# Finite Temperature Fine Lattice Simulation with DWQs

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DWQ@25

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# Contributors of main results

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1: YITP, 2: R-CCS, 3: Osaka, 4: KEK

# acknowledgements


- Codes used:
  - HMC
    - Grid / Regensburg (A64FX-tuned)
  - Measurements:
    - Hadrons / Grid
    - BQCD
    - Bridge++

- MEXT program

  成果創出加速プログラム

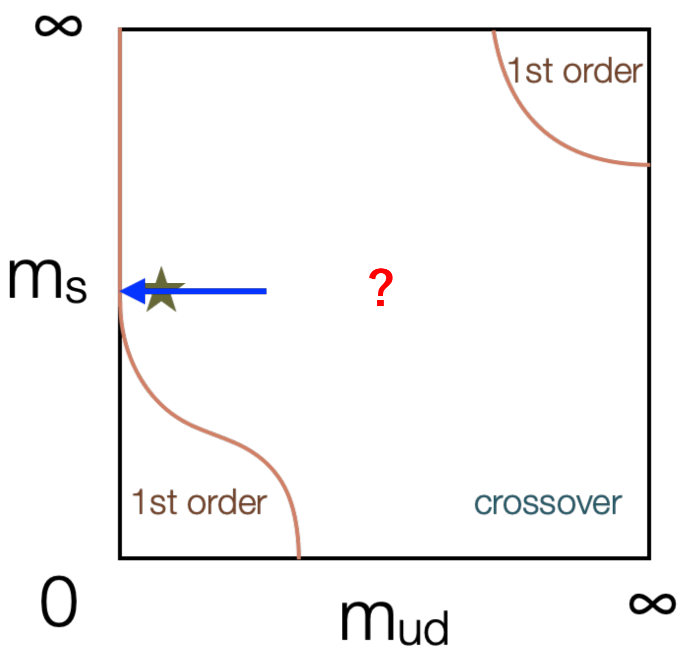
  Program for Promoting Researches on the Supercomputer Fugaku

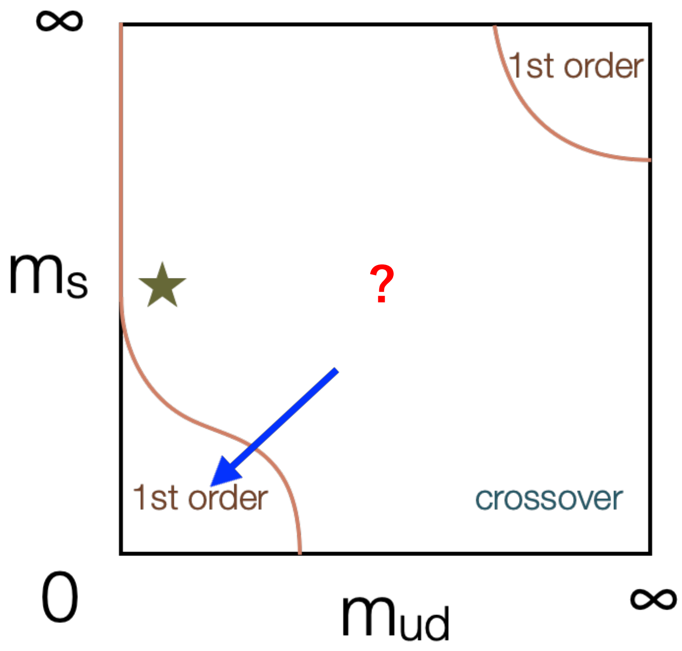
  - Simulation for basic science: from fundamental laws of particles to creation of nuclei

- Computers
  - Oakforest-PACS
  - Polaire and Grand Chariot at Hokkaido University
  - supercomputer Fugaku provided by the RIKEN Center for Computational Science

# Intro

- $N_f$=2+1 thermodynamic property
  - through chiral symmetric formulation
  - Order of the transition
  - (pseudo) critical temperature
  - Location of the phase boundary
  - Near the physical point
- Chiral symmetric formulation
  - Ideal to treat flavor SU(2) and $U(1)_A$ properly
  - Domain wall fermion (DWF) : practical choice
- DWF and chirality
  - Fine lattice needed
  - Aiming for a < 0.08 fm (eventually)
  - Current search domain:  0.08 < a < 0.12fm



# $N_f$=2 Möbius DWF

- Lessons learned
  - Chiral symmetry important for discussing
    - chiral, $U(1)_A$ problems
  - Reweighting to overlap essential
  - For reweighting to be successful for DW - OV
    - Fine lattice needed (efficiency of reweighting): $a \lesssim 0.1$ fm
    - Smoothness of configuration & smallness of $m_{res}$
  - For reweighting to be successful in general
    - Large volume is problematic
    - It may not work for further finer lattices
- Expectation
  - (Finer the lattice, smaller $m_{res}$)
  - DWF itself eventually becomes good enough
    - Except for a few very tricky quantities → detailed investigation needed

# $N_f$=2 Möbius DWF

- Action
  - Tree-level improved Symanzik gauge
  - stout-smeared, scale-factor 2 Shamir DWF
- So far studied
  - $U(1)_A$ and chiral symmerty
  - Topological charge
  - Chiral susceptibility
  - Spatial correlation of color singlet channels
- Simulation setup
  - Fix $\beta$
  - Fix $N_t$
  - Vary $m$

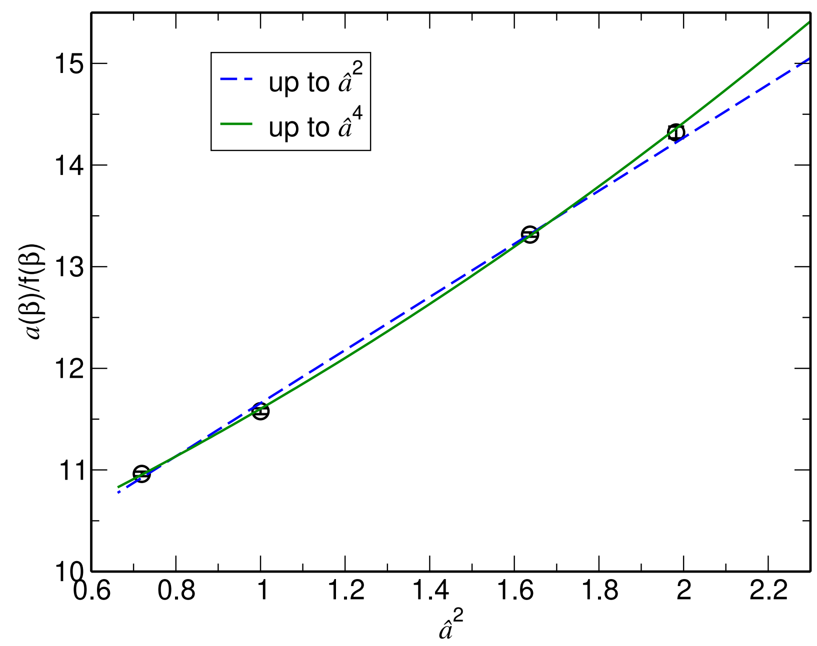
# $N_f$=2+1

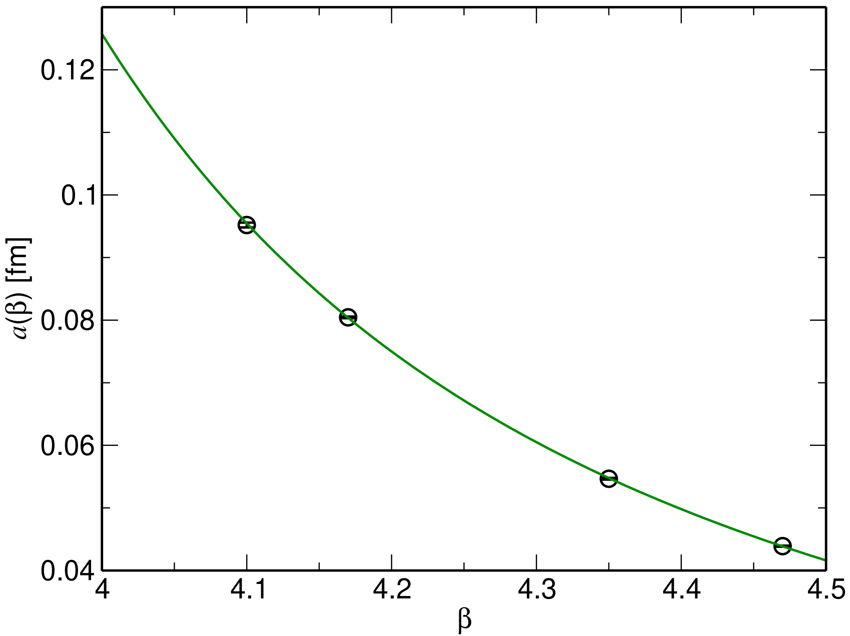
- Action: same as $N_f$=2
- Simulation setup (we follow most of the simulations by now)
  - Fix $\beta$
  - Fix $N_t$
  - Fix $m_s^{latt}$ near physical
  - Vary $m_l^{latt}$
  - Aiming to understand the role of chiral symmetry, $U(1)_A$, topology
- ↔ new setup: fix physics and vary T in this study
  - Line of Constant Physics
  - Aiming to study the (pseudo) criticality w/ fixed physics

See presentation by K. Suzuki at Lattice 2021

# $N_f$=2+1 Möbius DWF

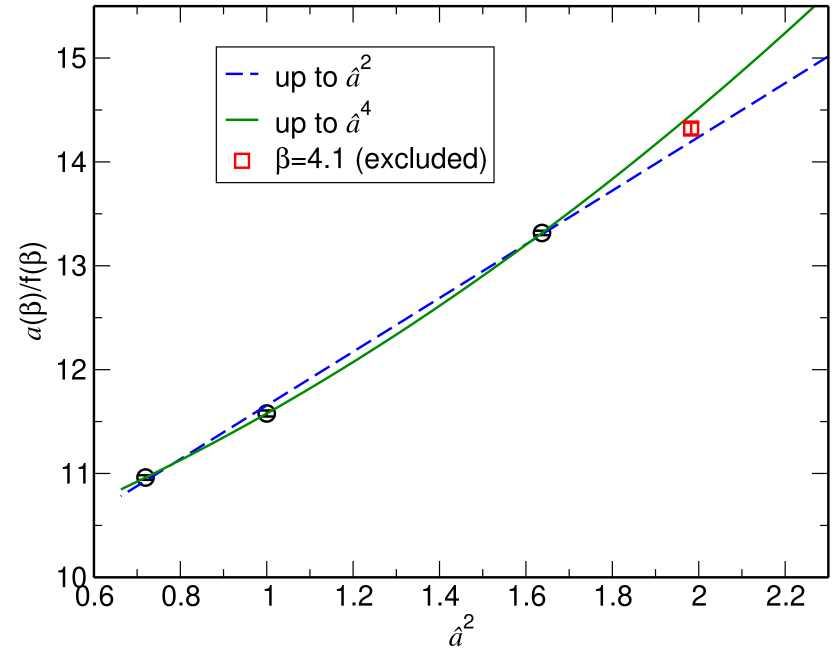
- $a(\beta)$
- Using
  - JLQCD T=0 lattices with $t_0$ meas.
    - $a$=0.080, 0.055, 0.044 fm (published)
    - $a$=0.095 fm (pilot study)
  - Parameterization of Edwards et al (1998)
    - $a = c_0 f(g^2)(1 + c_2 \hat{a}(g)^2 + c_4 \hat{a}(g)^4)$.
    - $$\hat{a}(g)^2 \equiv [f(g^2)/f(g_0^2)]^2,$$
      $$f(g^2) \equiv (b_0 g^2)^{-b_1/2b_0^2} \exp\left(-\frac{1}{2b_0 g^2}\right),$$
      $$b_0 = \frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right), \quad b_1 = \frac{1}{(4\pi)^4}\left(102 - \frac{38N_f}{3}\right),$$
    - Fit to $\hat{a}^4$ works well

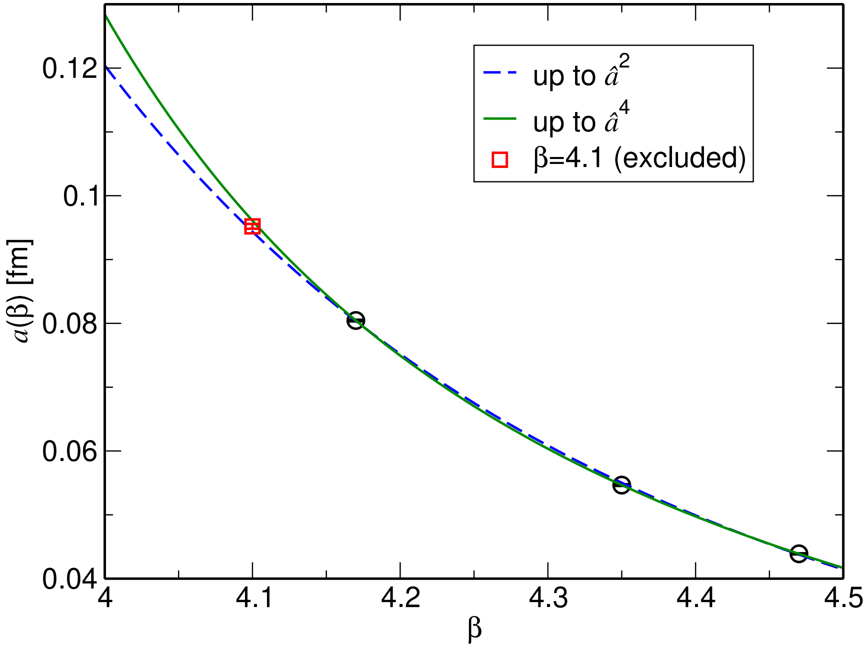


# $N_f$=2+1 Möbius DWF

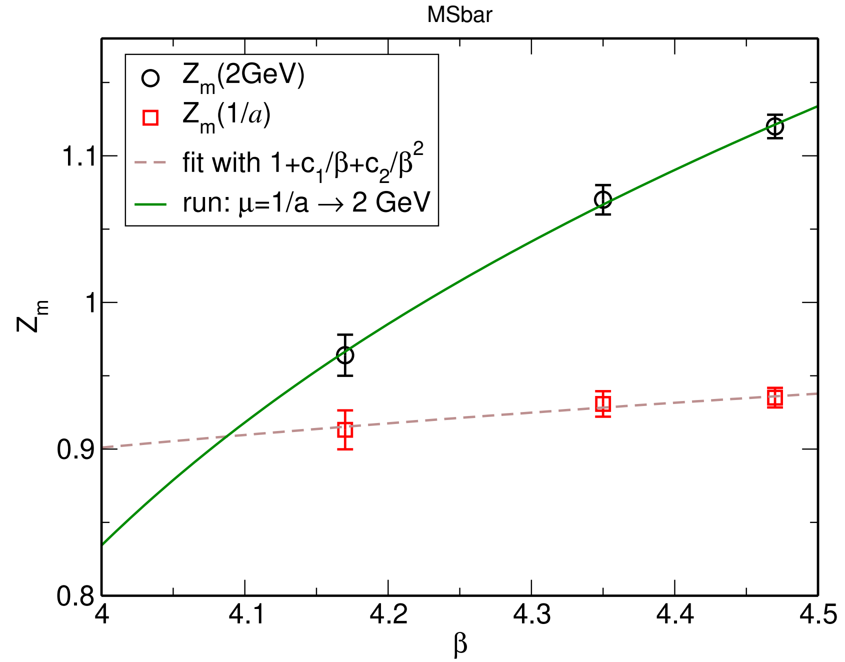
- $a(\beta)$ precision over the range
- Test excluding coarsest one
  - 1 % diff : fit <-> measurement @$\beta$=4.1
  - Difference $O(\hat{a}^4) - O(\hat{a}^2)$ fits: good measure of error (maybe overestimating)
- Full range fit
  - @ $\beta = 4.0$ error is ~ few %
  - The fit may be regarded as renormalized trajectory
    - Continuum limit will absorb the error

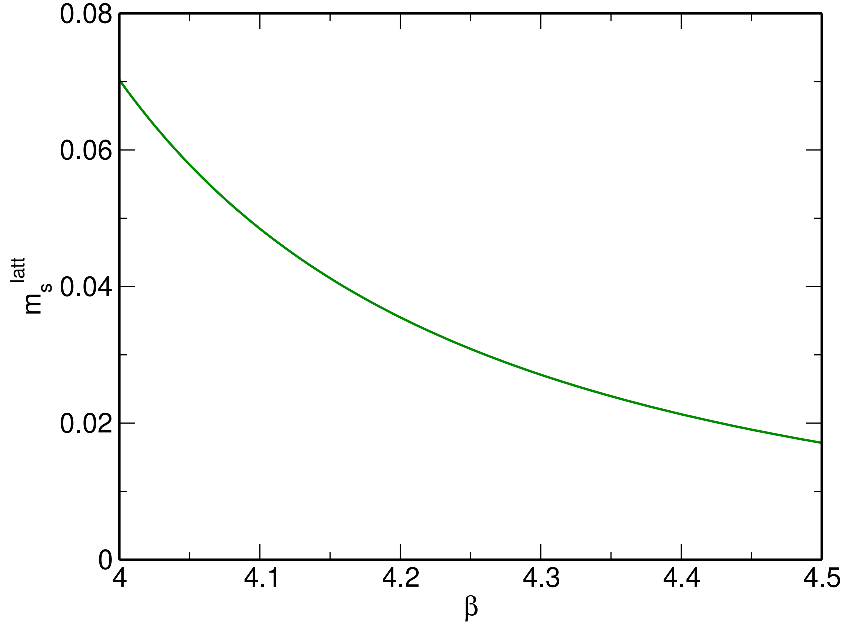


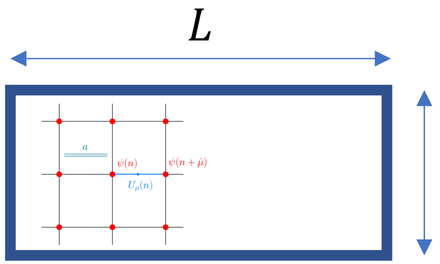



# $N_f$=2+1 Möbius DWF LCP

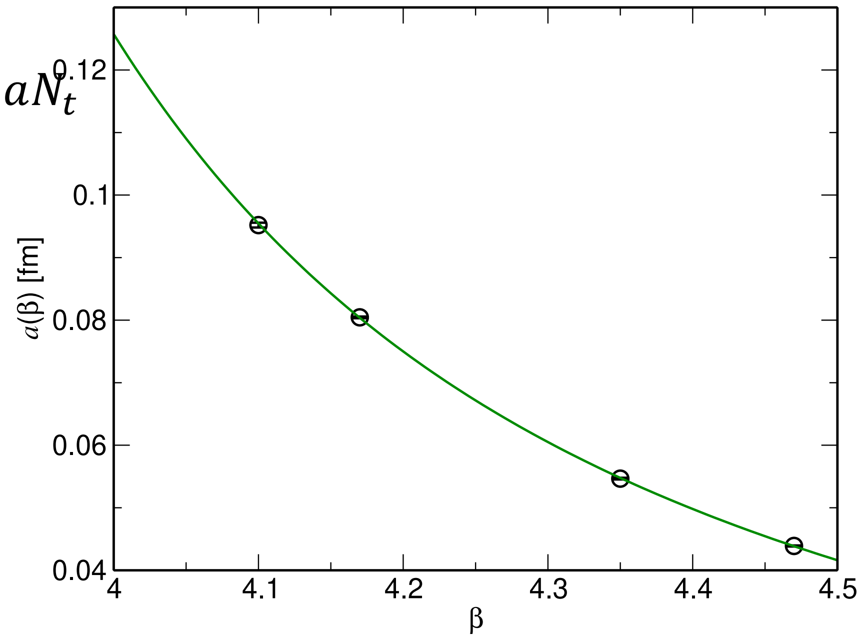
- Quark mass as function of $\beta$ [fixed physics]
- We use quark mass input
  - $m_s = 92\ MeV$ (MSb 2GeV)
  - $\frac{m_s}{m_{ud}} = 27.4$ (See for example FLAG 2019)
  - $m_q^R = Z_m \cdot (am_q^{latt}) \cdot a^{-1}(\beta)$
- Parameterizing $Z_m(\beta)$
  - Take $Z_m(2GeV)$ w/ NPR Tomii et al 2016
  - $Z_m(2GeV) \rightarrow Z_m(a^{-1})$ NNNLO pert.
    - No (large) $\log(a\mu)$
    - Should behave like $1 + d_1 g^2 + d_2 g^4 + \cdots$
  - Fit $Z_m(a^{-1})$ with $1 + c_1\beta^{-1} + c_2\beta^{-2}$
  - $Z_m(a^{-1}) \rightarrow Z_m(2GeV)$ NNNLO pert.

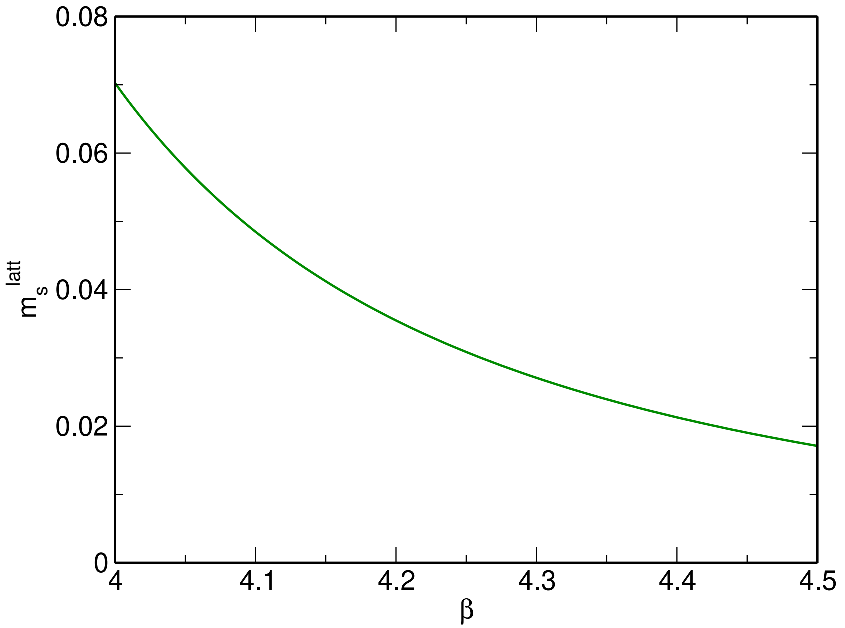


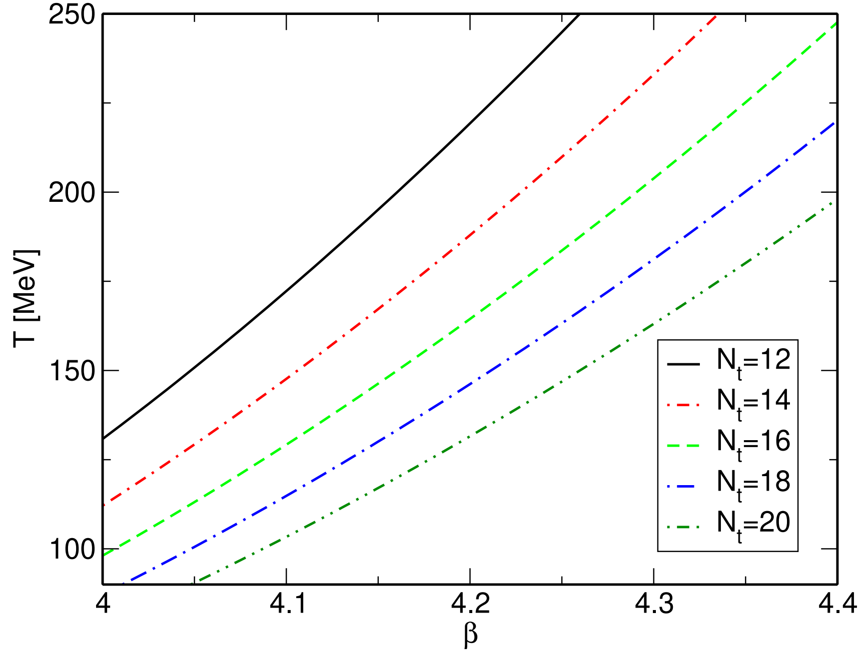



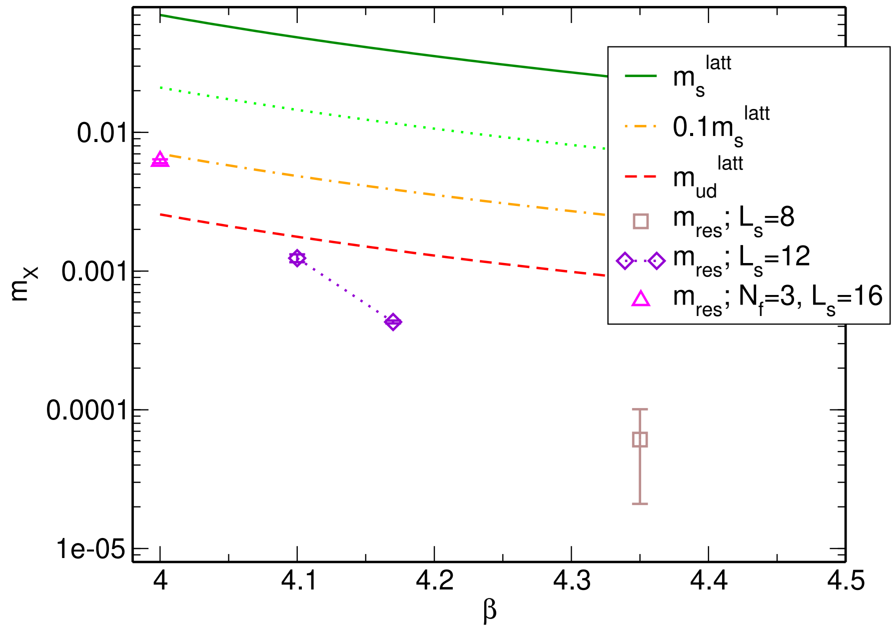

L
a
ψ(n)
ψ(n + μ̂)
U_μ(n)

$$\frac{1}{T} = aN_t$$


a(β) [fm]
β

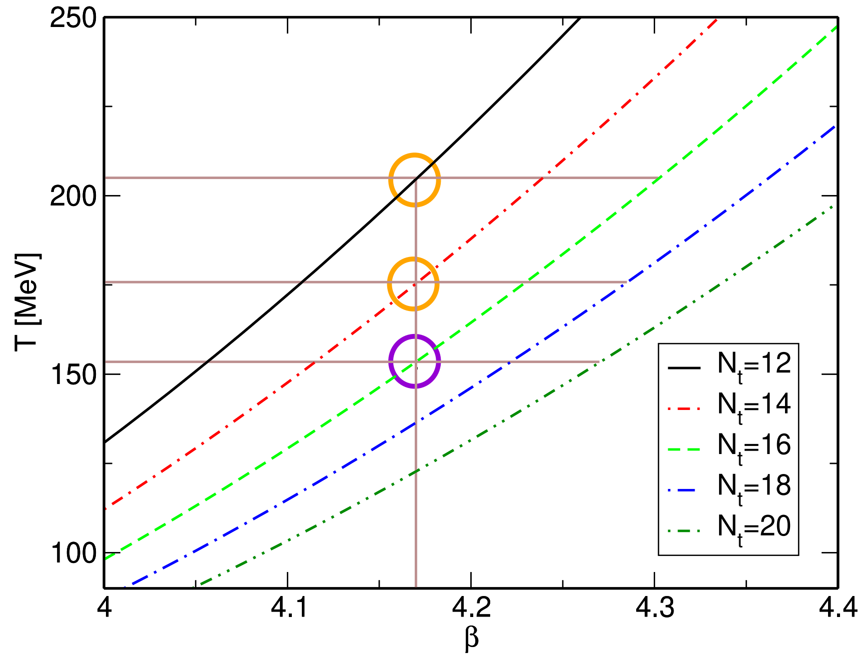

m_s^latt
β

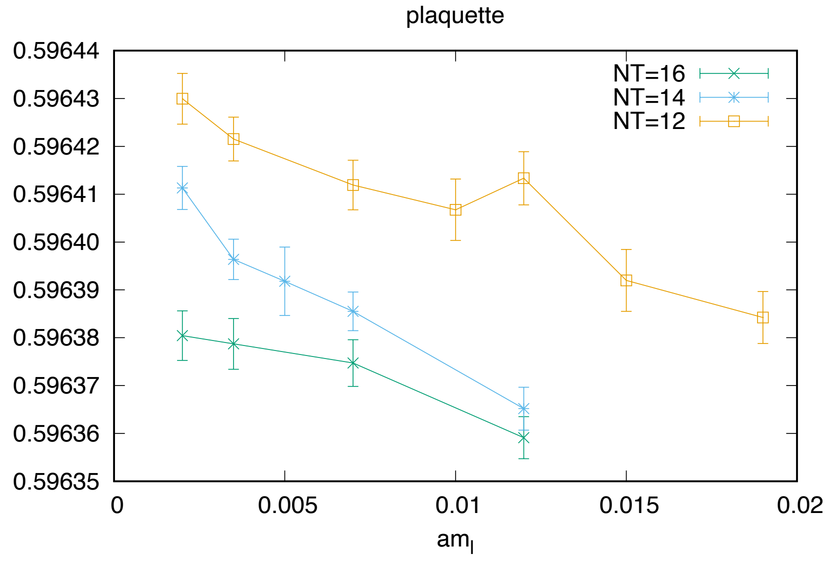

T [MeV]
β
N_t=12
N_t=14
N_t=16
N_t=18
N_t=20

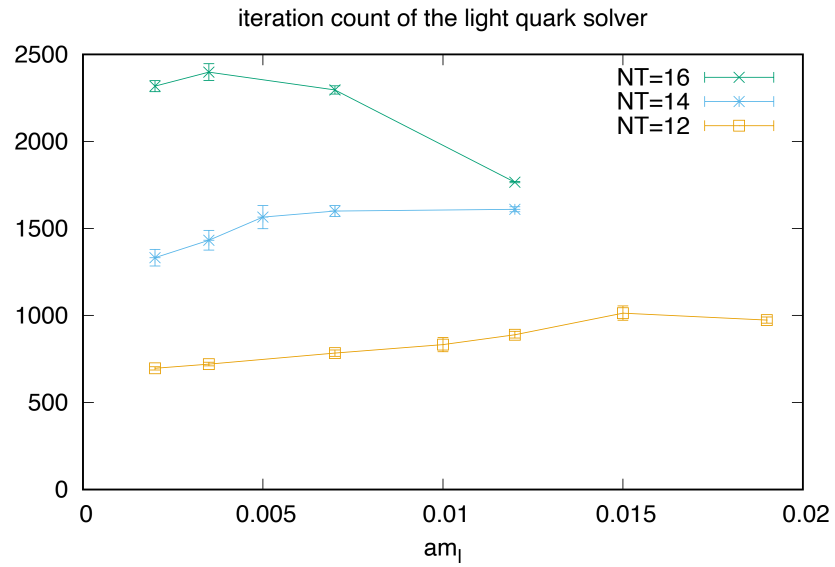

m_X
β
m_s^latt
0.1m_s^latt
m_ud^latt
m_res; L_s=8
m_res; L_s=12
m_res; N_f=3, L_s=16

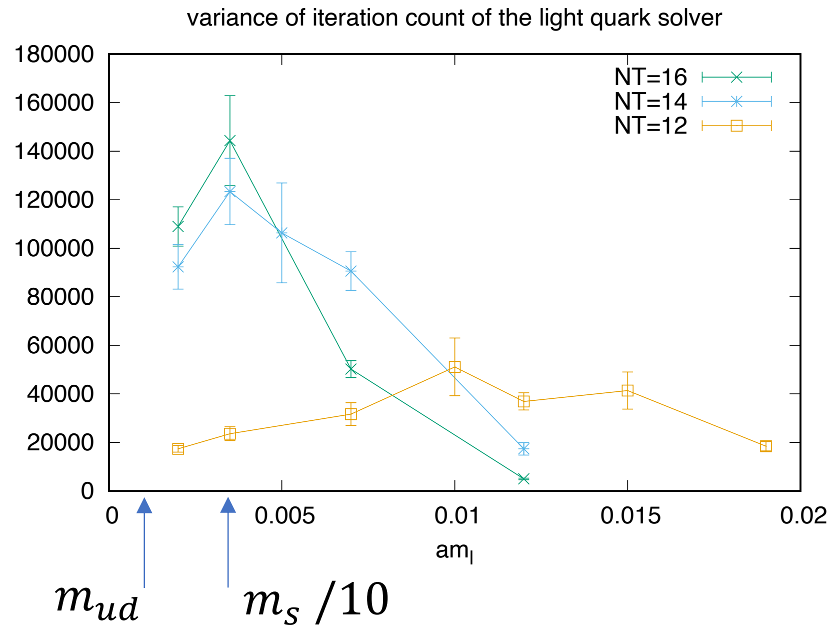
# Simulation range

- $T-\beta$ relation $\quad T = 1/(aN_t)$
- Information from fixed $\beta$ simulation
  - $N_t = 12: \ T \sim 200\ MeV$
  - $N_t = 14: \ T \sim 170\ MeV$
  - $N_t = 16: \ T \sim 150\ MeV$
  - $N_s = 32,\ L_s = 12:\ m_{res} \simeq 4\times10^{-4}$
  - $m_{ud}^{latt} = 0.0014,\ m_s^{latt} = 0.0388$

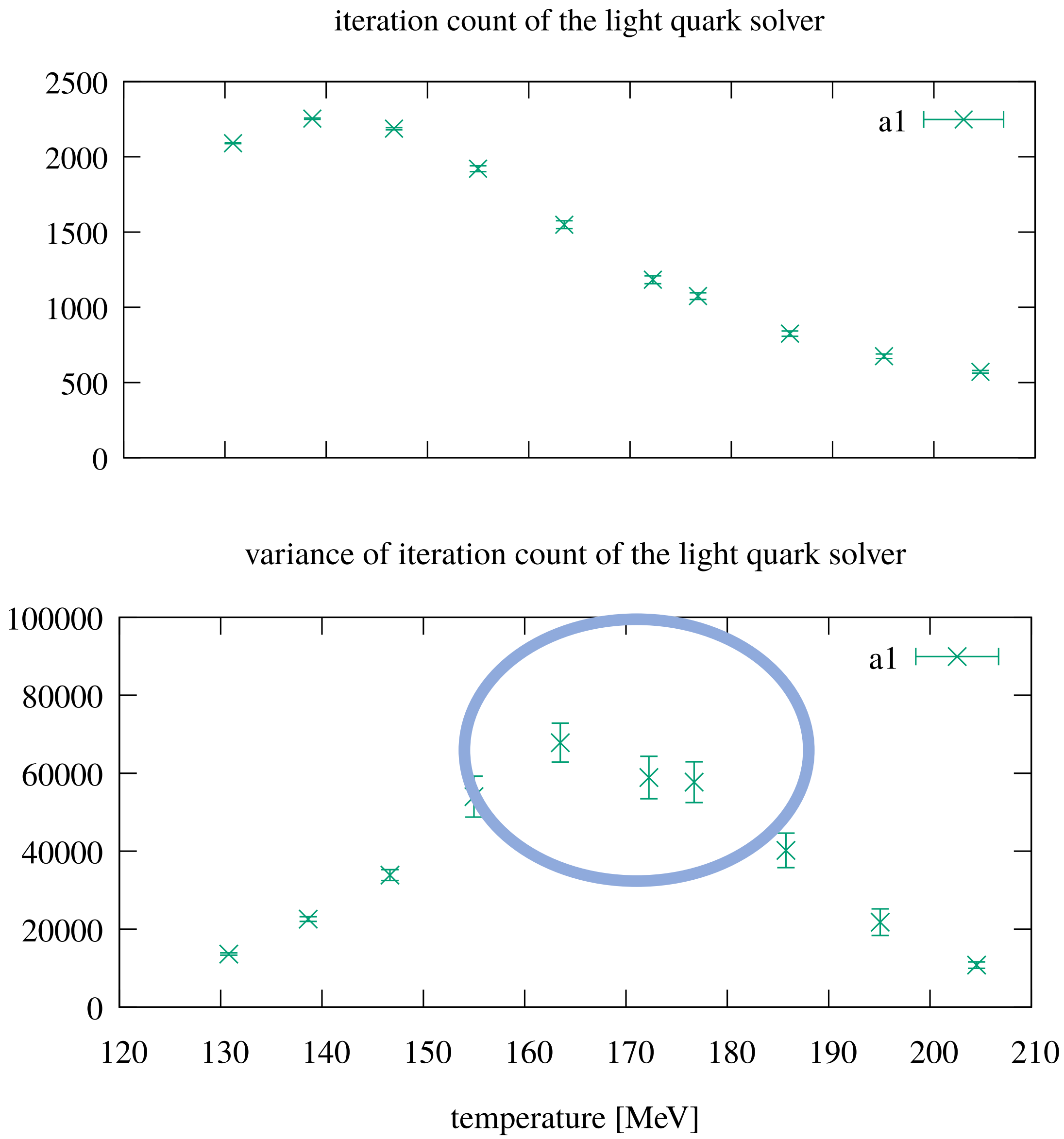


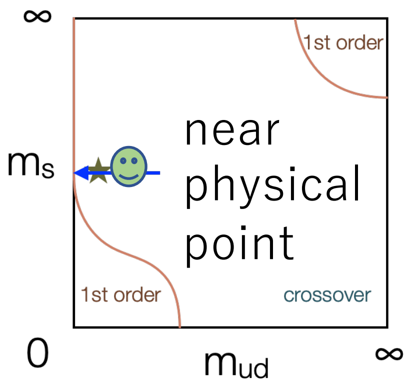



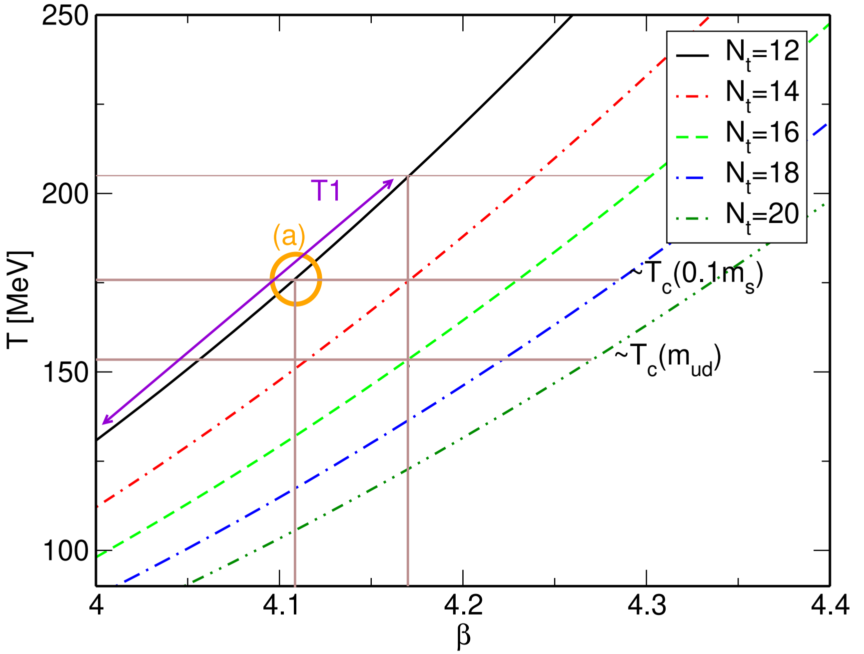



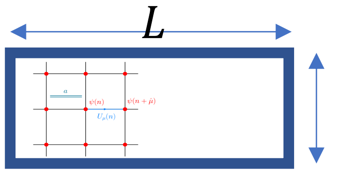



# Initial simulations on LCP



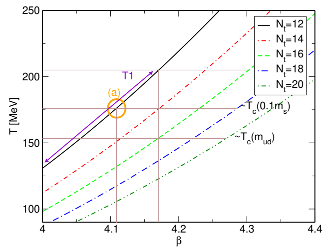

$$\frac{1}{T} = aN_t$$

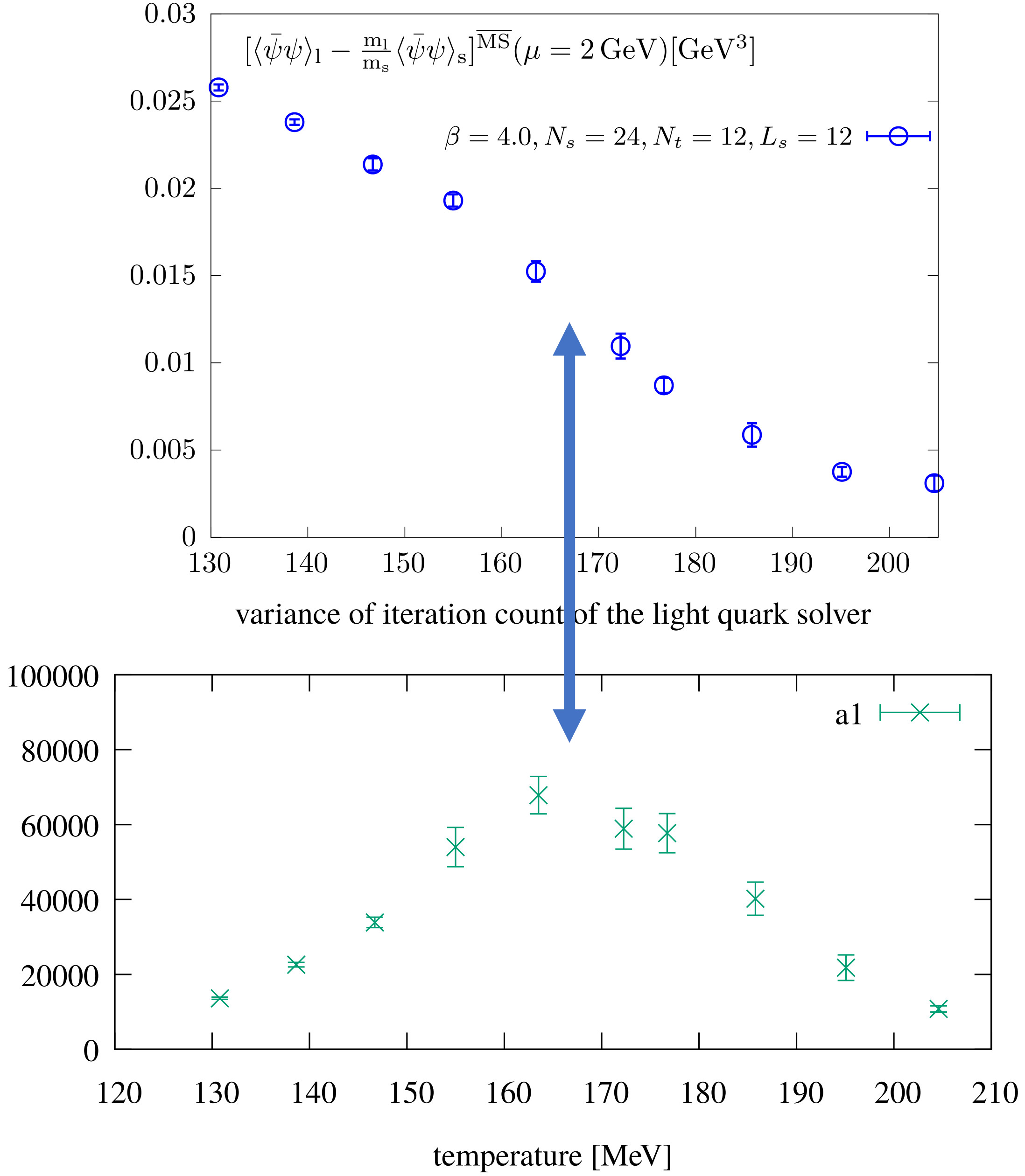
- $N_t = 12$ (T1)
- $m = 0.1m_s$ (a)
- $N_s = 24,\ L_s = 12$

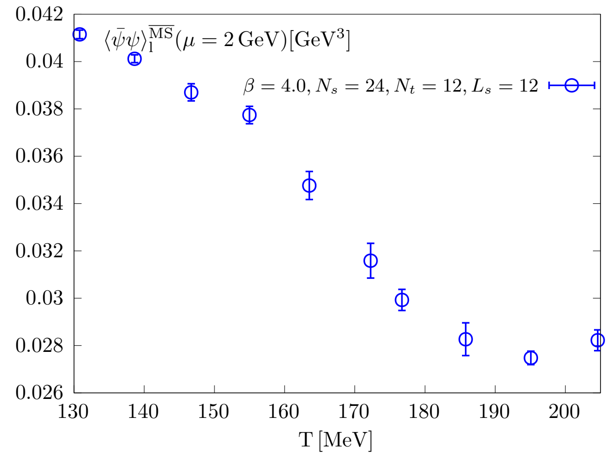
[I. Kanamori @ JPS 2021 fall]

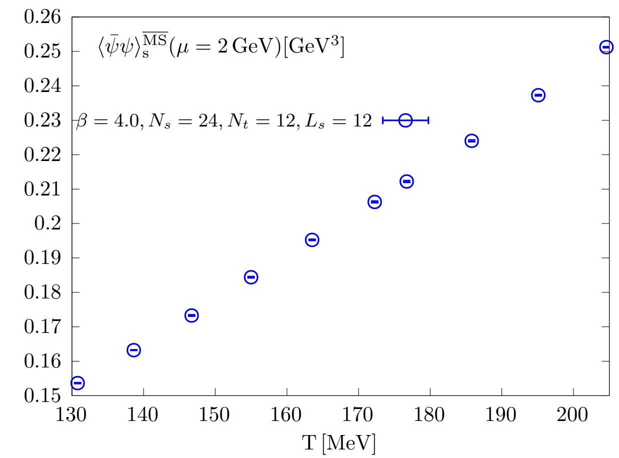
# Initial simulations on LCP

- $N_t = 12$ (T1)
- $m = 0.1 m_s$ (a)
- $N_s = 24,\ L_s = 12$

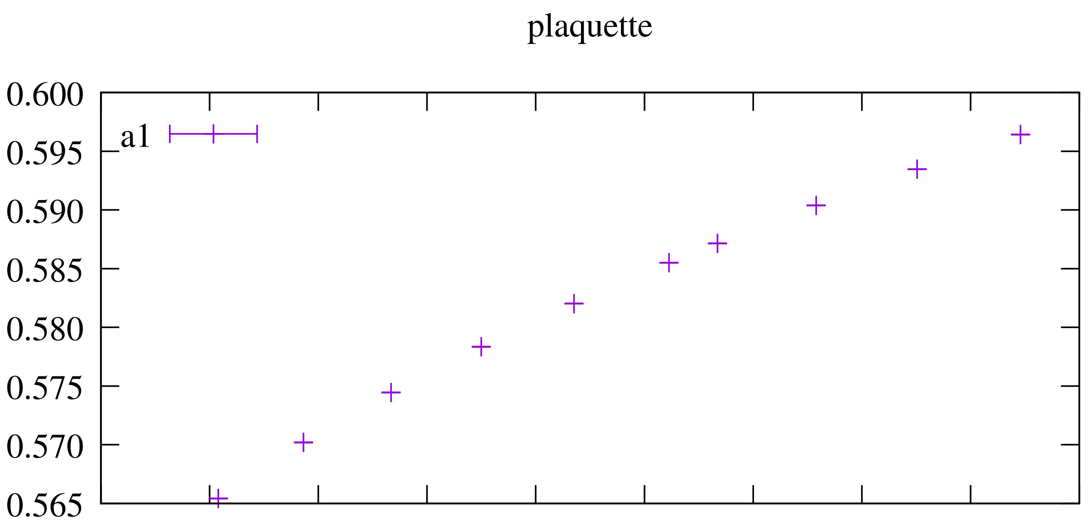


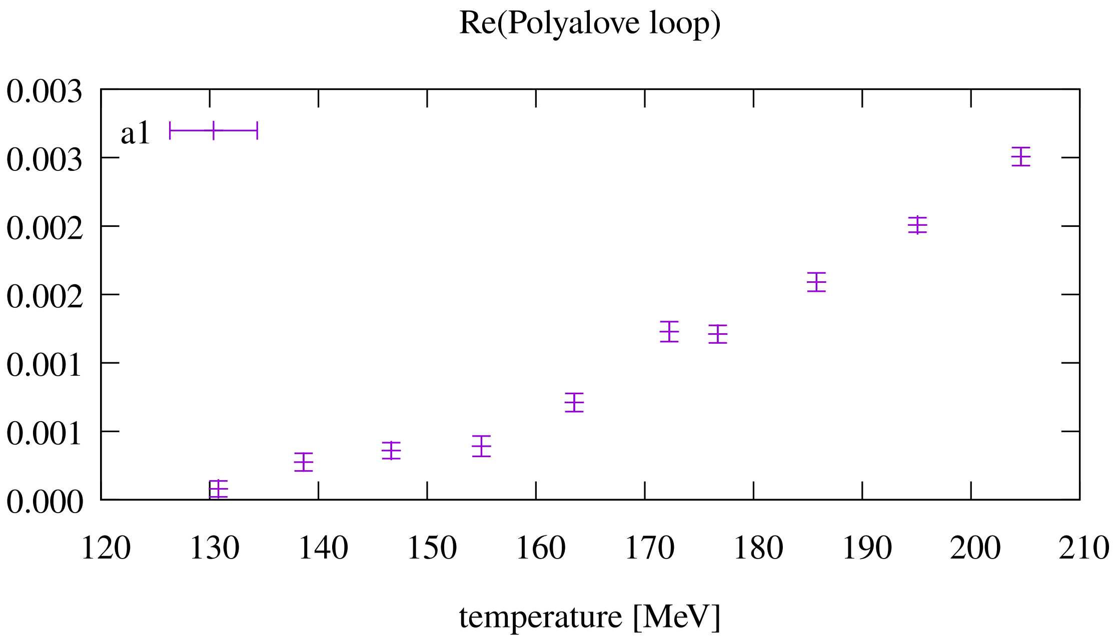


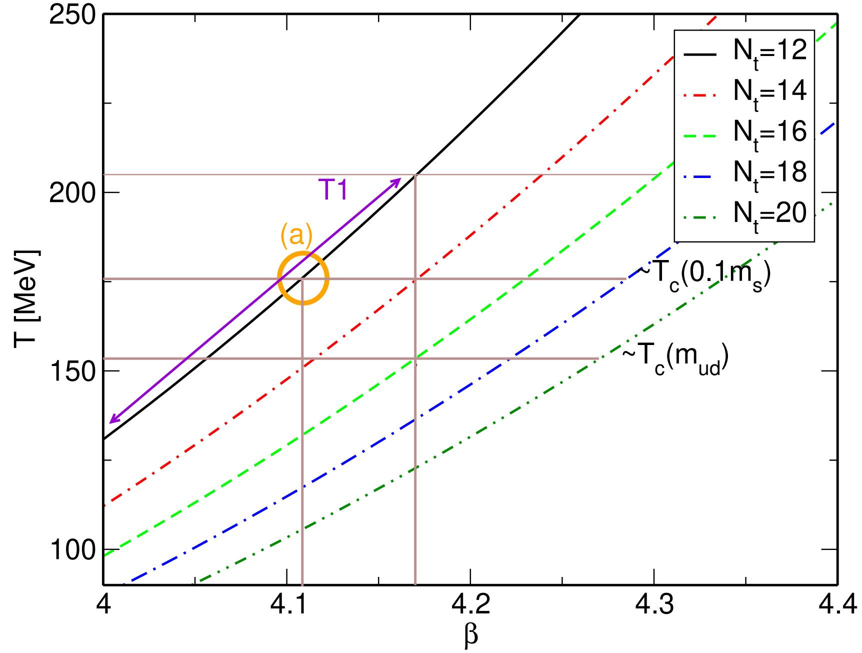



# Initial simulations on LCP

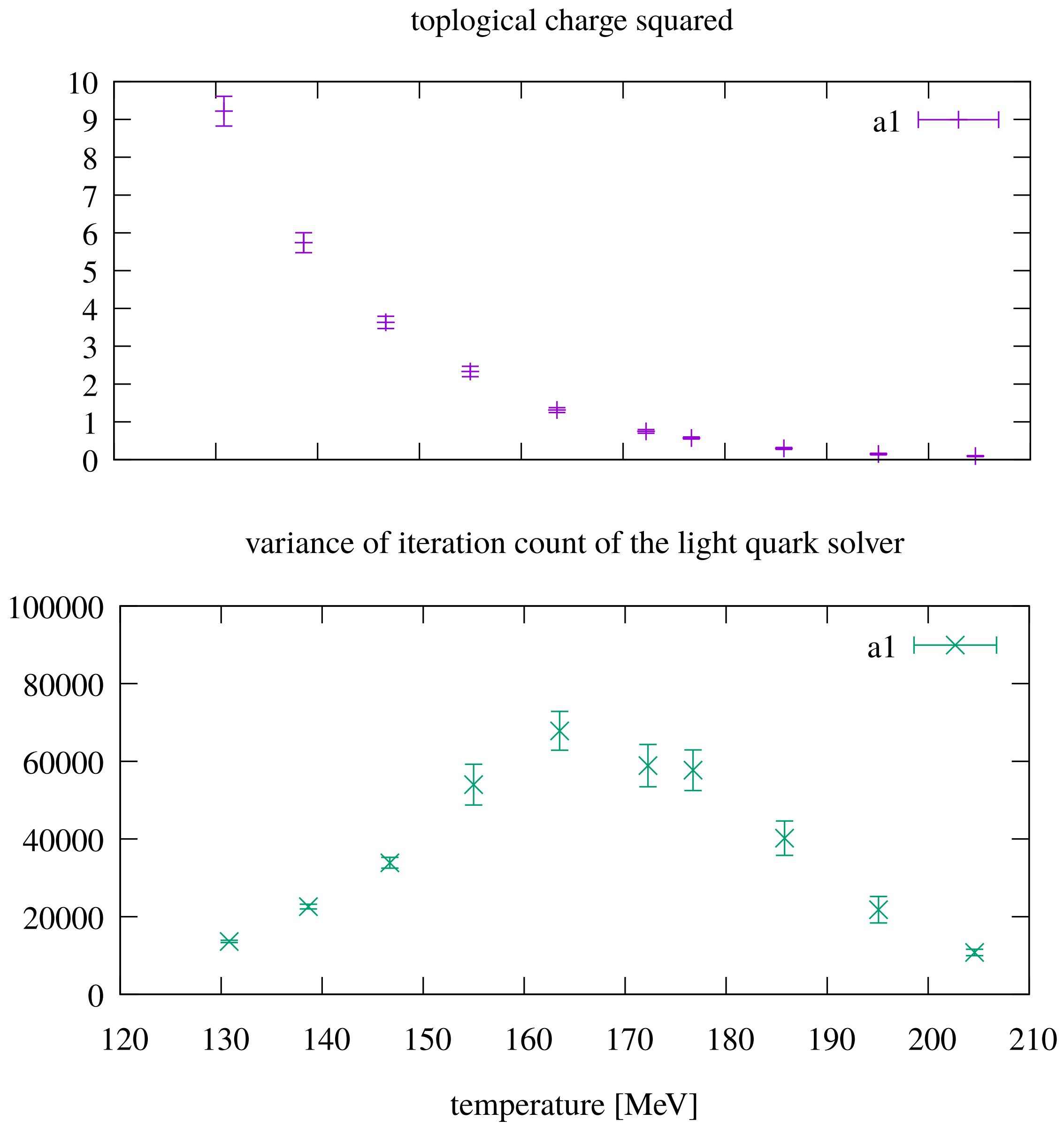


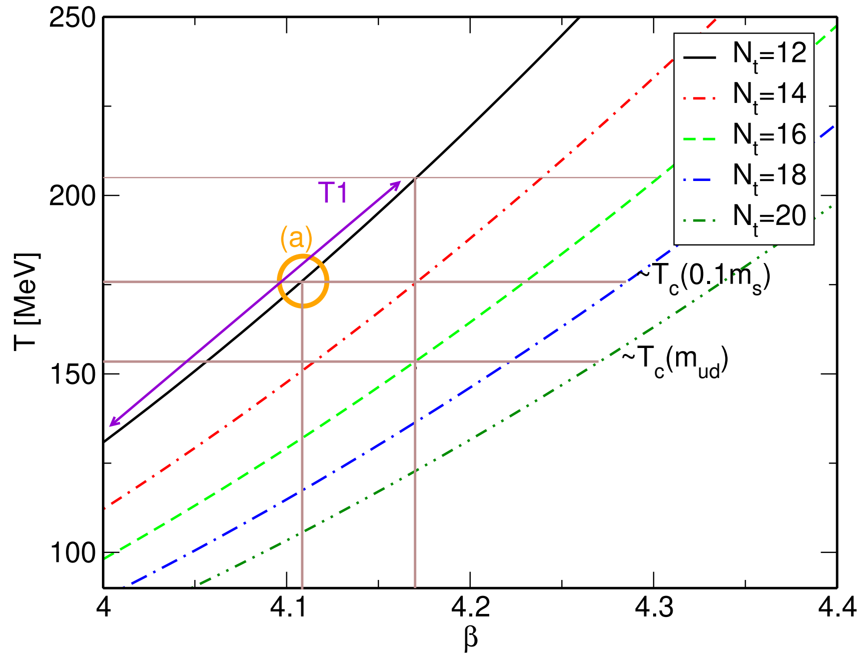




- $N_t = 12$ (T1)
- $m = 0.1 m_s$ (a)
- $N_s = 24, L_s = 12$

# Initial simulations on LCP

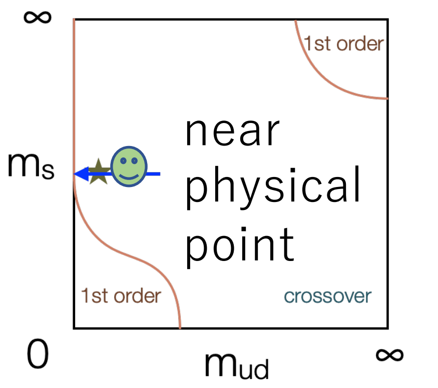


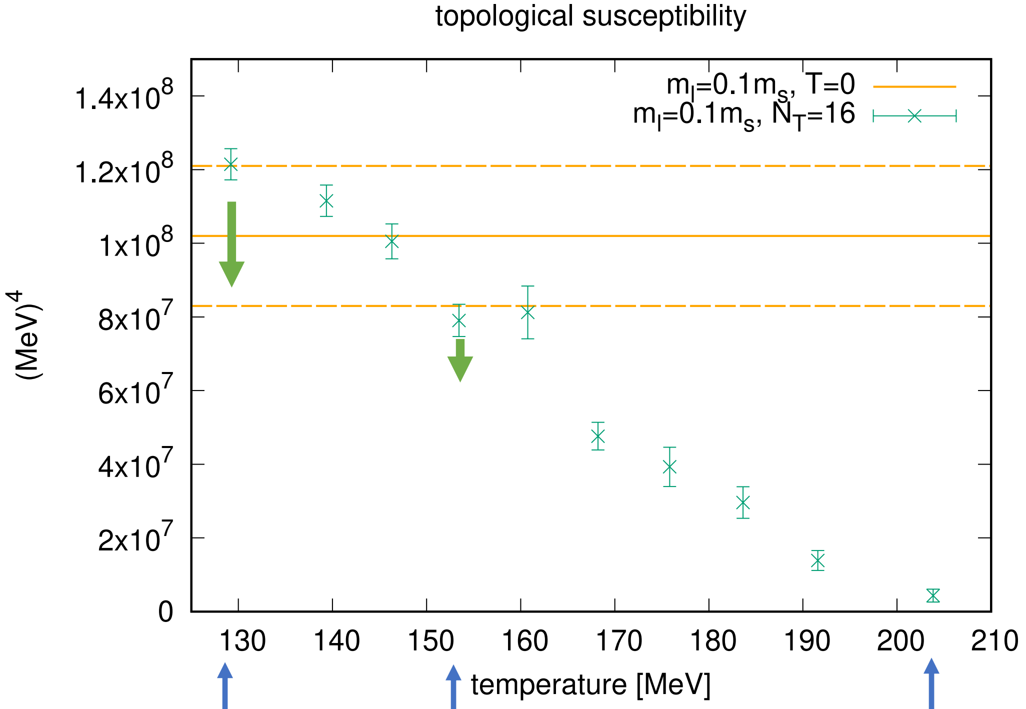




- $N_t = 12$ (T1)
- $m = 0.1m_s$ (a)
- $N_s = 24, L_s = 12$

Topological susceptibility with fully chiral fermions

Topological charge: gluonic definition with Wilson flow

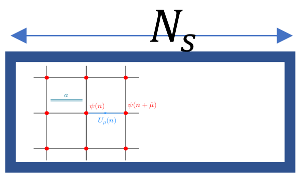


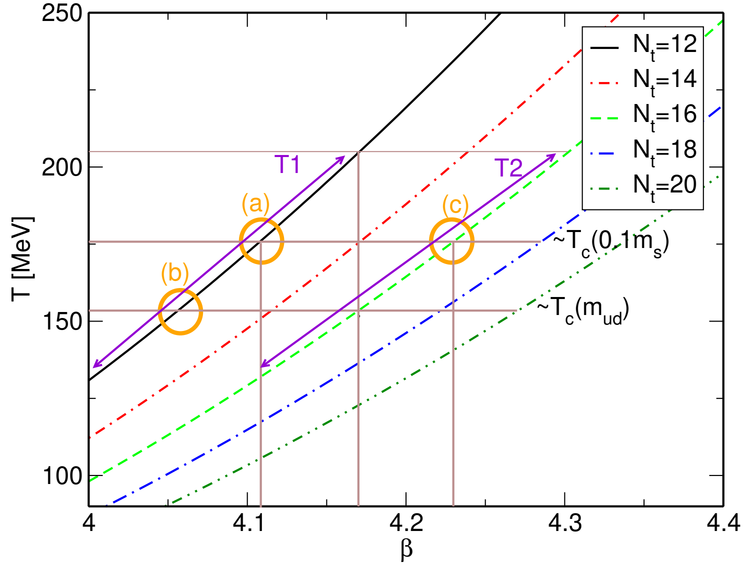

$\frac{m_{res}}{m_l} \simeq 0.26$ 0.011 (0.02)

Not a rigorous LCP: $m_{res}$ neglected

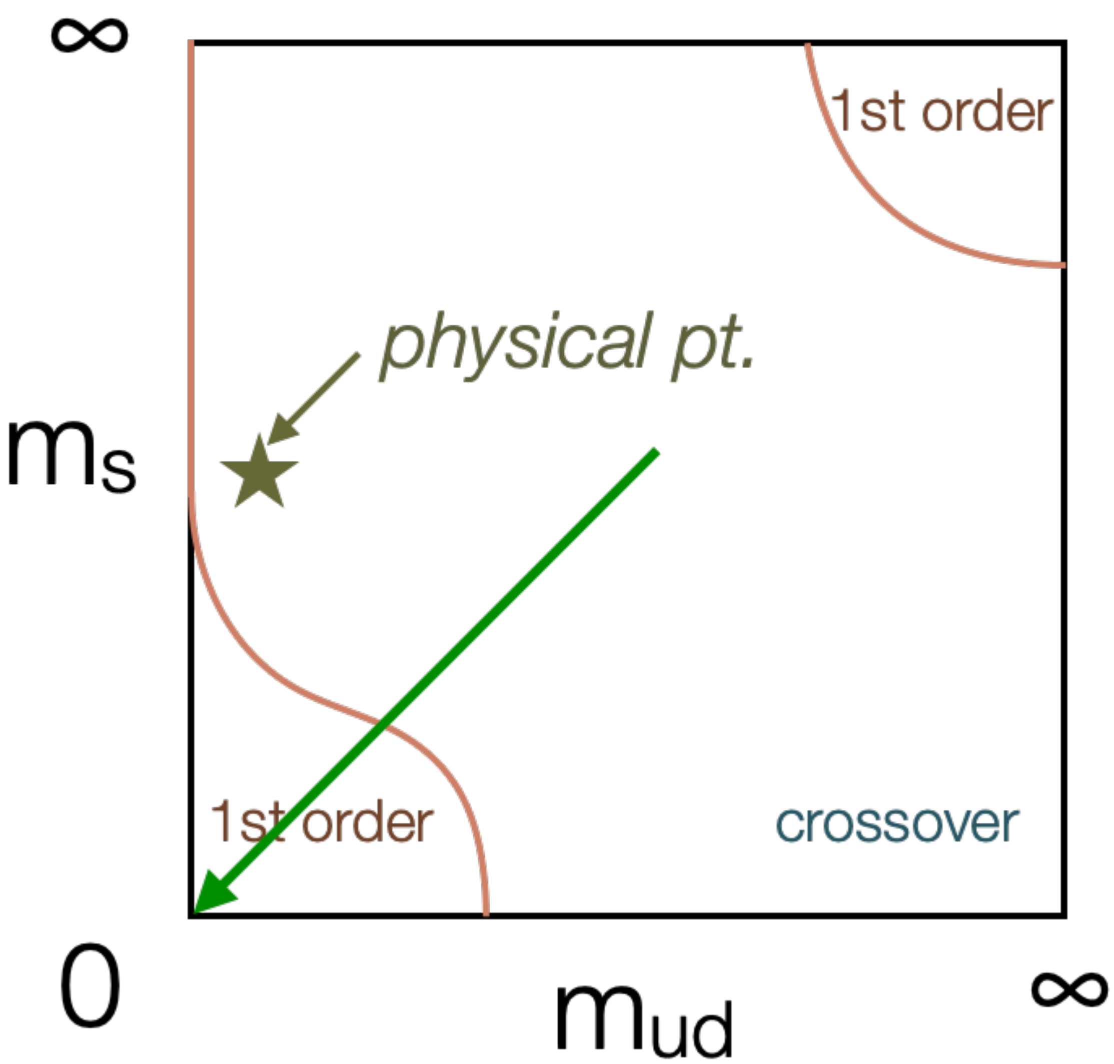
Correction required.



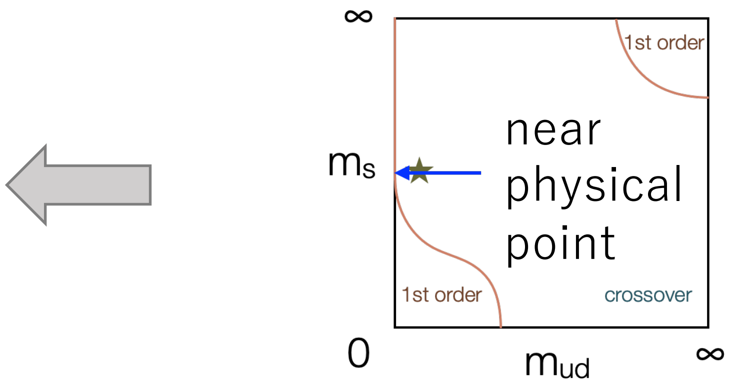

$\frac{1}{T} = aN_t$



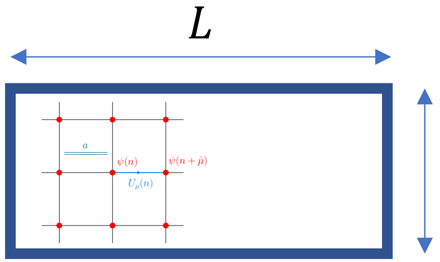

- $N_t = 16$ (T2)
- $m = 0.1m_s$ (c)
- $N_s = 32,\ L_s = 12$



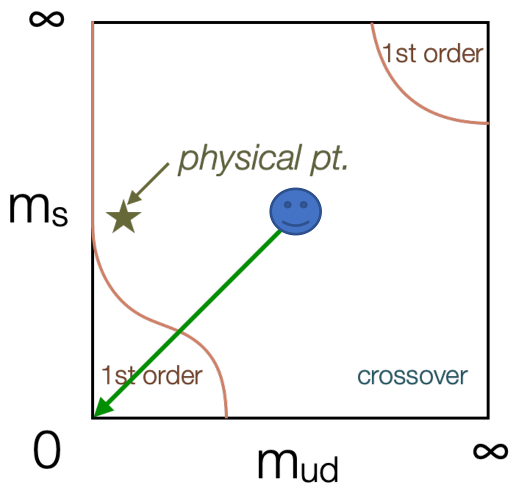
∞
1st order
physical pt.
$m_s$
1st order
crossover
0
$m_{ud}$
∞

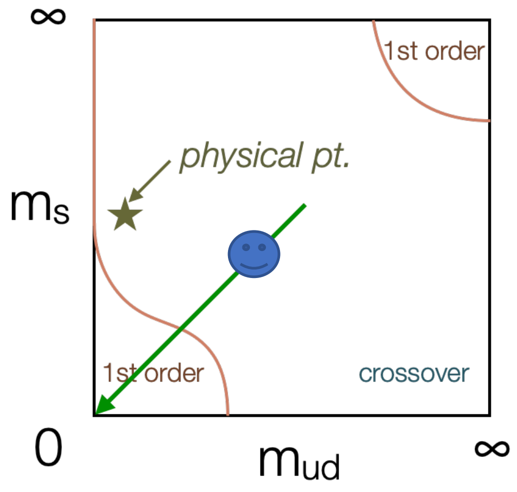



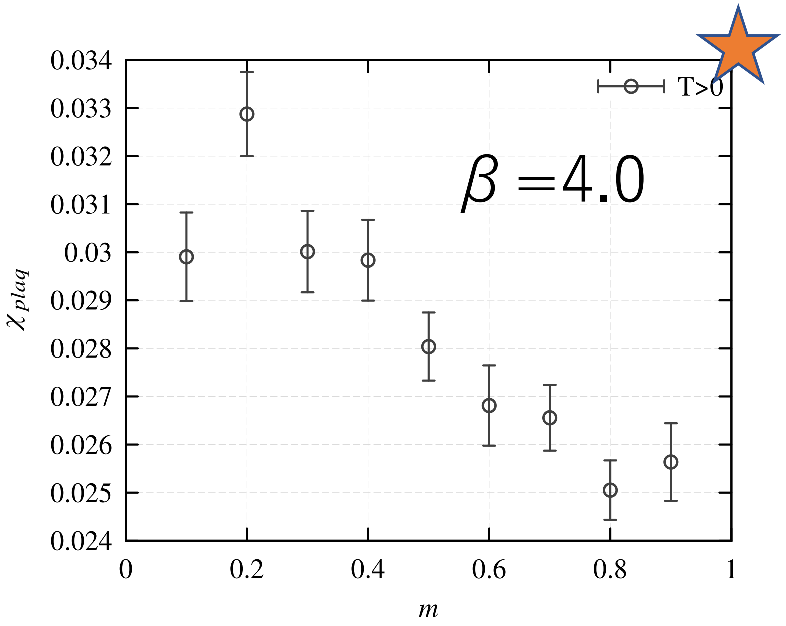
∞
1st order
near
physical
point
$m_s$
1st order
crossover
0
$m_{ud}$
∞



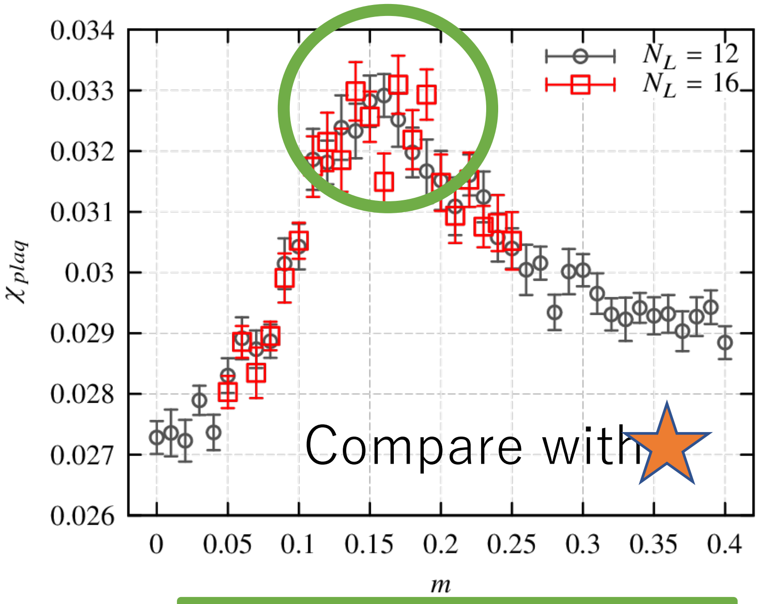

$$\frac{1}{T} = aN_t$$



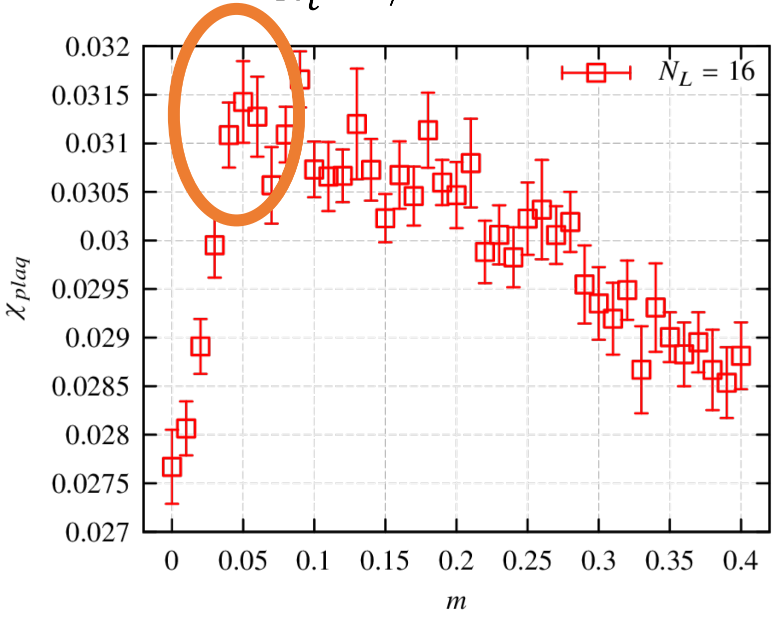




$N_t$=6, T=262 MeV




$N_t$=8, T=196 MeV

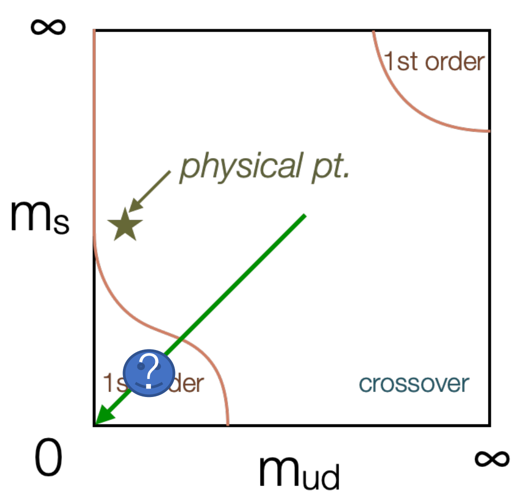



Cross-over established

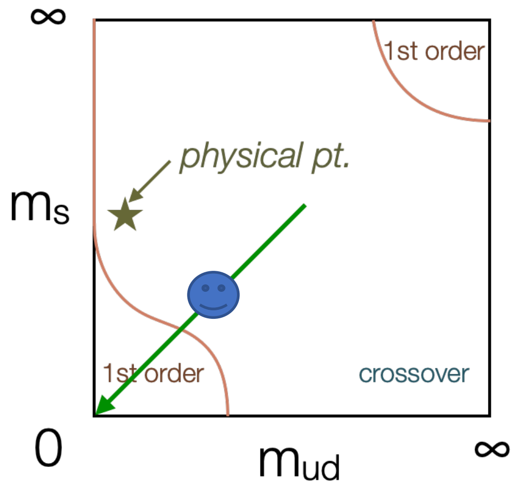
Kyushu Ito (hp200050)

「Fugaku」(hp210032)

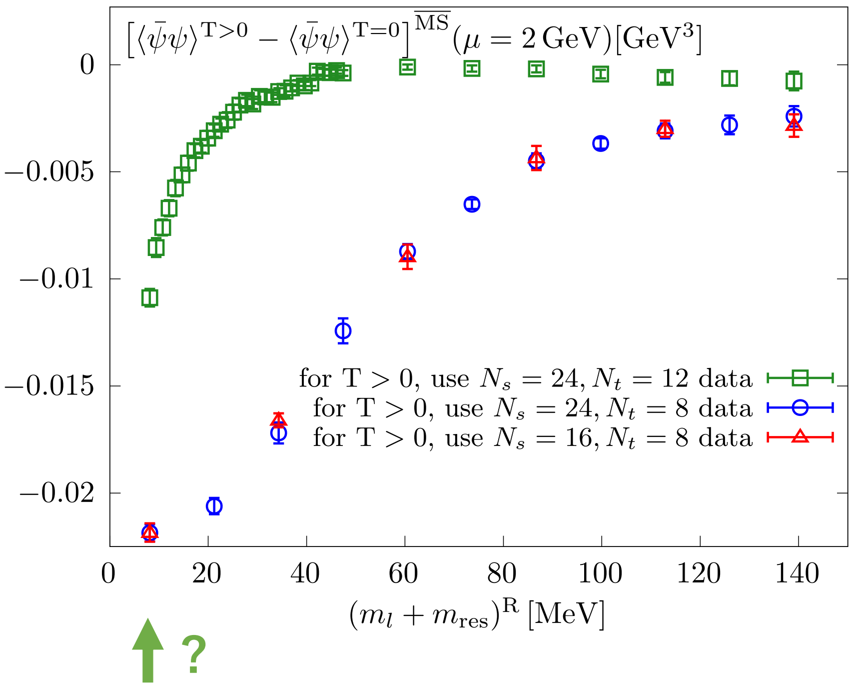
[Y. Nakamura @ JPS 2021 fall]



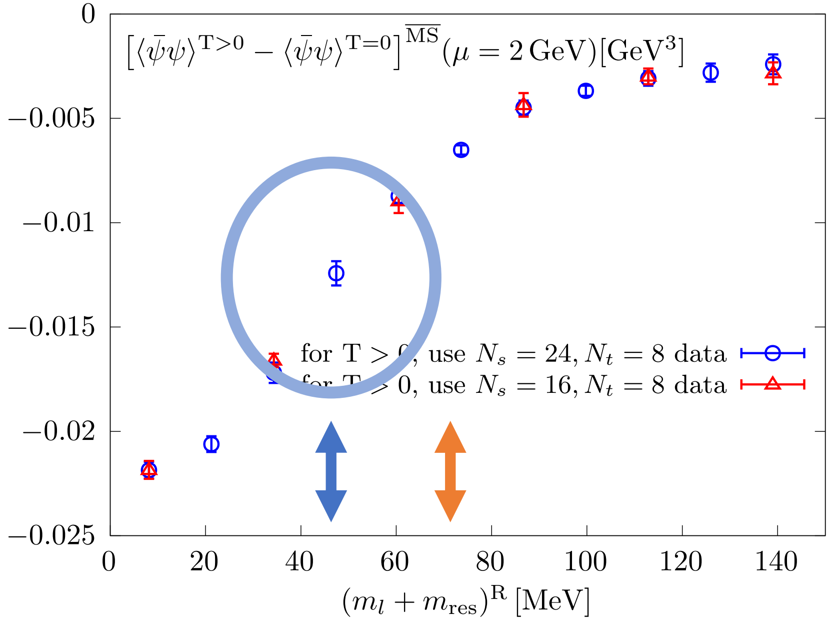

- ✓ Chiral order parameter
- ✓ Physical scale
- ✓ renormalize power divergence
- ✓ renormalize log divergence
- ✓ renormalize quark mass



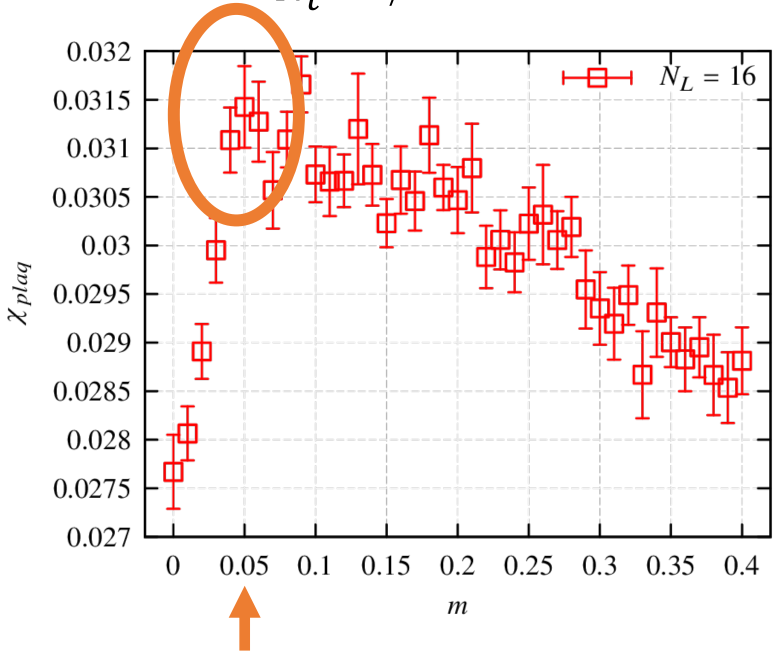

$N_t$=12, T=131 MeV



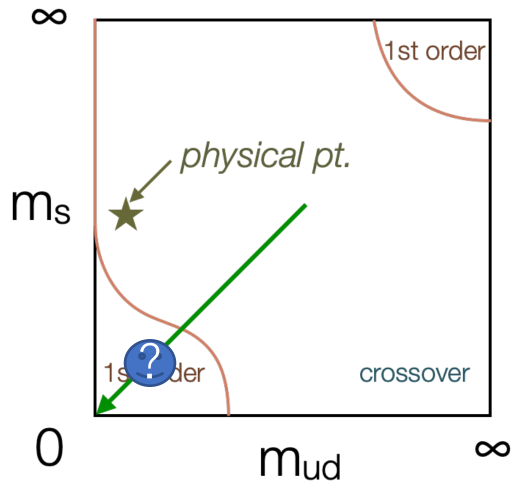

$N_t$=8, T=196 MeV




$N_t$=8, T=196 MeV



# Nf=3 phase structure is a hot topic




- Staggered computations suggest
  - cross over all the way m→0
- Recent HISQ results
  - Bielefeld-Bangalore arXiv/2111.12599
  - $T_c = 98^{+3}_{-6}$ MeV (Nt=8) chiral limit [2nd order]
  - $m_c$=0
- Wilson:
  - Tsukuba-RIKEN-Kanazawa: non-perturbatively O(a) improved
  - up to Nt=12: 1st order region exists
  - Continuum limit: only upper limit of $m_c$ ($m_\pi \lesssim$110 MeV) determined
  - Continuum limit: $T_E$ =134(3) MeV
- Our Möbius DWF:
  - Nt=12, $T$ =131 MeV, $m_c \simeq 2m_{ud} \rightarrow m_\pi = 190\ MeV$
  - Crossover / order of the transition yet to be determined.
- Tensions exist among different approaches

# Summary and outlook

- Summary
  - Nf=2+1
    - Möbius DWF simulation for T>0 with $N_t$=12, 16
      ↔ $N_t$=8 by HotQCD (2012)
    - Along the Line of Constant Physics
    - First simulations with $m = 0.1\, m_s$, $N_s/N_t$=2
      - Underway using Fugaku
    - some results begin to unfold
    - mres effects sizable for some quantities, especially topological susceptibility
  - Nf=3
    - Möbius DWF simulation at $\beta = 4.0$ ($a \simeq 0.12$ fm), T=131, 196, 262 MeV
    - Yet to resolve the tension between Wilson and staggered results
- Outlook
  - Nf=2+1
    - LCP: mres correction needed
    - Get closer to physical $m_{ud}$
    - Larger volume
  - Nf=3
    - Get closer to chiral limit and lower T

# Nf=2+1 Simulation plan

- T1-(a)
  - $N_t = 12$
  - $m = 0.1m_s$
  - $N_s = 24, L_s = 12$
  - Almost done
- T2-(c)
  - $N_t = 16$
  - $m = 0.1m_s$
  - $N_s = 32, L_s = 12$
  - Still underway
- T1-(b)
  - $N_t = 12$
  - $m \simeq m_{ud}$
  - $N_s = 24, L_s = 12$
  - Mass tuning is necessary
    - $m_{res} \simeq m_{ud}$