Leading Hadronic Contribution to the Muon g-2 with Twisted-Mass Quarks

BNL-HET & RBRC
Joint Workshop
"DWQ@25"

13th - 17th December

Davide Giusti

Universität Regensburg

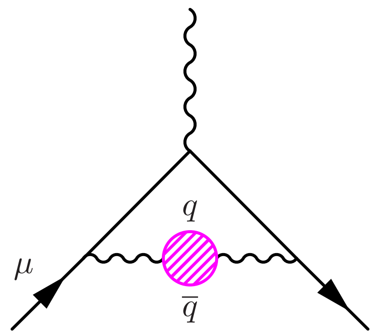
In collaboration with:

V. Lubicz, G. Martinelli, F. Sanfilippo and S. Simula

ArXiv:1707.03019; ArXiv:1808.00887; ArXiv:1901.10462;
ArXiv:1910.03874; ArXiv:2003.12086; ArXiv:2111.15329

Extended Twisted Mass Collaboration

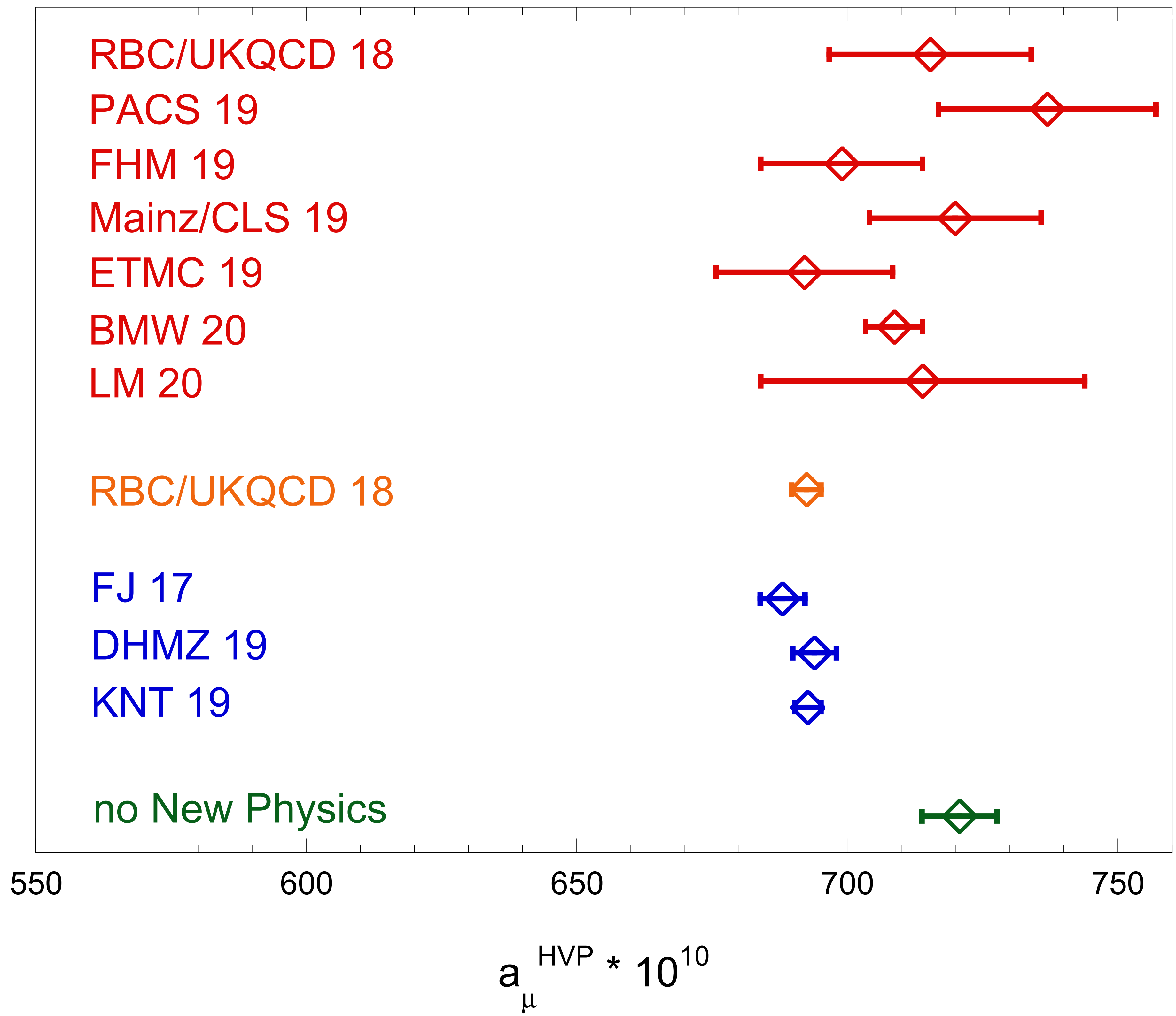
Hadronic Vacuum Polarization



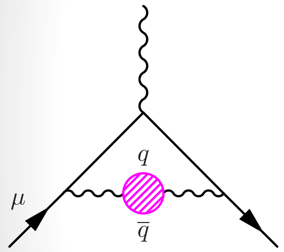
lattice data
100%

lattice + e^+e^-
~ 30% + 70%

e^+e^- data
100%



Muon anomalous magnetic moment from ETMC

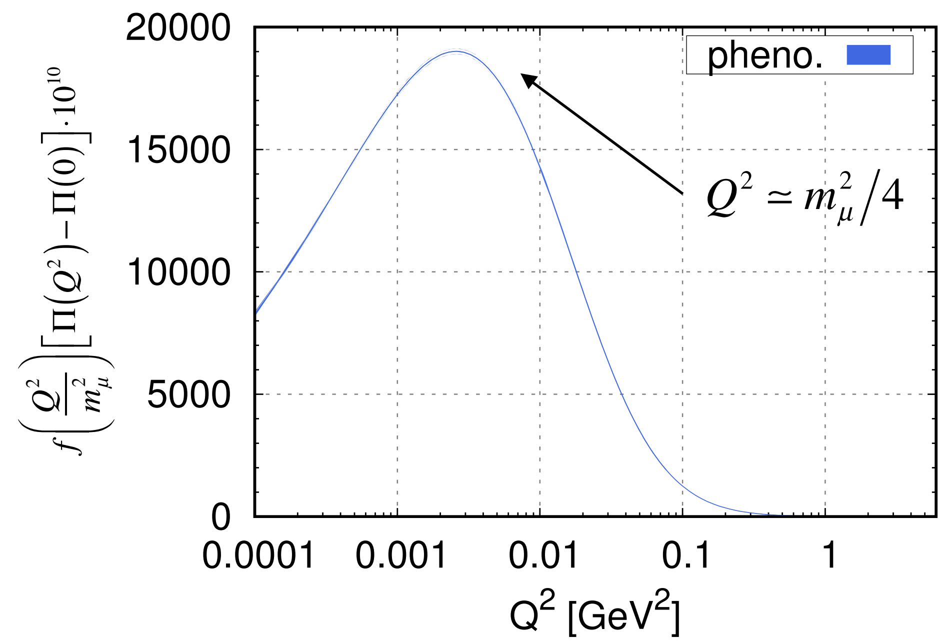


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi(Q^2)$$

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right) \left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972; T. Blum, 2002



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}_\ell(t)\, V(t)$$

D. Bernecker and H. B. Meyer, 2011

$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \, \langle J_i(\vec{x},t) J_i(0) \rangle$$

$$a_\ell^{\mathrm{HVP}} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} \tilde{f}_\ell(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} \tilde{f}_\ell(t) \frac{G_V^f}{2M_V^f} e^{-M_V^f t} \right\}$$

$t \le T_{data} < T/2$ (avoid bw signals)

$t > T_{data} > t_{min}$ (ground-state dom.)

quark-connected terms only

lattice data

local vector currents

analytic representation

Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)

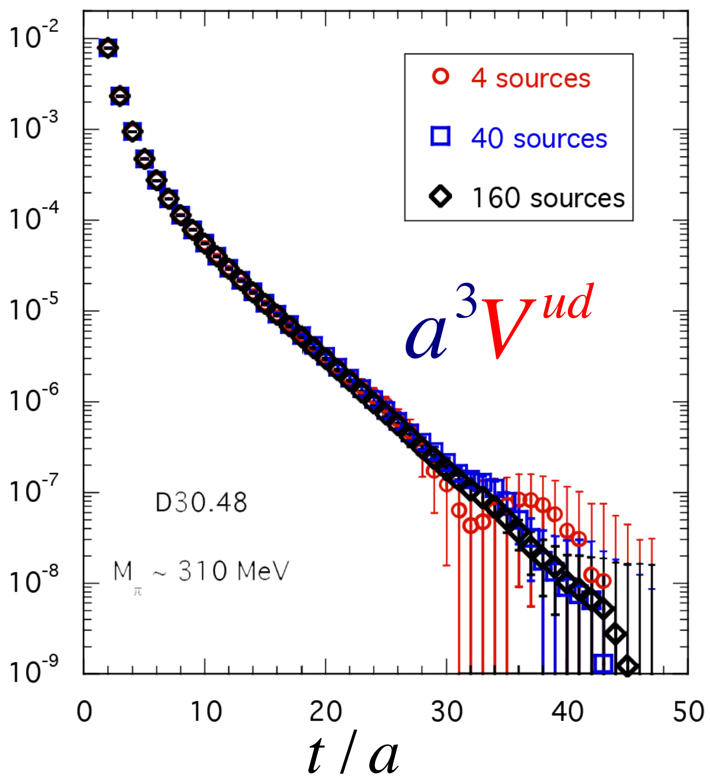
OS for *s* and *c* valence quarks

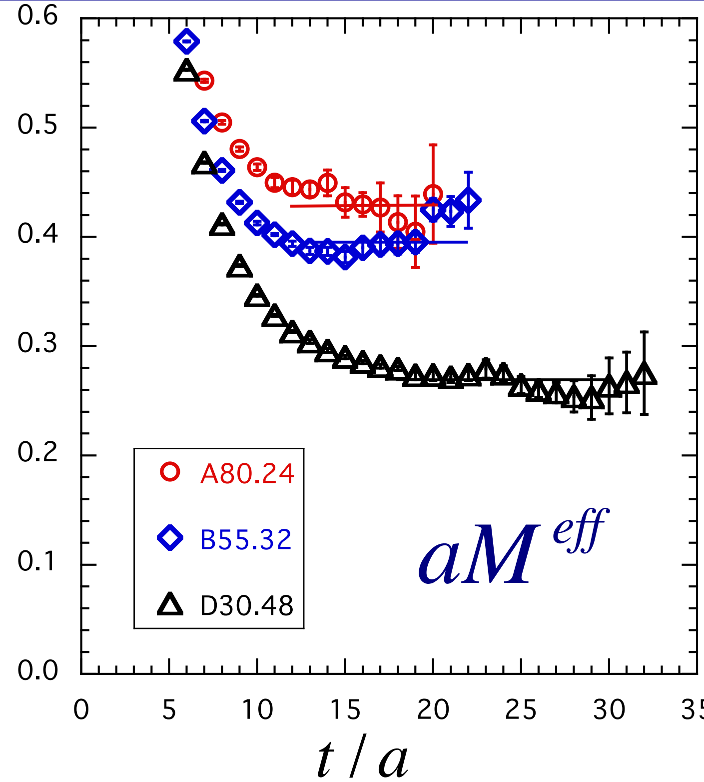
Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

Light quark contribution



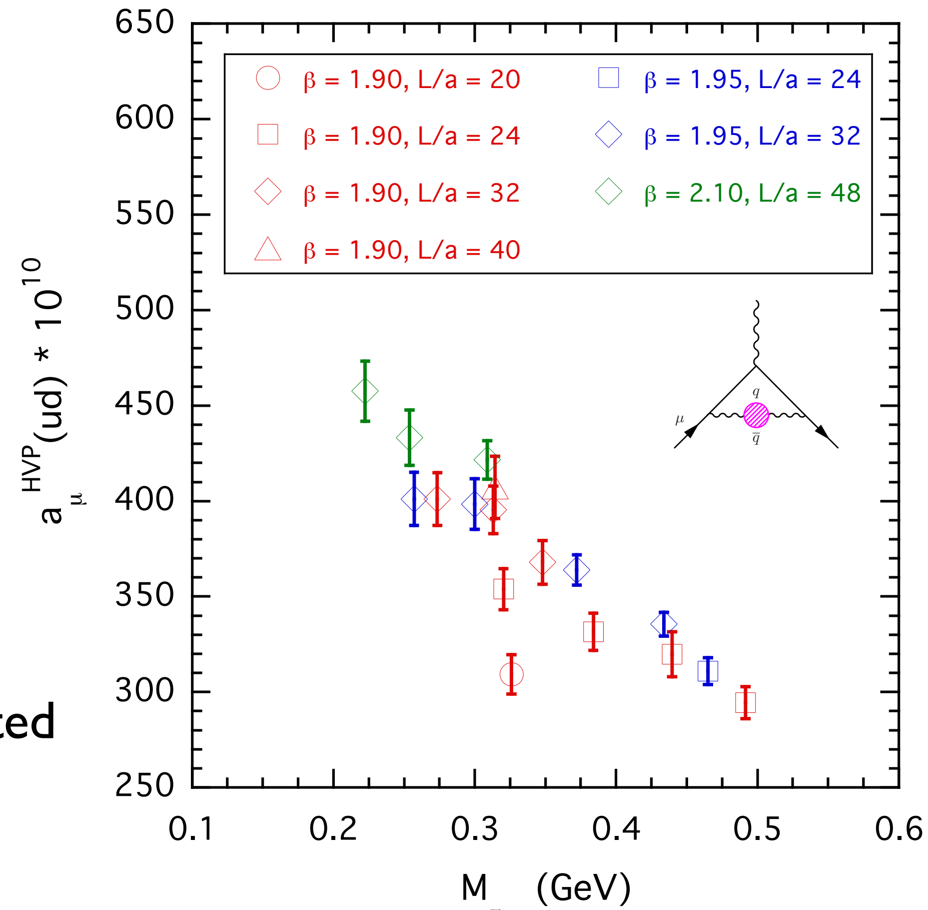


StN: $\propto e^{-(M_\rho - M_\pi)t}$

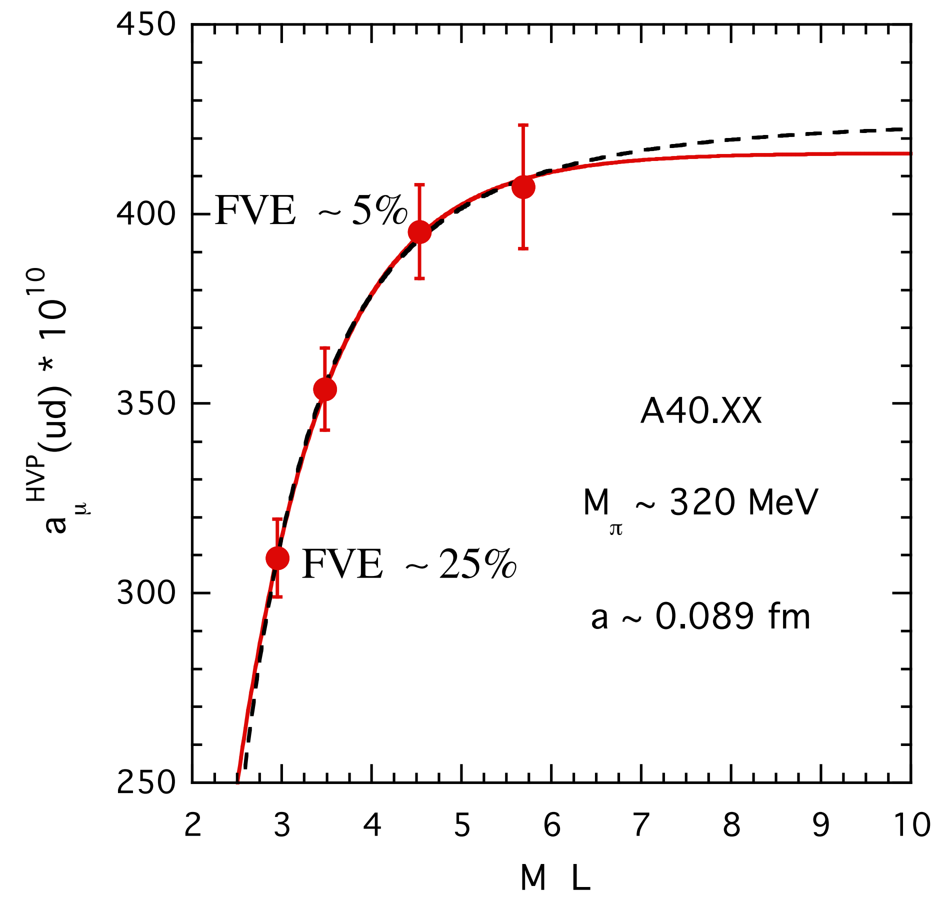
G. Parisi, 1984;
G. P. Lepage, 1989

160 stoch. sources / gauge conf.

DG *et al.*, 2018
PRD98(2018)114504



quark-connected
terms only



Correlator representation

$$V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)$$

low and intermediate time distances

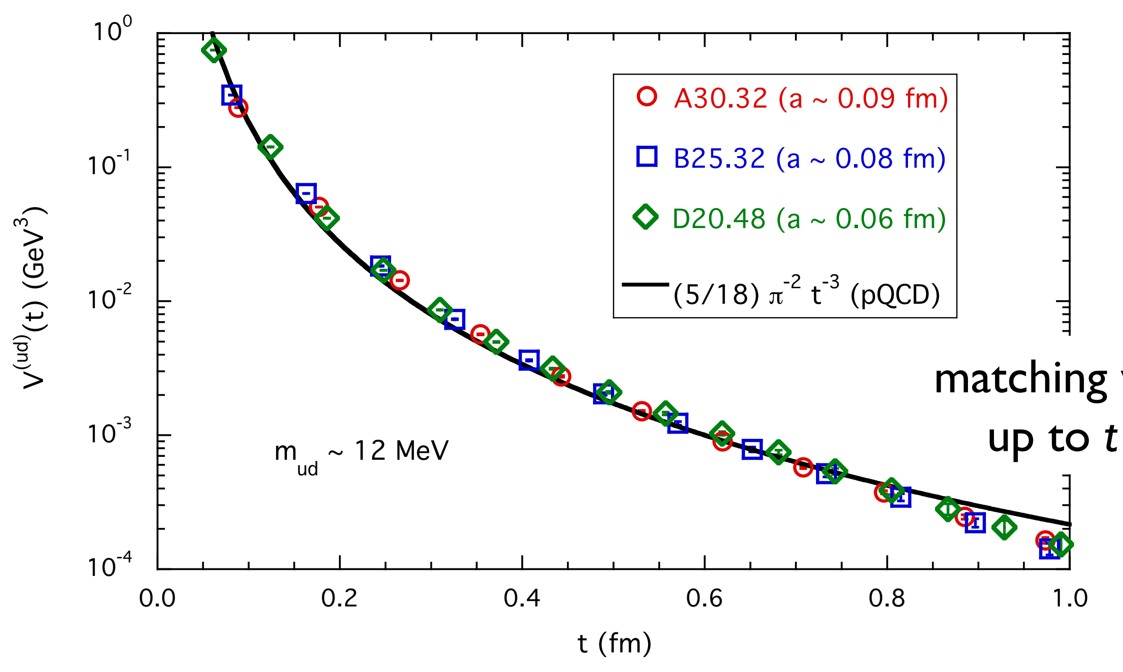
$$V_{dual}(t) \equiv \frac{1}{24\pi^2}\int_{s_{dual}}^{\infty} ds\sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

$$s_{dual} = \left(M_\rho + E_{dual}\right)^2 \qquad R_{dual} = 1 + O\left(\frac{m_{ud}^4}{s_{dual}^2}\right) + O(\alpha_s) + O(a^2)$$

$$V_{dual}(t) = \frac{5}{18\pi^2}\frac{R_{dual}}{t^3} e^{-(M_\rho + E_{dual})t}\left[1 + \left(M_\rho + E_{dual}\right)t + \frac{1}{2}\left(M_\rho + E_{dual}\right)^2 t^2\right]$$

quark-hadron duality *à la SVZ*

SVZ, 1979



matching with pQCD up to $t \sim 0.8$ fm

long time distances

$$V_{\pi\pi}(t) = \sum_n \nu_n |A_n|^2 e^{-\omega_n t}$$

M. Lüscher 1991

$$\omega_n = 2\sqrt{M_\pi^2 + k_n^2}$$

↘ Lüscher condition

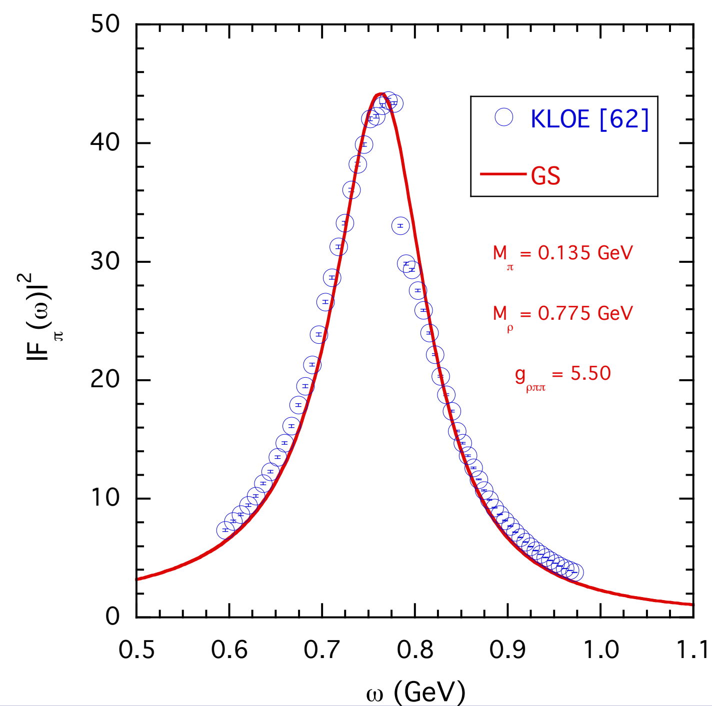
L. Lellouch and M. Lüscher, 2001

H.B. Meyer, 2011

$$|A_n|^2 \rightarrow |F_\pi(\omega_n)|^2$$

Gounaris-Sakurai parameterization

M_ρ, $g_{\rho\pi\pi}$ GS, 1968

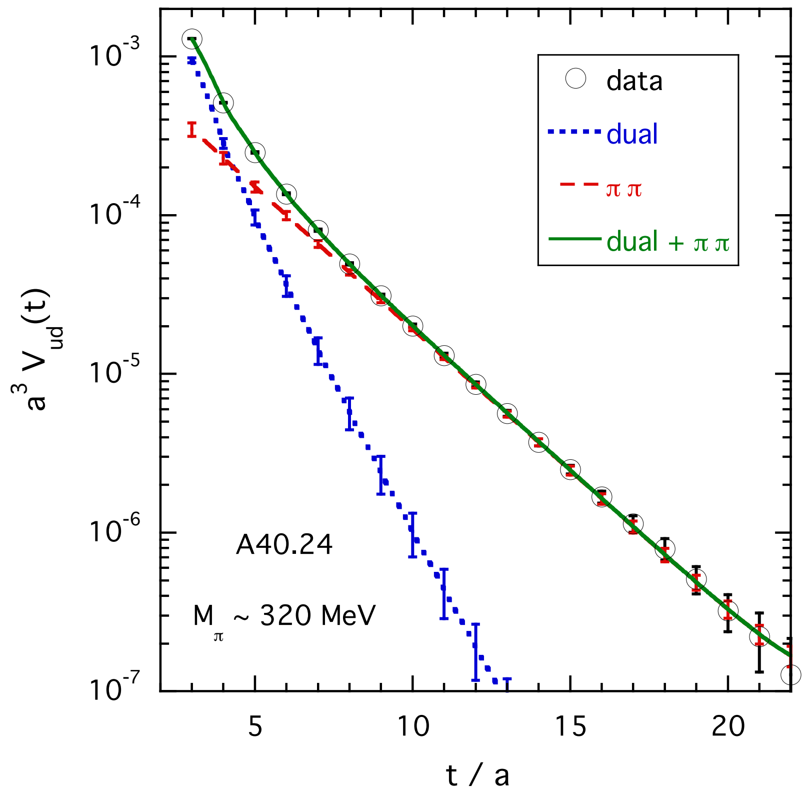


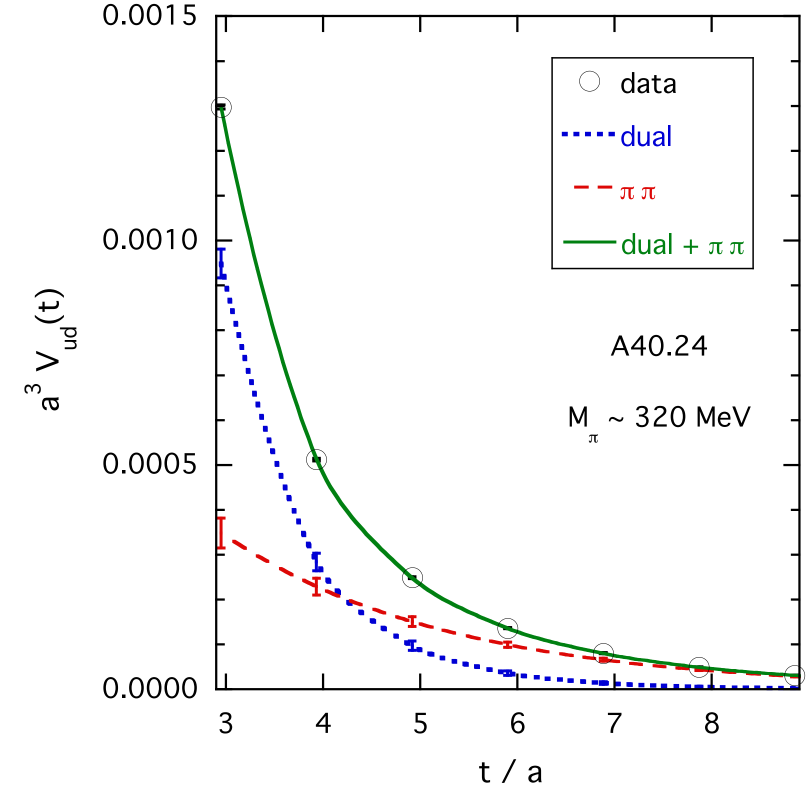
Subtraction of FVEs

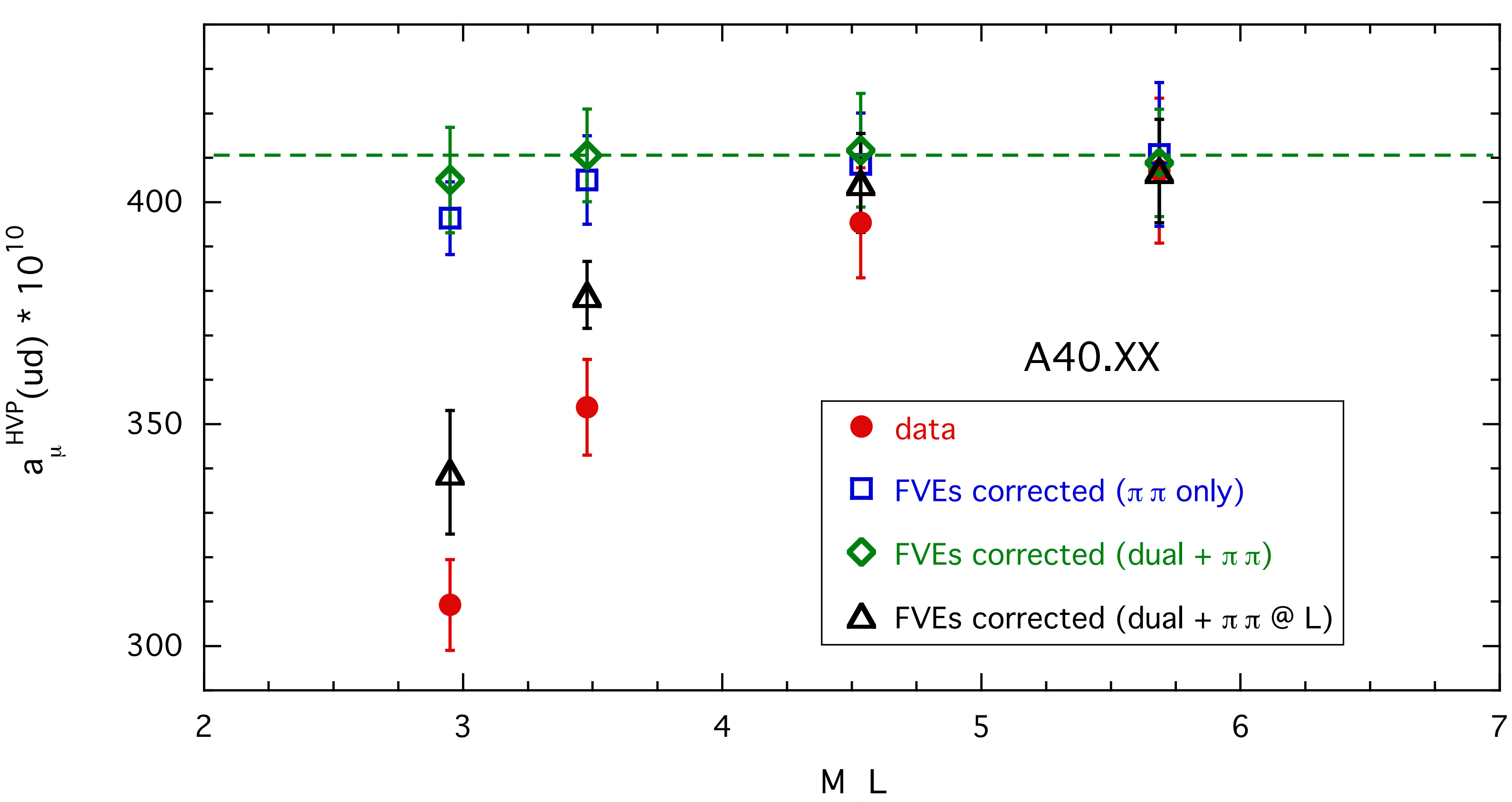
Accurate reproduction
for all the ETMC ensembles
$t \geq 0.2$ fm

$$R_{dual}, \frac{E_{dual}}{M_\pi}, g_{\rho\pi\pi}, \frac{M_\rho}{M_\pi}$$

π-π : 4 energy levels







$$a_\mu^{\rm HVP}(\infty) = a_\mu^{\rm HVP}(L) + \Delta_{\rm FVE} a_\mu^{\rm HVP}$$

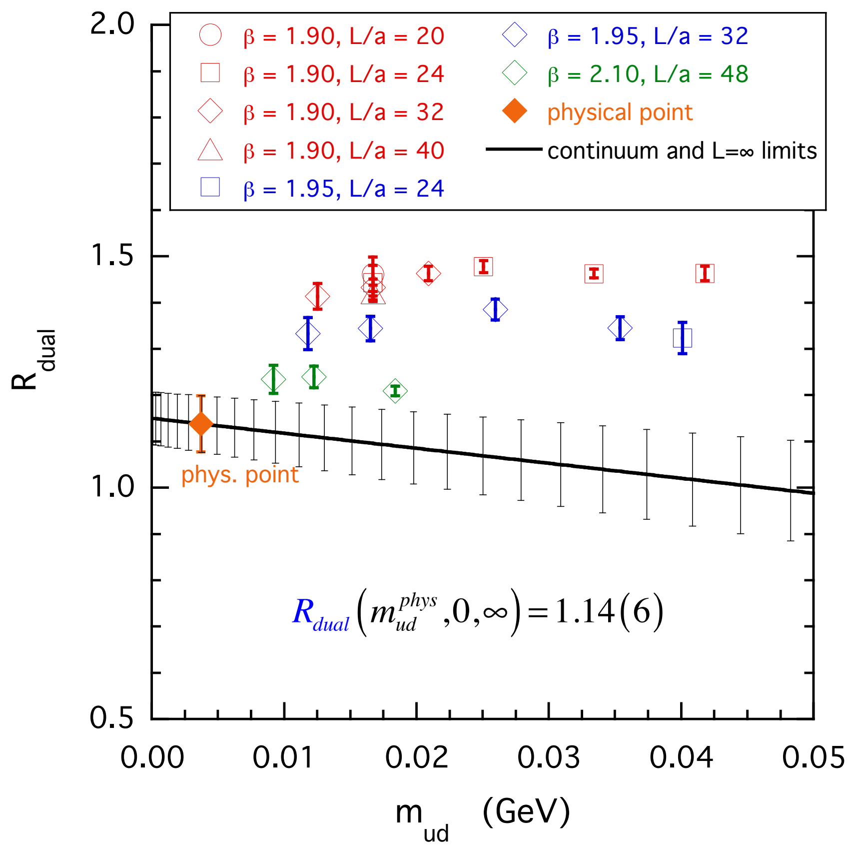
infinite-volume limit

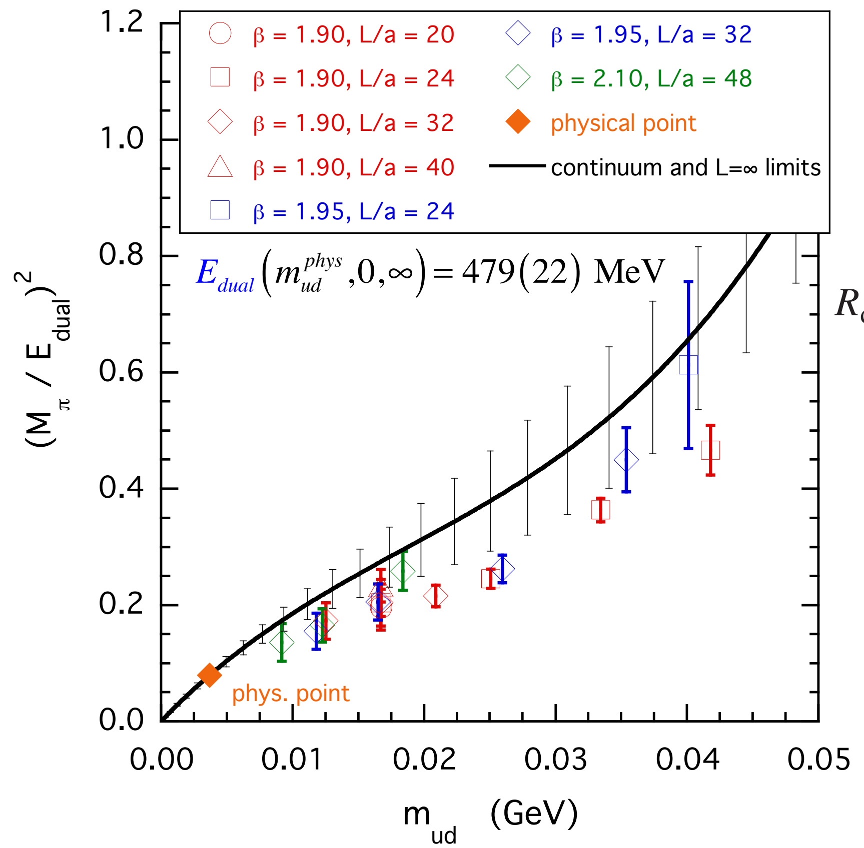
$$V_{dual}^{\infty}(t): \; R_{dual}^{\infty} \;\; M_\rho^{\infty} \;\; E_{dual}^{\infty}$$

$$V_{\pi\pi}^{\infty}(t) = \frac{1}{48\pi^2} \int_{2M_\pi^\infty}^{\infty} d\omega \; \omega^2 \left[1 - \frac{(2M_\pi^\infty)^2}{\omega^2} \right]^{3/2} \left| F_\pi^\infty(\omega) \right|^2 e^{-\omega t}$$

H. B. Meyer, 2011

Parameters

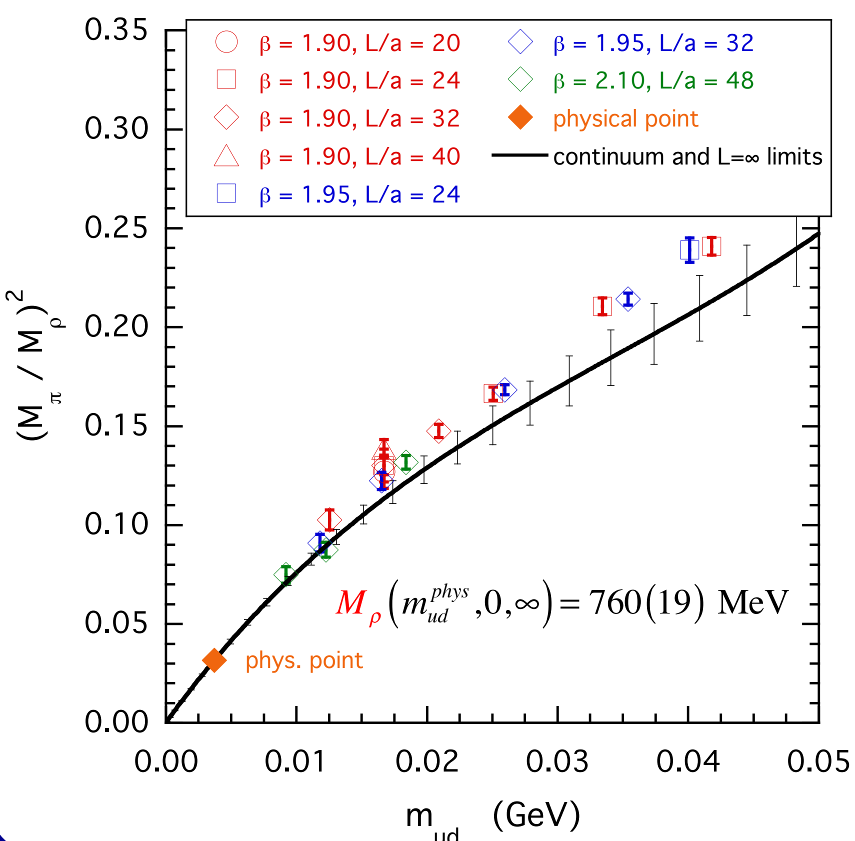


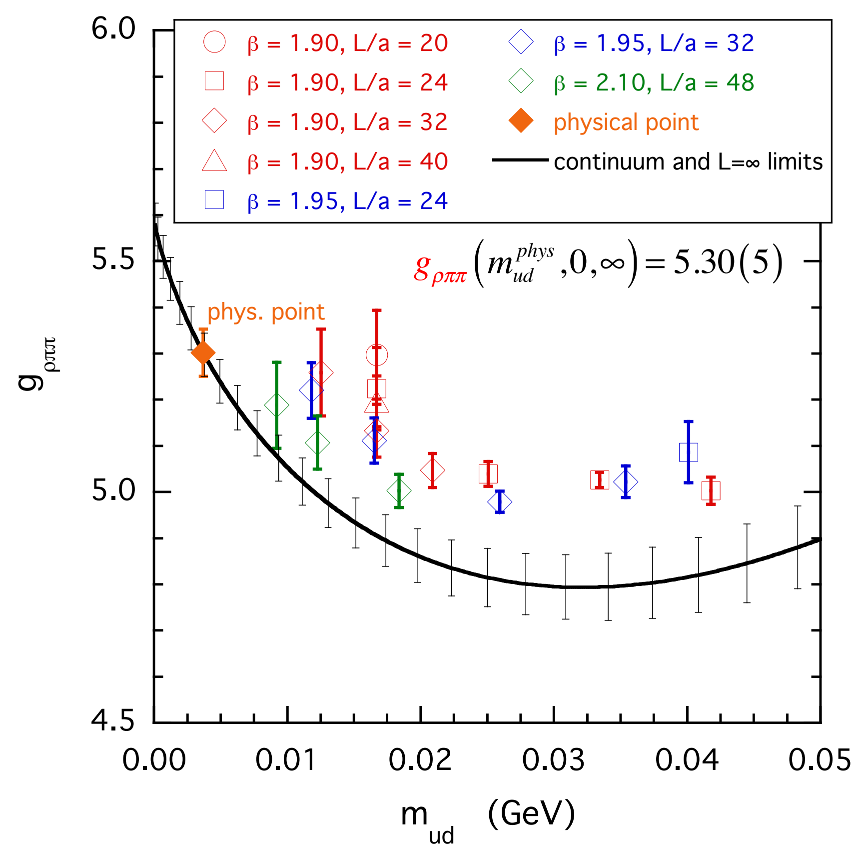


good control of FVEs

$$R_{\rm dual}(m_{ud}, a^2, L) = R_0[1 + R_1 m_{ud} + R_a a^2 + R_{am} a^2 m_{ud}] \times \left[1 + R_{\rm FVE}\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

$$\frac{M_\pi^2}{E_{\rm dual}^2}(m_{ud}, a^2, L) = E_0 m_{ud}[1 + E_1 m_{ud} + \xi\log(\xi) + E_2 m_{ud}^2 + E_a a^2]$$



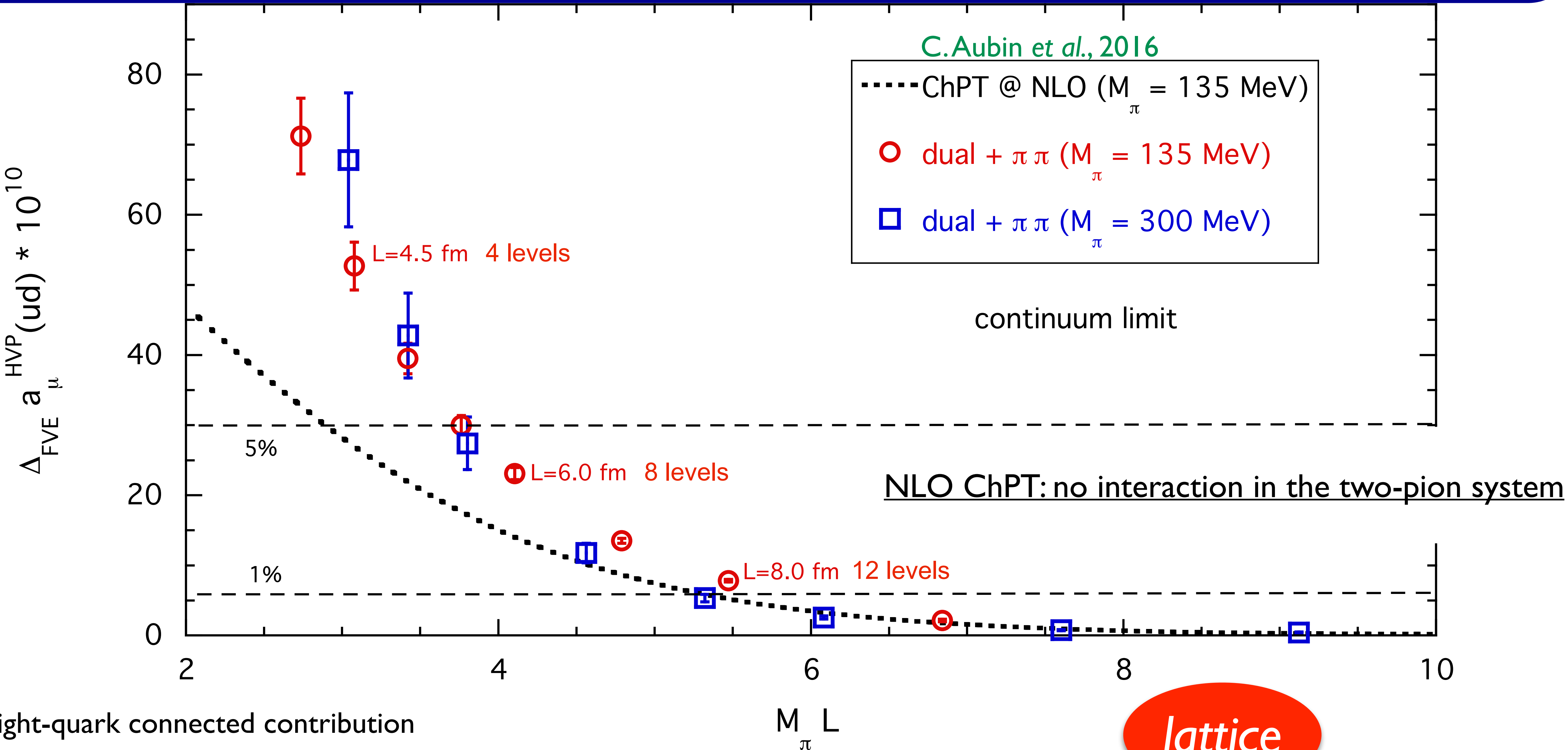


$$\frac{M_\pi^2}{M_\rho^2}(m_{ud}, a^2, L) = V_0 m_{ud}[1 + V_1 m_{ud} + \xi\log(\xi) + V_2 m_{ud}^2 + V_a a^2]$$

$$g_{\rho\pi\pi}(m_{ud}, a^2, L) = g_0[1 + g_1 m_{ud} + 2\xi\log(\xi) + g_a a^2] \times \left[1 + g_{\rm FVE}\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

$$M_\pi^2(m_{ud}, a^2, L) = 2B_0 m_{ud}[1 + P_1 m_{ud} + \xi\log(\xi) + P_2 m_{ud}^2 + P_a a^2] \cdot \left[1 + P_{\rm FVE}\xi \frac{e^{-ML}}{(ML)^{3/2}}\right]$$

FVEs correction @ $a^2 \to 0$



light-quark connected contribution

$M_\pi^{phys} L$	L (fm)	$\Delta_{\rm FVE}^{\rm lat}(L)/\Delta_{\rm FVE}^{\rm ChPT,NLO}(L)$
2.7	4.0	2.17 (17)
3.1	4.5	1.95 (13)
3.4	5.0	1.79 (10)
3.8	5.5	1.68 (8)
4.1	6.0	1.60 (6)
4.8	7.0	1.48 (4)
5.5	8.0	1.37 (5)

lattice

$$\frac{\Delta_{\rm FVE}^{\rm lat}}{\Delta_{\rm FVE}^{\rm ChPT,NLO}}(L = 5 \div 6\ {\rm fm}) \simeq 1.7(1)$$

$(5.4\ {\rm fm} \to 10.8\ {\rm fm})$

$$\frac{\Delta_{\rm FVE}^{\rm lat}}{\Delta_{\rm FVE}^{\rm ChPT,NLO}} \simeq 1.74(71)$$

PACS19

C.Aubin et al., 2019

NNLO ChPT

$$\frac{\Delta_{\rm FVE}^{\rm ChPT,NNLO}}{\Delta_{\rm FVE}^{\rm ChPT,NLO}}(L = 5 \div 6\ {\rm fm}) \simeq 1.4(2)$$

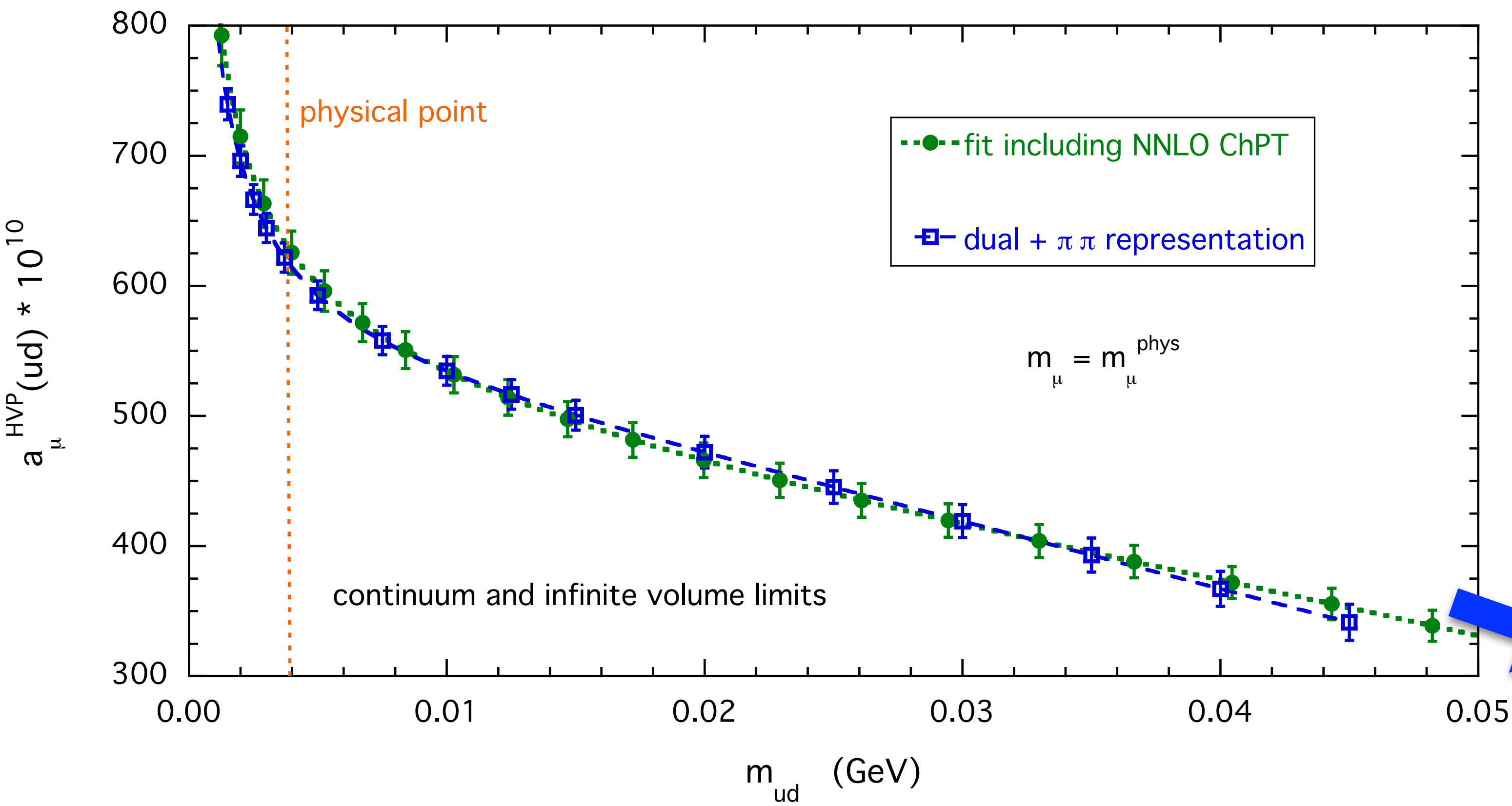
Extrapolation to M_π^{phys}

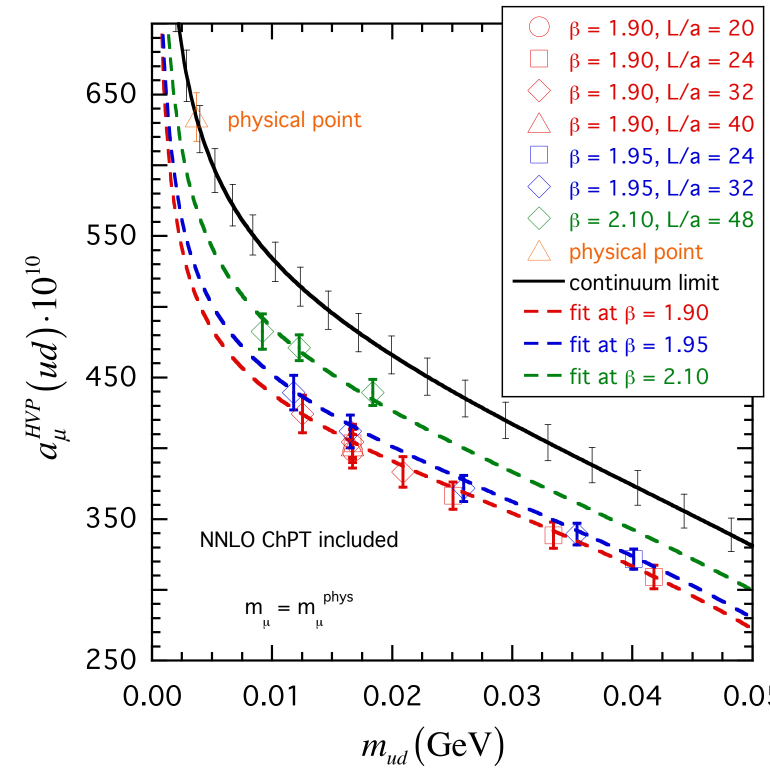
$a_\mu^{\mathrm{HVP}}(ud)$ diverges in the chiral limit ➡ $a_\mu^{\mathrm{HVP}}(ud) = \left\{ \left[a_\mu^{\mathrm{HVP}} \right]^{NLO} + \left[a_\mu^{\mathrm{HVP}} \right]^{NNLO}_{L_9, C_{93}} + A_0 + A_1 m_{ud} \right\} \left(1 + D_0 a^2 + D_1 a^2 m_{ud} \right)$

$\ln(m_{ud})$ LECs-independent

E. Golowich and J. Kambor, 1995; G. Amoros *et al.*, 2000
J. Bijnens and J. Refelors, 2016; M. Golterman *et al.*, 2017

Using the (dual + ***π-π***) analytic representation





The blue points do not contain chiral logs explicitly

Using the analytic representation $a_\mu^{\mathrm{HVP}}(ud)$ does not depend on the absolute scale setting

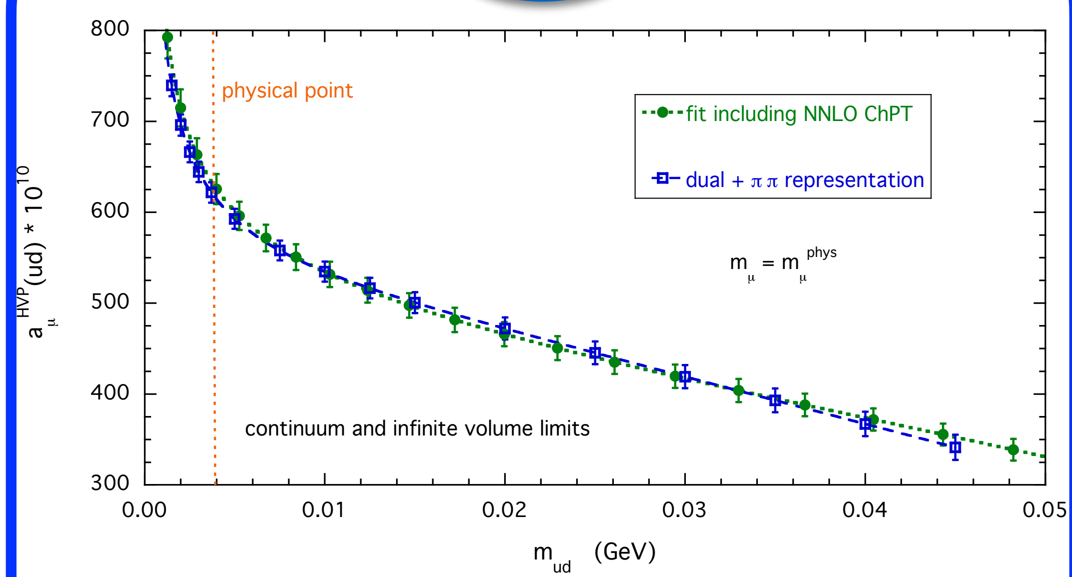
$$V_{dual+\pi\pi}(t) = M_\pi^3 \widetilde{V}\left(\tau_\pi; R_{dual}, \frac{E_{dual}}{M_\pi}, \frac{M_\rho}{M_\pi}, g_{\rho\pi\pi} \right)$$

$\tau_\pi = M_\pi t$ dimensionless quantities

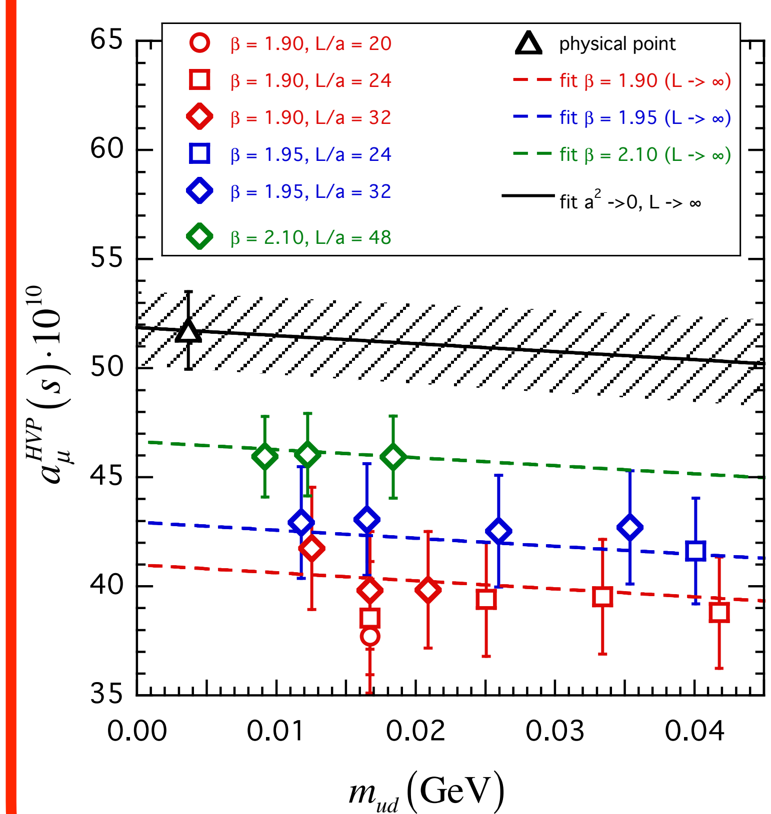
$$a_\ell^{\mathrm{HVP}}(ud) = 4\alpha_{em}^2 \int_0^\infty d\tau_\pi \widetilde{K}_\ell(\tau_\pi) \widetilde{V}(\tau_\pi)$$

udsc-quark contributions

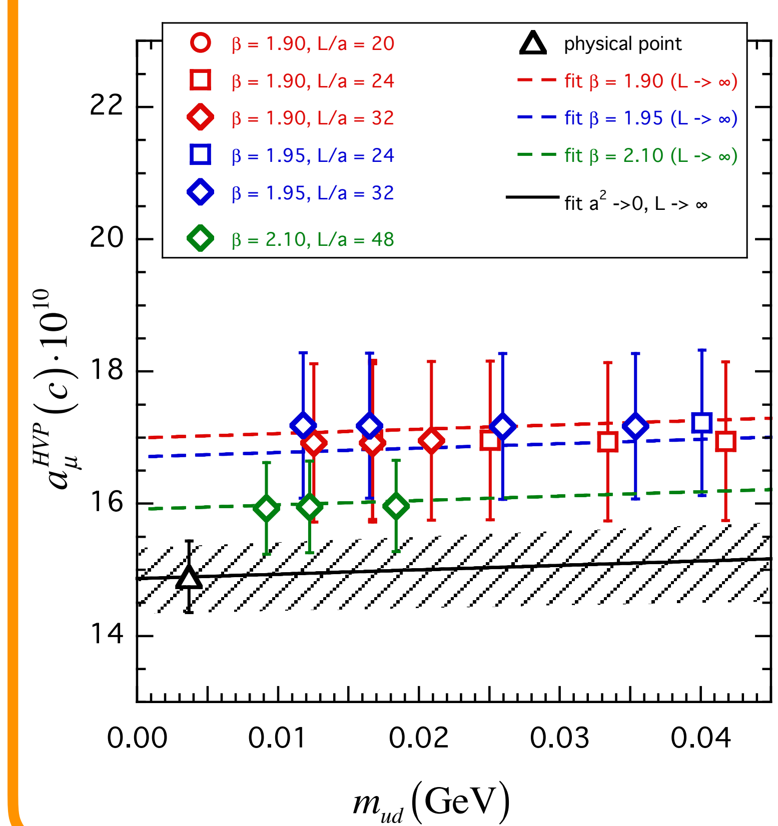
ud



s



c



$$a_\mu^{\rm HVP}(ud) = 629.1(11.5)(7.5)[13.7] \cdot 10^{-10}$$

$$a_\mu^{\rm HVP}(s) = 53.1(2.5) \cdot 10^{-10}$$

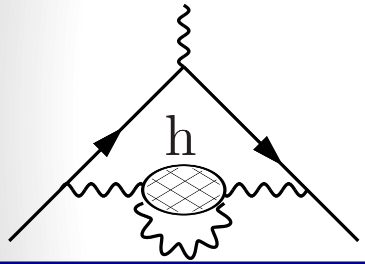
$$a_\mu^{\rm HVP}(c) = 14.75(56) \cdot 10^{-10}$$

DG *et al.*, 2018 — PRD98(2018)114504

DG and S. Simula, 2019 — ArXiv:1910.03874

DG *et al.*, 2017 — JHEP1710(2017)157

quark-connected terms only



LIB corrections

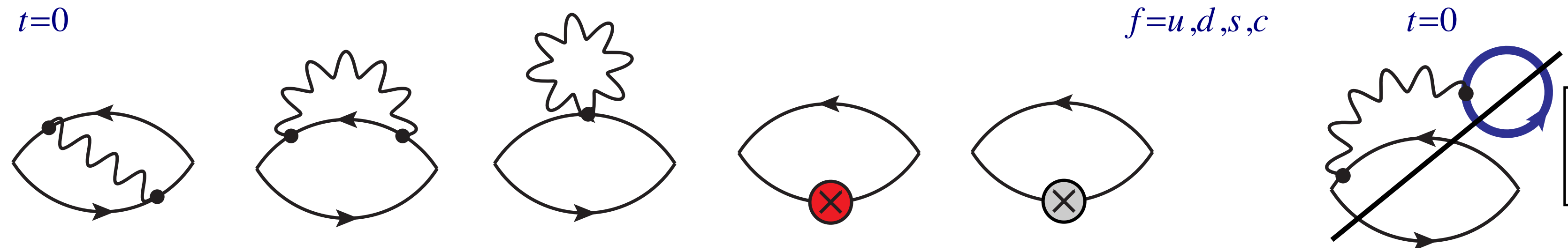
quark-connected terms only

$$\delta a_\ell^{\mathrm{HVP}} = \delta a_\ell^{\mathrm{HVP}}(QCD) + \delta a_\ell^{\mathrm{HVP}}(QED)$$

photon zero-mode: QED_L
M. Hayakawa and S. Uno, 2008

$$\delta a_\ell^{\mathrm{HVP}}(QCD) = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \sum_{t=0}^{\infty} \widetilde{f}_\ell(t)\, \delta V_f^{QCD}(t) \qquad \delta a_\ell^{\mathrm{HVP}}(QED) = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \sum_{t=0}^{\infty} \widetilde{f}_\ell(t)\, \delta V_f^{QED}(t)$$

RM123 method
G. M. de Divitiis *et al.*, 2012; 2013

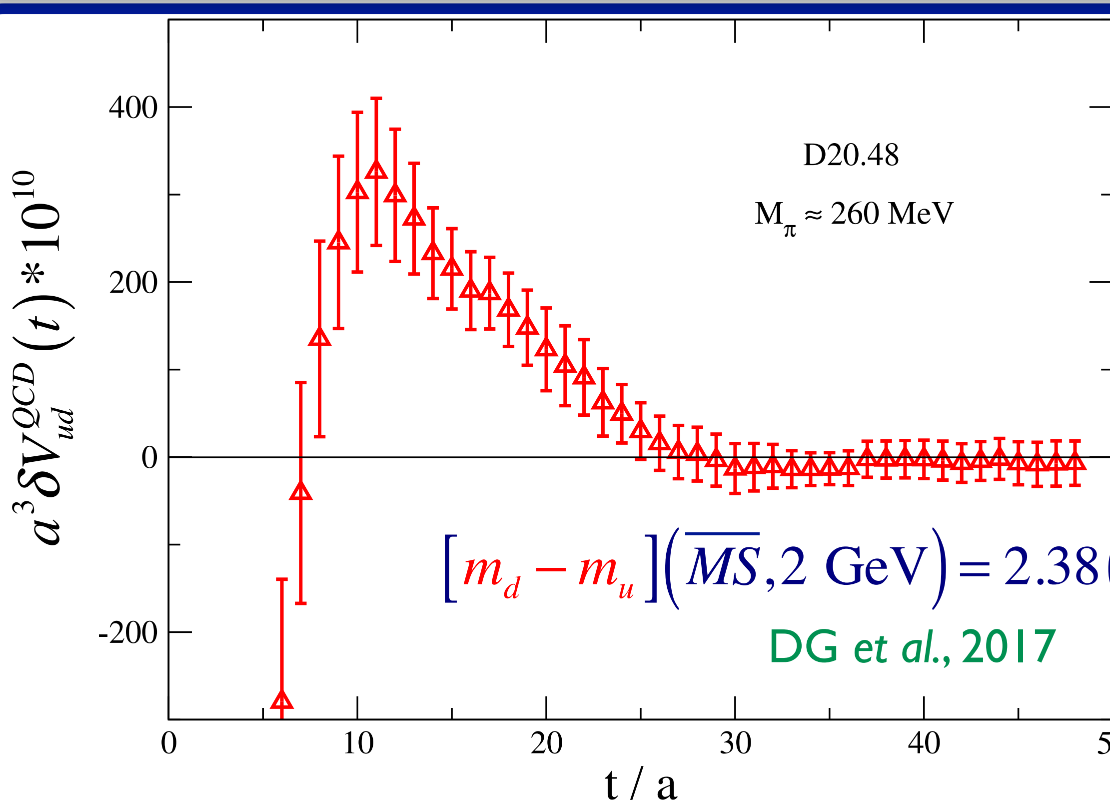


qQED approximation

isoQCD/IB separation: consistent prescriptions

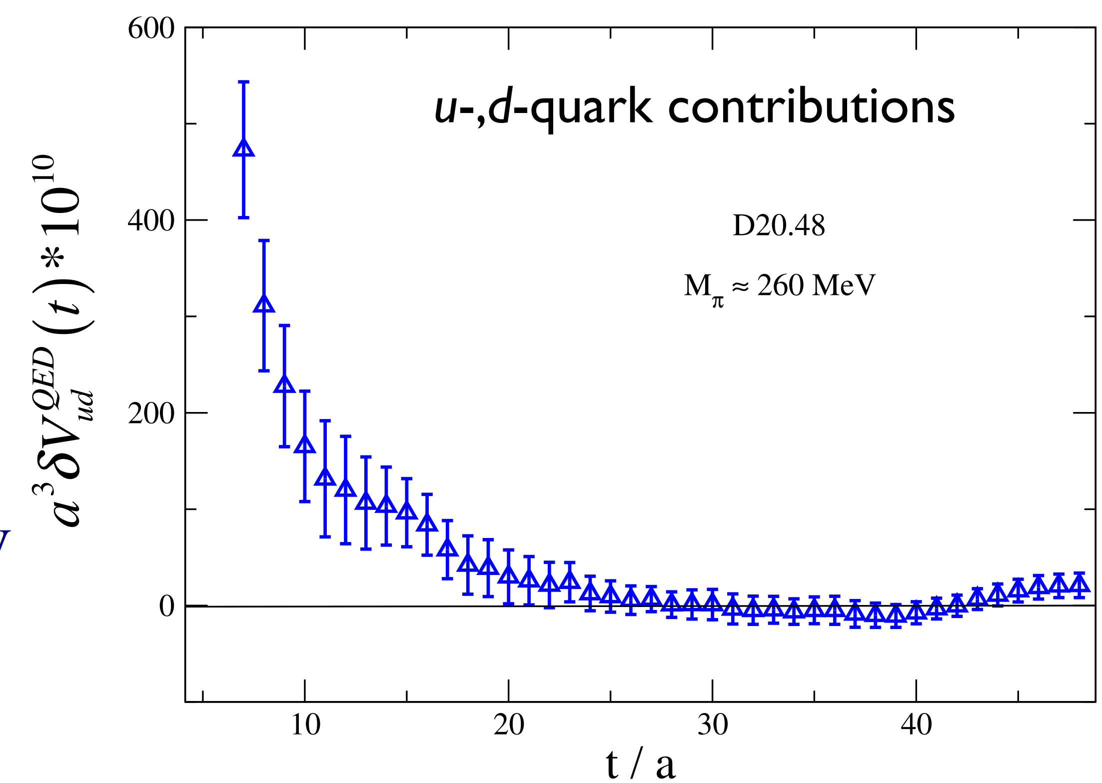
QCD/QED separation is scheme and scale dependent → $m_f\left(\overline{MS}, 2\ \mathrm{GeV}\right) = m_f^0\left(\overline{MS}, 2\ \mathrm{GeV}\right)$
J. Gasser *et al.*, 2003

What is QCD in the full QCD+QED theory? see M. Di Carlo *et al.*, 2019

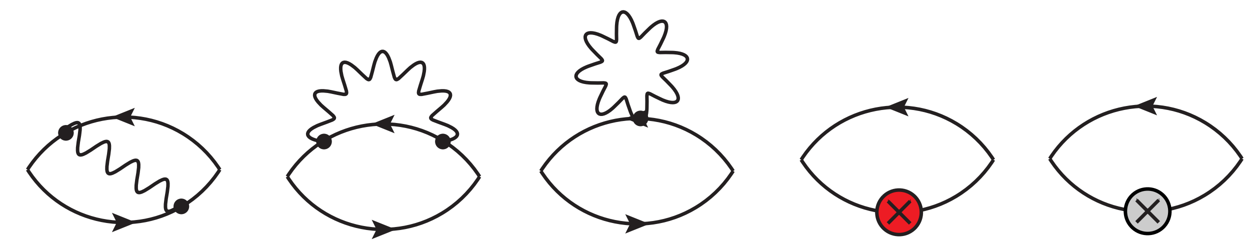


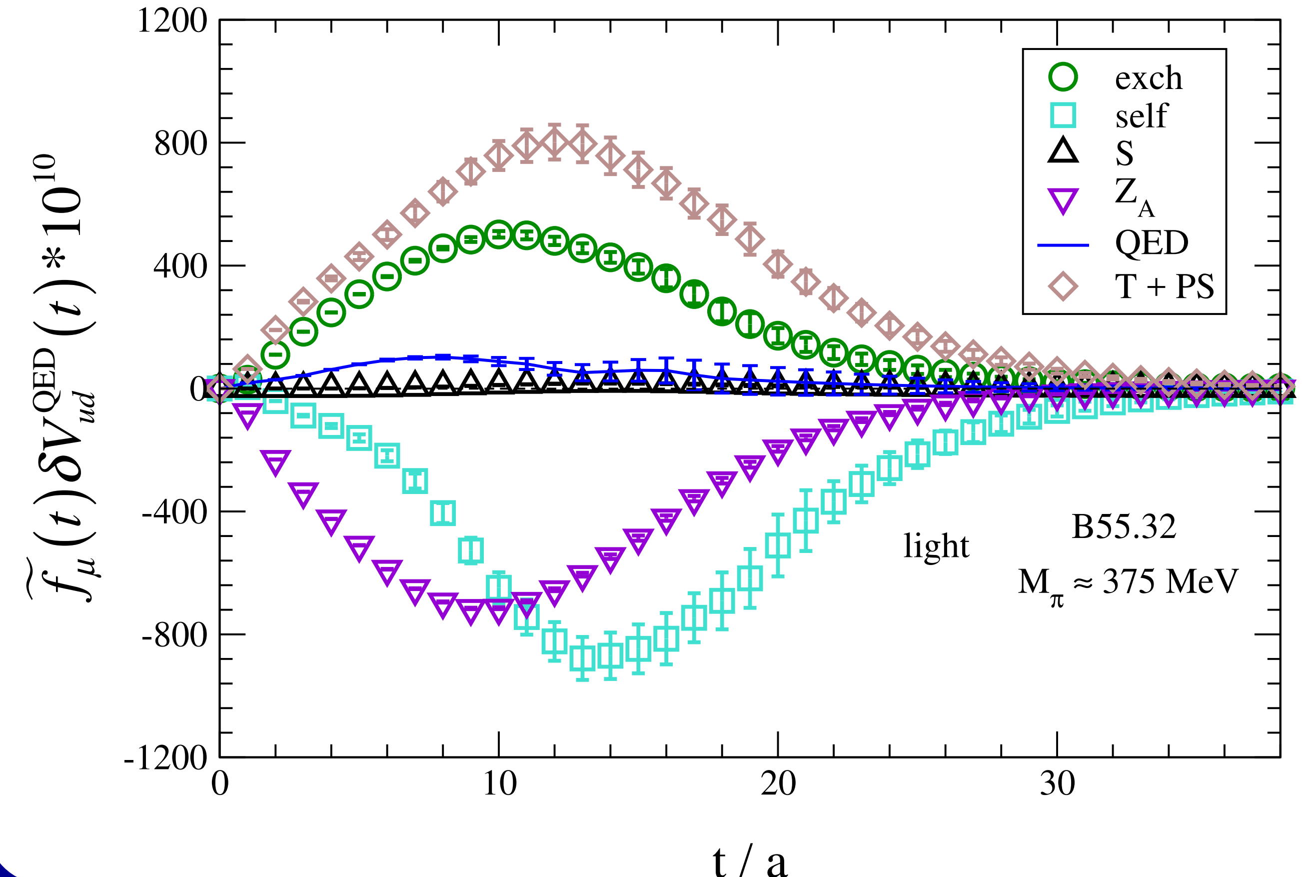
$$[m_d - m_u]\left(\overline{MS}, 2\ \mathrm{GeV}\right) = 2.38(0.18)\ \mathrm{MeV}$$

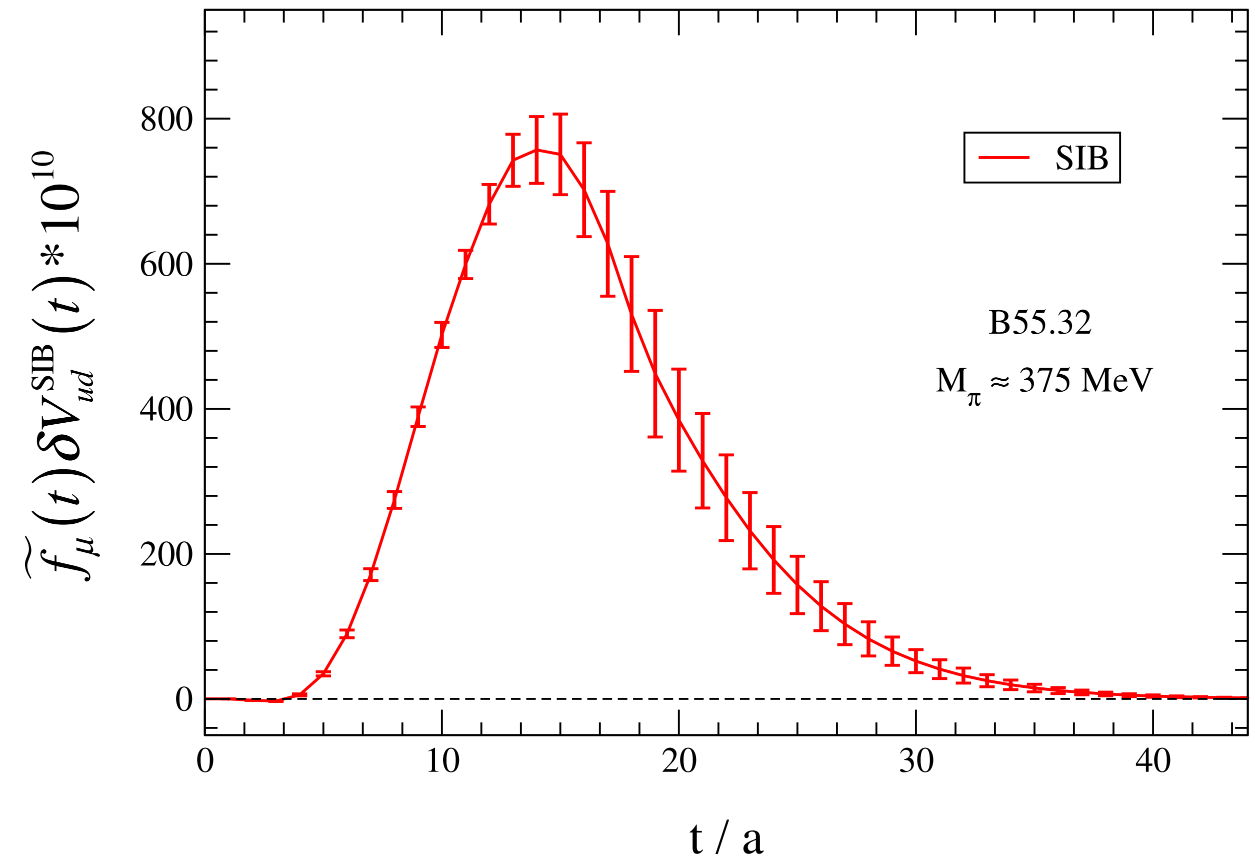
DG *et al.*, 2017



LIB corr.:







$$Z_A = Z_A^{(0)} \left(1 + \frac{\alpha_{em}}{4\pi} \delta Z_A^{QED} Z_A^{fact} \right) + O\left(\alpha_{em}^m \alpha_s^n\right)$$

$$\delta Z_A^{QED} = -15.7963\, q_f^2$$

perturbative estimate at LO

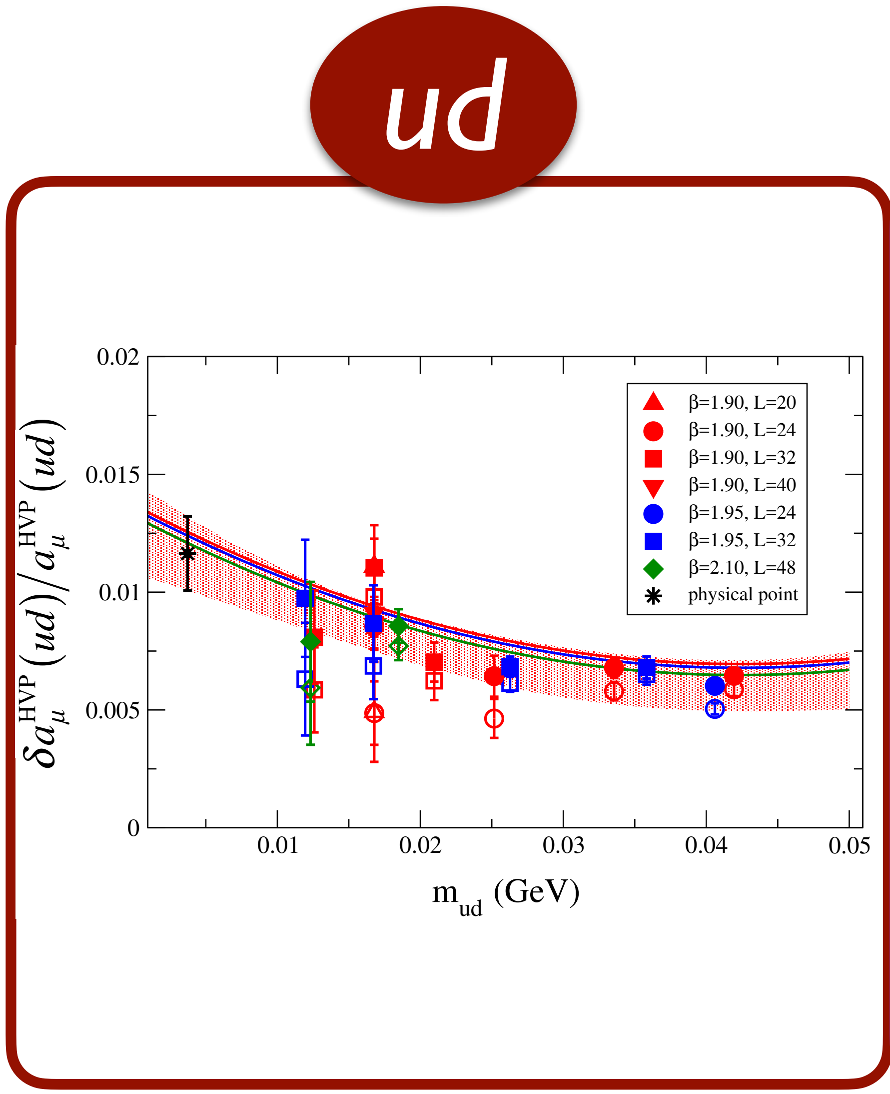
G. Martinelli and Y.-C. Zhang, 1982

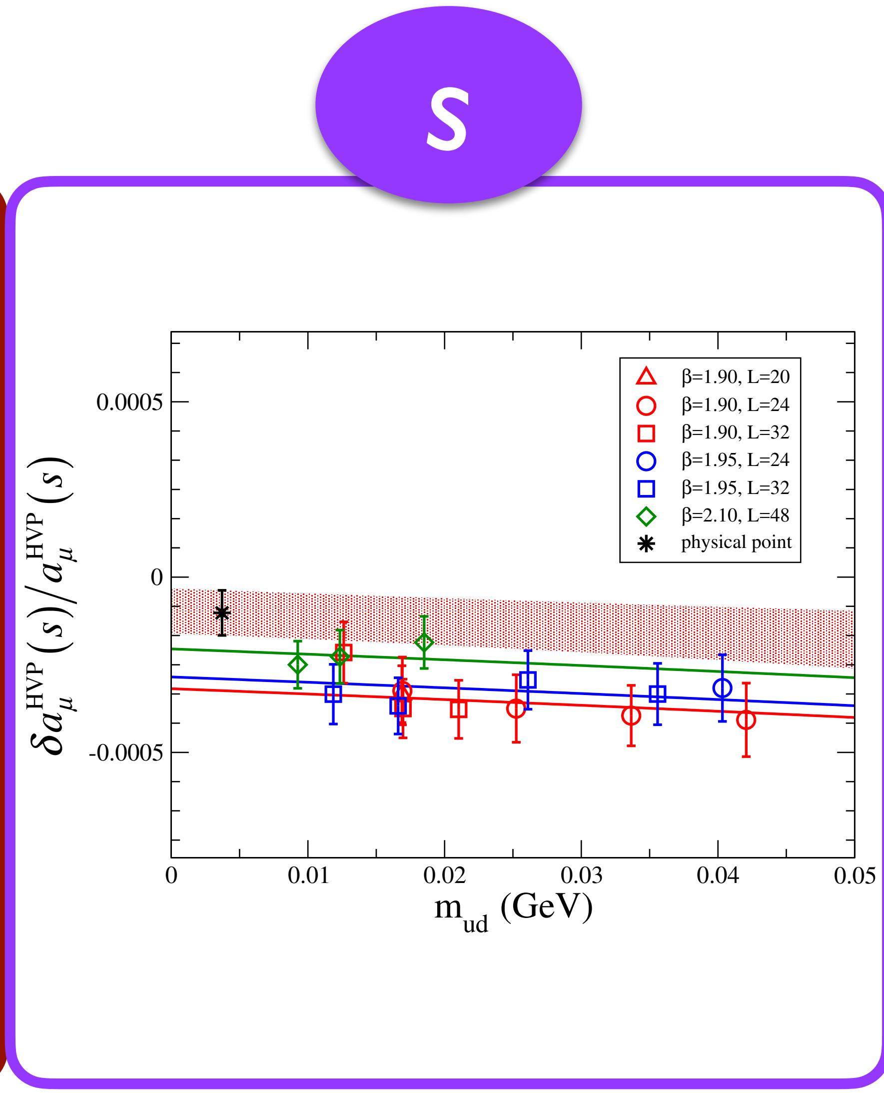
$$\delta V_f^{Z_A}(t) = \frac{\alpha_{em}}{4\pi} \delta Z_A^{QED} Z_A^{fact} \; V^f(t)$$

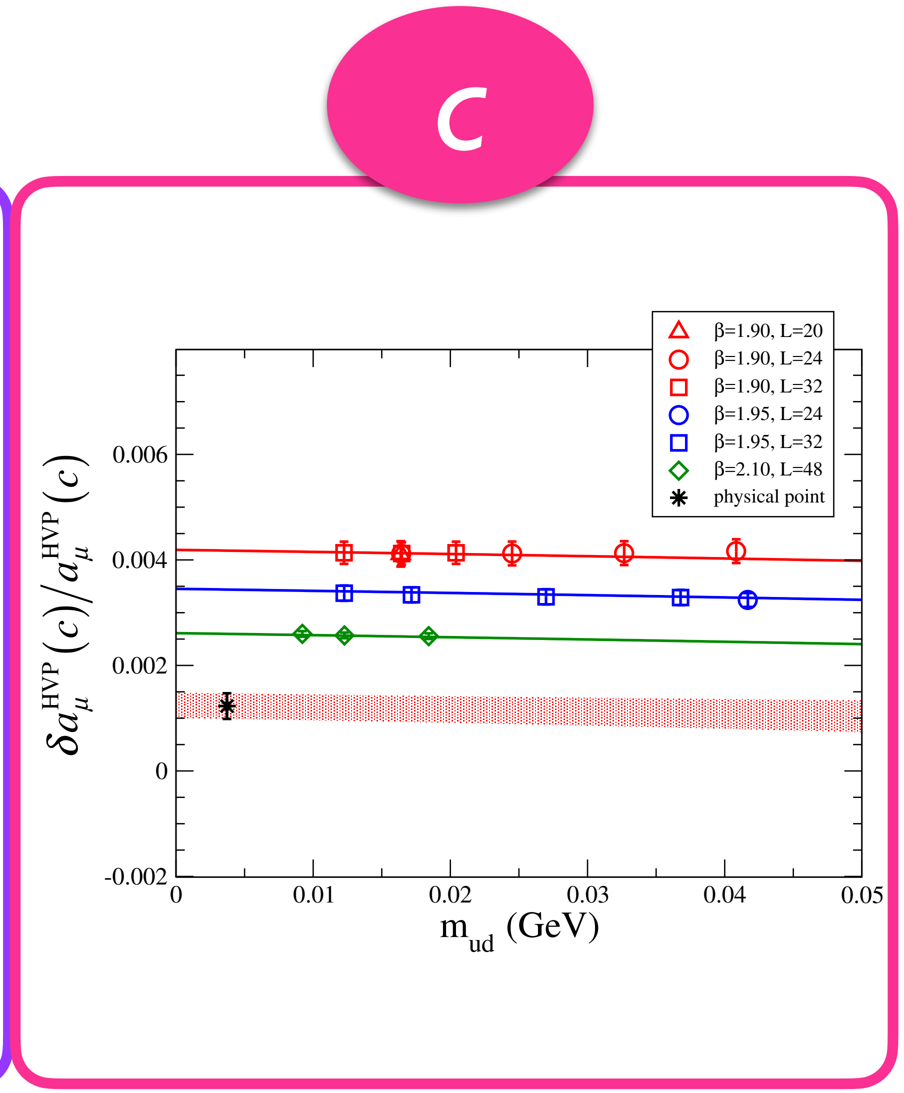
β	Z_m^{fact} (M1)	Z_A^{fact} (M1)	Z_m^{fact} (M2)	Z_A^{fact} (M2)
1.90	1.629 (41)	0.859 (15)	1.637 (14)	0.990 (9)
1.95	1.514 (33)	0.873 (13)	1.585 (12)	0.980 (8)
2.10	1.459 (17)	0.909 (6)	1.462 (6)	0.958 (3)

RI'-MOM @ $O\left(\alpha_{em}\alpha_s^n\right)$ DG *et al.*, 2019; M. Di Carlo *et al.*, 2019

LIB corr.: *udsc*-quark contr.







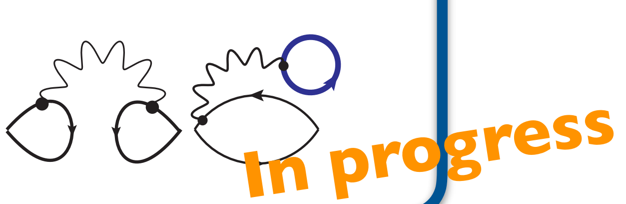
$\delta a_\mu^{\rm HVP}(ud) = 7.1(2.5)\cdot 10^{-10}$

$\delta a_\mu^{\rm HVP}(s) = -0.0053(33)\cdot 10^{-10}$

$\delta a_\mu^{\rm HVP}(c) = 0.0182(36)\cdot 10^{-10}$

DG *et al.*, 2019
PRD99(2019)114502

$$\delta a_\mu^{\rm HVP} = 7.1(2.6)(1.2)_{qQED+disc}\cdot 10^{-10}$$
$$= 7.1(2.9)\cdot 10^{-10}$$



a_e^{HVP} and a_τ^{HVP} : Lattice results

LO

ArXiv:1910.03874

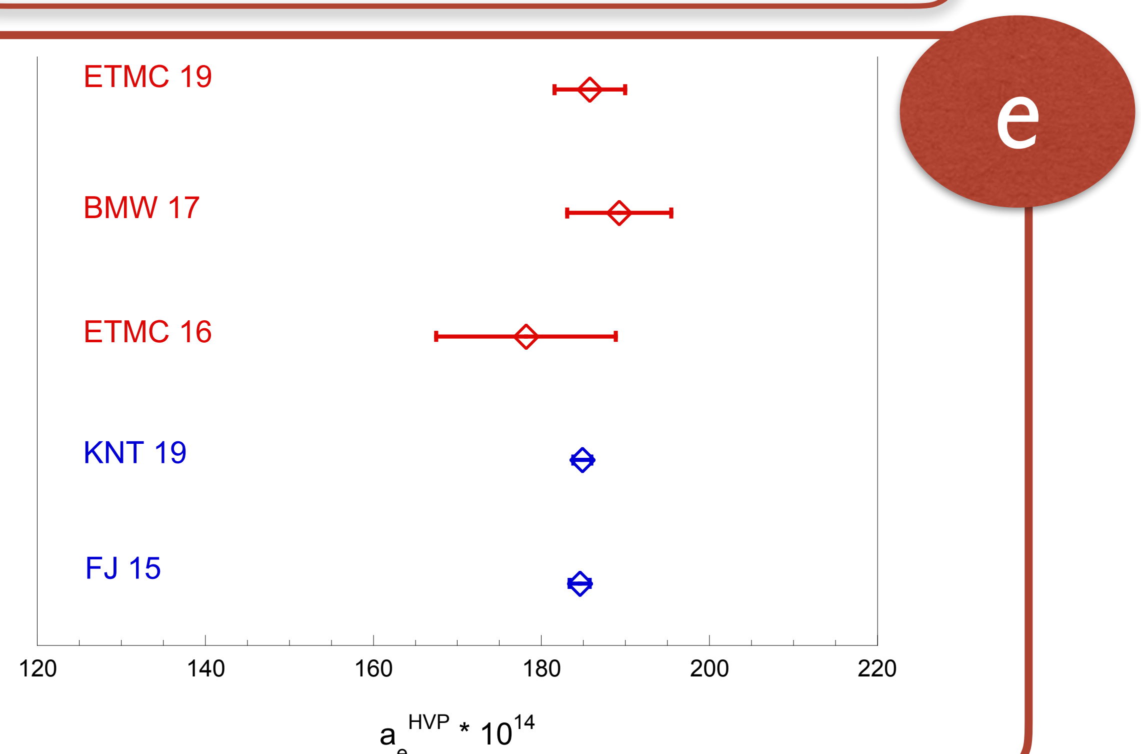
IB

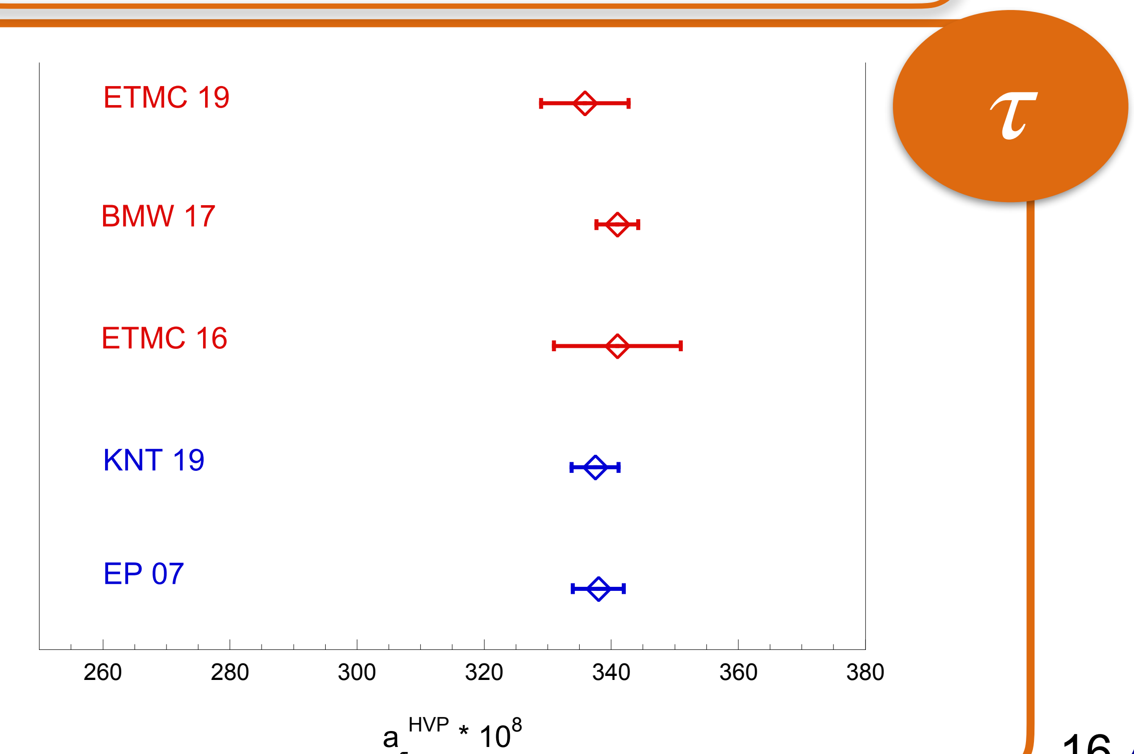
f	$a_e^{HVP}(f)\cdot 10^{14}$	$a_\tau^{HVP}(f)\cdot 10^{8}$
ud	170.7 (3.9)	273.3 (6.6)
s	13.5 (0.8)	36.2 (1.1)
c	3.5 (0.2)	25.8 (0.8)
disc BMW 17	−3.8 (0.4)	−2.4 (0.3)

f	$\delta a_e^{HVP}(f)\cdot 10^{14}$	$\delta a_\tau^{HVP}(f)\cdot 10^{8}$
ud	1.9 (0.8)	3.0 (1.1)
s	−0.002 (0.001)	0.001 (0.002)
c	0.004 (0.001)	0.032 (0.006)
total	1.9 (1.0)	3.0 (1.3)

$$a_e^{HVP} = 185.8(4.2)\cdot 10^{-14}$$

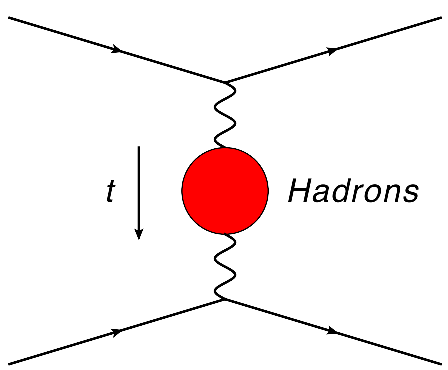
$$a_\tau^{HVP} = 335.9(6.9)\cdot 10^{-8}$$





MUonE experiment

MUonE



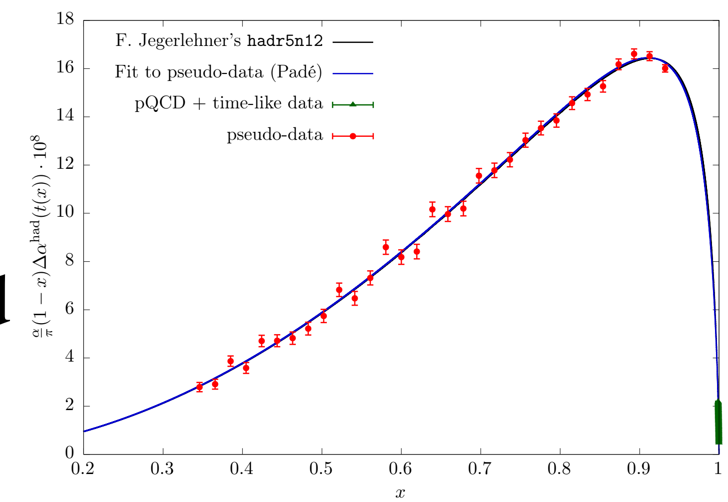
B. E. Lautrup *et al.*, 1972

$$a_\mu^{\rm HVP} = \frac{\alpha_{em}}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{em}^{\rm HVP}\left[t(x)\right]$$

$$\downarrow$$

$$\sigma(\mu e \to \mu e)$$

$$t(x) \equiv -\frac{x^2}{1-x} m_\mu^2$$



$x \in [0.93,\ 1]$ not experimentally reached

LQCD

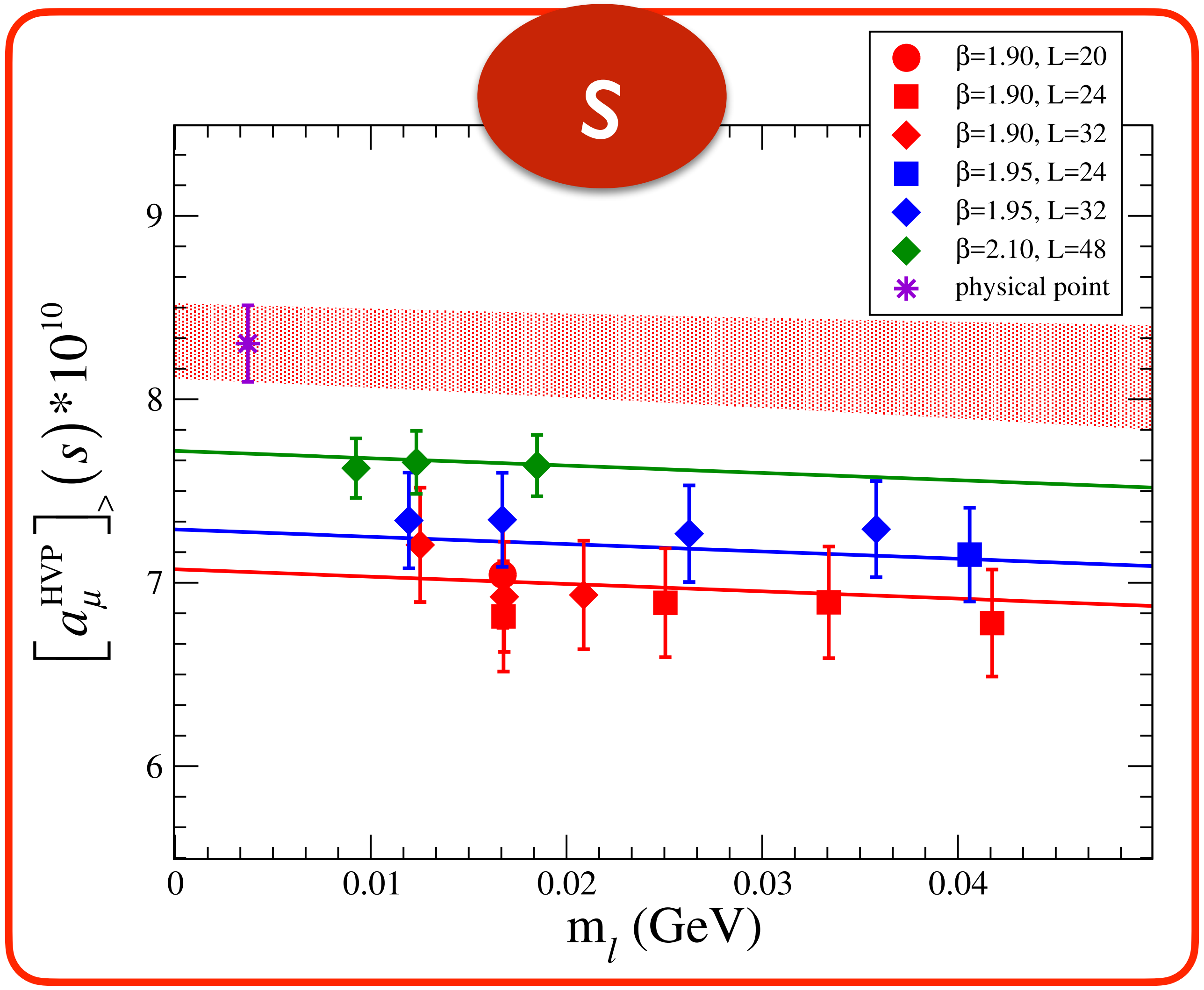
Using the (dual + π-π) repr.

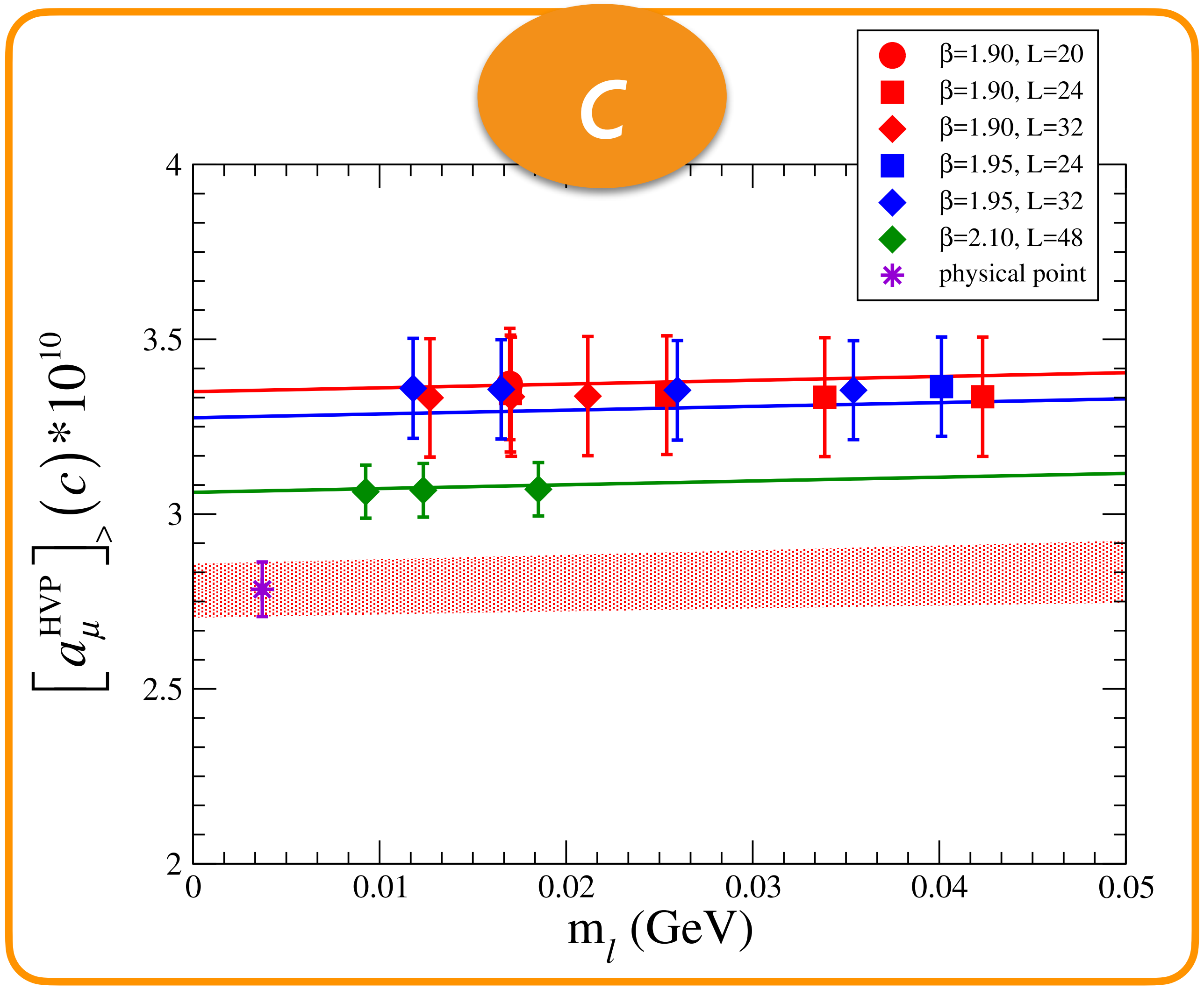
$$\left[a_\mu^{\rm HVP}\right]_> = 4\alpha_{em}^2 \int_0^\infty dt\ \left[\widetilde{f}_\mu(t)\right]_> V(t) \quad \rightarrow \quad \left[a_\mu^{\rm HVP}\right]_>(ud) = 81.2(1.7)\cdot 10^{-10}$$

quark-connected terms only

Uncertainty $\left(\simeq 2\cdot 10^{-10}\right)$ close to the experimental statistical target $(\simeq 0.3\%)$ of $\left[a_\mu^{\rm HVP}\right]_<$

MUonE





$$\left[a_\mu^{\mathrm{HVP}}\right]_>(s) = 8.30(21)_{stat}(32)_{syst}\cdot 10^{-10}$$

$$\left[a_\mu^{\mathrm{HVP}}\right]_>(c) = 2.785(78)_{stat}(68)_{syst}\cdot 10^{-10}$$

f	$[\delta a_\mu^{\mathrm{HVP}}]_>(f)\cdot 10^{10}$
ud	0.9 (0.3)
s	−0.0005 (0.0004)
c	0.0034 (0.0007)
total	0.9 (0.3)

$$\left[a_\mu^{\mathrm{HVP}}\right]_> = 92(2)\cdot 10^{-10}$$

Benchmark quantities

Windows

$$a_\mu^{\rm HVP} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f;t_0,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[1 - \Theta\left(t,t_0,\Delta\right)\right]$$

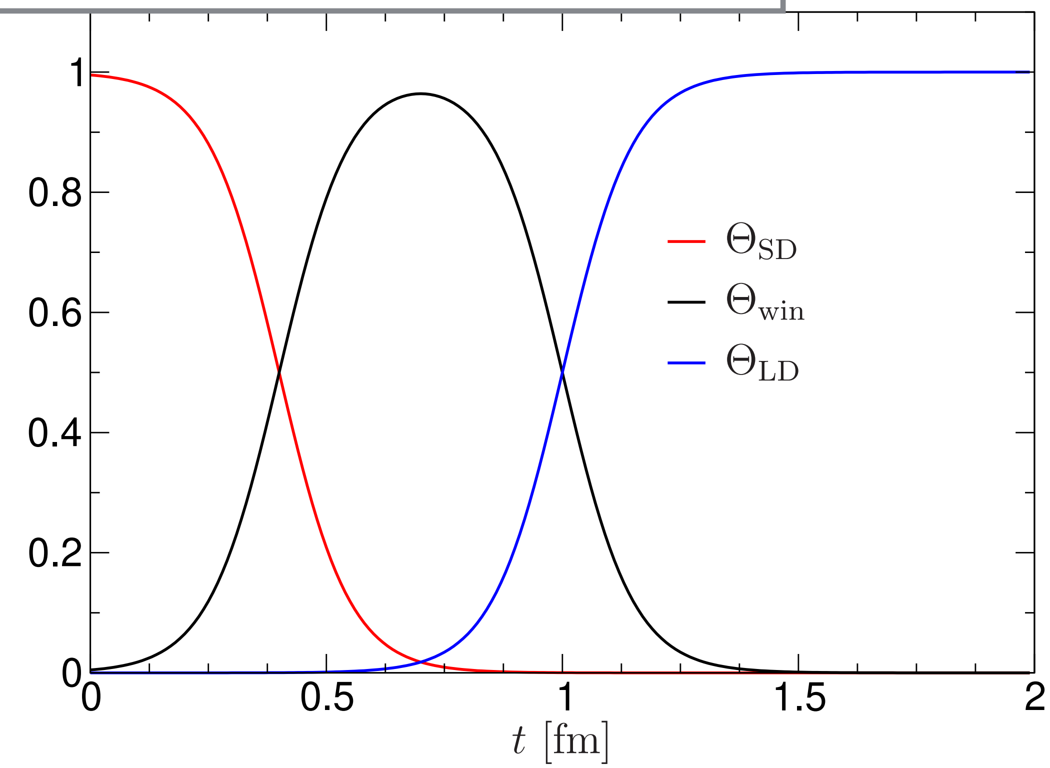
$$a_\mu^{W}(f;t_0,t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[\Theta\left(t,t_0,\Delta\right) - \Theta\left(t,t_1,\Delta\right)\right]$$

$$a_\mu^{LD}(f;t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t)\, \Theta\left(t,t_1,\Delta\right)$$

$$\Theta\left(t,t',\Delta\right) = \frac{1}{1+e^{-2(t-t')/\Delta}}$$

$$t_0 = 0.4\ \text{fm} \quad t_1 = 1.0\ \text{fm}$$

$$\Delta = 0.15\ \text{fm}$$



Effective lepton mass and effective windows

$$m_\mu^{eff} \equiv \left(m_\mu / X^{phys}\right) X$$

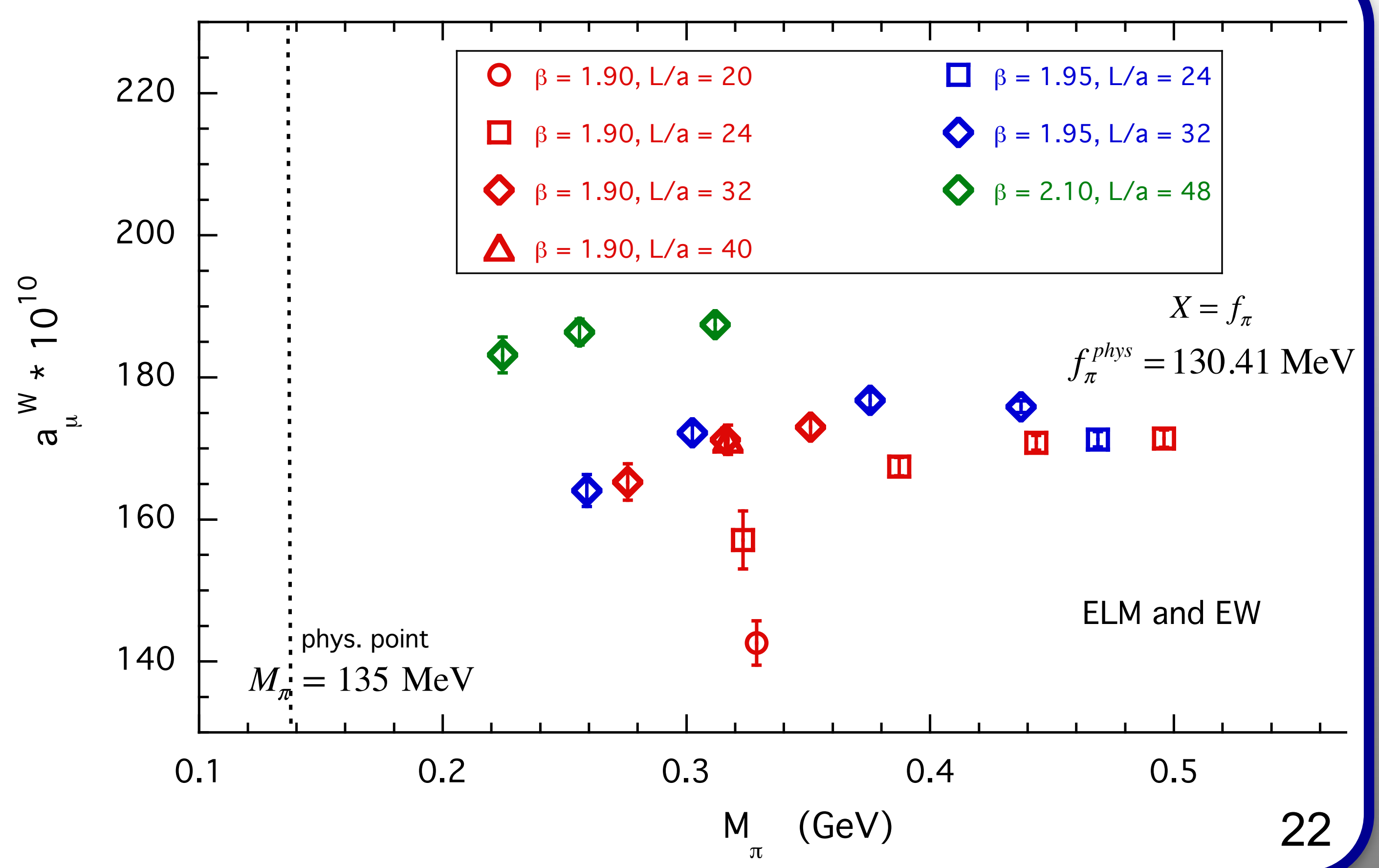
$$\Delta^{eff} \equiv \Delta X^{phys} / X$$

$$t_0^{eff} \equiv t_0 X^{phys} / X \qquad t_1^{eff} \equiv t_1 X^{phys} / X$$

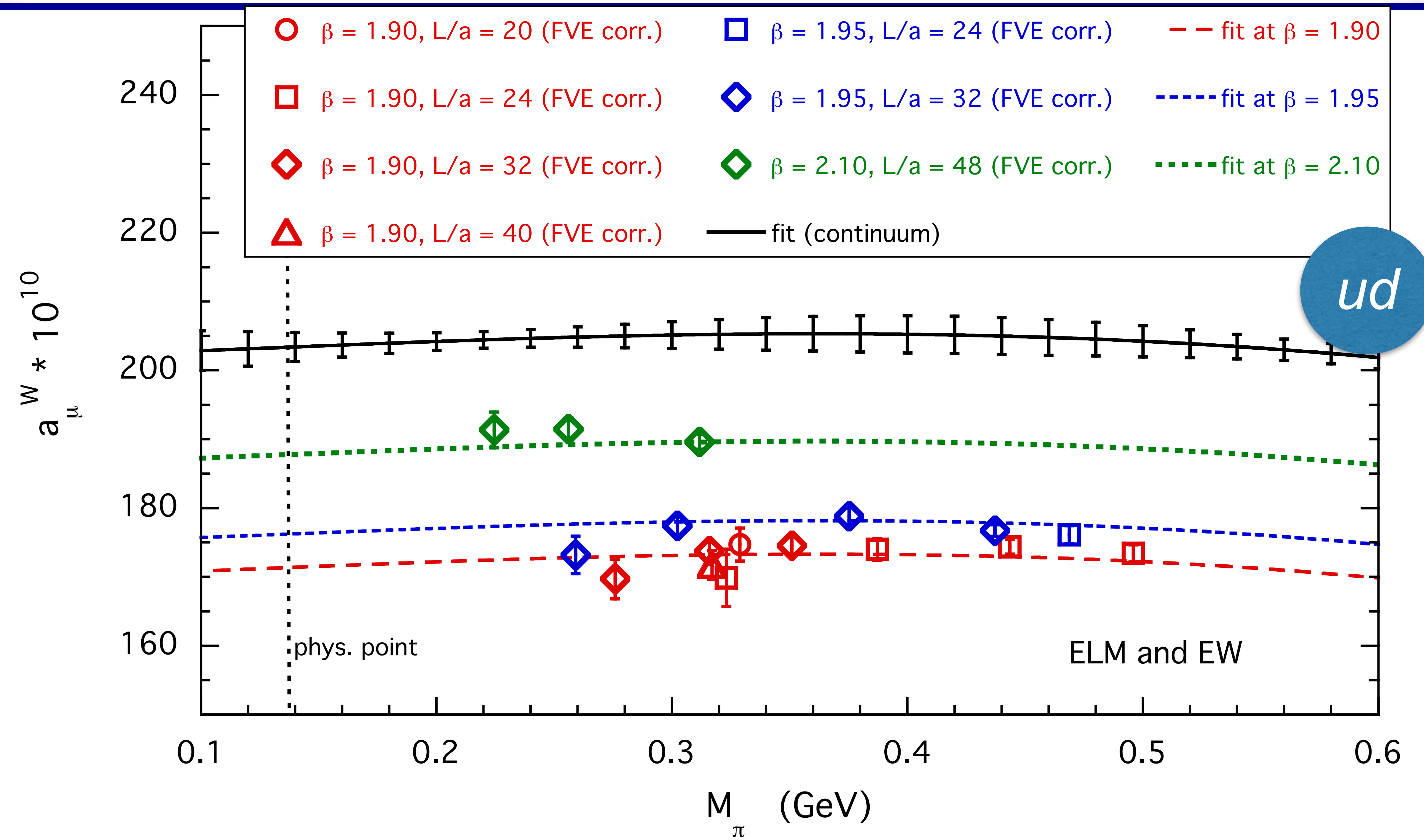
$$a_\mu^W\left(f; t_0^{eff}, t_1^{eff}, \Delta^{eff}\right) = 4\alpha_{em}^2 \frac{1}{m_\mu^2}\left(\frac{X^{phys}}{aX}\right)^2 \sum_{n=1}^{N_T} \widetilde{K}_\mu\left(m_\mu \frac{aX}{X^{phys}} n\right) a^3 V^f(an) \cdot\left[\Theta\left(aXn, t_0 X^{phys}; \Delta X^{phys}\right) - \Theta\left(aXn, t_1 X^{phys}; \Delta X^{phys}\right)\right]$$

$a_\mu^W(ud)$

- **Advantage**: uncertainty of the scale setting does not play any role
- For $X = f_\pi$ the pion mass dependence is mild
- Visible FVEs and large discretization effects



Intermediate window

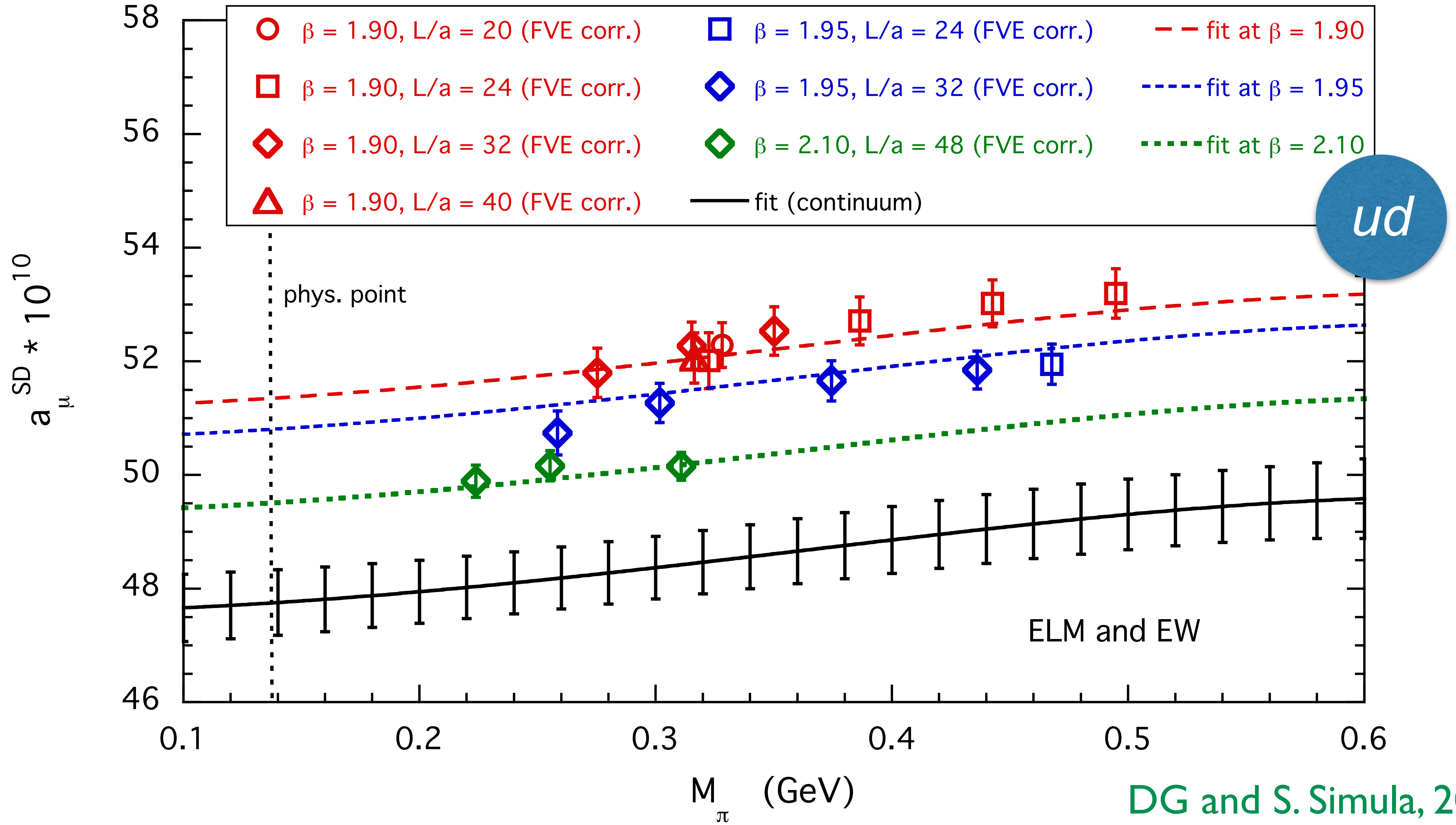


$$a_\mu^W(ud) = A_0\left[1 + A_{1\ell}M_\pi^2\log\left(M_\pi^2\right) + A_1 M_\pi^2 + A_2 M_\pi^4 + D_1 a^2\alpha_s^n(1/a) + D_2 a^4\right]$$
$$\cdot \left[1 + F M_\pi^2 e^{-M_\pi L}/(M_\pi L)^p\right]$$

DG and S. Simula, 2021
ArXiv:2111.15329

$$a_\mu^W(ud) = 202.2(2.0)_{stat}(0.4)_{chir}(1.5)_{disc}(0.7)_{FVE}[2.6]\cdot 10^{-10}$$

SD window



DG and S. Simula, 2021
ArXiv:2111.15329

$$a_\mu^{SD}(ud) = 48.21(0.56)_{stat}(0.10)_{chir}(0.50)_{disc}(0.25)_{FVE}[0.80] \cdot 10^{-10}$$

LD window

$$a_\mu^{LD}(ud) = 382.5(10.5)_{stat}(5.2)_{syst}[11.7] \cdot 10^{-10}$$

analytic representation

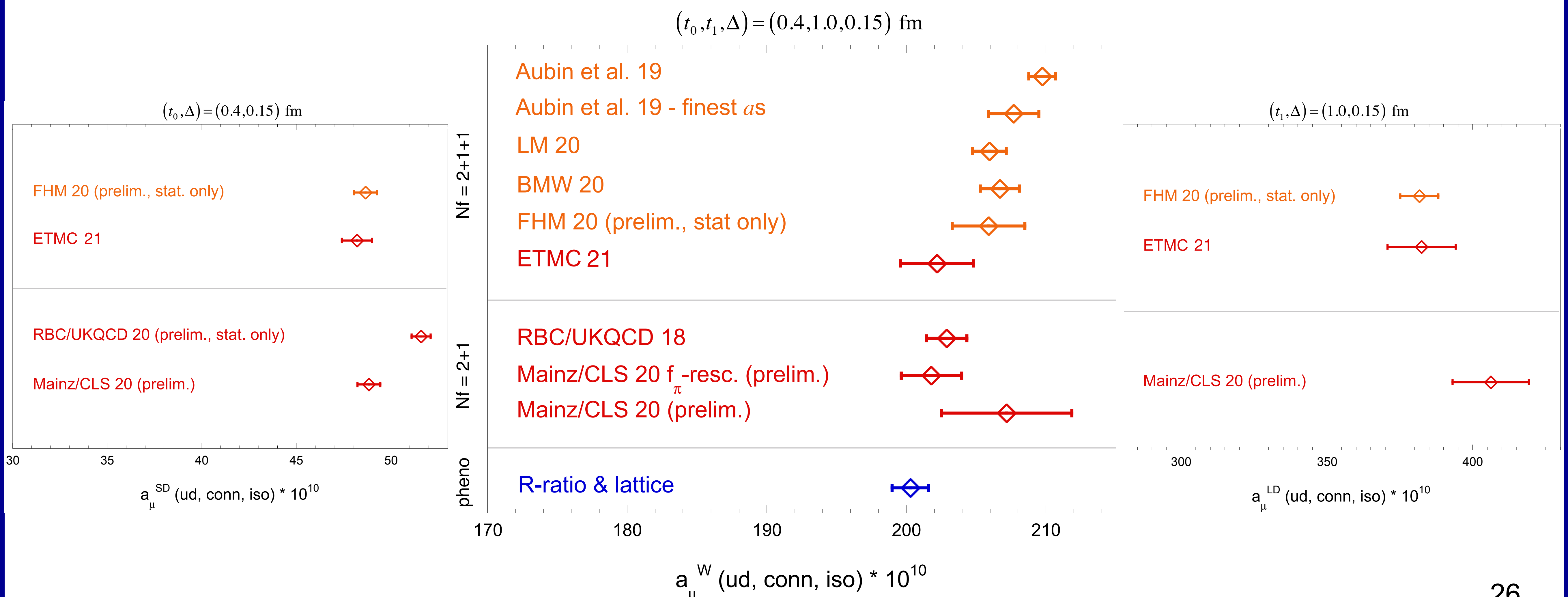
analytic representation ⟷ **data driven**

analytic representation	data driven
$a_\mu^{W}(ud) = 198.0(3.4)_{stat}(4.7)_{syst}[5.8] \cdot 10^{-10}$	$a_\mu^{W}(ud) = 202.2(2.0)_{stat}(1.7)_{syst}[2.6] \cdot 10^{-10}$
$a_\mu^{SD}(ud) = 48.6(1.8)_{stat}(1.0)_{syst}[2.0] \cdot 10^{-10}$	$a_\mu^{SD}(ud) = 48.21(0.56)_{stat}(0.57)_{syst}[0.80] \cdot 10^{-10}$

good consistency

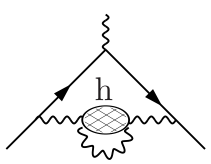
Summary: *ud* contribution

f	$a_\mu^{SD}(f) \cdot 10^{10}$	$a_\mu^{W}(f) \cdot 10^{10}$	$a_\mu^{LD}(f) \cdot 10^{10}$
ud	48.2 (0.8)	202.2 (2.6)	382.5 (11.7)



Intermediate window other contributions

f	$a_\mu^W(f) \cdot 10^{10}$
ud	202.2 (2.6)
s	26.9 (1.0)
c	2.81 (0.11)
IB	0.7 (0.4)
disc	−0.9 (0.2)



RBC/UKQCD 18 & BMW 20

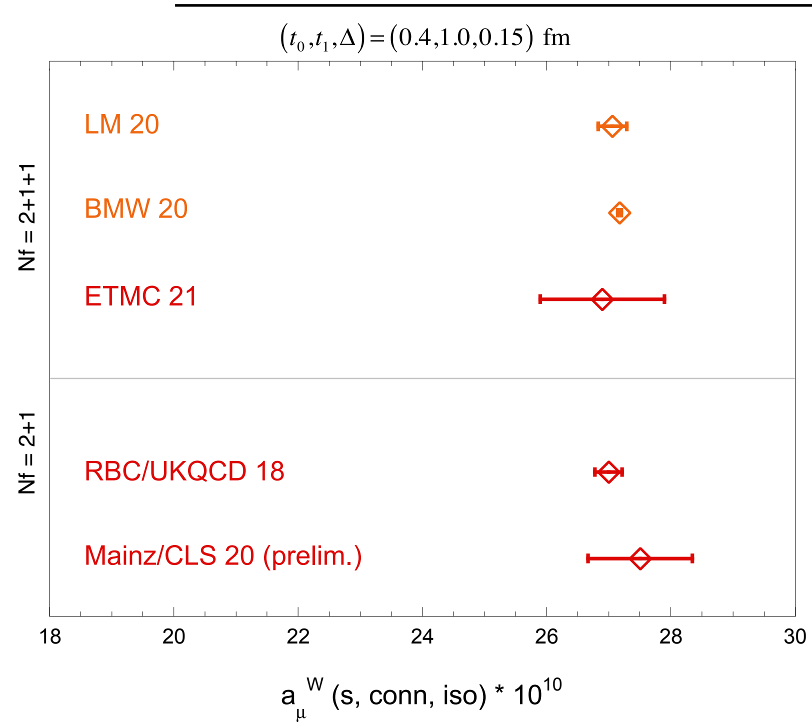
Lattice

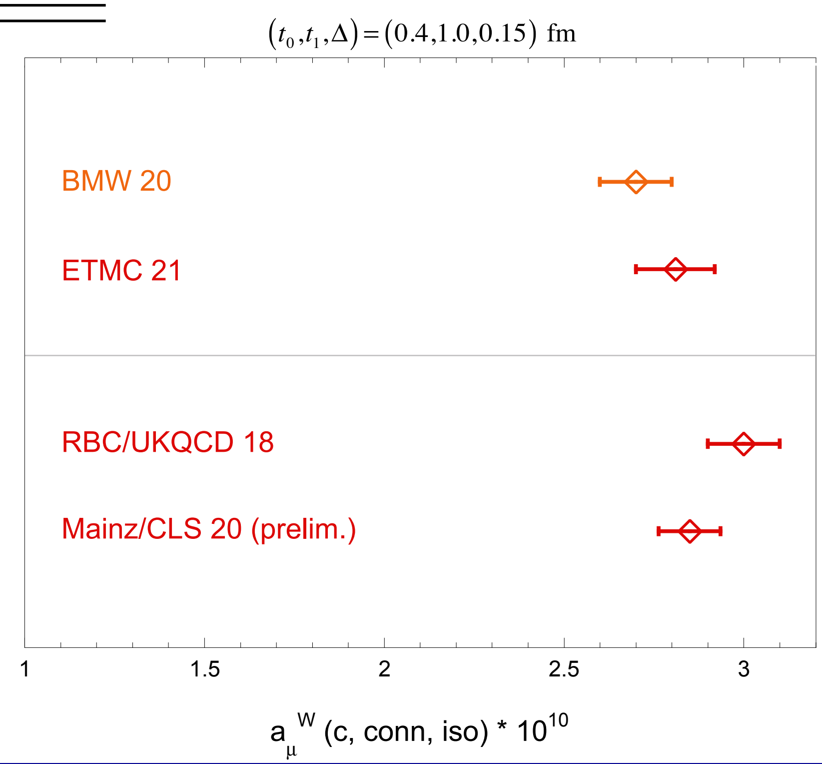
$$a_\mu^W = 231.7(2.8) \cdot 10^{-10}$$

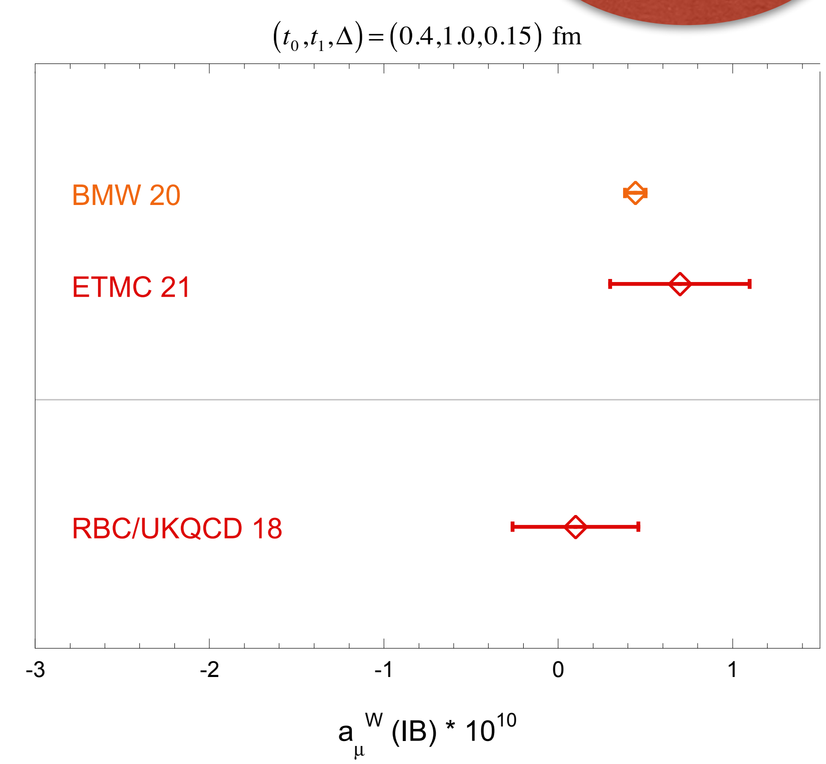
ArXiv:2111.15329

$$a_\mu^W = 229.7(1.3) \cdot 10^{-10}$$

R-ratio







Moments of the HVP function

light-quark connected contribution

$$\Pi^{ud}_{n+1} \equiv (-)^n \frac{(n+1)!}{(2n+4)!} \frac{18}{5} \int_0^\infty dt\ t^{2n+4} V^{ud}(t)$$

Total (incl. s,c-quak;disc.;IB)

$$\Pi_1^{tot} = 0.100(3)\ \mathrm{GeV}^{-2}$$

$\Pi_1^{tot} = 0.1000(30)\ \mathrm{GeV}^{-2}$ Sz. Borsanyi *et al.*, 2016

$\Pi_1^{tot} = 0.1000(23)\ \mathrm{GeV}^{-2}$ FHM19

(dual + π-π) representation

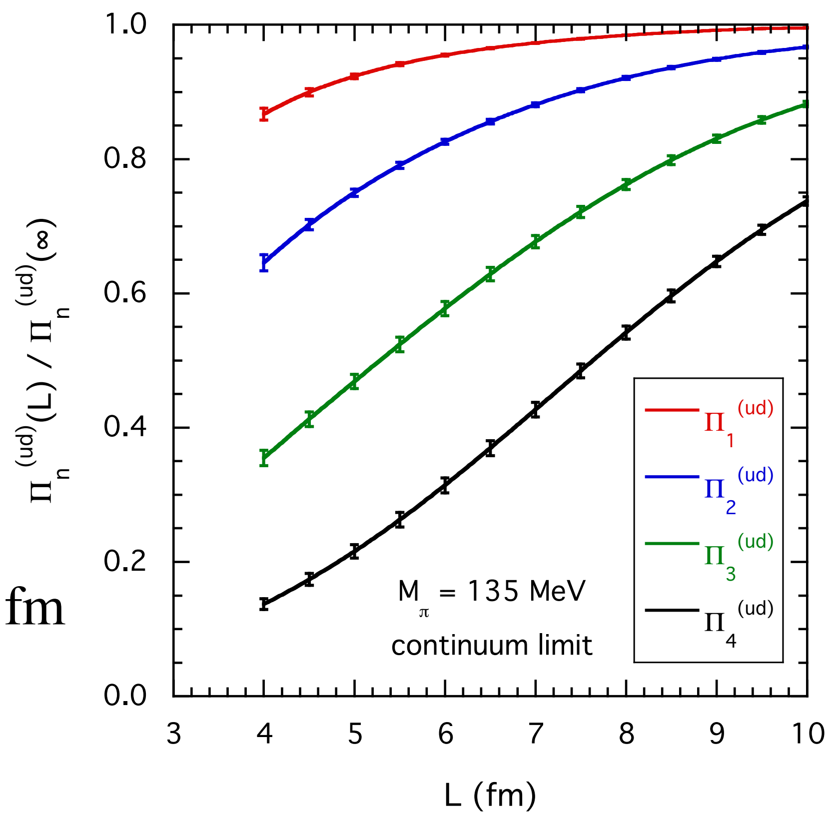
$\Pi_1^{ud} = 0.1642(33)\ \mathrm{GeV}^{-2}$ $\Pi_2^{ud} = -0.383(16)\ \mathrm{GeV}^{-4}$

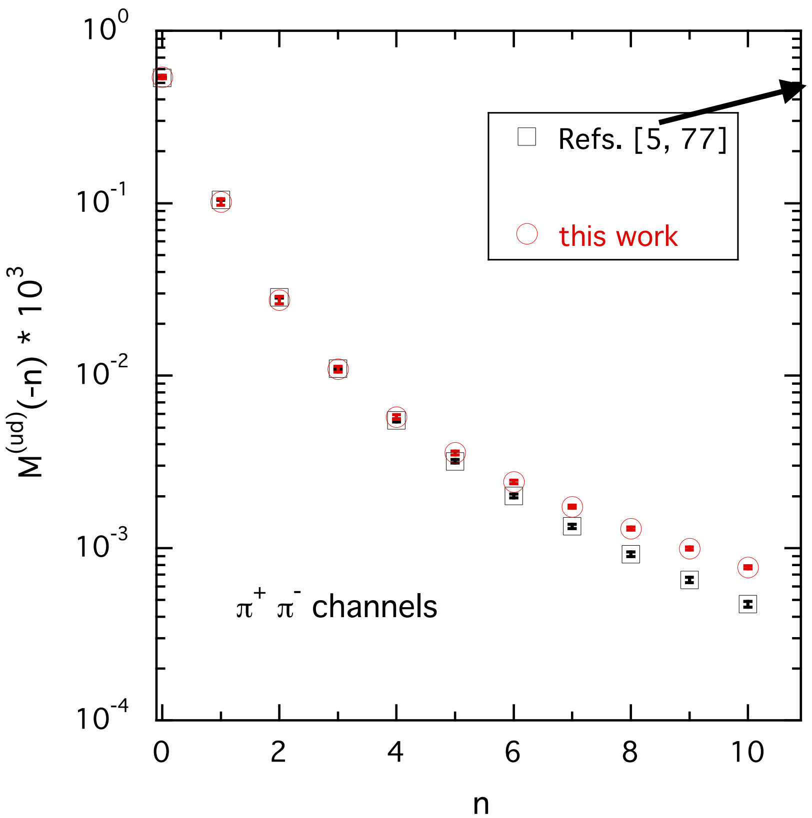
$\Pi_2^{ud} = -0.311(16)\ \mathrm{GeV}^{-4}$ Sz. Borsanyi *et al.*, 2016

$\Pi_3^{ud} = 1.394(65)\ \mathrm{GeV}^{-6}$ $\Pi_4^{ud} = -7.60(28)\ \mathrm{GeV}^{-8}$

for Π_2^{ud} $@L \sim 10$ fm

FVEs: $\simeq 3-4\%$





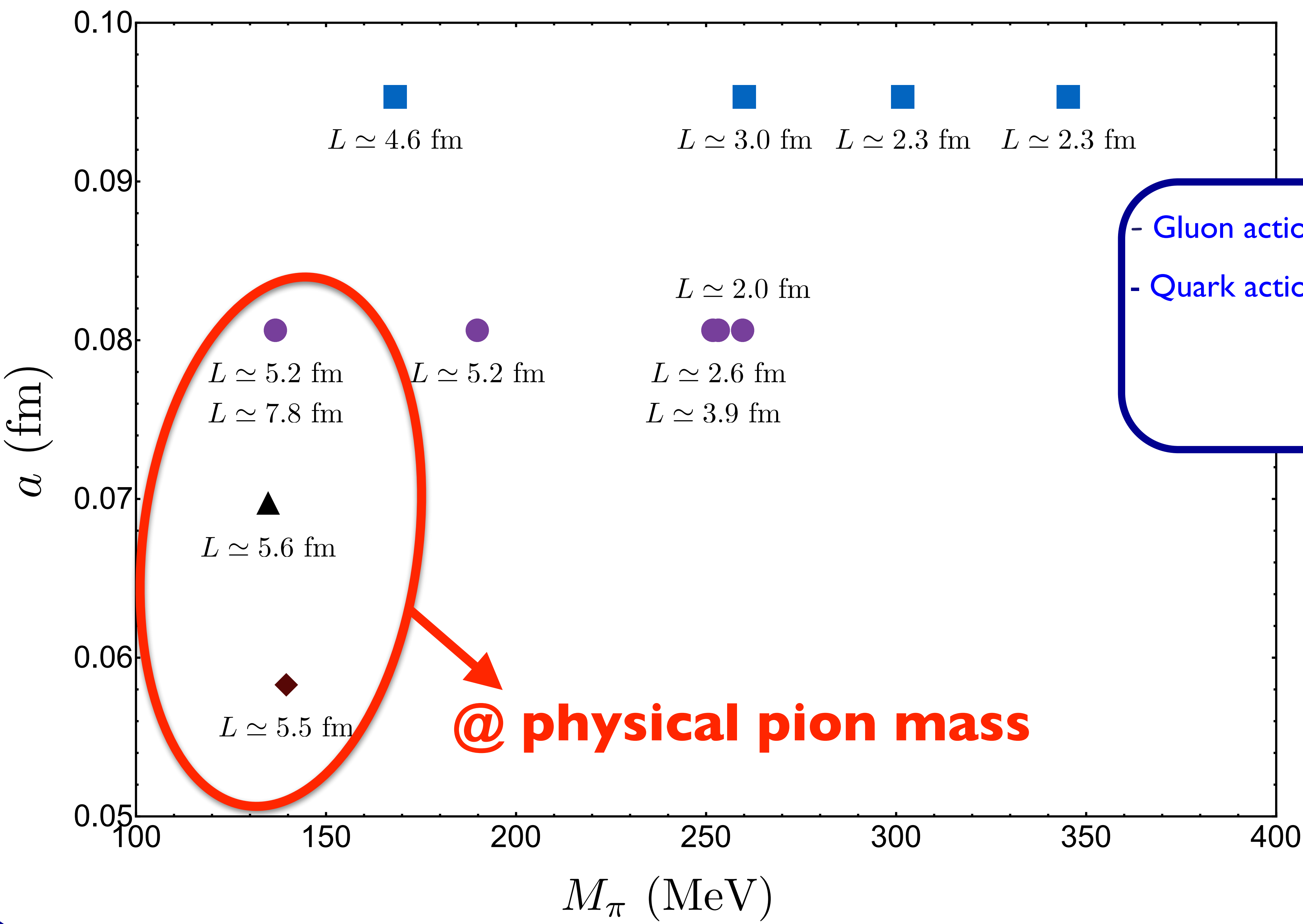
$\sigma(e^+e^- \to \pi^+\pi^-)$

A. Keshavarzi *et al.*, 2018

large time-distance behavior of $V^{ud}(t)$ reliably evaluated by using the repr.

New ETMC setup

We are generating new gauge field configurations with Nf=2+1+1 dynamical quarks, including ensembles at the physical pion mass



- Gluon action: **Iwasaki + clover**
- Quark action: twisted mass at maximal twist (automatically O(a) improved)
  OS for *s* and *c* valence quarks

Extended Twisted Mass Collaboration

Conclusions

- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) ➡ LQCD

- We have performed a first-principles lattice QCD+QED calculation of a_ℓ^{HVP}. Our results agree with recent determinations based on dispersive analyses.

$$a_e^{\mathrm{HVP}} = 185.8(4.2)\cdot 10^{-14} \qquad a_\mu^{\mathrm{HVP}} = 692.1(16.3)\cdot 10^{-10} \qquad a_\tau^{\mathrm{HVP}} = 335.9(6.9)\cdot 10^{-8}$$

DG and S. Simula, 2019

- Window contributions are sharp benchmark quantities. Our result for the intermediate window is in agreement with the R-ratio prediction.

$$a_\mu^W = 231.7(2.8)\cdot 10^{-10}$$

DG and S. Simula, 2021

In progress…

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation

- use of the new ETMC lattice setup @ the physical pion point