RBC/UKQCD: status & near-term prospects for the muon g-2 HVP contribution

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December 14, 2021 – DWQ @ 25

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RBC/UKQCD status 2018

PHYSICAL REVIEW LETTERS **121,** 022003 (2018)

Editors' Suggestion

Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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(Received 25 January 2018; published 12 July 2018)

We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_\mu^{\mathrm{HVP\ LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R-ratio data, we significantly improve the precision to $a_\mu^{\mathrm{HVP\ LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_\mu^{\mathrm{HVP\ LO}}$.

Pure lattice result and dispersive result with reduced $\pi\pi$ dependence (window method)

Aaron Meyer (BNL $\to$ LBNL) & Mattia Bruno (BNL $\to$ CERN $\to$ Milano) joined since this 2018 paper

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Window method (introduced in RBC/UKQCD 2018)

We also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

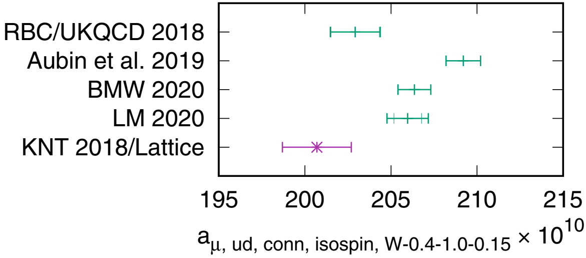
with

$$\begin{aligned} a_\mu^{\rm SD} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\ a_\mu^{\rm W} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\ a_\mu^{\rm LD} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\ \Theta(t, t', \Delta) &= \left[1 + \tanh\left[(t - t')/\Delta\right]\right]/2\,. \end{aligned}$$

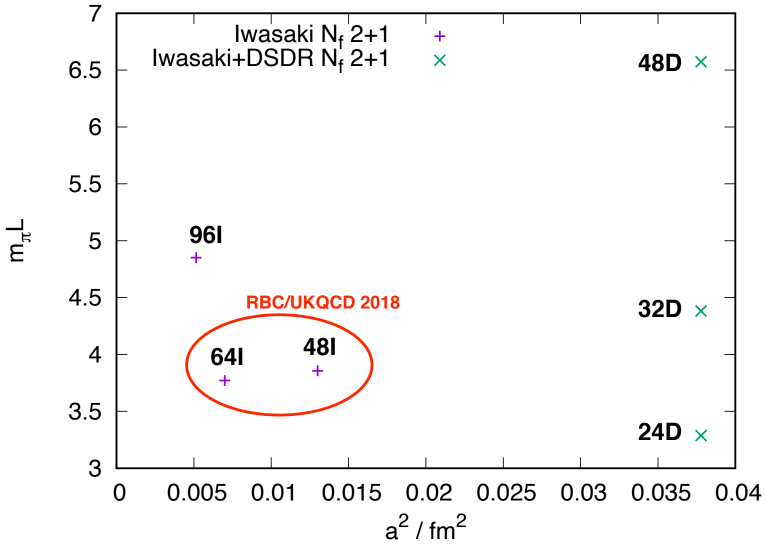
All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to {\rm had})$.

$a_\mu^{\rm W}$ has small statistical and systematic errors on lattice!

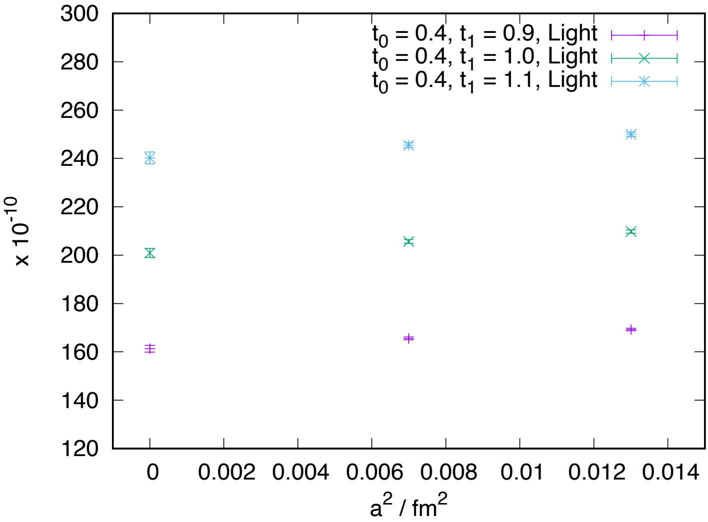
- In last few years, we reported on our progress for the complete result (improved bounding method, $(\pi\pi)_{I=1}$ phase shift study, improvements for disconnected/QED/SIB diagrams), this talk focuses entirely on progress on the Euclidean time window (RBC/UKQCD 2018) in the isospin symmetric limit with $t_0 = 0.4$ fm, $t_1 = 1.0$ fm, $\Delta = 0.15$ fm.
- This quantity promises reduced systematic lattice uncertainties, however, currently exhibits tensions between different lattice and R-ratio results:



Our extended list of ensembles, all with $m_\pi = 135 \pm 5$ MeV:

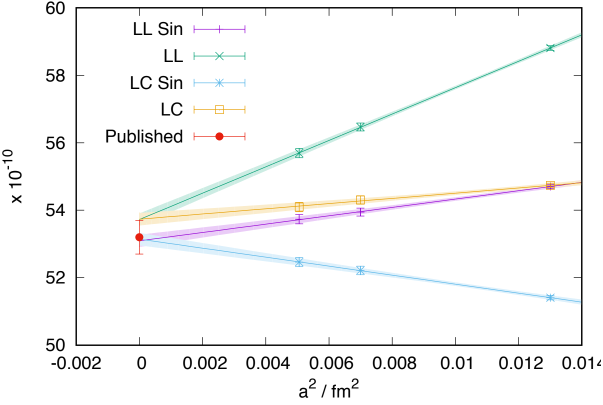


Previous continuum extrapolation mild (plot from RBC/UKQCD 18), will now check with even finer lattice spacing:



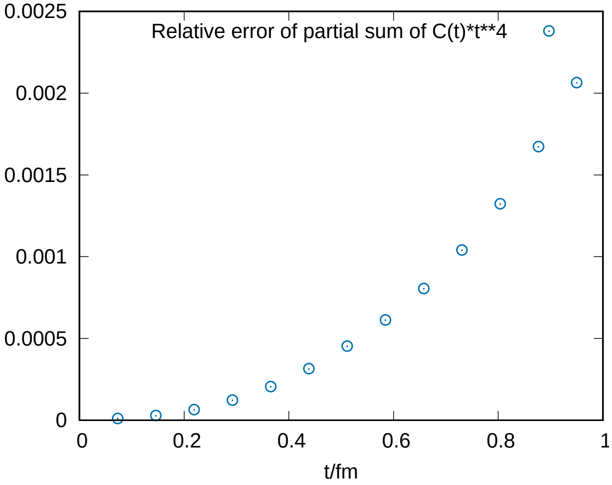
Previously already added $a^{-1} = 2.77$ GeV ensemble for strange quark:

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



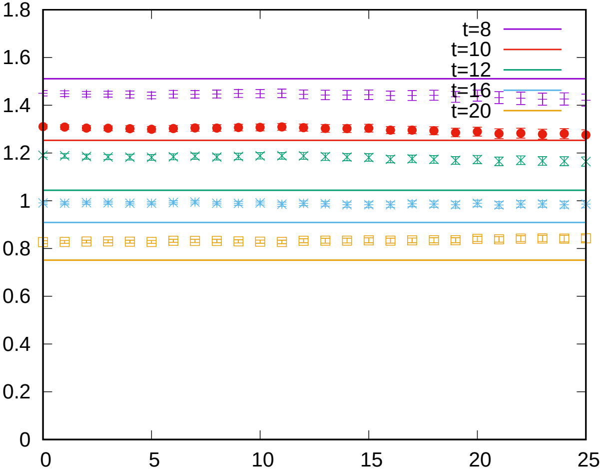
- For light quark use new 96I ensemble at physical pion mass. Data generated on Perlmutter and Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

We are still blinded, show only statistical relative error for new 96I:



Now full statistics (2x compared to what I showed at Lattice 21); 1 per mille statistical precision on finest lattice crucial for high-precision window tests

Master field error estimate versus simple Jackknife on 33 configurations on 96I:



y-axis: blinded noise, x-axis: distance cut for MF estimate

Other improvements relevant for window:

- 4x statistics on 48I and 64I
- Explicit calculation of parametric derivatives at physical point
- Concluding study of missing charm determinant ($2{+}1 \rightarrow 2{+}1{+}1$)
- All of the above based on master-field-type study, see next slides

Master-field calculation of gradients

For a local observable

$$O = \frac{1}{V} \sum_y O_y \tag{1}$$

we can define the truncated master-field covariance

$$\mathrm{Cov}_R(O, A) \equiv \frac{1}{V} \sum_{x,y,|y| \leq R} \left(\langle O_x A_{x+y} \rangle_\beta - \langle O_x \rangle_\beta \langle A_{x+y} \rangle_\beta \right) \tag{2}$$

such that, e.g., the β-derivative of O is given by

$$\frac{\langle O \rangle_{\beta+\varepsilon} - \langle O \rangle_\beta}{\varepsilon} = 6 \lim_{R \to \infty} \mathrm{Cov}_R(O, A)\,. \tag{3}$$

In practice use exponential approach to plateau for $R \to \infty$.

We isolate the dependence on sea-quark mass m of an observable O by studying

$$\langle O \rangle_m \equiv \frac{\int \det(D(m)) O P}{\int \det(D(m)) P} \tag{4}$$

with Dirac matrix $D(m)$ and residual weight P. Can show that

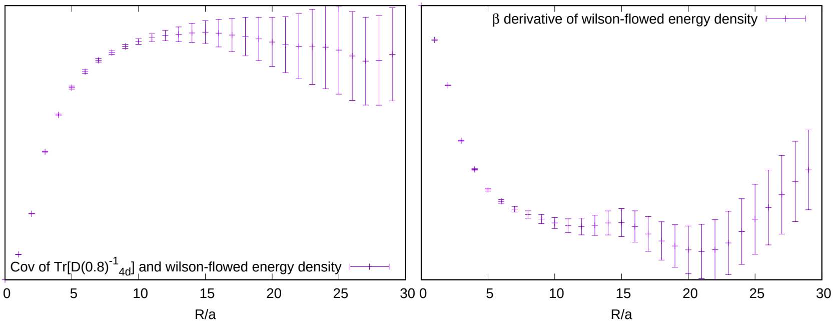
$$\frac{\langle O \rangle_{m+\varepsilon} - \langle O \rangle_m}{\varepsilon} = \mathrm{Cov}(O, \mathrm{Tr}[D_{4d}^{-1}(m)]) + \mathcal{O}(\varepsilon)\,. \tag{5}$$

Finally, for DWF an additional flavor enters as

$$\det(D(m) D^{-1}(1)) \tag{6}$$

such that for $m = 1$ the factor is trivial and we can view adding an additional flavor as changing the sea-quark mass down from $m = 1$ to the target value.

Example for wilson-flowed energy density (96I, $t_0 \approx 2$)



Computed in similar way also derivatives of, e.g., VV and PP correlators.

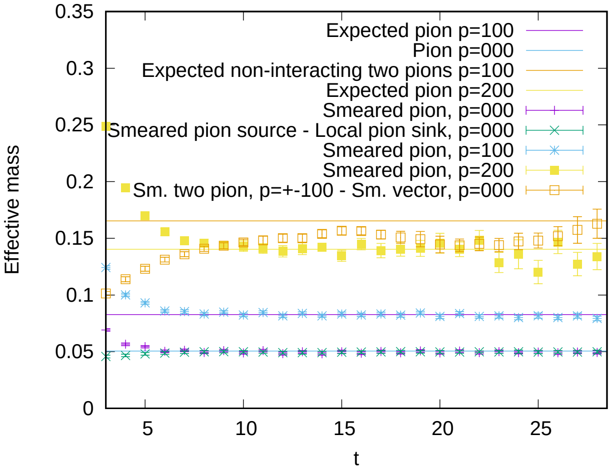
Blinding

- 3 analysis groups for ensemble parameters (not blinded)
- 6 analysis groups for vector-vector correlators (blinded, to avoid bias towards other lattice/R-ratio results)
- Blinded vector correlator $C_b(t)$ relates to true correlator $C_0(t)$ by

$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t) \tag{7}$$

with appropriate random b_0, b_1, b_2, different for each analysis group. This prevents complete unblinding based on previously shared data on coarser ensembles.

Preview of distillation data on finest 96I ensemble for improved bounding method:



Early science time on Perlmutter was/is crucial! Above is master field study with one configuration and limited measurements on it.

Conclusions and Outlook

- 4x statistics on 48I and 64I
- Third lattice spacing (96I) at $a^{-1} \approx 2.7$ GeV
- 2+1 $\rightarrow$ 2+1+1 and parametric gradients using master-field approach
- Still blinded, publish window results first
- For complete HVP analysis data set almost complete as well (still finishing distillation data on 96I, a lot of new data also on QCD+QED)