D→P Semileptonic Form Factors

with Highly Improved Staggered Quarks

William I. Jay — MIT

DWQ@25 Workshop, 15 Dec 2021

Outline:

- Experimental Motivation
- Theoretical background
- The all-HISQ campaign
- Preliminary results building on arXiv:2111.05184
- Outlook

PROCEEDINGS OF SCIENCE

B- and D-meson semileptonic decays with highly improved staggered quarks

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Fermilab Lattice and MILC Collaborations

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Many thanks to my friends and colleagues

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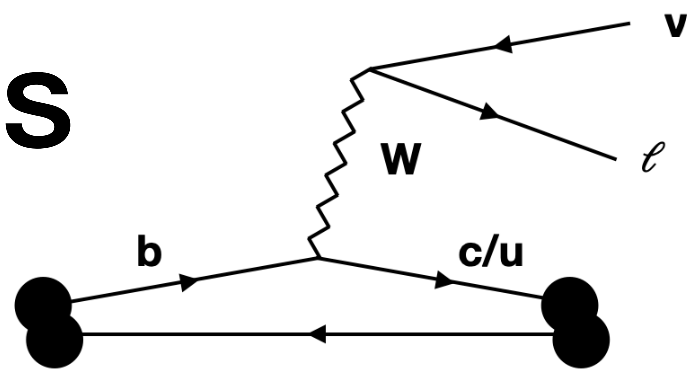
Experimental Motivation

Tensions: tree-level processes



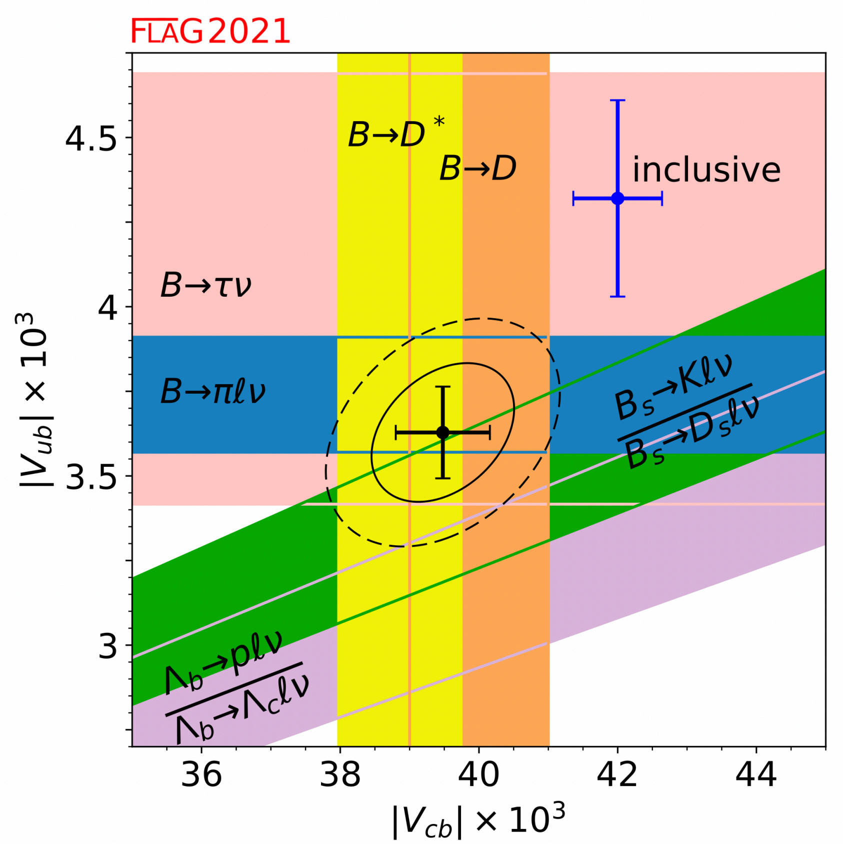
Inclusive vs. exclusive determinations of CKM matrix elements

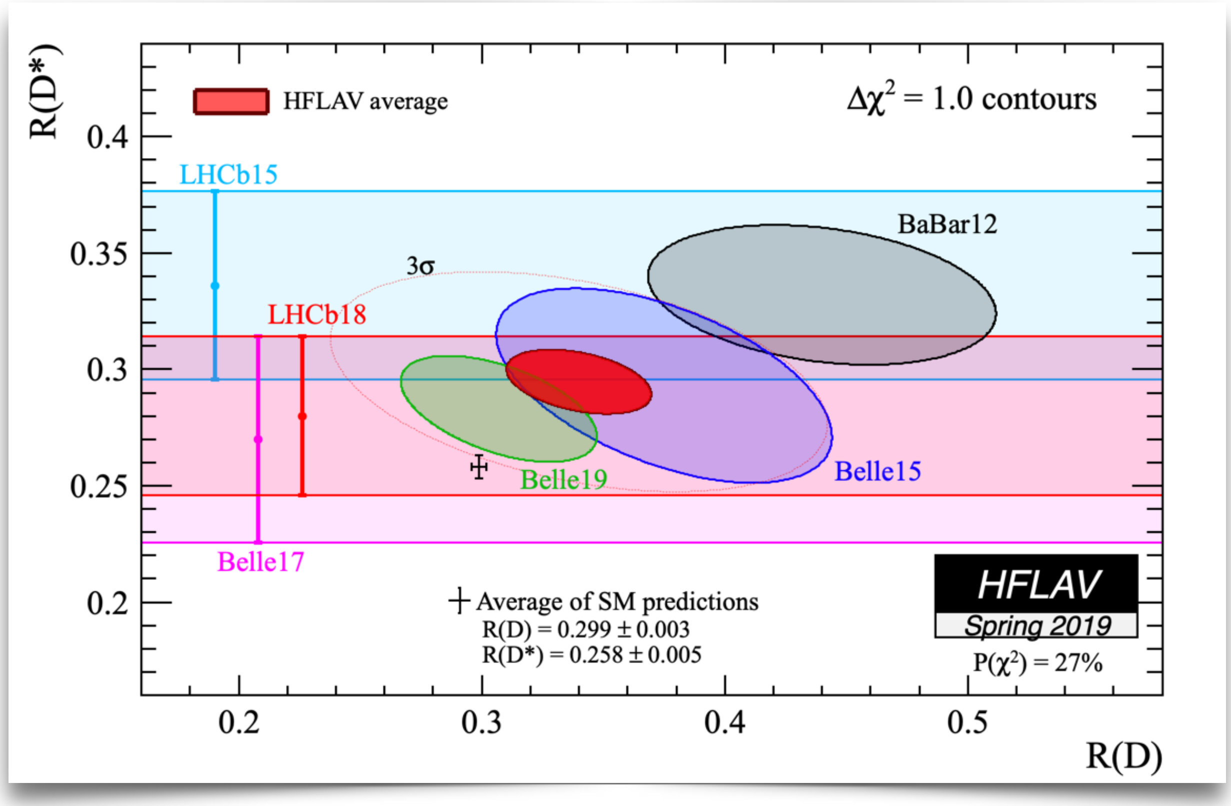
- $|V_{cb}|$ from B → $D^*\ell\nu$ ~ 3.3σ tension
- $|V_{cb}|$ from B → $D\ell\nu$ ~ 2.0σ tension
- $|V_{ub}|$ from B → $\pi\ell\nu$ ~ 2.8σ tension

- Tests of lepton universality, e.g.,

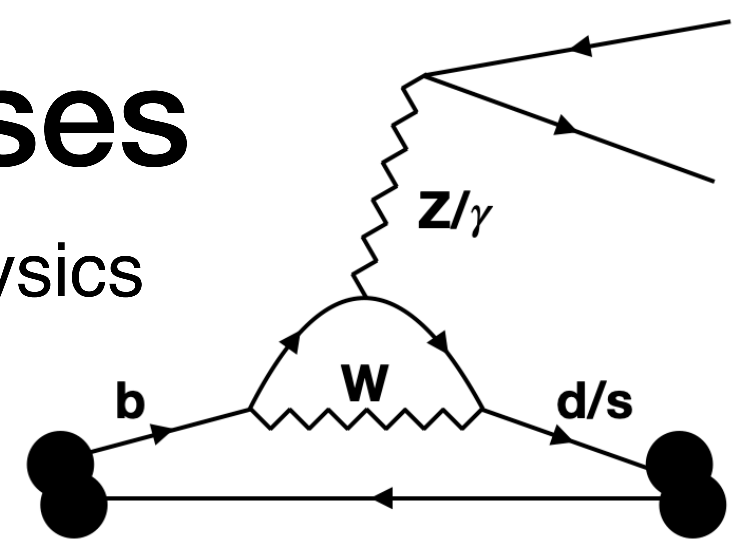
$$R(D) = \frac{\mathcal{B}(B \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to D\mu\bar{\nu}_\mu)}$$

- R(D) + R(D*) ~ 3.1σ





Tensions: loop-level processes



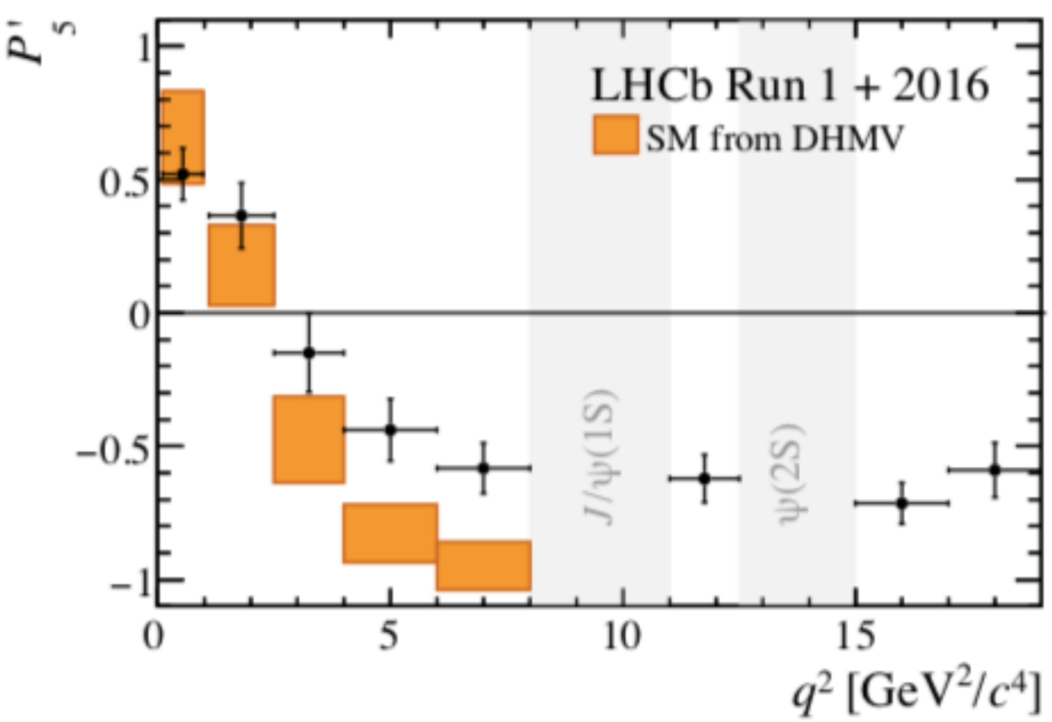
Rare processes are sensitive probes of high-scale physics

- New sources of CP violation?
- New RH currents?

$$B \to K\ell^+\ell^-$$

$$B \to K^*\ell^+\ell^-$$

Persistent tension (~3σ) with SM in angular distribution P_5'



PRL 125, 011802 (2020) arXiv:2003.04831

Tensions in tests of lepton universality.

LHCb: R_K ~ 3.1σ, R_{K^*} ~ 2.5σ

See talks from T. Iijima, M. Patel

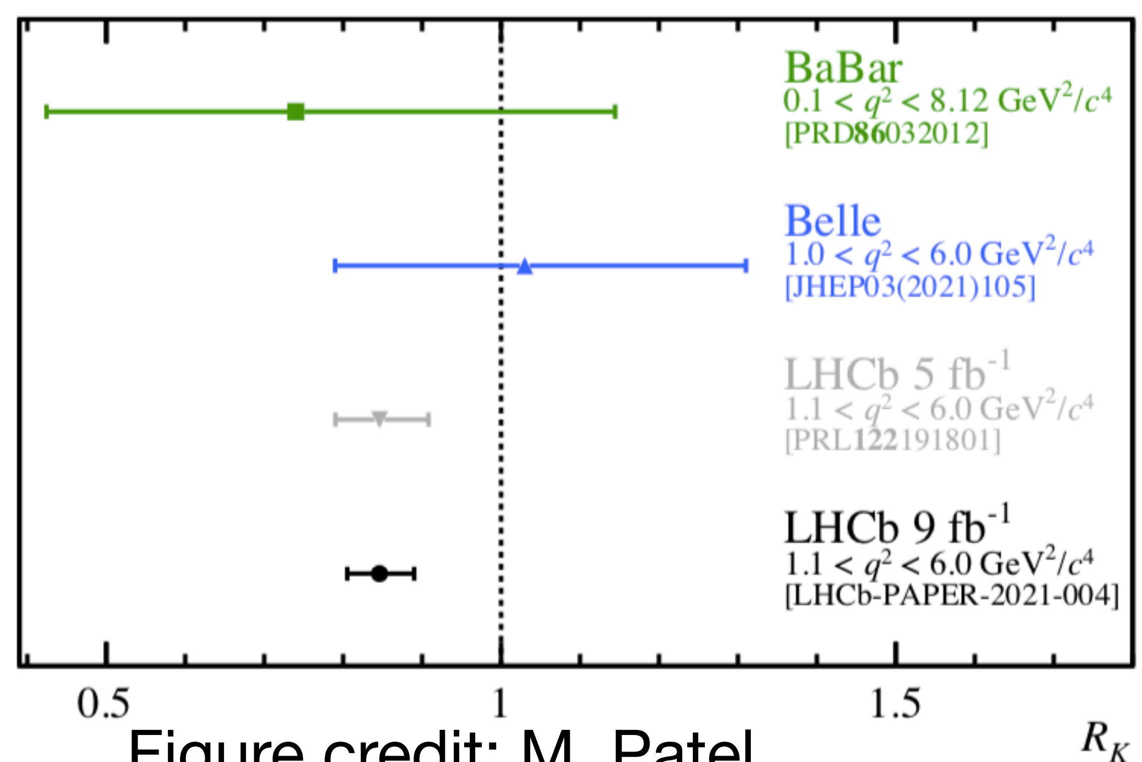
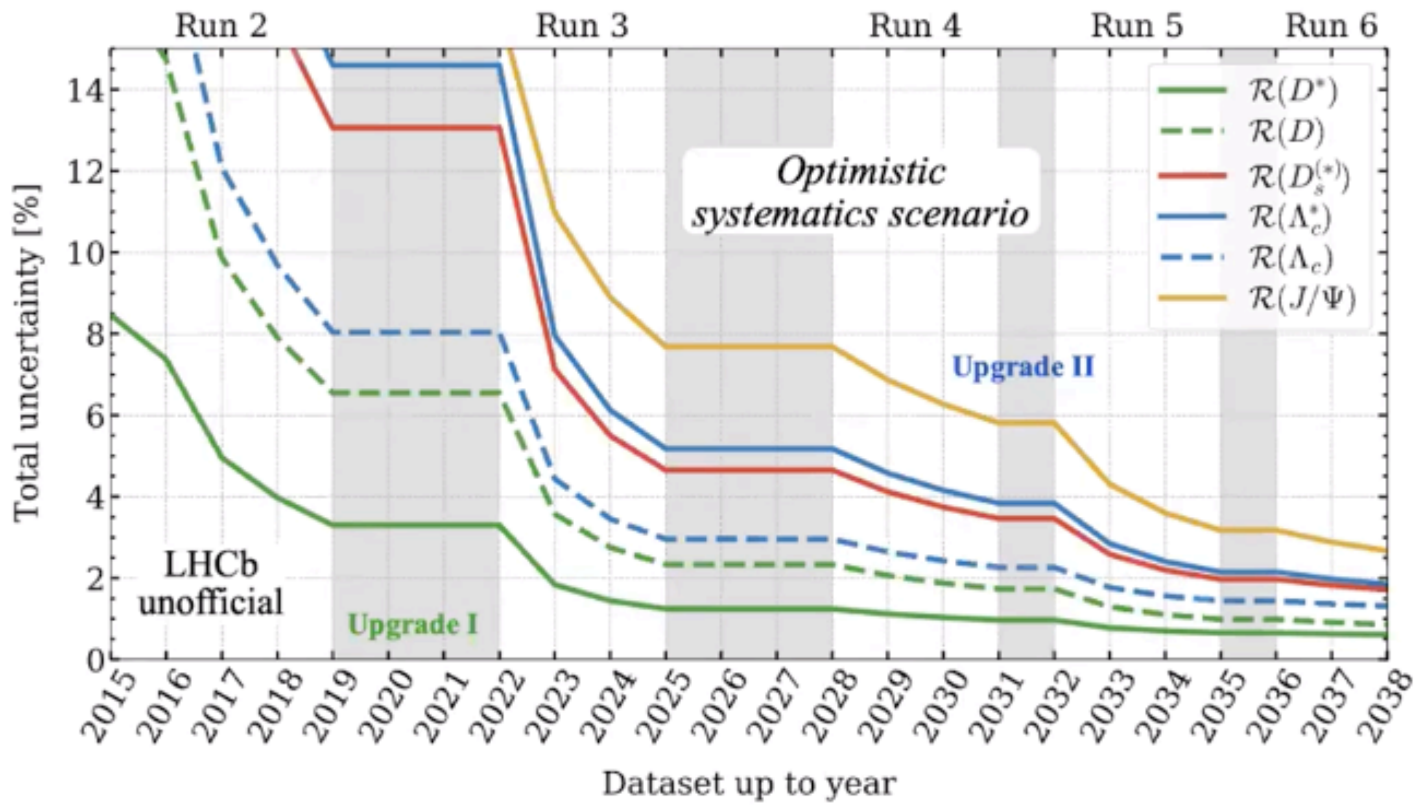


Figure credit: M. Patel

Improved theory is timely

- LHCb: pp at LHC
  - ~10^{12} b-hadrons to date (cf. ~10^{7} at LEP)
- Belle II: e^+e^- around $\Upsilon(4s)$ ~ 10.5 GeV
  - Goal: 50 ab^{-1} (50x Belle), roughly 215 fb^{-1} to date



Many exciting first measurements.
For example:

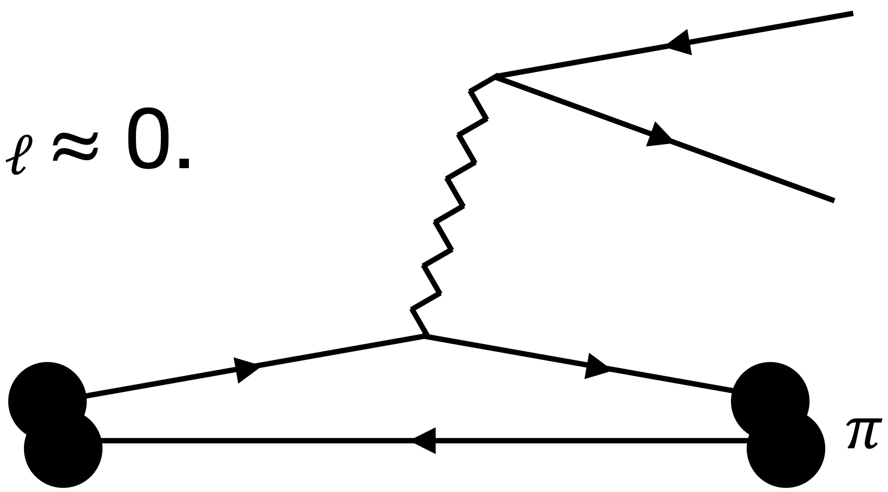
- BESIII: Form factors for $D_s \to K^{(*)} e \nu$
  - PRL 122, 061801 arXiv:1811.02911
- LHCb: Rare CKM suppressed $B \to \pi \mu^+ \mu^-$
  - JHEP 10 (2015) 034 arXiv:1509.00414

Theoretical background

Semileptonic decays: H→Pℓν

Theoretical preliminaries

- Consider the decay D→ $\pi\ell\nu$. Take $m_\ell \approx 0$.



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

$$\langle\pi| \mathcal{V}^\mu |D\rangle \equiv f_+(q^2)(p_D^\mu + p_\pi^\mu) + f_-(q^2)(p_D^\mu - p_\pi^\mu)$$

Or can equivalently decompose as:

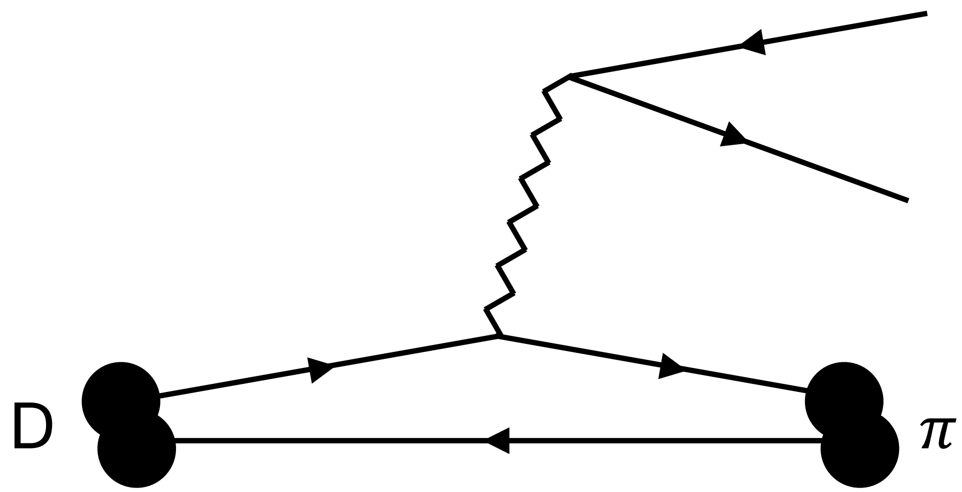
$$\langle\pi| \mathcal{V}^\mu |D\rangle \equiv \sqrt{2M_D}\left(v^\mu f_{||}(q^2) + p_\perp^\mu f_\perp(q^2)\right)$$

Semileptonic decays: H→Pℓν

Theoretical preliminaries

- Consider the decay D→ $\pi\ell$ν

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$



- When the lepton mass is *not* small, another form factor called $f_0(q^2)$ also contributes: $d\Gamma/dq^2 \ni$ (const) $\times$ $m_\ell^2 f_0(q^2)$

- $f_0(q^2)$ defined through yet another relation for $\langle\pi|V^\mu|D\rangle$

- Or, more simply via the Ward identity: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$

$$\frac{m_c - m_\ell}{M_\pi^2 - M_D^2} \langle\pi|\,\mathcal{S}\,|D\rangle = f_0(q^2)$$

Semileptonic decays: H→Pℓν

Theoretical preliminaries

- Relate the lattice and continuum currents via $\mathcal{J} = Z_J J$

- The form factors simplify in the rest frame of the D:

$$f_{\|} = Z_{V^0} \frac{\langle \pi | V^0 | D \rangle}{\sqrt{2M_D}}$$

$$f_{\perp} = Z_{V^i} \frac{\langle \pi | V^i | D \rangle}{\sqrt{2M_D}} \frac{1}{p_\pi^i}$$

$$f_0 = Z_S \frac{m_c - m_\ell}{M_D^2 - M_\pi^2} \langle \pi | S | D \rangle$$

$$f_+ = \frac{1}{\sqrt{2M_D}} \Big(f_{\|} + (M_D - E_\pi) f_{\perp} \Big)$$

We'll use lattice QCD to compute the matrix elements on the RHS

Semileptonic decays: H→Pℓν

Theoretical preliminaries

- Renormalization: the Z-factors $\mathcal{J} = Z_J J$
- Recall the Ward identity: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$
- In terms D→π matrix elements, this reads:

$$Z_{V^0}(M_D - E_\pi) \left\langle \pi \right| V^0 \left| D \right\rangle + Z_{V^i} \boldsymbol{q} \cdot \left\langle \pi \right| \boldsymbol{V} \left| D \right\rangle = Z_S (m_h - m_\ell) \left\langle \pi \right| S \left| D \right\rangle$$

- For the conserved lattice vector current, Z_V=1
    - Scalar density absolutely normalized: Z_S=1
    - For local vector currents, imposing the Ward identity provides a non-perturbative definition for Z_{V4}, Z_{Vi}

The all-HISQ campaign

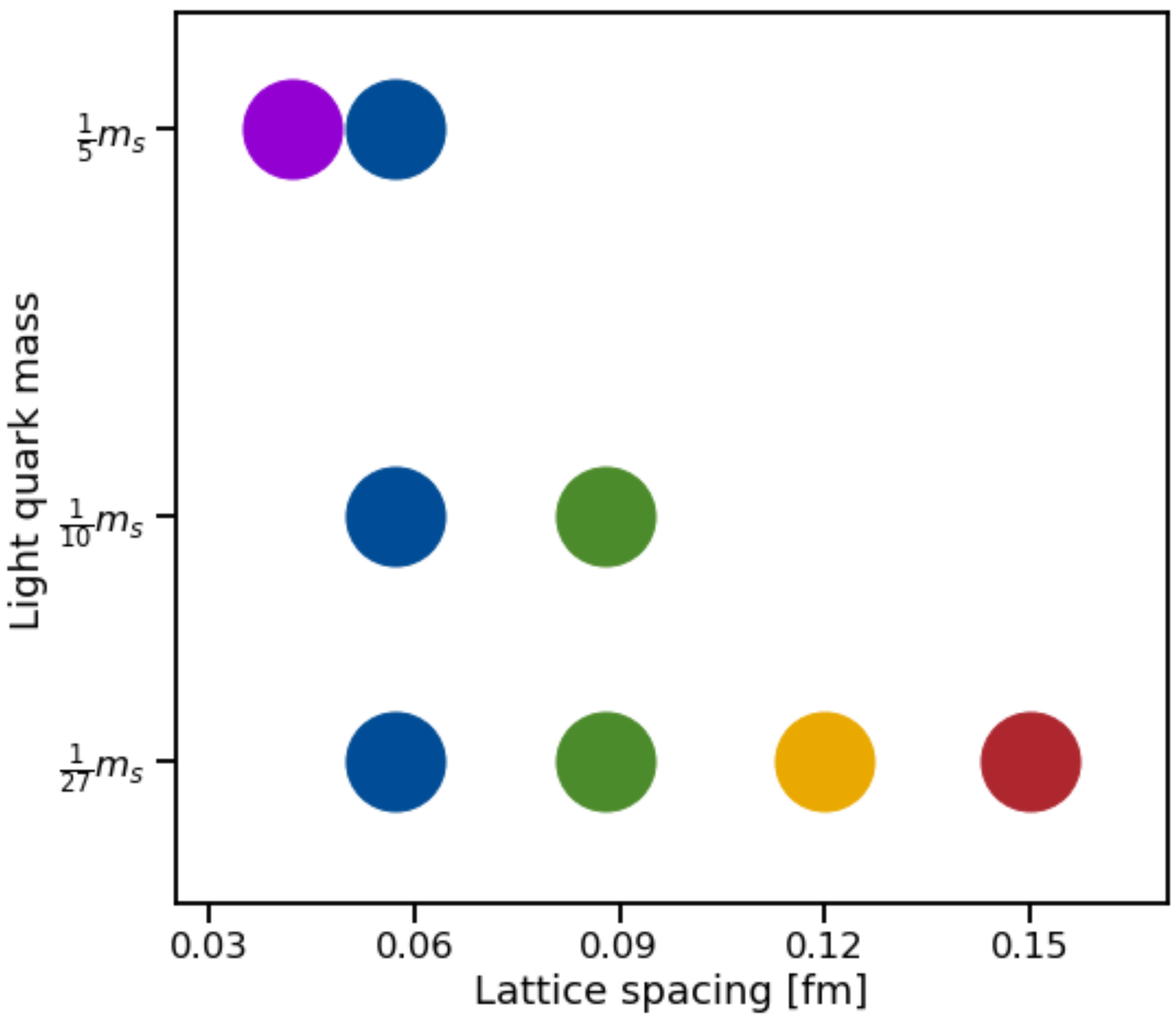
Scope: the all-HISQ campaign

- **Campaign goal**: precise ($\lesssim$ 1%) form factors for decays of B and D meson to pseudoscalars
  - D mesons: $D_{(s)} \to \pi$, K
  - B mesons: $B_{(s)} \to D_{(s)}$, π, K
  - Full set of scalar, vector, and tensor currents
- Ensembles: N_f =(2+1+1) dynamical sea quarks generated by the MILC collaboration
- Valence quarks:
  - Light and strange quarks match the sea
  - Heavy quarks: range from 0.9 m_c up to cutoff (ma~1)
- Eventual target: lattice spacings from 0.15 fm—0.03 fm
- **This talk**:
  - Preliminary results for decays of D mesons only
  - 5 lattice spacings: 0.15, 0.12, 0.09, 0.06, 0.042 fm
- **All 3pt functions are fully blinded**

Semileptonic decays: H→P$\ell\nu$

The setup: ensembles

- Initial focus on decays of D mesons: $D_{(s)} \to \pi/K\ \ell\nu$

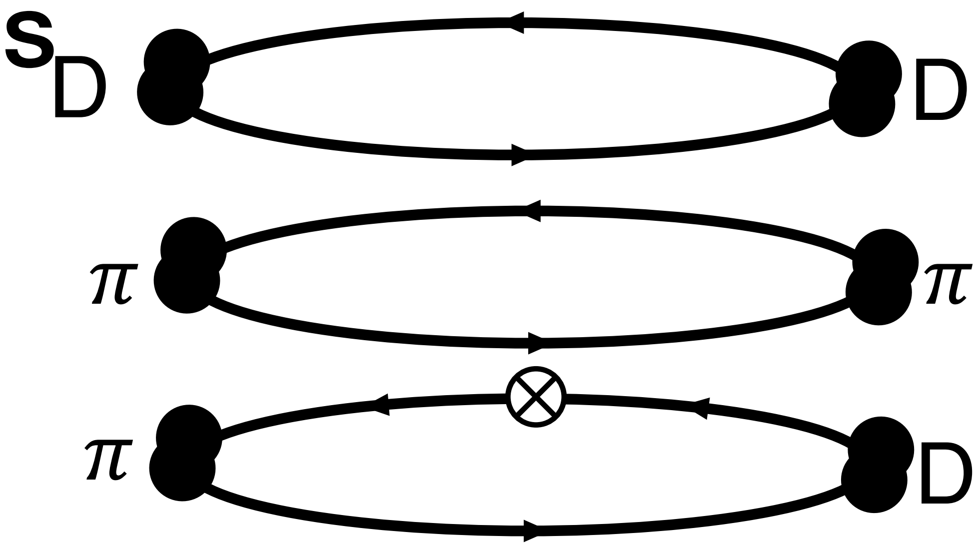


$\approx a$ [fm]	m_ℓ	m_h/m_c
0.15	physical	0.9, 1.0, 1.1
0.12	physical	0.9, 1.0, 1.4
0.088	physical	0.9, 1.0, 1.5, 2.0, 2.5
0.088	$0.1 \times m_s$	0.9, 1.0, 1.5, 2.0, 2.5
0.057	physical	0.9, 1.0, 1.1, 2.2, 3.3
0.057	$0.1 \times m_s$	0.9, 1.0, 2.0, 3.0, 4.0
0.057	$0.2 \times m_s$	0.9, 1.0, 2.0, 3.0, 4.0
0.042	$0.2 \times m_s$	0.9, 1.0, 2.0, 3.0, 4.0, 4.2

← **Physical-mass light quarks / pions**

Semileptonic decays: H→Pℓν

The setup: correlation functions



- Focus in today's talk: $D_{(s)} \to \pi/K\, \ell\nu$

$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0})\mathcal{O}_D(t,\boldsymbol{x})\rangle \longrightarrow \left|\langle 0|\,\mathcal{O}_D\,|D\rangle\right|^2 e^{-M_D t}$$

$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0})\mathcal{O}_\pi(t,\boldsymbol{x})\rangle \longrightarrow \left|\langle 0|\,\mathcal{O}_\pi\,|\pi\rangle\right|^2 e^{-E_\pi t}$$

$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0})J(t,\boldsymbol{y})\mathcal{O}_D(T,\boldsymbol{x})\rangle$$

$$\longrightarrow \langle 0|\,\mathcal{O}_\pi\,|\pi\rangle \langle \pi|\,J\,|D\rangle \langle D|\,\mathcal{O}_D\,|0\rangle\, e^{-E_\pi t} e^{M_D(T-t)}$$

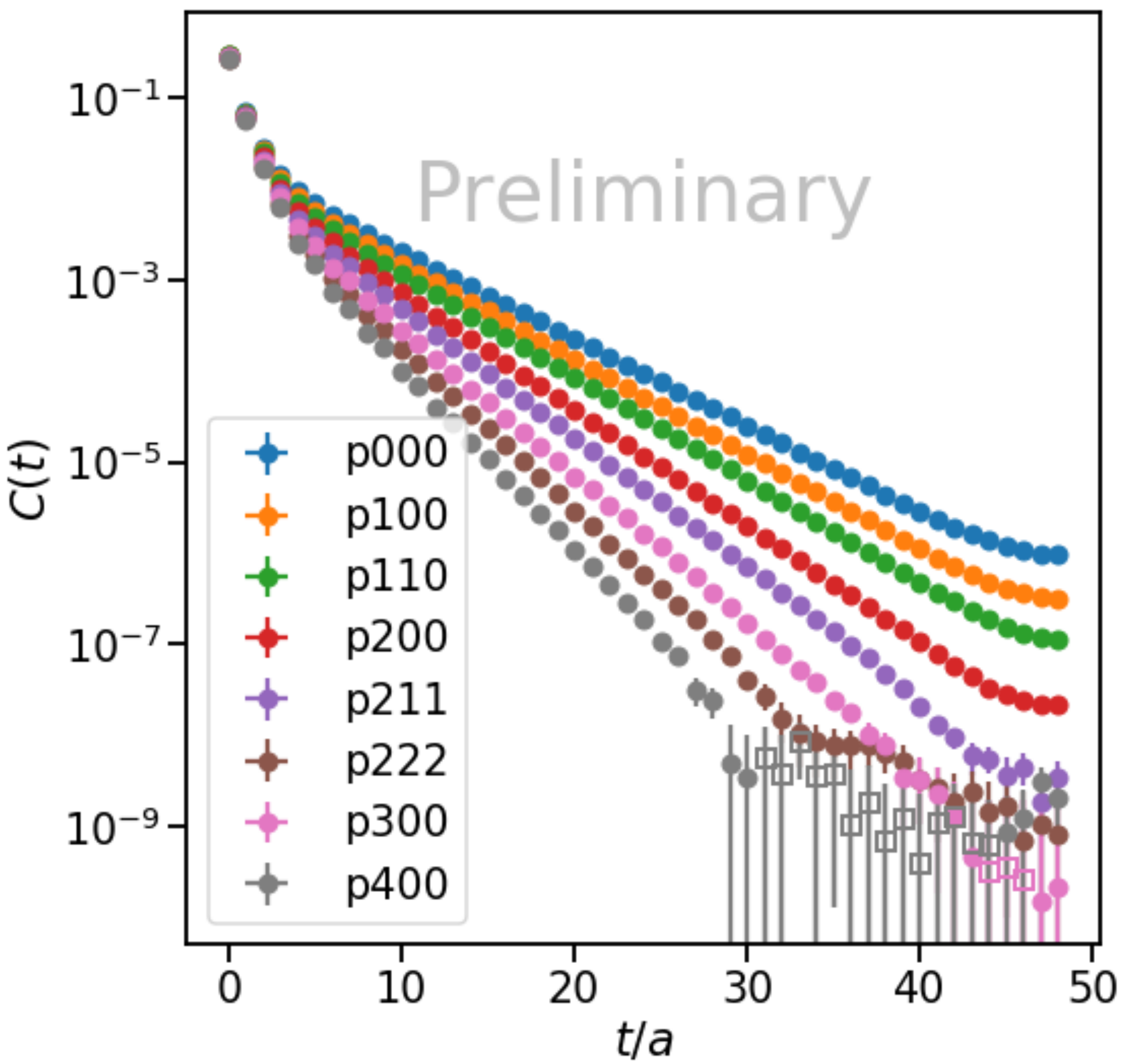
Semileptonic Decays

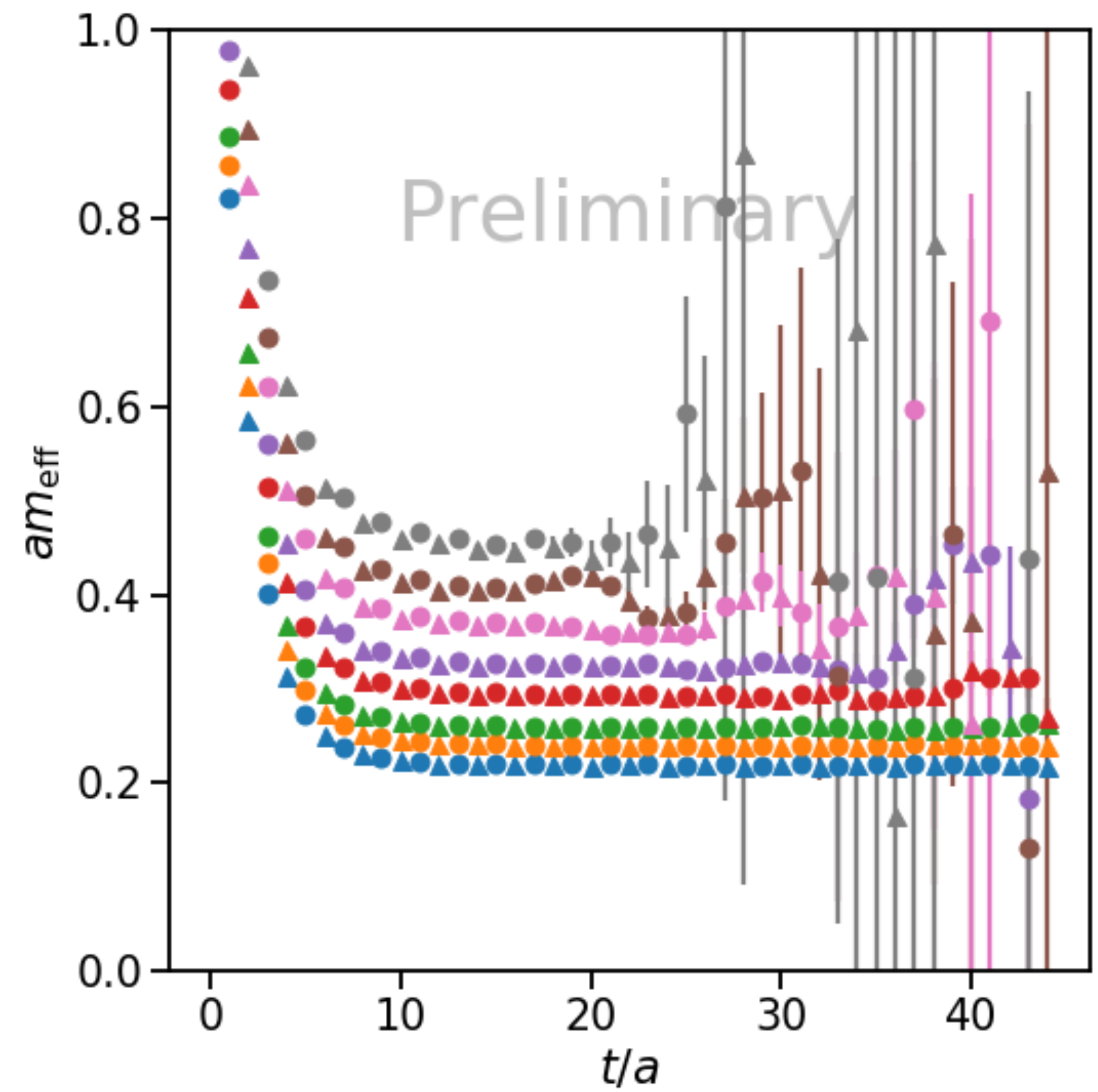
Preliminary results

Semileptonic decays: H→Pℓν

Results: Kaon 2pt functions at a ≈ 0.09 fm

- Effective mass: "$m_{eff} = \ln[C(t)/C(t+1)]$"

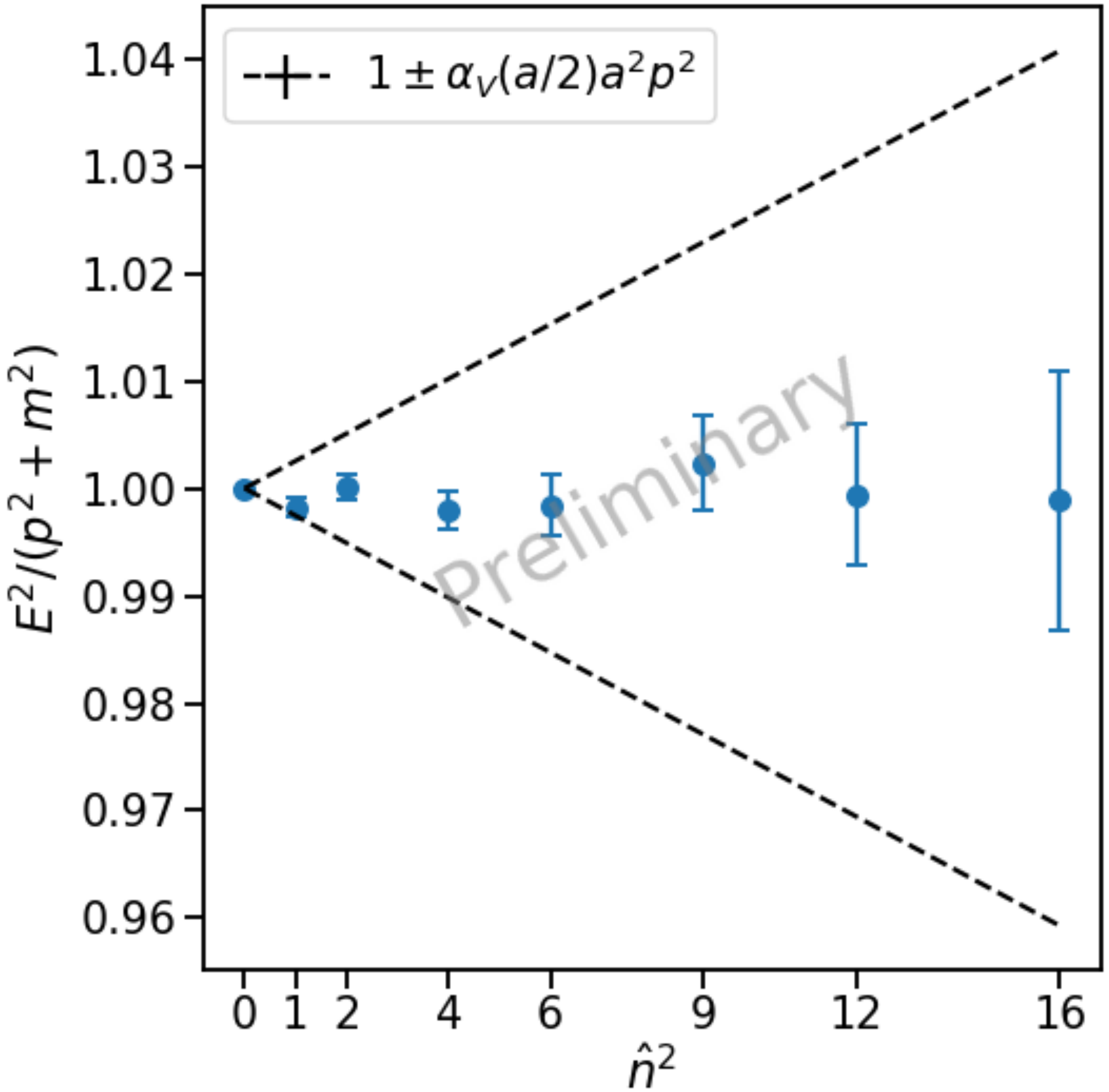




Semileptonic decays: H→Pℓν

Results: Kaon 2pt functions at a ≈ 0.09 fm

- Compare fit results to continuum dispersion: $E^2/(\mathbf{p}^2+m^2)$



Evidence that HISQ action is giving good control of discretization effects

Semileptonic decays: H→Pℓν

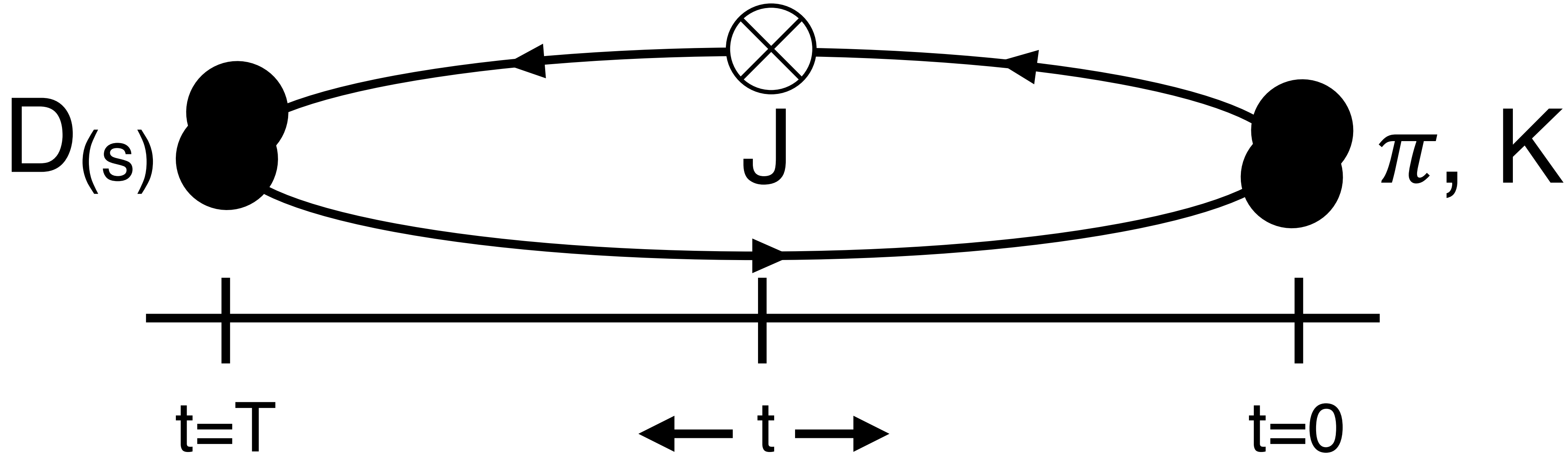
Extracting the matrix element

- As usual, the spectral decomposition reads:

$$C_3(t;T) = \langle \mathcal{O}_D(T) J(t) \mathcal{O}_\pi(0) \rangle$$

$$\sim \langle 0 | \mathcal{O}_D | D \rangle \underbrace{\langle D | J | \pi \rangle} \langle \pi | \mathcal{O}_\pi | 0 \rangle \, e^{-m_D(T-t)} e^{-m_\pi t}$$

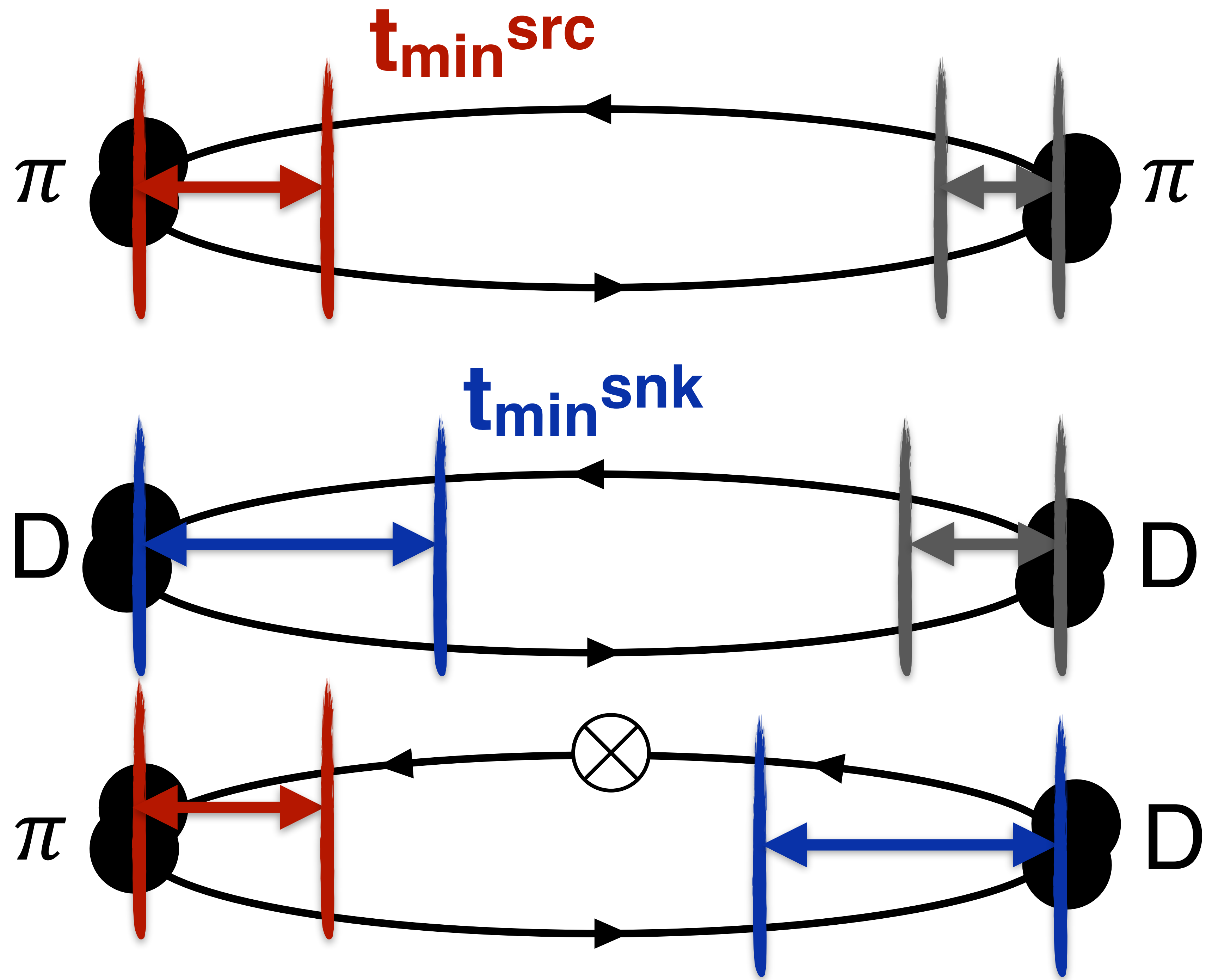
(bare) transition matrix element ~ desired form factor



Semileptonic decays: $H \to P \ell \nu$

Extracting the matrix element

- π: $(n+n_o)$ states
- D: $(m+m_o)$ states
- Fix distances t_{min}^{src} and t_{min}^{snk} in physical units for all correlators

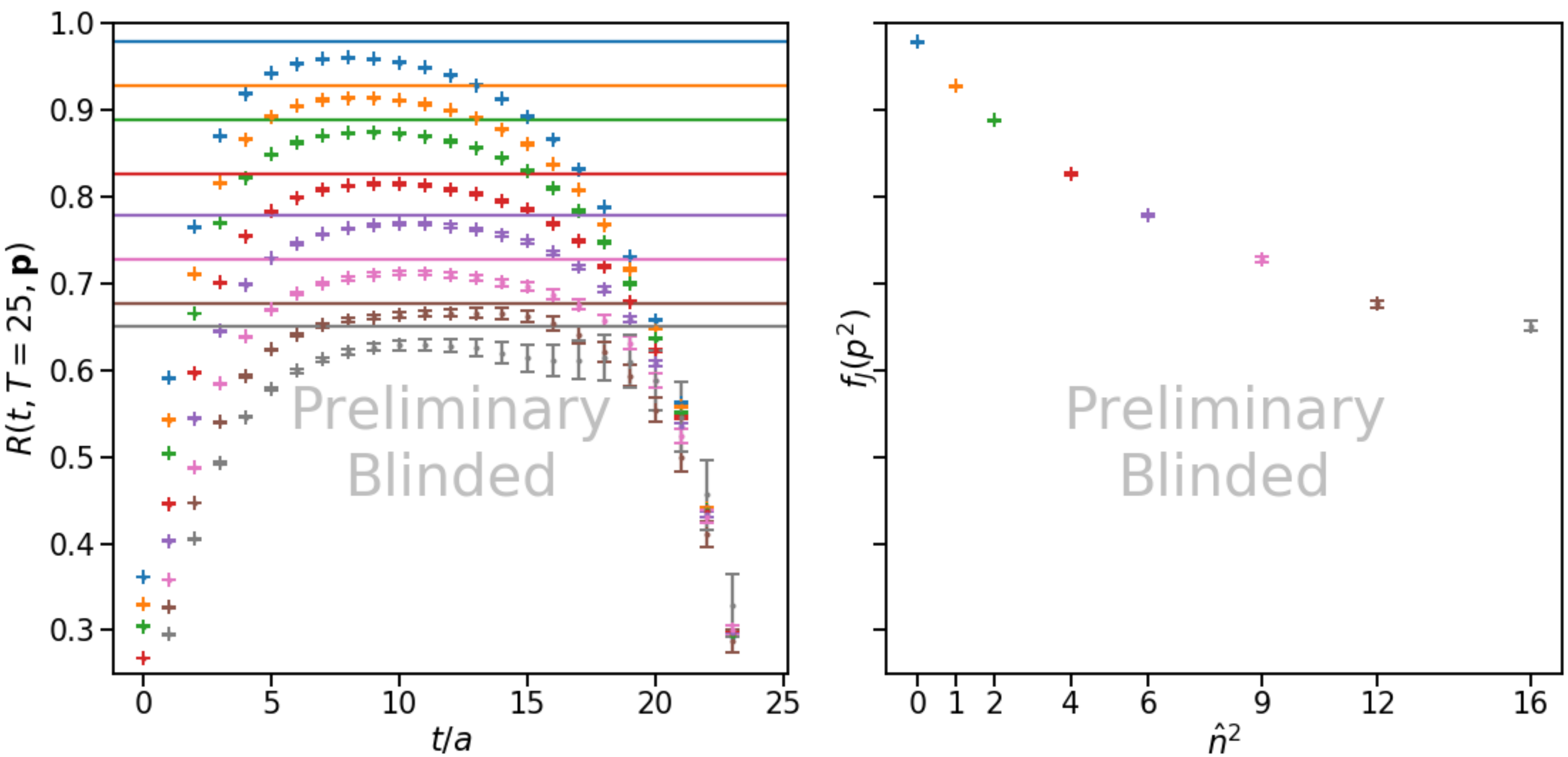


Semileptonic decays: H→Pℓν

Results: 3pt functions - f_0 for D_s to K

- A certain ratio is useful to isolate form factors visually:

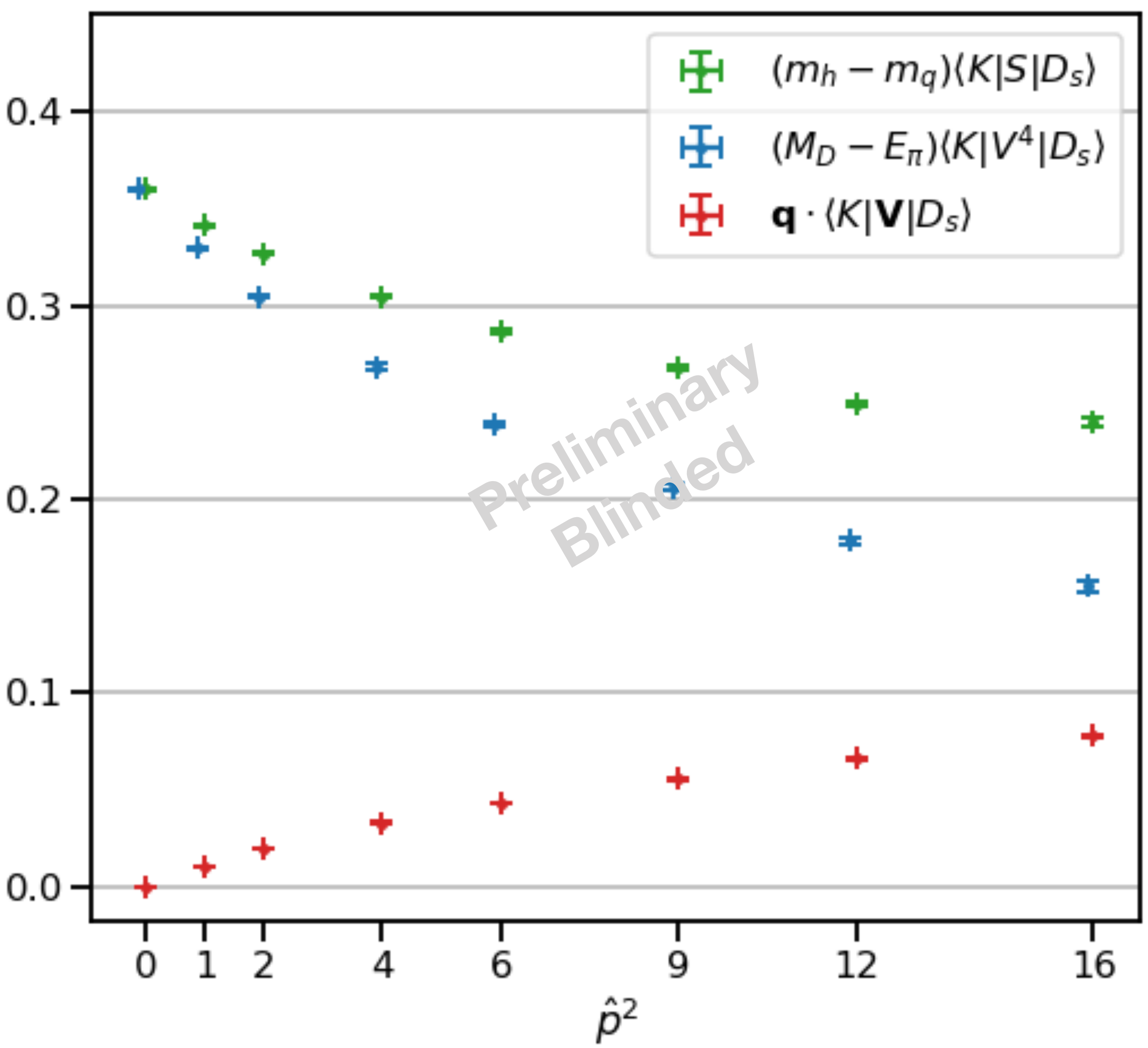
$$R^J(t,T,\boldsymbol{p}) \propto \frac{C_3^J(t,T,\boldsymbol{p})}{\sqrt{C_\pi(t,\boldsymbol{p})C_D(T-t)e^{-E_\pi t}e^{-M_D(T-t)}}} \longrightarrow f_J$$

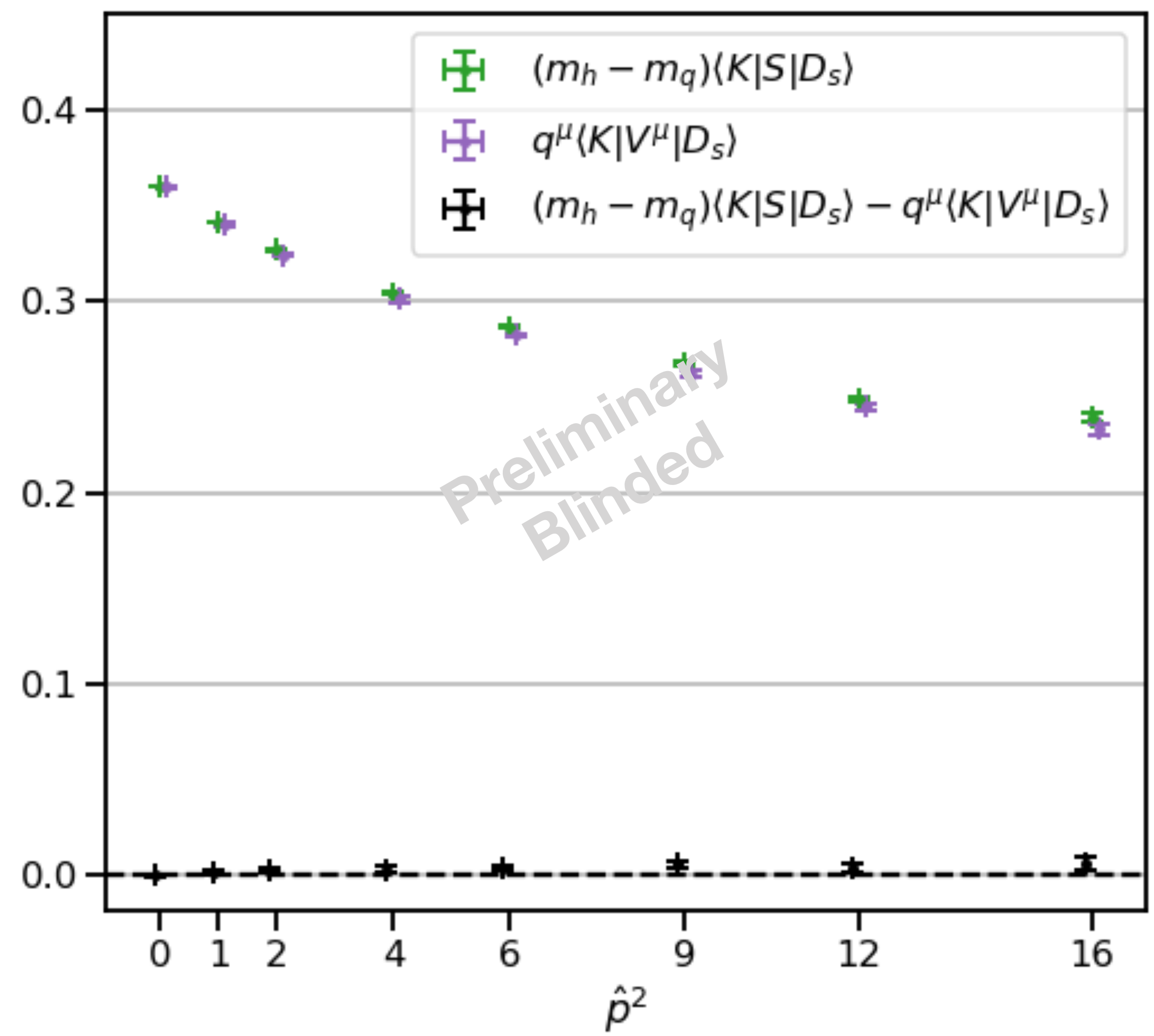


Semileptonic decays: H→Pℓν

Results: Renormalization

- Check Ward identity visually with bare matrix elements
- $(m_h - m_q)\langle K|S|D_s\rangle = (M_{D_s} - E_K)\langle K|V^4|D_s\rangle + \mathbf{q}\cdot\langle K|\mathbf{V}|D_s\rangle = q^\mu\langle K|V^\mu|D_s\rangle$

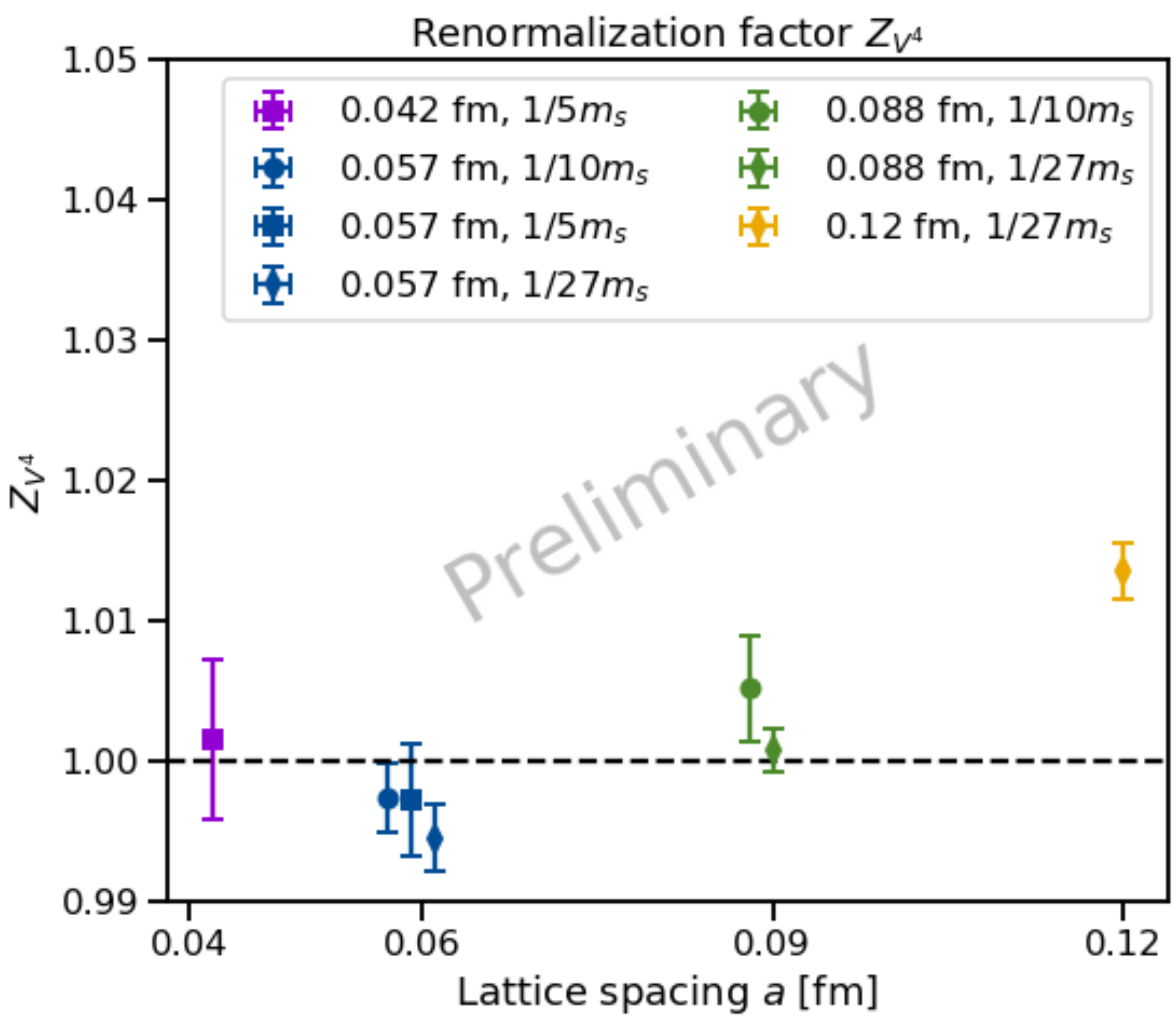


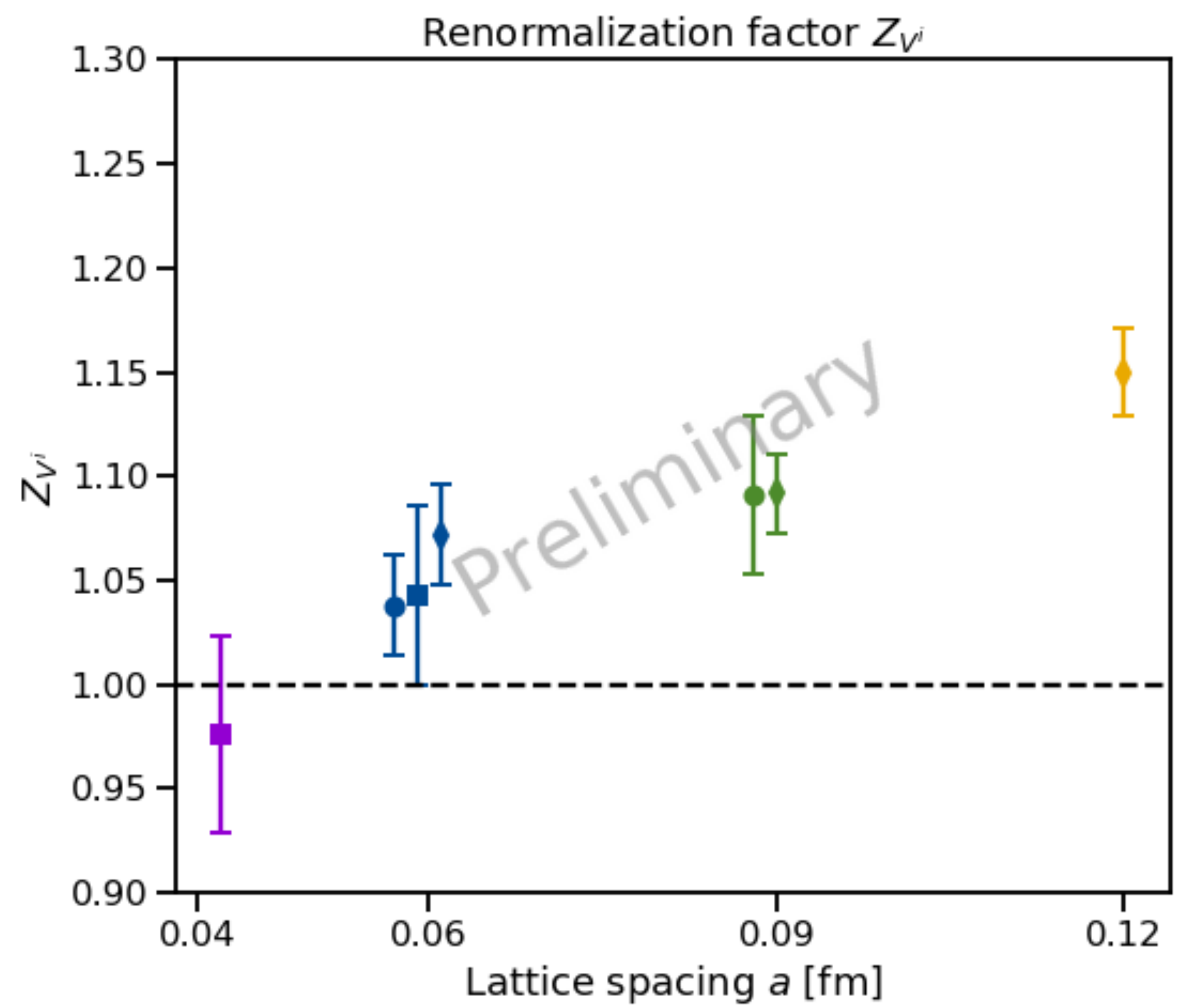


Semileptonic decays: H→P$\ell\nu$

Results: Renormalization

- Fit PCVC relation with Z_{V4}, Z_{Vi} as parameters:
- $(m_h - m_q)\langle K|S|D_s\rangle = Z_{V4}\,(M_{D_s} - E_K)\langle K|V^4|D_s\rangle + Z_{Vi}\,\mathbf{q}\cdot\langle K|\mathbf{V}|D_s\rangle$





Semileptonic decays: H→Pℓν

Results: Renormalized form factors

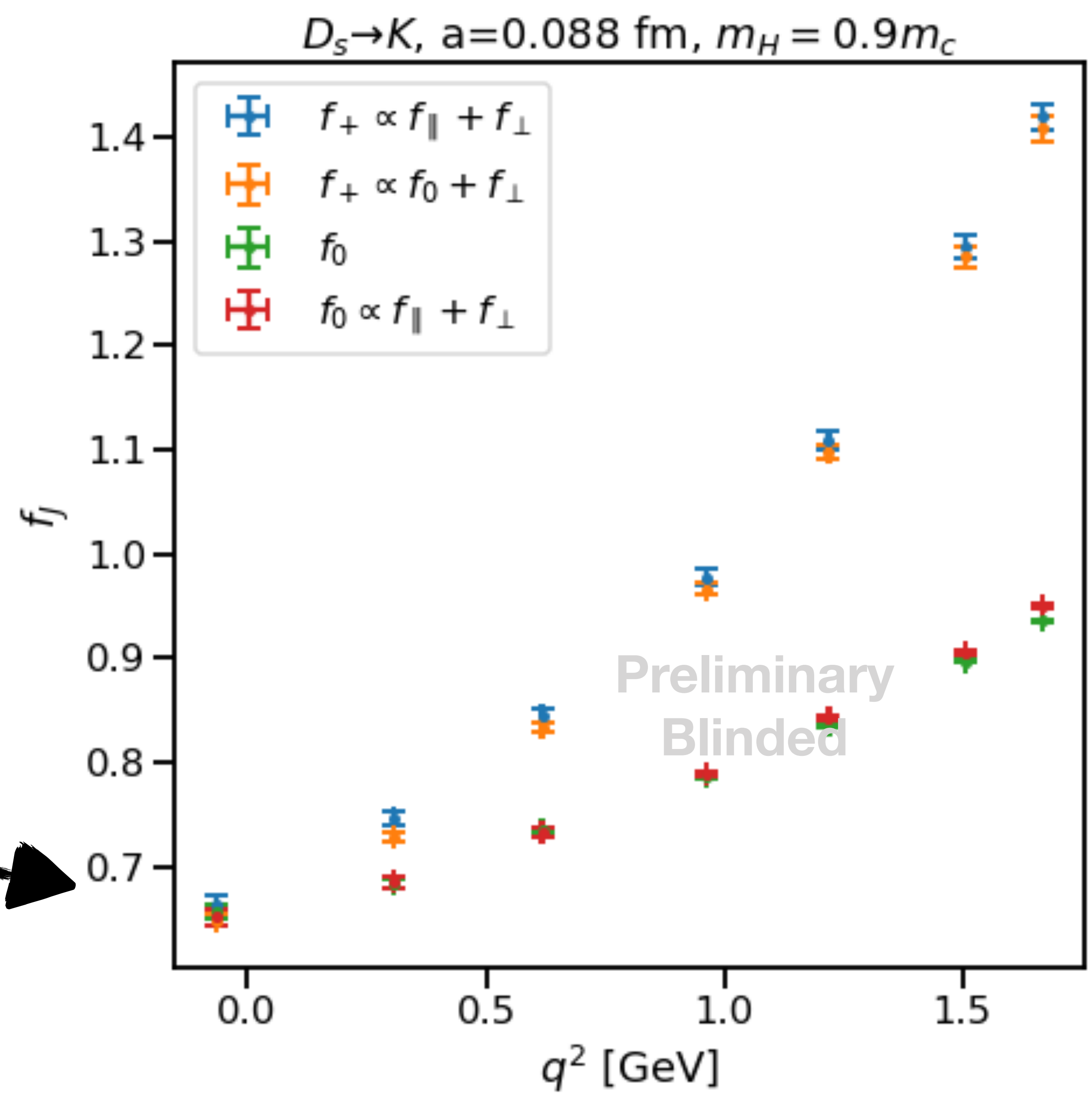
Standard f_+ via "$f_\parallel + f_\perp$"

Alternate f_+ via "$f_0 + f_\perp$"

f_0 via scalar matrix element

Standard f_0 via "$f_\parallel + f_\perp$"

Check kinematic identity:
$f_+(q^2) = f_0(q^2)$ at $q^2=0$ ✔



Global view of data for D→π

physical-mass pions

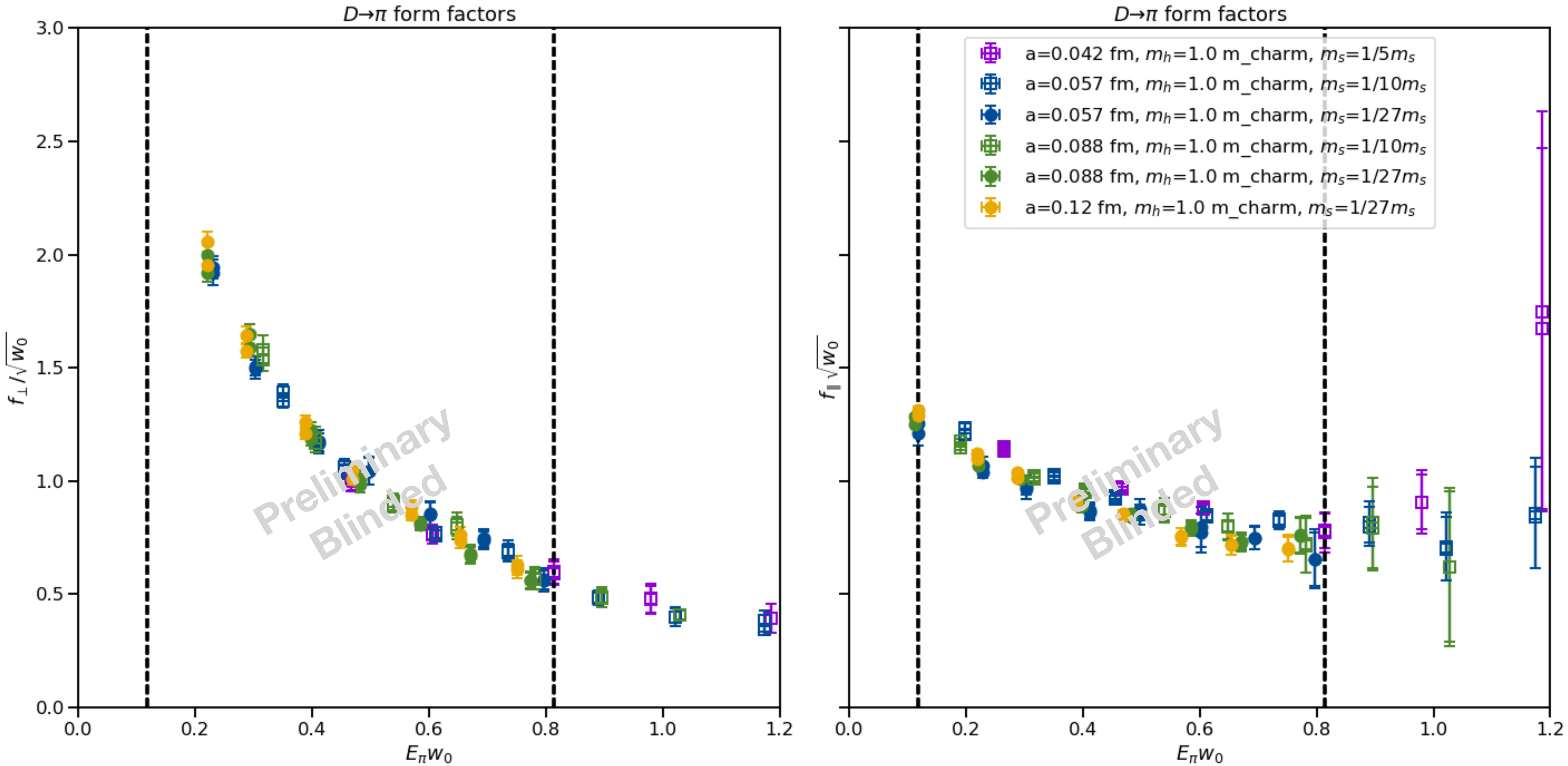
$m_l = 1/10\ m_s$ or $1/5\ m_s$

a = 0.09 fm

a = 0.12 fm

a = 0.04 fm

a = 0.06 fm



Global view of data for $D_s \to K$

physical-mass pions

m_l = 1/10 m_s or 1/5 m_s

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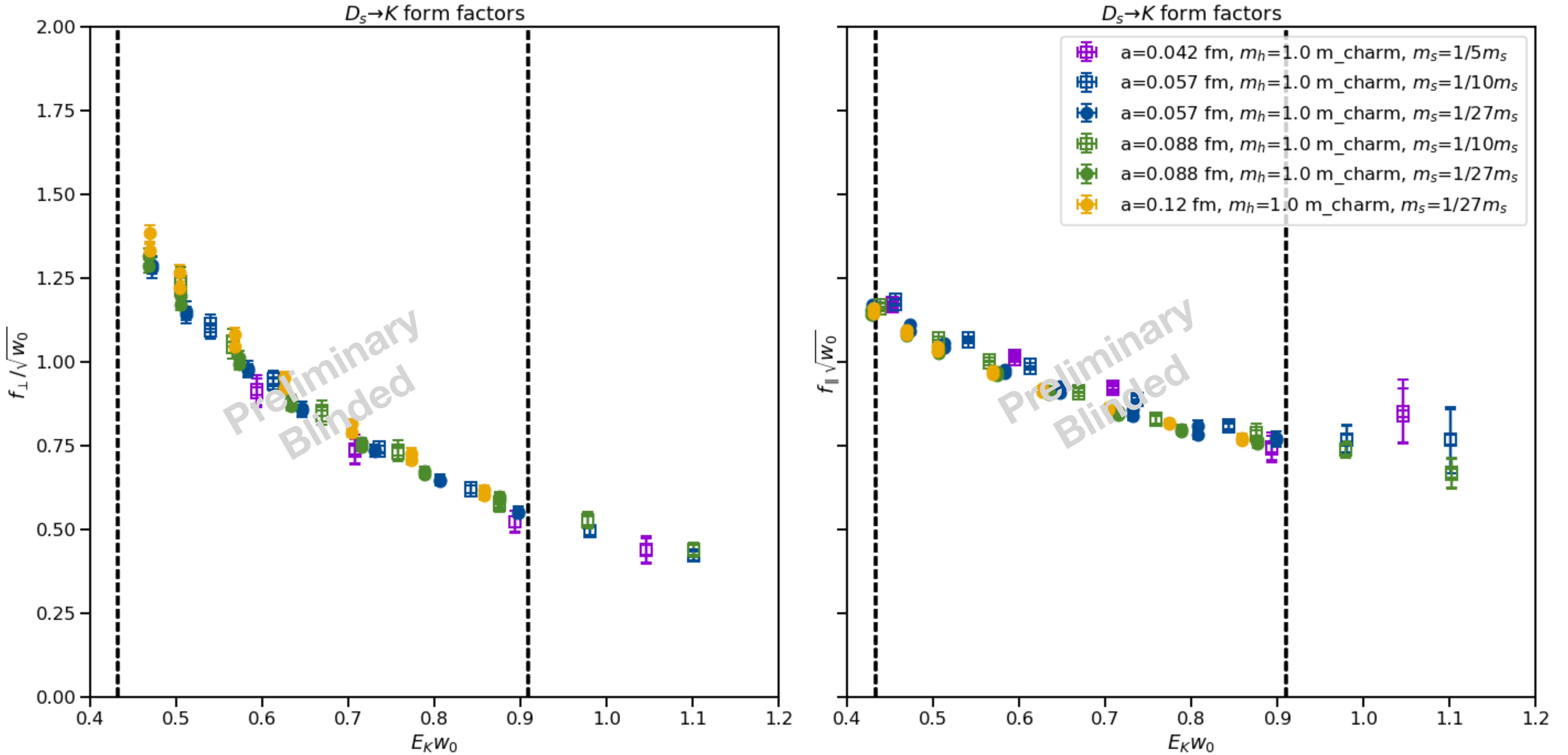
a = 0.12 fm

a = 0.04 fm

a = 0.06 fm



Chiral / continuum fits

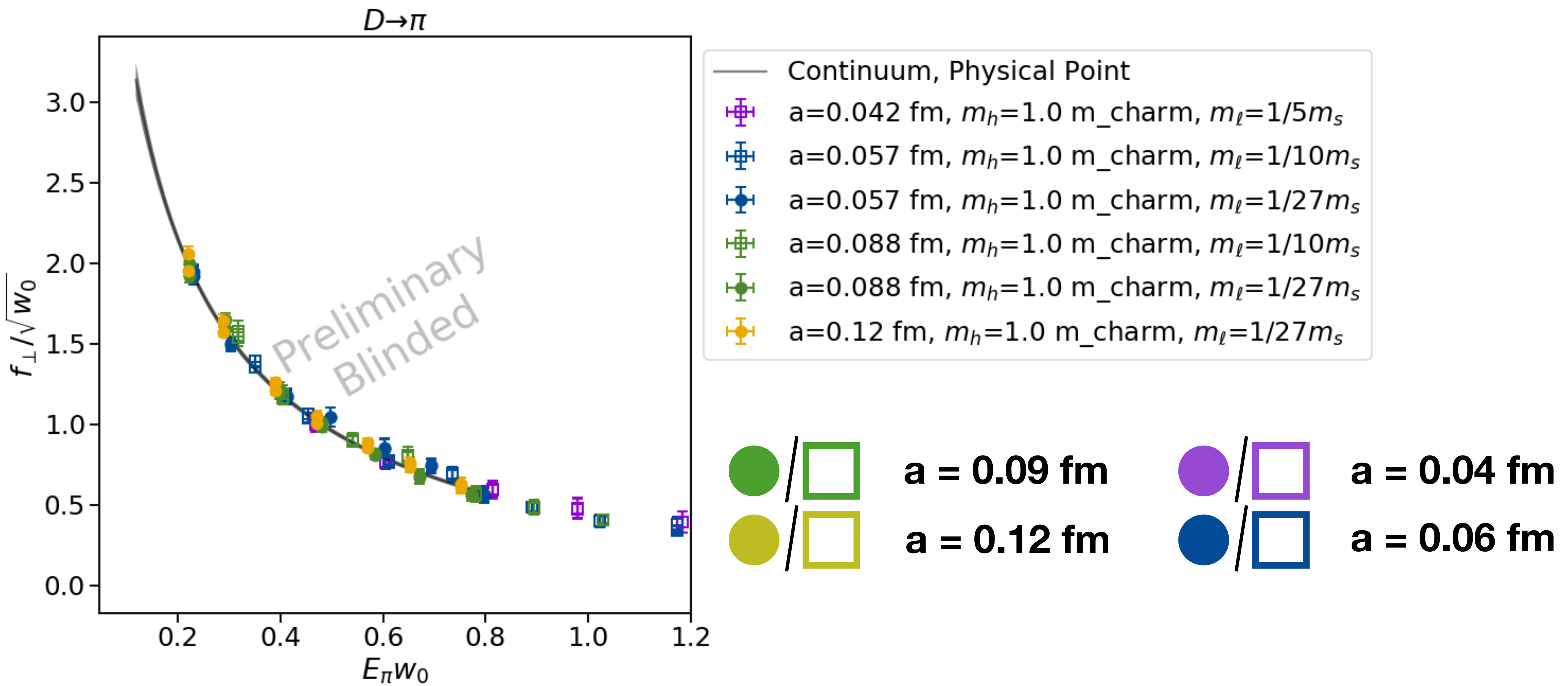
- Lattice-spacing dependence is quite mild—HISQ at work!
- With simulations at and above the physical pion mass, the chiral fits are ***interpolations***, not extrapolations
- HMRSχPT: the shape of the form factors can be modeled using EFT combining:
  - Chiral symmetry $$\Sigma = \exp(2i\phi/f)$$
  - HQET spin symmetry $$H^a = \frac{1+\not{v}}{2}\left[P^{*a}_{\mu}(v)\gamma^{\mu} - P^{a}(v)\gamma_5\right]$$
  - Embellishment from staggered fermions $$\frac{1}{16}\sum_{\Xi} M_{\Xi}^2 \log\left(\frac{M_{\Xi}^2}{\Lambda^2}\right)$$

Chiral / continuum fits

- Example: HMRSχPT, SU(2) theory, hard-pion limit
- All NLO logs, all NLO analytic terms, all NNLO analytic terms consistent with power counting
- Basically: $$f = \frac{\text{const}}{E + \Delta^* + \Sigma}\left(1 + \delta\text{logs} + \sum_i c_i \chi_i\right)$$

Chiral / continuum fits

- Example: HMRSχPT, SU(2) theory, hard-pion limit
- All data included in fit. Curve shown for physical q^2 only



Chiral / continuum fits

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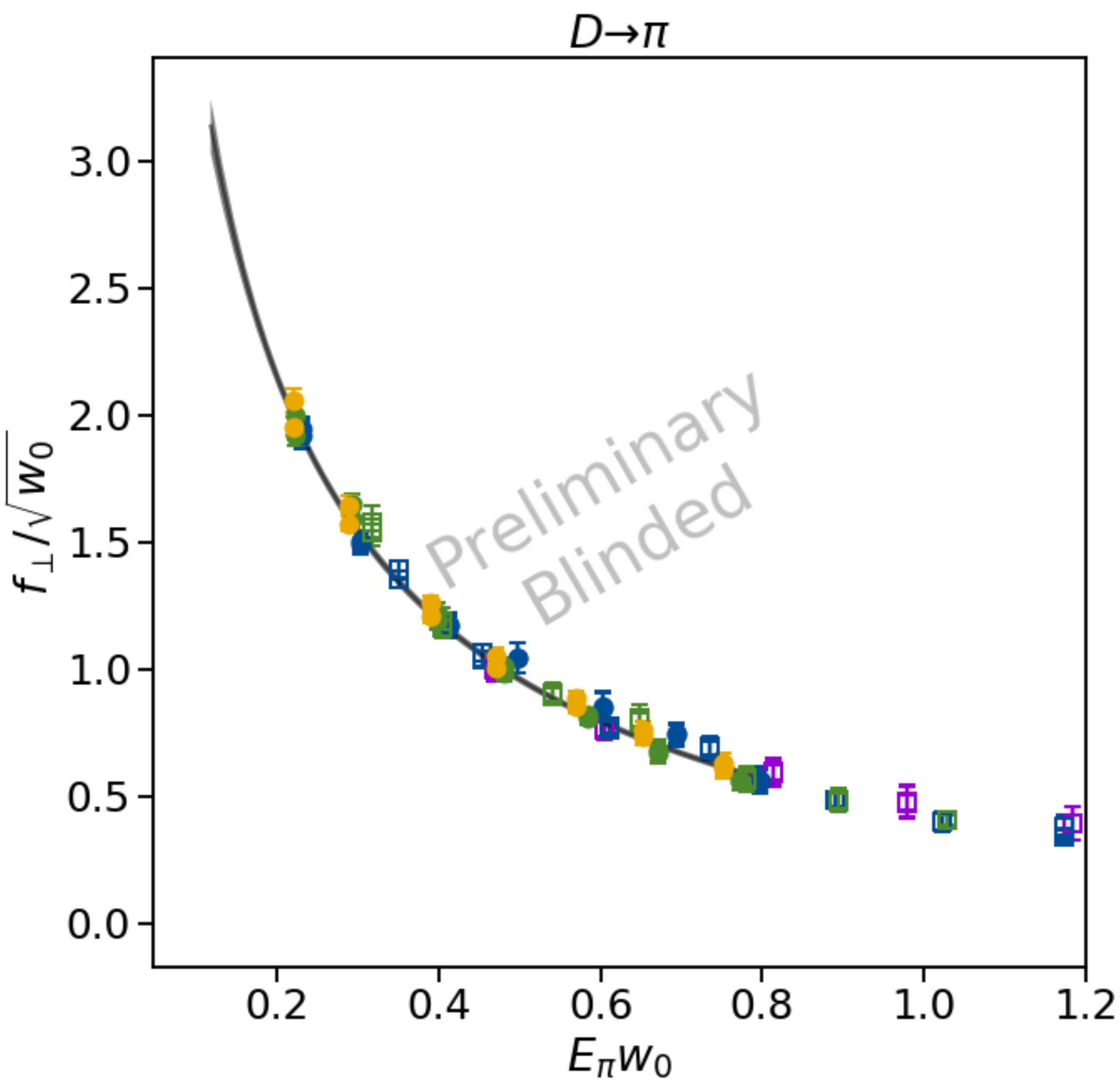
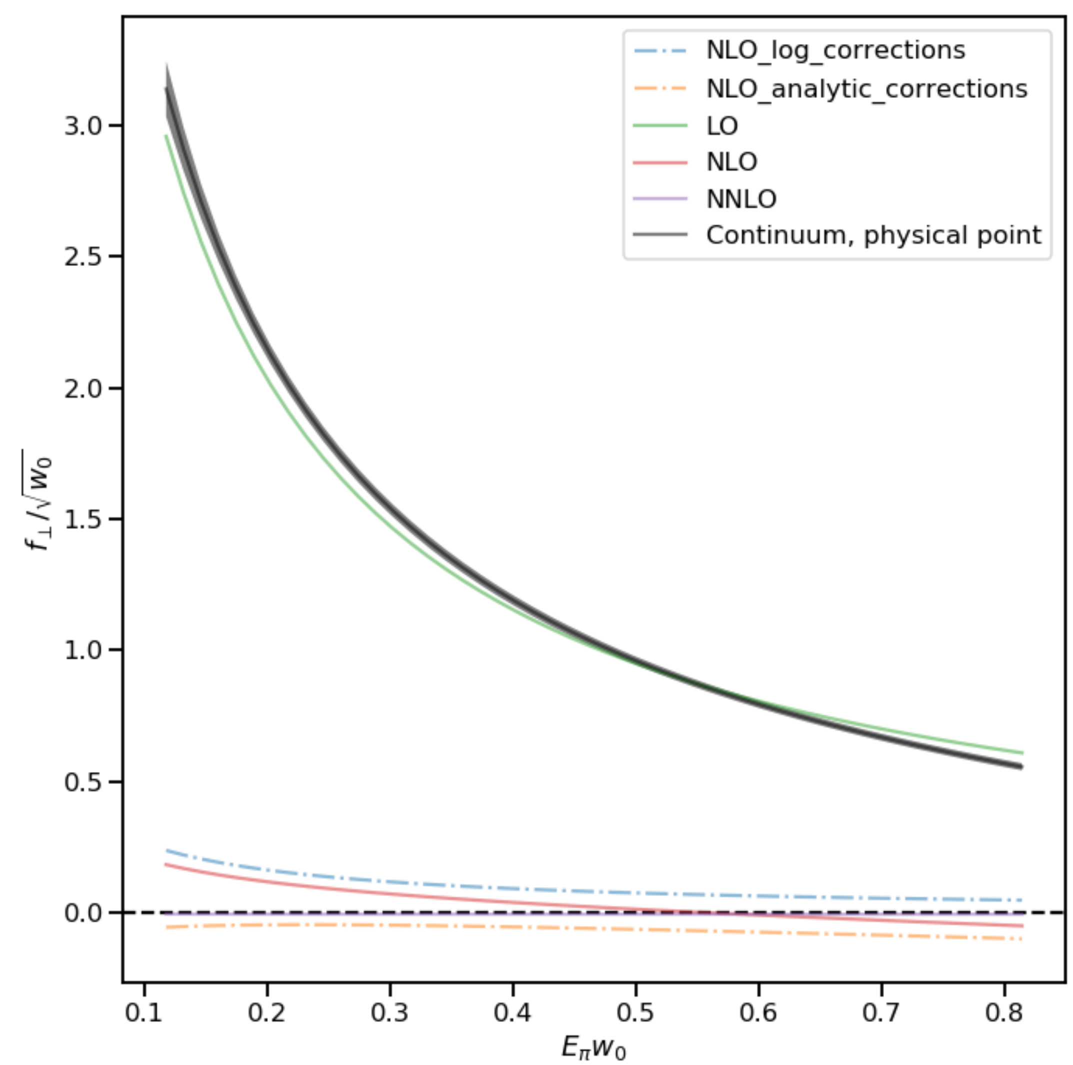
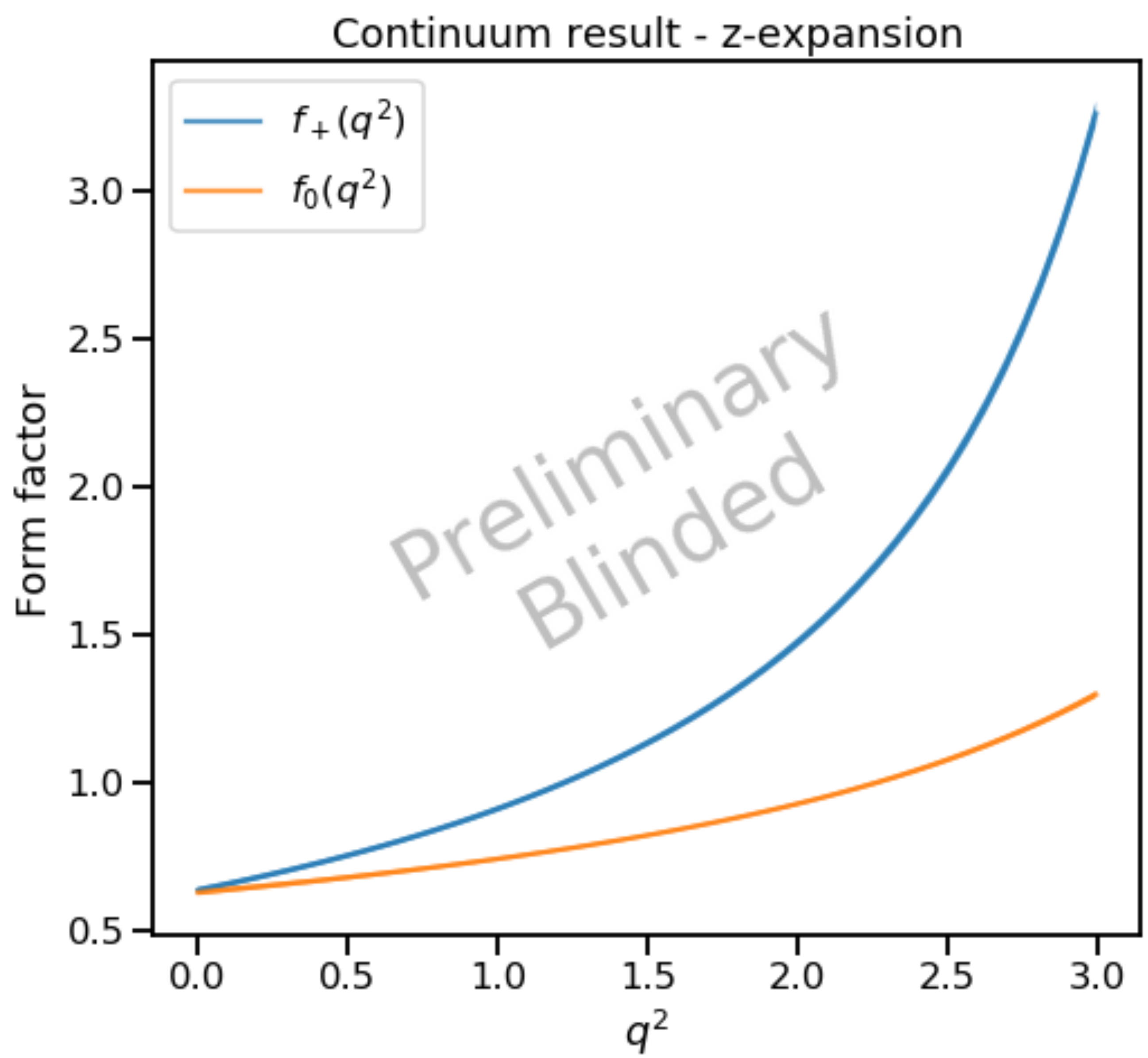




Chiral / continuum fits

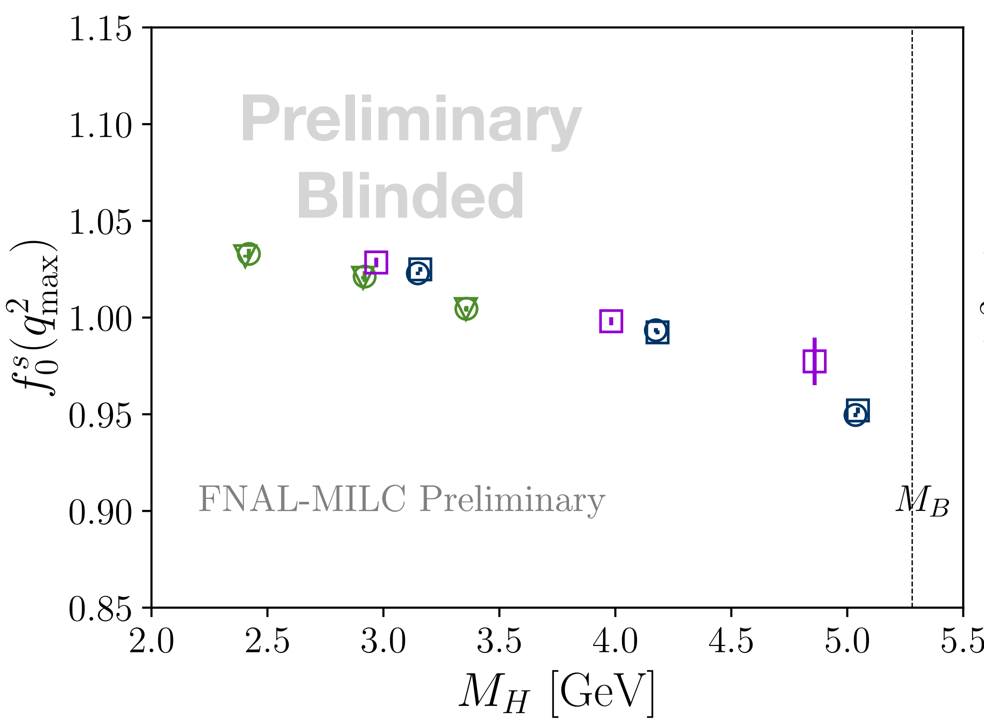
- Combine $f_{\parallel}$ and $f_{\perp}$ to obtain f_+ and f_0
- Express continuum, physical-point results using z-expansion
- Preliminary statistical precision ≲ 1.5%
- Other channels ($D_s \to K$, $D \to K$) broadly similar

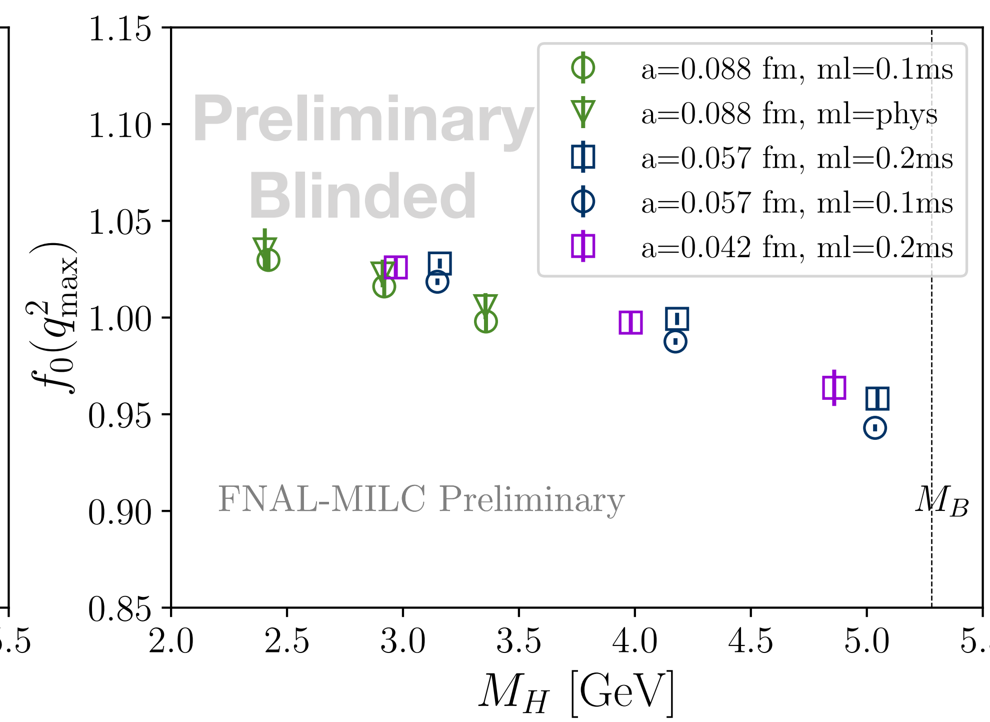
$D \to \pi$



Next steps

- In progress: finalizing analysis choices
- In progress: complete systematic error budget
- Near future: Unblinding, comparison to experimental results
- 2022: decays of B-mesons



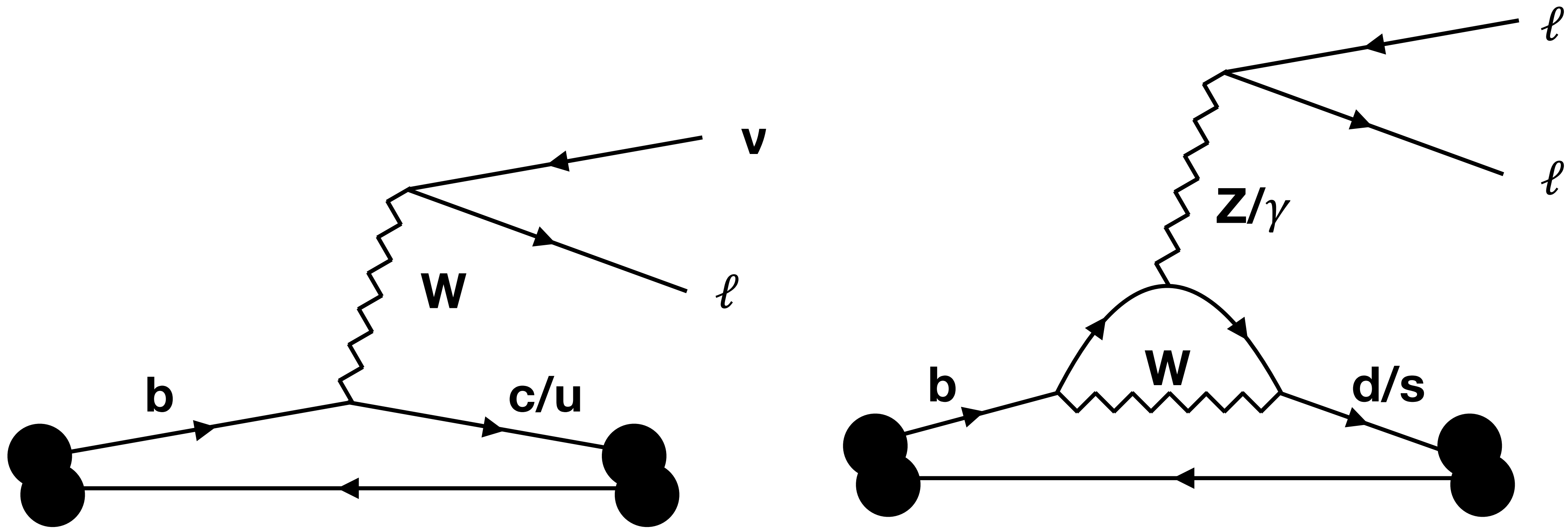


Summary

- Many exciting measurements from experimentalists working in quark flavor physics are on the horizon

- High-precision form factors from lattice QCD are needed to extract CKM matrix elements and test the Standard Model

- This winter, we plan to publish the first results on decays of D-mesons ($D\to\pi$, $D\to K$, $D_s\to K$). Preliminary results highlight the good statistical control afforded by the HISQ action



v
W
ℓ
b
c/u
ℓ
ℓ
Z/γ
W
b
d/s