Semileptonic form factors from JLQCD

KEK, SOKENDAI

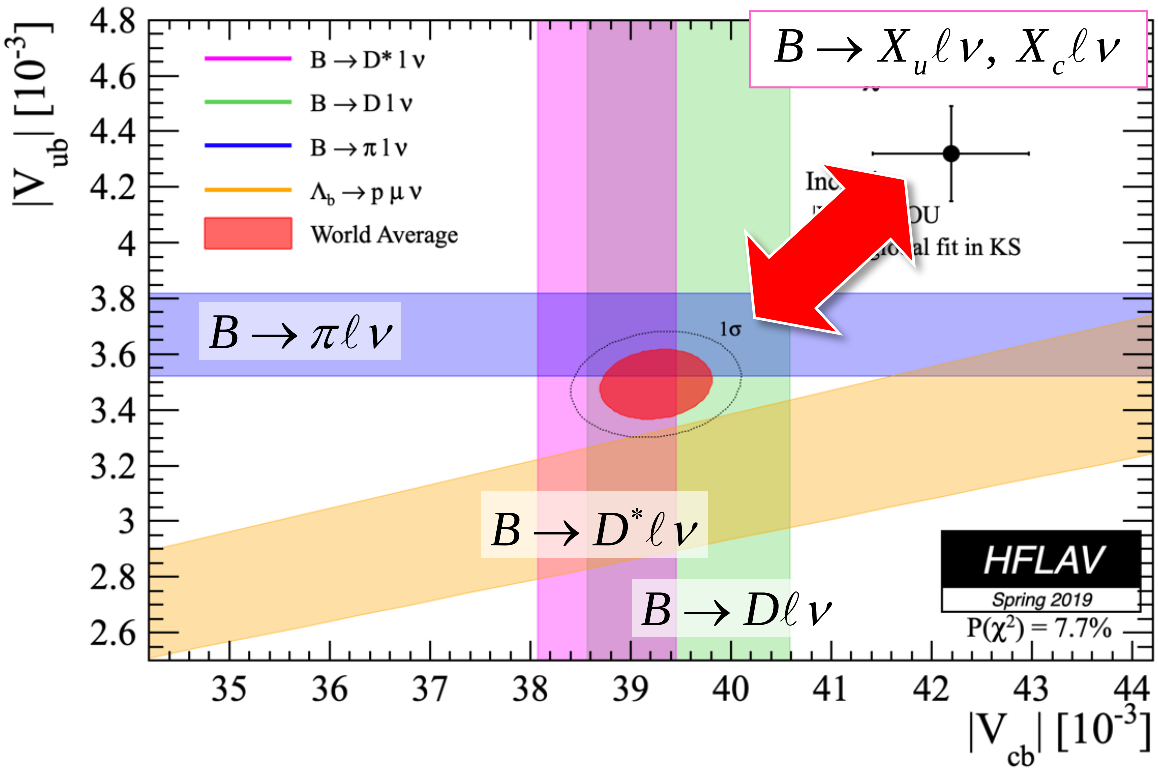
Takashi Kaneko (JLQCD Collaboration)

BNL-HET and RBRC Joint Workshop "DWQ@25"

December 15th, 2021, Zoom / BNL

introduction

semileptonic decays : probes of new physics $\Rightarrow$ talks by Iijima, Patel



exclusive vs inclusive

HFLAV '19

$|V_{ub}|$: 15%, 3σ tension

$|V_{cb}|$: 6-8%, 2-3σ tension

recent update
$\Rightarrow$ *e.g.* CKM 2021, ...

new physics? $\Leftrightarrow$ *e.g.* Crivellin-Pokorski '18 : $d_L^{qb} \partial^\nu \left(\bar{q} \sigma_{\mu\nu} P_L b \right) \Leftrightarrow \Gamma\left(Z \to b\bar{b} \right) \times 2$

need deeper understanding of th. and/or exp't uncertainties

– theory side : form factors (FFs) describing non-perturbative QCD effects

introduction

published studies : review by Flavor Lattice Averaging Group '21

$B \to \pi \ell \nu$

Collaboration	Ref.	N_f	publicati	continuum	chiral ext	finite vol	renormal.	heavy-qu
FNAL/MILC 15	[554]	2+1	A	★	○	★	○	✓
RBC/UKQCD 15	[555]	2+1	A	○	○	○	○	✓
HPQCD 06	[552]	2+1	A	○	○	○	○	✓

$M_\pi \gtrsim$ 165MeV, $a^{-1} \lesssim$ 4.5GeV

~ expr'tal error limited by statistics

⇒ to be largely reduced by Belle II

$B \to D \ell \nu$

Collaboration	Ref.	N_f	publicati	continuum	chiral ext	finite vol	renormal.	heavy-qu
HPQCD 15, HPQCD 17	[605, 607]	2+1	A	○	○	○	○	✓
FNAL/MILC 15C	[604]	2+1	A	★	○	★	○	✓
Atoui 13	[601]	2	A	★	○	★	—	✓

both 2 FFs @ zero & nonzero recoils

$B \to D^* \ell \nu$

Collaboration	Ref.	N_f	publicati	continuum	chiral ext	finite vol	renormal.	heavy-qu
HPQCD 17B	[609]	2+1+1	A	○	★	★	○	✓
FNAL/MILC 14	[603]	2+1	A	★	○	★	○	✓

only h_{A1} out of 4

only @ zero-recoil

Fermilab/MILC '21 : 1st calculation of all SM FFs at $w \neq 1$ (!)

many on-going : HISQ (, DWQ) @ $m_{ud,\mathrm{phys}}$, B_s, B_c decays ⇒ Lattice '21, DWQ@25

this talk

$B \to \pi\ell\nu$, $B \to D^{(*)}\ell\nu$ FFs from JLQCD

- simulation method
  - relativistic approach w/ DWQs
- $B \to D^{(*)}\ell\nu$
  - $B \to D^{*}\ell\nu$ @ non zero recoils
    - a comparison w/ Fermilab/MILC '21
  - $B \to D\ell\nu$ tensor FF
- $B \to \pi\ell\nu$
  - current status and future prospect

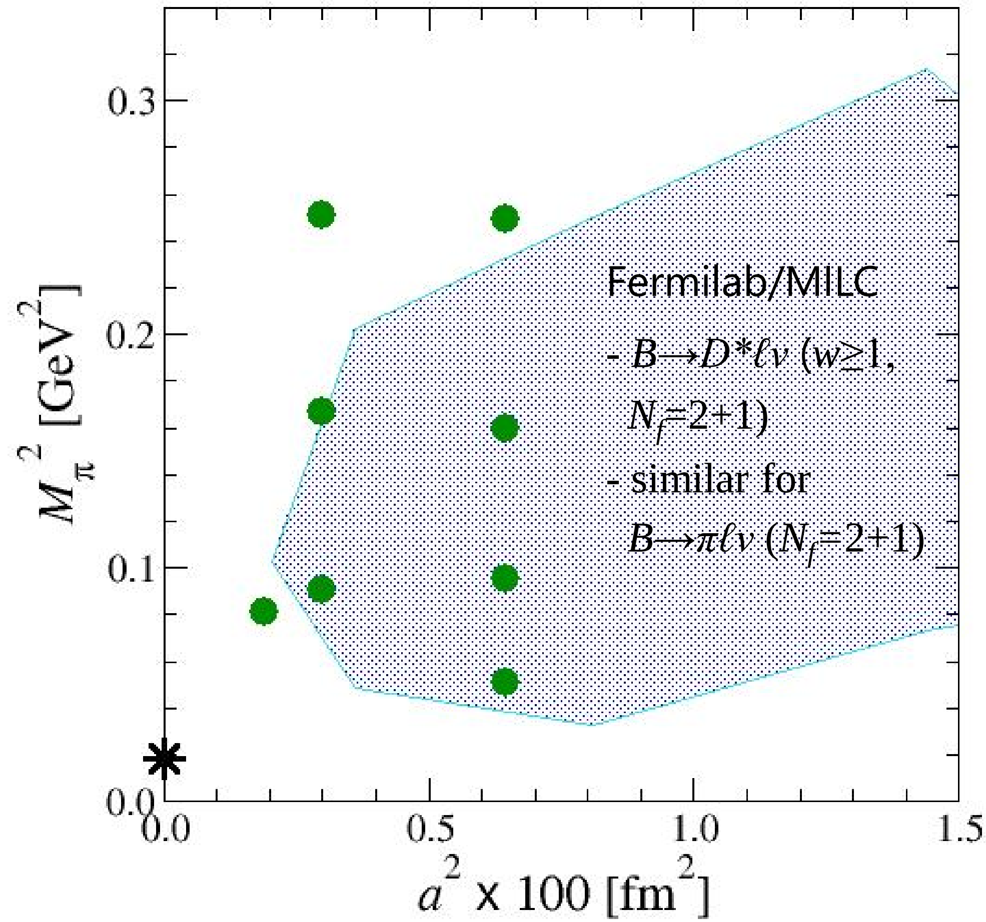
simulation method

gauge ensembles

Möbius DWQs + link smearing

simulation parameters

- $N_f = 2+1$
- $a^{-1} \lesssim 2.5, 3.6,$ 4.5 GeV $\sim m_b$
  + $m_{res} \leq$ a few MeV w/ $N_5 \sim O(10)$
  + no $O(a^{2n+1})$ errors
- $M_\pi \gtrsim 230$ MeV
  + $D^* \not\to D\pi$
  + chiral log from $a=0$ HMChPT
- $M_\pi L \gtrsim 4$
  + $M_\pi L = 3$ to directly check FVEs
- statistics < staggered-type



$a^{-1} \lesssim m_b \Rightarrow$ need a careful treatment of heavy quarks

“relativistic approach”

light and heavy DWQs

- no matching to EFT
- + chiral symmetry

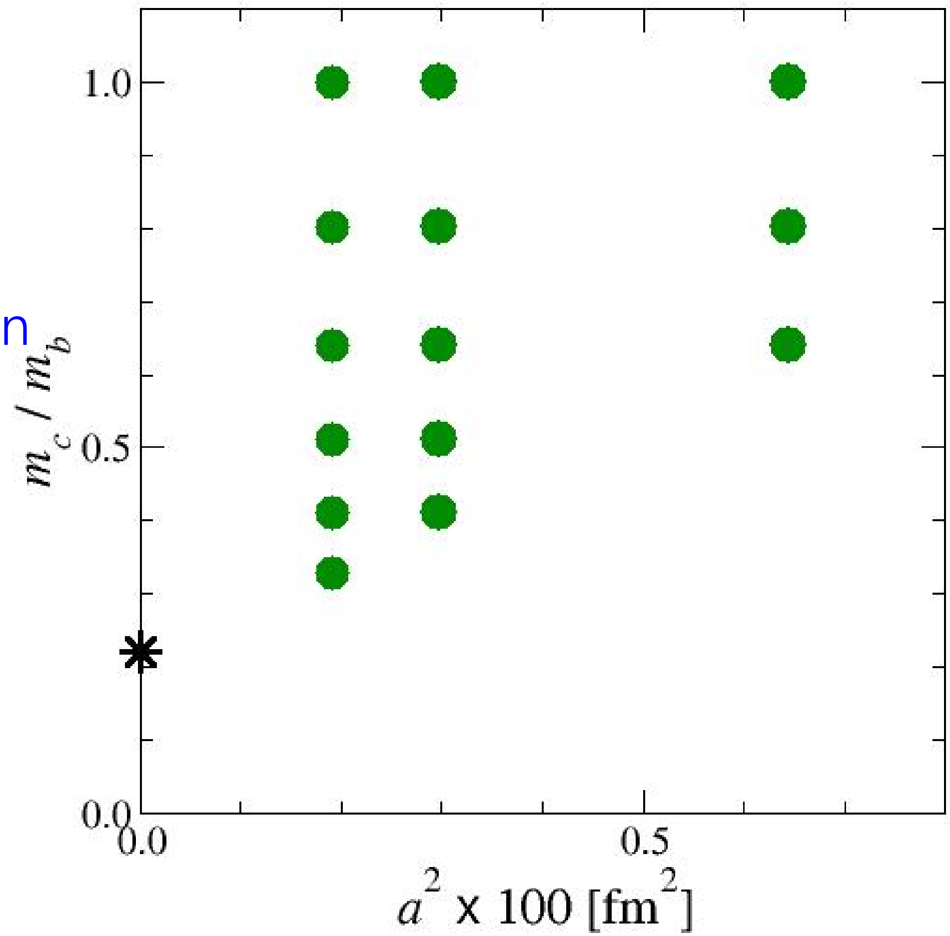
  $\Rightarrow$ not need explicit renormalization

  for SM FFs
- charm is OK : $m_c < a^{-1}$'s

  fixed to physical mass
- simulate 3 – 6 $m_b \leq 0.7a^{-1}$

  $\Rightarrow$ extrapolate to $m_{b,\mathrm{phys}}$

  $\Leftrightarrow$ m_b dependence not large



other studies : Fermilab, RHQ, HQET, NRQCD, ...

w/ systematics very different from other studies

$B \to D^{(*)} \ell \nu$ decays

$B \to D^{(*)}\ell\nu$ form factors

"relativistic" convention

$$\langle D(p')|V^\mu|\bar{B}(p)\rangle = f_+(p+p')^\mu + f_-(p-p')^\mu$$

$$\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle = ig\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p'_\beta p_\gamma,$$

$$\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle = f\epsilon^{*\mu} + (\epsilon^*\cdot p)\left[a_+(p+p')^\mu + a_-(p-p')^\mu\right]$$

"HQET" convention $|\mathrm{HQET}\rangle = |\mathrm{rel}\rangle/\sqrt{M}$

$$\langle D(v')|V^\mu|\bar{B}(v)\rangle = h_+(w)\,(v+v')^\mu + h_-(w)\,(v-v')^\mu$$

$$\langle D^*(v',\varepsilon')|V^\mu|\bar{B}(v)\rangle = ih_V(w)\,\epsilon^{\mu\nu\alpha\beta}\,\varepsilon'^*_\nu v'_\alpha v_\beta$$

$$\langle D^*(v',\varepsilon')|A^\mu|\bar{B}(v)\rangle = h_{A_1}(w)\,(w+1)\,\varepsilon'^{*\mu} - \left[h_{A_2}(w)\,v^\mu + h_{A_3}(w)\,v'^\mu\right]\varepsilon^*\cdot v$$

- 2 FFs for $B \to D\ell\nu$; 4 FFs for $B \to D^*\ell\nu$ w/ $\varepsilon_{D^*}p_{D^*} = 0$
- function of $q^2 = (p - p')^2 \leq q^2_{\max} = (M_B - M_{D(*)})^2$ and $w = v\,v' \geq 1$

ratio method (Hashimoto *et al.* '99)

e.g. 4 FFs for $B \to D^* \ell \nu$

$$\frac{\langle D^*(\boldsymbol{\varepsilon},\mathbf{0})|A_1^{(\mathrm{lat})}|B(\mathbf{0})\rangle\langle B(\mathbf{0})|A_1^{(\mathrm{lat})}|D^*(\boldsymbol{\varepsilon},\mathbf{0})\rangle}{\langle D^*(\boldsymbol{\varepsilon},\mathbf{0})|V_4^{(\mathrm{lat})}|D^*(\boldsymbol{\varepsilon},\mathbf{0})\rangle\langle B(\mathbf{0})|V_4^{(\mathrm{lat})}|B(\mathbf{0})\rangle} \Rightarrow \frac{|h_{A_1}(1)|^2}{F_V^{D^*}(1)\,F_V^{B}(1)} \Rightarrow h_{A_1}(1)$$

$$\frac{\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)|A_1^{(\mathrm{lat})}|B(\mathbf{0})\rangle\langle D^*(\boldsymbol{\varepsilon},\mathbf{0})|D^*(\boldsymbol{\varepsilon},\mathbf{0})\rangle}{\langle D^*(\boldsymbol{\varepsilon},\mathbf{0})|A_1^{(\mathrm{lat})}|B(\mathbf{0})\rangle\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)|D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)\rangle} \Rightarrow \frac{h_{A_1}(w)}{h_{A_1}(1)}$$

$$\frac{\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)V_1^{(\mathrm{lat})}B(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)A_1^{(\mathrm{lat})}B(\mathbf{0})\rangle} \Rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

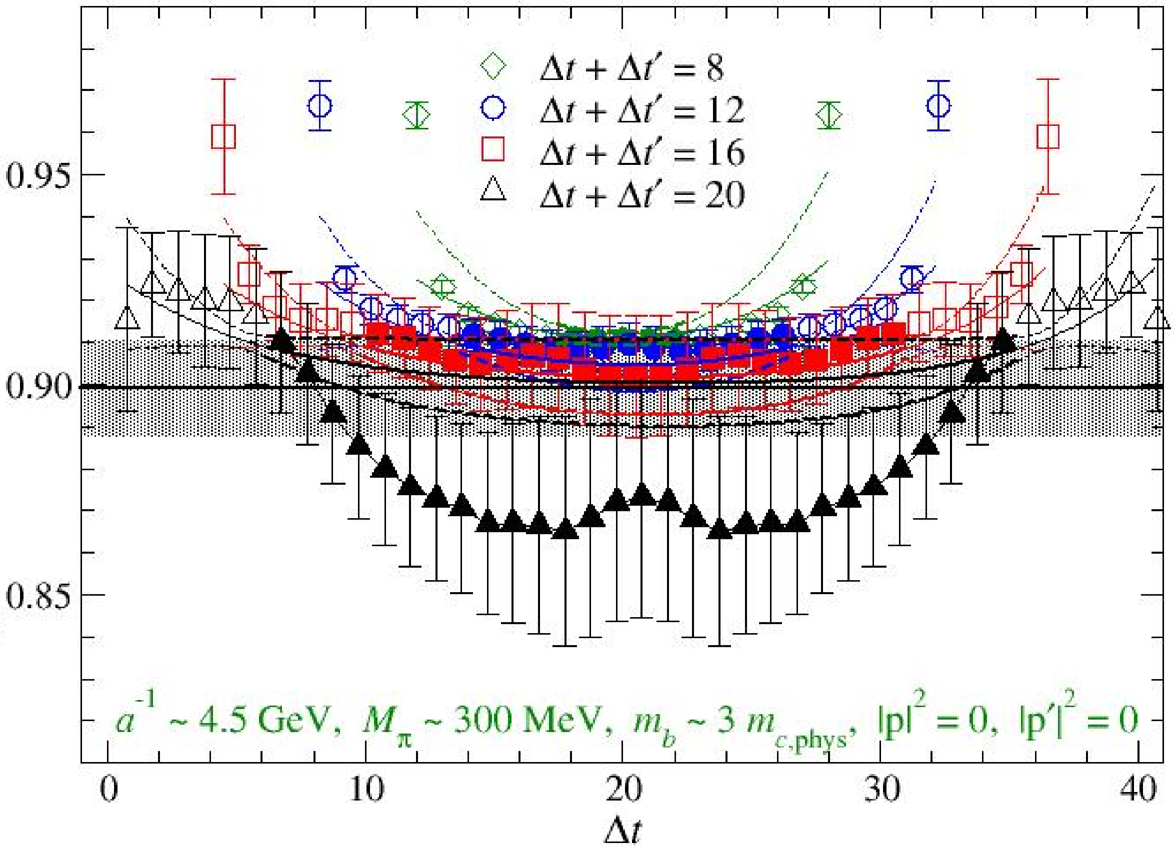
$$\frac{\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_{\not\perp})A_{1(4)}^{(\mathrm{lat})}B(\mathbf{0})\rangle}{\langle D^*(\boldsymbol{\varepsilon},\mathbf{p}_\perp)A_1^{(\mathrm{lat})}B(\mathbf{0})\rangle} \Rightarrow \frac{h_{A_{3(2)}}(w)}{h_{A_1}(w)}$$

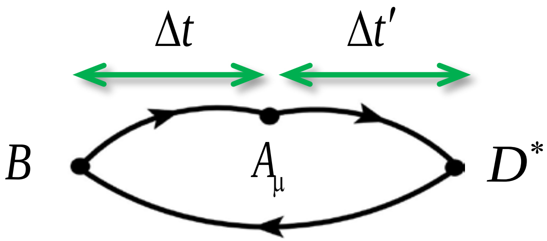
- B at rest
- $|\mathbf{p}|^2$ = 0,1,2,3,4
- $\boldsymbol{\varepsilon}\mathbf{p}_\perp = 0$
- $\boldsymbol{\varepsilon}\mathbf{p}_{\not\perp} \neq 0$

do not need explicit renormalization for SM FFs w/ DWQs

extracting FFs

$$\langle D^*|A_1|B\rangle\langle B|A_1|D^*\rangle/\langle D^*|V_4|D^*\rangle\langle B|V_4|B\rangle \;\rightarrow\; h_{A1}(1)$$





- 4 values of source-sink separation $\Delta t+\Delta t'$

  ~ 0.6 – 1.6 fm

- multi-exponential fit

$$\langle D^*|A|B\rangle\{1+c\,e^{-\Delta E_B \Delta t}+c' e^{-\Delta E_{D^*}\Delta t'}\}$$

- large $\Delta t+\Delta t'$ $\Rightarrow$ ground state saturation
- small $\Delta t+\Delta t'$ $\Rightarrow$ statistical accuracy: *e.g.* 1 - 2% for $h_{A1}(w)$

continuum + chiral extrapolation

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

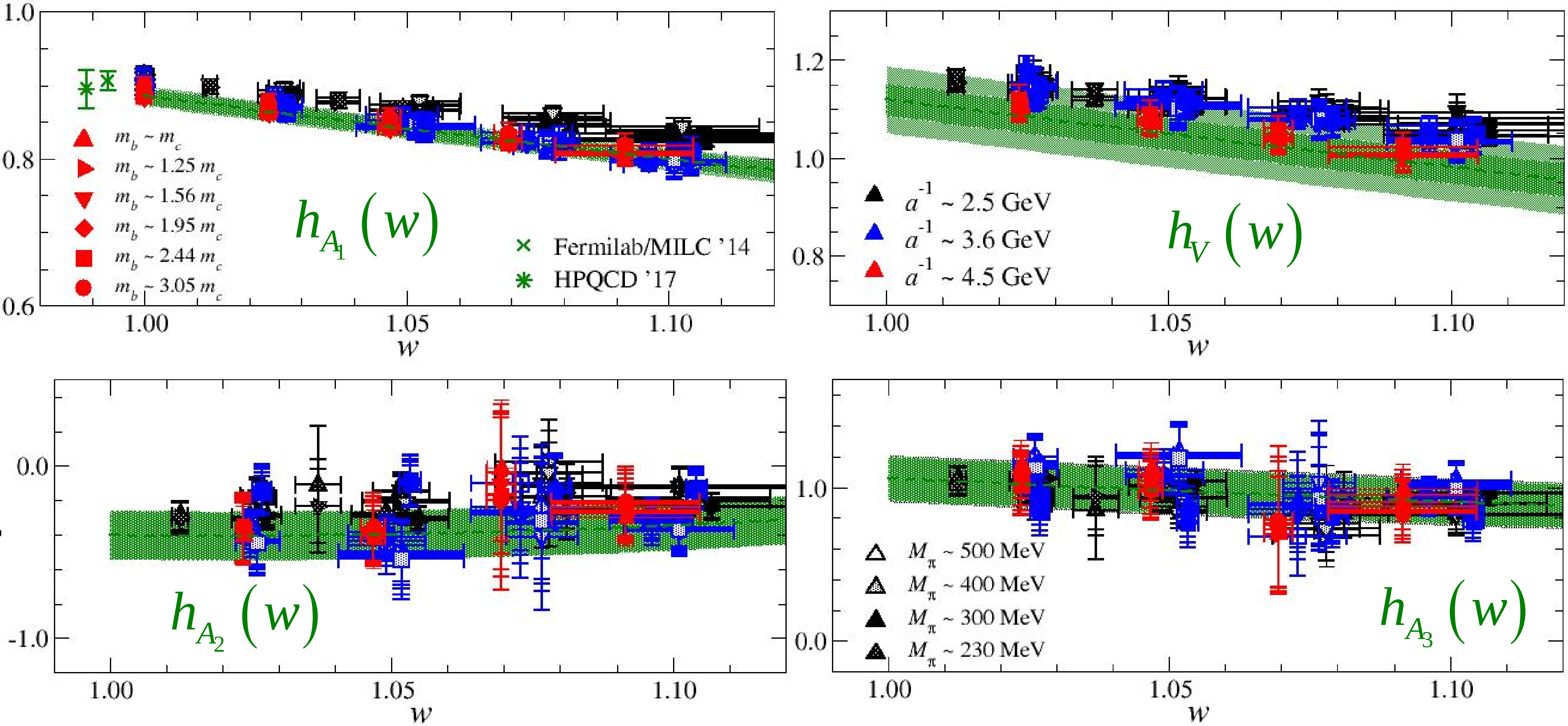
$$\frac{h_{A_1}(w)}{\eta_{A_1}} = c + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2}\Delta_c^2 b_{\log}\bar{F}_{\log}\left(M_\pi,\Delta_c,\Lambda_\chi\right)$$

$$+c_w\left(w-1\right)+d_w\left(w-1\right)^2+c_b\left(w-1\right)\varepsilon_b+c_\pi\xi_\pi+c_{\eta s}\xi_{\eta s}+c_a\xi_a+c_{am_b}\xi_{amb}$$

$$\varepsilon_b=\frac{\bar{\Lambda}}{2m_b},\quad \xi_\pi=\frac{M_\pi^2}{\left(4\pi f_\pi\right)^2},\quad \xi_{\eta s}=\frac{M_{\eta s}^2}{\left(4\pi f_\pi\right)^2},\quad \xi_a=\left(a\Lambda_{\mathrm{QCD}}\right)^2,\quad \xi_a=\left(am_b\right)^2$$

- singular correlation matrix $\Rightarrow$ SVD cut, shrinkage $\Leftrightarrow$ Fermilab/MILC
- one-loop radiative correction η_X is explicitly included (Neubert '92)
- $g_{D^*D\pi}$ = 0.53(8) (Fermilab/MILC '14) $\Rightarrow$ small systematic error
- ξ - expansion : better convergence for light quark obs. (JLQCD '08)
- $O((w-1)/m_b)$ for h_{A1}, h_+ $\Leftrightarrow$ Luke's theorem '90 ; include $O(1/m_b^2)$

$B \to D^* \ell \nu$ form factors

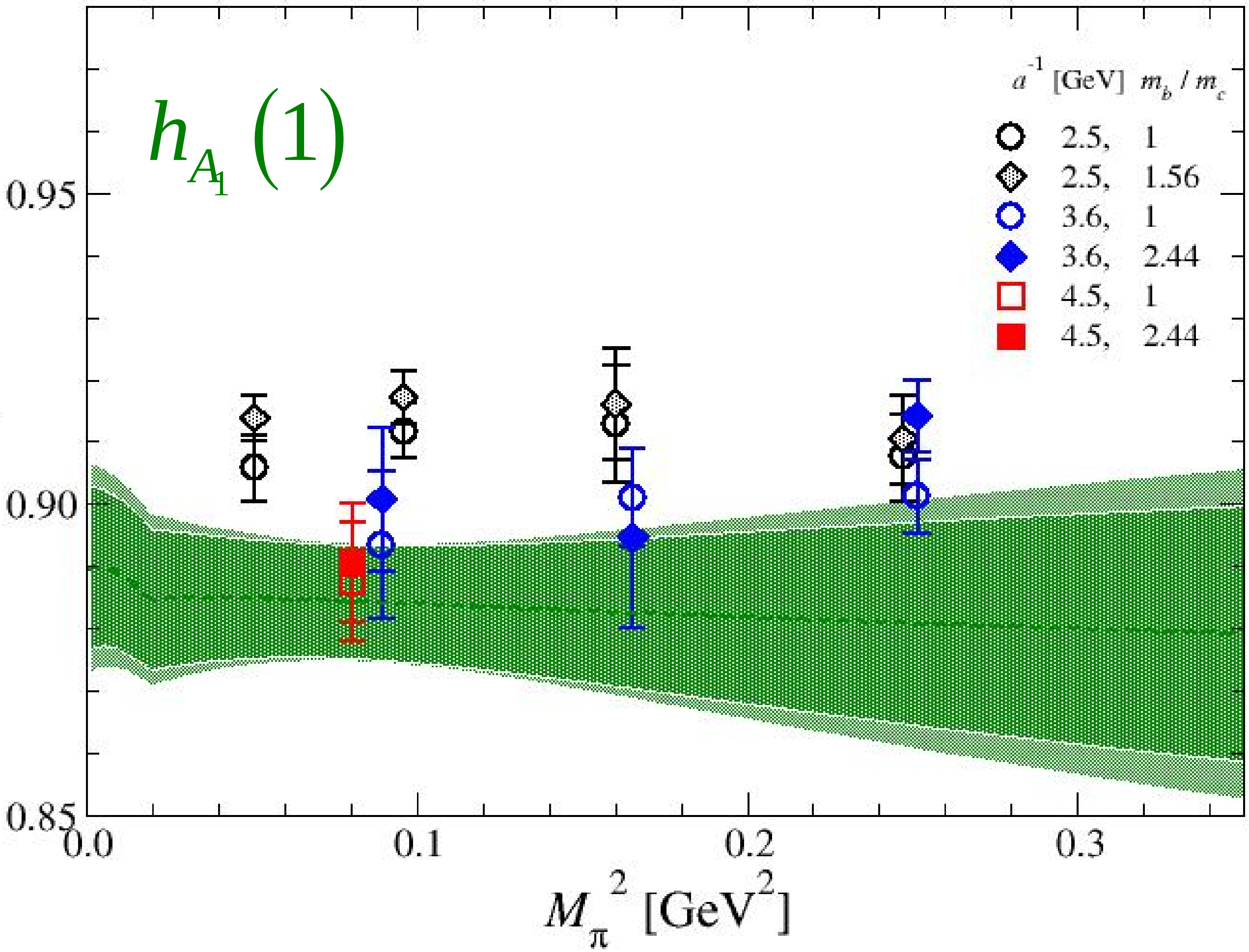


- mild dependence on a, M_π, m_s, m_b

  $\Rightarrow$ all coefficients $c_X \leq O(1)$; $\geq$ 50% error for c_π, $c_{\eta s}$, c_a [except h_{A1}]

- extrapolation : reasonably controlled w/ χ^2/d.o.f. ≤ 1

M_π dependence



- mild dependence
  - suppressed log
  - no valence π
- similar for other w , FFs

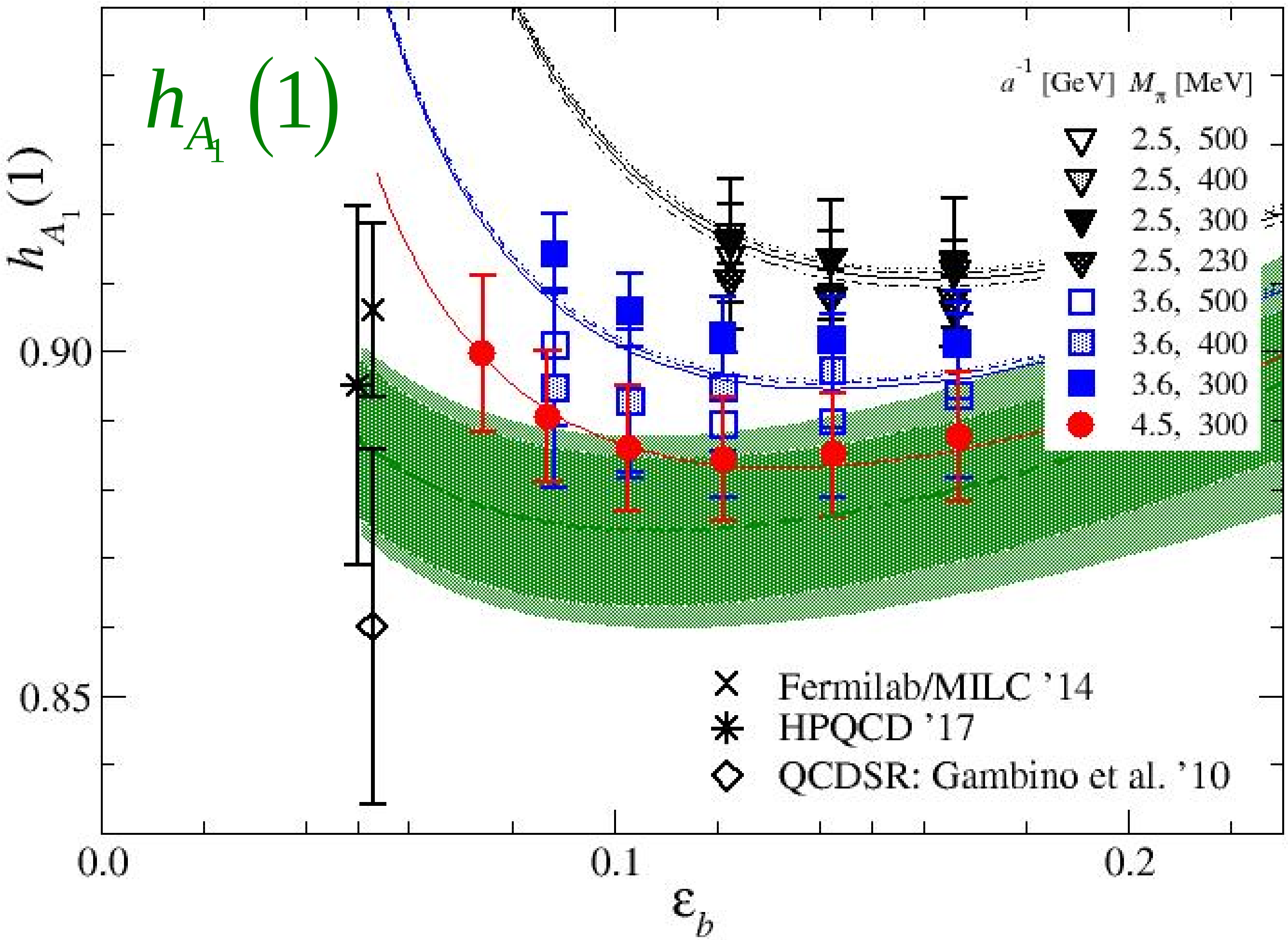
w/ $H^{(Q)}$, π

$$\sqrt{\Delta^2 - M_\pi^2}\ \ln\left[\frac{\Delta + \sqrt{\Delta^2 - M_\pi^2}}{\Delta - \sqrt{\Delta^2 - M_\pi^2}}\right]$$

$$\Delta = M_{D*} - M_D$$

- $D^* \to D\pi$ $\Rightarrow$ concave structure $<$ statistical accuracy
- mild dependence $\Rightarrow$ controlled extrapolation $\Leftrightarrow$ $B \to \pi \ell \nu$

m_b and a dependences



$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b} = \frac{\bar{\Lambda}}{M_{\eta_b}}$$

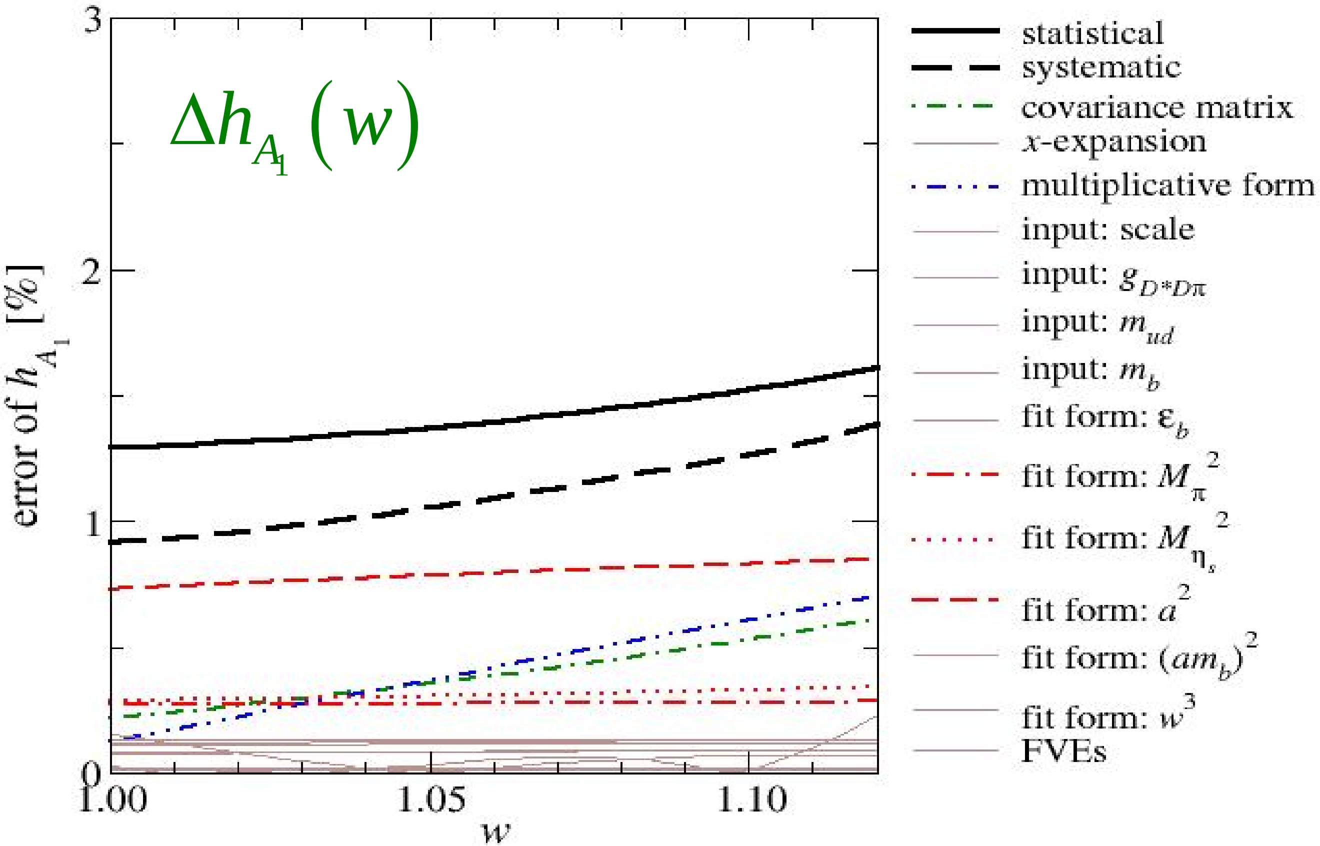
$\bar{\Lambda} = 0.5\text{GeV}$

- two $a \neq 0$ effects
  - $(a\Lambda)^{2n}$, $(am_c)^{2n}$
  - $(am_b)^{2n}$
- consistency w/ QCDSR (?)

  Gambino-Mannel-Uraltsev '10

- turn out to be a few % effects
- reasonably controlled extrapolation in ε_b and a $\Leftrightarrow$ smaller a ?

uncertainties



correlation matrix
extrapolation form

inputs
a, m_{q}, $g_{D^*D\pi}$

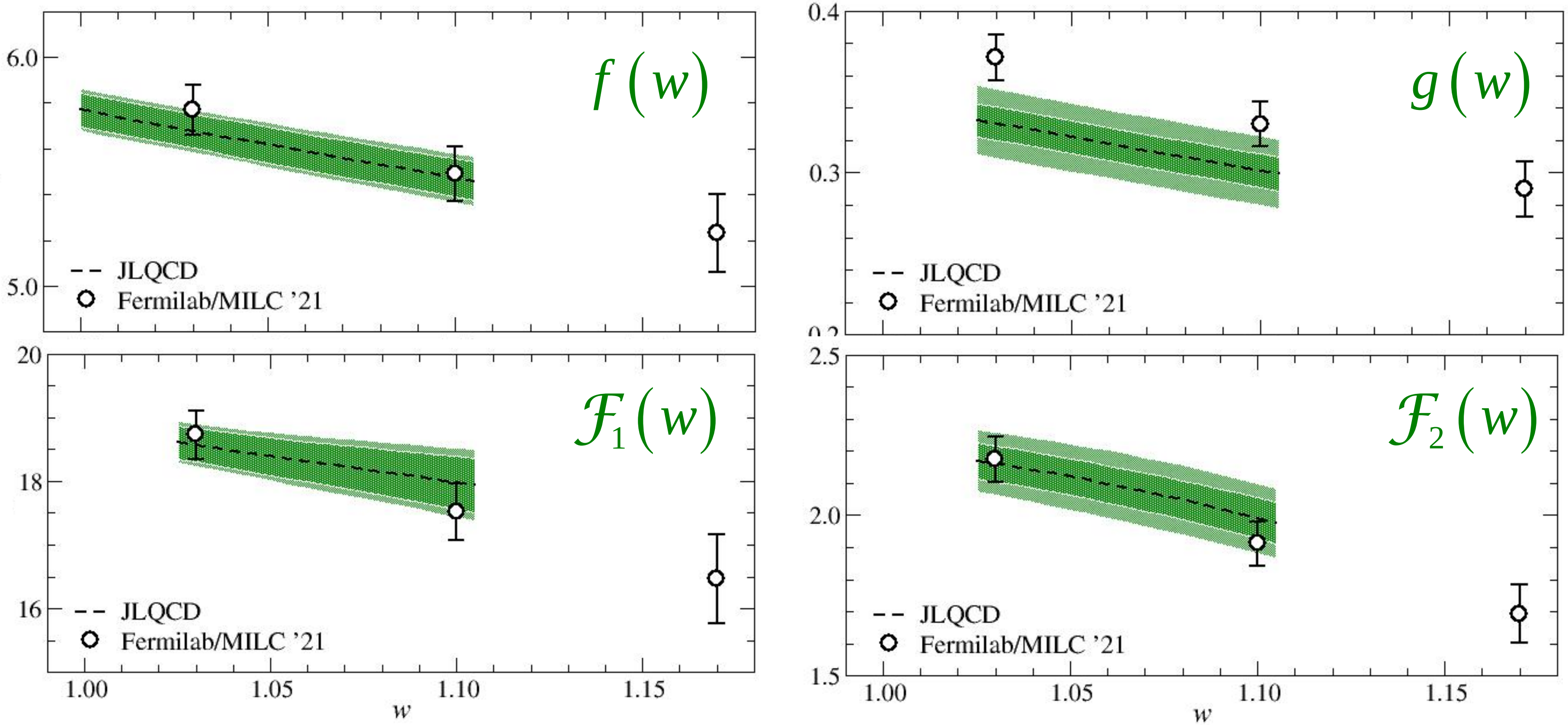
polynomial corr.
add higher order
remove poorly determined term

\- small by $2L$'s

- h_{A1} : largest uncertainties – statistics and discretization – but 1-2 %
- other FFs : larger and more dominant statistical error

synthetic data of FFs

synthetic data @ a=0 and physical m_q's



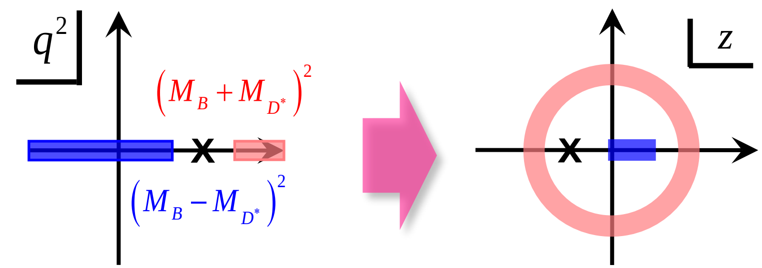
- relativistic FFs from HQET FFs
- w = 1.000, 1.050, 1.100 (f), 1.030, 1.065, 1.100 (other FFs)

  c.f. Fermilab/MILC : w = 1.03, 1.07, 1.17

BGL parameterization

Boyd-Grinstein-Lebed parameterization '97

$$F(w) = \frac{1}{P_F(z)\phi_F(z)} \sum_n^{n_F} a_n^F z^n, \quad z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$



- w = [1.0, 1.1], q^2 = [13.0, 10.7] $\rightarrow$ z =[0.000, 0.012]

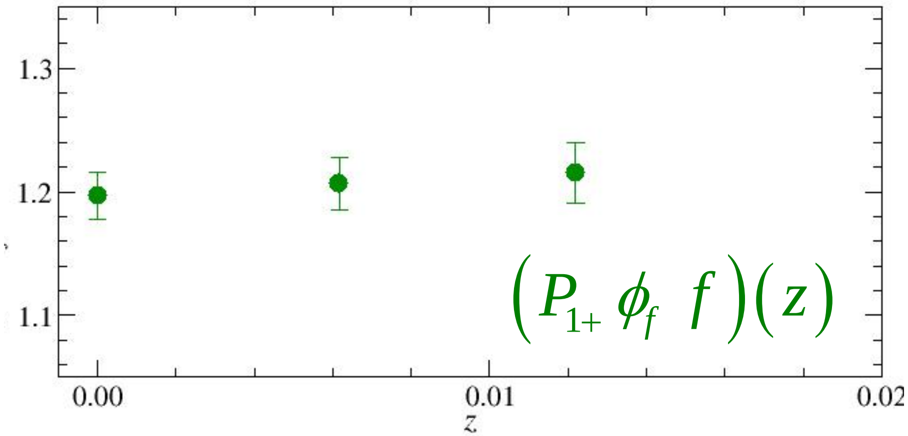
- Baschke factors P_F to factor out singularity
- outer functions ϕ_F leading to unitarity constraint $\Sigma\, |a^F{}_n|^2 \leq 1$

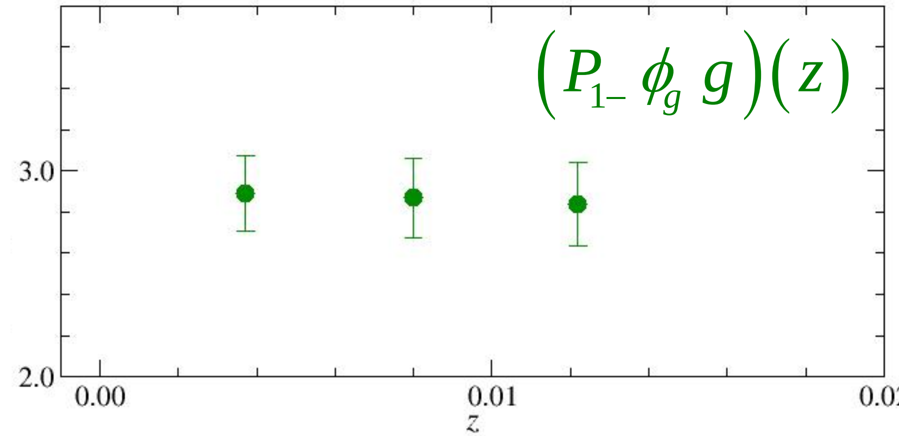
  resonance masses, derivatives of VPFs same as Bigi-Gambino-Schacht '17
- w/o unitarity constraint, but satisfied within errors
- w/ kinematical constraints @ $w = 1$, $w_{\rm max}$ (q^2=0)

  $\mathcal{F}_1(1) = (M_B\text{-}M_{D^*})\, f(1)$, $\mathcal{F}_2(w_{\rm max}) = (1+{\rm r})\, \mathcal{F}_1(w_{\rm max}) \,/\, M_B^2\, r(1+r)(1+ w_{\rm max})$

- test quadratic exapansion : $n_g, n_f, n_{\mathcal{F}1}, n_{\mathcal{F}2}$ = 2

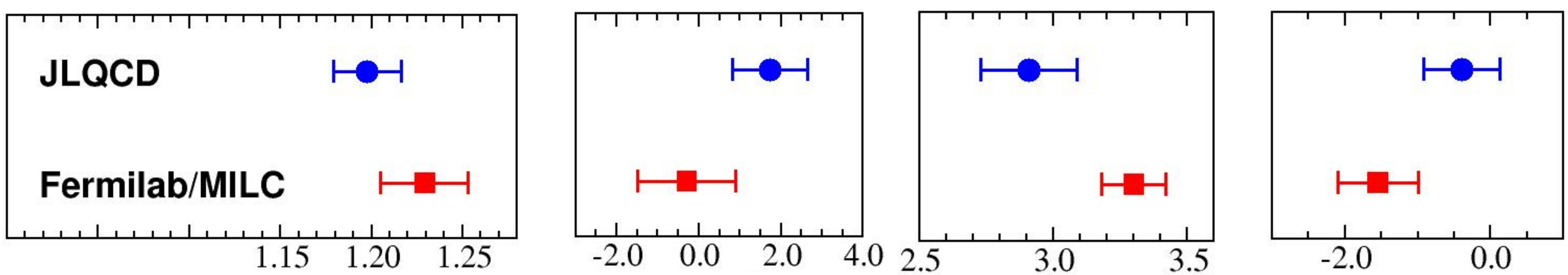
BGL parametrization of FFs





- regular parts : very mild dependence on z - quadratic or even linear fit is OK

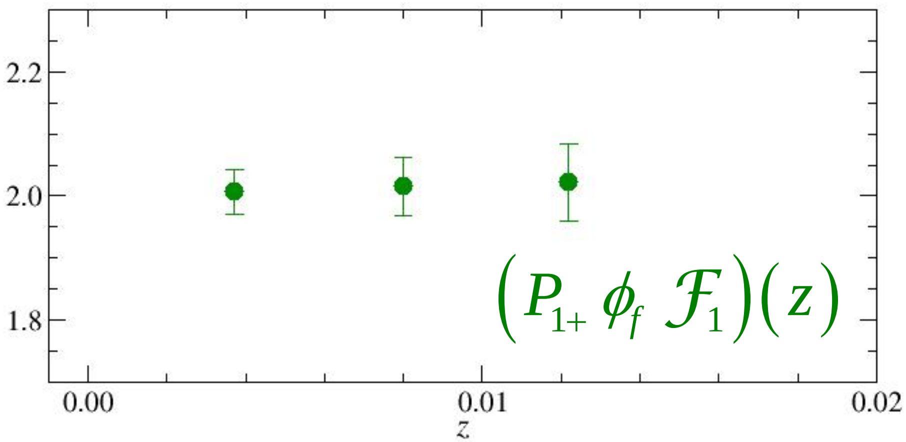
coefficients $a_0^f\times10^2$ $a_1^f\times10^2$ $a_0^g\times10^2$ $a_1^g\times10^2$

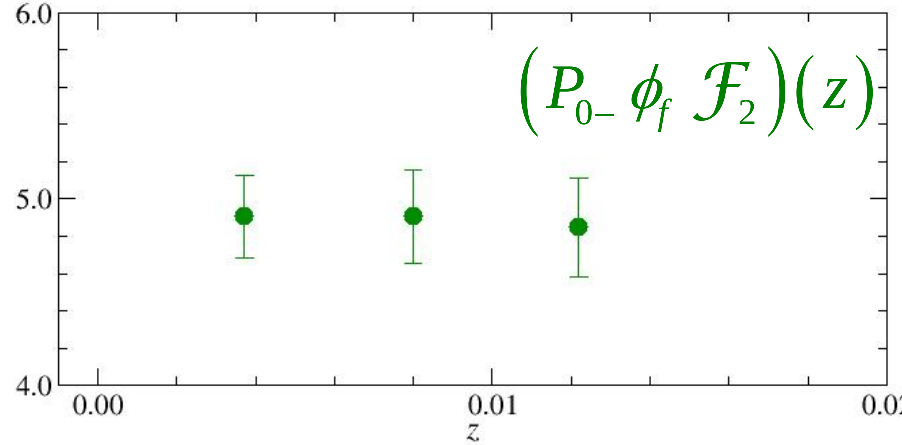


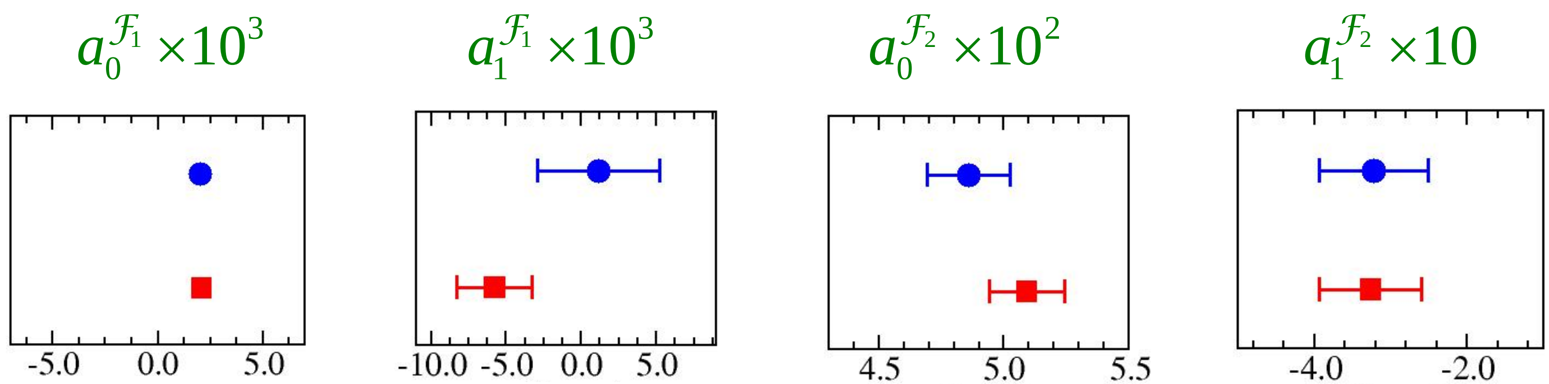
- better determined than pheno. : *e.g.* $\Delta a^g{}_0$ = 0.02 (lat) ⇔ 0.15 (Gambino et al. '19)
- consistent with Fermilab/MILC within 2σ or so in spite of different systematics

  tends to favor smaller normalization and slope for f and g (h_{A1} and h_V)

BGL parametrization of FFs





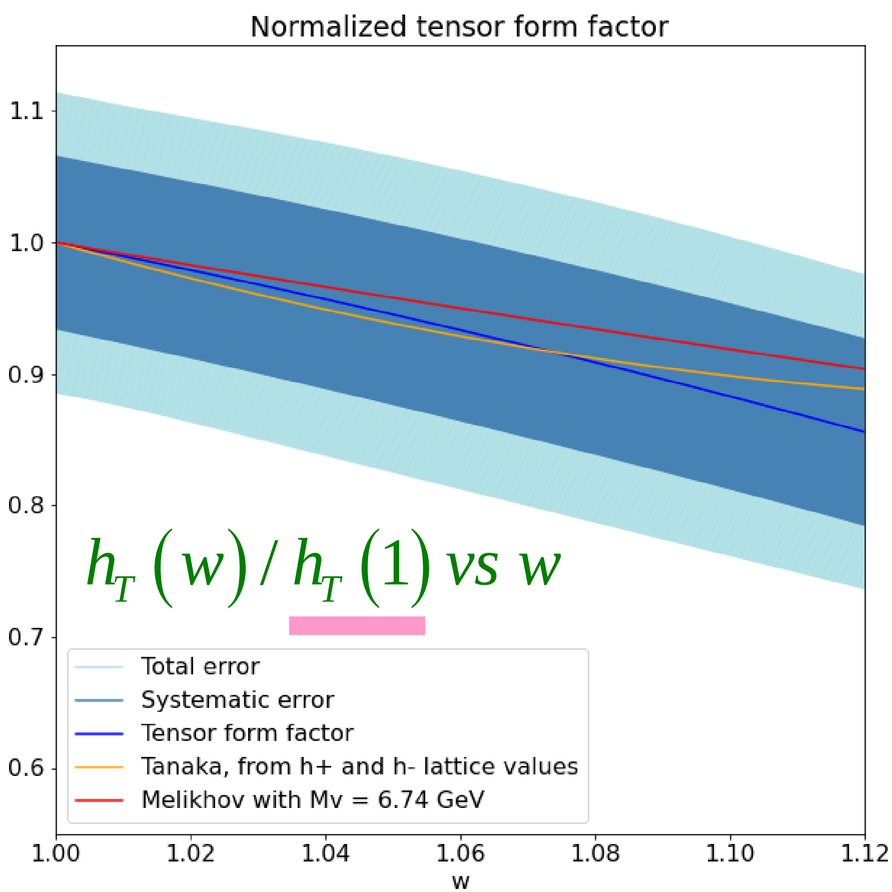


- $\approx$ 2σ consistency also for $\mathcal{F}_1$ and $\mathcal{F}_2$
- $\mathcal{F}_2$ for scalar contribution to $d\Gamma/dw \propto m_l^2$

  difficult from $B\to D^*\{e,\mu\}\nu$ $\Leftrightarrow$ useful for $B\to D^*\tau\nu$, $R(D^*)$
- analysis together w/ experimental data in progress

beyond SM

$B \to D\ell\nu$ tensor FF (M. Faur [Paris ENS, internship] + Kou + JLQCD)



$$\langle D(p') | T_{\mu\nu} | B(p) \rangle = i\left(v'^{\mu} v^{\nu} - v'^{\nu} v^{\mu}\right) h_T(w)$$

- ✓ extraction of tensor FF
- ✓ continuum chiral extrapolation
- ✓ systematic uncertainties
- renormalization in progress

T. Ishikawa @ Lattice 2021

- 10% stat. and 10% sys. errors
- consistent w/ phenomenology

useful input for BSM interpretation of B anomalies

$B \to \pi \ell \nu$ decay

$B \to \pi \ell \nu$ form factors

"relativistic" convention

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

"HQET" convention $|\mathrm{HQET}\rangle = |\mathrm{rel}\rangle / \sqrt{M}$

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = 2\left[f_1(v \cdot p_\pi)\, v^\mu + f_2(v \cdot p_\pi)\frac{p_\pi^\mu}{v \cdot p_\pi}\right]$$

Burdman, et al. '93

$$f_+ \propto f_2, \quad f_0 \propto f_1 + f_2 \quad (M_\pi, p_\pi \to 0)$$

JLQCD' measurements

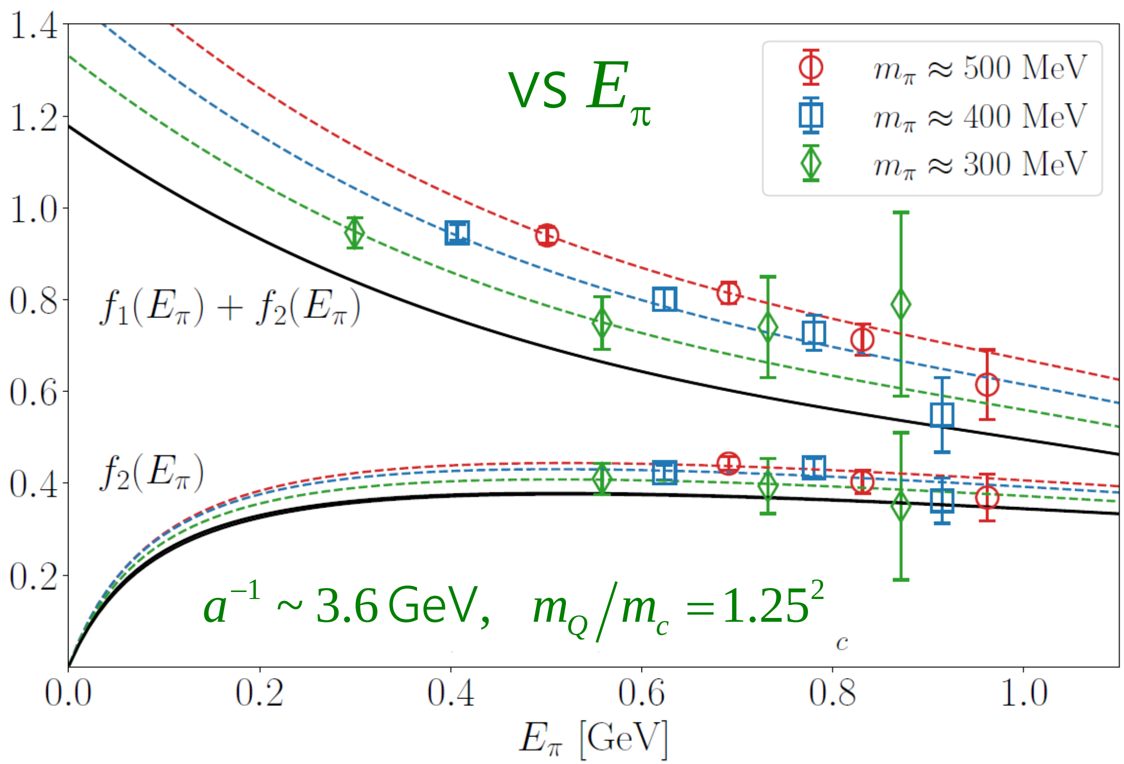
- B is at rest, $|\mathbf{p}_\pi|^2 = 0, 1, 2, 3$
- limited data set compared to $B \to D^{(*)}\ell\nu$
  - single source-sink separation $\Delta t + \Delta t'$
  - upto 3 m_Q's
  - …

continuum + chiral extrapolation

analysis led by B. Colquhoun (York), J. Koponen (Mainz)

as a function of E_π, M_π, m_Q, a

$$f_2(v\cdot p_\pi) = D_0\left[\frac{E_\pi}{E_\pi+\Delta_B}\left(1+D_{E_\pi}E_\pi\right)\right]\left(1+D_{\chi\log}\frac{\delta f^{B\to\pi}}{(4\pi f_\pi)^2}+D_{m_\pi^2}m_\pi^2\right)\left(1+\frac{D_{m_Q}}{m_Q}\right)$$
$$\times\left(1+D_{m_{s\bar s}^2}\delta m_{s\bar s}^2\right)\left(1+D_{a^2}a^2+D_{(am_Q)^2}(am_Q)^2\right).$$



FFs vs E_π

$a^{-1} \sim 3.6$ GeV, $m_Q/m_c = 1.25^2$

$M_\pi \sim 300, 400, 500$ MeV

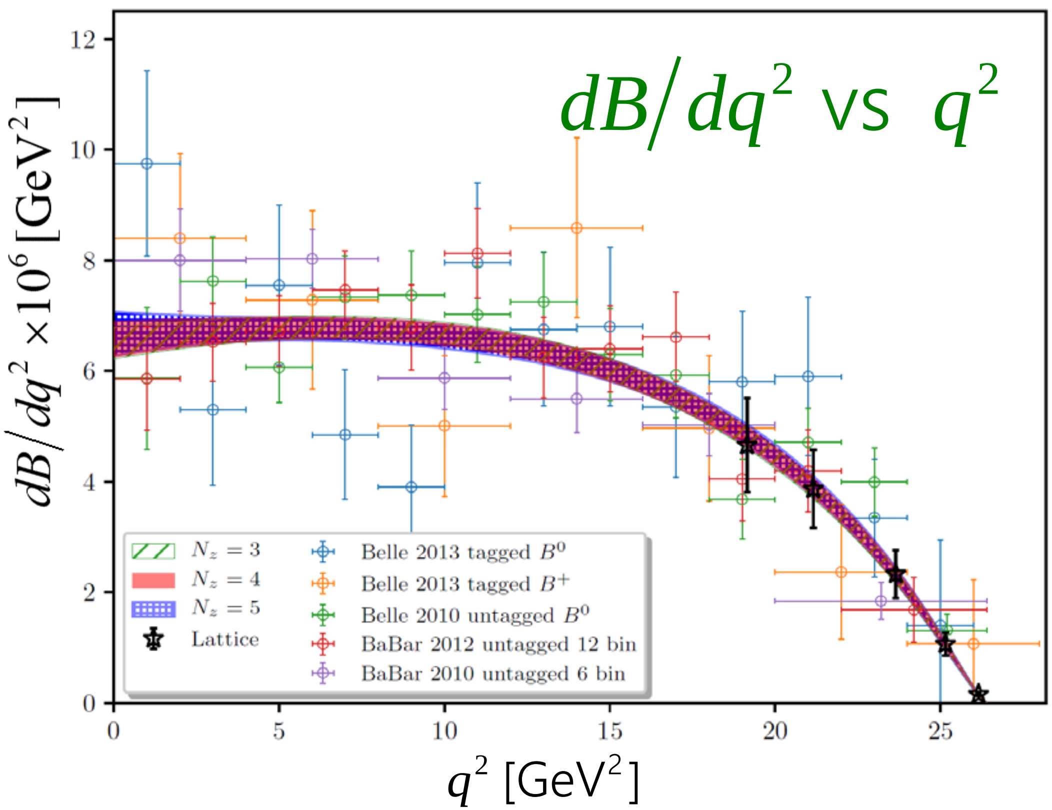
– reasonably described w/ χ^2/dof ~ 0.7

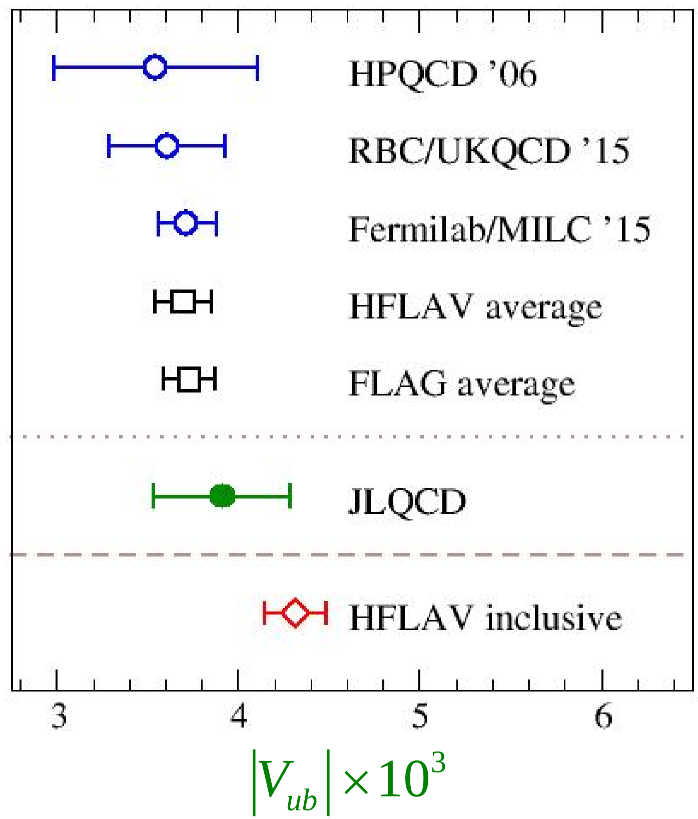
– less accurate @ larger E_π and smaller M_π

partial branching fraction and $|V_{ub}|$

Bourrely-Caprini-Lellouch (BCL) parameterization w/ Belle, BaBar data

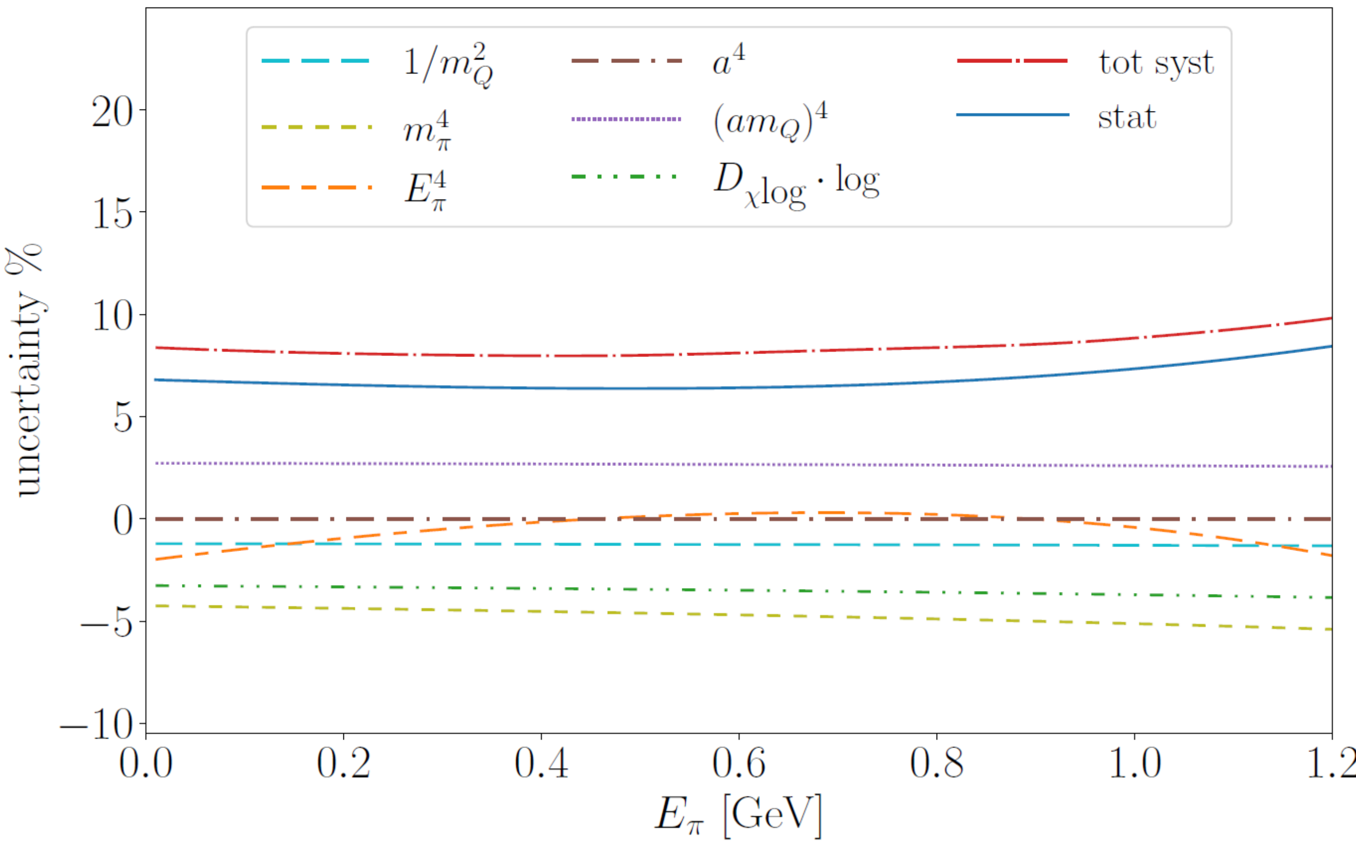
$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{N_z - 1} b_k^+ \left[z^k - (-1)^{k-N_z} \frac{k}{N_z} z^{N_z} \right] \qquad f_0(q^2) = \sum_{k=0}^{N_z} b_k^0 z^k$$





10% uncertainty : consistent w/ both previous excl. and incl.

uncertainties



uncertainties of f_2

total systematic

statistics

chiral extrap $+M_\pi^4$

- f_2, f_1+f_2 : largest uncertainties – statistics ~ 7%, chiral extrapolation ~ 5%
- a paper will be submitted to arXiv soon
- statistical accuracy to be improved

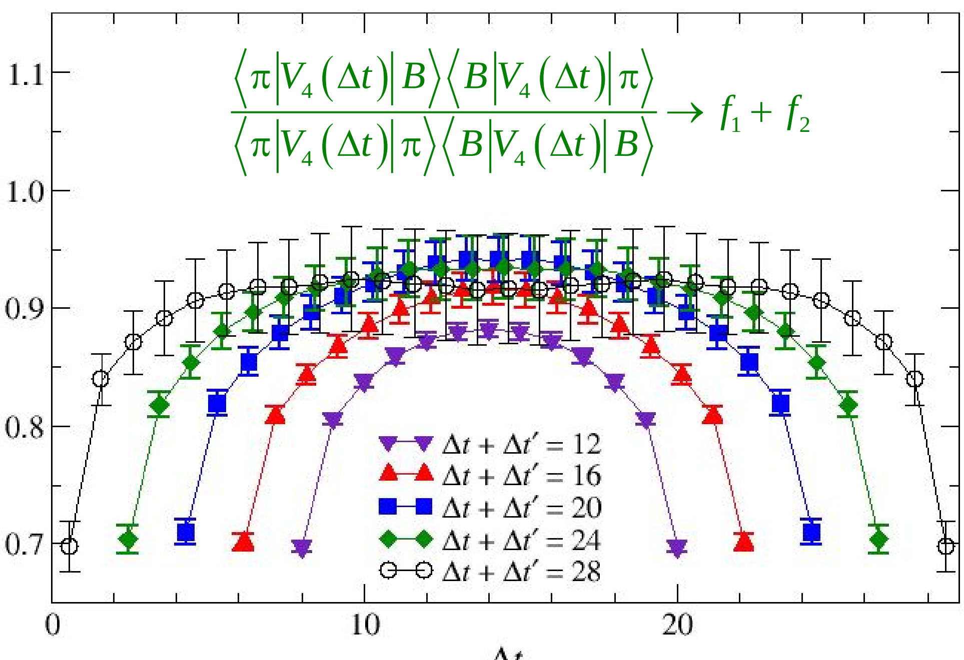
more data w/ Fugaku

Fugaku @ RIKEN R-CCS

A64FX x 158,976, 0.5 EFLOPS

+ Grid / Hadrons

GRID Hadrons



hope significant improvement of statistical accuracy

$\#(\Delta t + \Delta t') : 1 \to 5$

$\#t_{\mathrm{src}} = 1 - 2 \to 4$

$|\mathbf{p}_\pi|^2 \leq 3 \to 4$

$\#m_Q = 3 \to 6$

(and hopefully new ensemble(s)...)

Summary

semileptonic FFs from JLQCD

- relativistic lattice QCD w/ light and heavy DWQs
- $B \longrightarrow D^{(*)}\ell\nu$
  - mild a, m_q dependence $\Rightarrow$ controlled continuum – chiral extrap.
  - h_{A1} ~ 2% total accuracy
  - reasonable consistency w/ Fermilab/MILC '21
  - analysis w/ Belle, BaBar data in progress
- $B \longrightarrow \pi\ell\nu$
  - 10% accuracy for $|V_{ub}|$
  - being improved by Fugaku + Grid + Hadrons
- BSM FFs for NP model interpretation of B anomalies
  - $B \rightarrow D\ell\nu$ tenor FF / expect more in the future