Charged Meson Decays to $\ell\nu\gamma$

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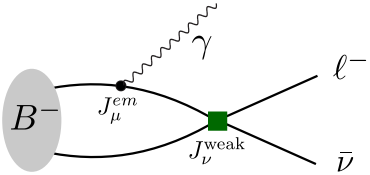
$B^- \to \ell^- \bar{\nu}\gamma$



- Hard photon removes helicity suppression $(m_\ell/m_B)^2$
- For large $E_\gamma^{(0)}$, simplest decay that probes the inverse moment of the B meson light-cone distribution amplitude

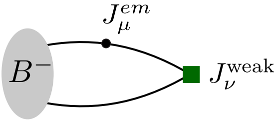
$$\frac{1}{\lambda_B} = \int_0^\infty \mathrm{d}\omega\ \frac{\Phi_{B+}(\omega)}{\omega}$$

- λ_B important input in QCD factorization approach to exclusive B decays, currently not well known
  [See e.g., Beneke, Braun, Ji, Wei, arXiv:1804.04962/JHEP 2018;

  Beneke, Buchalla, Neubert, Sachrajda, arXiv:hep-ph/9905312/PRL 1999]
- Belle: $\mathcal{B}(B^+ \to \ell^+ \nu\gamma) < 3.0 \times 10^{-6}$ $(E_\gamma^{(0)} > 1$ GeV$)$

  [arXiv:1810.12976/PRD 2018]

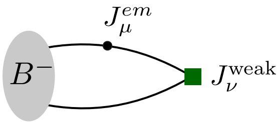
Hadronic Tensor and Form Factors



$$J_\mu^{em} = \sum_q e_q \bar{q}\gamma_\mu q, \qquad J_\nu^{weak} = \bar{u}\gamma_\nu(1-\gamma_5)b$$

$$T_{\mu\nu} = -i \int \mathrm{d}^4x \; e^{ip_\gamma\cdot x} \langle 0| \, \mathbf{T}\big(J_\mu^{em}(x) J_\nu^{weak}(0)\big) \, \big|B^-(\vec{\mathbf{p}}_B)\big\rangle$$

Hadronic Tensor and Form Factors



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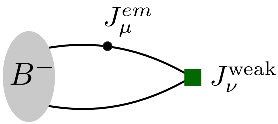
$$= \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i\big[-g_{\mu\nu}(v\cdot p_\gamma) + v_\mu (p_\gamma)_\nu\big] F_A - i\frac{v_\mu v_\nu}{(v\cdot p_\gamma)} m_B f_B$$

$$+ (p_\gamma)_\mu\text{-terms}$$

$$F_{A,SD} = F_A + f_B/E_\gamma^{(0)}, \qquad E_\gamma^{(0)} = p_B \cdot p_\gamma / m_B$$

Hadronic Tensor and Form Factors



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$$= \epsilon_{\mu\nu\tau\rho}p_\gamma^\tau v^\rho F_V + i\big[-g_{\mu\nu}(v\cdot p_\gamma) + v_\mu(p_\gamma)_\nu\big]F_A - i\frac{v_\mu v_\nu}{(v\cdot p_\gamma)}m_B f_B$$
$$+\,(p_\gamma)_\mu\text{-terms}$$

$$F_{A,SD} = F_A + f_B/E_\gamma^{(0)}, \qquad E_\gamma^{(0)} = p_B\cdot p_\gamma/m_B$$

Goal: Calculate $F_V, F_{A,SD}$ as a function of $E_\gamma^{(0)}$

Euclidean correlation function

(* all times are now Euclidean)

$$C_{3,\mu\nu}(t_{em}, t_H) = \int d^3x \int d^3y \; e^{-i\vec{\mathbf{p}}_\gamma \cdot \vec{\mathbf{x}}} e^{i\vec{\mathbf{p}}_H \cdot \vec{\mathbf{y}}} \langle J_\mu^{\text{em}}(t_{em}, \vec{\mathbf{x}}) J_\nu^{\text{weak}}(0) \phi_H^\dagger(t_H, \vec{\mathbf{y}}) \rangle$$

$$\phi_H^\dagger \sim \bar{Q}\gamma_5 u$$

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$$I_{\mu\nu}^{<}(T, t_H) = \int_{-T}^{0} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

$$I_{\mu\nu}^{>}(T, t_H) = \int_{0}^{T} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

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$$I_{\mu\nu}(T, t_H) = I^{<}(T, t_H) + I^{>}(T, t_H)$$

Euclidean correlation function

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$$C_{3,\mu\nu}(t_{em}, t_H) = \int d^3x \int d^3y \; e^{-i\vec{\mathbf{p}}_\gamma \cdot \vec{\mathbf{x}}} e^{i\vec{\mathbf{p}}_H \cdot \vec{\mathbf{y}}} \langle J^{\text{em}}_\mu(t_{em}, \vec{\mathbf{x}}) J^{\text{weak}}_\nu(0) \phi^\dagger_H(t_H, \vec{\mathbf{y}}) \rangle$$

$$\phi^\dagger_H \sim \bar{Q}\gamma_5 u$$

$$I^<_{\mu\nu}(T, t_H) = \int_{-T}^{0} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

$$I^>_{\mu\nu}(T, t_H) = \int_{0}^{T} dt_{em} e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)$$

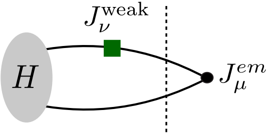
$$I_{\mu\nu}(T, t_H) = I^<(T, t_H) + I^>(T, t_H)$$

Show relation between $I_{\mu\nu}(T, t_H)$ and $T_{\mu\nu}$

→ compare spectral decompositions of both time orderings of $I_{\mu\nu}$ and $T_{\mu\nu}$

Euclidean spectral decomposition of $I^{>}_{\mu\nu}$

Time ordering: $t_{em} > 0$



$$T^{>}_{\mu\nu} = -\sum_n \frac{\langle 0|\, J_\mu^{em}(0)\,|n(\vec{\mathbf{p}}_\gamma)\rangle \, \langle n(\vec{\mathbf{p}}_\gamma)|\, J_\nu^{weak}(0)\,|H(\vec{\mathbf{p}}_H)\rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})}$$

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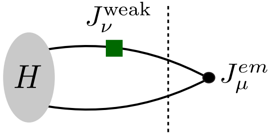

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$$I^{>}_{\mu\nu}(t_H, T) = \int_0^T dt_{em}\; e^{E_\gamma t_{em}} C_{\mu\nu}(t_{em}, t_H)$$

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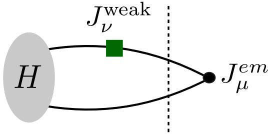

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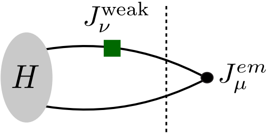
$$I^{>}_{\mu\nu}(t_H, T) = \int_0^T dt_{em}\; e^{E_\gamma t_{em}} C_{\mu\nu}(t_{em}, t_H)$$

$$= -\sum_m e^{E_m t_H} \frac{\langle m(\vec{\mathbf{p}}_H)|\, \phi^\dagger_H(0)\, |0\rangle}{2E_{m,\vec{\mathbf{p}}_H}}$$

$$\times \sum_n \frac{\langle 0|\, J^{em}_\mu(0)\, |n(\vec{\mathbf{p}}_\gamma)\rangle \langle n(\vec{\mathbf{p}}_\gamma)|\, J^{weak}_\nu(0)\, |m(\vec{\mathbf{p}}_H)\rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})} \left[1 - e^{(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})T}\right]$$

Euclidean spectral decomposition of $I_{\mu\nu}^{>}$

Time ordering: $t_{em} > 0$



$$T_{\mu\nu}^{>} = -\sum_n \frac{\langle 0 | J_\mu^{em}(0) | n(\vec{\mathbf{p}}_\gamma) \rangle \langle n(\vec{\mathbf{p}}_\gamma) | J_\nu^{weak}(0) | H(\vec{\mathbf{p}}_H) \rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})}$$

$$I_{\mu\nu}^{>}(t_H, T) = \int_0^T dt_{em}\; e^{E_\gamma t_{em}} C_{\mu\nu}(t_{em}, t_H)$$

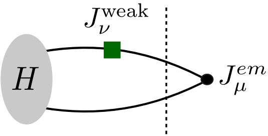
$$= -\sum_m e^{E_m t_H} \frac{\langle m(\vec{\mathbf{p}}_H) | \phi_H^\dagger(0) | 0 \rangle}{2E_{m,\vec{\mathbf{p}}_H}}$$

$$\times \sum_n \frac{\langle 0 | J_\mu^{em}(0) | n(\vec{\mathbf{p}}_\gamma) \rangle \langle n(\vec{\mathbf{p}}_\gamma) | J_\nu^{weak}(0) | m(\vec{\mathbf{p}}_H) \rangle}{2E_{n,\vec{\mathbf{p}}_\gamma}(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})} \left[1 - e^{(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})T}\right]$$

$t_H \to -\infty$ to achieve ground state saturation

Euclidean spectral decomposition of $I^{>}_{\mu\nu}$

Time ordering: $t_{em} > 0$



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$t_H \to -\infty$ to achieve ground state saturation

$T \to \infty$ to remove unwanted exponentials that come with intermediate states

Final relation

For $\mathbf{p}_\gamma \neq \mathbf{0}$,

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_H\to-\infty} \frac{-2E_H e^{-E_H t_H}}{\langle H(\vec{\mathbf{p}}_H)|\, \phi_H^\dagger \,|0\rangle} \underbrace{\int_{-T}^{T} dt_{em}\ e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)}_{I_{\mu\nu}(T, t_H)}$$

Calculating $I_{\mu\nu}(T, t_H)$

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_H\to-\infty} \frac{-2E_H e^{-E_H t_H}}{\langle H(\vec{\mathbf{p}}_H)|\, \phi_H^\dagger \,|0\rangle} \underbrace{\int_{-T}^{T} dt_{em}\ e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)}_{I_{\mu\nu}(T, t_H)}$$

Two methods to calculate $I_{\mu\nu}(T, t_H)$:

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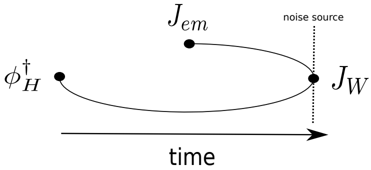
Two methods to calculate $I_{\mu\nu}(T, t_H)$:

1: 3d (timeslice) sequential propagator through $\phi_H^\dagger \to$ calculate $C_{3,\mu\nu}(t_{em}, t_H)$ on lattice, fixed t_H get all t_{em} for free



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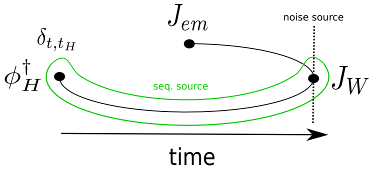

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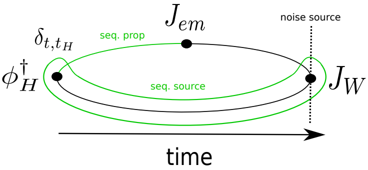

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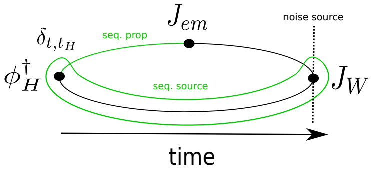
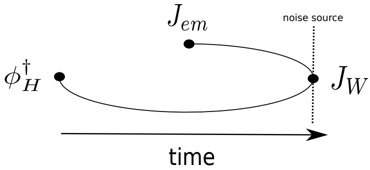
Two methods to calculate $I_{\mu\nu}(T, t_H)$:

1: 3d (timeslice) sequential propagator through $\phi_H^\dagger$ → calculate $C_{3,\mu\nu}(t_{em}, t_H)$ on lattice, fixed t_H get all t_{em} for free



2: 4d sequential propagator through J_μ^{em} → calculate $I_{\mu\nu}(T, t_H)$ on lattice, fixed T get all t_H for free



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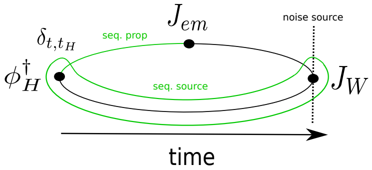

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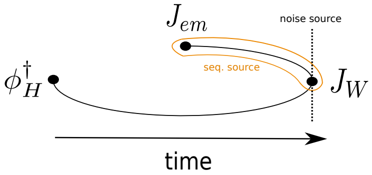

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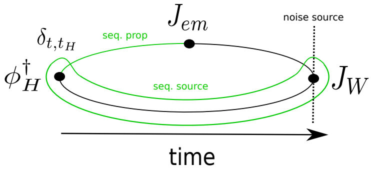

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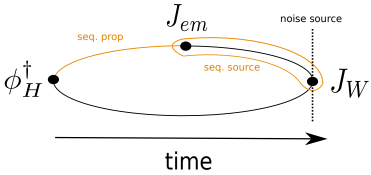

Past lattice studies

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_H\to-\infty} \frac{-2E_H e^{-E_H t_H}}{\langle H(\vec{\mathbf{p}}_H)|\, \phi_H^\dagger \,|0\rangle} \underbrace{\int_{-T}^{T} dt_{em}\ e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)}_{I_{\mu\nu}(T,t_H)}$$

- [1] we presented results at Lattice 2019 using 3d method
  - fitting to a constant looking for plateaus in T and t_H
  - all calculations were done in the rest frame of the meson
- [2] use 4d method to perform realistic physical calculation
  - set $T = N_T/2$ and fit to constant in t_H where data has plateaued
- [3] we presented results at Lattice 2021 comparing 3d/4d methods
  - some new data did not plateau
  - performed more complicated fits to take $\lim_{T\to\infty}$ and $\lim_{t_H\to-\infty}$ limits using 3d and 4d methods

[1] [Kane, Lehner, Meinel, Soni, arXiv:1907.00279]

[2] [Desiderio, Frezzotti, Garofalo, Giusti, Hansen, Lubicz, Martinelli, Sachrajda, Sanfilippo, Simula, Tantalo, PRD 2021, arXiv:2006.05358]

[3] [Kane, Giusti, Lehner, Meinel, Soni, arXiv:2110.13196]

Current work

Develop and compare fit methods to take $\lim_{T\to\infty}$ and $\lim_{t_H} \to -\infty$

- fitting only 4d sequential propagator data
  - calculate $I_{\mu\nu}(T, t_H)$ directly (K^-, D^+, D_s^+)
  - calculate $I^<_{\mu\nu}(T, t_H)$ and $I^>_{\mu\nu}(T, t_H)$ separately (D_s^+)
- fitting only 3d sequential propagator data (K^-, D^+, D_s^+)
- performing global fits to both 3d and 4d method data

Simulation parameters

So far we have considered:

$$K^- \to \gamma \ell^- \bar{\nu}, \qquad D^+ \to \gamma \ell^+ \nu, \qquad D_s^+ \to \gamma \ell^+ \nu$$

- RBC/UKQCD: $24^3 \times 64$, $m_\pi \approx 340$ MeV, $a \approx 0.11$ fm (K^-, D^+, D_s^+)
- RBC/UKQCD: $32^3 \times 64$, $m_\pi \approx 340$ MeV, $a \approx 0.11$ fm (K^-)
- Up/down/strange valence quarks: same domain-wall action as sea quarks
- Charm valence quarks: Möbius domain-wall with "stout" smearing
- Light quark deflation
  - $24^3 \times 64$ calculate lowest 400 eigenvectors
  - $32^3 \times 64$ calculate lowest 800 eigenvectors
- $\mathbb{Z}_2$ random wall sources at weak current location
- Neglect disconnected diagrams
- "Mostly nonperturbative" renormalization
- All-mode averaging with 16 sloppy and 1 exact samples per config

see [Kane, Giusti, Lehner, Meinel, Soni, arXiv:2110.13196] for more details

$D_s^+ \to \gamma \ell^+ \nu$ runs

RBC/UKQCD Ensembles: $24^3 \times 64$, $m_\pi \approx 340$ (MeV), $a \approx 0.11$(fm)

$D_s^+ \to \gamma \ell^+ \nu$ runs

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3d method:

$-t_{D_s}/a$	$\frac{L}{2\pi}\mathbf{p}_{D_s}$	$\lvert\frac{L}{2\pi}\mathbf{p}_\gamma\rvert^2$	# configs
$\{6, 9, 12\}$	(0,0,0)	{1,2,3,4}	25

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$\{6, 9, 12\}$	(0,0,0)	{1,2,3,4}	25

4d method:

T/a	$\frac{L}{2\pi}\mathbf{p}_{D_s}$	$\frac{L}{2\pi}\mathbf{p}_\gamma$	# configs
$\{6, 9, 12\}$	(0,0,0)	(0,0,1)	25
	(0,0,1)		
	(0,0,2)		
	(0,0,-1)		

Compare 4d methods

- fitting only 4d sequential propagator data
  - calculate $I_{\mu\nu}(T, t_H)$ directly (K^-, D^+, D_s^+)
  - calculate $I^<_{\mu\nu}(T, t_H)$ and $I^>_{\mu\nu}(T, t_H)$ separately (D_s^+)

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

Use fit ranges where data has plateaued in t_H, i.e. $t_H \to -\infty$

Include terms to fit
(1) unwanted exponential from first intermediate state

Fit form: 4d method

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Use fit ranges where data has plateaued in t_H, i.e. $t_H \to -\infty$

Include terms to fit
(1) unwanted exponential from first intermediate state

Fit form factors $F_V, F_{A,SD}$ directly instead of $I_{\mu\nu}$

$$F(t_H, T) = F + B_F^{<}\, e^{-(E_\gamma - E_H + E^{<})T} + B_F^{>}\, e^{(E_\gamma - E^{>})T}$$

■ fit parameters

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

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$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

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■ fit parameters

Only have three values of T, fitting multiple exponentials not possible
$\to$ Use broad Gaussian prior on $E^{>}$ exclude unphysical values

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

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■ fit parameters

Calculate time orderings separately:

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

Calculate time orderings separately:

Include terms to fit

(1) unwanted exponential from first intermediate state ($I^{<}_{\mu\nu}$ and $I^{>}_{\mu\nu}$)

(2) first excited state ($I^{>}_{\mu\nu}$ only)

Time ordering $t_{em} > 0$:

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

Calculate time orderings separately:

Include terms to fit

(1) unwanted exponential from first intermediate state ($I^{<}_{\mu\nu}$ and $I^{>}_{\mu\nu}$)

(2) first excited state ($I^{>}_{\mu\nu}$ only)

Time ordering $t_{em} > 0$:

$$F^{>}(t_H, T) = F^{>} + B_F^{>} e^{(E_\gamma - E^{>})T} + C_F^{>} e^{\Delta E t_H}$$

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

Calculate time orderings separately:

Include terms to fit

(1) unwanted exponential from first intermediate state ($I^{<}_{\mu\nu}$ and $I^{>}_{\mu\nu}$)

(2) first excited state ($I^{>}_{\mu\nu}$ only)

Time ordering $t_{em} > 0$:

$$F^{>}(t_H, T) = F^{>} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{(1)} + C_F^{>} \underbrace{e^{\Delta E t_H}}_{(2)}$$

Fit form: 4d method

Sum of both time orderings $I_{\mu\nu}(T, t_H) = I^{<}_{\mu\nu}(T, t_H) + I^{>}_{\mu\nu}(T, t_H)$

$$F(t_H, T) = F + B_F^{<} \underbrace{e^{-(E_\gamma - E_H + E^{<})T}}_{t_{em}<0} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{t_{em}>0}$$

■ fit parameters

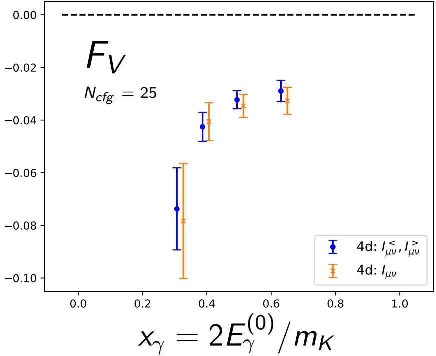
Calculate time orderings separately:
Include terms to fit
(1) unwanted exponential from first intermediate state ($I^{<}_{\mu\nu}$ and $I^{>}_{\mu\nu}$)
(2) first excited state ($I^{>}_{\mu\nu}$ only)

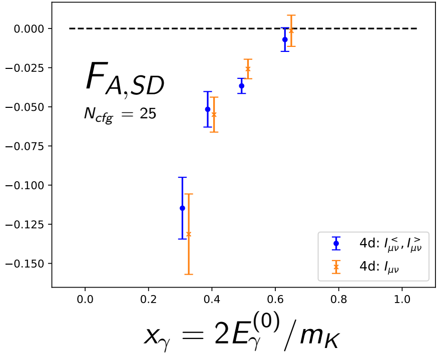
Time ordering $t_{em} > 0$:

$$F^{>}(t_H, T) = F^{>} + B_F^{>} \underbrace{e^{(E_\gamma - E^{>})T}}_{(1)} + C_F^{>} \underbrace{e^{\Delta E t_H}}_{(2)}$$

Look at preliminary data and fit results for $D_s^+ \to \gamma \ell^+ \nu_\ell$ form factors

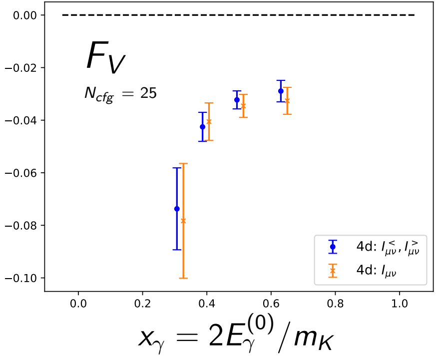
$D_s^+ \to \gamma \ell^+ \nu$: Compare 4d methods

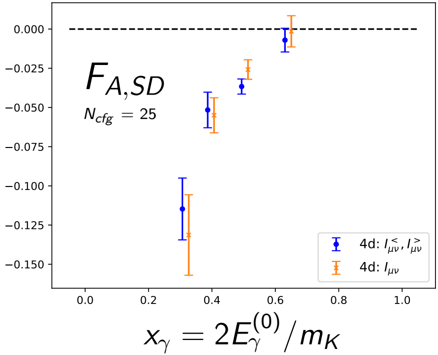




$$0 < x_\gamma < 1$$

$D_s^+ \to \gamma \ell^+ \nu$: Compare 4d methods





Calculating $I^>_{\mu\nu}$ and $I^<_{\mu\nu}$ separately

$\rightarrow$ remove prior on the fit parameter $E^>$

$\rightarrow$ smaller error bars

$\rightarrow$ more propagator solves needed

Compare 3d and 4d methods

- fitting only 4d sequential propagator data
  - calculate $I^{<}_{\mu\nu}(T, t_H)$ and $I^{>}_{\mu\nu}(T, t_H)$ separately
- fitting only 3d sequential propagator data
- performing global fits to both 3d and 4d method data

Fit form: 3d method

Include terms to fit
(1) unwanted exponential from first intermediate state
(2) first excited state

Fit form: 3d method

Include terms to fit
(1) unwanted exponential from first intermediate state
(2) first excited state

Time ordering $t_{em} < 0$:

$$F^{<}(t_H, T) = F^{<} + B_F^{<}\, e^{-(E_\gamma - E_H + E^{<})T} + C_F^{<}\, e^{\Delta E t_H}$$

■ fit parameters

Fit form: 3d method

Include terms to fit
(1) unwanted exponential from first intermediate state
(2) first excited state

Time ordering $t_{em} < 0$:

$$F^{<}(t_H, T) = F^{<} + B_F^{<}\, e^{-(E_\gamma - E_H + E^{<})T} + C_F^{<}\, e^{\overbrace{\Delta E\, t_H}^{(2)}}$$

■ fit parameters

Fit form: 3d method

Include terms to fit
(1) unwanted exponential from first intermediate state
(2) first excited state

Time ordering $t_{em} < 0$:

$$F^{<}(t_H, T) = F^{<} + B_F^{<} \overbrace{e^{-(E_\gamma - E_H + E^{<})T}}^{(1)} + C_F^{<} \overbrace{e^{\Delta E t_H}}^{(2)}$$

■ fit parameters

Fit form: 3d method

Include terms to fit
(1) unwanted exponential from first intermediate state
(2) first excited state

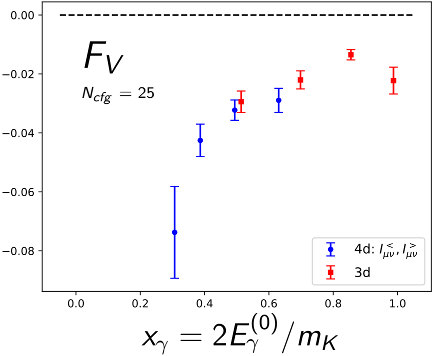
Time ordering $t_{em} < 0$:

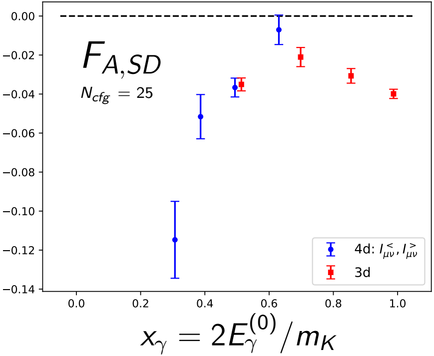
$$F^{<}(t_H, T) = F^{<} + B_F^{<} \overbrace{e^{-(E_\gamma - E_H + E^{<})T}}^{(1)} + C_F^{<} \overbrace{e^{\Delta E t_H}}^{(2)}$$

■ fit parameters

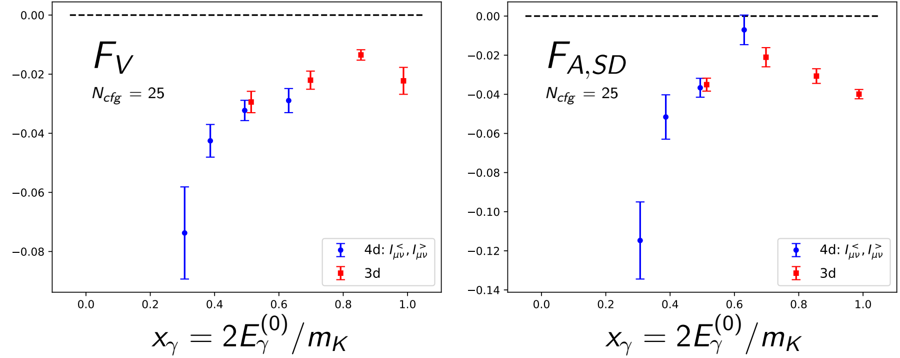
Look at preliminary data and fit results for $D_s^+ \to \gamma \ell^+ \nu_\ell$ form factors

$D_s^+ \to \gamma \ell^+ \nu_\ell$: Independent 3d and 4d analysis results



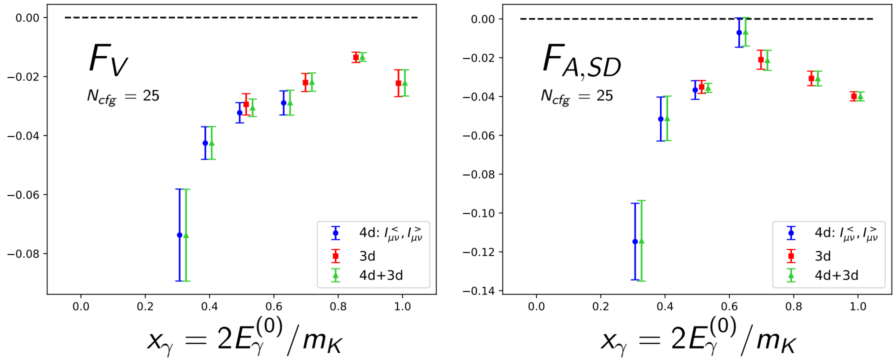


$D_s^+ \to \gamma \ell^+ \nu_\ell$: Independent 3d and 4d analysis results



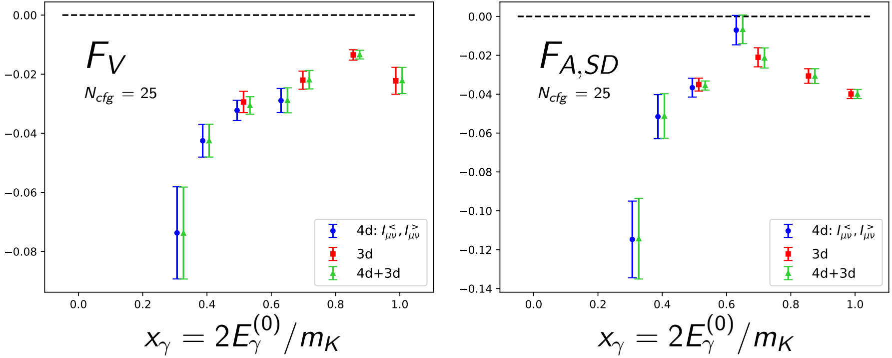
- 3d has similar or smaller uncertainties compared to 4d at same x_γ

$D_s^+ \to \gamma \ell^+ \nu_\ell$: Combined 3d and 4d analysis results



- 3d has similar or smaller uncertainties compared to 4d at same x_γ
- Combined fits have reduced uncertainties when 3d and 4d have data point at same x_γ

$D_s^+ \to \gamma \ell^+ \nu_\ell$: 3d and 4d analysis results



- 3d has similar or smaller uncertainties compared to 4d at same x_γ
- Combined fits have reduced uncertainties when 3d and 4d have data point at same x_γ
- Almost identical uncertainties otherwise

Final remarks

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_H\to-\infty} \frac{-2E_H e^{-E_H t_H}}{\langle H(\vec{\mathbf{p}}_H)|\,\phi_H^\dagger\,|0\rangle} \underbrace{\int_{-T}^{T} dt_{em}\ e^{E_\gamma t_{em}} C_{3,\mu\nu}(t_{em}, t_H)}_{I_{\mu\nu}(T,t_H)}$$

- Started calculations on finer lattice spacing RBC/UKQCD ensemble to investigate discretization effects ($32^3 \times 64$, $a^{-1} = 2.383(9)$GeV)
- Future plans:
  $\rightarrow$ Twisted boundary conditions to get to small photon energies $E_\gamma^{(0)}$
  $\rightarrow$ Realistic physical calculation for K and $D_{(s)}$, $B_{(s)}$ mesons

Backup slides

Solves required per configuration

$$3d : N_{\text{solves}} \sim 1 + N_{p_H} \times N_{p_\gamma} + 1(N_{t_{\text{sep}}} \times N_{p_H})$$

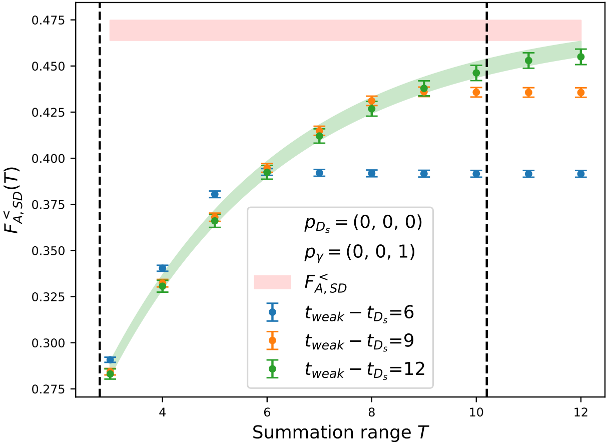
$$4d \ I_{\mu\nu} : N_{\text{solves}} \sim 1 + N_{p_H} \times N_{p_\gamma} + 4(N_T \times N_{p_\gamma})$$

$$4d \ I^{<}_{\mu\nu}, I^{>}_{\mu\nu} : N_{\text{solves}} \sim 1 + N_{p_H} \times N_{p_\gamma} + 8(N_T \times N_{p_\gamma})$$

$D_s^+ \to \gamma \ell^+ \nu$: 3d method $p_{D_s} = \frac{2\pi}{L}(0,0,0)$ $p_\gamma = \frac{2\pi}{L}(0,0,1)$

Time ordering $t_{em} < 0$:

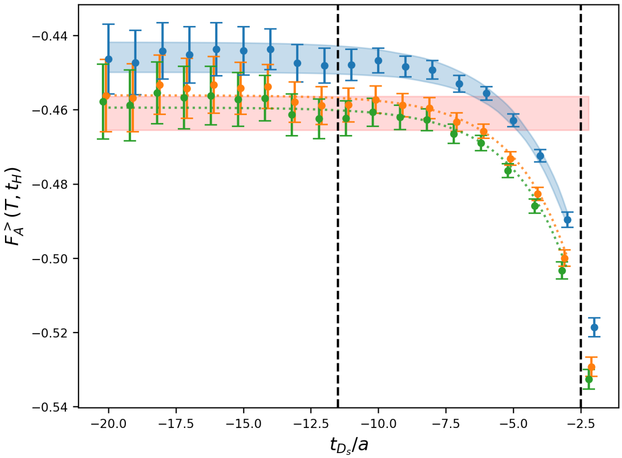
$$F^<_{A,SD}(t_H, T) = F^<_{A,SD} + B^<_{F_{A,SD}} e^{-(E_\gamma - E_{D_s} + E^<_A)T} + C^<_{F_{A,SD}} e^{\Delta E t_H}$$



$D_s^+ \to \gamma\ell^+\nu_\ell$: 4d method $p_{D_s,z} = \frac{2\pi}{L}(-1)$ $p_{\gamma,z} = \frac{2\pi}{L}(1)$

Time order $t_{em} > 0$:

$$F_A^>(t_H, T) = F_A^> + B_{F_A}^< e^{-(E_\gamma - E_K + E_A^<)T} + C_{F_A}^> e^{\Delta E t_H}$$



Euclidean spectral decomposition of $I_{\mu\nu}$

Time ordering $t_{em} < 0$: (for large negative t_B)

$$I^{<}_{\mu\nu}(t_B, T) = \int_{-T}^{0} dt_{em}\; e^{E_\gamma t} C_{3,\mu\nu}(t_{em}, t_B)$$

(* all times are now Euclidean)

$$= \langle B(\vec{\mathbf{p}}_B)|\, \phi_B^\dagger(0)\, |0\rangle \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{E_B t_B}$$

$$\times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_B-\vec{\mathbf{p}}_\gamma}} \frac{\langle 0|\, J_\nu^{weak}(0)\, |n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle \langle n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)|\, J_\mu^{em}(0)\, |B(\vec{\mathbf{p}}_B)\rangle}{E_\gamma + E_{n,\vec{\mathbf{p}}_B-\vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B}}$$

$$\times \left[1 - e^{-(E_\gamma + E_{n,\vec{\mathbf{p}}_B-\vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B})T}\right]$$

Require $E_\gamma + E_{n,\vec{\mathbf{p}}_B-\vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B} > 0$ to get rid of unwanted exponential

States $|n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle$ has same flavor quantum numbers as B meson

$\rightarrow E_{n,\mathbf{p}_B-\mathbf{p}_\gamma} \geq E_{B,\mathbf{p}_B-\mathbf{p}_\gamma} = \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2}$

For $\mathbf{p}_\gamma \neq 0$, $|\mathbf{p}_\gamma| + \sqrt{m_n^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2} > \sqrt{m_B^2 + \mathbf{p}_B}$ is automatically satisfied

Euclidean spectral decomposition of $I_{\mu\nu}$

Time ordering: $t_{em} > 0$ (for large negative t_B)

$$
\begin{aligned}
I^{>}_{\mu\nu}(t_B, T) &= \int_0^T dt_{em}\ e^{E_\gamma t} C_{\mu\nu}(t_{em}, t_B) \qquad \text{(* all times are now Euclidean)} \\
&= -\langle B(\vec{\mathbf{p}}_B)|\, \phi_B^\dagger(0)\, |0\rangle \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{E_B t_B} \\
&\times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_\gamma}} \langle 0|\, J_\mu^{em}(0)\, |n(\vec{\mathbf{p}}_\gamma)\rangle \langle n(\vec{\mathbf{p}}_\gamma)|\, J_\nu^{weak}(0)\, |B(\vec{\mathbf{p}}_B)\rangle \\
&\times \frac{1}{E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma}} \left[1 - e^{(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})T} \right]
\end{aligned}
$$

Require $E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma} < 0$

Because the states $\left|n(\mathbf{p}_\gamma)\right\rangle$ have mass, $\sqrt{m_n^2 + \mathbf{p}_\gamma^2} > |\mathbf{p}_\gamma|$ is automatically satisfied

Time orderings