$a_\mu^{\mathrm{HVP,LO}}$ with 2+1+1 HISQ fermions: Status and Prospects from FNAL-HPQCD-MILC

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University of Illinois

DWQ@25 Workshop
hosted by BNL & RBRC (virtual)
13 - 17 December 2021

Fermilab Lattice, HPQCD, MILC g-2 group

Subgroup of members of the three collaborations actively engaged in the g-2 projects:

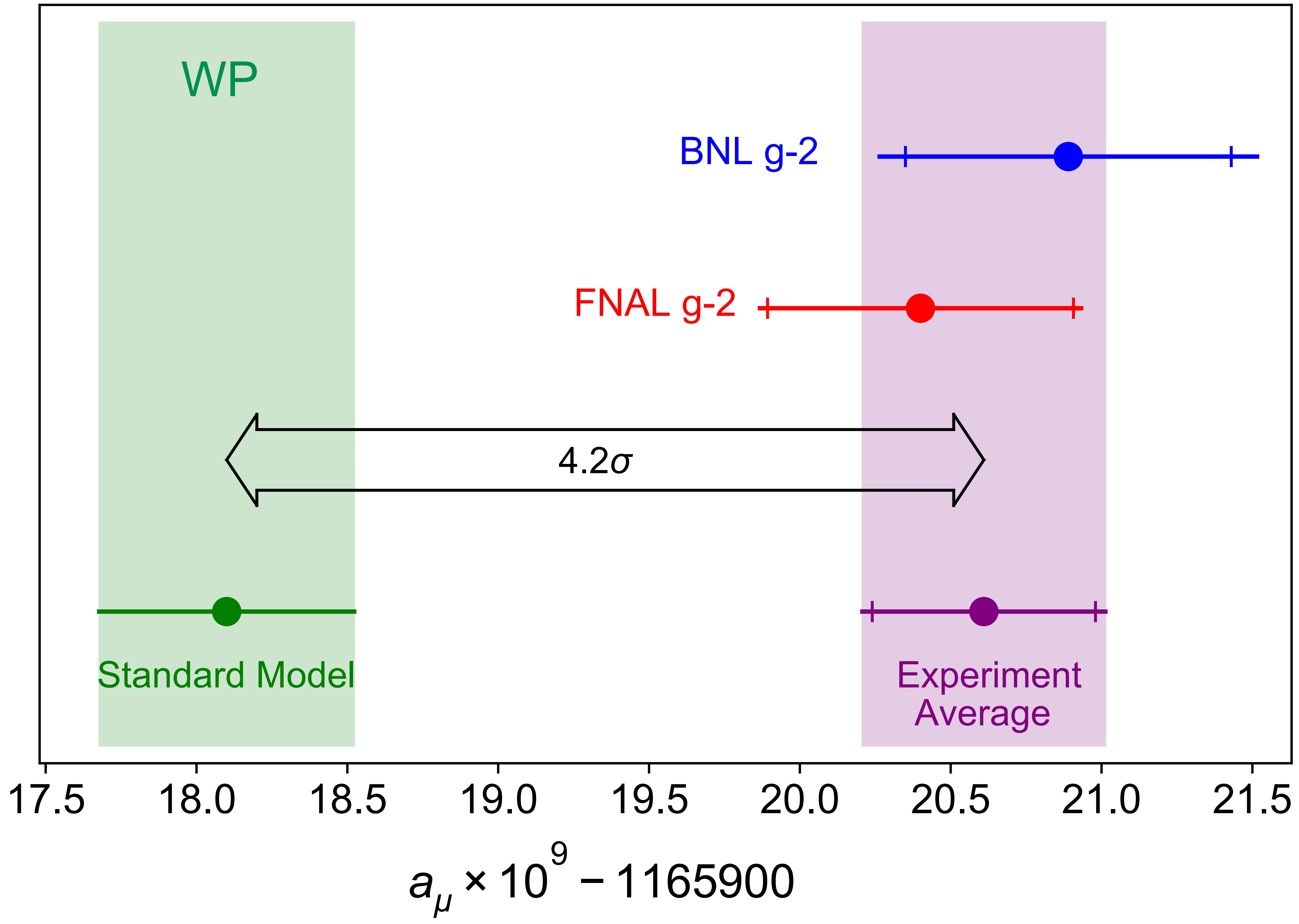
- Alexei Bazavov
- Carleton DeTar
- A. El-Khadra
- Elvira Gámiz
- Steve Gottlieb
- Andreas Kronfeld
- Shaun Lahert
- Michael Lynch
- Yin Lin (Ω baryon mass)
- Ethan Neil
- Curtis Peterson
- Jim Simone
- Alex Vaquero
- Ruth Van de Water

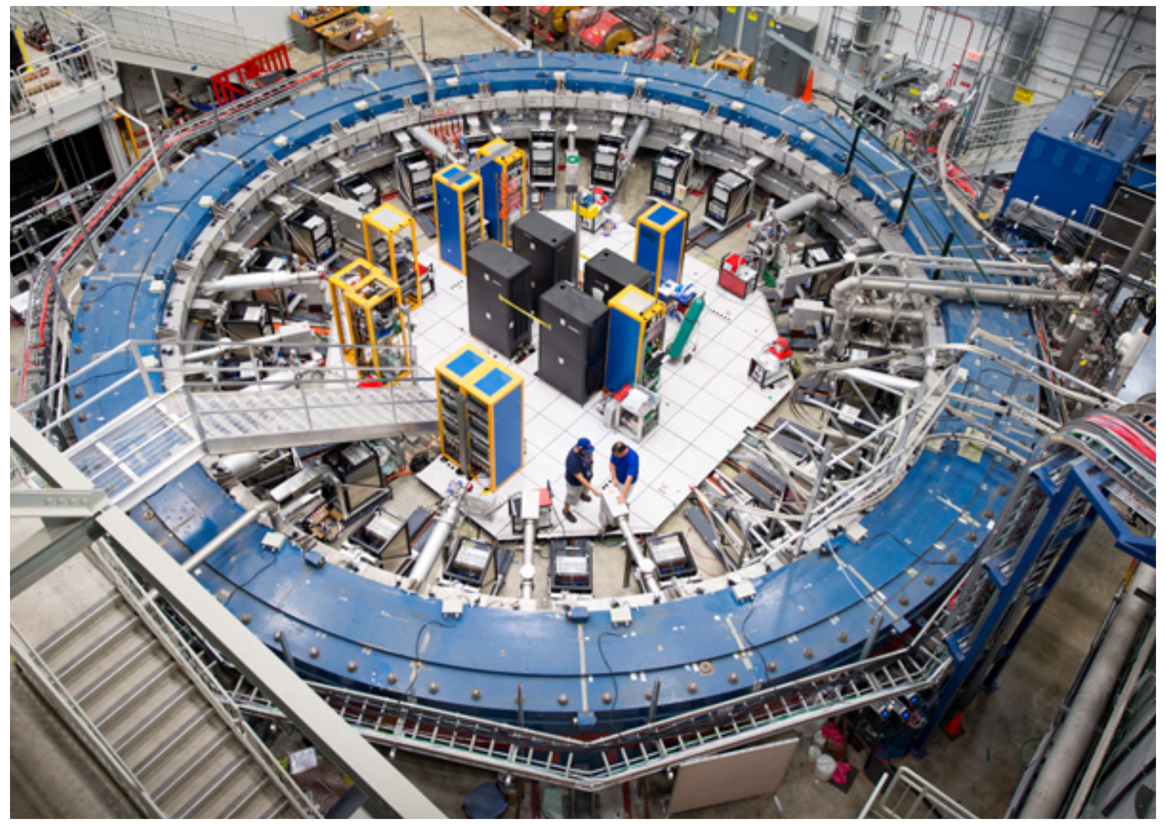
HPQCD

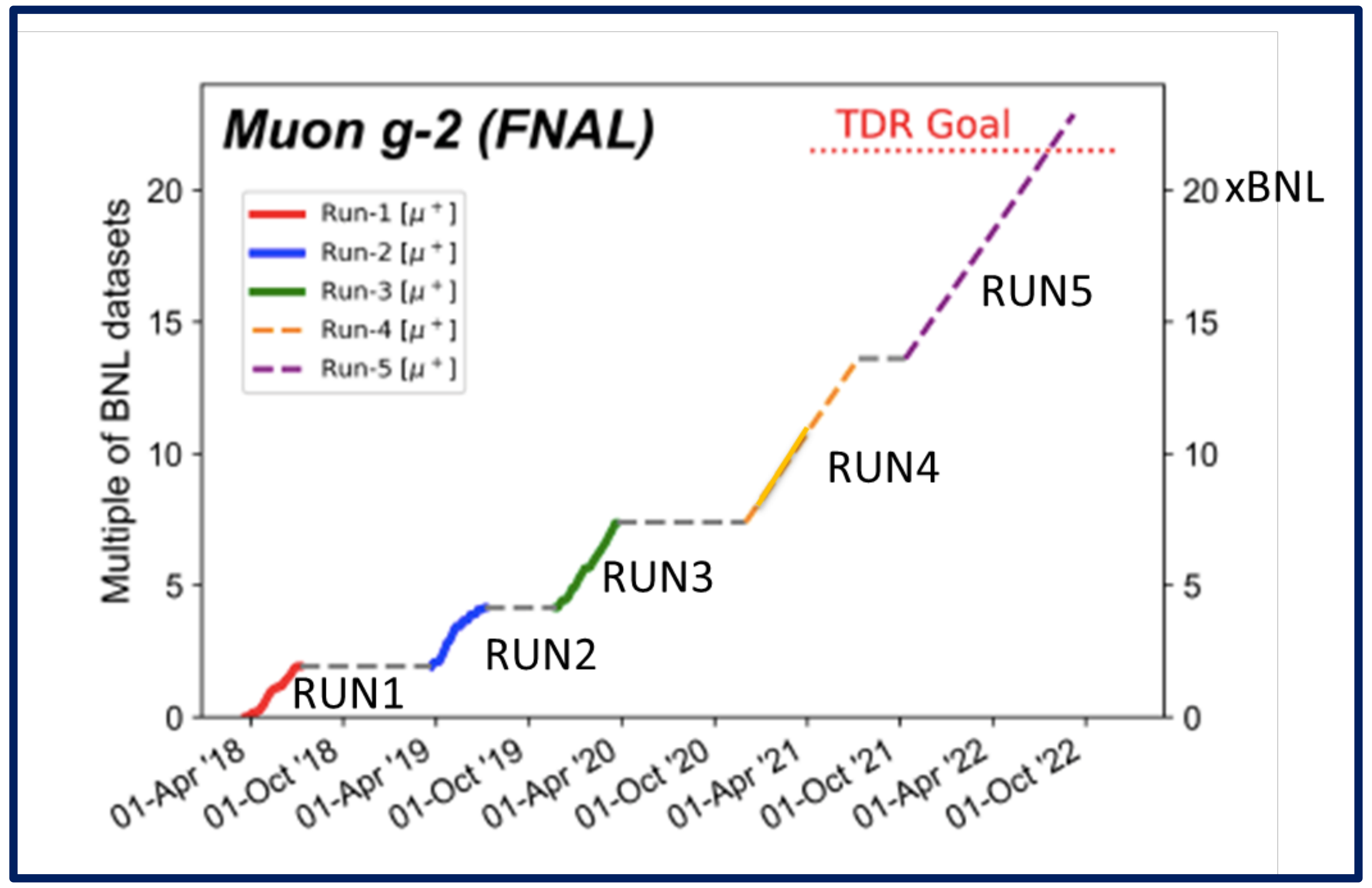
- Christine Davies
- Dan Hatton
- Peter Lepage
- Craig McNeile
- Gaurav Ray

Muon g-2: experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021.
  [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Analysis of runs 2 and 3 is now underway.



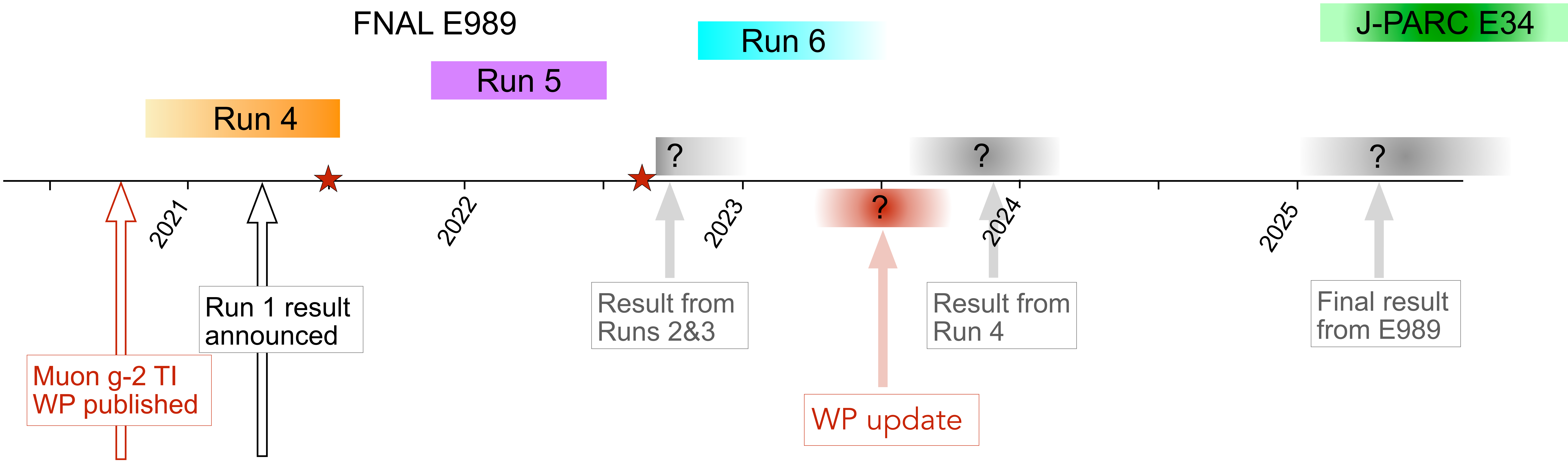




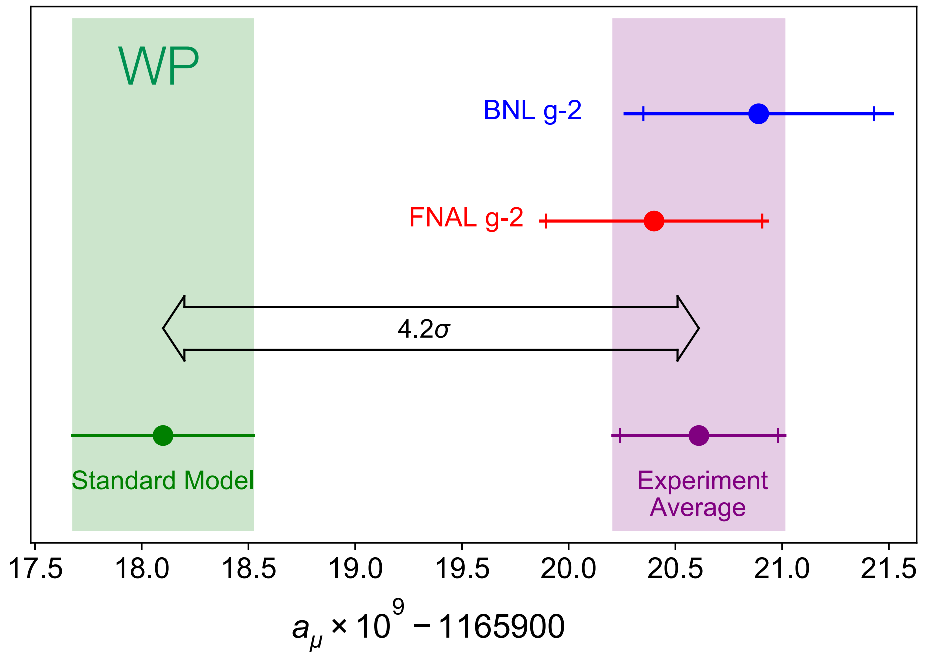
Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
- 1st White Paper published in 2020 (132 authors, 82 institutions, 21 countries)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- 2nd White Paper (~2023): First discussions @ KEK meeting in June 2021
  expect to develop a concrete plan (outline, authors) @ Higgs Centre workshop

Timeline




Physics Reports
The anomalous magnetic moment of the muon in the Standard Model



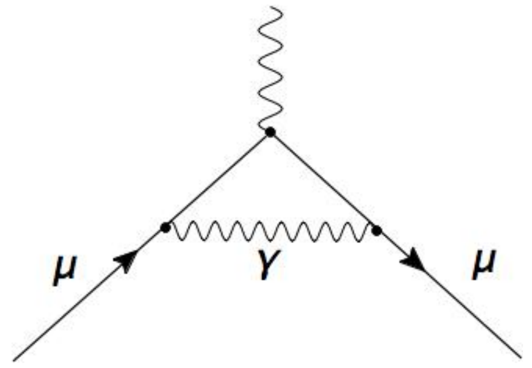
Theory Initiative:

- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ plan to update WP with new SM predictions (~ 2023)

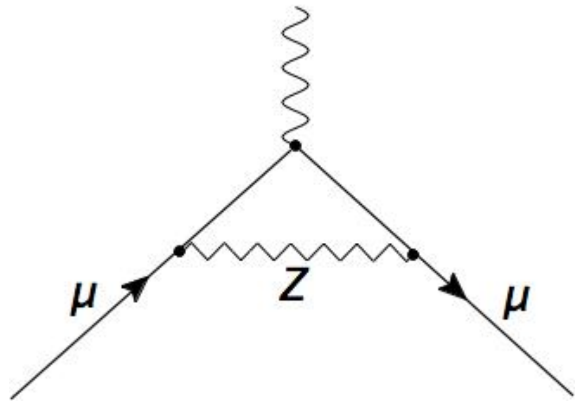
★ TI workshops: Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

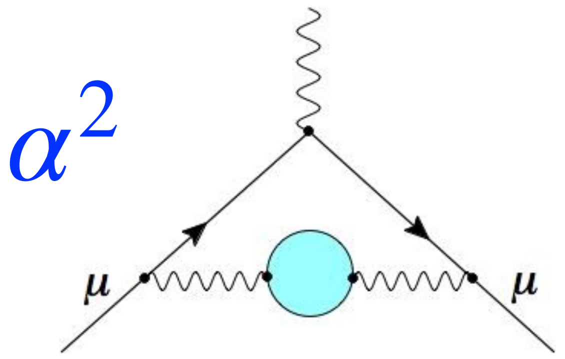
QED 

$+\ldots$ $\quad 116\,584\,718.9\,(1) \times 10^{-11}$ $\quad$ 0.001 ppm

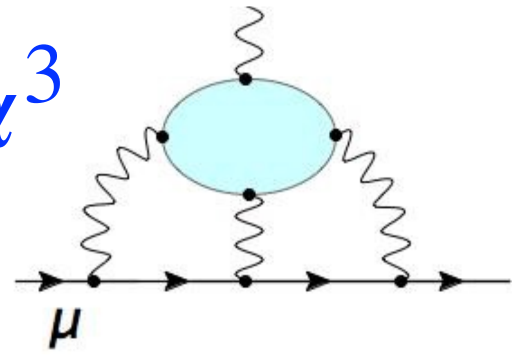
EW 

$+\ldots$ $\quad 153.6\,(1.0) \times 10^{-11}$ $\quad$ 0.01 ppm

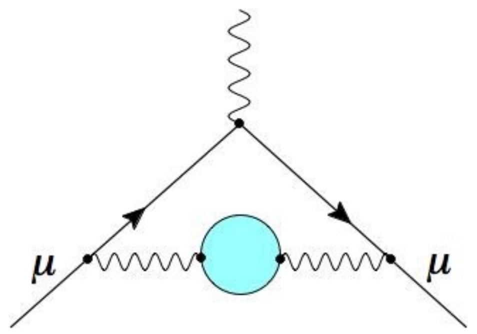
Hadronic corrections

HVP α^2 

$+\ldots$ $\quad 6845\,(40) \times 10^{-11}$ [0.6%] $\quad$ 0.34 ppm

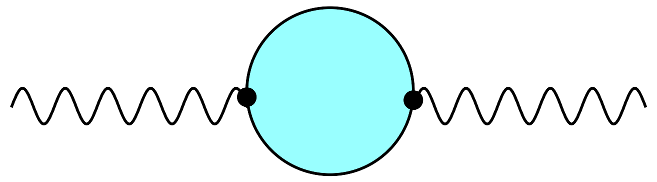
HLbL α^3 

$+\ldots$ $\quad 92\,(18) \times 10^{-11}$ [20%] $\quad$ 0.15 ppm



Lattice HVP: Introduction

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972;
E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

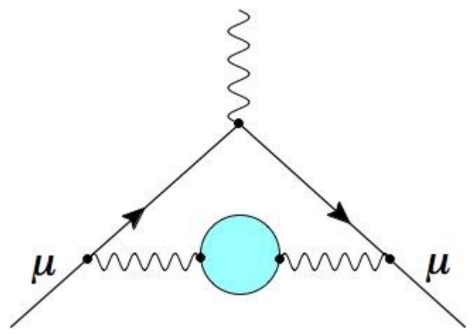
  Compute correlation function: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$

  and $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right)\right]$

[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

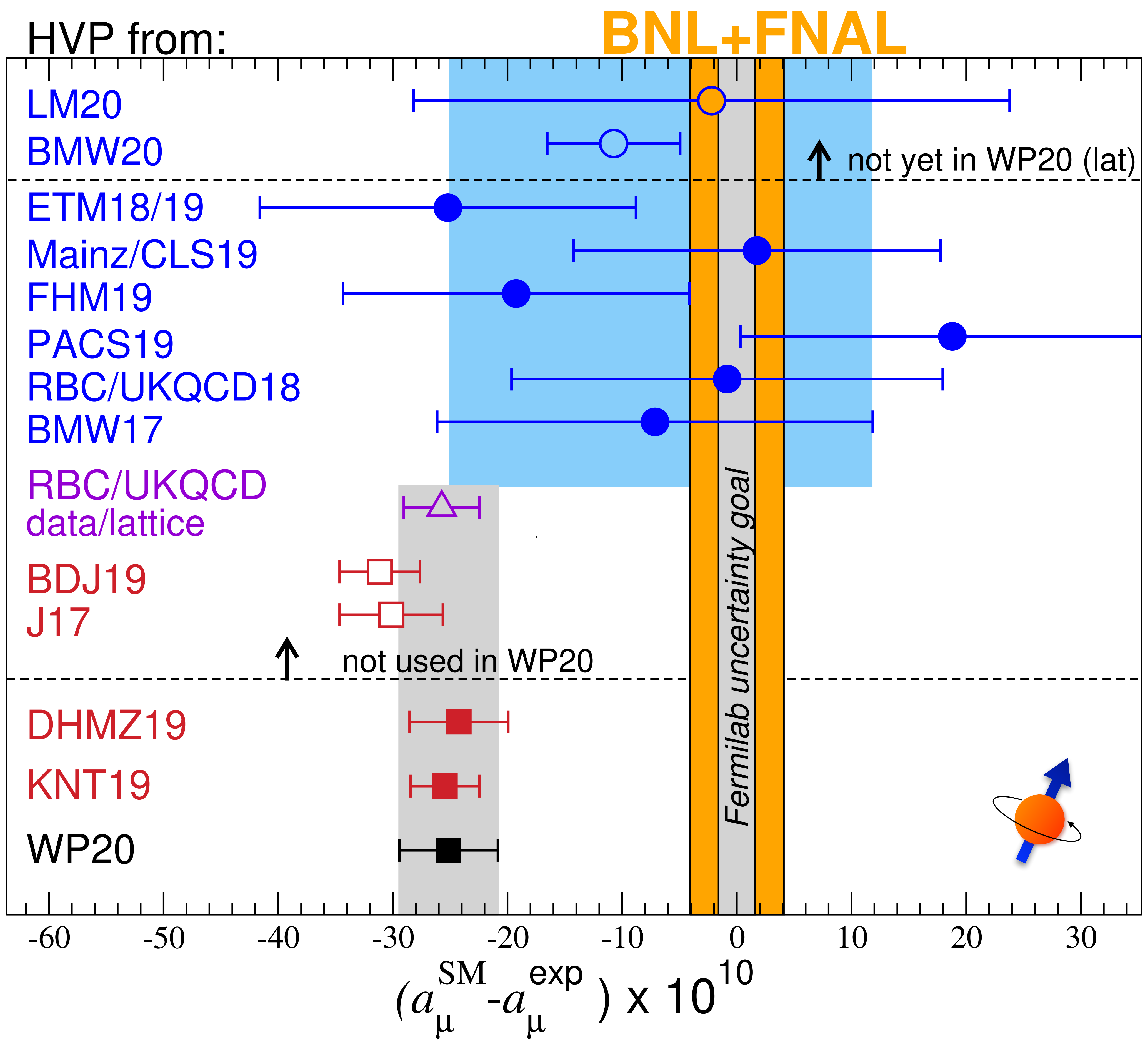
Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$



HVP: Comparison

$$a_\mu^{SM} \quad a_\mu^{HVP} + [a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{HLbL}]$$

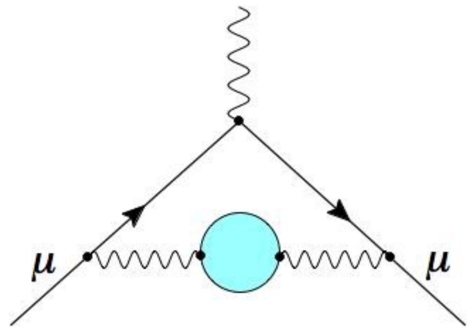


Lattice QCD + QED

hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints



HVP: Comparison

$$a_\mu^{\text{SM}} \quad \leftarrow \quad a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

FNAL-HPQCD-MILC:

[Chakraborty et al, arXiv:1710.11212, 2018 PRL]

[Davies et al, arXiv:1902.04223, 2020 PRD]

Updates in:

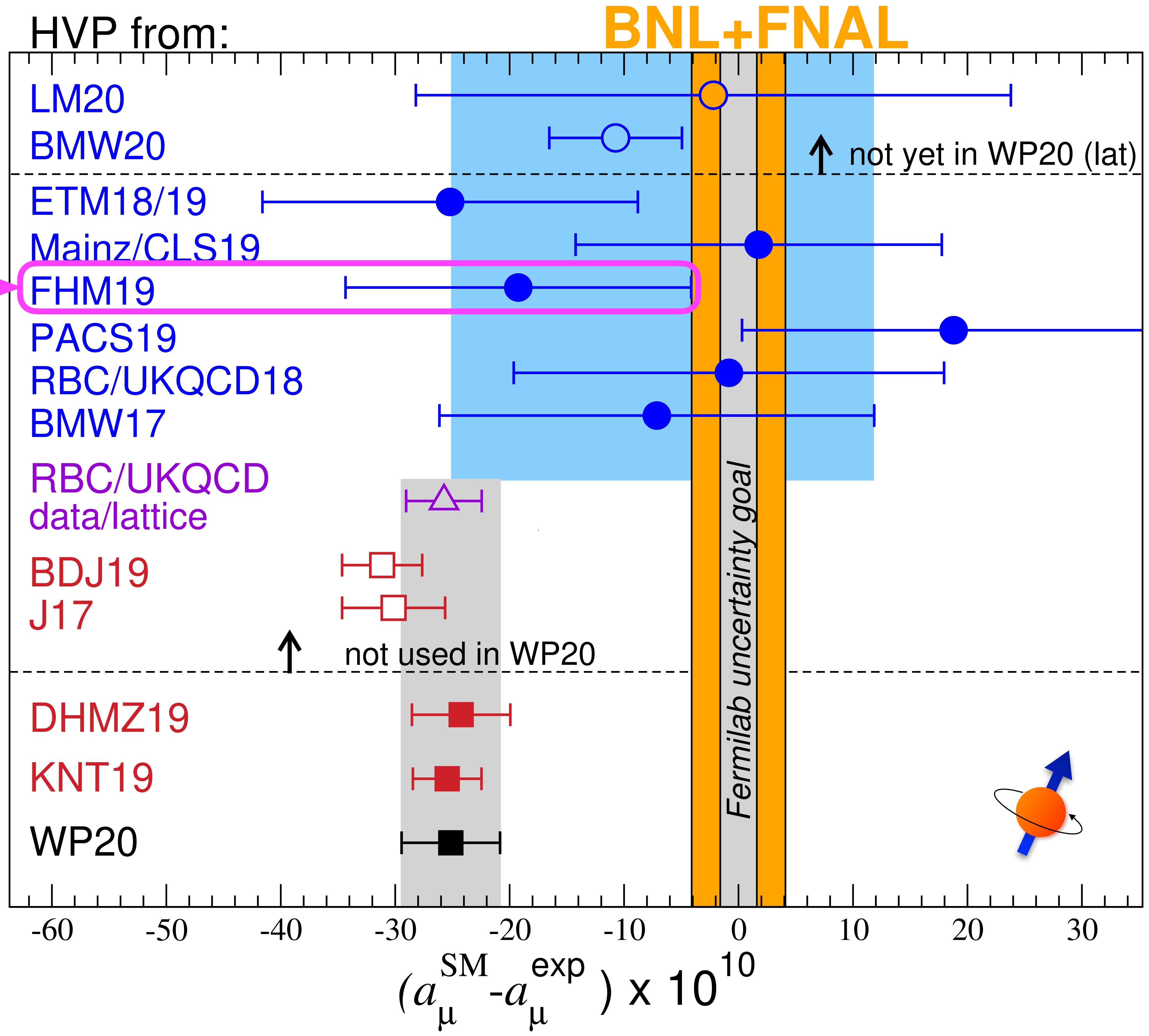
[C. DeTar et al, arXiv:1912.04382, Lattice 2019]

[S. Lahert @ Lattice 2021]

[C. McNeile @ Lattice 2021]

Ongoing program:

- Reduce uncertainties on connected light quark contribution
- Disconnected contribution
- Strong IB corrections
- QED corrections



Lattice QCD + QED

hybrid: combine data & lattice

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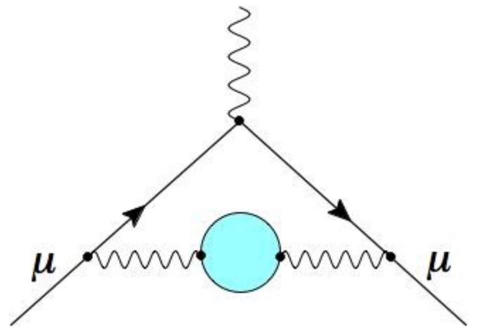
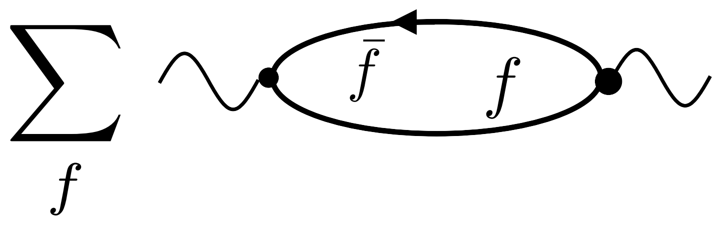
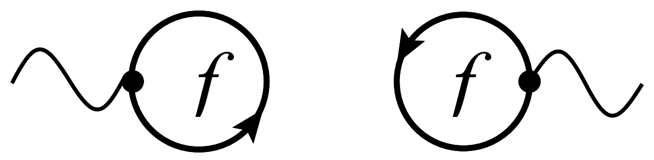


FNAL-HPQCD-MILC summary

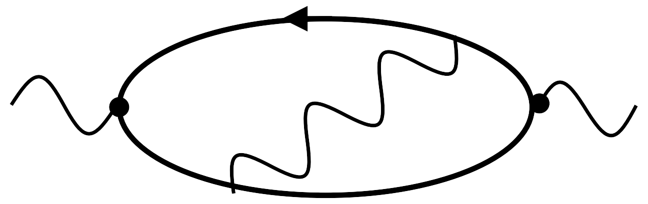
[Davies et al, arXiv:1902.04223, 2020 PRD]



- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total

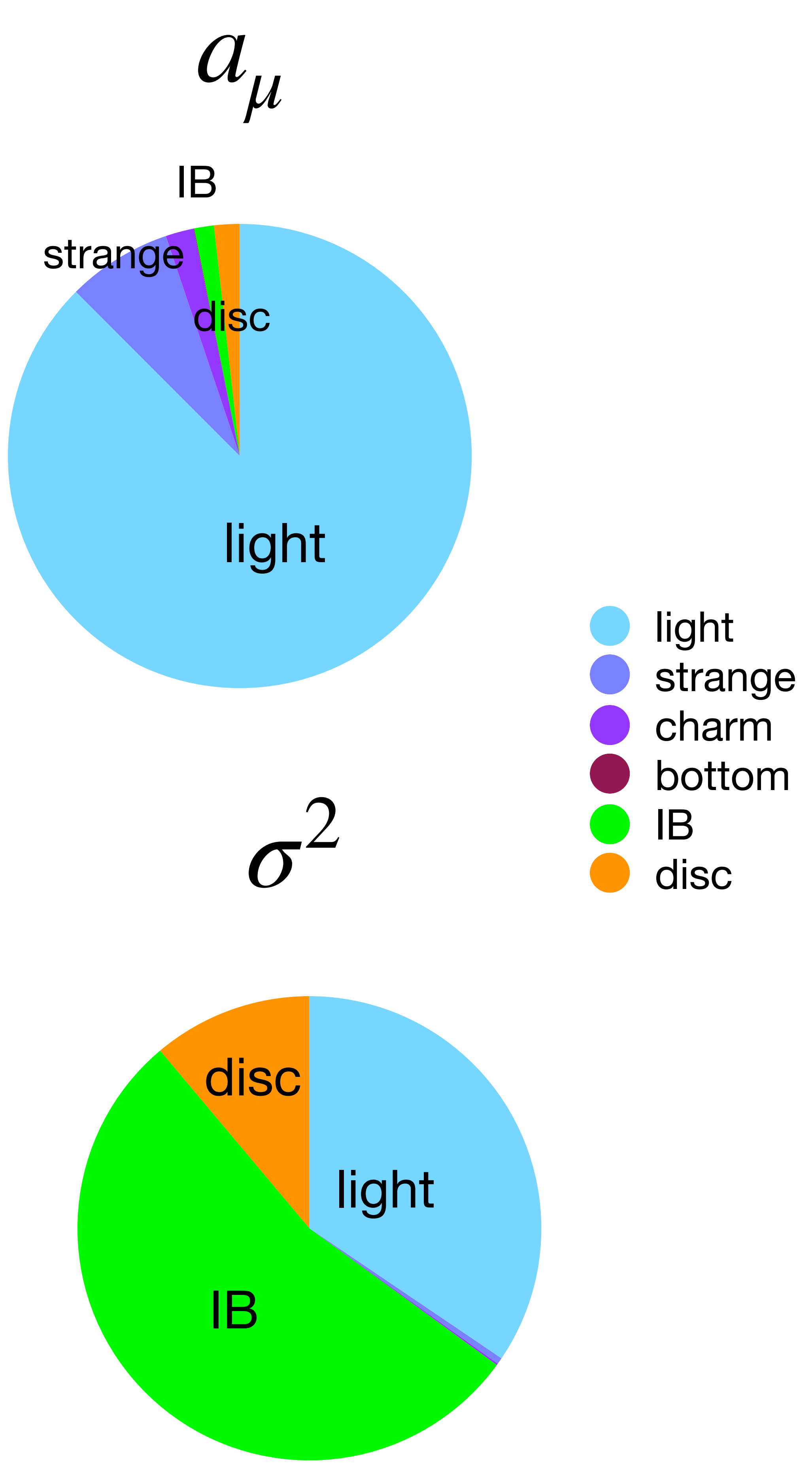


- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total

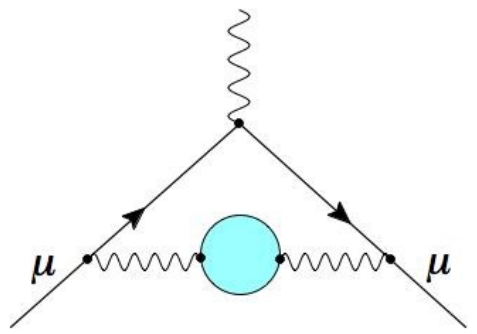


+ …

- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total

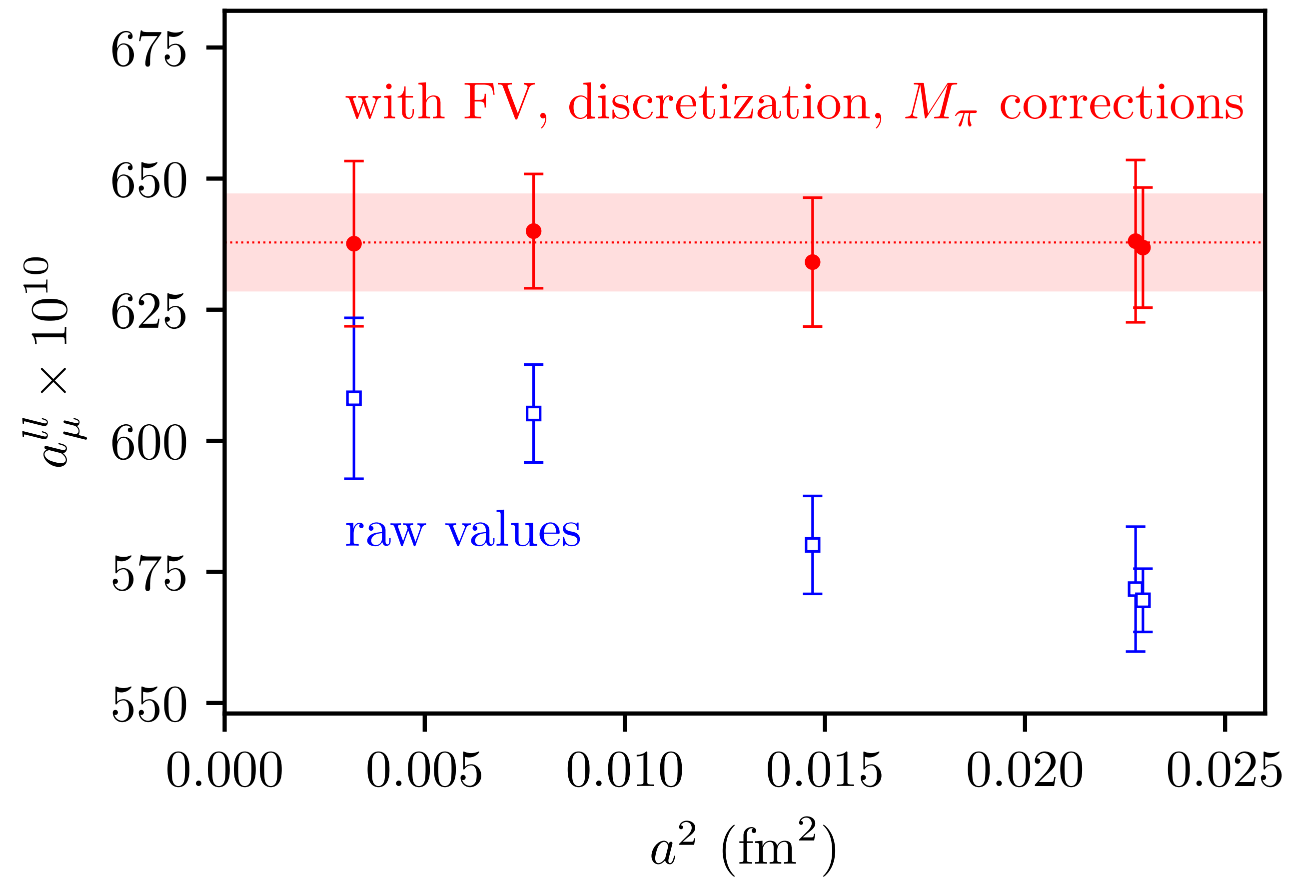


$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$



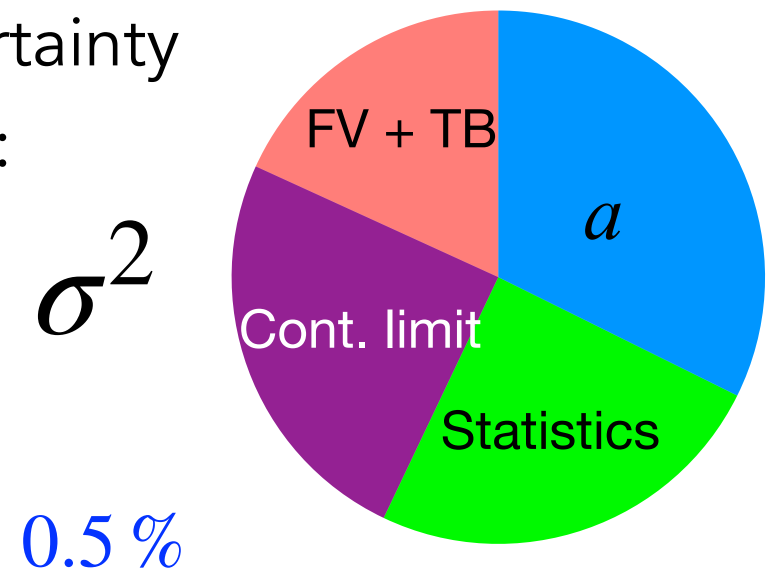
Light-quark connected contribution

[Davies et al, arXiv:1902.04223, 2020 PRD]



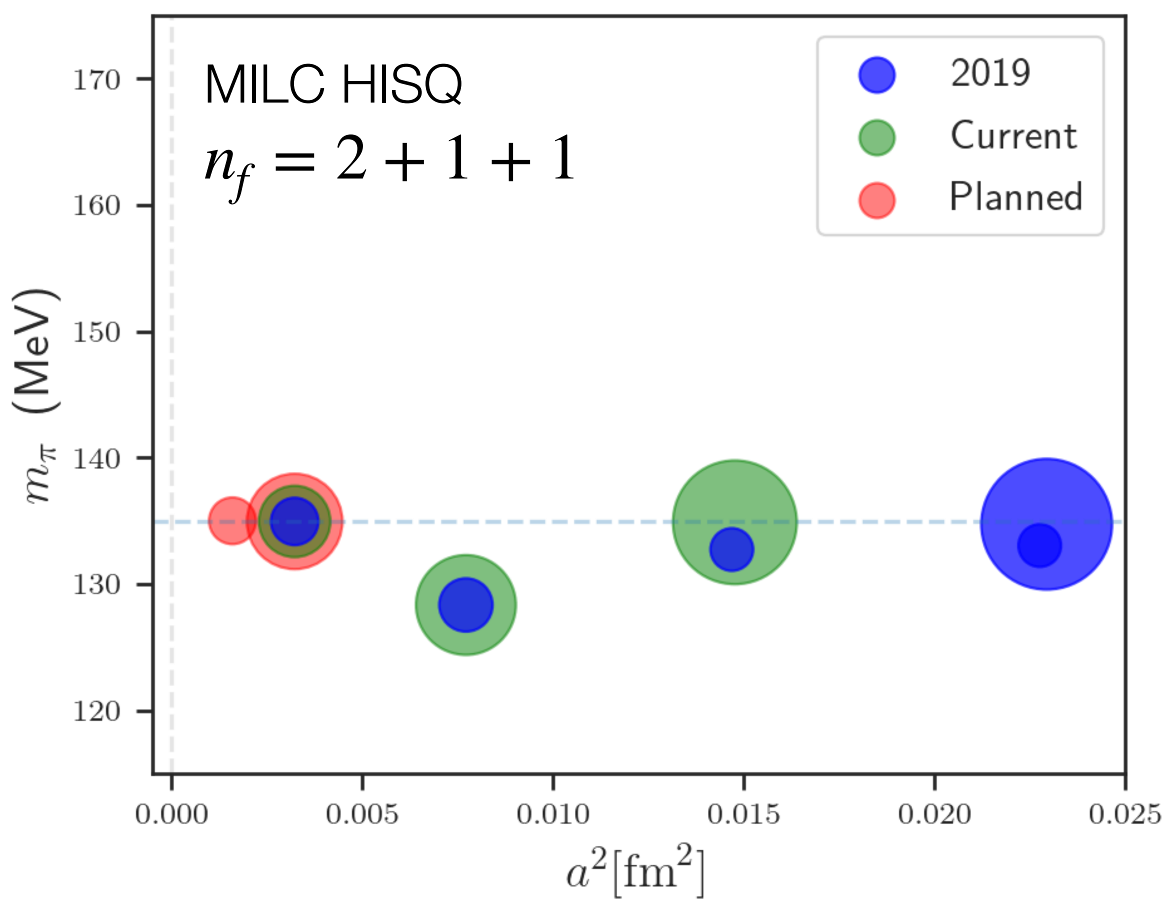
raw ➠ corrected:

- FV + taste-breaking corrections using chiral model
- adjust for small differences between simulation M_π and M_{π^0}
- $1.4\,\%$ total uncertainty
- error breakdown:



- ultimate goal: $\lesssim 0.5\,\%$

Statistics, continuum limit:



- MILC HISQ ensembles with (near) physical mass light quarks
- increasing statistics at 3 finest ensembles
- plan to add data at a 5th lattice spacing, $a \approx 0.042$ fm

FV corrections:

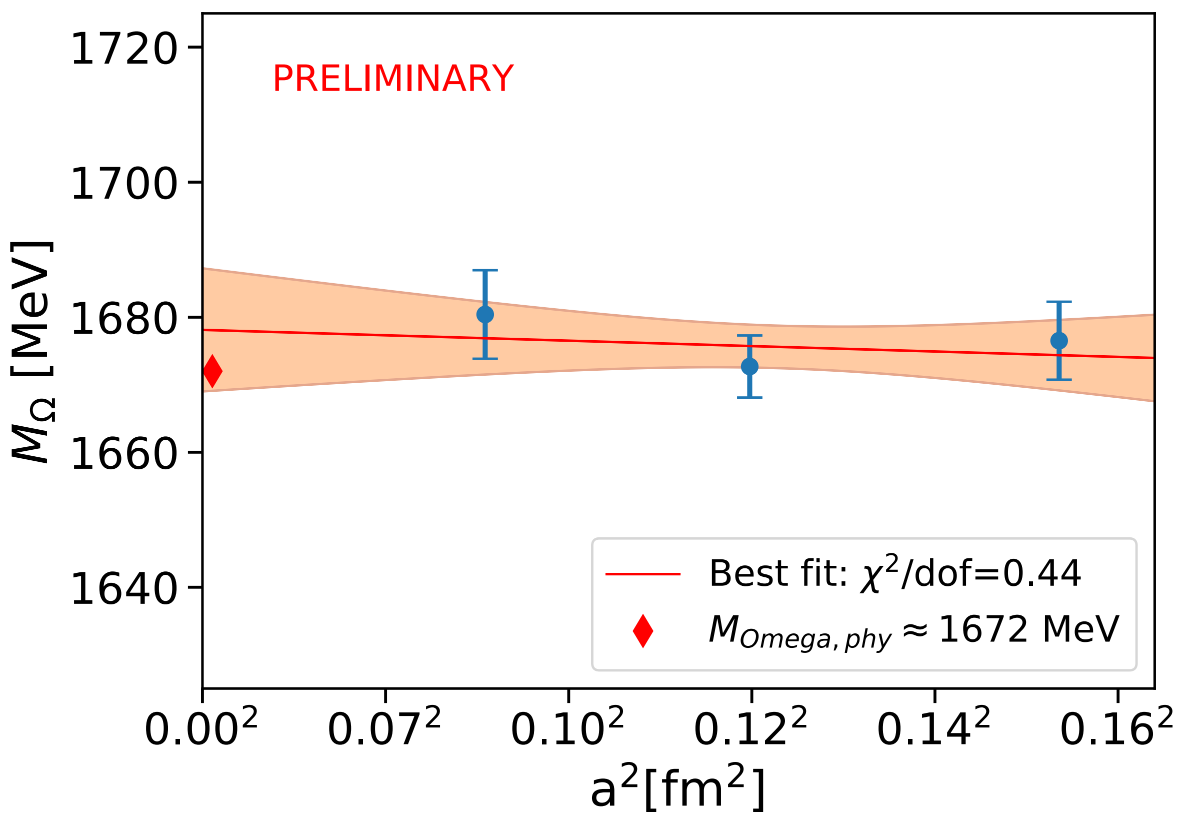
- Compare FV corrections using different approaches (chiral model, NNLO ChPT, HP, GS)

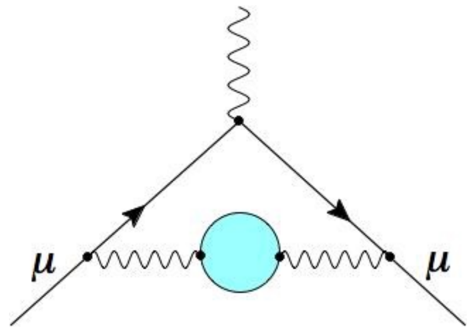
Lattice spacing uncertainty w_0 (fm):

- current ~0.5% error based on f_π

Ongoing work:

- Refining calculation of f_π on MILC ensembles (Claude Bernard)
- Refining calculation of relative scale (Alexei Bazavov)
- Computing Ω baryon mass (Yin Lin)
  - extending pilot project [Yin Lin, arXiv:1912.00028]
  - adding data at $a \approx 0.06$ fm
  - increasing statistics





Windows in Euclidean time

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

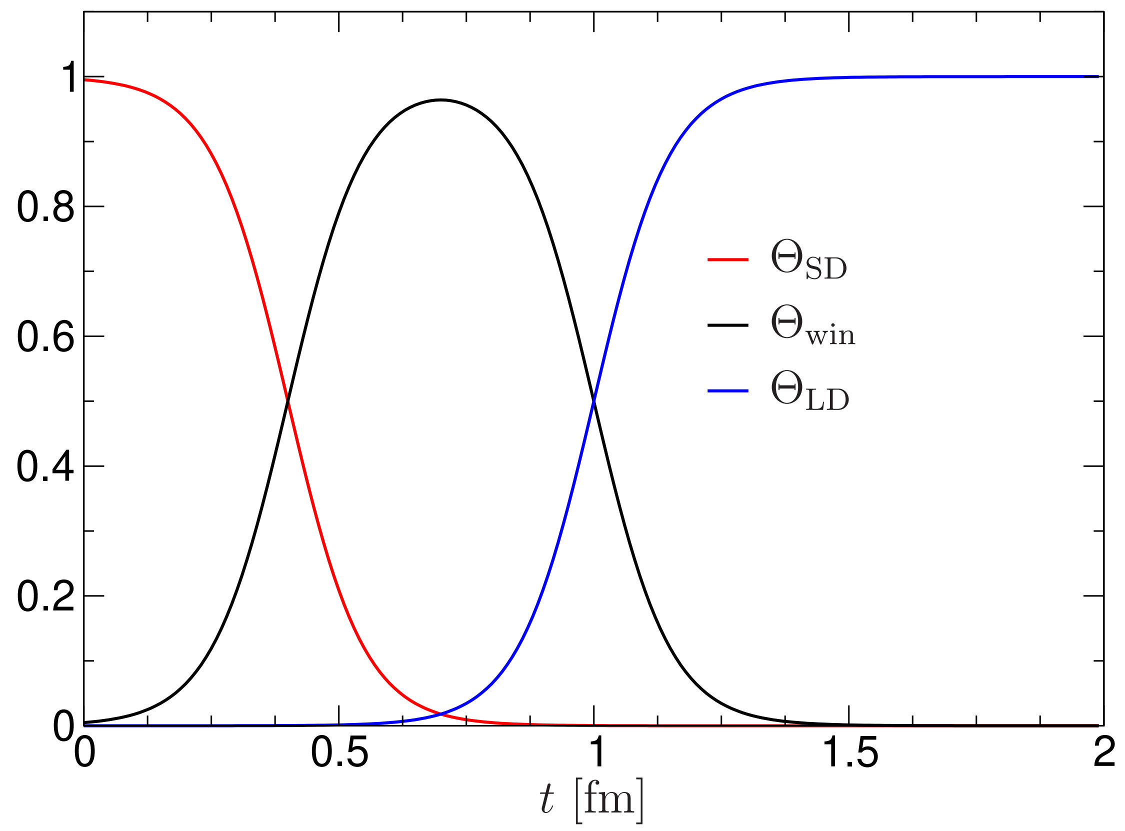
- Use windows in Euclidean time to consider the different time regions separately [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD) $t: 0 \rightarrow t_0$

Intermediate (W) $t: t_0 \rightarrow t_1$

Long Distance (LD) $t: t_1 \rightarrow \infty$

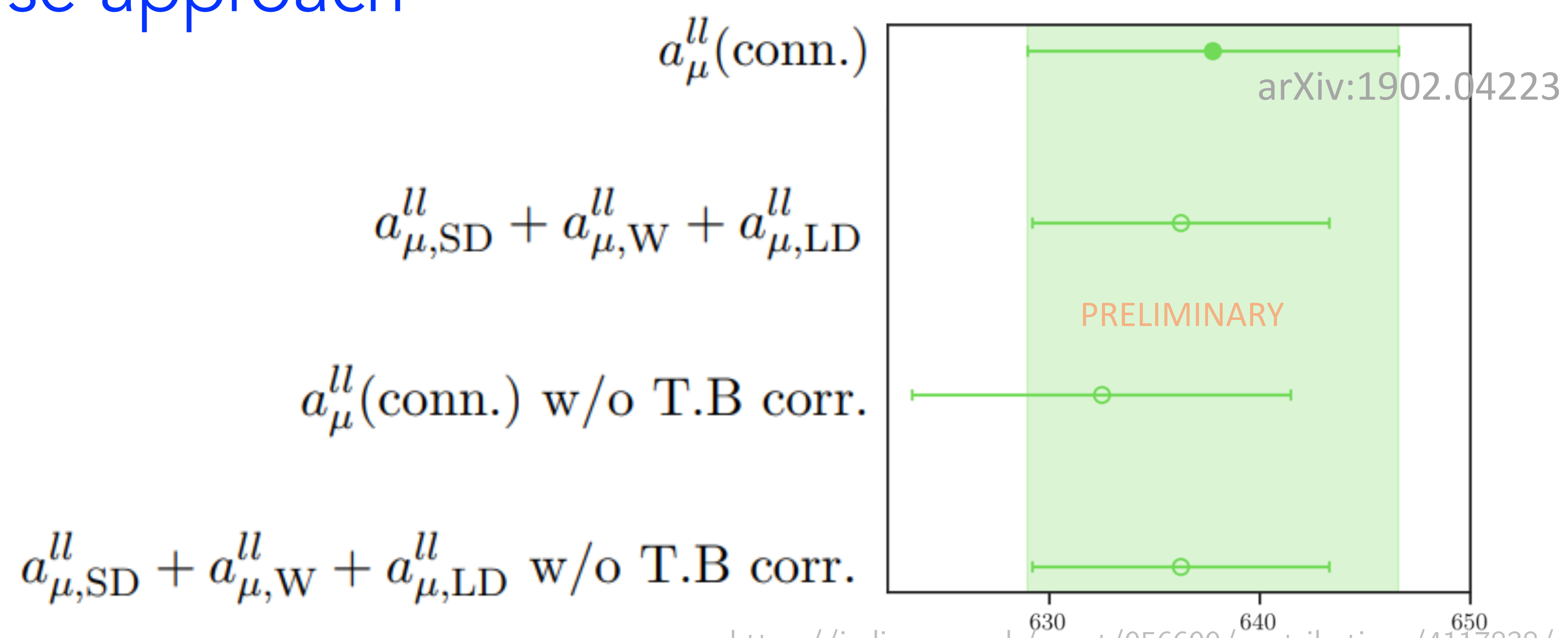
$$t_0 = 0.4\,\mathrm{fm},\ t_1 = 1.0\,\mathrm{fm}$$



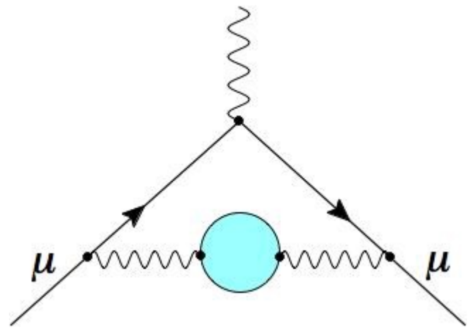
- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine:

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

Shaun Lahert @ HVP 2020 workshop

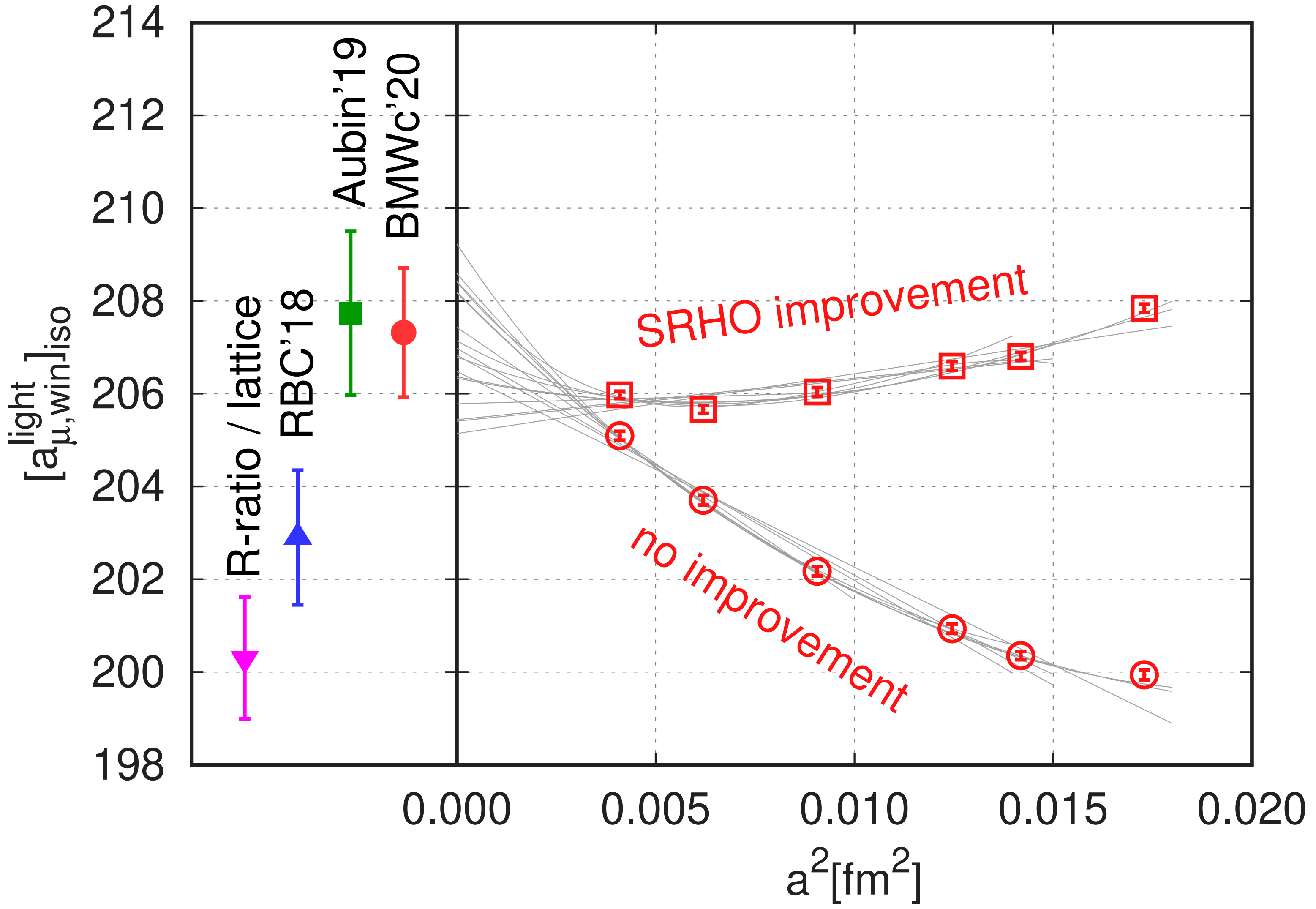


https://indico.cern.ch/event/956699/contributions/4117838/



Intermediate window (ud)

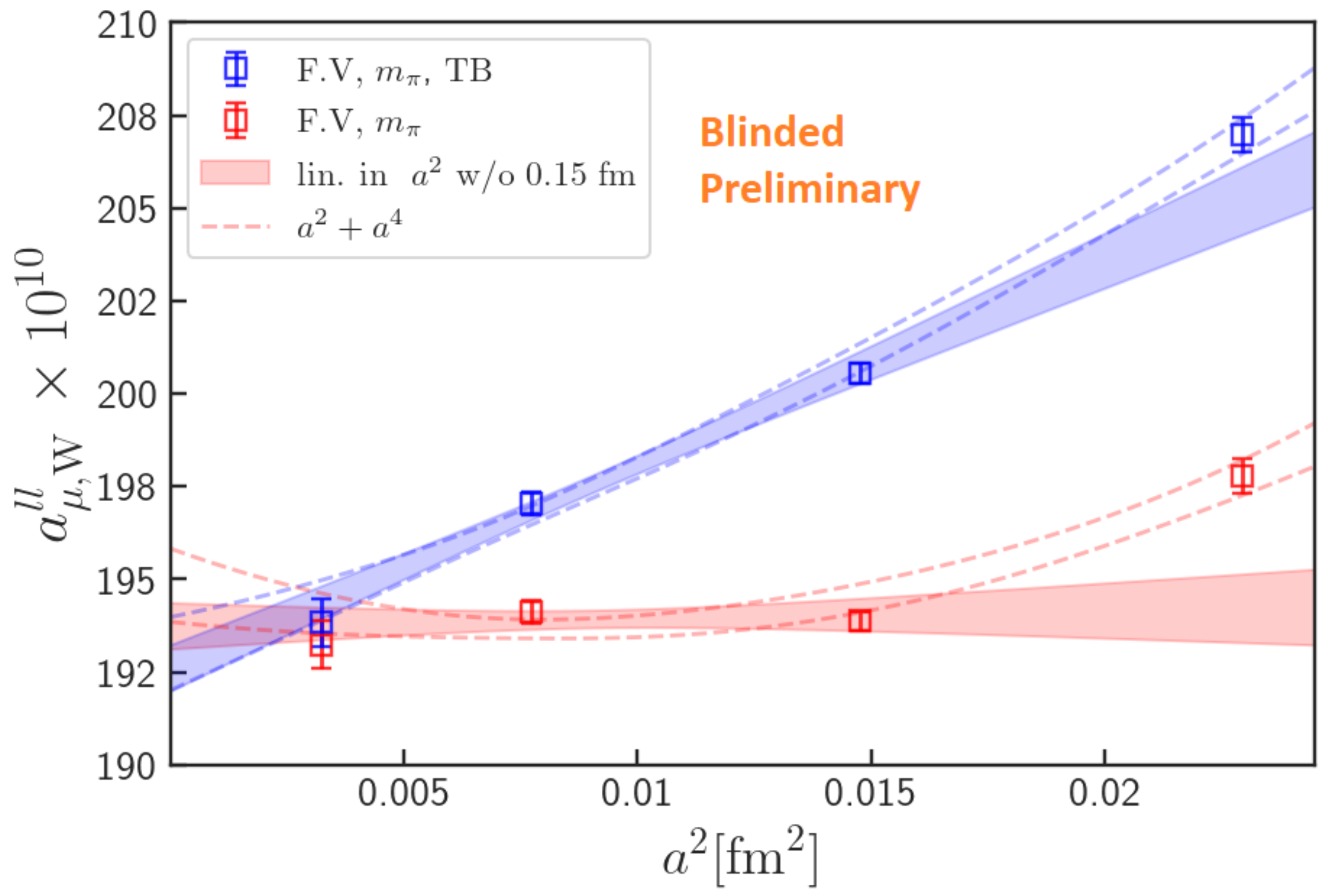
BMWc [S. Borsanyi et al, arXiv:2002.12347, Nature 2021]



-3.7 σ tension with data-driven evaluation

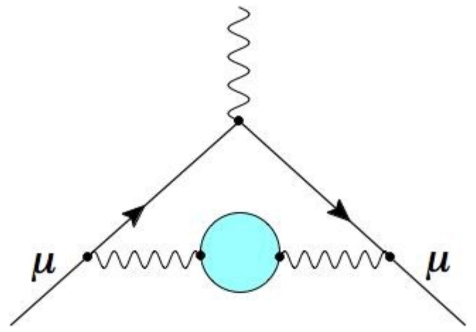
-2.2 σ tension with RBC/UKQCD18

Shaun Lahert @ Lattice 2021 (updated)



Ongoing work:

- continuum limit with & without taste-breaking corrections
- varying functional forms for continuum extrapolations
- FV corrections using different approaches

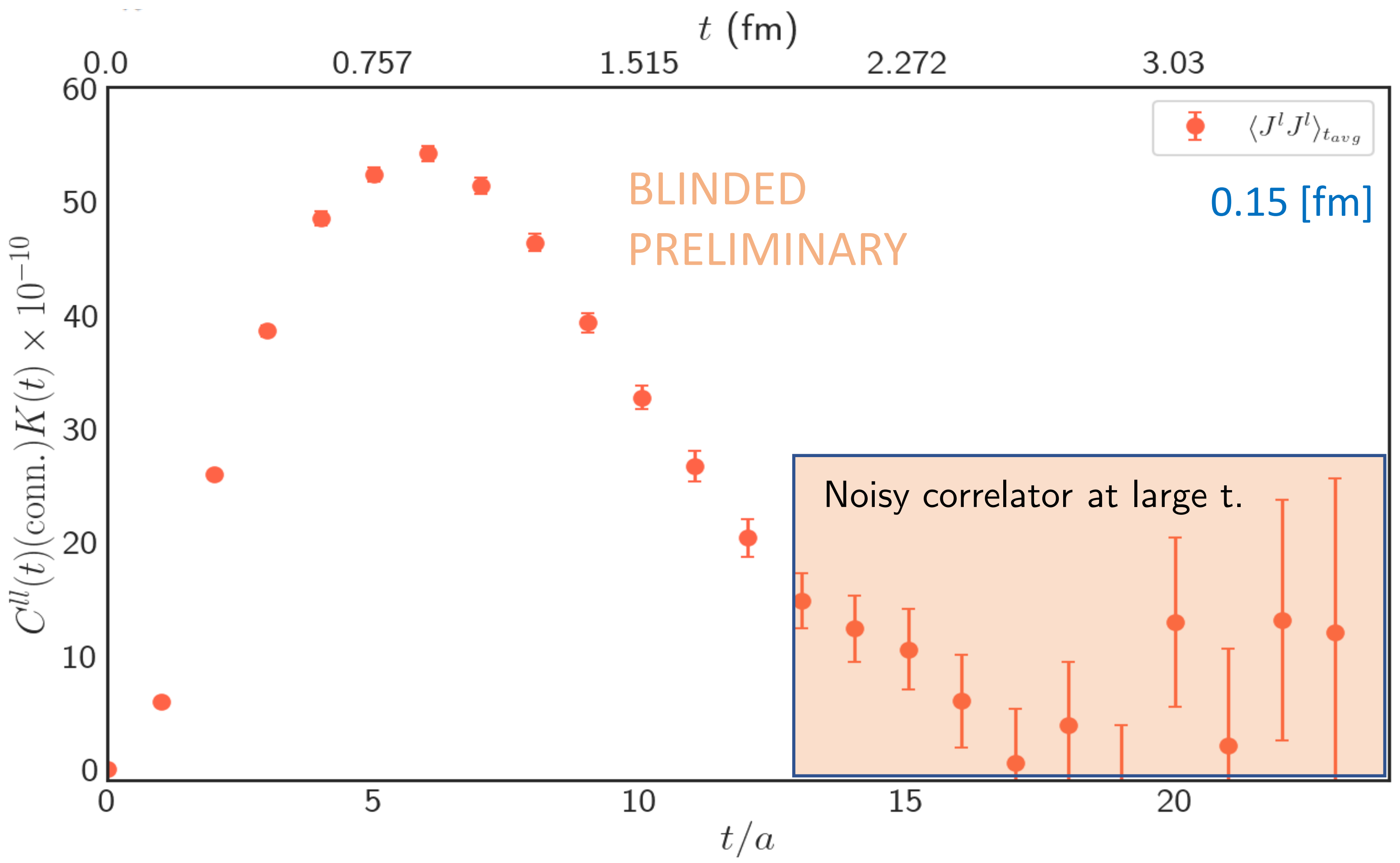


Long-distance tail (ud)

Shaun Lahert @ Lattice 2021

$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\,j_i(0,0)\rangle$$

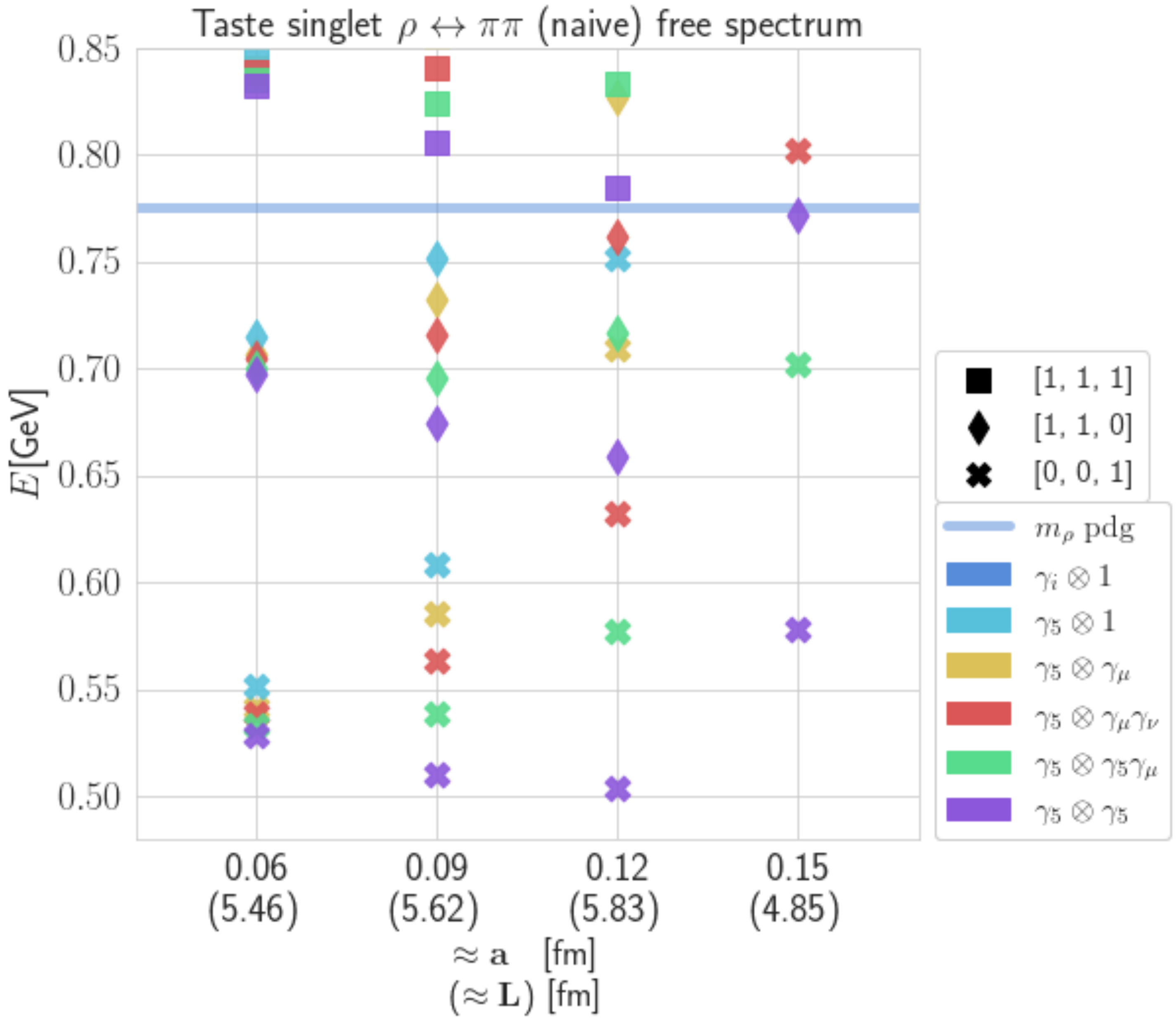
- Growth of statistical noise at large t:

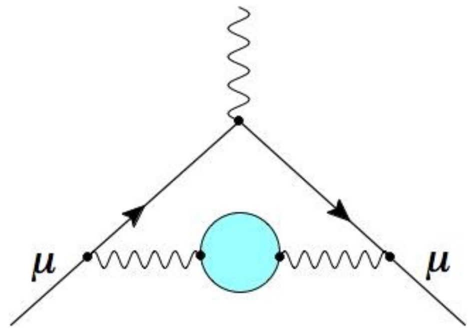


- Spectral reconstruction:
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ First LQCD calculation with staggered multi-pion operators
  - ✦ Construct matrix of correlators (2,3,4-point functions):

$$\mathbf{C}(t) = \begin{pmatrix} C(t)_{J,\tilde{J}\to J,\tilde{J}} & C(t)_{J,\tilde{J}\to\pi\pi} \\ C(t)_{\pi\pi\to J,\tilde{J}} & C(t)_{\pi\pi\to\pi\pi} \end{pmatrix}$$

  - ✦ Use GEVP to obtain energies and amplitudes for $\pi\pi$ states
  - ✦ Reconstruct vector-current correlator





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Shaun Lahert @ Lattice 2021

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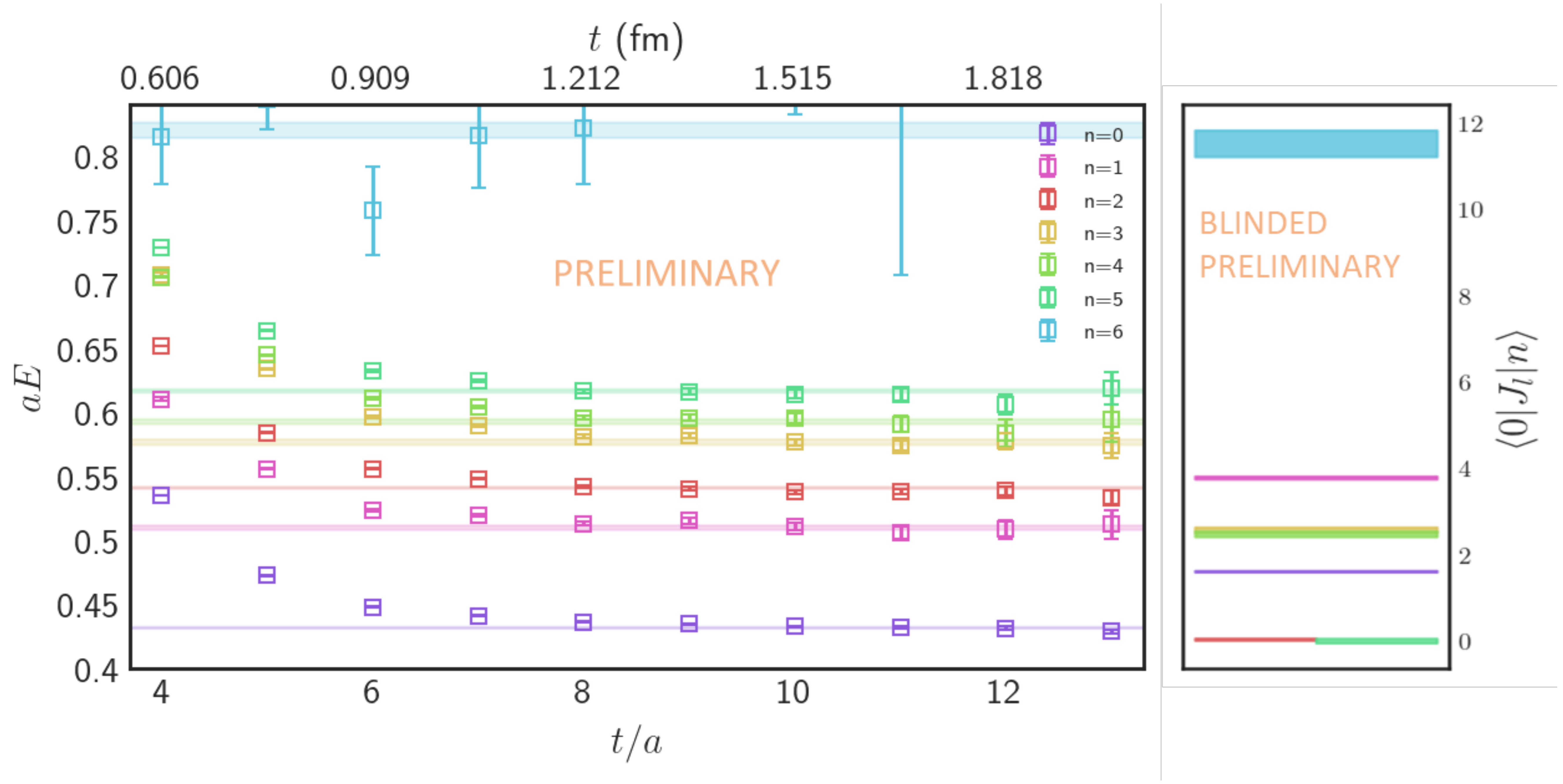
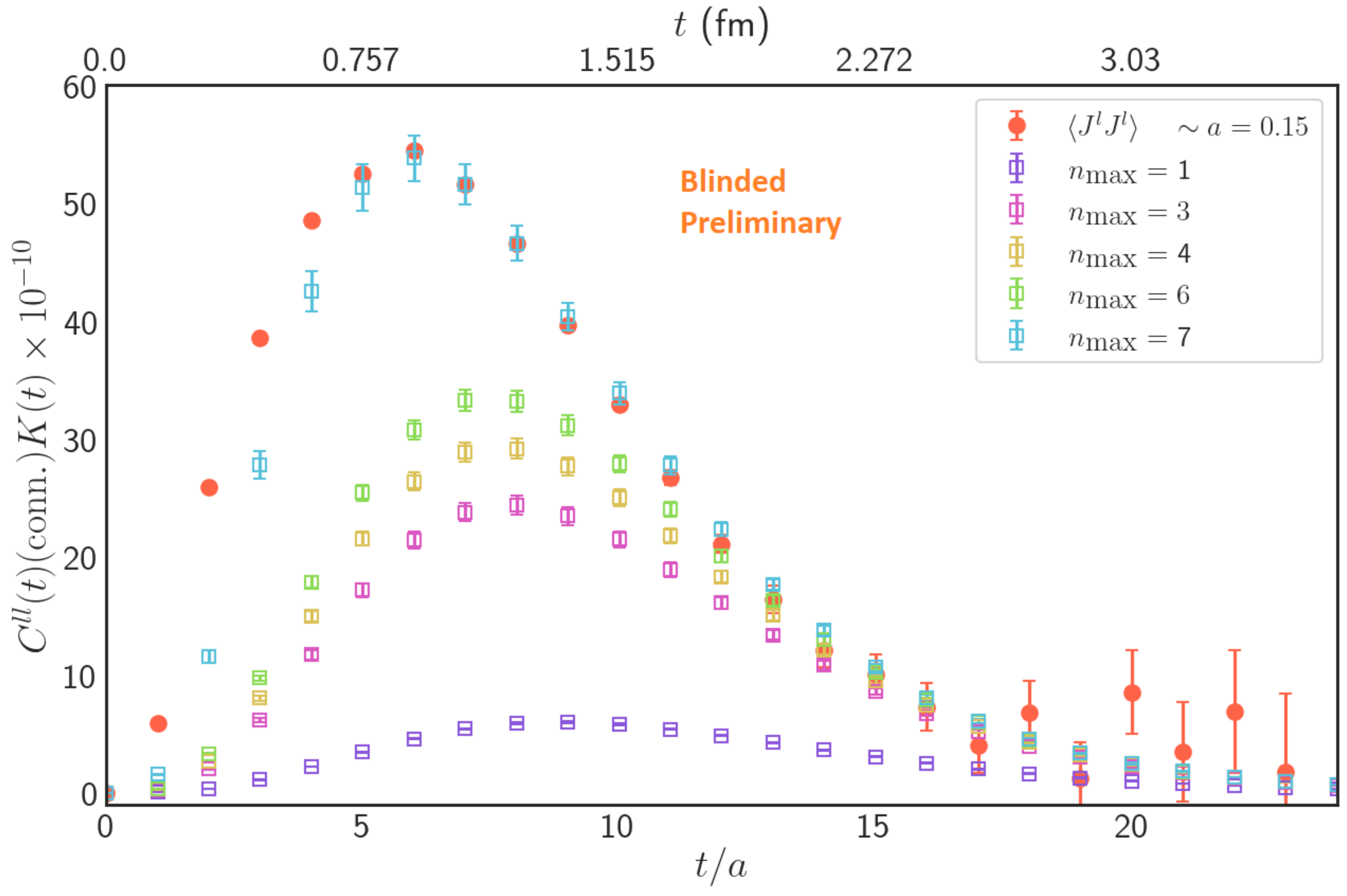
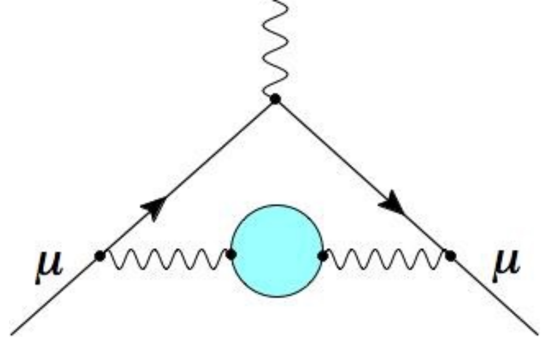


✦ Reconstruct vector current correlator at large t

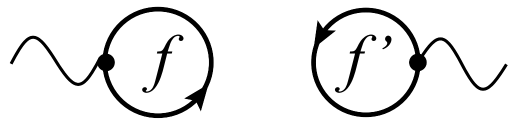


~four-fold reduction in statistical uncertainty compared to bounding method



Disconnected contribution

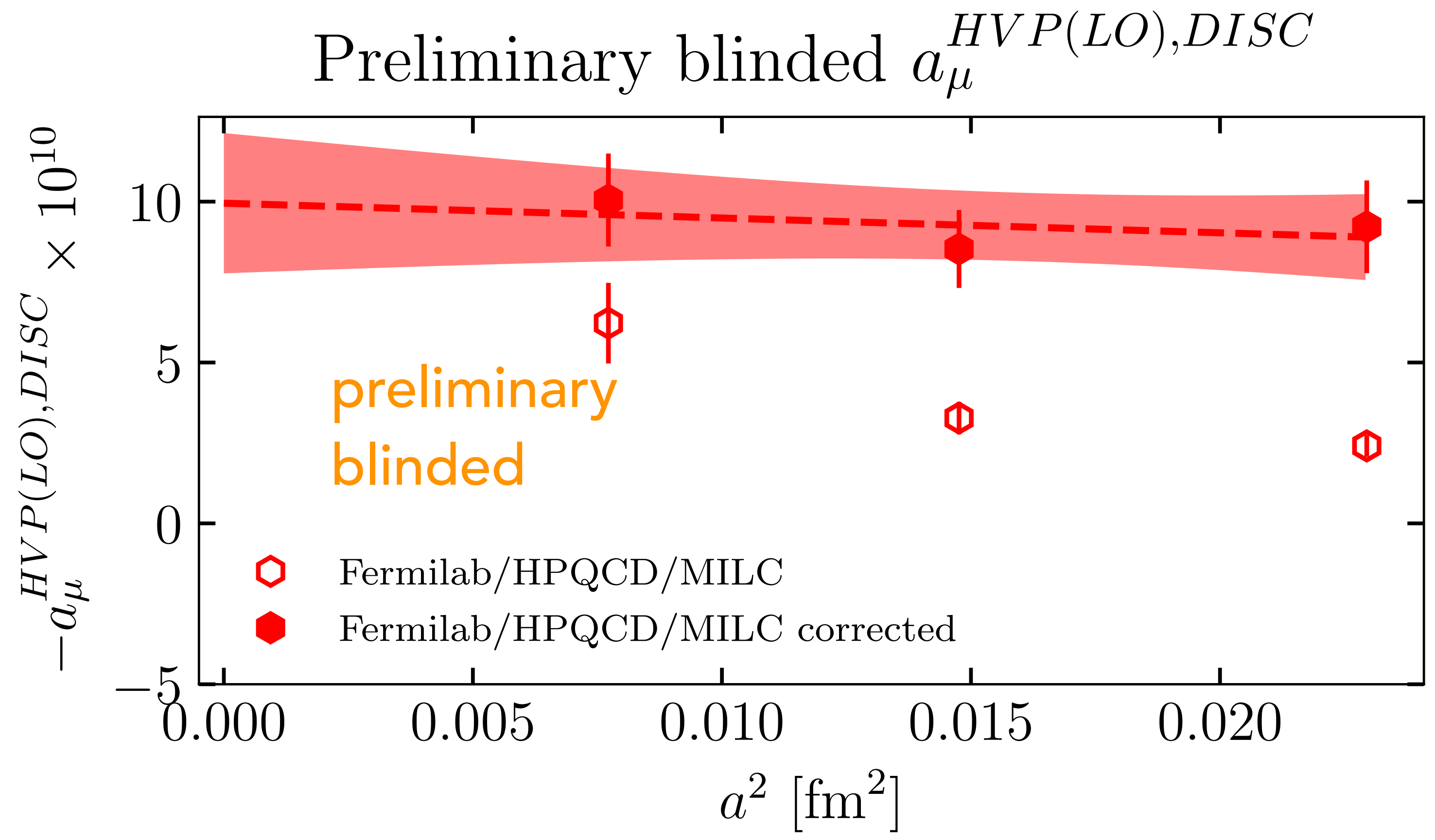
Craig McNeile @ Lattice 2021

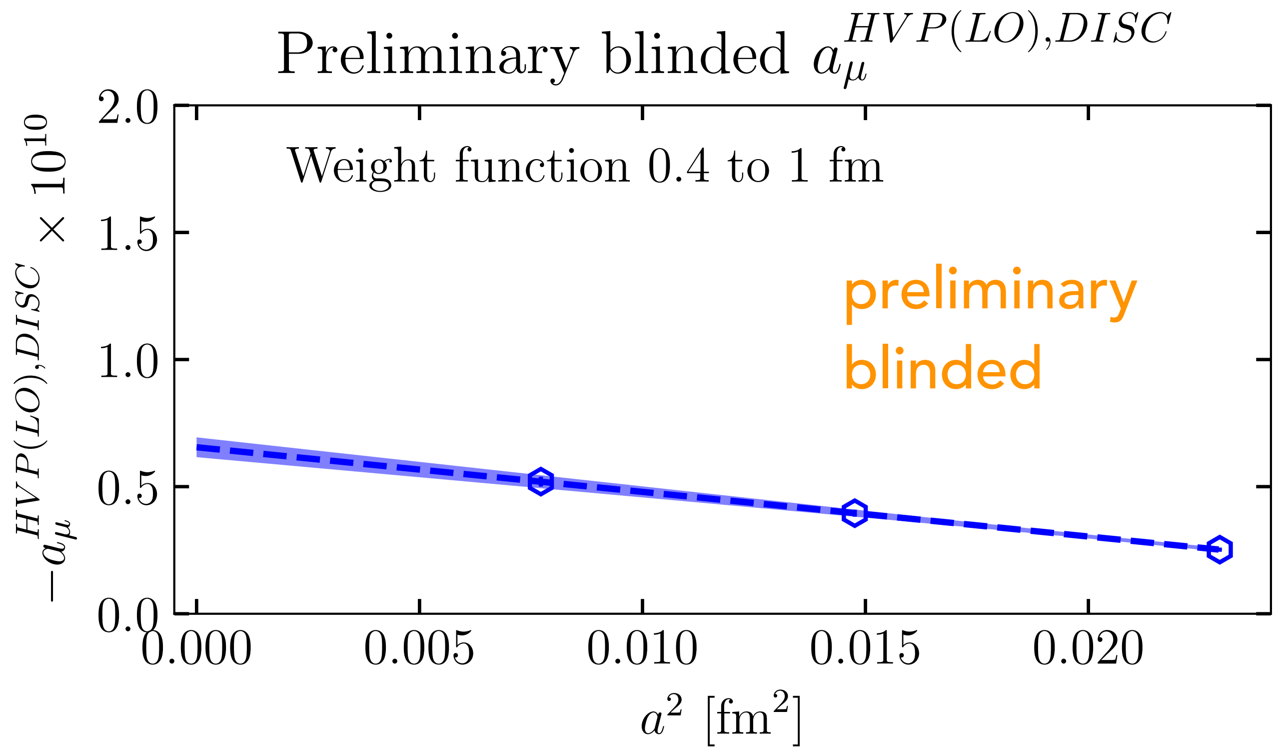


a fm	m_π MeV	L fm	Eigenmodes	N_{meas}
0.15	134.7	4.8	300	1692
0.12	134.9	5.8	-	787
0.09	128.3	5.8	1000	271

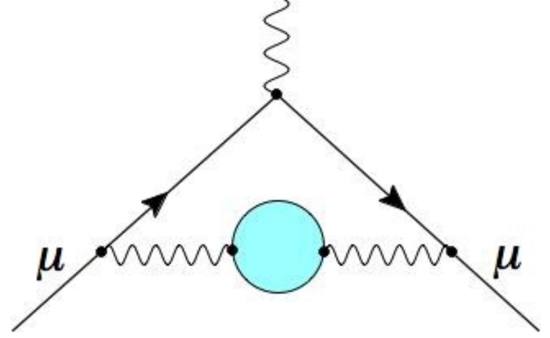
Ongoing work:

- computing SIB correction of disconnected contribution
- increasing statistics
- continuum limit with & without taste-breaking corrections
- varying functional forms for continuum extrapolations
- FV corrections using different approaches



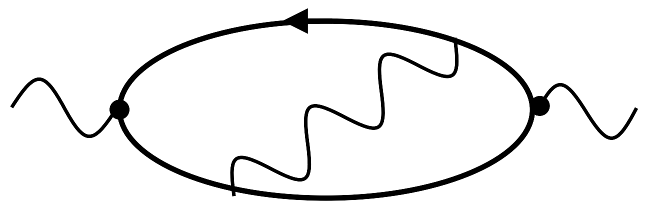


No taste-breaking corrections



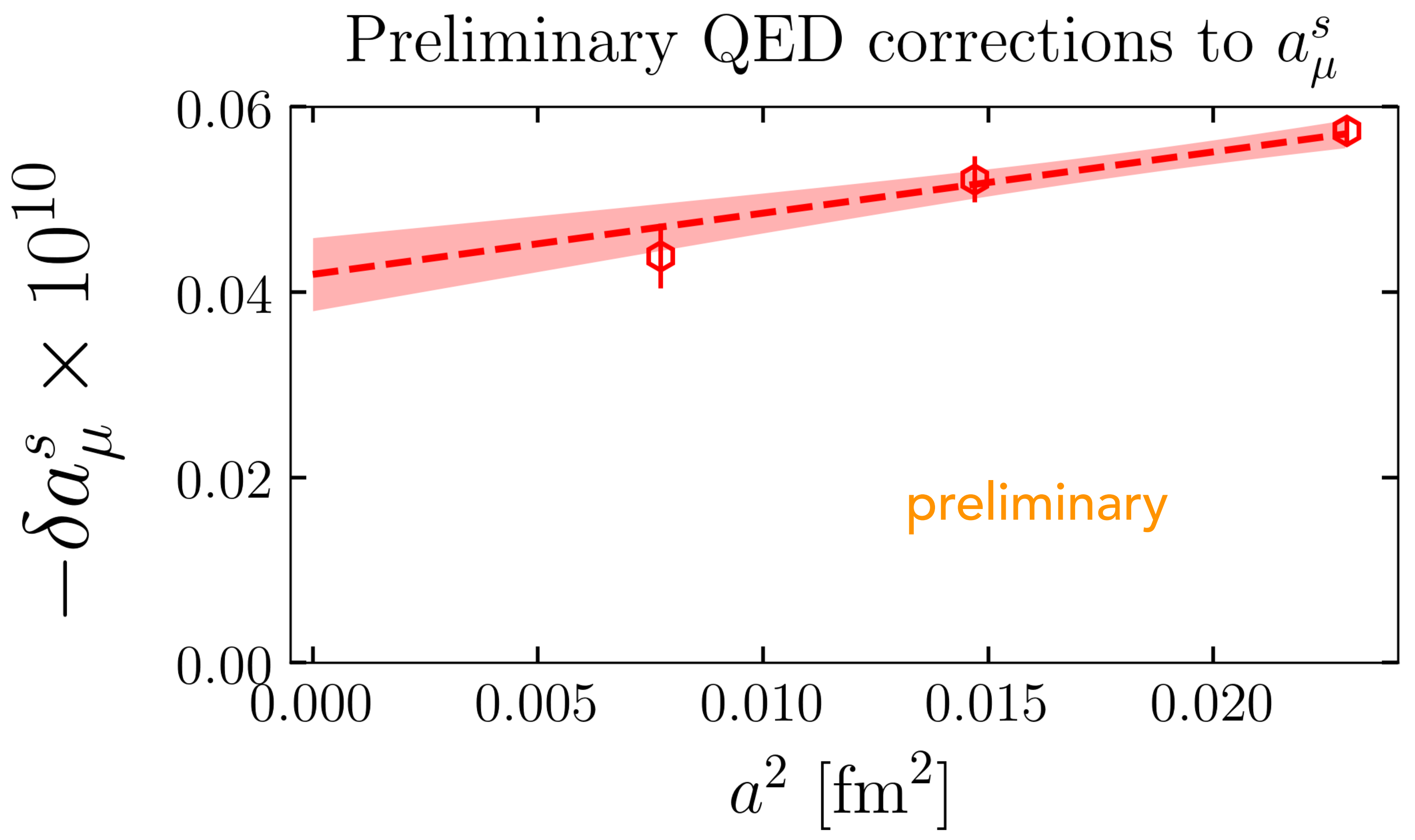
SIB + QED corrections

Craig McNeile @ Lattice 2021



$+ \ldots$

$$\delta a_\mu^s = a_\mu^s[QCD + qQED] - a_\mu^s[QCD]$$



Set-up for QED corrections: (Gaurav Ray)

- Quenched QED_L study
- Physical mass ensembles at 3 lattice spacings (0.15, 0.12, 0.09 fm)
- Range of valence quark masses: light → strange

Ongoing work:

- Increasing statistics
- retune quark masses
- varying functional forms for continuum extrapolations
- dynamical QED+QCD ensemble @ 0.15 fm being generated
- disc. QED corrections

Ongoing work SIB corrections: (Curtis Peterson)

- follow-up to previous work [Chakraborty et al, arXiv:1710.11212, 2018 PRL]:
- analyzing data @ 0.12fm
- add data at finer lattice spacings
- use results to study M_π dependence

Summary and Outlook

- Ultimate goal: $a_\mu^{\rm HVP,LO}$ at $\lesssim 0.5\,\%$
- Blind analysis of dominant contributions
- light-quark connected contribution is needed with commensurate precision:
  - Increasing statistics at finest lattice spacings
  - Reduce scale setting uncertainty (Ω baryon mass, refined f_π)
  - Refine estimates of FV and other corrections
  - Reconstruction of $\pi\pi$ tail
  - Add data at a fifth lattice spacing
- Intermediate window: analysis in progress (~early 2022)
- Disconnected contribution (includes SIB): goal $< 10\,\%$
  - Increasing statistics, refining analysis
- QED corrections:
  - Quenched QED_L study at three lattice spacings (phys. mass ensembles)

Farah Willenbrock

Appendix

Staggered two-pion operators

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Staggered two-pion operators

Staggered pion operator

$$\pi^-(\vec{p})_\xi = \sum_x e^{i\vec{p}\cdot\vec{x}} \bar{u}(x)\gamma_5 \otimes \gamma_\xi d(x)$$

Zero momentum taste irreps:

Scalar & Pseudo-scalar (1D)
Vector & Pseudo-vector (3D)
Tensor (3D+3D)

Non-zero momentum taste irreps:

1D irreps are Wilson like.
3D irreps can split (1D+2D) based on momentum & taste direction alignment.

Two-pion operators

$$\mathcal{O}_{\pi\pi}(0) = \sum_{\substack{\xi_1,\xi_2 \\ \vec{p}\in\{p\}^\star}} CG_{\text{stag, iso.}}\, \pi(\vec{p})_{\xi_1}\pi(-\vec{p})_{\xi_2}$$

Clebsch-Gordon's from irreducible representations of staggered lattice symmetry group.

$$SU_I(2) \times \left(T_N^3 \rtimes \{\Xi_\mu, C_0\} \rtimes \left\{\tilde{R}_{ij}, I_S\right\}\right) = SU(2) \times \left(Z_N^3 \rtimes \Gamma_{4,1} \rtimes W_3\right)$$

Wigner's method twice ~ little groups of little groups.

$$CG = \text{unitarity} : \sum_{g\in G} \left[D^{(\pi_1)}(g) \times D^{(\pi_2)}(g)\right] AD^{(J)}(g)^\dagger$$

-Sakata, J.Math.Phys. 15 (1974) 1702-1709

Taste scalar vector current "one-link" -> taste diagonal two pion states.

Operator	Momentum (back-to-back)
$J_l, \tilde{J}_l$	
$O_{\pi\pi}^{\otimes\gamma_5}$	$[0,0,1], [1,1,0]$
$O_{\pi\pi}^{\otimes\gamma_{5x/y}}, O_{\pi\pi}^{\otimes\gamma_{5z}}$	$[0,0,1]$
$O_{\pi\pi}^{\otimes\gamma_{xt/yt}}, O_{\pi\pi}^{\otimes\gamma_{zt}}$	$[0,0,1]$

$$\mathbf{C}(t) = \begin{pmatrix} C_{J_l,\tilde{J}_l\to J_l,\tilde{J}_l}(t) & C_{J_l,\tilde{J}_l\to\pi\pi}(t) \\ C_{\pi\pi\to J_l,\tilde{J}_l}(t) & C_{\pi\pi\to\pi\pi}(t) \end{pmatrix}$$

$$\mathbf{C}(t)v = \lambda \mathbf{C}(t_0)\, v.$$

Solve GEVP:

$\chi_n = (v_n)_i \mathcal{O}_i$ with best overlap with state n.

$$\langle \chi_n \chi_n^\dagger \rangle = \sum_n Z_n^2 e^{-E_n t}$$

$$\langle \chi_n J_l^\dagger \rangle = \sum_n Z_n \langle 0 | J_l | n \rangle e^{-E_n t}$$