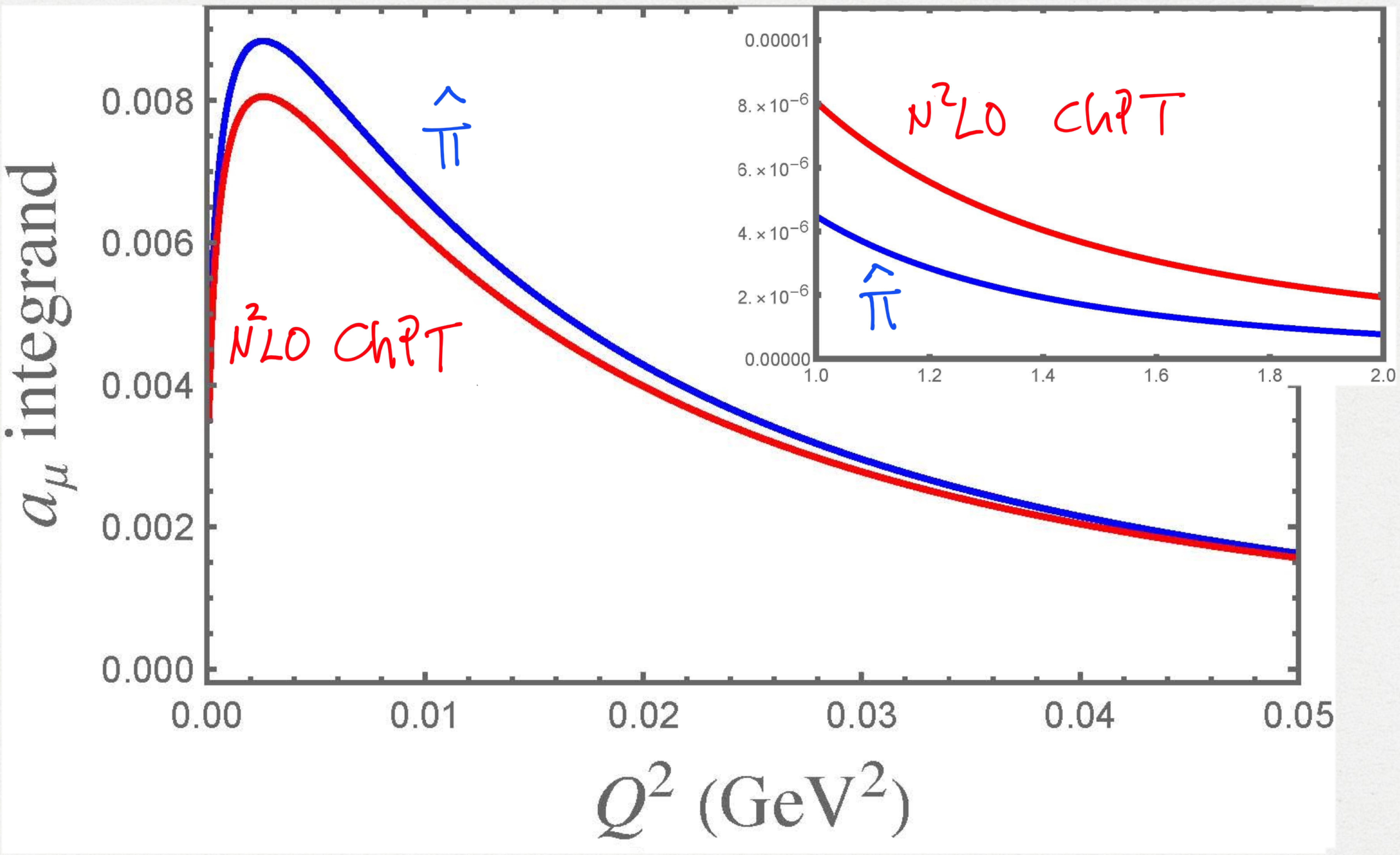
For $k \le 2$, LEC's = those of ChPT

For $k \geqslant 3$, new Op's: e.g.

$$\frac{\alpha^2 m_\mu^3}{(4\pi f_\pi)^4}\,\bar\mu\,\sigma^{\alpha\beta}F_{\alpha\beta}\mu\,\mathrm{Tr}\left[Q_L U Q_R U^\dagger\right] \qquad (k=3)$$

ChPT vs. a_μ^{HVP} : CONTINUUM (I)

- Compare a_μ^{HVP} integrand from ChPT, with $\hat{\Pi}(Q^2)$ from $R(e^+e^- \to had)$



- ChPT uses

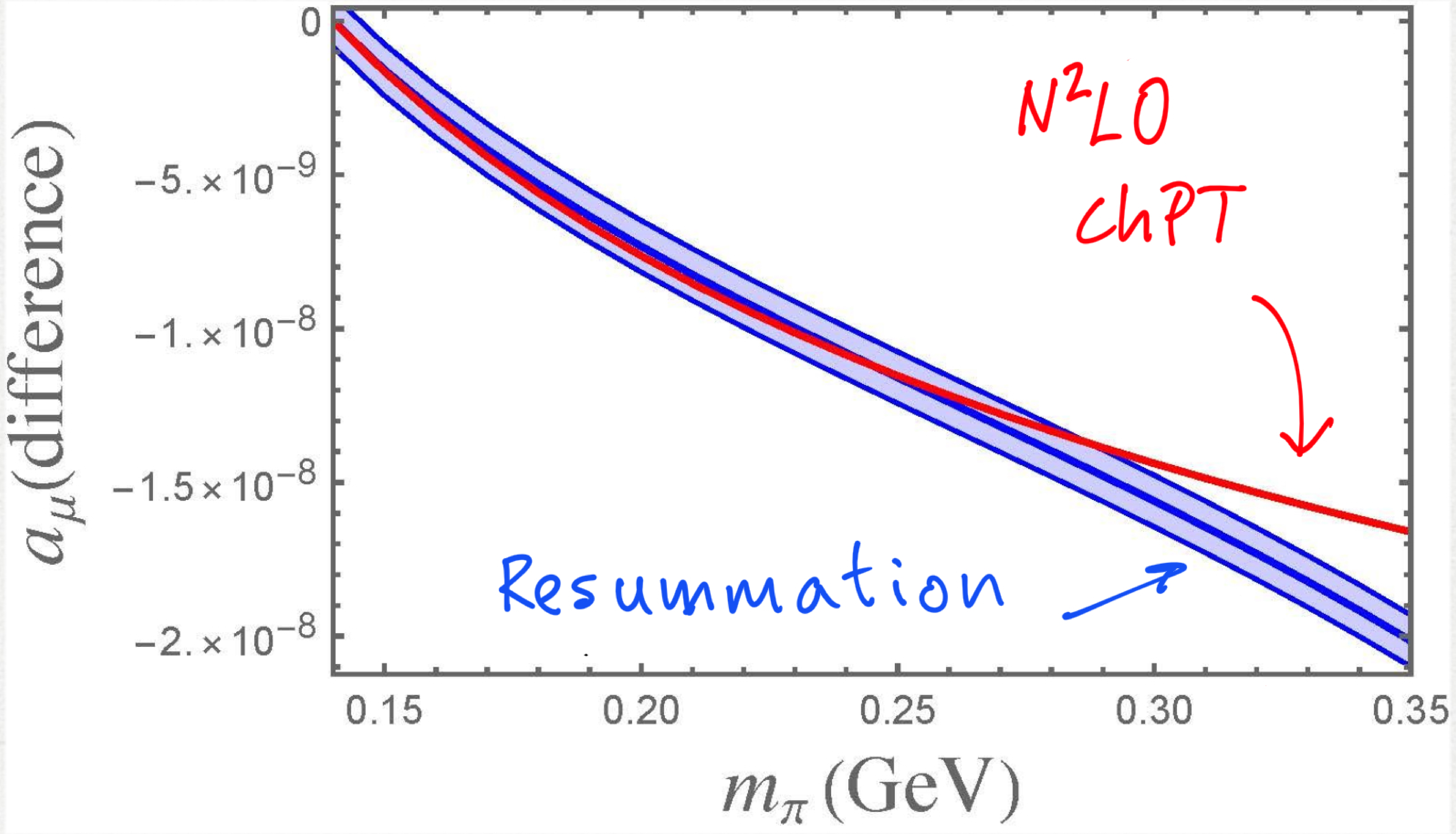
$\ell_6(M_\rho)$, $C_{56}(M_\rho)$

$\downarrow$

not very well known

ChPT vs. a_μ^{HVP} : CONTINUUM (II)

- Study m_π difference : $a_\mu^{HVP}(m_\pi) - a_\mu^{HVP}(m_\pi^{phys})$



- Colangelo et al. (2110.05493 [hep.ph]):
  IAM ⊕ Omnes resummation of ChPT
  (in agreement with exp. data.)

$\Rightarrow N^2LO$ ChPT works up to $m_\pi \lesssim 250$ MeV

(In the difference, LEC c_{56} cancels out)

— N^2LO ChPT useful for lattice corrections —

a_μ^{HVP} ON THE LATTICE

$$a_\mu^{HVP}(T) = \sum_{t=-T/2}^{+T/2} w(t)\, C(t) \quad ; \quad C(t) = \frac{1}{3}\sum_{\vec{x},i} \langle j_i(\vec{x},t)\, j_i(0)\rangle$$

Bernecker et al. EPJA 47, 148

$$\hat{\Pi}(q^2) = \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{t^2}{2} \right) C(t)$$

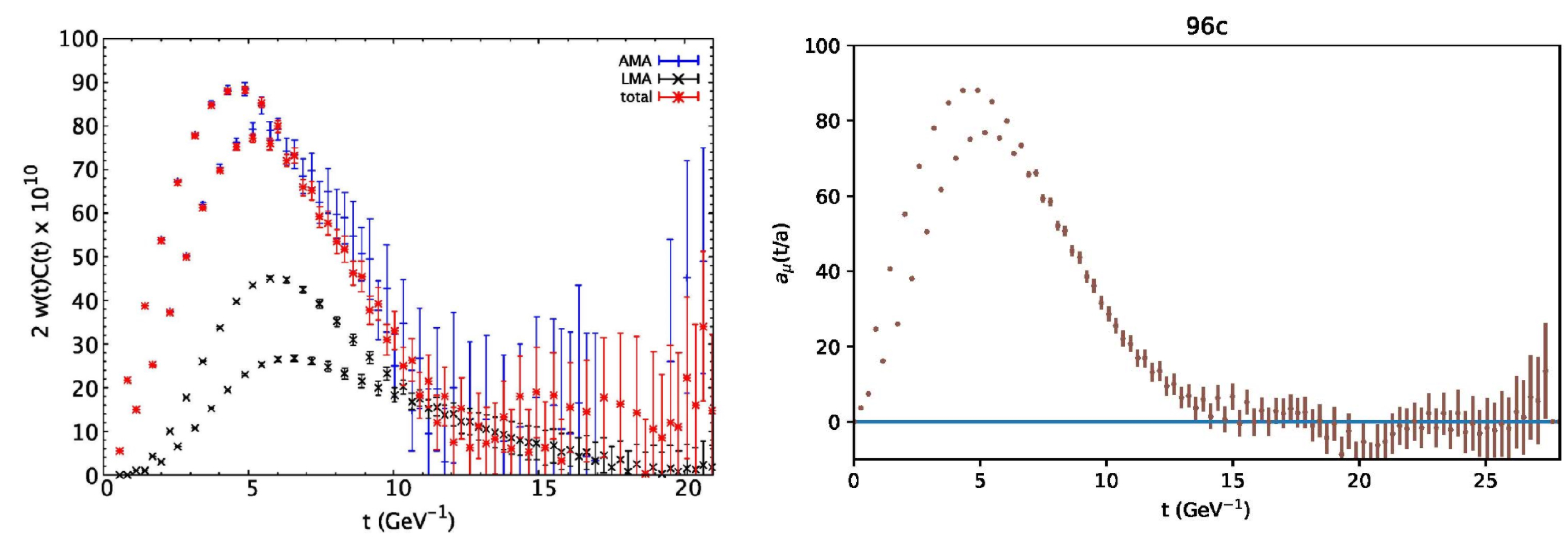
- 2+1+1 staggered (light quark connected)

1. Lattice Data

m_π(MeV)	heaviest taste (MeV)	a(fm)	L^3	T	L(fm)	$m_\pi L$
134	153	0.0568	96^3	192	5.46	3.71
130	211	0.0879	64^3	96	5.62	3.69
133	326	0.1212	48^3	64	5.82	3.91
134	418	0.1510	48^3	64	7.25	4.93
133	418	0.1515	32^3	48	4.85	3.27

Table 1. Different lattice ensembles. All from MILC, except the 48^3($a = 0.1510$ fm), which is from CalLat.

Compare summand for original 96^3 data vs. new data:



$C(t)$ in SChPT

BMW / Borsanyi et al. Nature ('21)
Aubin et al. (in preparation)

- $$C^{(t>0)}_{ChPT}(t) = \frac{1}{48L^3}\sum_{\vec{p},X}\frac{\vec{p}^{\,2}}{E_X^2(p)}\, e^{-2tE_X(p)} \otimes$$ NLO

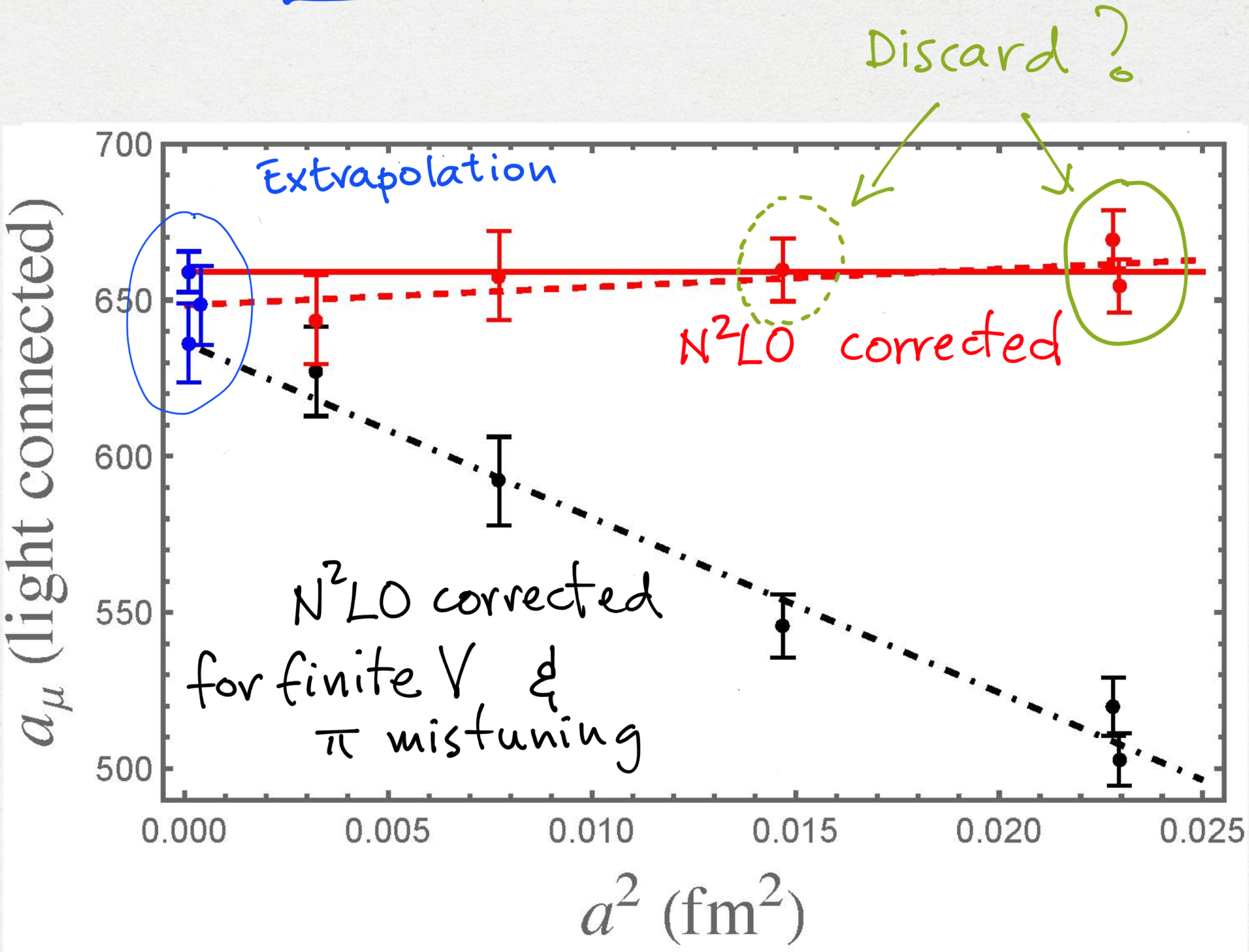
$$\otimes\left(1 - \frac{1}{4f_\pi^2L^3}\sum_{\vec{k}}\frac{1}{2E_Y(k)} - \frac{16\, l_6(\vec{p}^{\,2}+m_X^2)}{f_\pi^2} + \frac{1}{24f_\pi^2L^3}\sum_{\vec{q},Y}\frac{\vec{q}^{\,2}}{E_Y(q)}\frac{1}{\vec{q}^{\,2}-\vec{p}^{\,2}+m_Y^2-m_X^2+i\epsilon}\right)$$ N^2LO

(Nontrivial sums to evaluate numerically, but l_6 term dominates)

$C_{ChPT}(t)$ will be used for corrections:

- Finite volume
- Taste breaking
- Pion retuning

a_μ^{HVP} in ChPT



constant fit

linear a^2 fit

a_μ	$96^3 - 64^3$	$96^3 - 48^3$	$96^3 - 32^3$
lattice	13(18)	59(16)	103(15)
NLO ChPT	20	28	65
NNLO ChPT	28	75	114

TABLE 11. Differences of a_μ between different lattices. All number in units of 10^{-10}.

Good p-values ~ 0.4

To discriminate $\Longrightarrow$ smaller a (and better statistics)

a_μ^{HVP} with a window (0.4 - 1.0 fm)

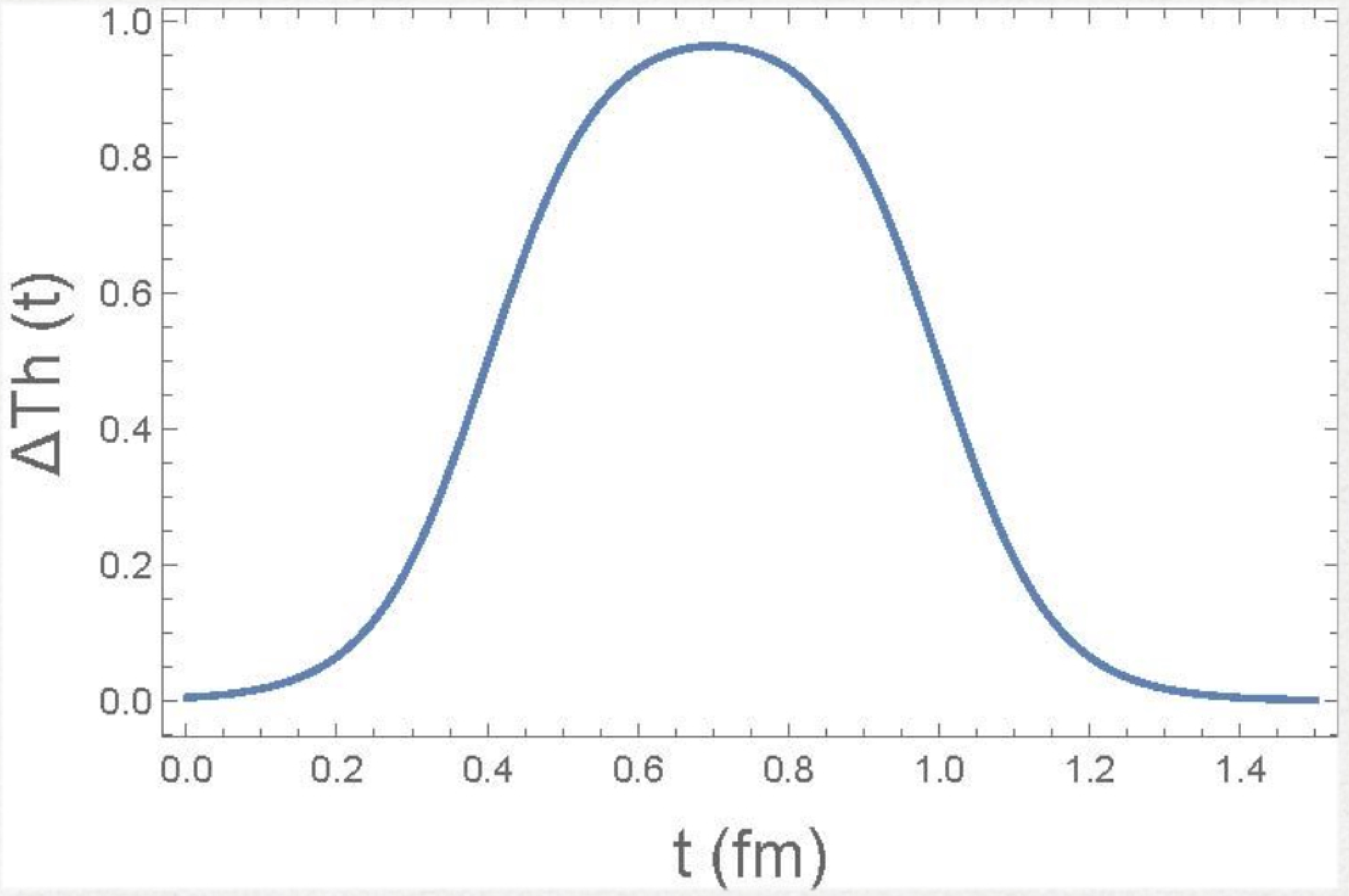
RBC/UKQCD Blum et al. PRL 121 ('18) 022003

- Window : $\Theta(t,t',\Delta) = \frac{1}{2}\left(1 + \tanh\frac{t-t'}{\Delta}\right)$; $\Delta Th(t) = \Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta)$

Study $a_\mu^{W} = 2\sum_{t=0}^{T/2} w(t)\, C(t)\, \Delta Th(t)$

NO EFT !

$t_0 = 0.4$ fm , $t_1 = 1.0$ fm, $\Delta = 0.15$ fm



Window 0.4–1.0 fm	$96^3 - 64^3$	$96^3 - 48^3$	$96^3 - 32^3$
lattice	1.42(54)	4.49(66)	5.43(79)
NLO ChPT	2.28	6.47	9.98
NNLO ChPT	6.67	21.08	35.88
SRHO (no FV)	1.38	5.26	9.44

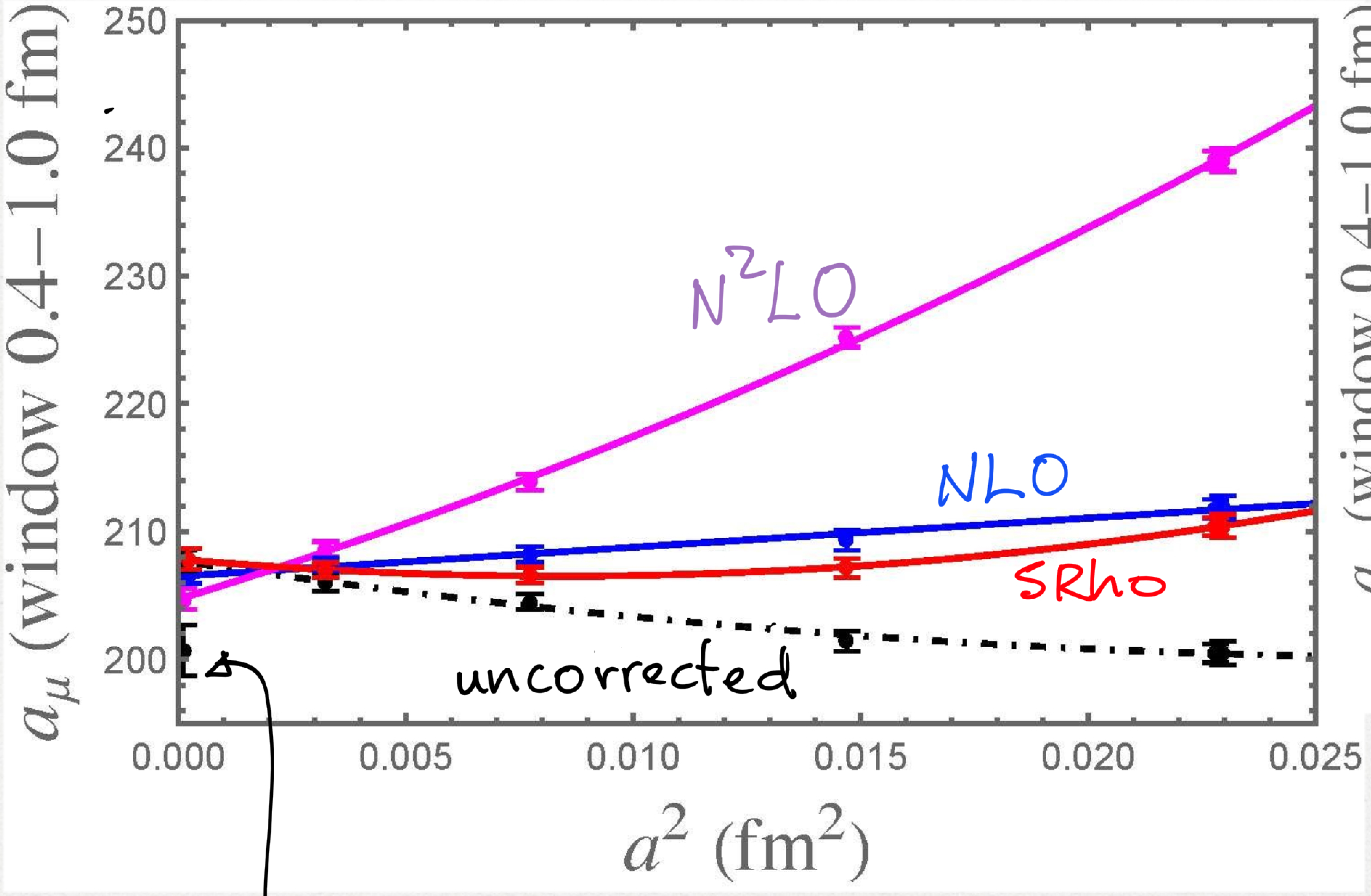
TABLE 12. Differences of 0.4–1.0 fm window between different lattices. NNLO FV corrections only take ℓ_6 contribution into account. FV corrections are not yet included in the SRHO-based corrections. All number in units of 10^{-10}.

HPQCD/ Chakraborty et al. PRD 96 ('17) 034516 window (0.4 - 1.0 fm)

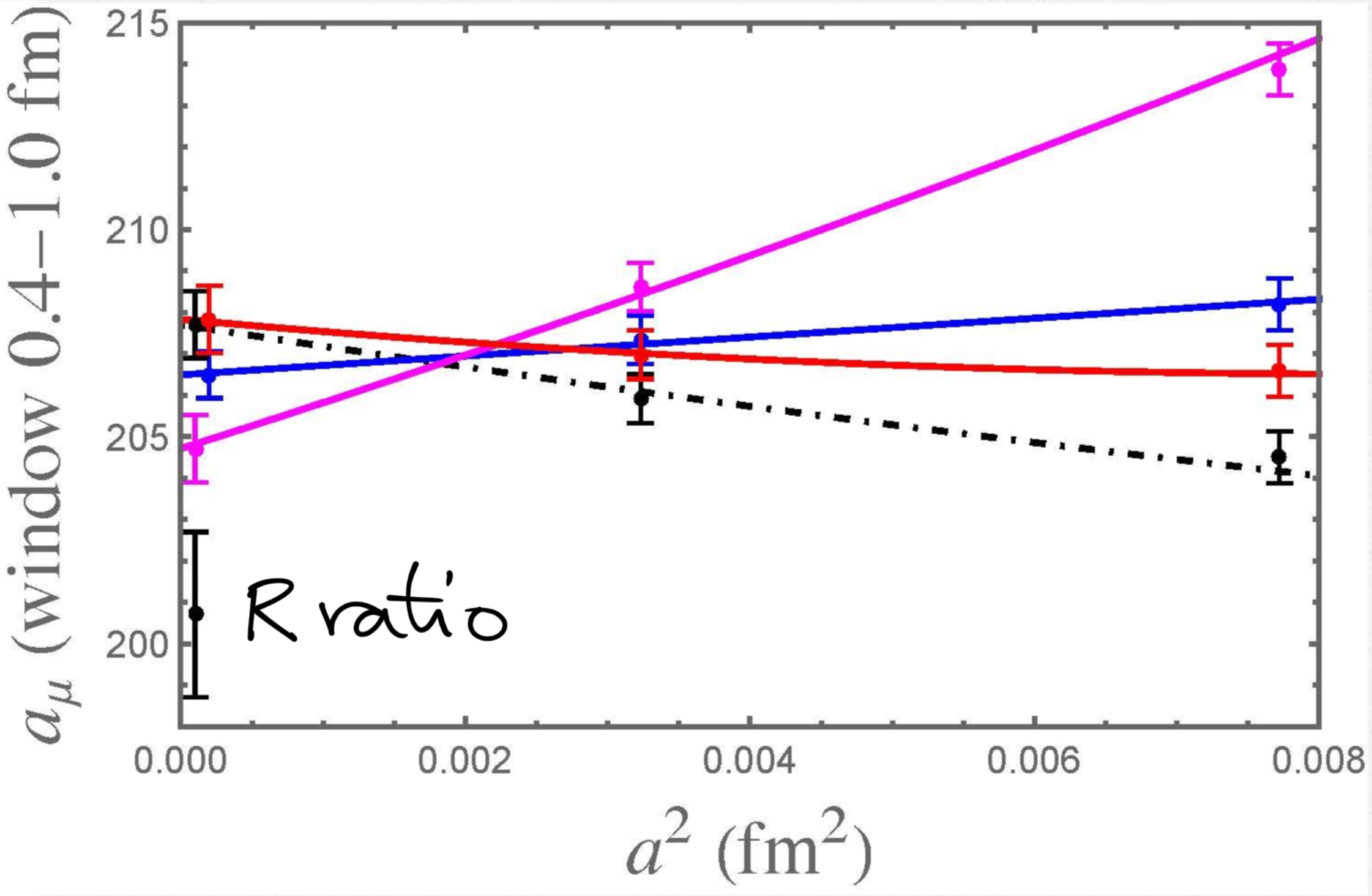
(cfr. spectrum in B/U slides)

All fits nonlinear in a^2 (including SRho), except NLO.

GOOD: small lattice errors   BAD: no reliable calculation of corrections



Rratio

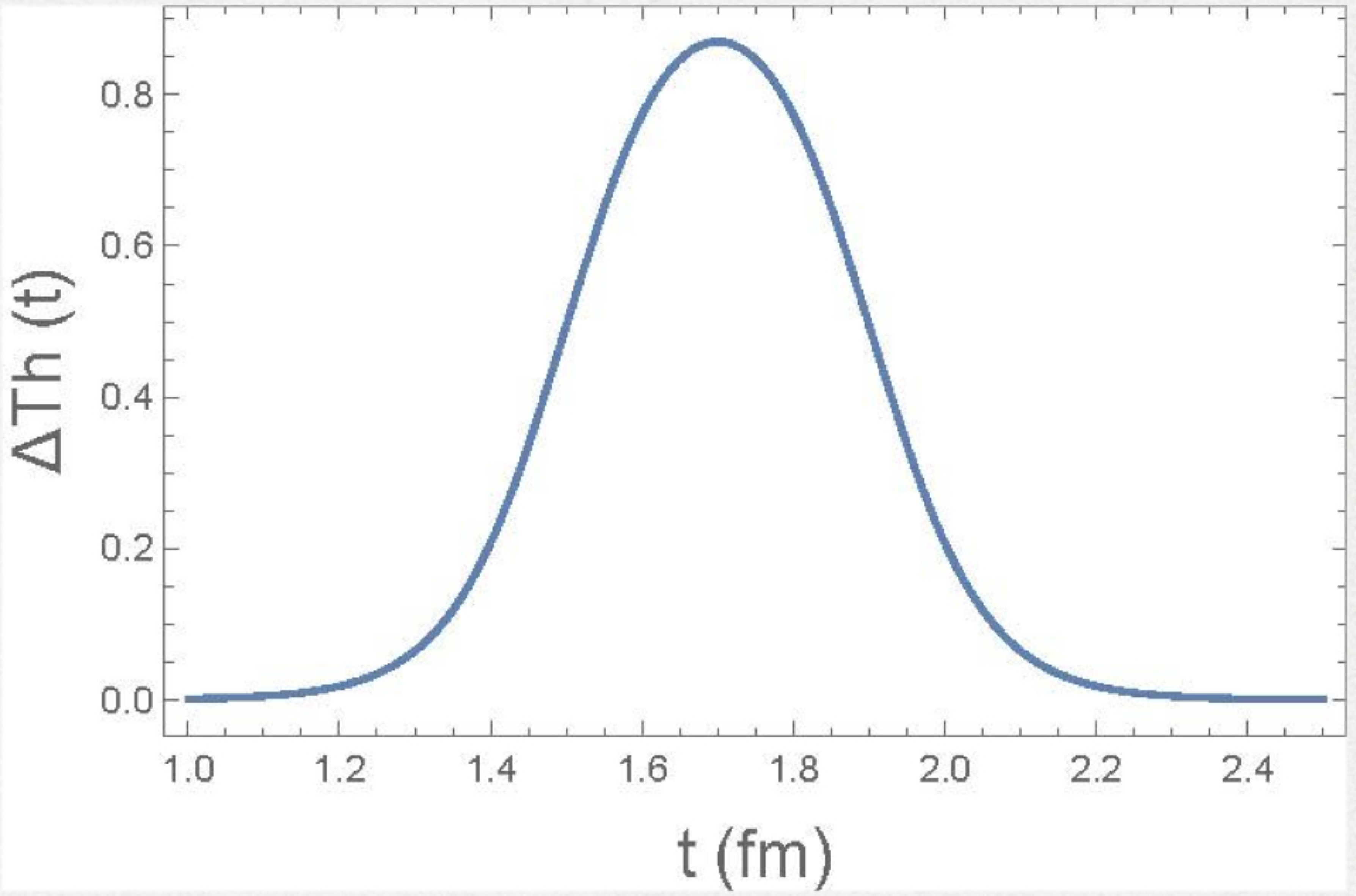


(SRho − R ratio) $\approx$ 3.3 σ

Good p-values $\gtrsim$ 0.3

Why not a new window? (1.5 – 1.9 fm)

☆ $t_0 = 1.5$ fm, $t_1 = 1.9$ fm, $\Delta = 0.15$ fm



EFT reliable!

(Do not need SRho...)

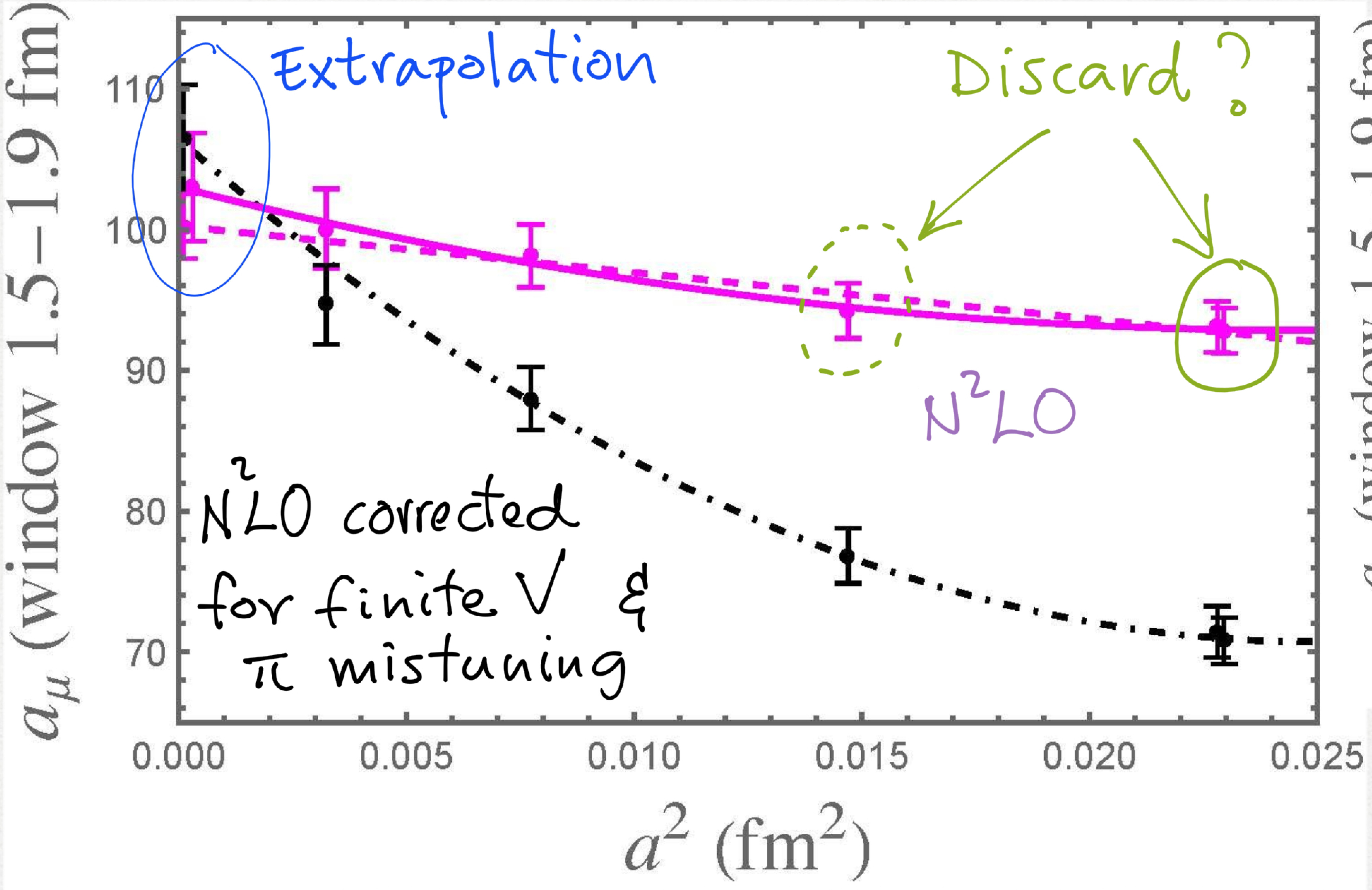
Window 1.5–1.9 fm	$96^3 - 64^3$	$96^3 - 48^{3\prime}$	$96^3 - 32^3$
lattice	6.67(2.95)	17.84(2.79)	23.89(2.59)
NLO ChPT	2.10	4.99	6.66
NNLO ChPT	4.74	12.02	16.67
SRHO (no FV)	8.00	20.92	30.32

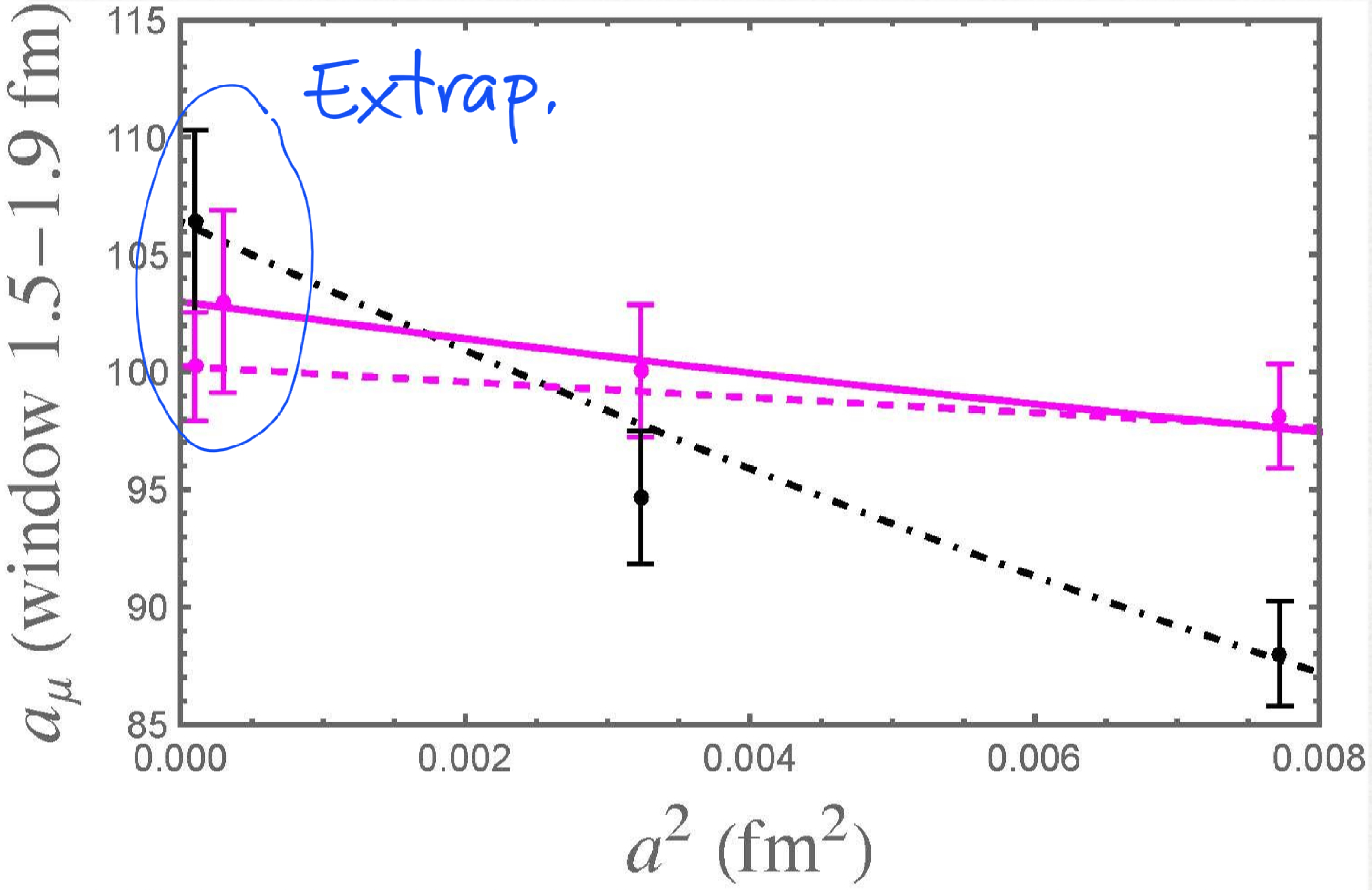
TABLE 13. Differences of 1.5–1.9 fm window between different lattices. NNLO FV corrections only take ℓ_6 contribution into account. FV corrections are not yet included in the SRHO-based corrections. All number in units of 10^{-10}.

Relative lattice errors for this window about half of those for a_μ^{HVP}.

new window (1.5 – 1.9 fm)

- - - - linear in a^2
——— quadratic in a^2





Very good p-values ~ 0.8

To discriminate $\Longrightarrow$ smaller a

(and better statistics)

CONCLUSIONS

- EFT treatment valid for a_μ^{HVP} (if masses small enough).
  (For staggered, a must be small enough so that taste splittings $\lesssim 150$ MeV if the exact pion is physical)
- new window (1.5–1.9 fm) better than (0.4–1.0 fm) if EFT to be used for correcting data in continuum extrapolation.
- We are not in the linear a^2 regime with these lattices. Need smaller a's. (Please quote taste splittings.)
- What about a^2 effects beyond taste splittings? (cfr. DWF)
- Models should be avoided in a "first-principles" calculation.

BACK-UP

SLIDES

SPECTRUM EXTRAPOLATION $a \to 0$

Taste spectrum nonlinear in a^2.