Data-driven approach to hadronic contributions to $(g-2)_\mu$

Gilberto Colangelo

b UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

DWQ@25, 14.12.2021

Outline

Introduction: present status of $(g-2)_\mu$

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

- Consequences of the BMW result

Hadronic light-by-light contribution to $(g-2)_\mu$

- Short-distance constraints

Conclusions and Outlook

Outline

Introduction: present status of $(g-2)_\mu$

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$
- Consequences of the BMW result

Hadronic light-by-light contribution to $(g-2)_\mu$
- Short-distance constraints

Conclusions and Outlook

Present status of $(g-2)_\mu$, experiment vs SM

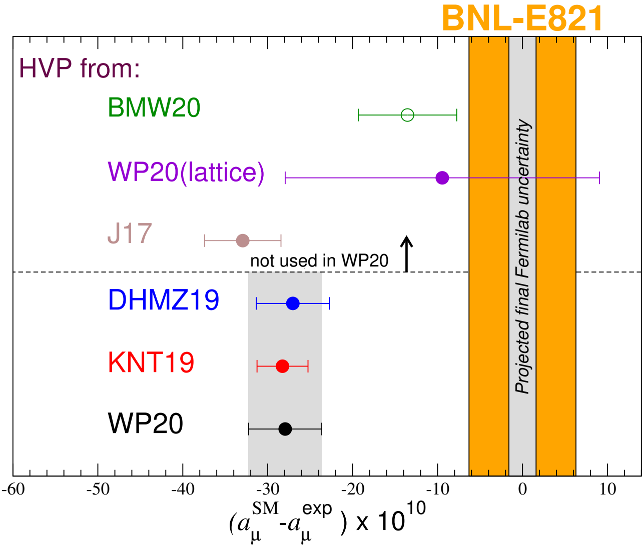
$$a_\mu(BNL) = 116\,592\,089(63) \times 10^{-11}$$

$$a_\mu(FNAL) = 116\,592\,040(54) \times 10^{-11}$$

$$a_\mu(Exp) = 116\,592\,061(41) \times 10^{-11}$$

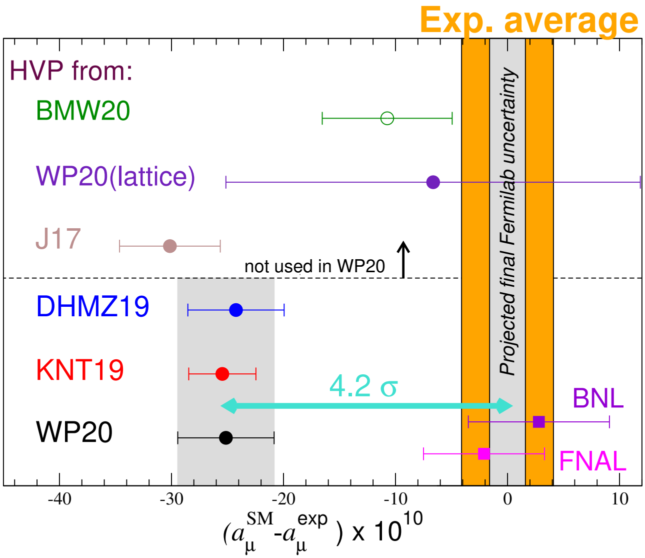
Present status of $(g-2)_\mu$, experiment vs SM

Before the Fermilab result



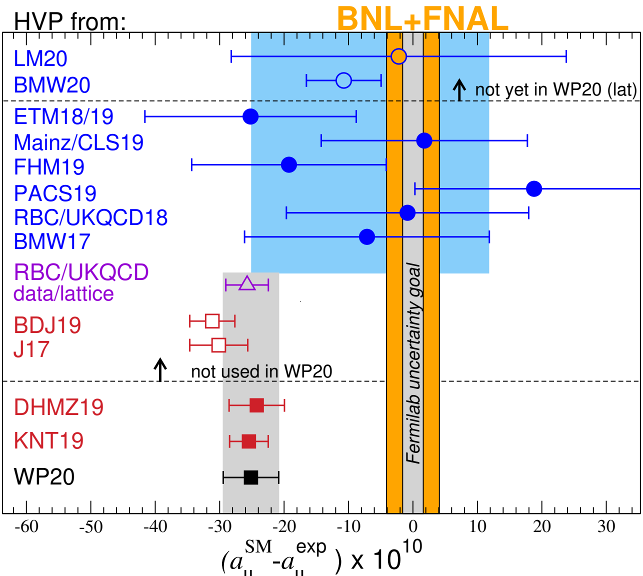
Present status of $(g-2)_\mu$, experiment vs SM

After the Fermilab result



Present status of $(g-2)_\mu$, experiment vs SM

After the Fermilab result



White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)

White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice BMW(20), *udsc*)	7075(55)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)

White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier (vice-chair)
Aida El-Khadra (chair)
Martin Hoferichter
Laurent Lellouch
Christoph Lehner (vice-chair)
Tsutomu Mibe (J-PARC E34 experiment)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Workshops:

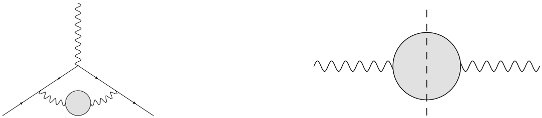
- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Lattice HVP workshop, virtual, 16-20 November 2020
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021
- Fifth plenary meeting, Higgs Center, Edinburgh, 5-9 Sept. 2022

White Paper executive summary (my own)

- QED and EW known and stable, negligible uncertainties
- HVP dispersive: consensus number, conservative uncertainty (KNT19, DHMZ19, CHS19, HHK19)
- HVP lattice: consensus number, $\Delta a_\mu^{\rm HVP,latt} \sim 5\,\Delta a_\mu^{\rm HVP,disp}$ (Fermilab-HPQCD-MILC18,20, BMW18, RBC/UKQCD18, ETM19,SK19, Mainz19, ABTGJP20)
- HVP BMW20: central value $\to$ discrepancy $< 2\sigma$; $\Delta a_\mu^{\rm HVP,BMW} \sim \Delta a_\mu^{\rm HVP,disp}$ published 04/21 $\to$ not in WP
- HLbL dispersive: consensus number, w/ recent improvements $\Rightarrow$ $\Delta a_\mu^{\rm HLbL} \sim 0.5\,\Delta a_\mu^{\rm HVP}$
- HLbL lattice: single calculation, agrees with dispersive ($\Delta a_\mu^{\rm HLbL,latt} \sim 2\,\Delta a_\mu^{\rm HLbL,disp}$) $\to$ final average (RBC/UKQCD20)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$

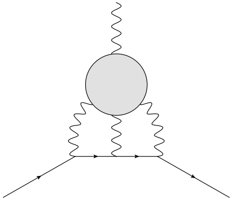


  - unitarity and analyticity $\Rightarrow$ dispersive approach
  - $\Rightarrow$ direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$
  - e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
  - alternative approach: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



- earlier: hadronic models
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- lattice QCD is becoming competitive

(Mainz, RBC/UKQCD)

Outline

Introduction: present status of $(g-2)_\mu$

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

- Consequences of the BMW result

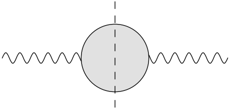
Hadronic light-by-light contribution to $(g-2)_\mu$

- Short-distance constraints

Conclusions and Outlook

HVP contribution: Master Formula

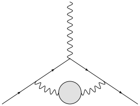
Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\left[\bar{\Pi}(q^2) = \frac{q^2}{\pi}\int ds \frac{\mathrm{Im}\bar{\Pi}(s)}{s(s-q^2)}\right] \Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)



$\Leftrightarrow$

$$a_\mu^{\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty} \frac{ds}{s} K(s)R(s)$$

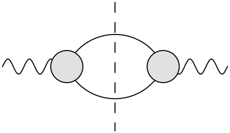
$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

The 2π contribution

For HVP the unitarity relation is simple and looks the same for all possible intermediate states, like 2π



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \pi^+\pi^-)$$

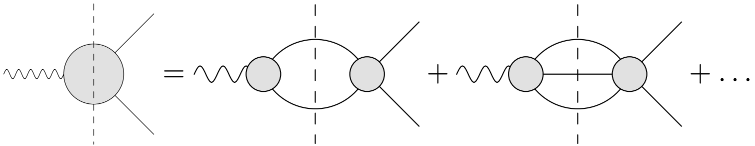
which implies

$$\bar{\Pi}_{2\pi}(q^2) == \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} dt \frac{\alpha\sigma_\pi(t)^3|F_V^\pi(t)|^2}{12t(t-q^2)}$$

de Trocóniz, Ynduráin (01,04), Leutwyler, GC (02,03), Anthanarayan et al. (13,16)

The pion vector form factor $F_V^\pi(t)$ also satisfies a dispersion relation

Omnès representation including isospin breaking



Omnès representation including isospin breaking

- Omnès representation

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

- Split elastic ($\leftrightarrow \pi\pi$ phase shift, δ_1^1) from inelastic phase

$$\delta = \delta_1^1 + \delta_{\rm in} \quad \Rightarrow \quad F_V^\pi(s) = \Omega_1^1(s)\Omega_{\rm in}(s)$$

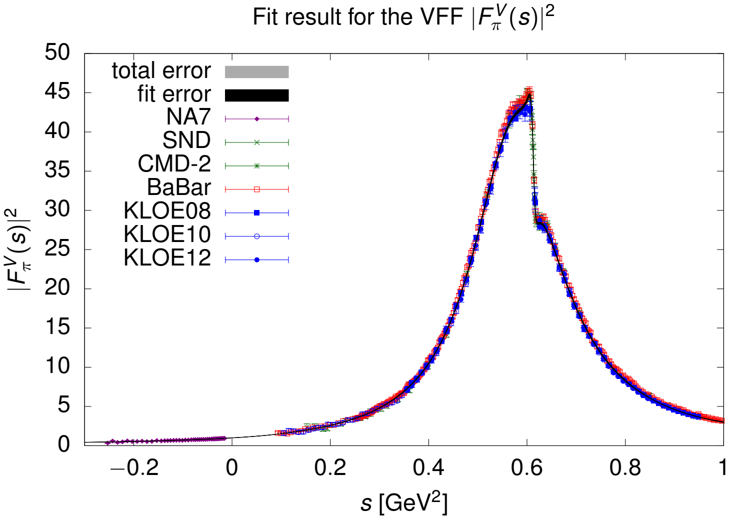
Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right), \; r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}} \Rightarrow s_{\rm in} = (M_\pi + M_\omega)^2$$

- $\rho-\omega$–mixing $\qquad F_V(s) = \Omega_{\pi\pi}(s)\cdot\Omega_{\rm in}(s)\cdot G_\omega(s)$

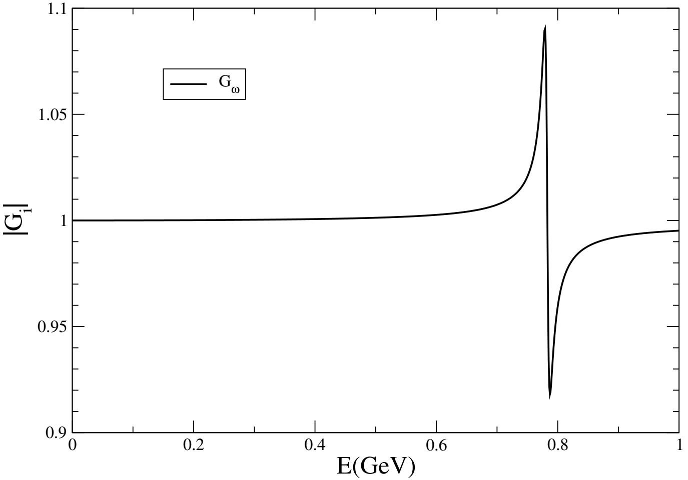
$$G_\omega(s) = 1 + \epsilon\frac{s}{s_\omega - s} \qquad \text{where} \qquad s_\omega = (M_\omega - i\Gamma_\omega/2)^2$$

Fit results

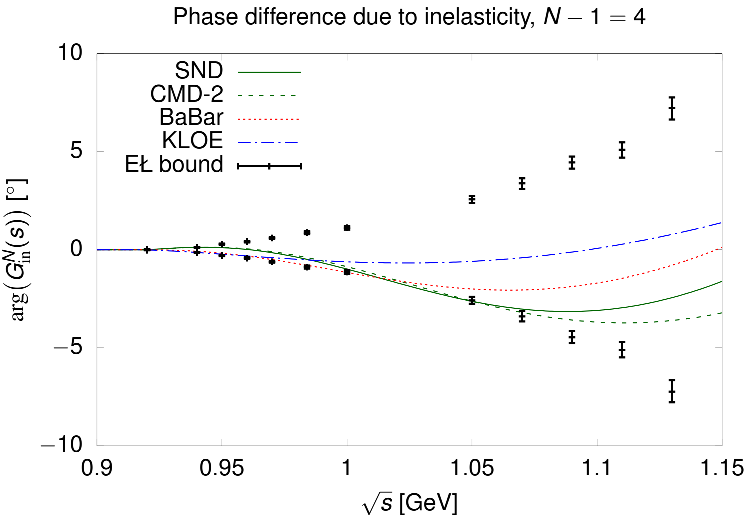


Fit result for the VFF $|F_\pi^V(s)|^2$
total error
fit error
NA7
SND
CMD-2
BaBar
KLOE08
KLOE10
KLOE12
$|F_\pi^V(s)|^2$
s [GeV^2]
0
5
10
15
20
25
30
35
40
45
50
−0.2
0
0.2
0.4
0.6
0.8
1

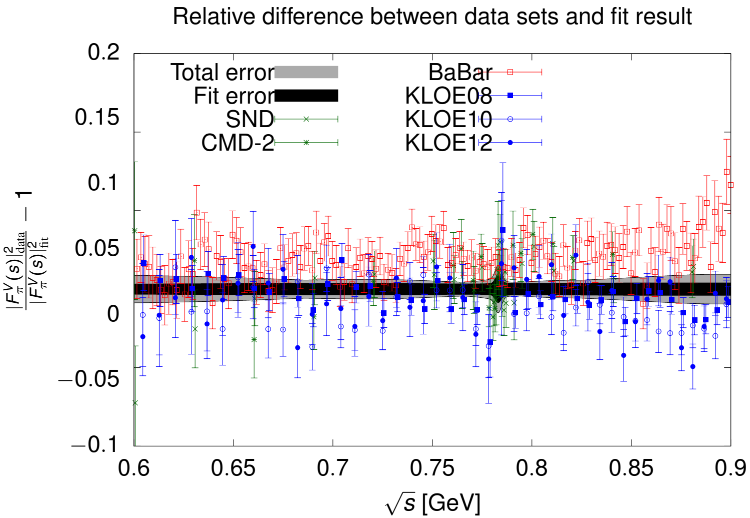
Fit results



Fit results

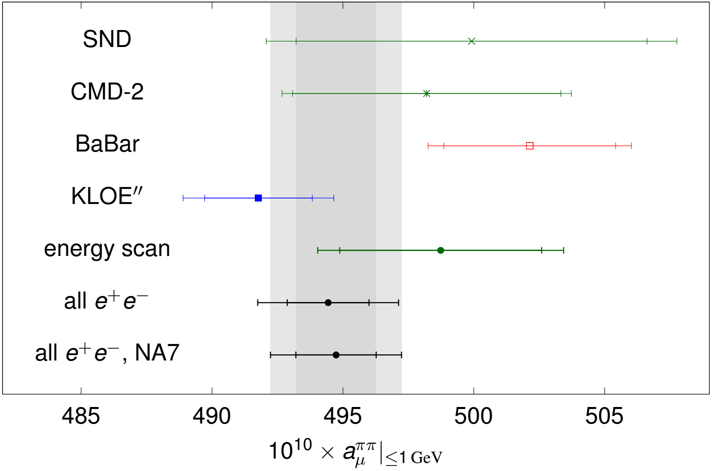

Phase difference due to inelasticity, $N-1=4$
SND
CMD-2
BaBar
KLOE
EŁ bound
$\arg(G_{\text{in}}^N(s))$ [°]
$\sqrt{s}$ [GeV]
10
5
0
−5
−10
0.9
0.95
1
1.05
1.1
1.15

Fit results


Relative difference between data sets and fit result
Total error
Fit error
SND
CMD-2
BaBar
KLOE08
KLOE10
KLOE12
$\frac{|F_\pi^V(s)|^2_{\text{data}}}{|F_\pi^V(s)|^2_{\text{fit}}} - 1$
$\sqrt{s}$ [GeV]
0.2
0.15
0.1
0.05
0
−0.05
−0.1
0.6
0.65
0.7
0.75
0.8
0.85
0.9

Results for $(g-2)_\mu$

Result for $a_\mu^{\pi\pi}|_{\leq 1\,\mathrm{GeV}}$ from the VFF fits to single experiments and combinations



2π: comparison with the dispersive approach

The 2π channel can itself be described dispersively $\Rightarrow$ more constrained theoretically Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
$\leq$ 0.6 GeV		110.1(9)	110.4(4)(5)	108.7(9)
$\leq$ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq$ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq$ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq$ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.3(1)
$\leq$ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$\left[\sqrt{0.1}, \sqrt{0.95}\right]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

WP(20)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}$$

WP(20)

Consequences of the BMW result

→ talk by K. Szabo

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

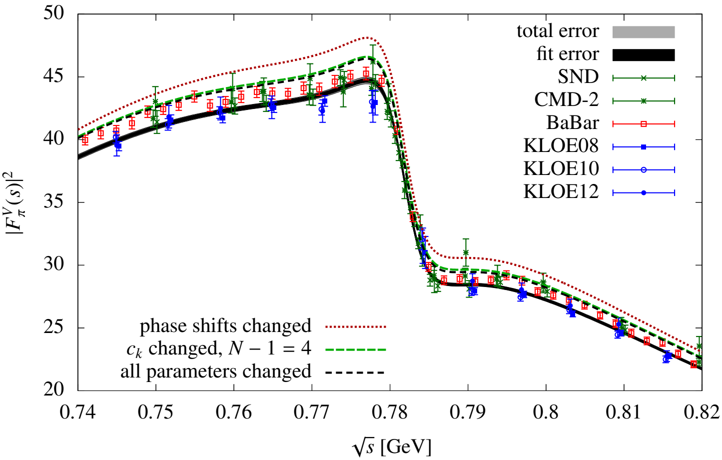
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad \Delta\sigma(e^+e^- \to \text{hadrons})$
- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (more weight at high energy)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$: size depends on the energy range of $\Delta\sigma(e^+e^- \to \text{hadrons})$
- a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below $\sim$ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \mathrm{hadrons})$ below 1 GeV?

- Below $1-2$ GeV only one significant channel: $\pi^+\pi^-$
- Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- $F_\pi^V(s)$ parametrization which satisfies these
  $\Rightarrow$ small number of parameters GC, Hoferichter, Stoffer (18)
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad$ shifts in these parameters
  analysis of the corresponding scenarios GC, Hoferichter, Stoffer (21)

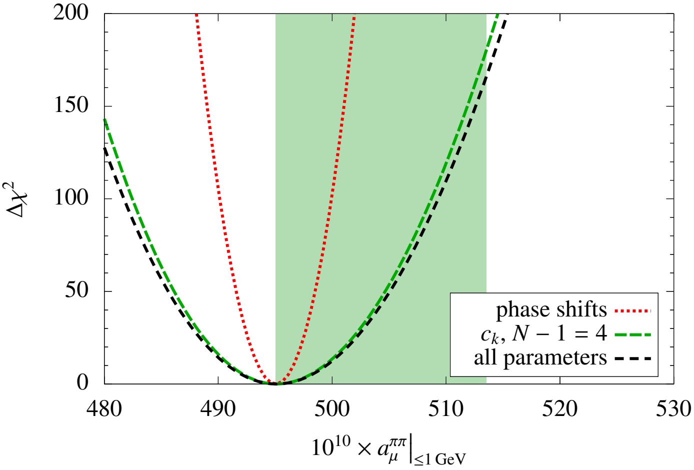
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

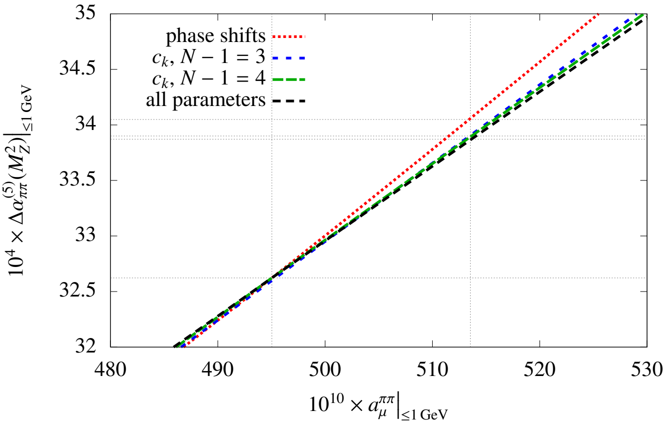
Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

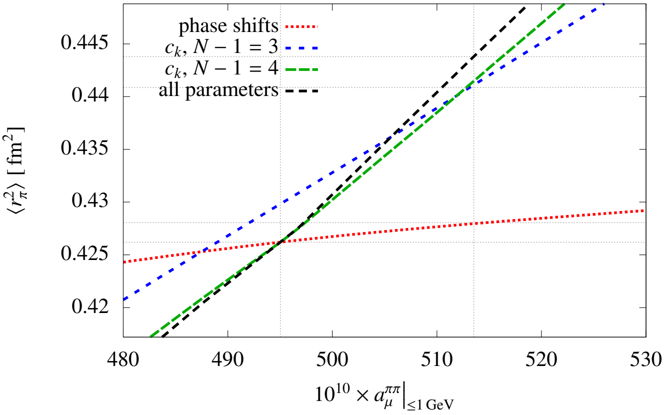
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

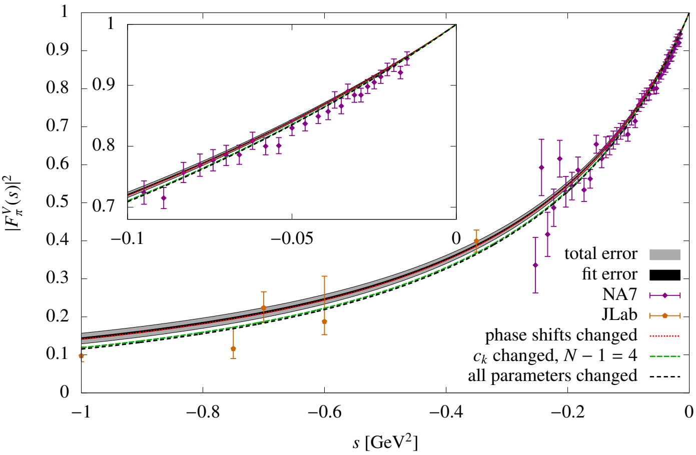
Changes in $\sigma(e^+e^- \to \mathrm{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

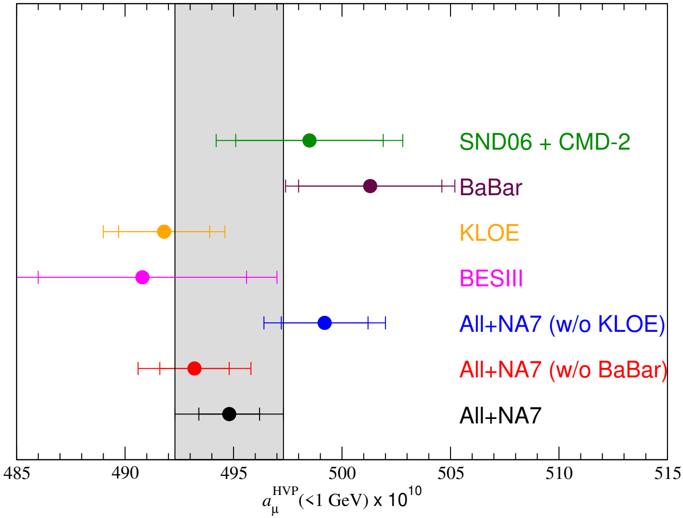
$$\langle r_\pi^2 \rangle = \begin{cases} 0.429(4)\mathrm{fm}^2 & \mathrm{CHS(18)} \\ 0.436(5)(12)\mathrm{fm}^2 & \chi\mathrm{QCD(20)} \end{cases}$$

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?

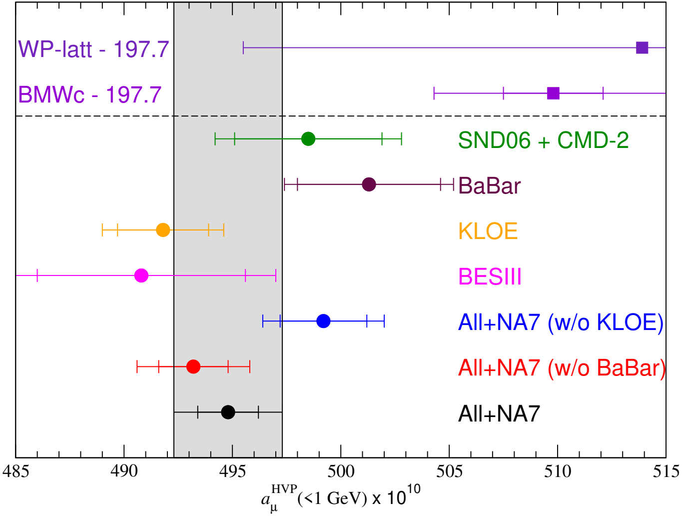


GC, Hoferichter, Stoffer (21)

BMW vs individual $\pi^+\pi^-$ experiments

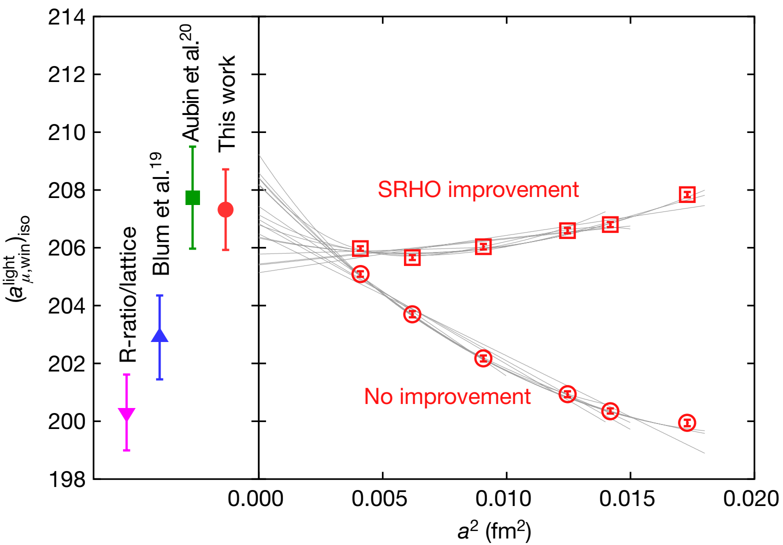


BMW vs individual $\pi^+\pi^-$ experiments



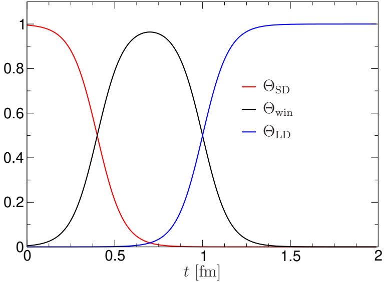
BMW vs individual $\pi^+\pi^-$ experiments

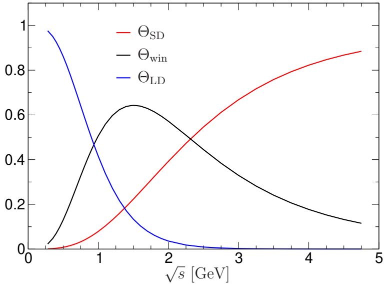
Article



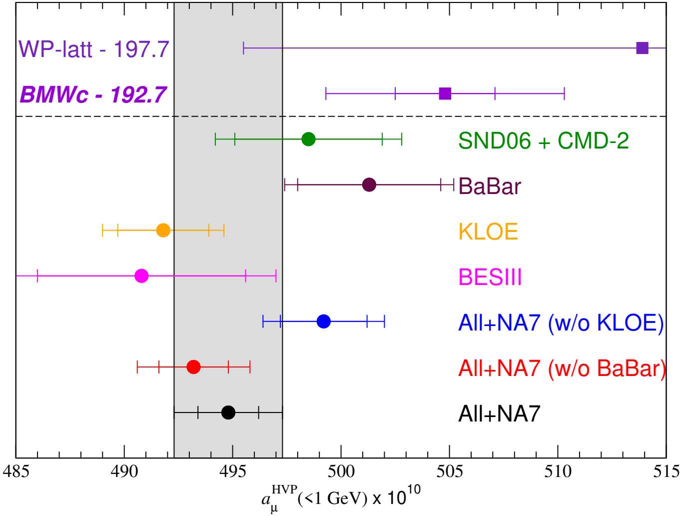
BMW vs individual $\pi^+\pi^-$ experiments

Weight functions for the window quantities





BMW vs individual $\pi^+\pi^-$ experiments



a_μ^{win} suggests that $\sim 5 \times 10^{-10}$ must come from above 1 GeV

GC, Hoferichter, Stoffer

Outline

Introduction: present status of $(g-2)_\mu$

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$
- Consequences of the BMW result

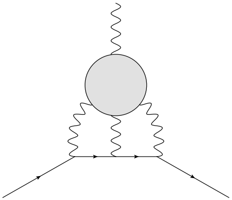
Hadronic light-by-light contribution to $(g-2)_\mu$
- Short-distance constraints

Conclusions and Outlook

Calculating the HLbL contribution

The HLbL contribution is a very complex quantity

- 4-point function of em currents in QCD



- early on, it has been calculated with models

  Hayakawa-Kinoshita-Sanda/Bijnens-Pallante-Prades (96), Knecht, Nyffeler (02), Melnikov, Vainshtein (04)

- a data-driven approach, like for HVP, has only recently been developed and used

  GC, Hoferichter, Procura, Stoffer=CHPS (14,15,17), Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- lattice QCD is becoming competitive RBC/UKQCD (20), Mainz (21)

Different model-based evaluations of HLbL

Jegerlehner-Nyffeler 2009

Contribution	BPaP(96)	HKS(96)	KnN(02)	MV(04)	BP(07)	PdRV(09)	N/JN(09)
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
" " + subl. in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

Legenda: B=Bijnens Pa=Pallante P=Prades H=Hayakawa K=Kinoshita S=Sanda Kn=Knecht N=Nyffeler M=Melnikhov V=Vainshtein dR=de Rafael J=Jegerlehner

- large uncertainties (and differences among calculations) in individual contributions
- pseudoscalar pole contributions most important
- second most important: pion loop, *i.e.* two-pion cuts (*Ks are subdominant*)
- heavier single-particle poles decreasingly important

Advantages of the dispersive approach

- model independent
- unambiguous definition of the various contributions
- makes a data-driven evaluation possible (in principle)
- if data not available: use theoretical calculations of subamplitudes, short-distance constraints etc.
- First attempts: GC, Hoferichter, Procura, Stoffer (14), Pauk, Vanderhaeghen (14)

  [Schwinger sum rule: Hagelstein, Pascalutsa (17)]

The HLbL tensor

HLbL tensor:

$$\Pi^{\mu\nu\lambda\sigma} = i^3 \int dx \int dy \int dz\, e^{-i(x\cdot q_1 + y\cdot q_2 + z\cdot q_3)} \langle 0 | T\{ j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0) \} | 0 \rangle$$

$$q_4 = k = q_1 + q_2 + q_3 \qquad k^2 = 0$$

General Lorentz-invariant decomposition:

$$\Pi^{\mu\nu\lambda\sigma} = g^{\mu\nu} g^{\lambda\sigma} \Pi^1 + g^{\mu\lambda} g^{\nu\sigma} \Pi^2 + g^{\mu\sigma} g^{\nu\lambda} \Pi^3 + \sum_{i,j,k,l} q_i^\mu q_j^\nu q_k^\lambda q_l^\sigma \Pi^4_{ijkl} + \ldots$$

consists of 138 scalar functions $\{\Pi^1, \Pi^2, \ldots\}$, but in $d = 4$ only 136 are linearly independent

Eichmann *et al.* (14)

Constraints due to gauge invariance? (see also Eichmann, Fischer, Heupel (2015))

$\Rightarrow$ Apply the Bardeen-Tung (68) method+Tarrach (75) addition

Gauge-invariant hadronic light-by-light tensor

Applying the Bardeen-Tung-Tarrach method to $\Pi^{\mu\nu\lambda\sigma}$ one ends up with:

GC, Hoferichter, Procura, Stoffer ≡ CHPS (2015)

- 43 basis tensors (BT) — in $d=4$: 41=no. of helicity amplitudes
- 11 additional ones (T) — to guarantee basis completeness everywhere
- of these 54 only 7 are distinct structures
- all remaining 47 can be obtained by crossing transformations of these 7: manifest crossing symmetry
- the dynamical calculation needed to fully determine the HLbL tensor concerns these 7 scalar amplitudes

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i$$

HLbL contribution: Master Formula

$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

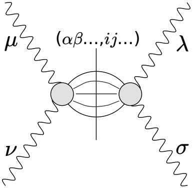
Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. CHPS (15)

- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment:
  imaginary parts are related to measurable subprocesses

“Amenable to a dispersive treatment”



+ crossed diagrams

$$\mathrm{Im}\,\Pi^{\mu\nu\lambda\sigma} = \sum_{\alpha\beta...,ij...} \Gamma^{\mu\nu\alpha\beta...}_{ij...} \Gamma^{\lambda\sigma\alpha\beta...\,\star}_{ij...}$$

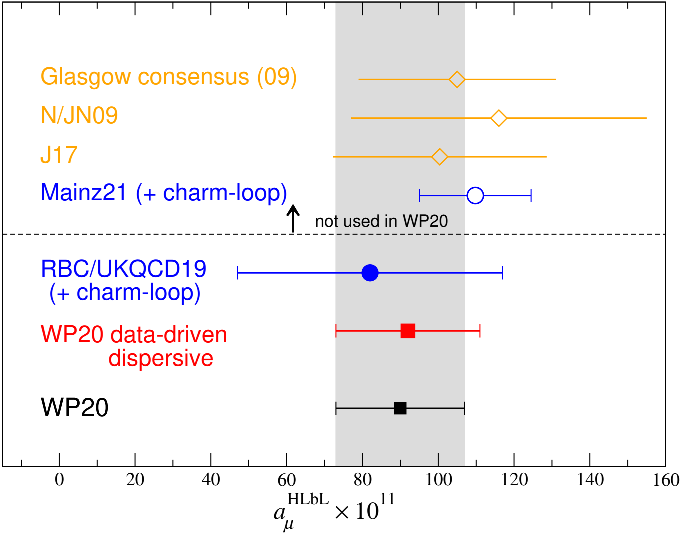
- projection on the BTT basis for $\Pi^{\mu\nu\lambda\sigma}$ $\Rightarrow$ DR for Π_i
- result for $\Pi^{\mu\nu\lambda\sigma}$ (and a_μ) depends on the basis choice unless a set of sum rules is satisfied CHPS 17
- even for single-particle intermediate states this is in general not the case, other than for pseudoscalars

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} −1(3) (scalars + tensors)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

- significant reduction of uncertainties in the first three rows
  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid et al. (18), Gerardin, Meyer, Nyffeler (19)
- $1-2$ GeV resonances affected by basis ambiguity and large uncertainties Danilkin, Hoferichter, Stoffer (21)
- asymptotic region recently addressed, Melnikov, Vainshtein (04), Nyffeler (09), WP still work in progress Bijnens et al. (20,21), Cappiello et al. (20), Leutgeb, Rebhan (19,21)

Situation for HLbL



Recent activity on SDCs (mainly post WP)

- calculation of (non-)perturbative corrections to the OPE

  Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

- tower of excited pseudoscalars (Regge model)

  GC, Hagelstein, Hoferichter, Laub, Stoffer (19)

- tower of axial-vectors (holographic QCD model)

  Leutgeb, Rebhan (19,21), Cappiello, Catà, D'Ambrosio, Greynat, Iyer (20)

- solution based on interpolants

  Lüdtke, Procura (20)

- general considerations, comparison of model solutions

  Knecht (20), Masjuan, Roig, Sánchez-Puertas (20), GC, Hagelstein, Hoferichter, Laub, Stoffer (21)

Recent activity on SDCs (mainly post WP)

- calculation of (non-)perturbative corrections to the OPE

  Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

- tower of excited pseudoscalars (Regge model)

  GC, Hagelstein, Hoferichter, Laub, Stoffer (19)

- tower of axial-vectors (holographic QCD model)

  Leutgeb, Rebhan (19,21), Cappiello, Catà, D'Ambrosio, Greynat, Iyer (20)

- solution based on interpolants

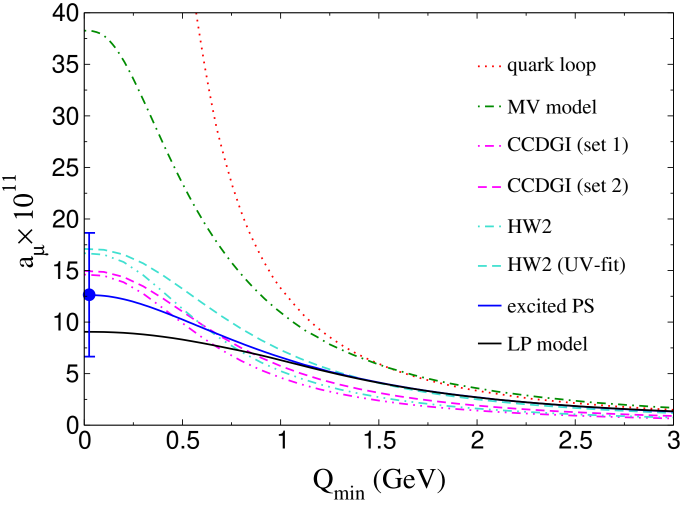
  Lüdtke, Procura (20)

- general considerations, comparison of model solutions

  Knecht (20), Masjuan, Roig, Sánchez-Puertas (20), GC, Hagelstein, Hoferichter, Laub, Stoffer (21)

Numerical comparison of LSDC solutions for $a_\mu^{\rm HLbL}$

GC, Hagelstein, Hoferichter, Laub, Stoffer (21)



Comments on the contribution of axial vectors

- like all resonances besides pseudoscalars, axial vectors affected by basis ambiguity
- model calculations: large spread, $\Rightarrow$ axial-vector contributions might potentially be large (transverse SDC)

	$a_\mu^{\text{axials}}[a_1, f_1, f_1']$
– Melnikov, Vainshtein (04)	$22(5) \times 10^{-11}$
– Pauk, Vanderhaeghen (14)(only f_1, f_1')	$6.4(2.0) \times 10^{-11}$
– Jegerlehner (17)	$7.6(2.7) \times 10^{-11}$
– Roig, Sánchez-Puertas (20)	$0.8(^{+3.5}_{-0.8}) \times 10^{-11}$
– hQCD models (contribution only to T amplitudes)	
Leutgeb, Rebhan (19,21)	$\sim 17 \times 10^{-11}$
Cappiello et al. (20)	$\sim 14 \times 10^{-11}$

- model-independent treatment of axials particularly urgent

Recent work on axial-vector contributions

- New basis free of kinematic singularities for axials

  GC, Hagelstein, Hoferichter, Laub, Stoffer (21)

- Asymptotic behaviour of TFF of axial vectors

  Hoferichter, Stoffer (20)

- Analysis of phenomenological and asymptotic constraints on a VMD model for TFF of axial vectors

  Zanke, Hoferichter and Kubis (21)

- hQCD models with $m_q \neq 0$, including phenomenological and asymptotic constraints

  Leutgeb, Rebhan (21)

  Large contributions confirmed. hQCD models successful so far $\Rightarrow$ this needs to be understood

Outline

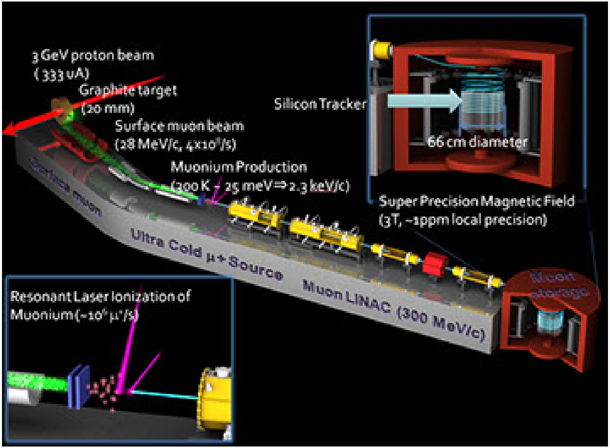
Conclusions

- The WP provides the current status of the SM evaluation of $(g-2)_\mu$: 4.2σ discrepancy with experiment (w/ FNAL)
- Evaluation of the HVP contribution based on the dispersive approach: 0.6% error $\Rightarrow$ dominates the theory uncertainty
- Recent lattice calculation [BMW(20)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  If confirmed $\Rightarrow$ discrepancy with experiment $\searrow$ below 2σ
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(20), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ potential 7σ discrepancy
- Improvements on the SM theory/data side:
  - HVP data-driven:
    Other e^+e^- experiments are available or forthcoming:
    SND, BaBar, Belle II, BESIII, CMD3 $\Rightarrow$ Error reduction
    MuonE will provide an alternative way to measure HVP
  - HVP lattice:
    More calculations w/ precision $\sim$ BMW are awaited
    Difference to data-driven evaluation must be understood
  - HLbL data-driven: goal of $\sim$ 10% uncertainty within reach
  - HLbL lattice: RBC/UKQCD $\Rightarrow$ similar precision as Mainz.
    Good agreement with data-driven evaluation.

Future: Muon $g-2$/EDM experiment @ J-PARC



Credit: J-PARC