Neutrino Physics from LQCD with Domain Wall Fermions

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DWQ @ 25

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California Lattice Collaboration

Outline

- Introduction

  Neutrino Oscillation, QE Form Factors

- Lattice QCD Calculation

  $\mathring{g}_A(Q^2)$ summary, computation, implications

- Conclusions

Introduction

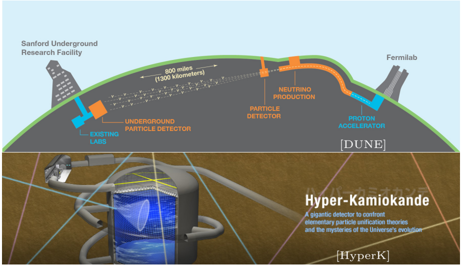
Neutrino Oscillation Expt

Next-gen ν oscillation effort

O($1b) high-precision expts

Measurements —

δ_{CP}	:	CP violation
$\mathrm{sign}[\Delta m^2_{13}]$	:	mass hierarchy



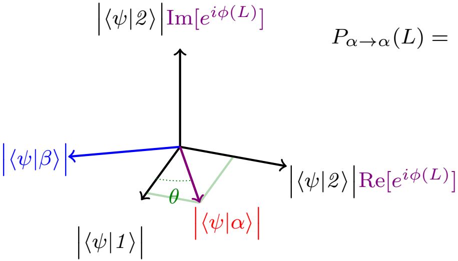
δ_{CP} — last missing parameter in PMNS matrix

$\Longrightarrow$ matter-antimatter asymm; 3-flavor unitarity (sterile ν)

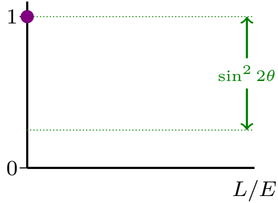
mass hierarchy — $m_3 > m_{1,2}$?

$\Longrightarrow$ appx symmetry $m_1 \sim m_2$? ; lower limit $0\nu\beta\beta$ decay rate ?

Two Flavor Neutrino Oscillation



$$P_{\alpha\to\alpha}(L) = \left|\langle\alpha|\psi_L\rangle\right|^2$$



creation $\rightarrow$ flavor eigenstate (α,β) propagation $\rightarrow$ *mass eigenstate* (*1*,*2*)

$$|\psi_L\rangle = e^{-im_1^2 L/4E}\Big(\cos\theta|1\rangle + e^{-i\phi(L)}\sin\theta|2\rangle\Big), \quad |\psi_0\rangle = |\alpha\rangle$$

$$\phi(L) = \Delta m^2(L/4E), \quad \Delta m^2 \equiv m_2^2 - m_1^2$$

"Survival probability": $\left|\langle\alpha|\psi_L\rangle\right|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\phi(L)}{2}$

"Appearance probability": $\left|\langle\beta|\psi_L\rangle\right|^2 = \quad + \sin^2 2\theta \sin^2 \frac{\phi(L)}{2} = 1 - \left|\langle\alpha|\psi_L\rangle\right|^2$

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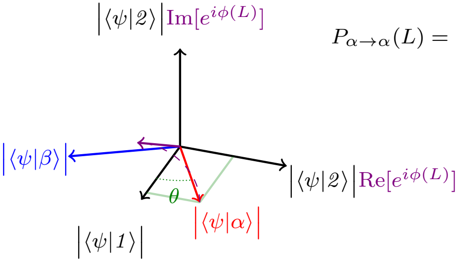

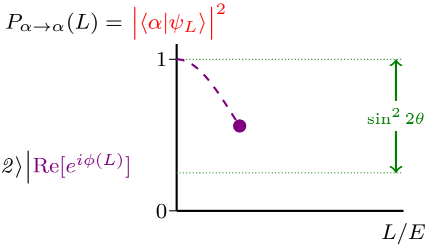

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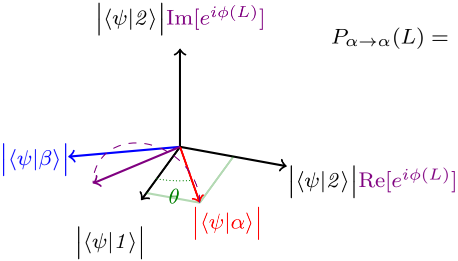

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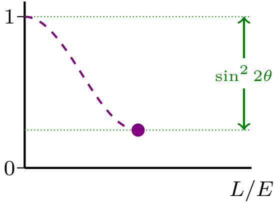

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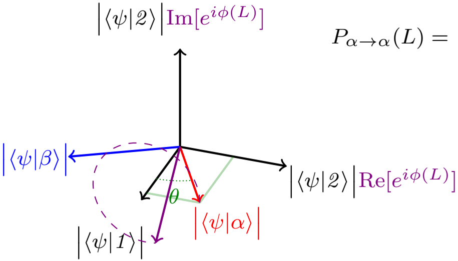

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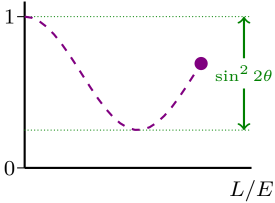

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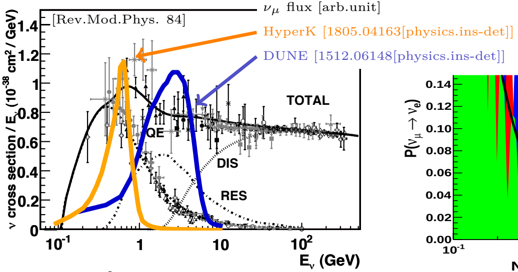
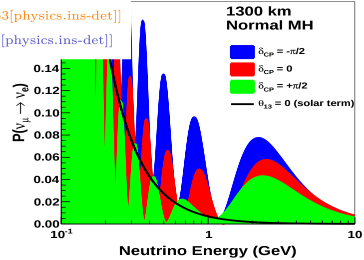
Near/Far Detector





$$N_{\text{event}} \sim \int dE_\nu \underbrace{\Phi}_{\text{flux}} \otimes \underbrace{\sigma}_{\text{xsec}} \left(\otimes \underbrace{\epsilon}_{\text{eff}}\right)$$

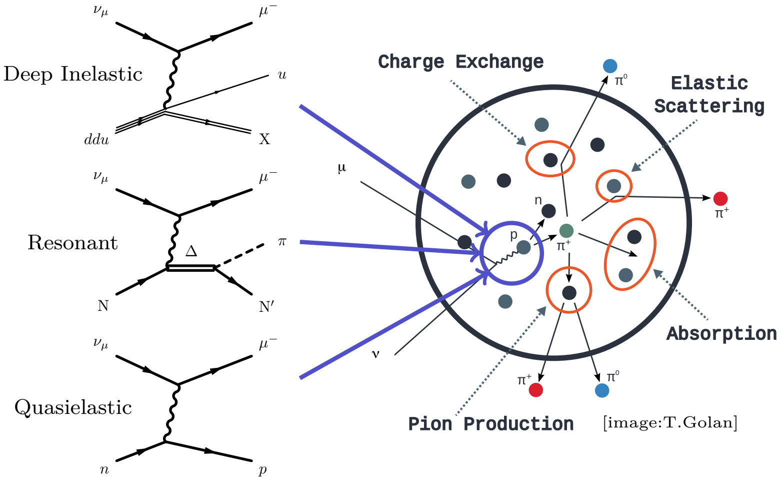
$P_{\text{disapp}}(E_\nu) \sim N_{\text{far}}/N_{\text{near}}$ fit to oscillation model

- ν beam not monoenergetic $\Longrightarrow$ $\Phi(E_\nu) \neq \delta(E_\nu - E_{\text{beam}})$
- E_ν not known event-by-event $\Longrightarrow$ reconstruct E_ν from measurement
- beam dynamics $\Longrightarrow$ different efficiency @ near, far

High precision expt needs robust, precise cross sections

Neutrino Interactions in Nuclear System

Many interaction topologies: primary & intranuclear rescattering



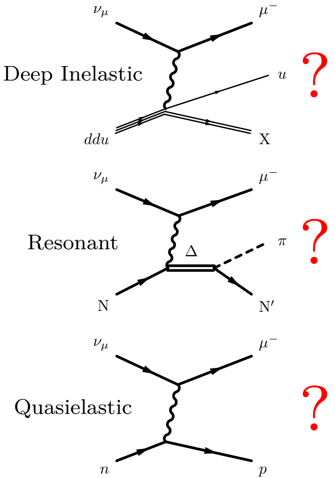
[image:T.Golan]

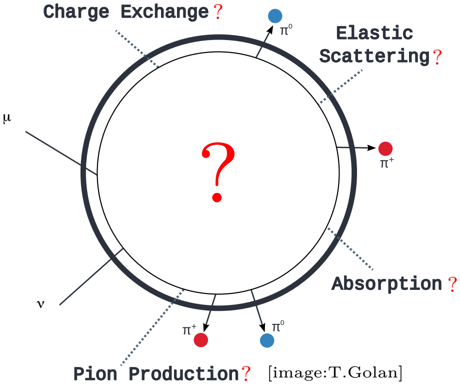
Neutrino Interactions in Nuclear System

Many interaction topologies: primary & intranuclear rescattering

Only particles that leave nucleus are detected

QE dominates low E_ν: HyperK mostly QE, DUNE equal mixture

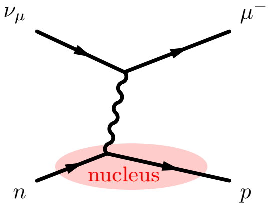




[image:T.Golan]

Quasielastic Form Factors

QE w/ Impulse appx.: interact with "free" nucleon in nucleus



$$\mathcal{M} = \langle \ell | J^\mu | \nu_\ell \rangle \langle N' | J_\mu | N \rangle$$

$$\langle N'(p') | J_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

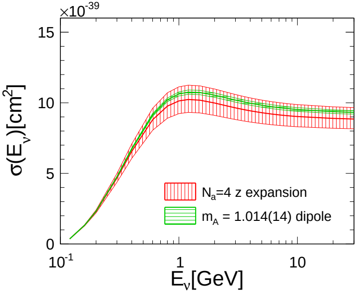
- F_1, F_2: constrained by eN scattering
- F_P: subleading in xsec; PCAC $\Longrightarrow \propto F_A$
- F_A: poorly constrained by expt, obtained from large-A data

In expt, both μ and p measured

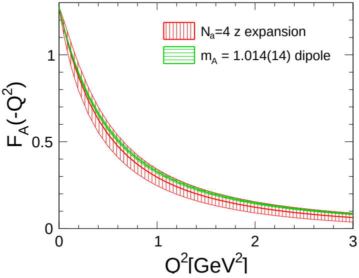
Free nucleon FFs are starting point; full nuclear cross section complicated

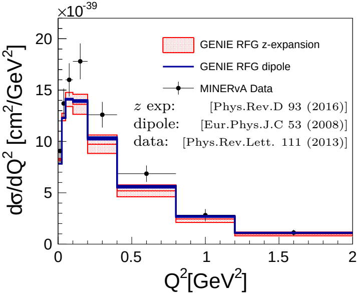
QE F_A from z Expansion

- Dipole $F_A(Q^2) = g_A/(1+Q^2/m_A^2)^2$
  strict Q^2 shape,
  inconsistent w/ QCD
- Dipole ansatz underestimates
  FF uncertainty $\times O(10)$



- Nucl. xsec uncertainty from FF
  same size as data-MC tensions
- Source of tensions unclear
  btw. nucleon/nuclear





Lattice QCD

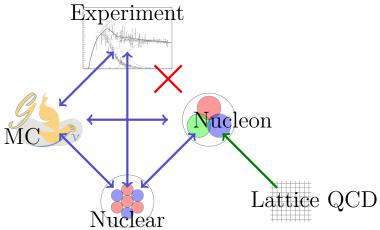
LQCD as Disruptive Technology

Ideal: Modern high stats ν-D_2 scattering bubble chamber expt

Some community push, safety concerns

$\Longrightarrow$ LQCD as a alternative/complement to expt

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

Nucleon amplitudes on solid theoretical ground,
more robust inputs to nuclear model/EFT

Axial Form Factor: $g_A(Q^2 > 0)$

Radius as metric to assess $g_A(Q^2)$

$$r_A^2 = -(6/g_A) dF_A/dQ^2 \Big|_{Q^2=0}$$

$$r_{A,\text{dipole}}^2 = (1\ \text{GeV}^2/m_A^2) \times 0.466\text{fm}^2$$

Agreement btw LQCD r_A^2 & expt good,

but r_A^2 anchored to $Q^2 = 0$,

has 50% uncertainty

Despite r_A^2 agreement,

LQCD consistently w/ slow Q^2 falloff

$\Longrightarrow$ dipole insufficient

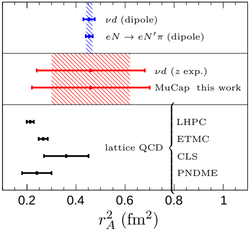
Excited state contaminations?

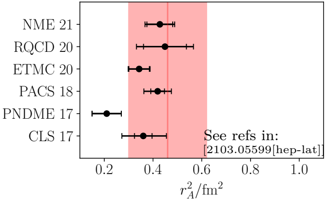
Chiral-continuum or FV extrap?

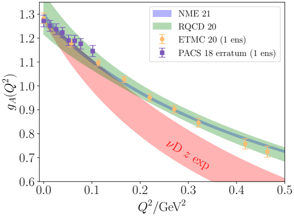
Expt incorrect?

$\Longrightarrow$ No answer yet

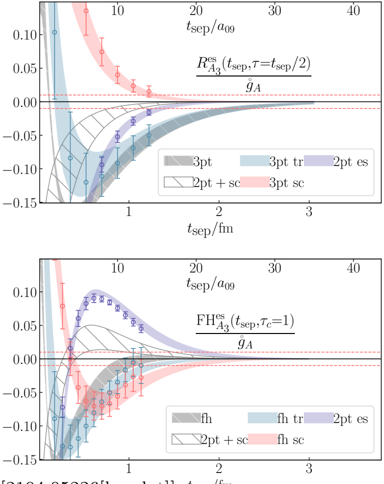
[Rept.Prog.Phys 81 (2018)]







Excited States in g_A



[2104.05226[hep-lat]]

Compare traditional 3pt
vs Feynman-Hellman

$a \approx 0.09$ fm, $M_\pi \approx 310$ MeV

Detailed excited state fit w/ 5 states

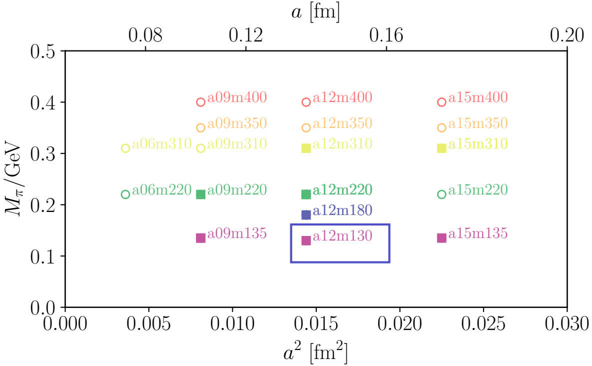
2pt excited state (blue) &
3pt "scattering" ($n \to n$, red)
contrib. similar, approx. cancellation

Remnant contamination dominated
by "transition" ($m \to n$, violet)

Fits to many small t_{sep} better than
fits with few large t_{sep}

Expect excited states to be problematic

CallLat $g_A(Q^2)$ Computation



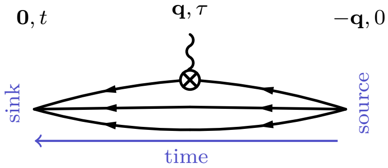
- Open circle: g_A computation
- Filled square: existing $g_A(Q^2)$

Domain wall valence on HISQ sea

Data on many M_π, a, L for full error budget

$\implies$ Only a12m130 in this talk

Computation Setup



For illustration: ratio of correlators

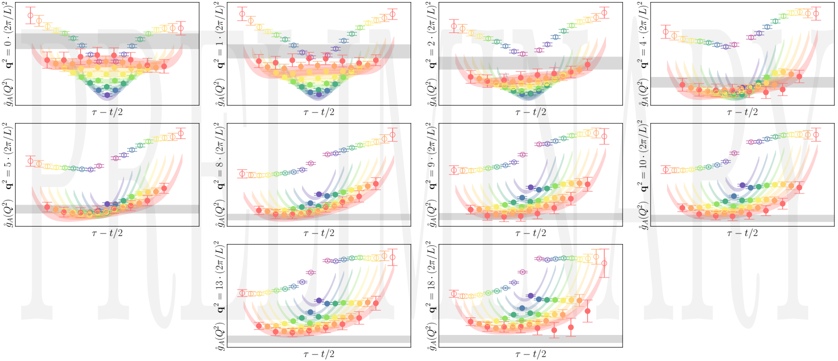
$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\mathrm{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\mathrm{pt}}(t-\tau,\mathbf{0})C^{2\mathrm{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\mathrm{pt}}(\tau,\mathbf{0})}{C^{2\mathrm{pt}}(t,\mathbf{0})} \frac{C^{2\mathrm{pt}}(t-\tau,\mathbf{q})}{C^{2\mathrm{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_{\mathbf{q}}(E_{\mathbf{q}}+M)}} \left[-\frac{q_z^2}{2M} \mathring{\tilde{g}}_P(Q^2) + (E_{\mathbf{q}}+M)\mathring{g}_A(Q^2) \right]$$

$Q^2 = |\mathbf{q}|^2 - (E_{\mathbf{q}} - M)^2$

$\mathcal{A}_z$ w/ $q_z = 0 \implies \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_{\mathbf{q}}+M}{2E_{\mathbf{q}}}}\mathring{g}_A(Q^2)$ no induced pseudoscalar

Three-Point Function Ratios



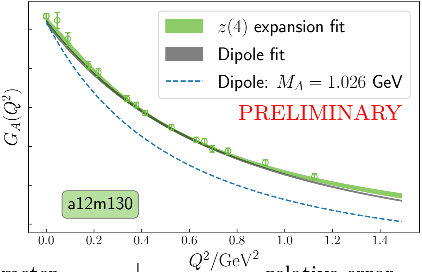
Global 3-state exponential fit $\times$ 10 momenta w/ $|q_i| \leq 3 \cdot (2\pi/L)$

- colors: $t/a \in \{3, ..., 12\}$
- gray band: fit $\mathring{g}_A(Q^2)$
- vertical: $\sqrt{2E_{\mathbf{q}}/(E_{\mathbf{q}} + M)}\mathcal{R}_{\mathcal{A}_z}(t_{\text{sep}}, \tau, \mathbf{q}) \to \mathring{g}_A(Q^2)$

Large range of source-sink separations, lots of momenta

Curvature to $\mathbf{q}^2 = 0 \implies$ opposite sign contribs

Axial Form Factor Fit



parameter source	relative error g_A/g_V	r_A^2/fm^2	$m_{A,\mathrm{dipole}}/\mathrm{GeV}$
$z(4)$ expansion	1.6%	18%	8.8%
dipole	1.6%	5.7%	2.8%
PDG	0.2%		
[Phys.Rev.D 93 (2016)]		48%	24%

Agreement with g_A from PDG, low r_A^2 compared to D_2 (like other LQCD)

Dipole underestimates r_A^2 uncertainty by factor ~ 3,
Will get worse with increased Q^2 range

Using data up to $(qL/2\pi)^2 = 18$, data for up to 108

Expt Implications

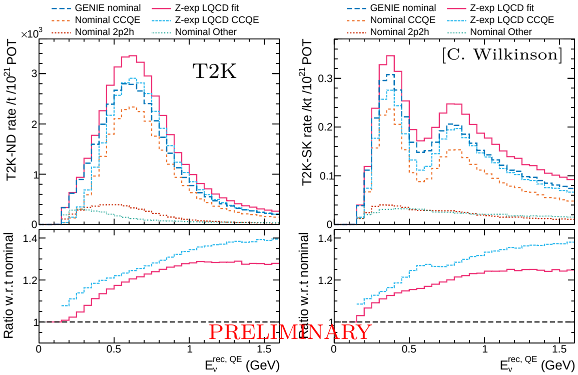
What if $g_A(Q^2)$ falloff is slow (LQCD is right)?

LQCD $g_A(Q^2)$:

$\Longrightarrow$ significant increase in norm

$\Longrightarrow$ suppression/ enhancement of oscillation max

$\Longrightarrow$ will shift oscillation peak



Other interaction mechanisms will be tuned to compensate for differences

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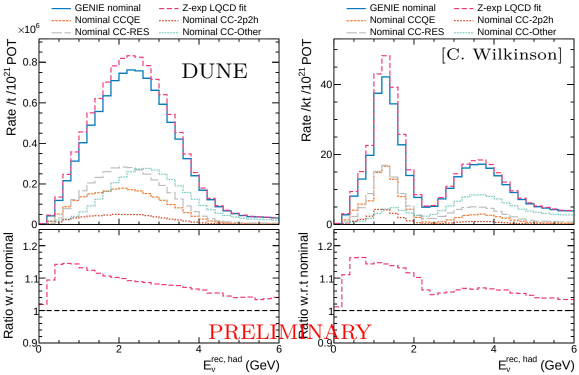

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DUNE at higher E_ν: changes to oscillation max opposite of T2K

Outlook

Nucleon axial form factor is a vital ingredient for neutrino oscillation studies

- Dipole ansatz is overconstrained by data, underestimates uncertainties
- $g_A(Q^2)$ lacking precision: current estimates from D_2 data are $\delta r_A^2 / r_A^2 \sim 50\%$
- First principles LQCD calculations at physical M_π give $\times 3$ improvement
- Early in analysis, many more data available than used
- First step in a series of calculations targeting weak matrix elements
- More to come!

Thanks for your attention!

Backup

Dipole Axial Form Factor

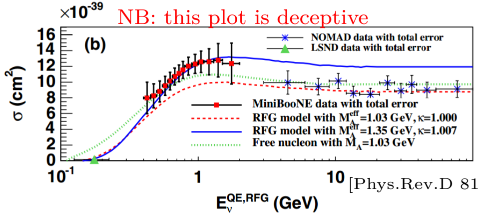
Most widely used: dipole ansatz [Phys.Rept.3 (1972),261]

$$F_A^{\text{dipole}}(Q^2) = g_A \left(1 + Q^2/M_A^2\right)^{-2}$$

Large variation in M_A ("axial mass problem"):

- $M_A = 1.026 \pm 0.021$ [J.Phys.G 28 (2002) R1-R35]
- $M_A^{\text{eff}} = 1.35 \pm 0.17$ [Phys.Rev.D 81 (2010)]

NB: this plot is deceptive



[Phys.Rev.D 81 (2010)]

- M_A^{eff}: nuclear modeling & nucleon FF entangled
- Expts: different selection criteria, sensitivity, MC model, ...

Goal: isolate nucleon FF, then address modeling

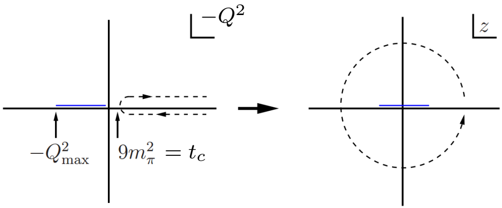
z Expansion

Want model independence, Q^2 expansion only good for $Q^2 \ll 1$

Conformal mapping: [Phys.Rev.D 84 (2011)]

$$z(-Q^2; t_0, t_c) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad t_c = 9m_\pi^2$$

$-Q^2 \le 0$ kinematically allowed $\quad \rightarrow \quad |z| < 1$



- Long history w/ flavor physics & CKM determination ($\lesssim$ 1971)
- Model independent: motivated by analyticity of QCD
- $|z|^k$, $|a_k| \rightarrow 0$ as $k \rightarrow \infty$
- Truncate at finite $k_{\rm max}$, use sum rules to regulate large-Q^2

Axial Charge: $g_A(Q^2 = 0)$

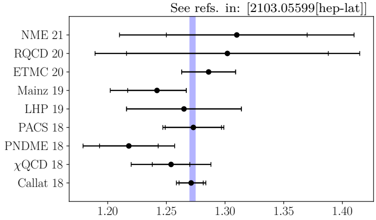
g_A is benchmark for
nucleon matrix elements in LQCD

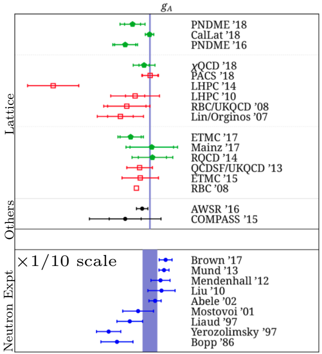
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

Lots of activity since 2018,
consistent agreement with PDG
full error budgets available





[Eur.Phys.J.A 55 (2019)]