The $B_{(s)} \to D_{(s)}^{(*)} \ell\nu$ Form-factors and the LUV R-ratios from Fermilab Lattice-MILC Collaboration

Alejandro Vaquero

University of Utah

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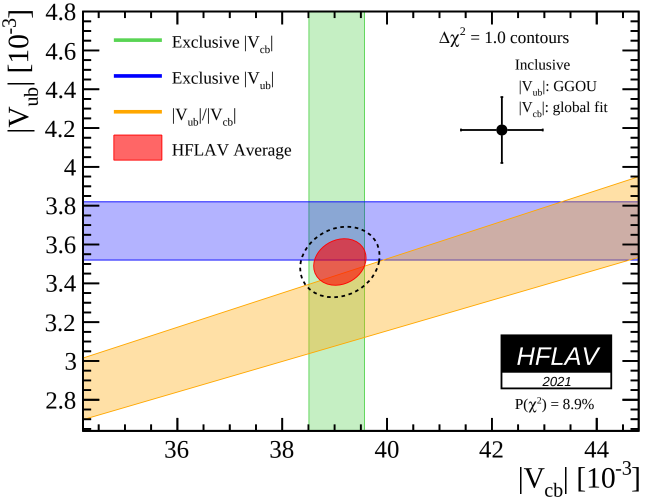
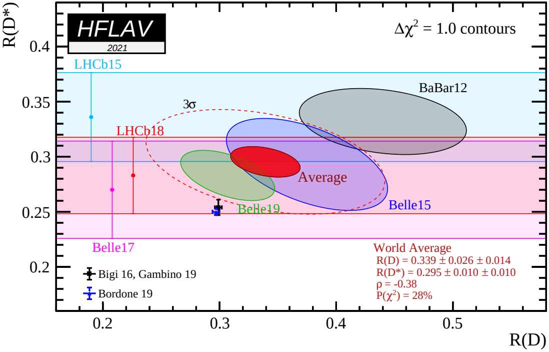
Motivation: $|V_{ub}|$ vs $|V_{cb}|$



Current status of $|V_{ub}|$ vs $|V_{cb}|$ (HFLAV 2021)

Motivation: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma$ tension with the SM

The V_{cb} matrix element: Calculating $R(D^*)$

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right]\times|V_{cb}|^2$$

- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative $\to$ LQCD
- Since $K_2(w,0)=0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
  - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw\,\left[K_1(w,m_\tau)\,|\mathcal{F}(w)|^2 + K_2(w,m_\tau)\,|\mathcal{F}_2(w)|^2\right]\times \cancel{|V_{cb}|^2}}{\int_1^{w_{\mathrm{Max}}} dw\,\left[K_1(w,0)\,|\mathcal{F}(w)|^2\right]\times\cancel{|V_{cb}|^2}}$$

- $|V_{cb}|$ cancels out

Form factors in LQCD: Heavy quarks

Heavy quark treatment in Lattice QCD

- For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
- For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
  - In this work $am_c \sim 0.15 - 0.6$, but $am_b > 1$

Need special actions and ETs to describe the bottom quark

- Relativistic HQ actions (these works $\rightarrow$ FermiLab)
- Non-Relativistic QCD (NRQCD)

If the action is improved enough, one can treat the bottom as a light quark

- Highly improved action AND small lattice spacing
- Use unphysical values for m_b and extrapolate

The discretization errors needn't disappear **as long as we keep them under control**

Form factors in LQCD: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu*}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu*}\left[g^{\mu\nu}\,(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\,(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w))\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Form factors in LQCD: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w+1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

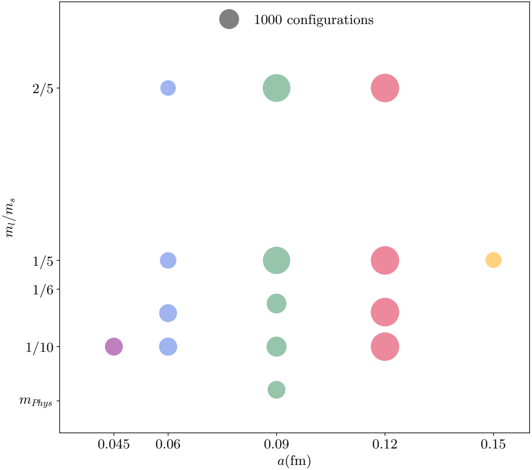
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\left|\mathcal{F}\right|^2 = \frac{1-2wr+r^2}{12m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Previous analysis: Ensembles

- Using 15 $N_f = 2+1$ MILC ensembles of asqtad sea quarks
- The heavy quarks are treated using the Fermilab action

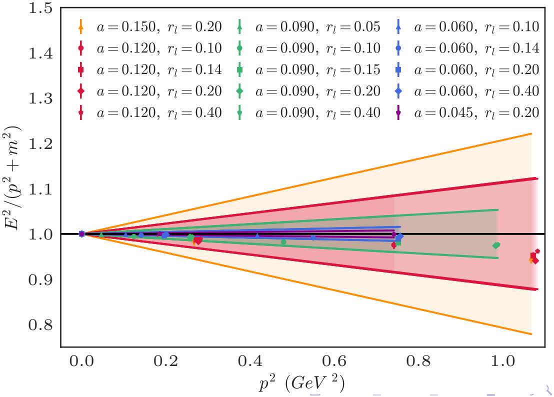


Previous analysis: Systematics in the two-point function fits

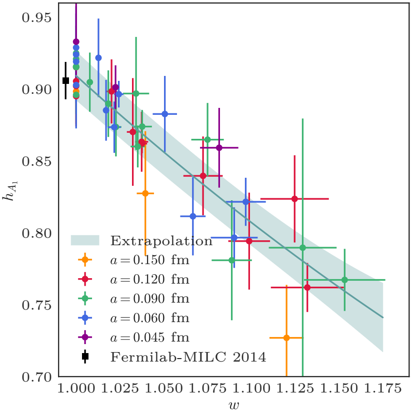
- Heavy quark discretization effects break the dispersion relation
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

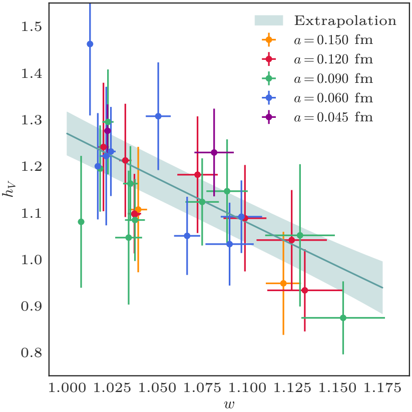
$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- Deviations from the continuum expression measure the size of the discretization errors
- As long as the discretization errors are within expected bounds, this is all right
- Data for B meson only at rest $\longrightarrow$ Ok in the past



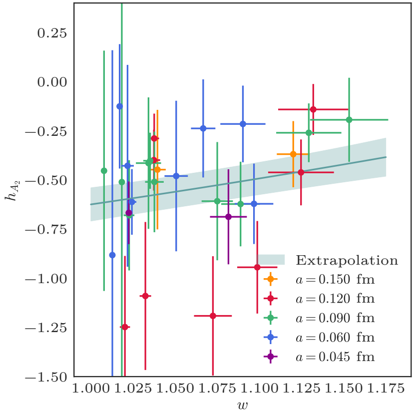
Previous analysis: Chiral-continuum extrapolation

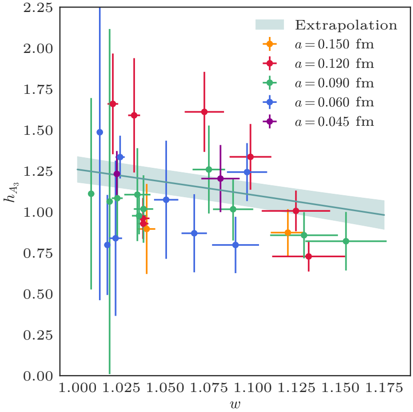




- Combined fit $p - \text{value} = 0.96$
- $h_{A_1}(1) = 0.909(17) \qquad h_V(1) = 1.270(46)$

Previous analysis: Chiral-continuum extrapolation





- Combined fit $p - \text{value} = 0.96$
- $h_{A_2}(1) = -0.624(85) \qquad h_{A_3}(1) = 1.259(79)$

Previous analysis: Error budget

Source	$h_V\,(\%)$	$h_{A_1}\,(\%)$	$h_{A_2}\,(\%)$	$h_{A_3}\,(\%)$
Chiral-continuum fit error	4.2	2.0	17.4	6.9
(Statistics)	(3.7)	(1.2)	(16.9)	(6.3)
(LQ and HQ discretization)	(2.6)	(1.3)	(9.7)	(4.4)
(Chiral-continuum extrapolation)	(0.8)	(0.9)	(1.7)	(0.5)
(Matching)	(0.3)	(0.2)	(1.7)	(0.5)
(HQ mistuning)	(0.0)	(0.0)	(1.7)	(0.0)
LQ mistuning	0.0	0.0	0.1	0.0
Scale settings	0.0	0.0	0.2	0.1
Isospin effects	0.1	0.2	1.2	0.5
Finite volume	-	-	-	-
Total error	4.2	2.0	17.4	6.9

Errors at $w = 1.11$

- **The discretization errors are one of the most important contributions to the final error**

Previous analyis: The z expansion

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL)

Phys. Rev. Lett. 74 (1995) 4603-4606

Phys.Rev. D56 (1997) 6895-6911

Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN)

Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + c z^2 - d z^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

Previous analysis: The z expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

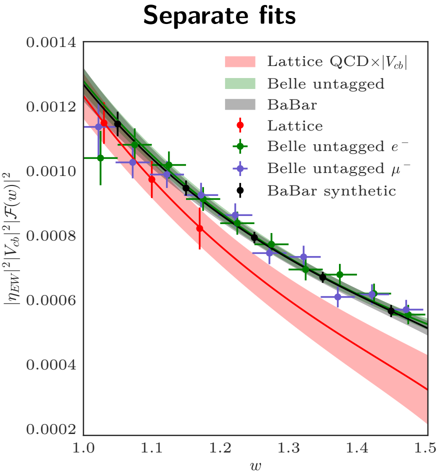
$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

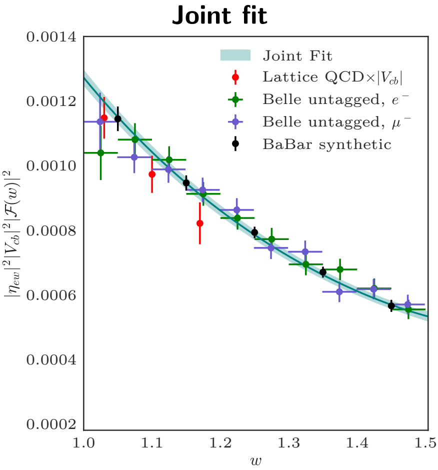
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\mathrm{Max}}) = (1+r) \mathcal{F}_2(z = z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

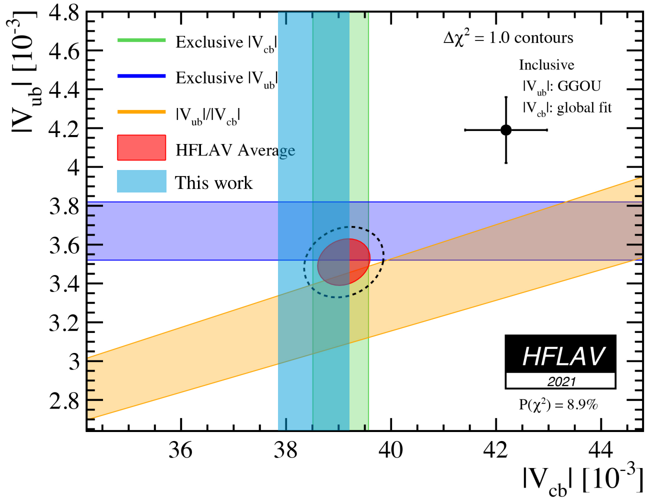
Previous analysis: z expansion fits





Unblinded, final result $|V_{cb}| = 38.57(78) \times 10^{-3}$

Previous analysis: Update of $|V_{ub}|$ vs $|V_{cb}|$



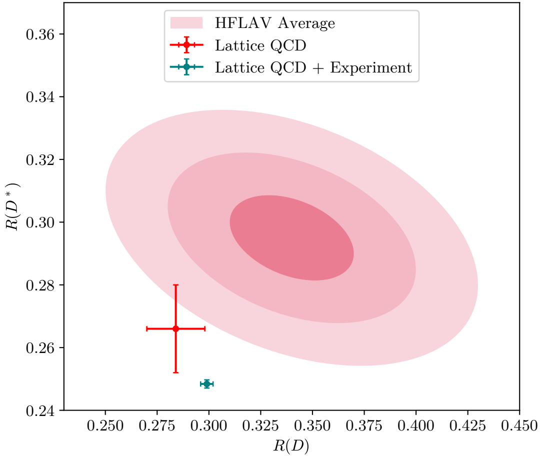
The $|V_{cb}|$ puzzle remains

Previous analysis: $R(D^*)$ in context

No constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.266(14)$ $\quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2484(13)$

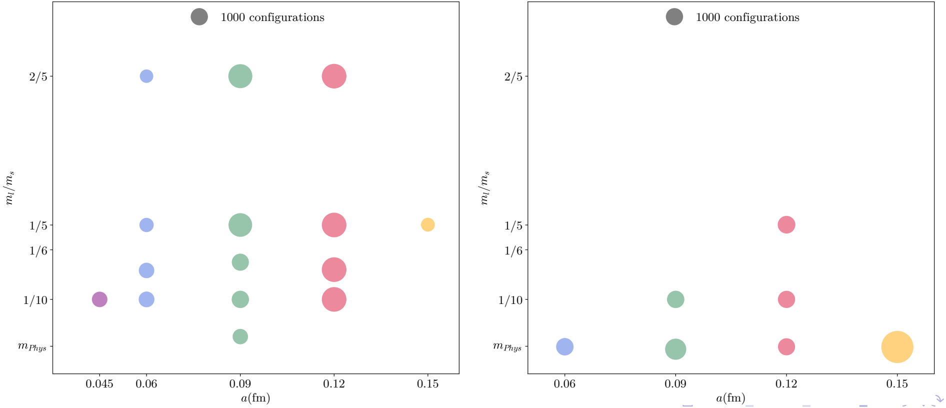
W/ constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.274(10)$ $\quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2492(12)$

Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801



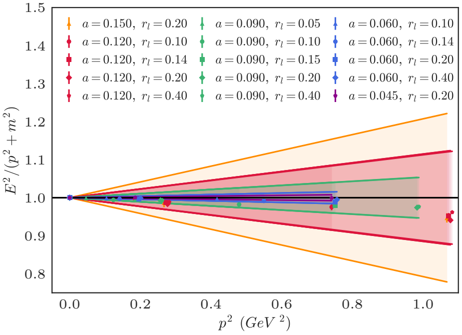
Current analysis: Ensembles

- Using 7 $N_f = 2+1+1$ MILC ensembles of HISQ quarks
- The heavy quarks b and c are treated using the Fermilab action
  - Reduce systematics by using the same action for valence b and c HPQCD, Phys.Rev.D 97, 054502 (2018)
- Momenta available for the form factors: 000, 100, 110, 200, 211, 300, 400.

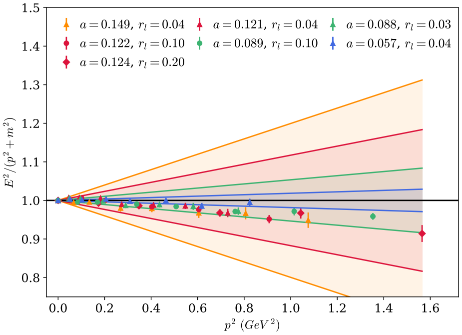


Current analysis: Expectations from ongoing two-point analysis

ASQTAD D^* meson

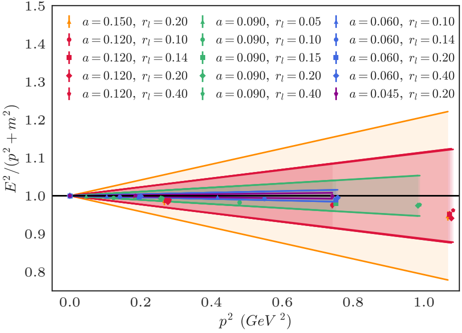


HISQ B meson

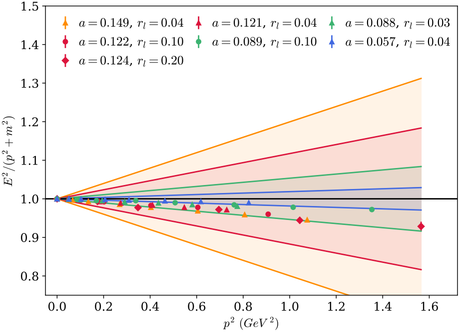


Current analysis: Expectations from ongoing two-point analysis

ASQTAD D^* meson

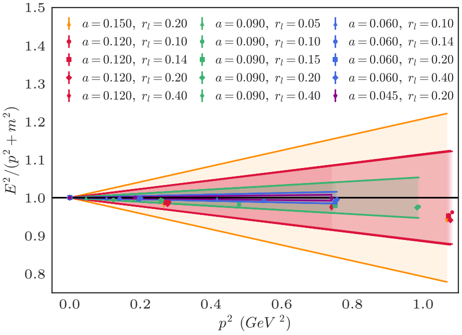


HISQ B_s meson

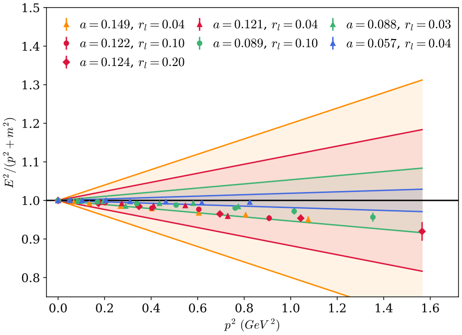


Current analysis: Expectations from ongoing two-point analysis

ASQTAD D^* meson

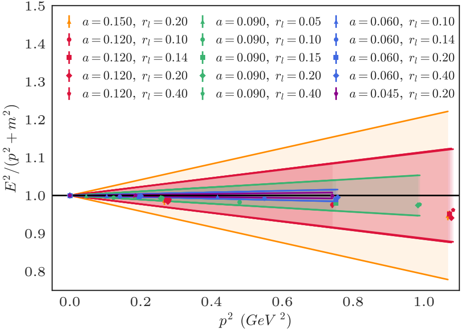


HISQ D meson

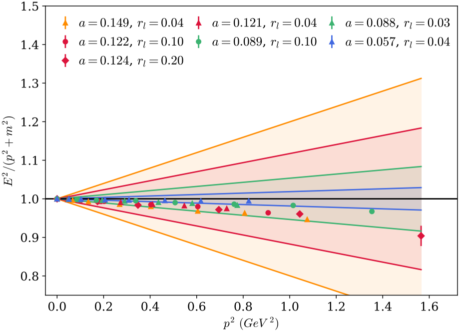


Current analysis: Expectations from ongoing two-point analysis

ASQTAD D^* meson



HISQ D_s meson



Current analysis: Improvements over the old analysis

- The smaller number of ensembles is compensated with the higher number of momenta per ensemble
- All lattice spacings have an ensemble at the physical pion mass
  - Expect a sizable reduction in the statistical and the chiral-continuum extrapolation errors
- Our preliminary analysis shows cleaner correlators with HISQ
  - Better control of excited states should reduce statistical errors wrt the previous analysis
- Performing a $B_{(s)} \to D^*_{(s)} \ell\nu$ combined analysis allows us to obtain correlated results in the $R(D_{(s)}) - R(D^*_{(s)}$ plane
- Analysis performed in close coordination with the $B \to \pi$ and $B_{(s)} \to K$, in order to obtain correlated results for $|V_{ub}|$ and $|V_{cb}|$

Conclusions

- The FNAL Lattice-MILC has finished the first unquenched calculation of the $B \to D^* \ell \nu$ form factors at non-zero recoil
  - The inclusive-exclusive tension in the determination of $|V_{cb}|$ remains unsolved
  - Results show $R(D^*)$ very close to the **theoretical prediction**
  - Main sources of errors: statistics and discretization errors
- A second analysis, much more ambitious, is currently in progress
  - Includes more decays
  - Correlate analysis with $B_{(s)}$ to light
  - Expect better statistical errors and some improvements in discretization errors

Thank you for your attention

BACKUP SLIDES