Semileptonic Form Factors for $B \to \pi \ell \nu$ decays

Ryan Hill

J. Flynn, A. Jüttner, A. Soni, J. T. Tsang, O. Witzel
RBC-UKQCD

15$^{\mathrm{th}}$ December 2021
DWQ@25

THE UNIVERSITY of EDINBURGH

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner (Southampton)

Columbia University
Norman Christ
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

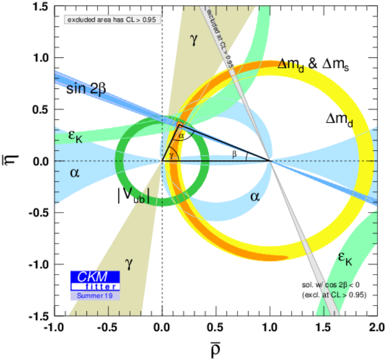
University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda

University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Motivation

- Test unitarity of CKM matrix
- 2-3σ discrepancy between exclusive ($B \to \pi \ell \nu$) and inclusive ($B \to X_u \ell \nu$)
- Lepton universality ratio predictions



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Goal

- Differential $B \to \pi \ell \nu$ decay rate:

$$\underbrace{\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}}_{\text{Experiment}} = \underbrace{|V_{ub}|^2}_{CKM} \times \left(\kappa_1 \underbrace{|f_+(q^2)|^2}_{non-pert.} + \kappa_2 \underbrace{|f_0(q^2)|^2}_{non-pert.} \right)$$

$$q^\mu \text{ — } p_B^\mu - p_\pi^\mu$$
$$\kappa \text{ — Known factors}$$

- Requires a theoretical computation of the form factors:

$$\langle \pi(\vec{p}) | \mathcal{V}^\mu | B \rangle = 2 f_+(q^2) \left(p_B^\mu - \frac{p_B \cdot q}{q^2} q^\mu \right) + f_0(q^2) \left(\frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right)$$

$$\mathcal{V}^\mu = \bar{u} \gamma^\mu b$$

Quark Actions

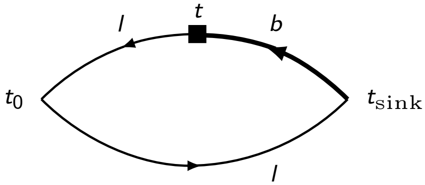
- RHQ Action for b quarks, Columbia interpretation
  [Christ et al. PRD 76 (2007) 074505] [Lin and Christ PRD 76 (2007) 074506]
  - Builds on original Fermilab action [El-Khadra et al. PRD 55 (1997) 3933]
  - Related to Tsukuba interpretation [S. Aoki et al. PTP 109 (2003) 383]
  - Clover action with anisotropic clover term
  - Uses 3 parameters $(m_0 a, c_p, \zeta)$ that can be non-pertubatively tuned to remove $\mathcal{O}((m_0 a)^n)$, $\mathcal{O}(\vec{p}a)$, $\mathcal{O}((\vec{p}a)(m_0 a)^n)$ errors [PRD 86 (2012) 116003]
- Shamir DWF for l, s
- Use improvement terms to get one-loop improved **vector current**

Ensembles

	$L^3 \times T \;/\; a^4$	a^{-1} / GeV	m_π / MeV
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1S	$48^3 \times 96$	2.77	270
C0*	$48^3 \times 96$	1.73	139

- 2+1f ensembles: Degenerate light quark
- Sea quarks: Shamir domain-wall fermions
- **F1S** ensemble: new for this analysis
- *Physical pion ensemble C0 planned for future inclusion.

Strategy



- Quark propagator $(t_0 \xrightarrow{l} t)$
- Sequential propagator $(t_0 \xrightarrow{l} t_{\rm sink} \xrightarrow{b} t)$
- Contract propagators at t
- Smeared point sources
- Project B meson to rest frame; momentum given to final state

Strategy

- Start by finding the parallel and perpendicular form factors $f_\parallel$ and $f_\perp$
- Simpler to relate to lattice data in the rest frame of the B-meson:

$$f_\parallel = \frac{\langle \pi | \mathcal{V}^0 | B \rangle}{\sqrt{2M_B}} \qquad f_\perp p^i = \frac{\langle \pi | \mathcal{V}^i | B \rangle}{\sqrt{2M_B}}$$

- Neatly separates into spatial and temporal components
- Can form linear combinations to find f_0 and f_+:

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 + E_\pi^2}\left[(M_B - E_\pi) f_\parallel(q^2) + (E_\pi^2 - M_\pi^2) f_\perp(q^2)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(q^2) + (M_B - E_\pi) f_\perp(q^2)\right]$$

Matrix Element Ratio

- Make plateau fits to the ratio of 3pt and 2pt functions:

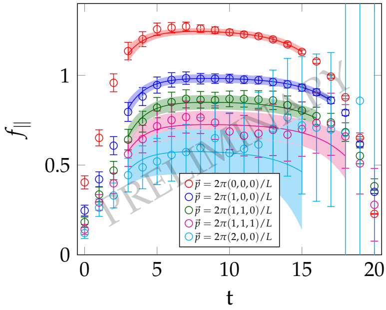
$$\langle \pi^{(0)} | V^\mu | B^{(0)} \rangle = \lim_{0 \ll t \ll t_{\mathrm{sink}}} \frac{C_3^\mu(t)}{\sqrt{C_2^\pi(t) C_2^B(t_{\mathrm{sink}} - t)}} \sqrt{\frac{4 E_\pi^{(0)} E_B^{(0)}}{e^{-tE_\pi^{(0)}} e^{-(t_{\mathrm{sink}} - t) E_B^{(0)}}}}$$

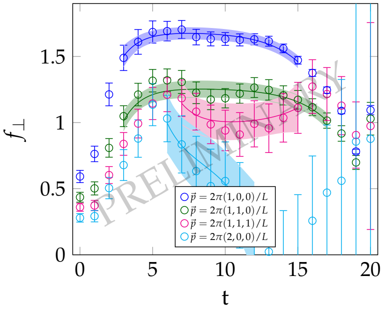
- Alternatively, include excited states fitting additional terms

$$\langle \pi^{(1)} | V^\mu | B^{(0)} \rangle \propto \sqrt{e^{-t(E_\pi^{(1)} - E_\pi^{(0)})}}$$

$$\langle \pi^{(0)} | V^\mu | B^{(1)} \rangle \propto \sqrt{e^{-(t_{\mathrm{sink}} - t)(E_B^{(1)} - E_B^{(0)})}}$$

Excited state fits on the C1 ensemble:





$B \to \pi$ Chiral Continuum Fits

- Extrapolate to physical pion mass and zero lattice spacing simultaneously
- Use NLO hard-pion SU(2) HMχPT [PRD 67 (2003) 054010]

$$f(M_\pi, E_\pi, a) = g(E_\pi, \Delta)\left(c_0 \left(1 + \frac{\delta f(M_\pi)}{(4\pi f_\pi)^2}\right) + c_1 \frac{M_\pi^2}{\Lambda^2} + c_2 \frac{E_\pi}{\Lambda} + c_3 \left(\frac{E_\pi}{\Lambda}\right)^2 + c_4 \left(a\Lambda\right)^2 \right)$$

where

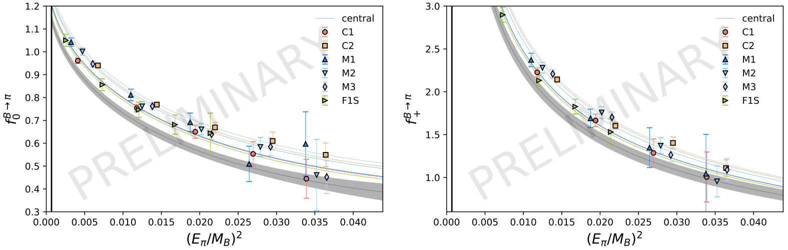
$\Lambda = 1$ GeV,

$\delta f(M_\pi) = -\frac{3}{4}(3g_b^2 + 1)\left(M_\pi^2 \log\left(\frac{M_\pi^2}{\Lambda^2}\right) + \mathrm{FVE}\right)$

$g_+(E_\pi, \Delta_+) = \frac{\Lambda}{E_\pi + \Delta_+}$; $\Delta_+ = M_{B^*} - M_B \approx 45.2$ MeV [Pole term]

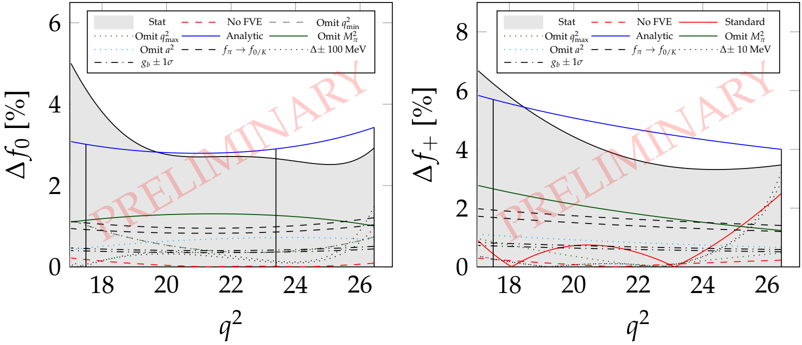
$g_0(E_\pi, \Delta_0) = \frac{\Lambda}{E_\pi + \Delta_0}$; $\Delta_+ = M_{B^*(0^+)} - M_B \approx 305$ MeV [Pole term]

$B \to \pi$ Chiral Continuum Fits



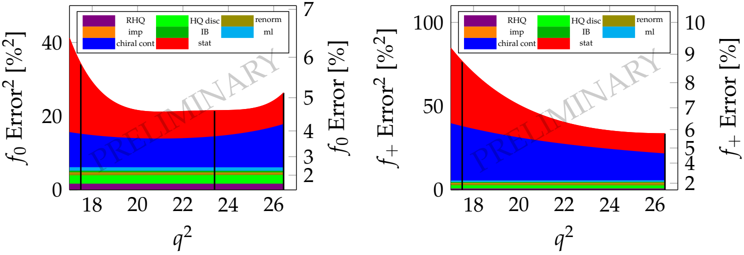
- Continuum form factor given by $f(M_\pi^{\text{phys}}, E_\pi, a = 0)$
- Need to assess the systematic error on the continuum results
- Include variations on the continuum fit ansatz as a systematic

Fit Variation Systematic



- Assess percentage deviations of possible alternatives the preferred fit
- Take largest deviation at reference q^2 values, which are then used in synthetic data points for the q^2-extrapolation

Total Error Budgets

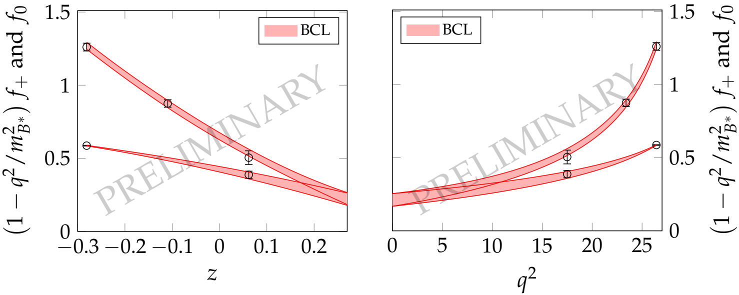


- Substantially reduced statistical errors over prior 2015 analysis
- Combine systematics at reference q^2 values with statistics and pass on to the q^2 extrapolation

z-expansions

- z-expansions given the kinematic constraint that $f_0 = f_+$ at $q^2 = 0$
- Can be compared with experimental data to determine $|V_{ub}|$, lepton-universality ratios
- z-expansions shown are BCL fits, also look at BGL

[PRD 79 (2009) 013008] [PRL 74 (1995) 4603]



- Test lepton flavour universality with the R ratio

$$R(\pi) = \frac{\int_{q^2=m_\tau^2}^{q^2=q^2_{\max}} dq^2 \frac{d\Gamma(B\to\pi\tau\overline{\nu}_\tau)}{dq^2}}{\int_{q^2=m_\mu^2}^{q^2=q^2_{\max}} dq^2 \frac{d\Gamma(B\to\pi\mu\overline{\nu}_\mu)}{dq^2}}$$

- Also consider the alternative R-ratio proposed by Isidori and Sumensari [Isidori & Sumensari, Eur. Phys. J. C 80 (2020)]

$$R^{\rm new}(\pi) = \frac{\int_{q^2=q^2_{\min}}^{q^2=q^2_{\max}} dq^2 \frac{d\Gamma(B\to\pi\tau\overline{\nu}_\tau)}{dq^2}}{\int_{q^2=q^2_{\min}}^{q^2=q^2_{\max}} \frac{\omega_\tau(q^2)}{\omega_\mu(q^2)} dq^2 \frac{d\Gamma(B\to\pi\mu\overline{\nu}_\mu)}{dq^2}}$$

- Common integration range $q^2_{\min} \geq m_\tau^2$

  [Freytis *et al.* Phys. Rev. D 92, 054018 (2015)]

  [Bernlochner & Ligeti, Phys. Rev. D 95, 014022 (2017)]

  [Soni, PoS ICHEP2020, 436 (2021)]

- Equal weights $\omega_\ell(q^2)$ in the integrand

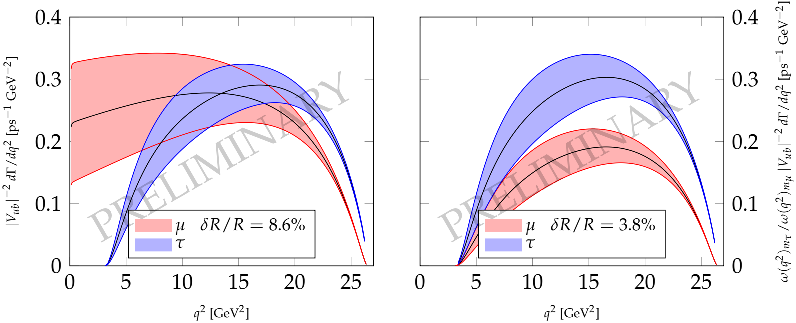
$$\frac{d\Gamma(B\to\pi\ell\overline{\nu})}{dq^2} = \phi(q^2)\omega_\ell(q^2)\left[F_V^2(q^2) + \left(F_S^\ell(q^2)\right)^2\right]$$

$$\phi(q^2) = \eta_{\rm EW}\frac{G_F^2|V_{ub}|^2}{24\pi^3}|\vec{k}|$$

$$\omega_\ell(q^2) = \left(1 - \frac{m_\ell^2}{q^2}\right)^2\left(1 + \frac{m_\ell^2}{2q^2}\right)$$

$$F_V^2 = \vec{k}^2\left|f_+(q^2)\right|^2$$

$$\left(F_s^\ell\right)^2 = \frac{3}{4}\frac{m_\ell^2}{m_\ell^2 + 2q^2}\frac{\left(m_B^2 - m_\pi^2\right)^2}{m_\pi^2}\left|f_0(q^2)\right|^2$$



- "Standard" (left) and "New" (right) integrands for the R-ratio
- New ratio definition exhibits a reduction in relative uncertainty

Summary

- Updates to RBC-UKQCD 2015 results in the pipeline
- More precisely determined lattice spacing and inclusion of third lattice spacing via F1S ensemble reduces errors
- More details available in conference proceedings: arXiv:1912.09946, arXiv:2012.04323
- Next step: Internal cross-checks

Related talks:

- $B_s \to K\ell\nu$, $B_s \to D_s\ell\nu$: Jonathan Flynn @ 11:00 EST today