Form factors for semileptonic $B_s \longrightarrow K$ and $B_s \longrightarrow D_s$ decays

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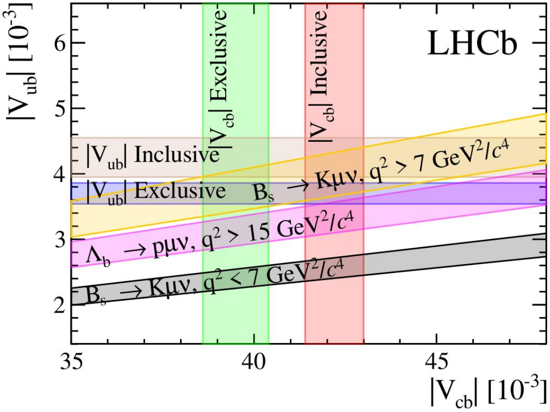
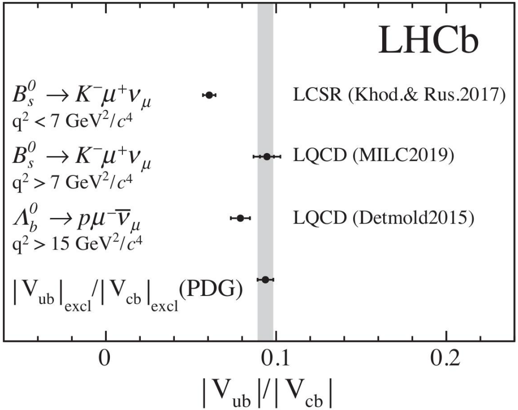
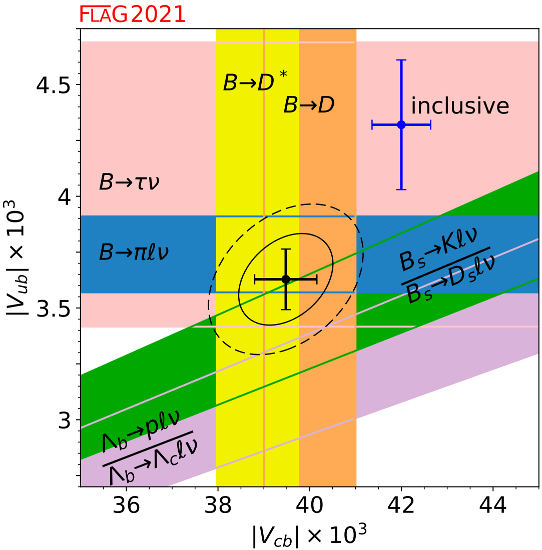
Motivation: SM parameters $|V_{ub}|$, $|V_{cb}|$





[LHCb first $B_s \to K\mu\nu$ 2021[1], Khodjamirian & Rus 2017[2], Fermilab/MILC 2019[3], Detmold et al 2015[4], PDG[5]]

Motivation: SM parameters $|V_{ub}|$, $|V_{cb}|$



$$R_{\mathrm{BF}}(B_s,\mathrm{high}) = \frac{\int_{7\,\mathrm{GeV}^2}^{q^2_{\max}} \frac{d\mathcal{B}(B_s\to K\mu\bar{\nu}_\mu)}{dq^2}\,dq^2}{\mathcal{B}(B_s\to D_s\mu\bar{\nu}_\mu)}$$

$$= 3.25(28)\times 10^{-3} \quad \text{[LHCb21[1]]}$$

$$FF_K(\mathrm{high}) = 4.02(31)\ \mathrm{ps}^{-1} \quad \text{[FLAG21[6]]}$$

$$FF_{D_s}(\mathrm{all}) = 9.15(37)\ \mathrm{ps}^{-1} \quad \text{[HPQCD19[7]]}$$

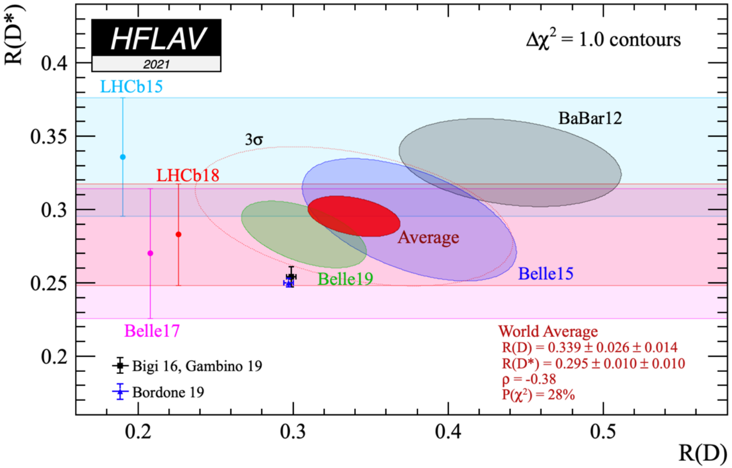
From

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 = R_{\mathrm{BF}}(B_s,\mathrm{high})\,\frac{FF_{D_s}(\mathrm{all})}{FF_K(\mathrm{high})}$$

find

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{\mathrm{FLAG21}} = 0.0860(37)^{\mathrm{latt}}(38)^{\mathrm{expt}}$$

Motivation: lepton flavour universality via $R(D^{(*)})$



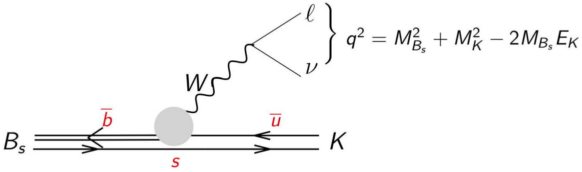
$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}}$$

HFLAV 2021:
3.4σ discrepancy

[HFLAV[8], Bigi[9], Gambino[10], Bordone[11], BaBar[12,13], Belle[14–17], LHCb[18–20]]

Pseudoscalar to pseudoscalar semileptonic decay

Semileptonic decay of pseudoscalar meson B_s of mass M and momentum p to pseudoscalar meson P of mass m and momentum k, with $q = p - k$



$$\frac{d\Gamma(B_s \to P\ell\nu)}{dq^2} = \eta \frac{G_F^2 |V_{xb}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 |\vec{k}|}{(q^2)^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \vec{k}^2 \, |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} \frac{(M^2 - m)^2}{M^2} |f_0(q^2)|^2 \right]$$

Form factors f_+ and f_0 from decomposition

$$\langle P(k) | \mathcal{V}^\mu(0) | B_s(p) \rangle = 2 f_+(q^2) \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu$$

where $\mathcal{V}^\mu = \bar{x}\gamma^\mu b$, with $x = u$ or c

Lattice setup

- Subset of 6 RBC/UKQCD 2+1-flavour DWF and Iwasaki gauge field ensembles
  - Three lattice spacings $a \sim 0.11, 0.08, 0.07\,\mathrm{fm}$, with $433\,\mathrm{MeV} > M_\pi < 267\,\mathrm{MeV}$
  - Light and strange quarks: Shamir DWF, $M_5 = 1.8$
  - Lattice spacings from combined RBC/UKQCD analysis [21–23]
- Bottom and charm quarks
  - Bottom: RHQ [Christ, Li, Lin[24,25]; Columbia variant of Fermilab action[26]] with three nonpt-tuned parameters $(m_0 a, c_P, \zeta)$ [27]
  - Charm: Möbius DWF, $M_5 = 1.6$[22,23,28]
    - 3 masses below m_c^{phys} on C ensembles
    - 2 masses which bracket m_c^{phys} on M and F

- Relate continuum and lattice currents: $\mathcal{V}_\mu$ and V_μ[29,30]

$$\langle P|\mathcal{V}_\mu|B_s\rangle = Z_{V_\mu}^{bx}\langle P|V_\mu|B_s\rangle$$

$$Z_{V_\mu}^{bx} = \rho_{V_\mu}^{bx}\sqrt{Z_V^{xx} Z_V^{bb}}$$

$$V_0 = V_0^0 + c_t^3 V_0^3 + c_t^4 V_0^4$$

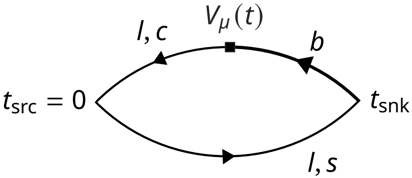
$$V_i = V_i^0 + c_s^1 V_i^1 + c_s^2 V_i^2 + c_s^3 V_i^3 + c_s^4 V_i^4$$

  - $\rho_{V_\mu}^{bx}$ and $c_{t,s}^n$ computed perturbatively [Lehner]
  - Z_V^{bb} from $Z_V^{bb}\langle B_s|V_0|B_s\rangle = 2M_{B_s}$
  - Z_V^{xx} from $Z_V^{xx} = Z_A^{xx} + O(am_{\mathrm{res}})$

Extracting form factors

Calculate

$$f_{\parallel}(E) = \frac{\langle P|\mathcal{V}^0(0)|B_s\rangle}{\sqrt{2M_{B_s}}}, \quad f_{\perp}(E) = \frac{\langle P|\mathcal{V}^i(0)|B_s\rangle}{k^i\sqrt{2M_{B_s}}}$$



from which

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_P^2}\left[(M_{B_s} - E)f_{\parallel}(E) + (E^2 - M_P^2)f_{\perp}\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_{\parallel}(E) + (M_{B_s} - E)f_{\perp}\right]$$

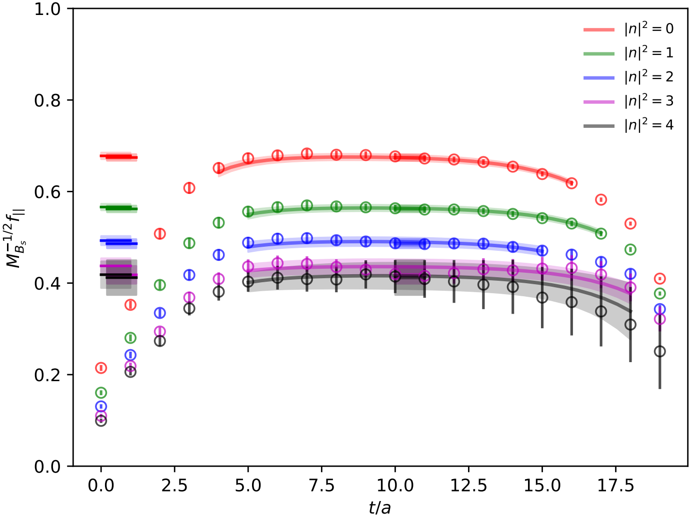
Extract from a correlator ratio (written here for $t_{\mathrm{src}} = 0$)

$$R_{3,\mu}(t, t_{\mathrm{snk}}, \vec{k}) = \frac{C_{3,\mu}(t, t_{\mathrm{snk}}, \vec{k})}{\sqrt{C_2^P(t, \vec{k})C_2^{B_s}(t_{\mathrm{snk}} - t, \vec{0})}}\sqrt{\frac{2E_P}{e^{-E_P t - M_{B_s}(t_{\mathrm{snk}} - t)}}}$$

$$f_{\parallel}^{\mathrm{bare}}(\vec{k}) = \lim_{0 \ll t \ll t_{\mathrm{snk}}} R_{3,0}(t, t_{\mathrm{snk}}, \vec{k})$$

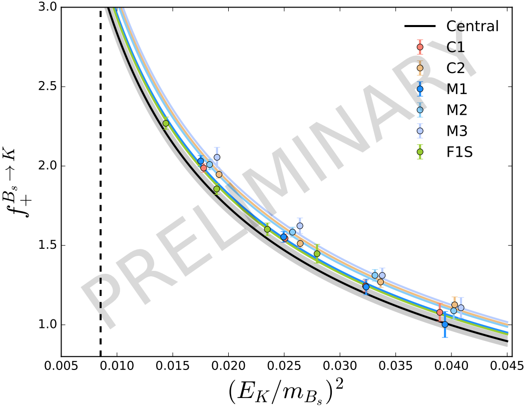
$$f_{\perp}^{\mathrm{bare}}(\vec{k}) = \lim_{0 \ll t \ll t_{\mathrm{snk}}} \frac{1}{p_P^i} R_{3,i}(t, t_{\mathrm{snk}}, \vec{k})$$

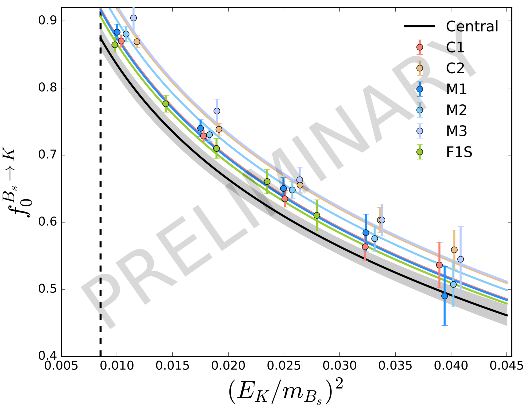
$B_s \to K$ form factor extraction



$f_{\parallel}$ on coarse (C1) ensemble

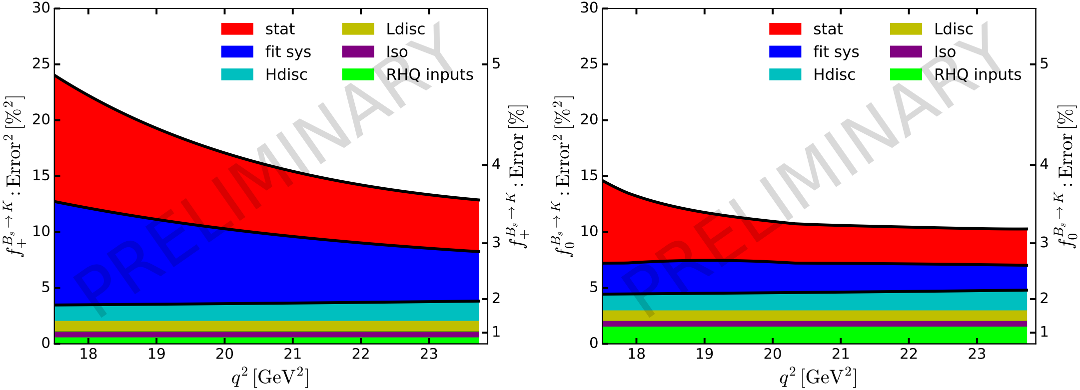
$B_s \longrightarrow K$ chiral-continuum extrapolation





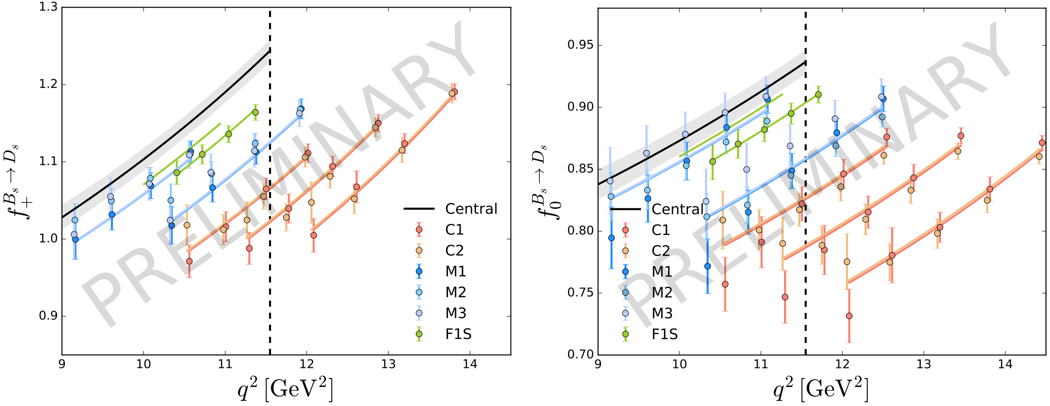
$$f^{B_s\to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K+\Delta}\left[c_0\left(1+\frac{\delta f(M_\pi^s)-\delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right)+c_1\frac{\Delta M_\pi^2}{\Lambda^2}+c_2\frac{E_K}{\Lambda}+c_3\frac{E_K^2}{\Lambda^2}+c_4(a\Lambda)^2\right]$$

$B_s \longrightarrow K$ cumulative error budget



▸ fit sys

$B_s \to D_s$ charm inter-/extrapolation and chiral-continuum extrapolation

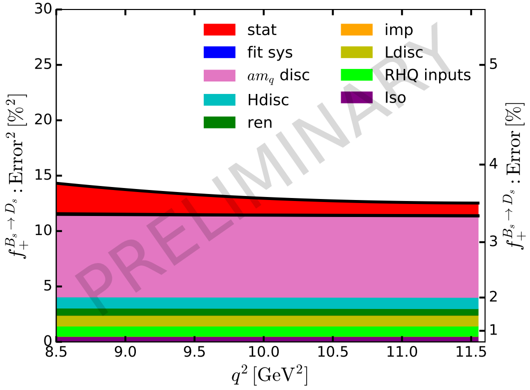


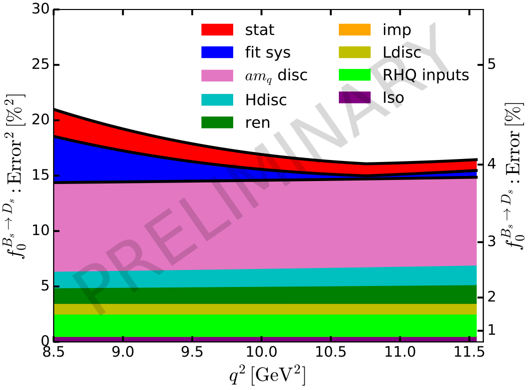
$$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + \sum_{j=1}^{n_{D_s}} c_{1j}\, h\left(\frac{M_{D_s}}{\Lambda}\right)^j + c_2(a\Lambda)^2\right] P_{a,b}(q^2/M_{B_s}^2)$$

$$h\left(\frac{M_{D_s}}{\Lambda}\right) = \frac{M_{D_s}}{\Lambda} - \frac{M_{D_s}^{\text{phys}}}{\Lambda}$$

$$P_{a,b}(x) = \frac{1+\sum_{i=1}^{a} a_i x^i}{1+\sum_{i=1}^{b} b_i x^i}$$

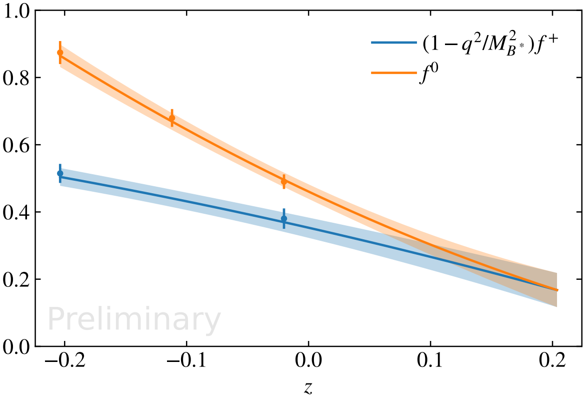
$B_s \to D_s$ cumulative error budget





fit sys

z-fits



- Do z-fits after χ-ctm extrapolation
- Use BGL[31–37] and BCL[38] for $B_s \longrightarrow K$ and $B_s \longrightarrow D_s$
- Example shown is BCL fit for $B_s \longrightarrow K$, with $f^+(q^2)(1-q^2/m_{B^*}^2)$ (lower) and $f^0(q^2)$ (upper) plotted

dispersion relation for current-current correlator
+
2-ptcl piece of complete set of states
+
z-transformation

$$\frac{1}{2\pi i}\oint \frac{dz}{z}\,|\phi(z)f(z)|^2 \leq \chi$$

unit circle

outer fn and Blaschke factors

form factor

from current-current correlator

$$z(t;t_*,t_0) = \frac{\sqrt{t_*-t}-\sqrt{t_*-t_0}}{\sqrt{t_*-t}+\sqrt{t_*-t_0}} \qquad t = q^2$$

$\boxed{\text{z-fits}}$

$\dfrac{\phi(z)f(z)}{\sqrt{\chi}}$ analytic inside unit circle

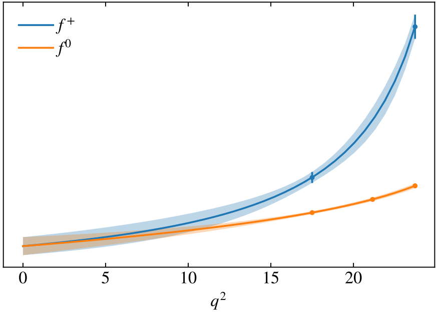
$$\frac{\phi(z)f(z)}{\sqrt{\chi}} = \sum_{n=0}^{\infty} a_n z^n \qquad \leftarrow \text{BGL}$$

with $\displaystyle\sum_{n=0}^{\infty} |a_n|^2 \leq 1$

$\boxed{\text{dispersive bounds}}$

combine constraint with known values of form factor

$f(q_i^2) = f_i \qquad i = 1, 2, \dots, N$



[BGL[32,35,36], Bourrely et al[31], Lellouch[34], Di Carlo et al[39], Martinelli et al[40–42]]

R ratios for LFU tests

$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar\nu_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\ell\bar\nu_\ell)}{dq^2}}$$

$$R^{\text{new}}(P) = \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)}\to P\tau\bar\nu_\tau)}{dq^2}}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)}\to P\ell\bar\nu_\ell)}{dq^2}}$$

- Adopt idea proposed for $B_{(s)} \longrightarrow V$ decays [Isidori–Sumensari[43]]
  - Common integration range; $q^2_{\min} \geq m_\tau^2$ [Freytsis et al[44], Bernlochner et al[45], Soni[46]]
  - *Same* weights for vector parts in integrands for τ and ℓ
- Write

$$\frac{d\Gamma(B_{(s)}\to P\ell\nu)}{dq^2} = \Phi(q^2)\omega_\ell(q^2)\left[F_V^2 + (F_S^\ell)^2\right]$$

$$\Phi(q^2) = \eta \frac{G_F^2 |V_{xb}|^2}{24\pi^3} |\vec{k}|$$

$$\omega_\ell(q^2) = \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left(1 + \frac{m_\ell^2}{2q^2}\right)$$

$$F_V^2 = \vec{k}^2 |f_+(q^2)|^2$$

$$(F_S^\ell)^2 = \frac{3}{4} \frac{m_\ell^2}{m_\ell^2 + 2q^2} \frac{(M^2 - m^2)^2}{M^2} |f_0(q^2)|^2$$

R ratios for LFU tests

$$R(P) = \frac{\int_{m_\tau^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}} \qquad R^{\text{new}}(P) = \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{d\Gamma(B_{(s)} \to P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)} \to P\ell\bar{\nu}_\ell)}{dq^2}}$$

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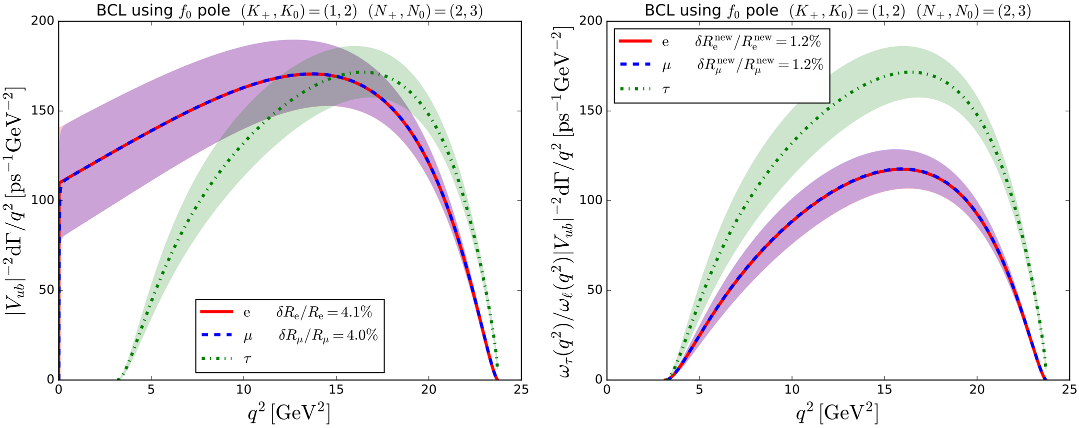
  - *Same* weights for vector parts in integrands for τ and ℓ
- Write

$$\frac{d\Gamma(B_{(s)} \to P\ell\nu)}{dq^2} = \Phi(q^2)\omega_\ell(q^2)\left[F_V^2 + (F_S^\ell)^2\right]$$

- If drop scalar contribution, $(F_S^\ell)^2$, in denominator $(m_\ell^2/2q^2 \leq m_\mu^2/2m_\tau^2 = 0.002)$ expect

$$R^{\text{new,SM}}(P) \approx 1 + \frac{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \Phi(q^2)\omega_\tau(q^2)(F_S^\tau)^2}{\int_{q^2_{\min}}^{q^2_{\max}} dq^2 \, \Phi(q^2)\omega_\tau(q^2)F_V^2}$$

R ratios: old and new definitions for $B_s \longrightarrow K$



- Integrands for old (left) and new (right) R ratios for $B_s \longrightarrow K$
- Reduction in relative error with new definition

Outlook

- Finalising results
- Additional lattice spacing since 2015 $B_s \longrightarrow K$ and added $B_s \longrightarrow D_s$
- q^2 extrapolations: synthetic data points, functional matching, dispersive bounds
- Subsequently $B \longrightarrow D\ell\nu$, vector final states, rare decays, B_c decays, $\ldots$

Backup

Motivation: Marcella Bona UT fit update EPS HEP 26 July 2021[47]

V_{cb} and V_{ub}

from FLAG 2019 arXiv:1902.08191

$|V_{cb}|$ *(excl)* = $(39.09 \pm 0.68)\ 10^{-3}$

$|V_{cb}|$ *(incl)* = $(42.16 \pm 0.50)\ 10^{-3}$

from Bordone et al.
arXiv:2107.00604

~2.8σ discrepancy

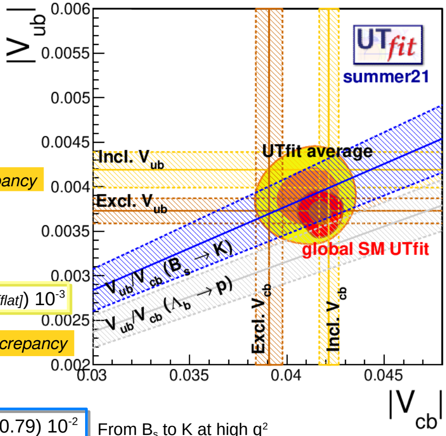
from FLAG 2019 arXiv:1902.08191

$|V_{ub}|$ *(excl)* = $(3.73 \pm 0.14)\ 10^{-3}$

$|V_{ub}|$ *(incl)* = $(4.19 \pm 0.17 \pm 0.18$ *[flat]*$)\ 10^{-3}$

from GGOU HFLAV 2021
adding a flat uncertainty
covering the spread
of central values

~1.5σ discrepancy



$|V_{ub} / V_{cb}|$ *(LHCb)* = $(9.46 \pm 0.79)\ 10^{-2}$ From B_s to K at high q^2

$|V_{ub} / V_{cb}|$ *(LHCb)* = $(7.9 \pm 0.6)\ 10^{-2}$ From Λ_b, excluded following FLAG guidelines

[LHCb first $B_s \to K\mu\nu$ result[1]]

Ensembles

	L	T	L_s	a^{-1}/GeV	am_l	am_s^{sea}	M_π/MeV	# cfgs	# sources
C1	24	64	16	1.785	0.005	0.040	340	1636	1
C2	24	64	16	1.785	0.010	0.040	433	1419	1
M1	32	64	16	2.383	0.004	0.030	302	628	2
M2	32	64	16	2.383	0.006	0.030	362	889	2
M3	32	64	16	2.383	0.008	0.030	411	544	2
F1S	48	96	12	2.785	0.002144	0.02144	267	98	24

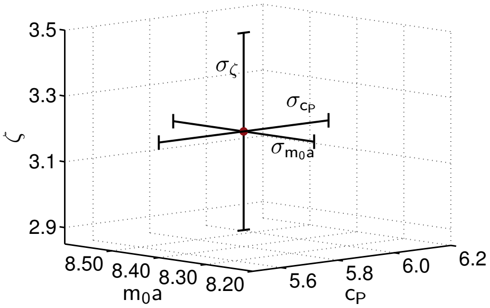
◂ setup

Nonperturbative RHQ parameter tuning

- Start with educated guess for m_0a, c_P and ζ
- At seven sets of nearby parameter values, evaluate

$$\begin{aligned} \text{spin-averaged mass} \quad & \bar{M} = (M_{B_s} + 3M_{B_s^*})/4 \\ \text{hyperfine splitting} \quad & \Delta_M = M_{B_s^*} - M_{B_s} \\ \text{ratio} \quad & M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}} \end{aligned}$$

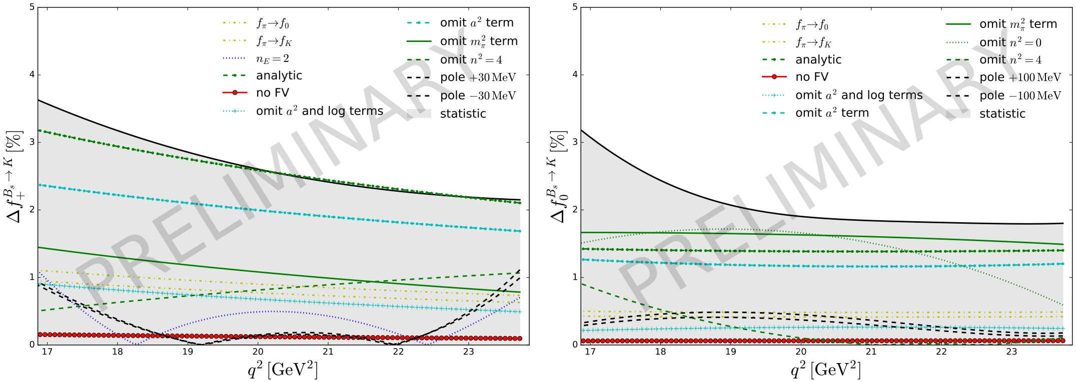
- Check for linear relation parameters $\leftrightarrow$ observables
- Obtain tuned parameters for physical b quarks which make $\bar{M}$ and Δ_M agree with experiment and $M_1 = M_2$



[PRD86 (2012) 116003[27]]

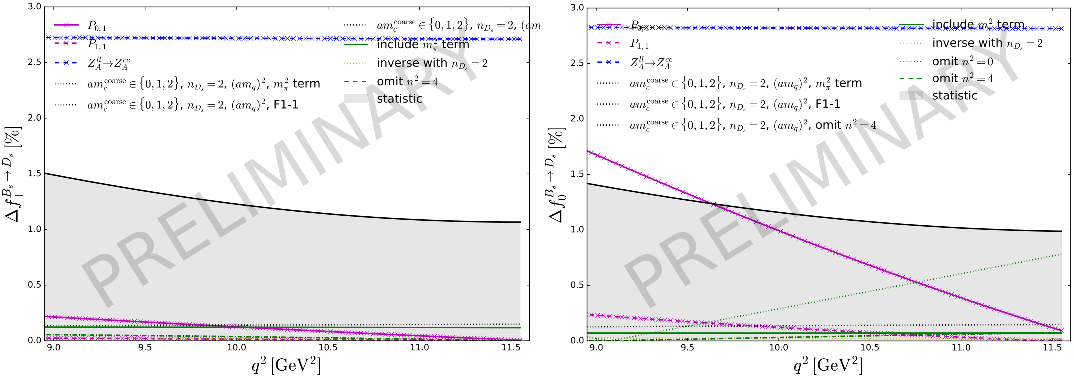
◂ setup

Chiral-continuum fit systematics: $B_s \longrightarrow K$



$\Delta f_i = |f_i^{\text{pref}} - f_i^{\text{alt}}|/f_i^{\text{pref}}$ for $i = 0, +$, for form-factor central values under fit variations.
Shaded band shows statistical uncertainty of preferred fit.

Chiral-continuum fit systematics: $B_s \to D_s$



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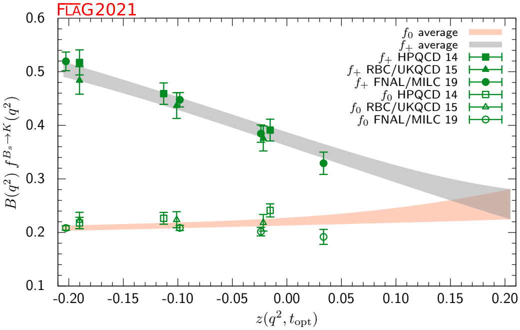
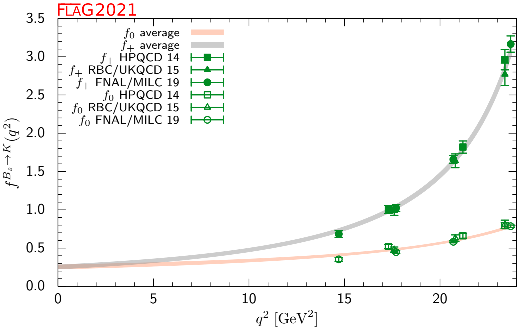
Shaded band shows statistical uncertainty of preferred fit.

$B_s \longrightarrow K\ell\nu$ form factors





FLAG 2021[6] with results from HPQCD[48], RBC/UKQCD[49], FNAL/MILC[3]

References I

[1] R Aaij et al. (LHCb), First observation of the decay $B_s^0 \to K^- \mu^+ \nu_\mu$ and measurement of $|V_{ub}|/|V_{cb}|$, Phys. Rev. Lett. **126**, 081804 (2021), arXiv:2012.05143 [hep-ex].

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