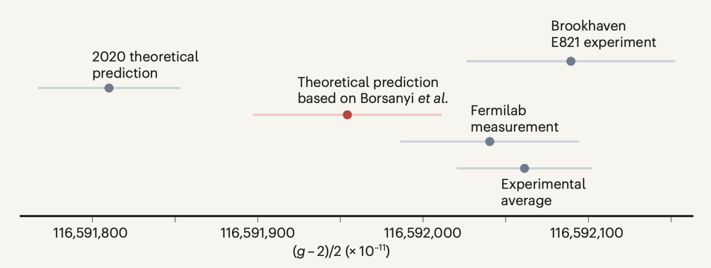
The hadronic vacuum polarisation contribution to the muon g-2 from lattice QCD with O(a) improved Wilson fermions

Harvey Meyer
Johannes Gutenberg University Mainz

"DWQ@25", BNL-HET & RBRC joint online workshop, 14 December 2021

$(g-2)$ of the muon: current status



[White Paper 2006.04822 (Phys.Rept); Borsanyi et al. 2002.12347 (Nature); Muon (g-2) collaboration 2104.03281 (PRL)].

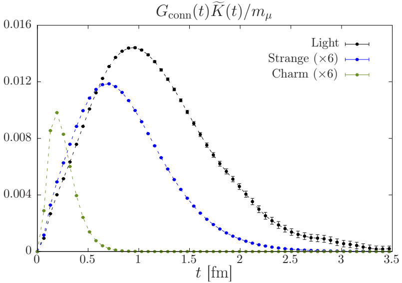
Figure from HM, *Thrill of the Magnetic Moment*, Nature News & Views.

- Currently, the theory prediction uncertainty is dominated by the hadronic vacuum polarisation contribution.
- The tension between the Borsanyi et al. lattice calculation and the pheno-based White Paper prediction should be scrutinized.

Computing $a_\mu^{\rm hvp}$ from lattice-QCD current-current correlators $G(t)$

$$G(t) \equiv \int d^3x \langle j_3(t,\vec{x})\, j_3^\dagger(0)\rangle = \int_0^\infty d\omega\, \omega^2 \frac{R_{e^+e^-\to{\rm hadrons}}(\omega^2)}{12\pi^2}\, e^{-\omega t}.$$

$a = 0.064{\rm fm}:$ $m_\pi = 200\,{\rm MeV}$ $m_\pi = 130\,{\rm MeV}$ Mainz/CLS 1904.03120.



$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \widetilde{K}(t) G(t), \qquad \text{where} \qquad \widetilde{K}(t) \sim \begin{cases} \frac{\pi^2}{9} m_\mu^2 t^4 & t \ll m_\mu^{-1} \\ 2\pi^2 t^2 & t \gg m_\mu^{-1}. \end{cases}$$

- The tail of the integrand is affected by statistical fluctuations, by finite-size effects and depends strongly on the pion mass due to the $\pi\pi$ channel.

Contribution of an intermediate window of Euclidean time

- A full lattice calculation of $a_\mu^{\rm hvp}$ at the subpercent level requires control over a vast number of contributions and effects.
- $\Rightarrow$ community strategy: perform comparisons of particularly precisely determined subcontributions or closely related quantities.

$$a_\mu^W = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \widetilde{K}(t)\cdot \mathrm{window}_{[t_0,t_1]}(t)\cdot G(t), \qquad t_0 = 0.4\,\mathrm{fm},\ t_1 = 1.0\,\mathrm{fm}.$$

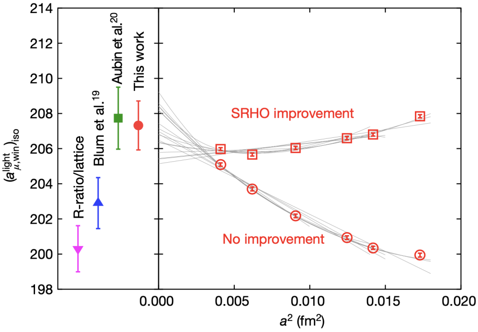


Figure from BMW, 2002.12347 (Nature).

The Mainz-CLS effort on a_μ^{hvp}

Last refereed publication: 1904.03120 (PRD).

People involved in current project:

Marco Cè, Antoine Gérardin, Georg von Hippel, Ben Hörz, Renwick Hudspith, HM, Kohtaroh Miura, Daniel Mohler, Konstantin Ottnad, Srijit Paul, Andreas Risch, Teseo San José, Hartmut Wittig.

Recent presentations of the ongoing Mainz-CLS calculation:

- Antoine Gérardin, online workshop "Muon g-2 theory initiative workshop in memoriam Simon Eidelman", KEK, Japan, 28 June - 3 July 2021.
- Hartmut Wittig, (online) Lattice conference 2021, MIT, July 26-30, 2021.

Mainz-CLS computational setup

$N_f = 2 + 1$ flavours of $\mathrm{O}(a)$-improved Wilson fermions; tree-level improved Lüscher-Weisz action

Six values of the lattice spacing: $a = 0.099, 0.085, 0.077, 0.065, 0.050, 0.039\,\mathrm{fm}$

22 Ensembles; $\gtrsim 35\,000$ gauge configurations

Volumes: $m_\pi L > 4$

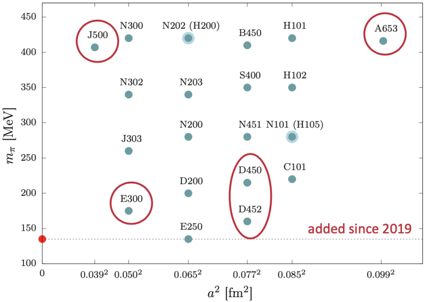
Chiral trajectory: $\mathrm{Tr}\, M_q \approx$ const

Time-momentum representation:

$$a_\mu^{\mathrm{hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)$$

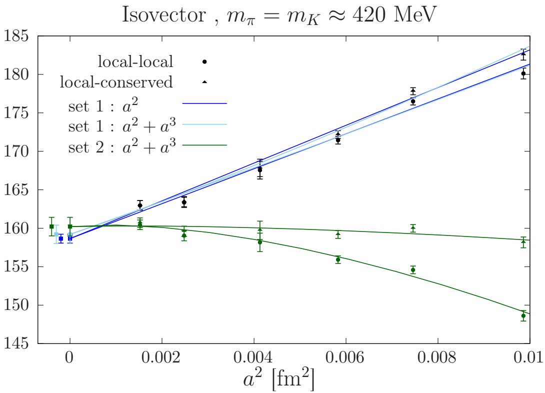
Local and point-split vector currents

→ simultaneous continuum extrapolations



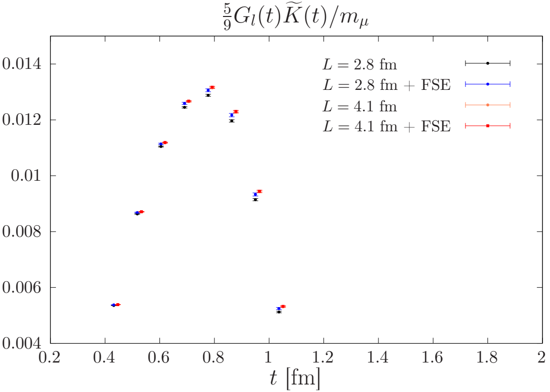
Slide from H. Wittig, LAT21.

$a_\mu^{\rm win}$ from Mainz-CLS: control over cutoff effects at $m_\pi > m_\pi^{\rm phys}$



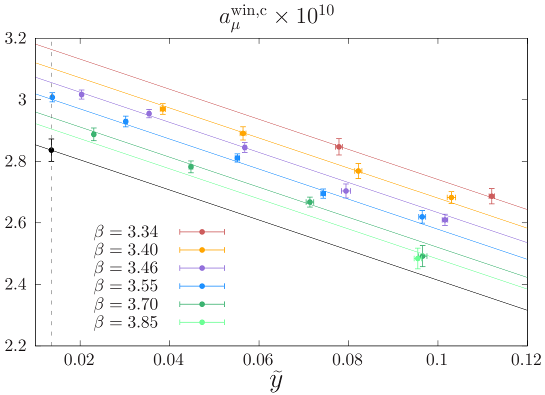
- study at $m_\pi = m_K \simeq 420\,$MeV:
  6 lattice spacings in the range $0.039 < a[\text{fm}] < 0.1$.
- choice of improvement coefficients (green: Münster 2010.09539, black: Mainz 1811.08209) does not appear to matter after continuum extrapolation
- similar study has been performed at $m_\pi \simeq 350\,$MeV.

$a_\mu^{\rm win}$: control over finite-size effects



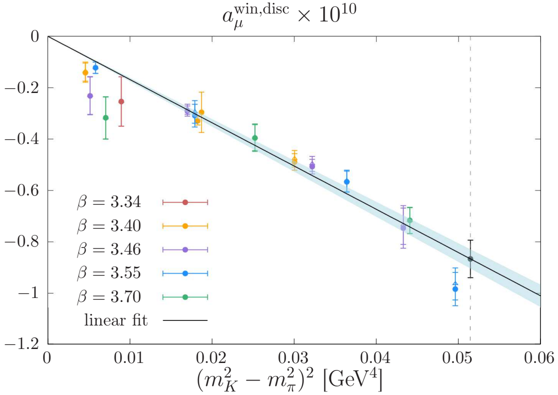
- Direct test at $m_\pi \simeq 280\,\text{MeV}$, $a = 0.086\,\text{fm}$: $m_\pi L = 3.9$ vs. $m_\pi L = 5.9$.
- From the lattice QCD calculation: $\Delta a_\mu^{\rm win} = 3.8(6)$
- Estimate using Hansen-Patella prediction (2004.03935) with VMD form factor: $\Delta a_\mu^{\rm win} = 2.44(12)$ (error from uncertainty on m_π and $m_{\rm VMD}$)
- in the full $a_\mu^{\rm hvp}$, we estimated $\Delta a_\mu^{\rm hvp} = 14.7(1.4)$ from scalar-QED + LLM method in 1904.03120.

Charm contribution: chiral & continuum extrapolation



- $\tilde{y} = m_\pi^2/(16\pi^2 f_\pi^2)$
- highly sensitive to the scale determination
- moderate cutoff effects in the local-conserved discretization.

Disconnected contribution: chiral extrapolation



- Taylor expansion around the SU(3)$_\mathrm{f}$ point works remarkably well (NB. a_μ^{win} is finite in the chiral limit)
- discretisation effects hard to discern.

Scale setting

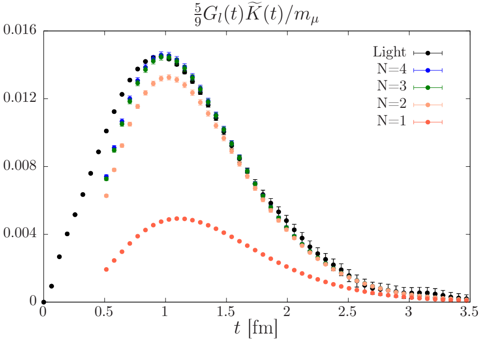
- We are using the absolute scale setting: $\sqrt{t_0} = 0.415(4)(2)$ fm from Bruno et al. 1608.08900 (PRD), which is based on the pion and kaon decay constants, $f_K + \frac{1}{2} f_\pi$.
- An update based on an extended set of ensembles is in progress (Ben Strassberger's talk at LAT21, 2112.06696).
- The preliminary central value of a[fm] has gone down by about 1.5σ. This would tend to reduce $a_\mu^{\rm hvp}$, since the value of $a \cdot m_\mu$ to be used is reduced and $a_\mu^{\rm hvp}$ is roughly proportional to m_μ^2.
- In parallel: independent scale determination from baryon octet masses (Alex Segner's talk at LAT21).

For the window quantity $a_\mu^{\rm win}$:

- the (u, d, s) contribution to $a_\mu^{\rm win}$ has a reduced sensitivity to the scale, as compared to the case of $a_\mu^{\rm hvp}$.
- the charm contribution however has an enhanced sensitivity, due to its short-distance character.

Possible strategies to improve control over the long-distance tail

1. Auxiliary calculation of the (discrete, finite-volume) spectrum of $\pi\pi$ states and their coupling to the e.m. current.
   The low-lying states saturate the correlator at long distances.



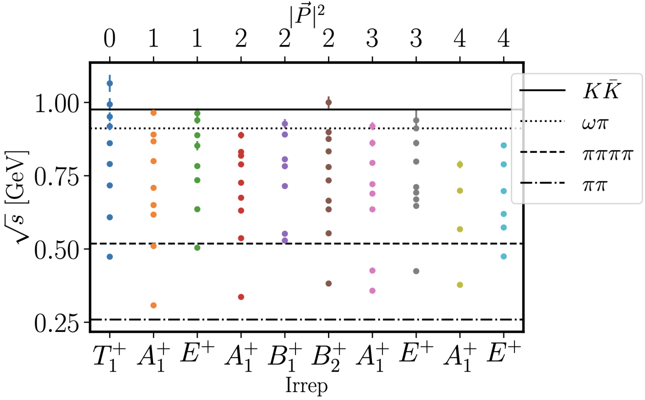
D200:
$L = 4.1\,\text{fm}$
$m_\pi = 200\,\text{MeV}$
Fig. from 1904.03120.

See also RBC/UKQCD 2019; Fermilab-HPQCD-MILC 2021.

2. 'all-to-all' propagators using the low eigenmodes of the Dirac operator
   RBC collaboration 1801.07224 (PRL); BMW collaboration 2002.12347 (Nature).
3. approximate factorization of the QCD path integral with bias correction.
   Dalla Brida et al., 2007.02973.

Mainz-CLS is currently pursuing strategies 1. and 2. in parallel.

Spectroscopy in the ρ meson channel at $m_\pi \simeq 130\,$MeV



- two-pion interpolating operators and $\bar{\psi}\gamma_i\psi$, $\bar{\psi}\gamma_i\gamma_0\psi$
- many states present ($L = 6.2$fm); role of 4π states remains to be investigated (see also work by Aaron Meyer).

Talk by Srijit Paul at LAT21.

Conclusion

- For a_μ^{hvp}, consistency checks between different methodologies, as well as between different lattice collaborations are crucial.
- Mainz preprint on the 'intermediate window' a_μ^{win} expected in 6 to 8 weeks.
- Major update on a_μ^{hvp} from Mainz-CLS mainly depends on our success at controlling the tail in the isovector channel.