ℓN inclusive scattering cross section

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Neutrinos

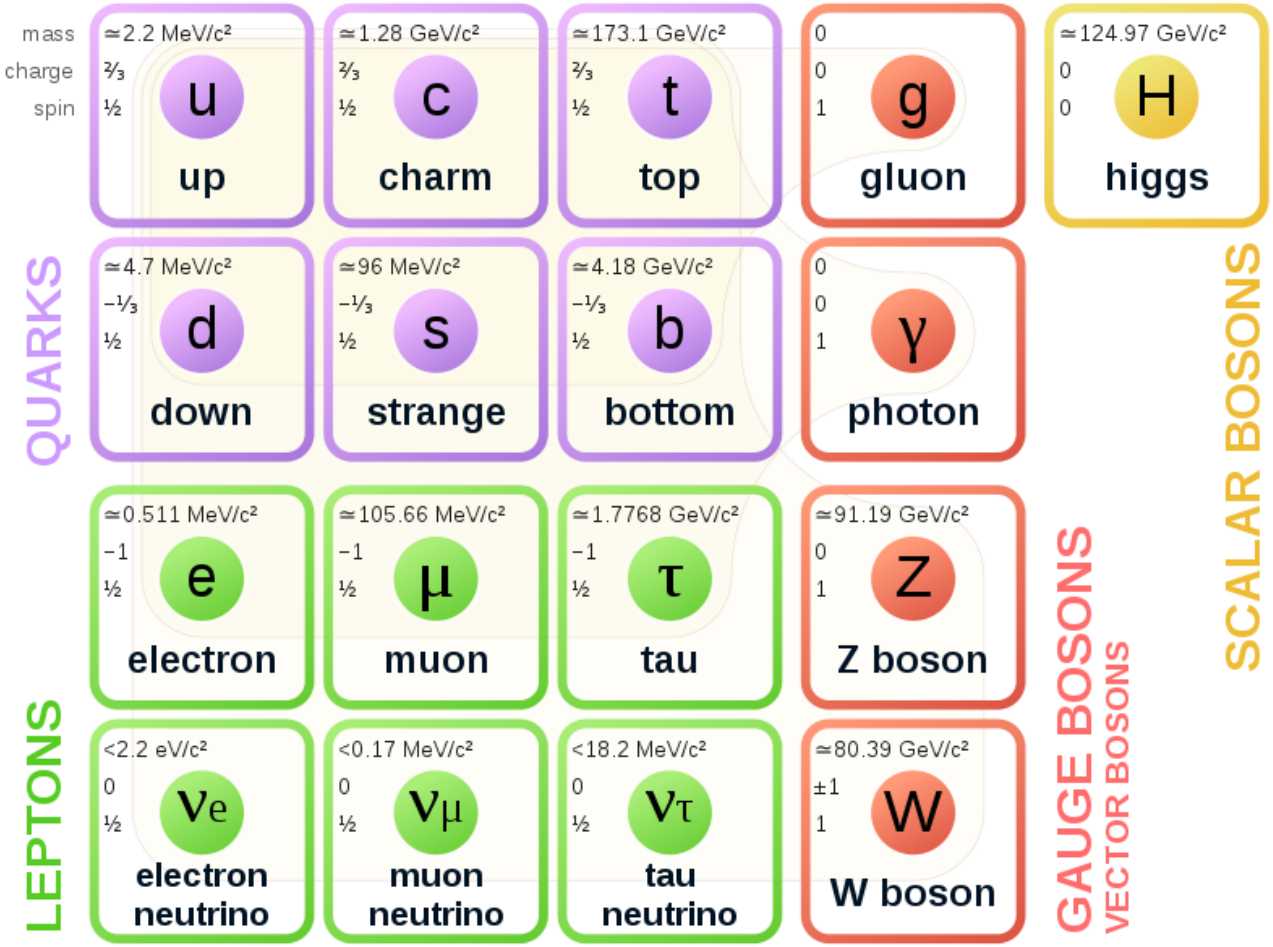


Figure: SM Elementary particles

Neutrino ν

- has tiny mass
- oscillates in flavor space
- Majorana or Dirac

Neutrino oscillation experiments

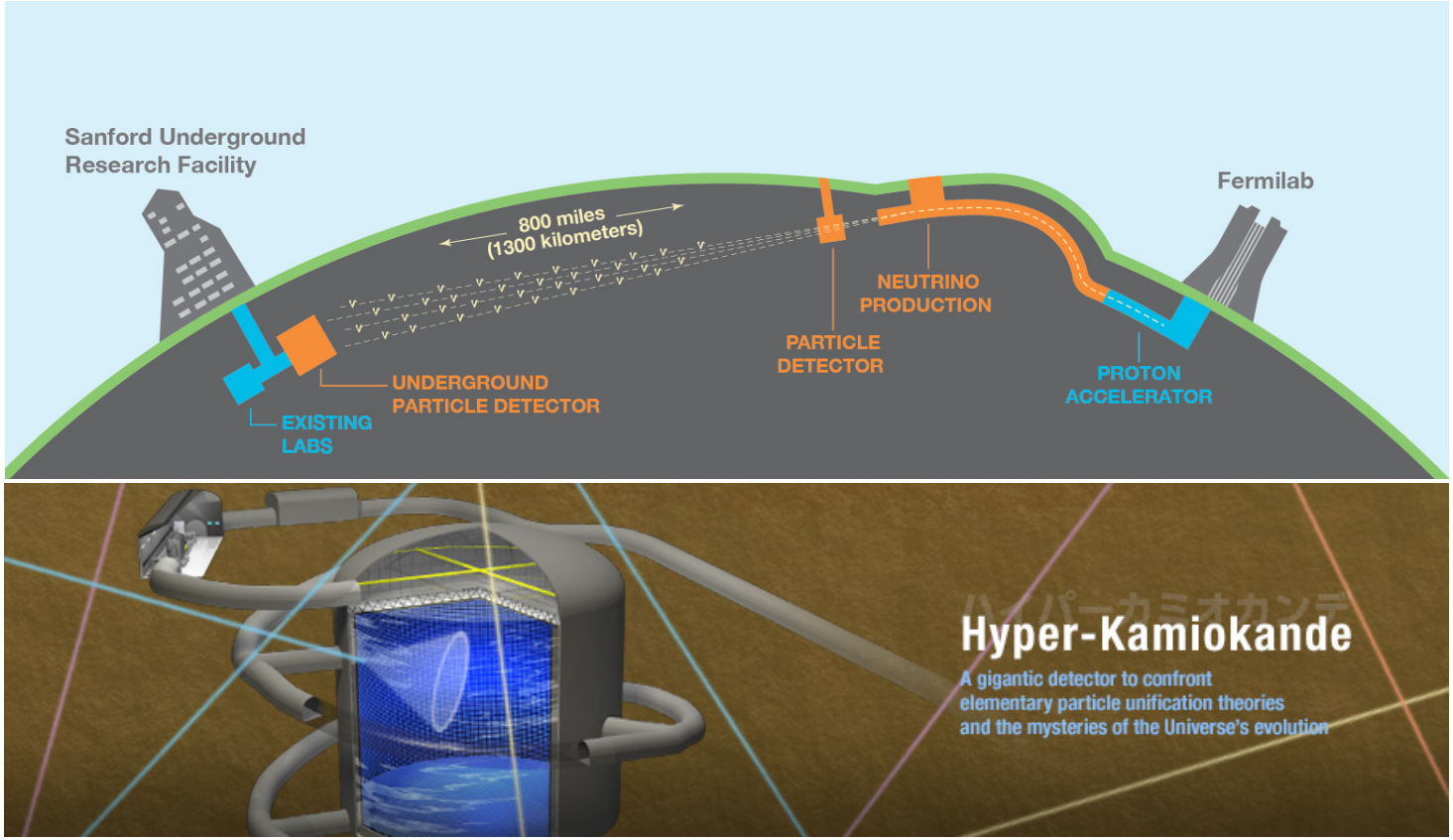


Figure: (Top) DUNE experiments (Bottom) Hyper Kamiokande

- Neutrino-Nucleus cross section is needed!

Neutrino oscillation experiments

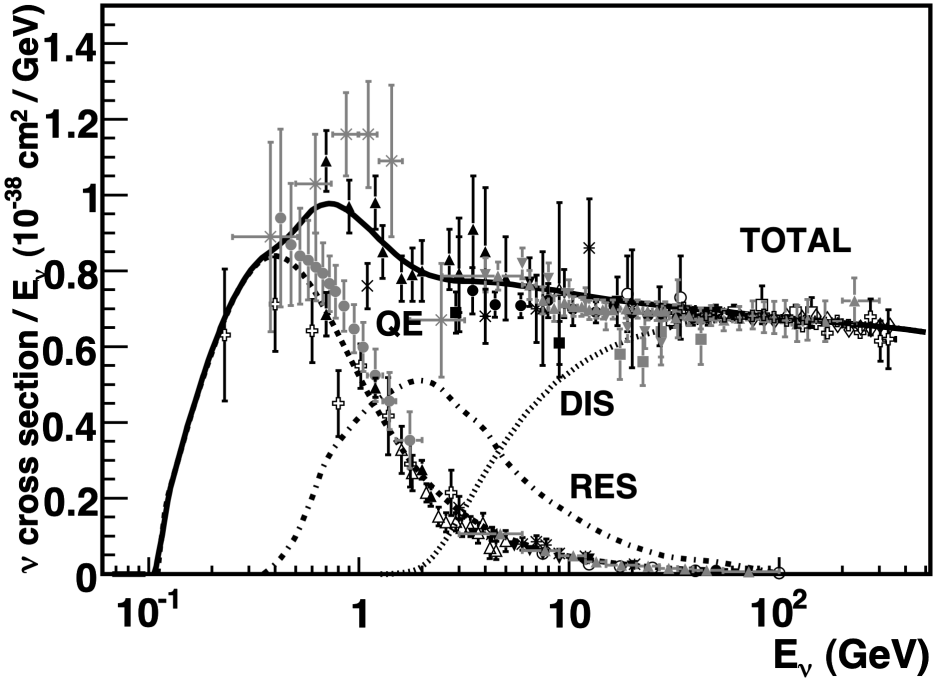
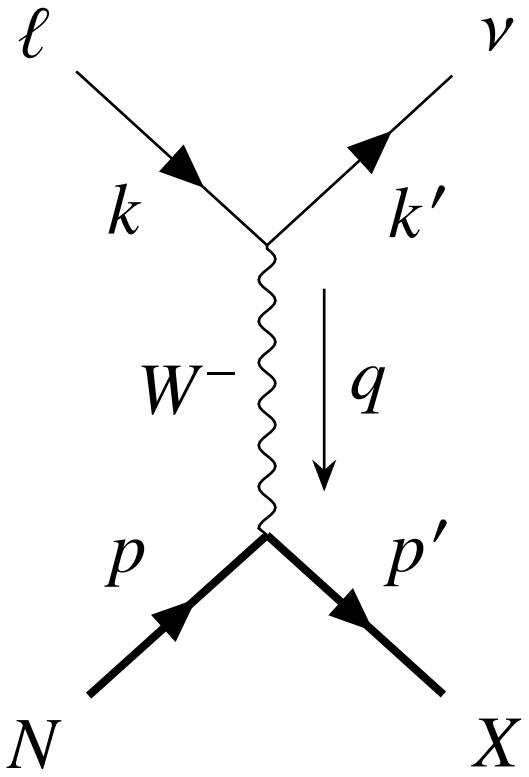


Figure: Neutrino cross sections across energy scales [4]

- Deep-Inelastic (DIS) $\langle N|J_\mu J_\nu|N\rangle$
- Resonance Region (RES) $\langle \text{res.}|J_\mu J_\nu|N\rangle$
- Quasi-Elastic (QE) $\langle N|J_\mu|N\rangle$

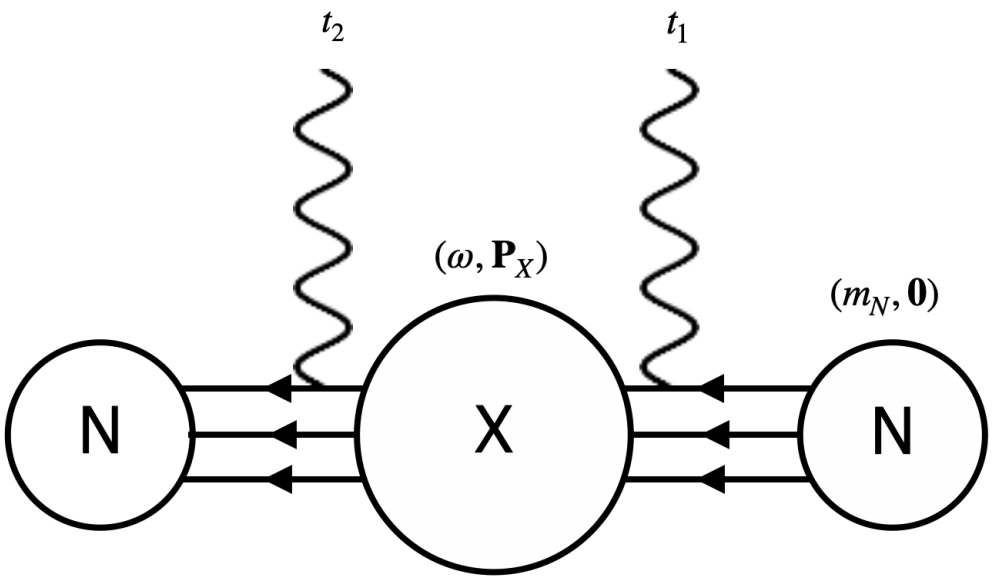
USQCD white paper [5]

Neutrino cross section



- Vector FF, Axial FF (Numerous lattice Groups)
- Hadronic Tensor (χQCD , QCDSF)
- Total Cross sections

Neutrino cross section



$$\rho(\omega, \mathbf{P}_X) = \sum_{X(\mathbf{P}_X)} \delta(\omega - E_{X(\mathbf{P}_X)}) \, |\langle X(\mathbf{P}_X) | J | N \rangle|^2$$

$$\sigma = \int d^3\mathbf{q} \int_0^\infty d\omega K(\omega; \boldsymbol{q}) \rho(\omega; \boldsymbol{q})$$

Lattice Setup

Lattice Params

$L_x^3 \times L_t$	gauge	fermion	a^{-1}	β	am_f
$16^3 \times 32$	Iwasaki	2+1-flavor DWF	1.73(3) GeV	2.13	0.01
L_s(Zmobius)	m_π	$m_\pi L$	Z_A [1]	$N_{\rm cfg}$	$N_{\rm exact}$
32(10)	420(10) MeV	3.69	0.7162(2)	140	1

Table: Lattice ensemble parameters. Ensemble is generated by RBC/UKQCD [2].

Total Cross section

The final Goal $\sigma = \int d^3\mathbf{q} \int_{\sqrt{m_N^2+\boldsymbol{q}^2}}^{m_N+|\boldsymbol{q}|} d\omega K(\omega;\boldsymbol{q})\rho(\omega;\boldsymbol{q})$

What we have	$C_{\mu\nu}(t;\boldsymbol{q}) = \left\langle N(\mathbf{0}) \middle\vert \tilde{J}_\mu^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}_\nu(\boldsymbol{q}) \middle\vert N(\mathbf{0}) \right\rangle$
	$= \int_0^\infty d\omega \rho_{\mu\nu}(\omega;\boldsymbol{q})e^{-\omega t}$
What we learned	$\rho_{\mu\nu} = \left\langle N(\mathbf{0}) \middle\vert \tilde{J}_\mu^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}_\nu(\boldsymbol{q}) \middle\vert N(\mathbf{0}) \right\rangle$
What we want to have	$\int_0^\infty d\omega K(\omega;\boldsymbol{q})\rho(\omega;\boldsymbol{q}) = \left\langle N(\mathbf{0}) \middle\vert \tilde{J}_\mu^\dagger(-\boldsymbol{q})K(\hat{H};\boldsymbol{q})\tilde{J}_\nu(\boldsymbol{q}) \middle\vert N(\mathbf{0}) \right\rangle$

$\Rightarrow$ Approximate $K(\omega;\boldsymbol{q})$!

$$K(\hat{H};\boldsymbol{q}) \simeq k_0(\boldsymbol{q}) + k_1(\boldsymbol{q})e^{-\hat{H}} + k_2(\boldsymbol{q})e^{-2\hat{H}} + \cdots + k_N(\boldsymbol{q})e^{-N\hat{H}},$$

*Chebyshev approximation $\quad \bar{K}^I(\omega) \simeq \frac{c_0^*(\mathbf{q})}{2} + \sum_{j=1}^N c_j^*(\mathbf{q})T_j^*(z)$

Kernel function

Total Cross section formula

$$\frac{d^2\sigma}{dQ^2 d\omega} = \frac{G_F^2 \omega}{4\pi E^2 m_N} L_{\mu\nu} W^{\mu\nu}$$

Kernel function $K_{\mu\nu} = \frac{G_F^2 \omega}{4\pi E^2 m_N} L_{\mu\nu}$

Leptonic Tensor $L_{\mu\nu} = 2\left[k'_\mu k_\nu + k_\mu k'_\nu - k' \cdot k g_{\mu\nu} + i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta\right]$

Approximation for each terms with different dependencies on ω.

$$\begin{aligned} K^{\mu\nu}(\omega, \mathbf{q}) &= \frac{G_F^2 \omega}{4\pi E^2 m_N} L_{\mu\nu}(E, \mathbf{q}) \\ &= C^2_{\mu\nu}(\omega) \bar{K}^{l=2}(\omega) \cdot \omega^2 + C^1_{\mu\nu}(\omega) \bar{K}^{l=1}(\omega) \cdot \omega + C^0_{\mu\nu}(\omega) \bar{K}^{l=0}(\omega) \end{aligned}$$

Smoothing kernel function

$$K^l(\omega) = e^{2\omega t_0}\omega^l \times \theta_\sigma\left(m_N + |\boldsymbol{q}| - \omega\right)$$

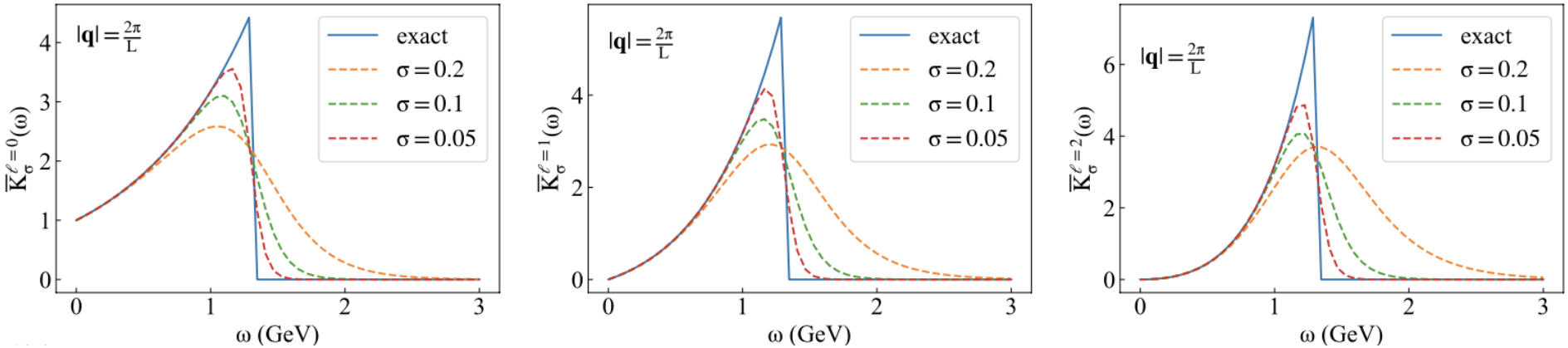


Figure: $K^l(\omega; \mathbf{q})$ is smoothed with the width σ.

Need $\sigma = .05$ to be accurate enough.

Chebyshev approximation of smoothed kernel function

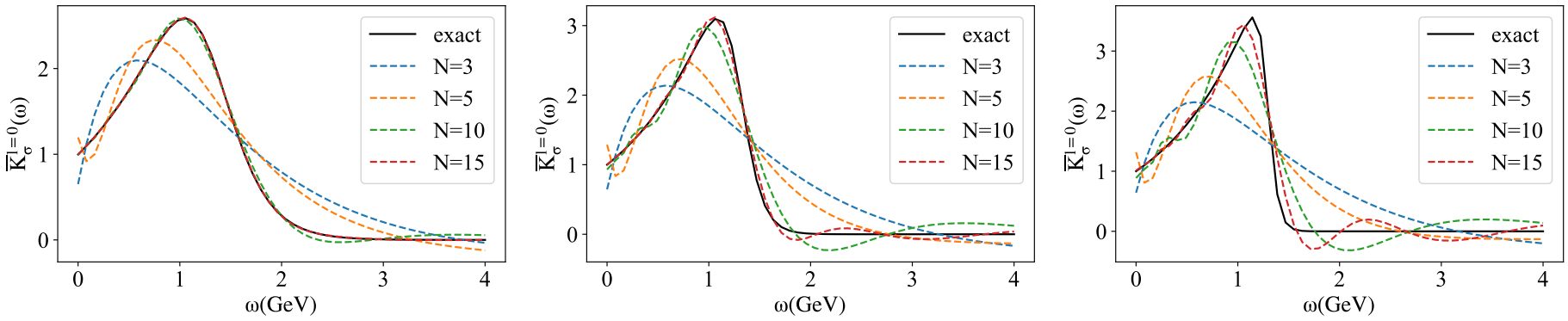


Figure: Chebyshev approximation of $\sigma = 0.2$ smoothed kernel function to order N.

Need $N = 15$ to be almost exact.
Need $N = 10$ to control the systematic error.
$N = 3$ choice, the approximation suffers 20% systematic uncert. with model spectral function.

Systematic uncertainty from Chebyshev approximation

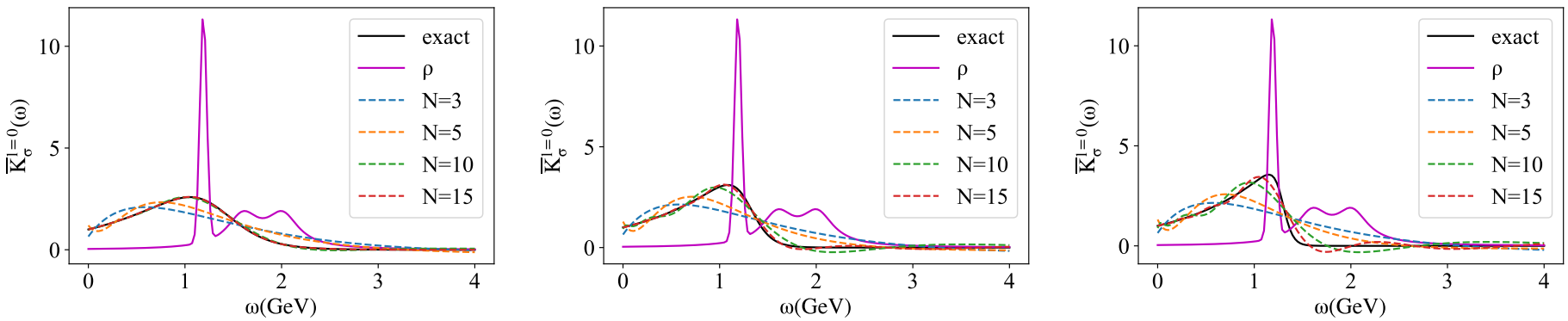


Figure: Example spectral function

For larger transfer momentum, the systematic error reduces as the sigmoid function shifts to the right.

Chebyshev approximation of smoothed kernel function

Model spectral function

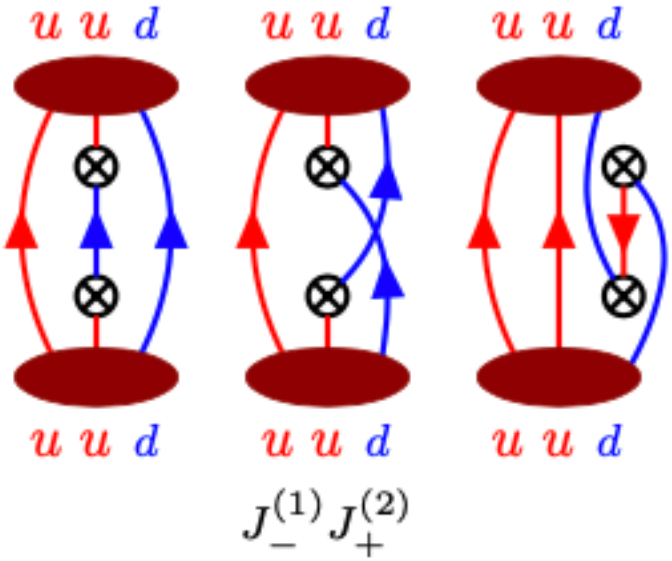
$$\rho(\omega) = \frac{1}{\sigma_1\sqrt{\pi}} e^{-\left(\frac{\omega - E_N}{\sigma_1}\right)^2} + \frac{1}{\pi\sigma_2}\left(\frac{1}{\left[1 + (\omega - m_N - m_\pi)^2/\sigma_2^2\right]} + \frac{1}{\left[1 + (\omega - m_N - 2m_\pi)^2/\sigma_2^2\right]}\right),$$

where $\sigma_1 = .05, \sigma_2 = .2$

Table: Systematic Uncertainty (Relative uncert.)

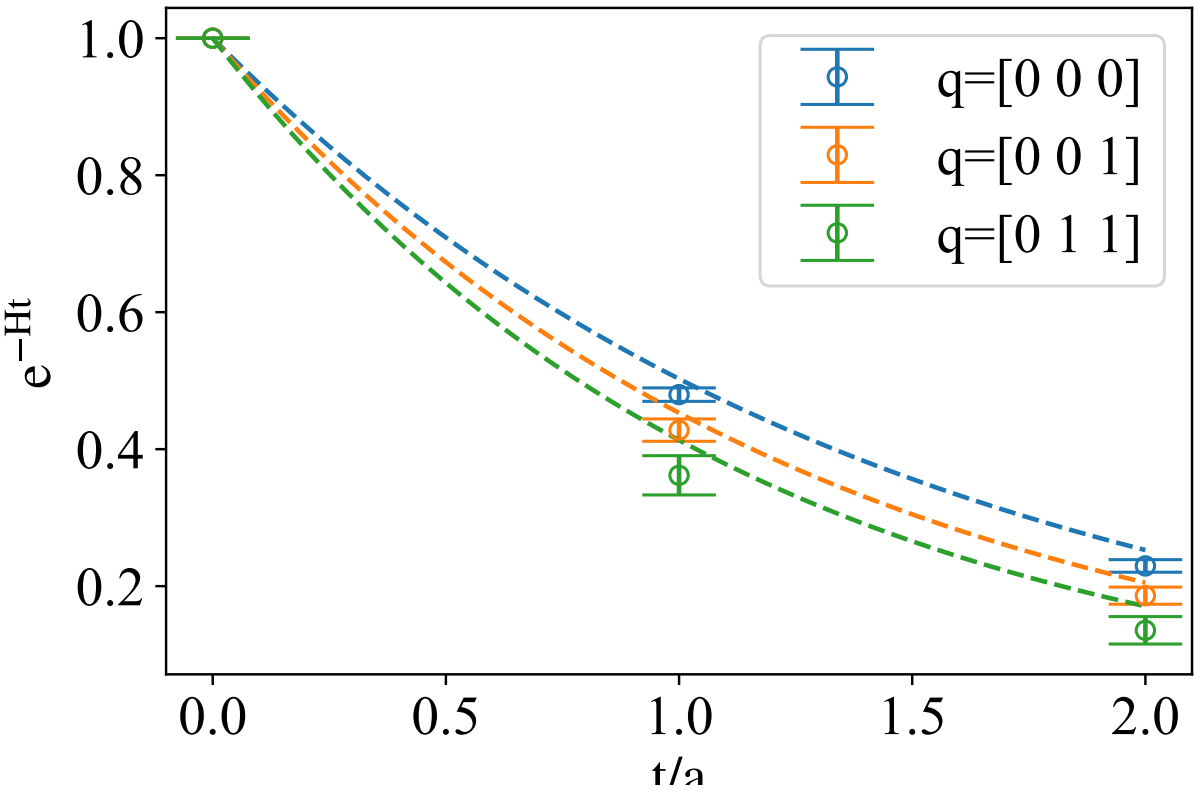
	[0 0 0]	[0 0 1]	[0 1 1]
N=3	17.8%	44 %	34 %
N=5	22 %	7 %	8 %
N=10	22 %	6 %	1.8%
N=15	13%	1.4%	6.4%

Contraction



- Source-sink separation = 8
- First current inserted in t = 2
- current = V, A (+,-,U,D)
- **p** = [000], **q** = [000], [001], [011]

Time evolution with $e^{-\hat{H}t}$



$$\frac{C_{\mu\nu}(t+2t_0)}{C_{\mu\nu}(2t_0)} = \frac{\left\langle \psi_\mu(\mathbf{q}) \left| e^{-\hat{H}t} \right| \psi_\nu(\mathbf{q}) \right\rangle}{\left\langle \psi_\mu(\mathbf{q}) \mid \psi_\nu(\mathbf{q}) \right\rangle} = \left\langle (e^{-\hat{H}})^t \right\rangle$$

Comparison with experiments

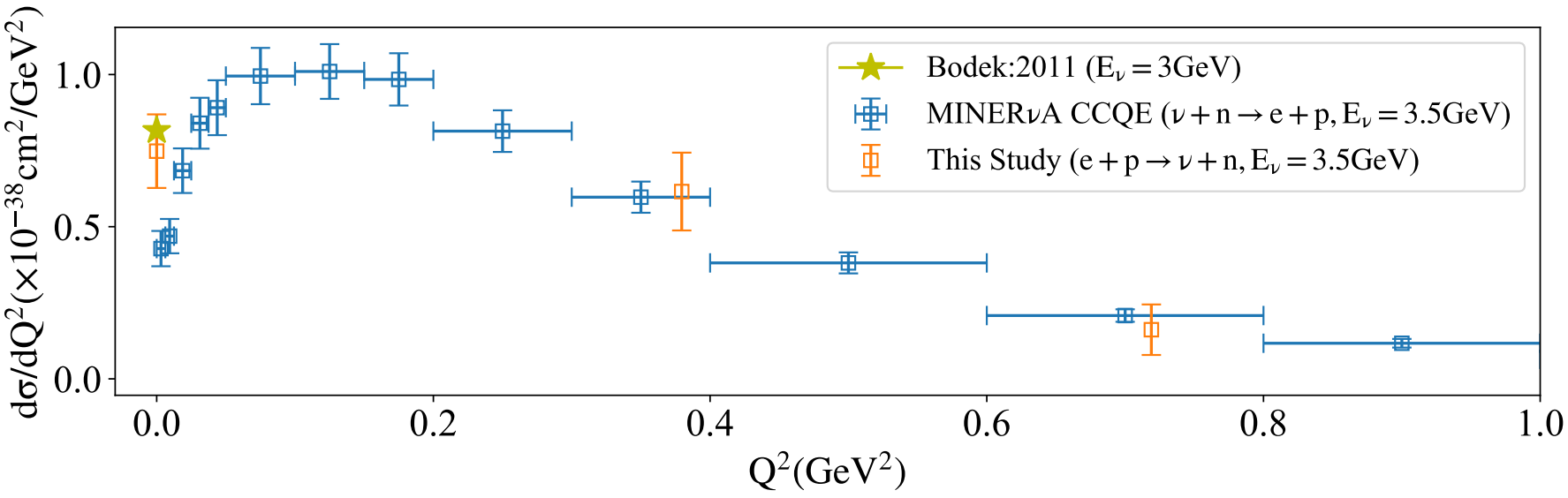


Figure: Comparison of differential cross section with experimental result from MINERνA [6] and model calculation by Bodek [3].

- Systematic uncertainty is not incorporated in the plot
- resonance region appears further away

Possible Extensions

- Higher precision
  Can access higher order Chebyshev using larger temporal axis lattice ensemble
- Resonance Region (RES)
  Use a higher momenta giving $Q^2 > 1\text{GeV}^2 \rightarrow 1\pi$ production
- β decay loop contribution (γW box)

$$\square_{\gamma W}^{VA} = \frac{3\alpha}{2\pi} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} M_3^{(0)} \left(1, Q^2\right)$$

- NC process
  Can access the neutral current processes

Conclusion

- Computed CC process differential cross section on $16^3 \times 32$ lattice
- Small momenta transfer range $Q^2 = 0., 0.37, 0.78\text{GeV}^2$
- Heavy nucleon (~ 1.2 GeV)
- Needs further investigation with deeper momenta transfer
- A few possible extensions exist

References

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