Jefferson Lab
Thomas Jefferson National Accelerator Facility

Non-pertrubative studies of parton distribution functions

December 16, 2021

Online DWQ@25 BNL

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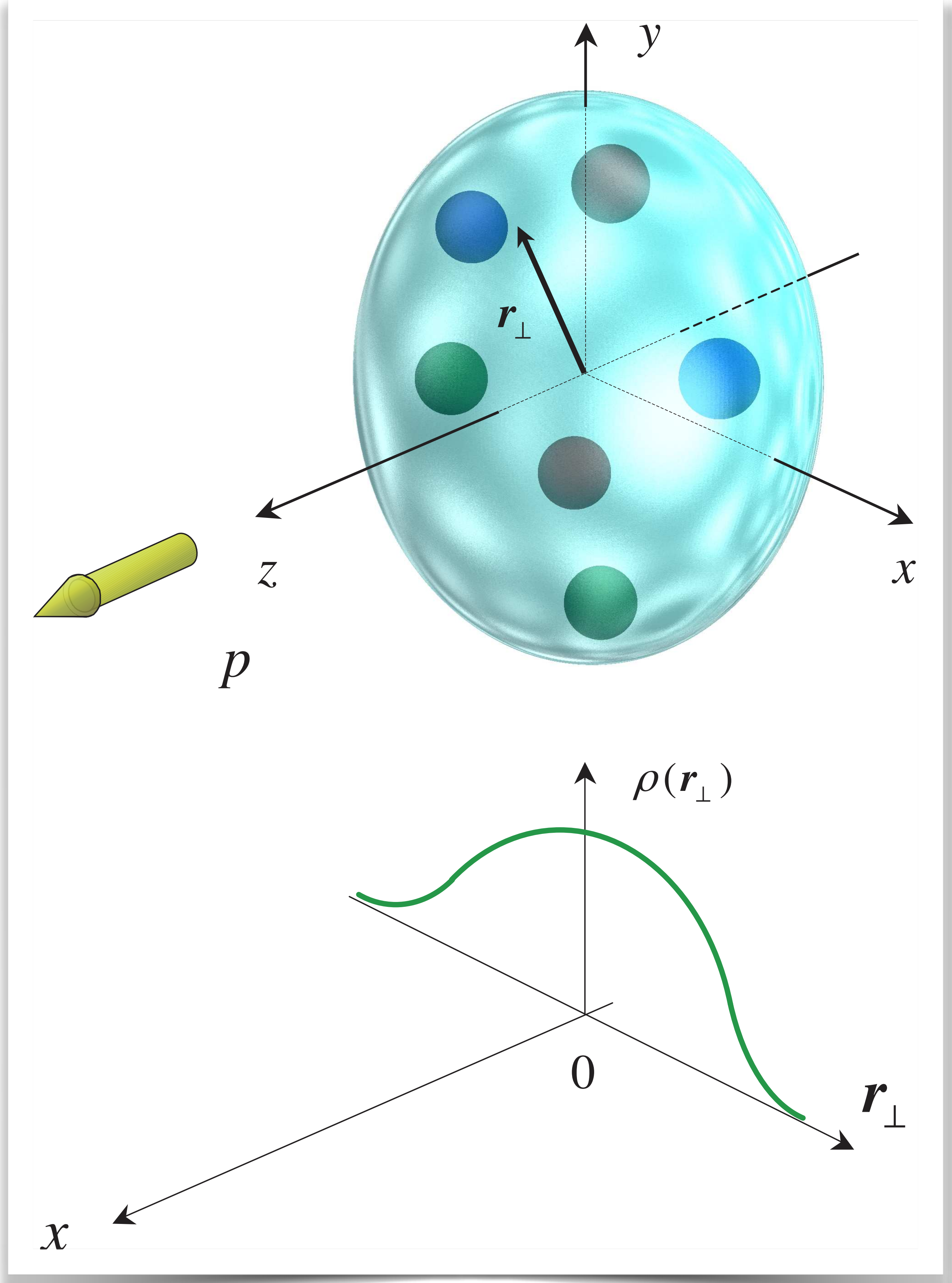
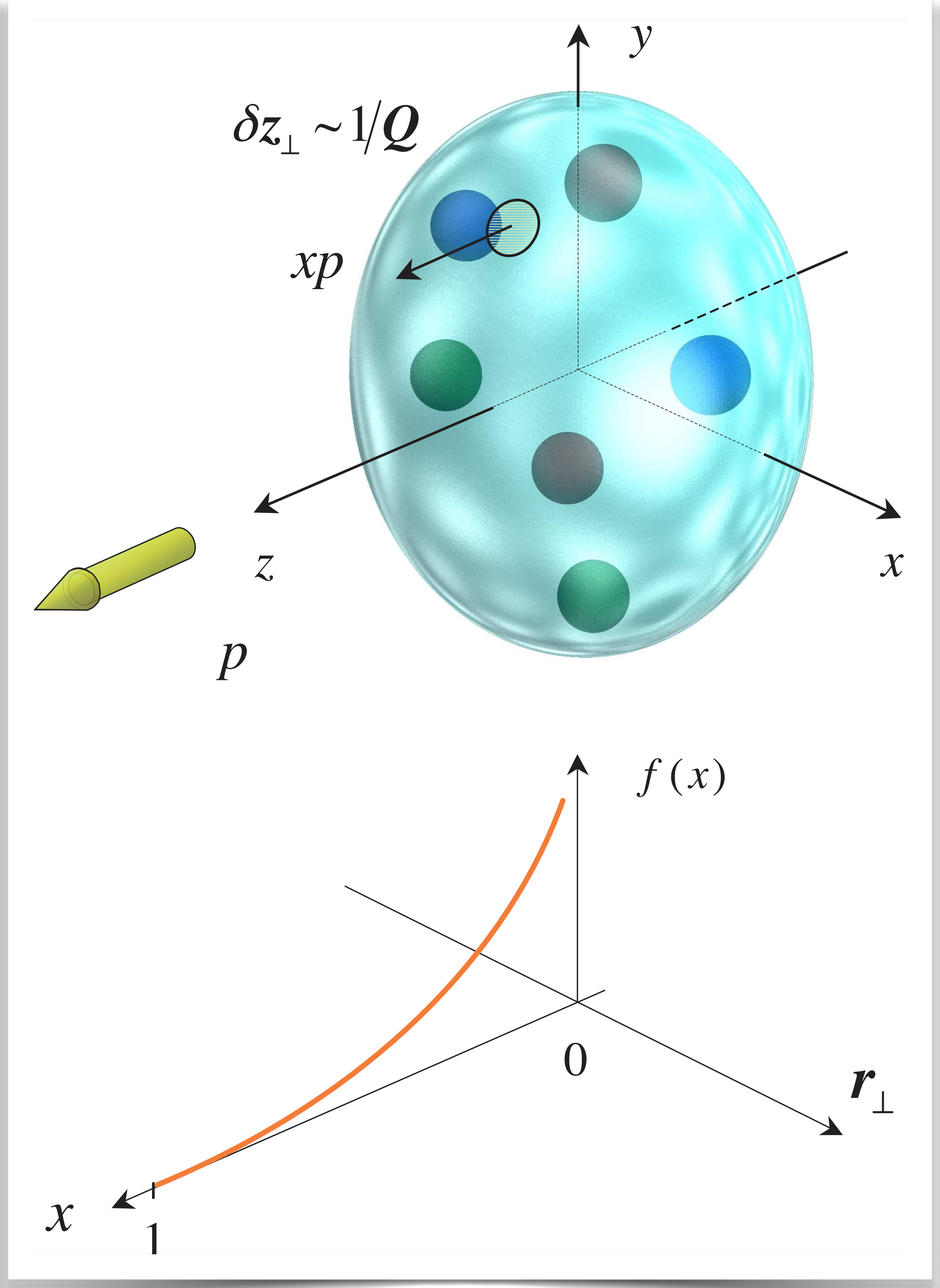
Kostas Orginos

CENTER for NUCLEAR FEMTOGRAPHY

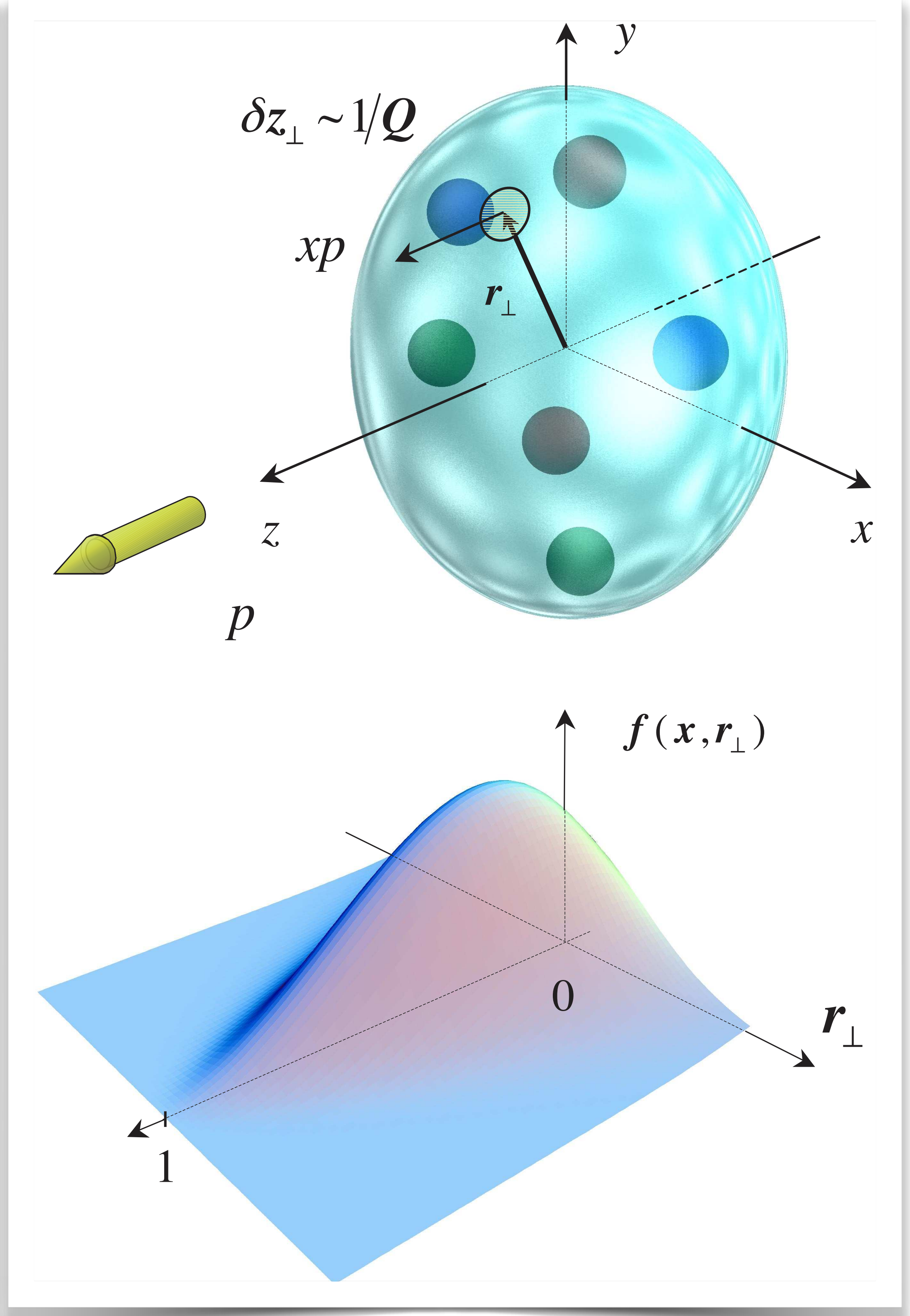
X. Ji, D. Muller, A. Radyushkin (1994-1997)



Form Factors



Parton Distribution functions



Generalized Parton Distribution functions

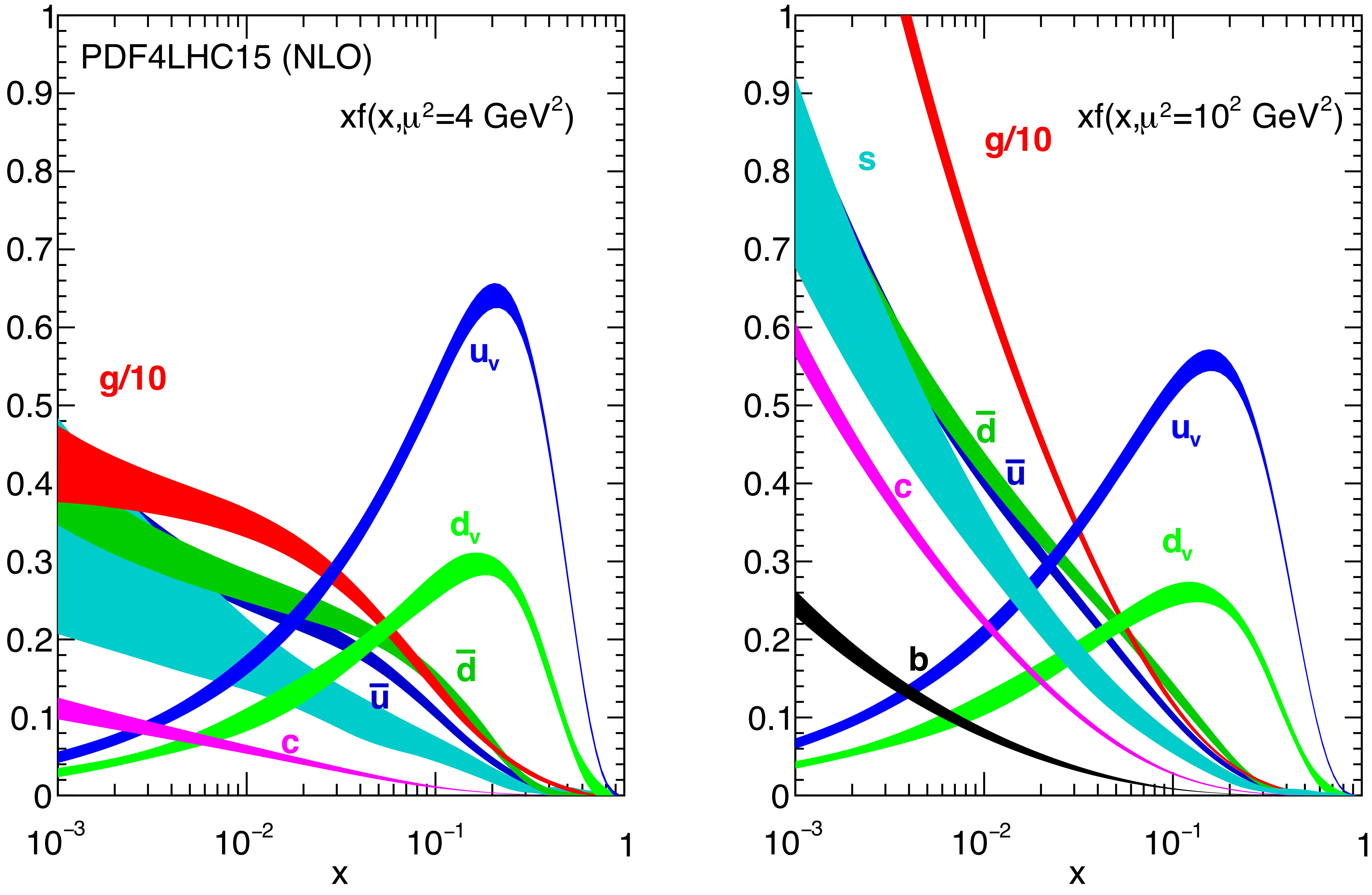
A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang[1,2], A. N. Nicholson[1,3,4], E. Rinaldi[1,5,6], E. Berkowitz[6,7], N. Garron[8], D. A. Brantley[1,6,9], H. Monge-Camacho[1,9], C. J. Monahan[10,11], C. Bouchard[9,12], M. A. Clark[13], B. Joó[14], T. Kurth[1,15], K. Orginos[9,16], P. Vranas[1,6] & A. Walker-Loud[1,6]*

Nature volume 558, pages 91–94 (2018)

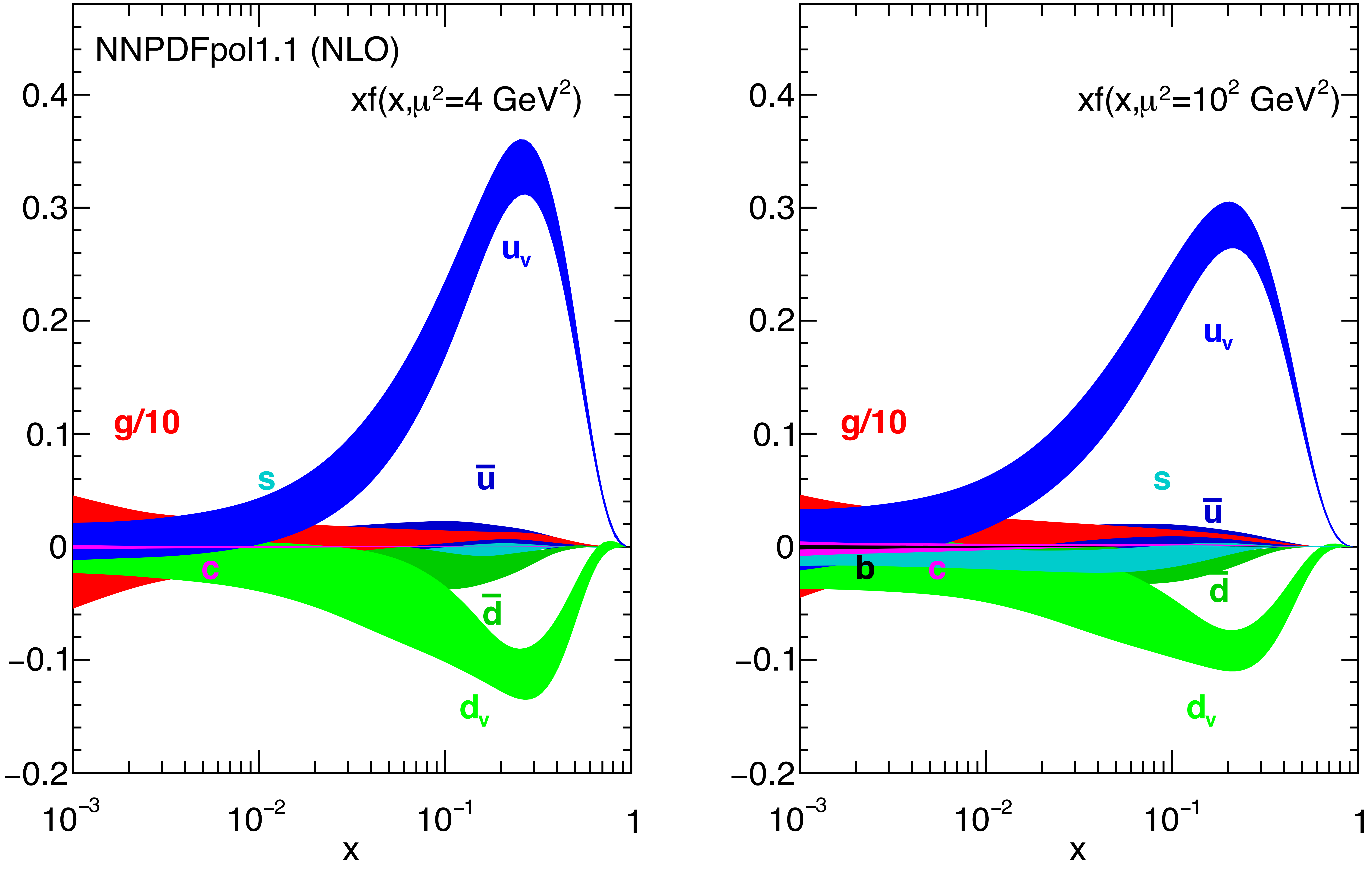
$$g_A = 1.271 \pm 0.013$$

Determination of Parton distribution functions from Experiment



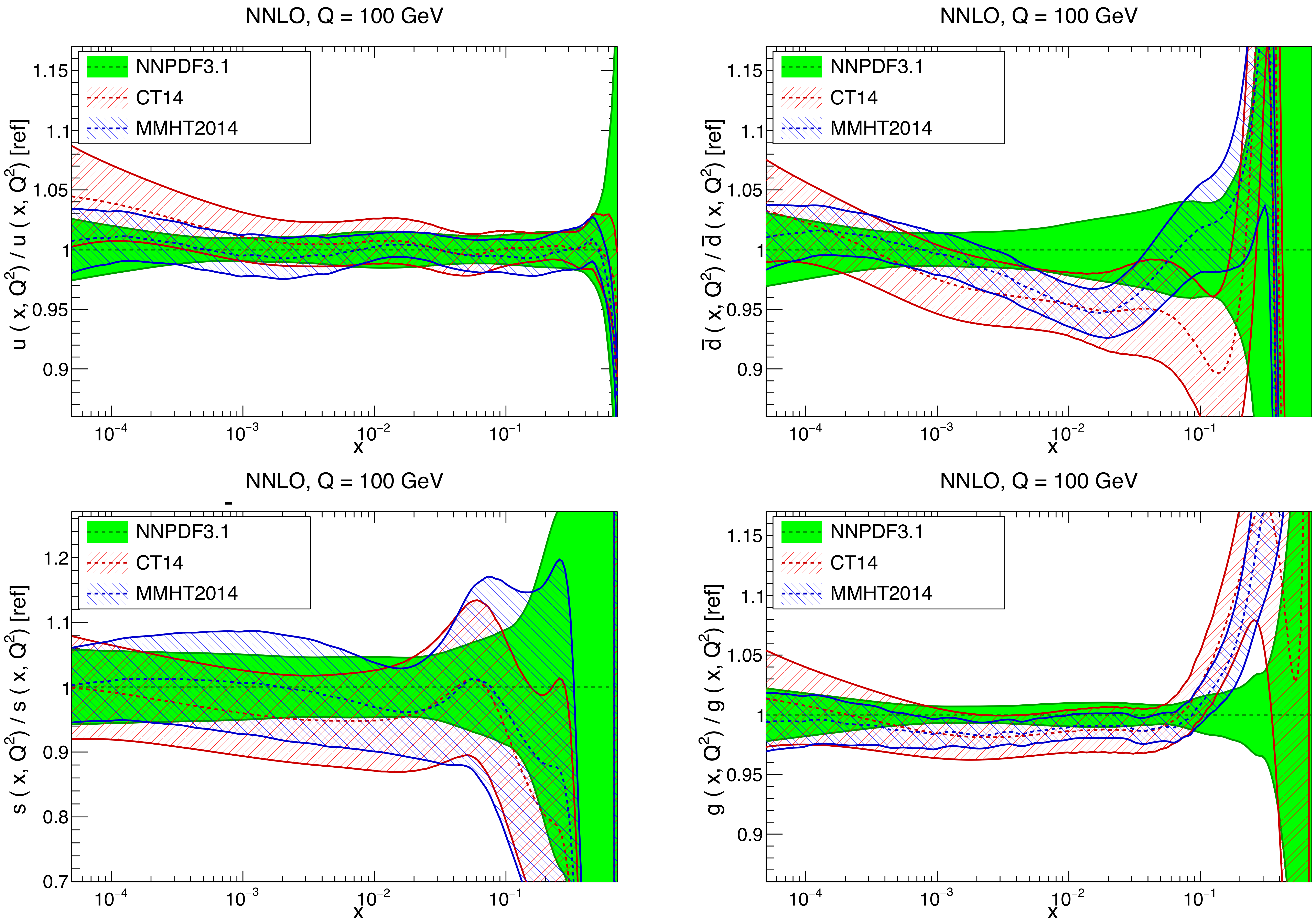
Fits to experimental data

Determination of Parton distribution functions from Experiment



Fits to experimental data

Determination of Parton distribution functions from Experiment

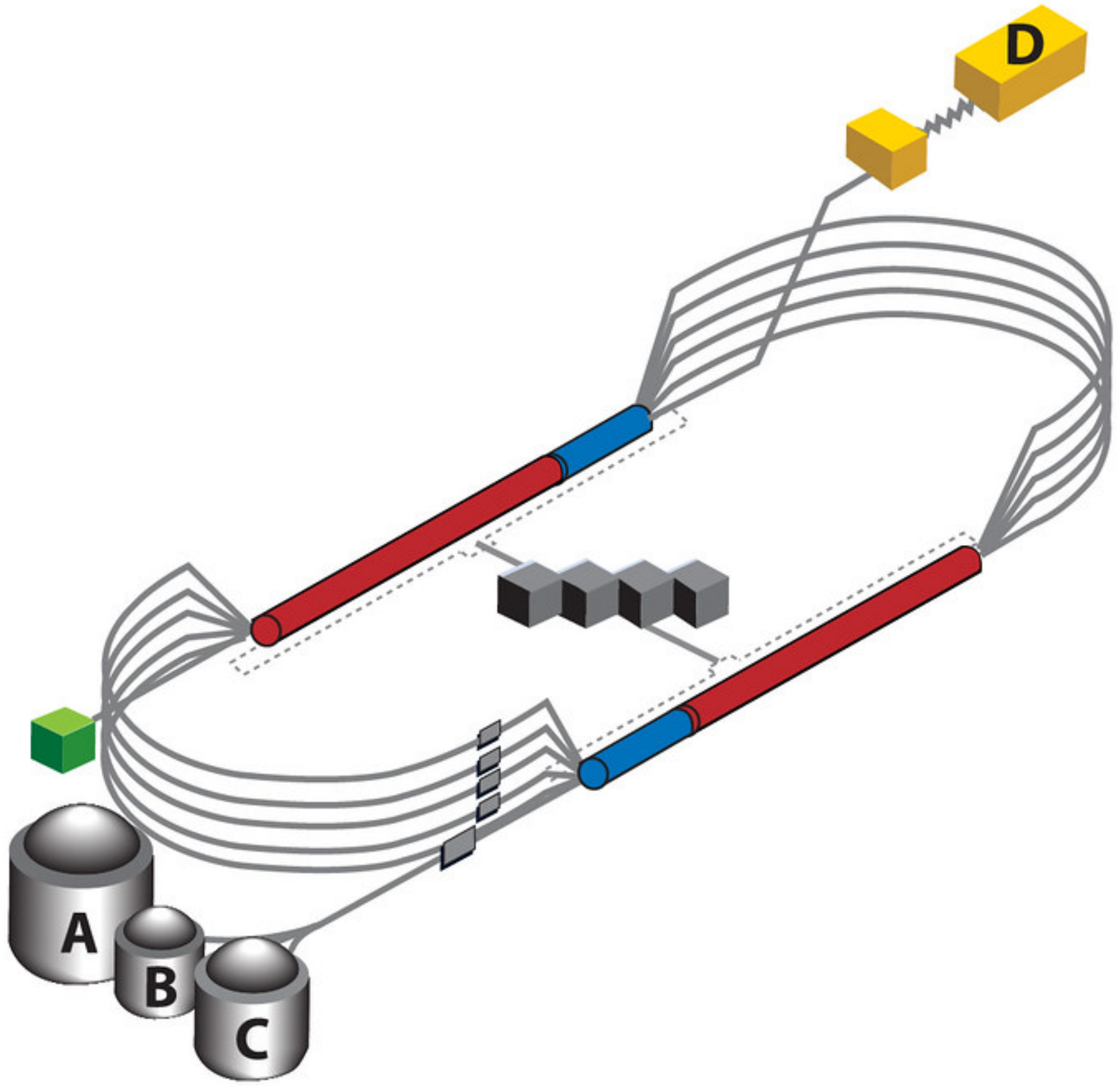


Parton distributions and lattice QCD calculations: a community white paper

arXiv:1711.07916

JLab 12 GeV

Generalized Parton Distributions



The Electron-Ion Collider

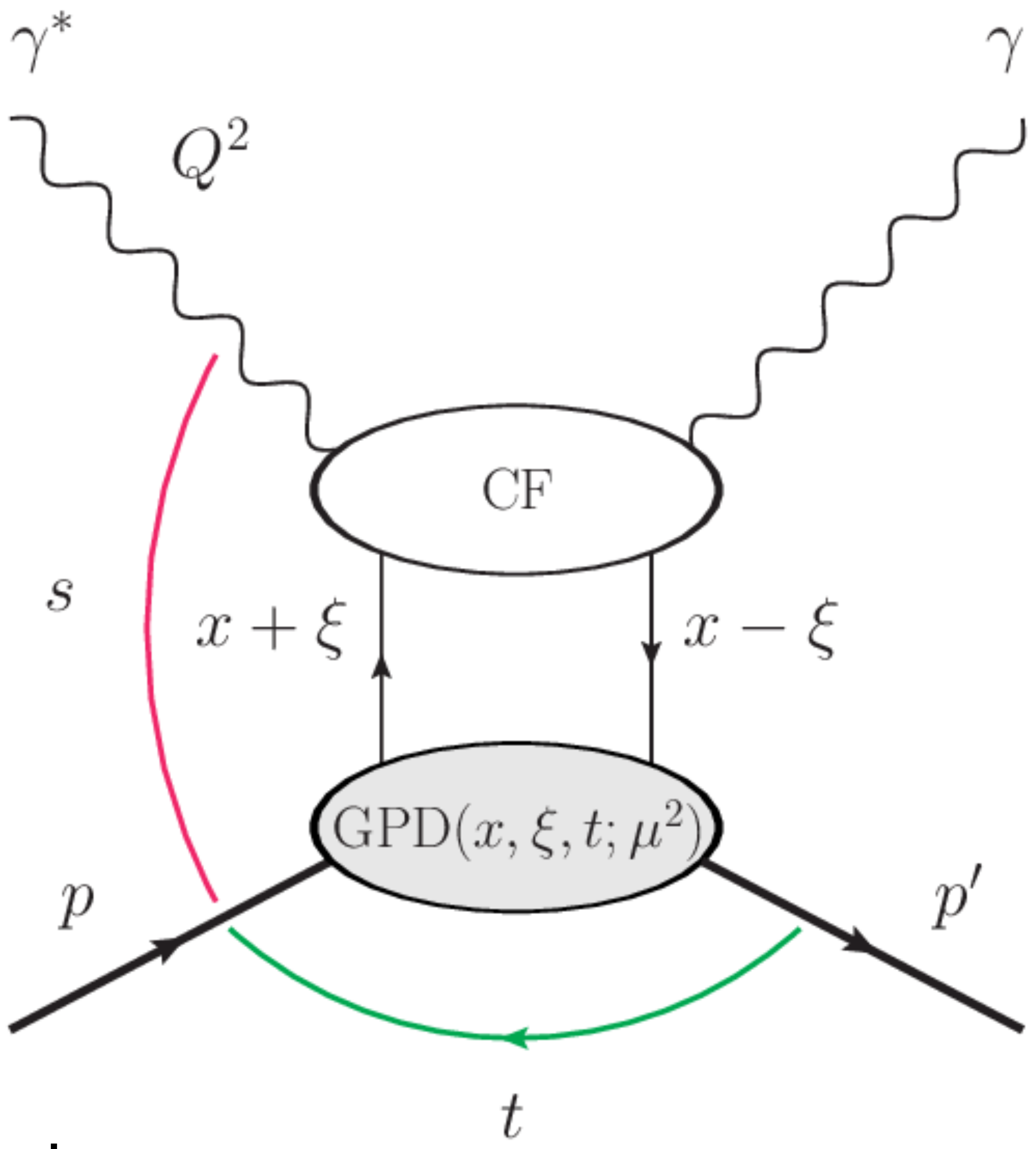
A machine that will unlock the secrets of the strongest force in Nature

The computers and smartphones we use every day depend on what we learned about the atom in the last century. All information technology—and much of our economy today—relies on understanding the electromagnetic force between the atomic nucleus and the electrons that orbit it. The science of that force is well understood but we still know little about the microcosm within the protons and neutrons that make up the atomic nucleus. That's why Brookhaven Lab is building a new machine—an Electron-Ion Collider, or EIC—to look *inside* the nucleus and its protons and neutrons.

The EIC will be a particle accelerator that collides electrons with protons and nuclei to produce snapshots of those particles' internal structure—like a CT scanner for atoms. The electron beam will reveal the arrangement of the quarks and gluons that make up the protons and neutrons of nuclei. The force that holds quarks together, carried by the gluons, is the strongest force in Nature. The EIC will allow us to study this "strong nuclear force" and the role of gluons in the matter within and all around us. What we learn from the EIC could power the technologies of tomorrow.

taken from https://www.bnl.gov/eic/

DVCS factorization



$$\mathcal{F}(\xi,t,Q^2)=\int_{-1}^{1}dx\,C(x,\xi,a_s(\mu),Q/\mu)G(x,\xi,t,\mu)$$

Ill-defined inverse problem —-> Lattice QCD computations are essential

Hadron Structure in Euclidean Space

Go beyond moments

- Goal: Compute full x-dependence (generalized) parton distribution functions (GPDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
  - Power divergent mixing limits us to few moments
- X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations quickly became available
- Older approaches based on the hadronic tensor

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

K-F Liu et al Phys. Rev. Lett. 72 (1994) , Phys. Rev. D62 (2000) 074501
Detmold and Lin 2005
M. T. Hansen et al arXiv:1704.08993.
UKQCD-QCDSF-CSSM Phys. Lett. B714 (2012), arXiv:1703.01153

Pseudo-PDFs

An alternative point of view

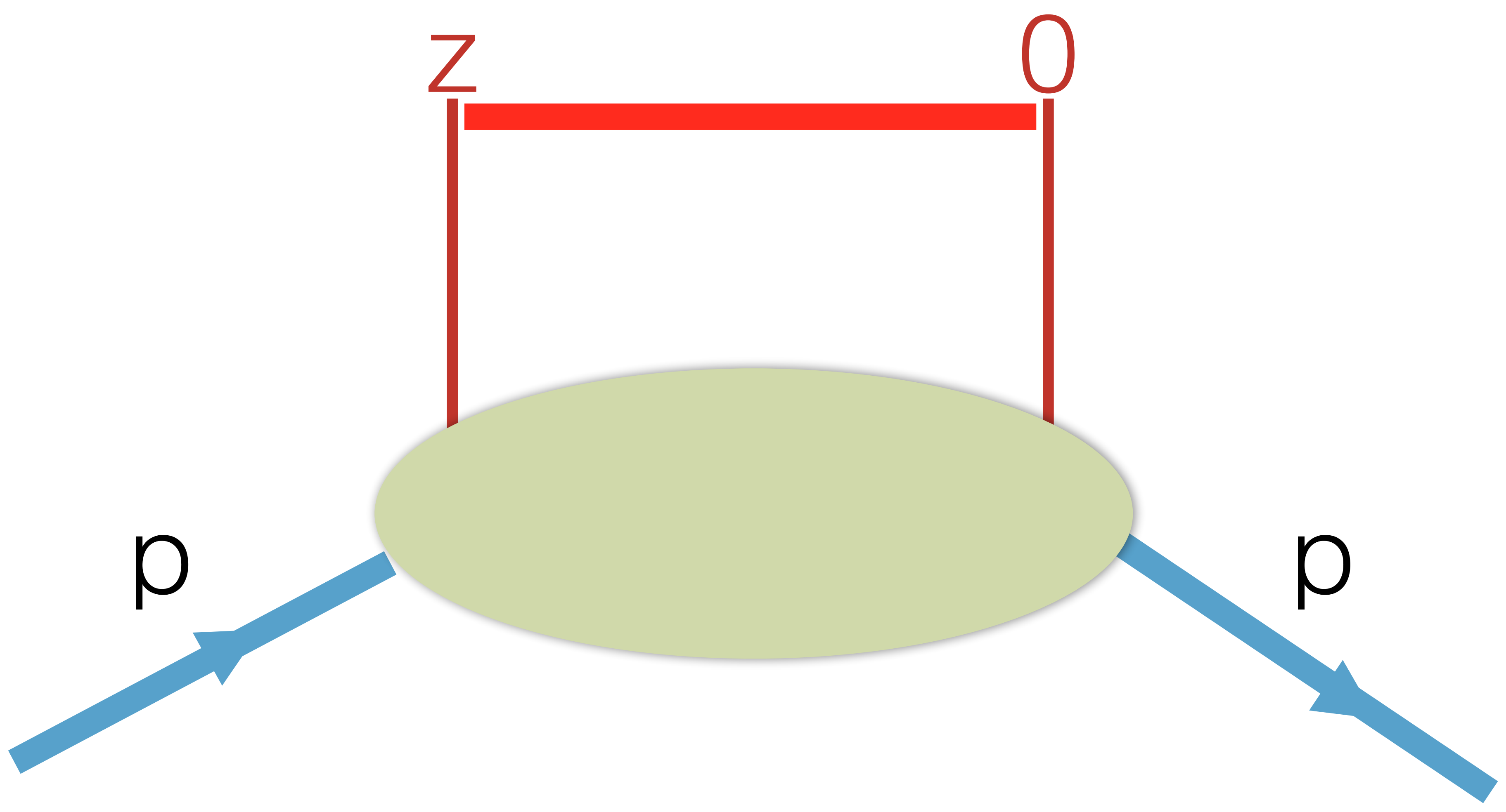
A. Radyushkin Phys.Lett. B767 (2017)

Unpolarized PDFs proton:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_\mu\, A^\mu_\alpha(z')T_\alpha\right]$$

space-like separation of quarks



Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha\mathcal{M}_p(-(zp),-z^2) + z^\alpha\mathcal{M}_z(-(zp),-z^2)$$

A. Radyushkin Phys.Lett. B767 (2017)

Pseudo-PDFs

Connection to light-cone PDFs

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp), -z^2) + z^\alpha \mathcal{M}_z(-(zp), -z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$
$$p = (p_+, 0, 0)$$
$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+ \mathcal{M}_p(-p_+ z_-, 0)$$

Definition of PDF:

$$\mathcal{M}_p(-p_+ z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+ z_-}$$

Lorentz invariance allows for the computation of invariant form factors in any frame

Use equal time kinematics for LQCD

A. Radyushkin Phys.Lett. B767 (2017)

Lattice QCD calculation:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

Choose $\begin{array}{l} p = (p_0, 0, 0, p_3) \\ z = (0, 0, 0, z_3) \end{array}$ γ^0

On shell equal time matrix element
computable in Euclidean space

Briceno *et al* arXiv:1703.06072

Obtaining only the relevant

$$\mathcal{M}_p(\nu, z_3^2) = \frac{1}{2p_0}\mathcal{M}^0(z_3, p_3)$$

$$\mathcal{P}(x, -z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, \mathcal{M}_p(\nu, -z^2)e^{-ix\nu}$$

the pseudo-PDF $x \in [-1, 1]$

Radyusking Phys.Lett. B767 (2017) 314-320

Choosing γ^0 was also suggested also by M. Constantinou at GHP2017 based
on an operator mixing argument for the renormalized matrix element

Alexandrou *et al* arXiv:1706.00265

Collinear singularity at $-z^2 \to 0$

Matching to $\overline{MS}$

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\mathcal{Q}(\alpha\nu, \mu) \;+ \mathcal{O}(z^2\Lambda^2_{qcd})$$

$\mathcal{Q}(\nu, \mu)$ is called the Ioffe time PDF

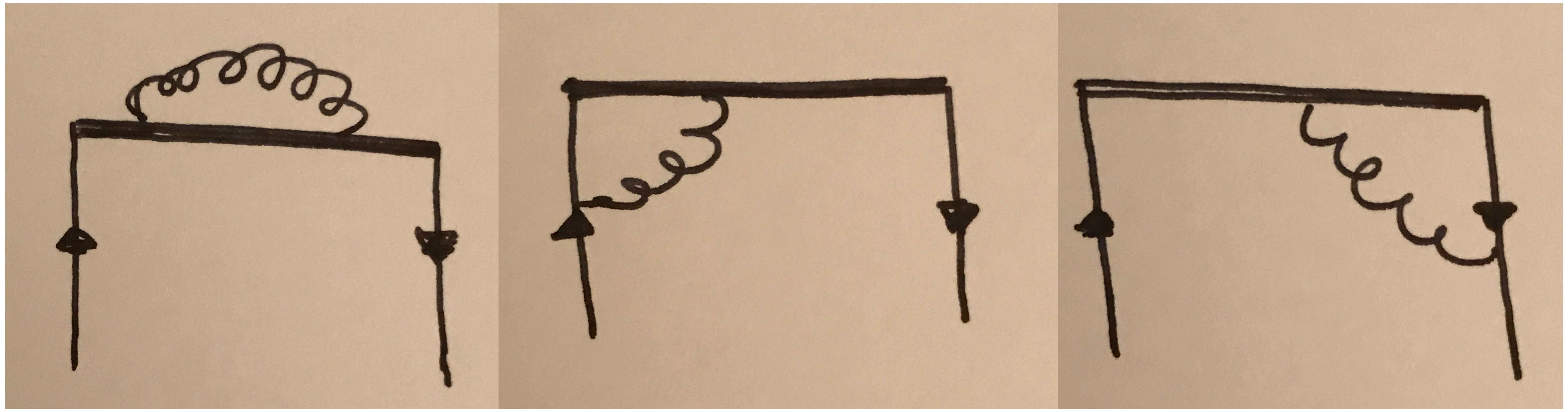
V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^1 dx\, e^{-ix\nu} f(x, \mu)$$

Calculation of the matching Kernel

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

Ioffe time $-z \cdot p = \nu$



One loop calculation of the UV divergences results in

$$\mathcal{M}^0(z,P,a) \sim e^{-m|z|/a}\left(\frac{a^2}{z^2}\right)^{2\gamma_{end}}$$

after re-summation of one loop result resulting exponentiation

- J.G.M.Gatheral,Phys.Lett.133B,90(1983)
- J.Frenkel, J.C.Taylor,Nucl.Phys.B246,231(1984),
- G.P.Korchemsky, A.V.Radyushkin,Nucl.Phys.B283,342(1987).

UV divergences appear multiplicatively

Consider the ratio $$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

UV divergences will cancel in this ratio resulting a renormalization group invariant (RGI) function

The collinear divergences at $z_3^2=0$ limit only appear in the numerator

The lattice regulator can now be removed

$\mathfrak{M}^{cont}(\nu, z_3^2)$ Universal independent of the lattice

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

Continuum limit matching to $\overline{MS}$ computed at 1-loop

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k \,.$$

$$\mathcal{K}(x\nu, z^2\mu^2) = \cos(x\nu) - \frac{\alpha_s}{2\pi} C_F \left[\ln(e^{2\gamma_E+1} z^2\mu^2/4)\tilde{B}(x\nu) + \tilde{D}(x\nu)\right]$$

$$\tilde{B}(x) = \frac{1-\cos(x)}{x^2} + 2\sin(x)\frac{x\mathrm{Si}(x)-1}{x} + \frac{3-4\gamma_E}{2}\cos(x) + 2\cos(x)\left[\mathrm{Ci}(x) - \ln(x)\right]$$

$$\tilde{D}(x) = x\mathrm{Im}\left[e^{ix}{}_3F_3(111;222;-ix)\right] - \frac{2-(2+x^2)\cos(x)}{x^2}$$

Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005
A. Radyushkin Phys.Lett. B767 (2017)

However on the Lattice after expanding in lattice spacing we have

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\rm cont}(\nu, z^2) + \sum_{n=1} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\rm QCD})^n R_n(\nu)\,.$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k\,.$$

Ioffe time $-z \cdot p = \nu$

- All coefficient functions respect continuum symmetries
- Lattice spacing corrections to higher twist effects are ignored
- On dimensional ground a/z terms must exist
- Additional O(a) effects (last term)

The inverse problem to solve: Obtain q(x,μ) from the lattice matrix elements

see discussion in J. Karpie *et al JHEP* 04 (2019) 057 and L. DelDebio *et al JHEP* 02 (2021) 138

Exploration of various methods for LO matching

Exploration of the NNPDF approach applied to lattice data

Our inverse problem

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\rm cont}(\nu,z^2) + \sum_{n=1}\left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\rm QCD})^n R_n(\nu)\,.$$

$$\mathrm{Re}\,\mathfrak{M}(\nu,z^2) = \int_0^1 dx\,\mathcal{K}_R(x\nu,\mu^2 z^2)q_-(x,\mu^2) + \mathcal{O}(z^2)$$

$$\mathrm{Im}\,\mathfrak{M}(\nu,z^2) = \int_0^1 dx\,\mathcal{K}_I(x\nu,\mu^2 z^2)q_+(x,\mu^2) + \mathcal{O}(z^2)\,,$$

- Obtain the PDF from a limited set of matrix elements obtained from lattice QCD
- z^2 is a physical length scale sampled on discrete values
- z^2 needs to be sufficiently small so that higher twist effects are under control
- v is dimensionless also sampled in discrete values
- the range of v is dictated by the range of z and the range of momenta available and is typically limited
- Parametrization of unknown functions

Sample data

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

ID	a(fm)	M_π(MeV)	β	$c_{\rm SW}$	κ	$L^3 \times T$	$N_{\rm cfg}$
$\widetilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

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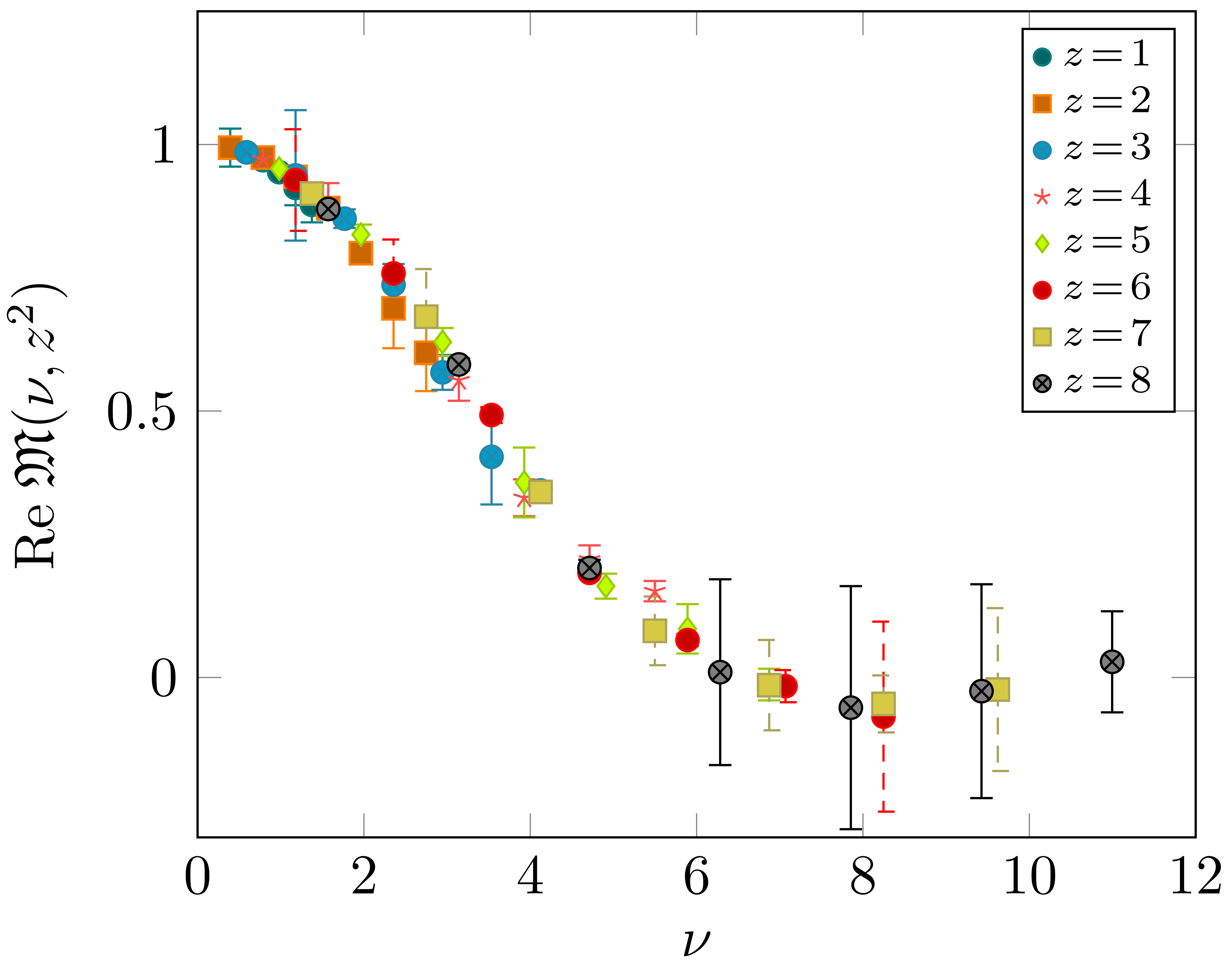

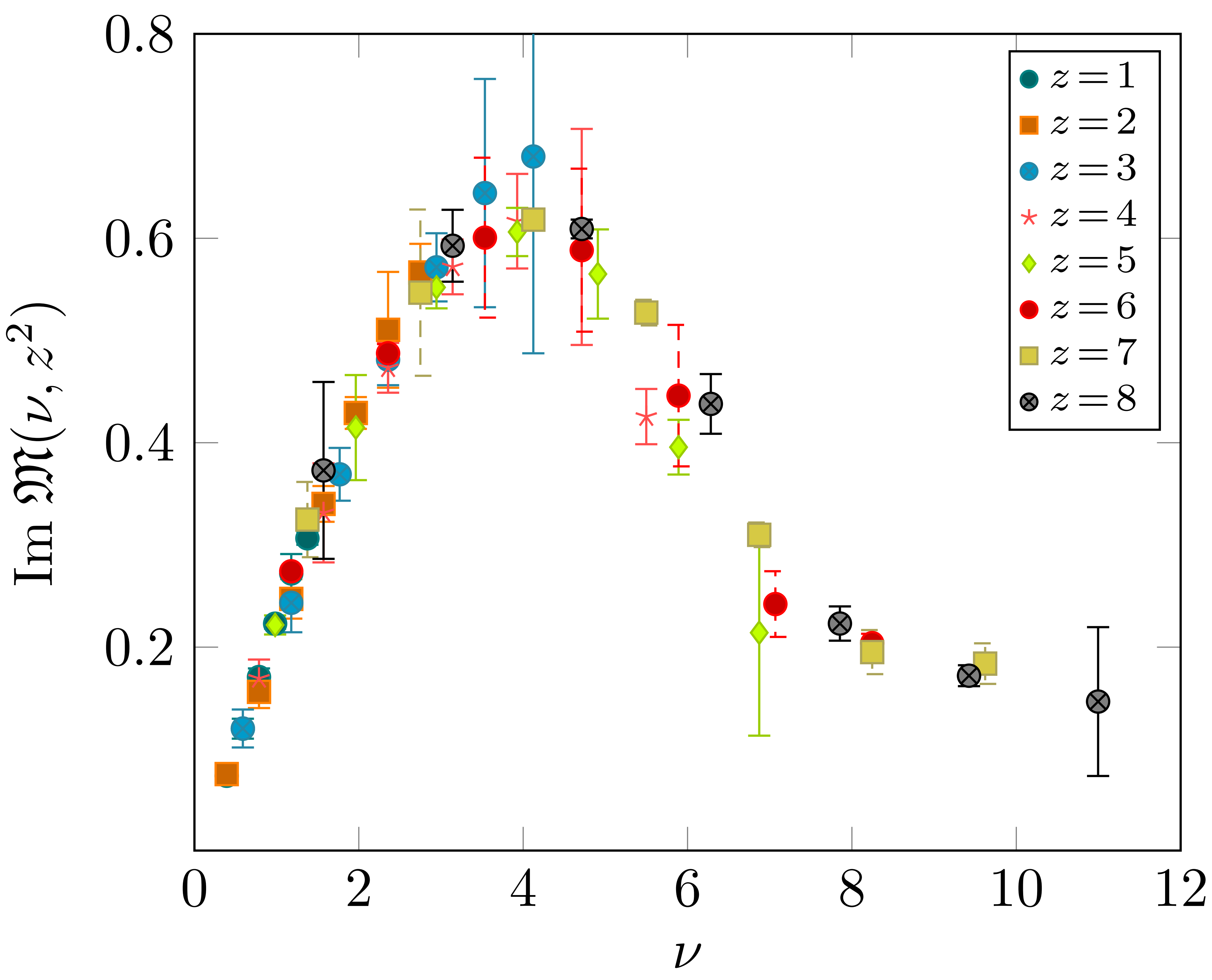

a= 0.075fm

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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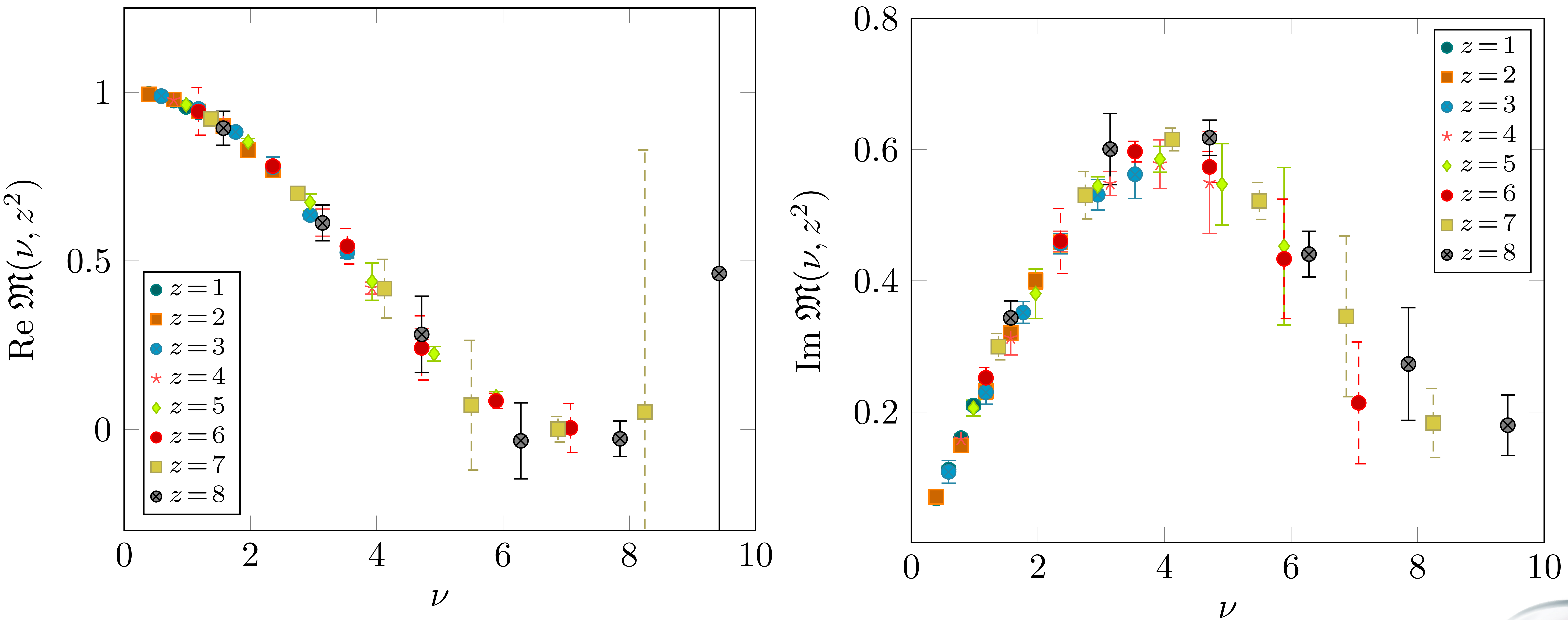

a= 0.065fm

arXiv:2105.13313 **[hep-lat]** **J. Karpie** *et. al.*

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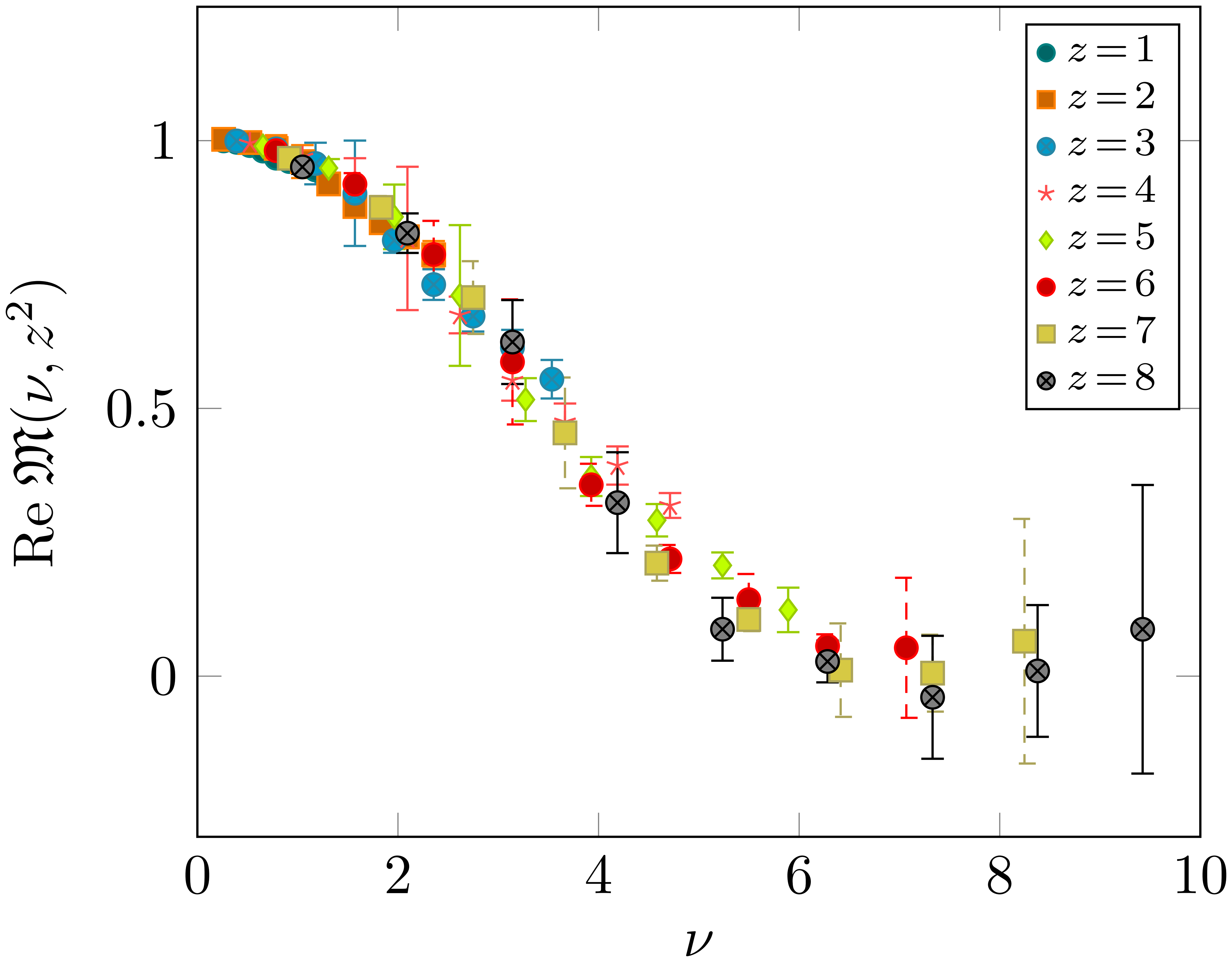

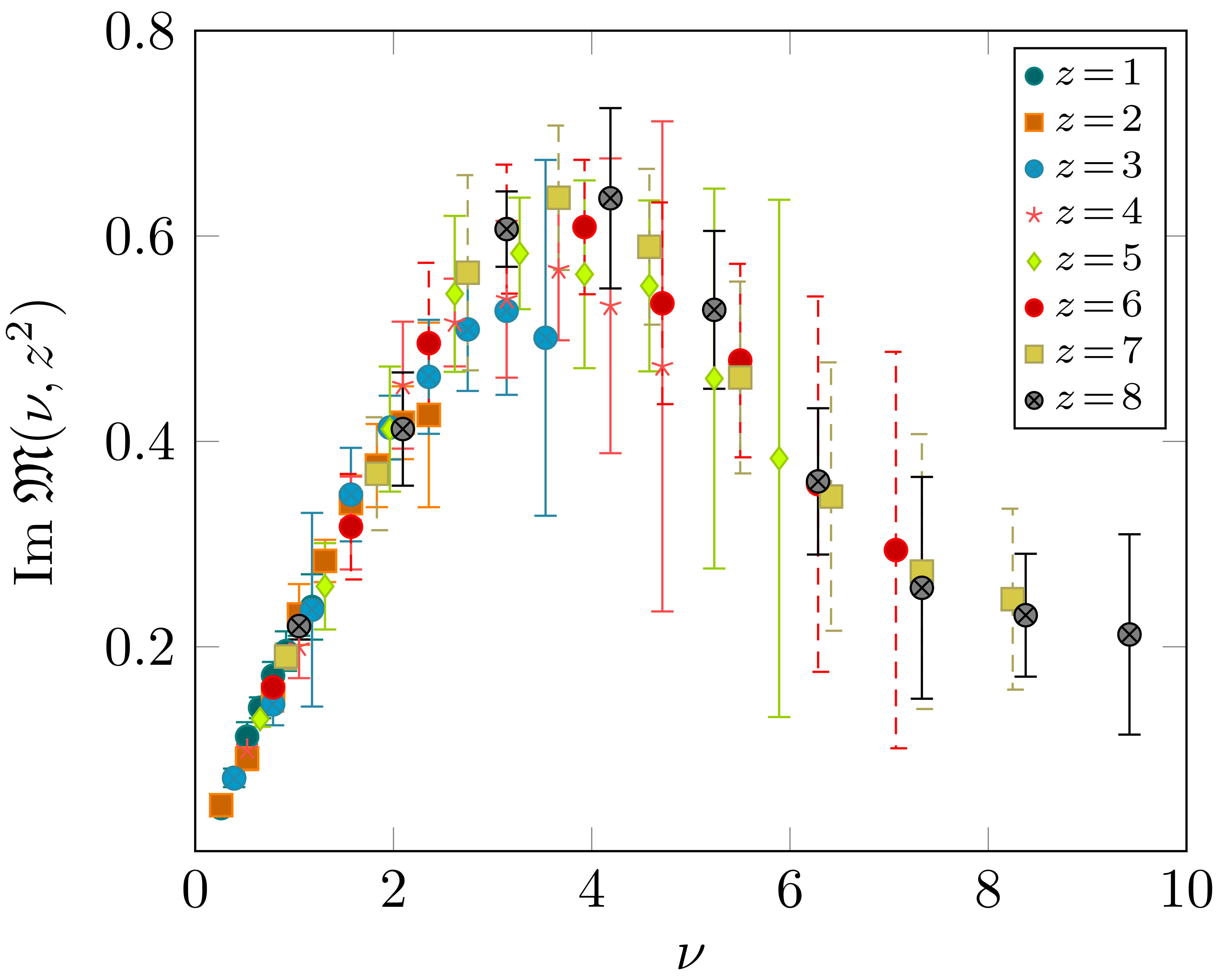

a= 0.048fm

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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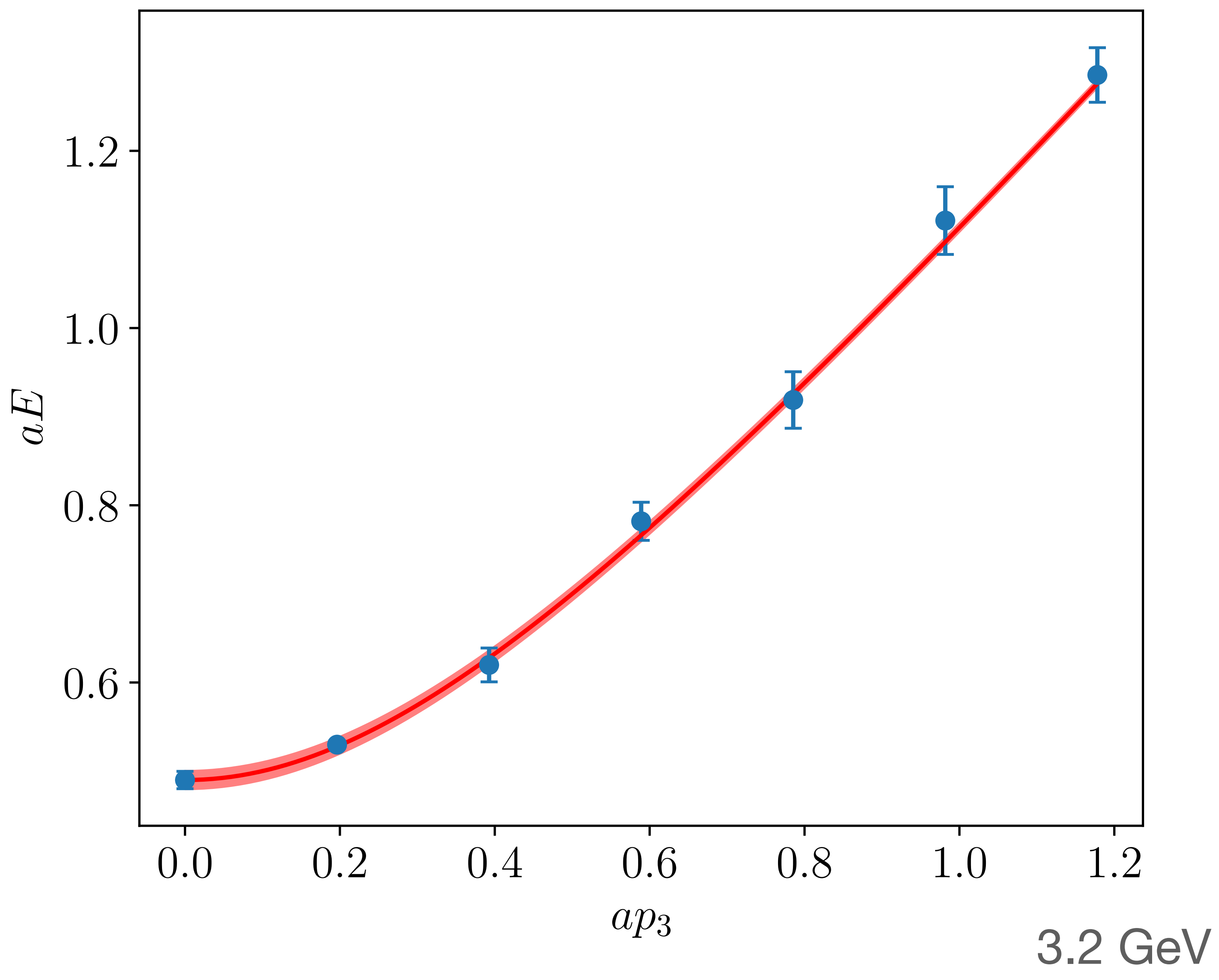
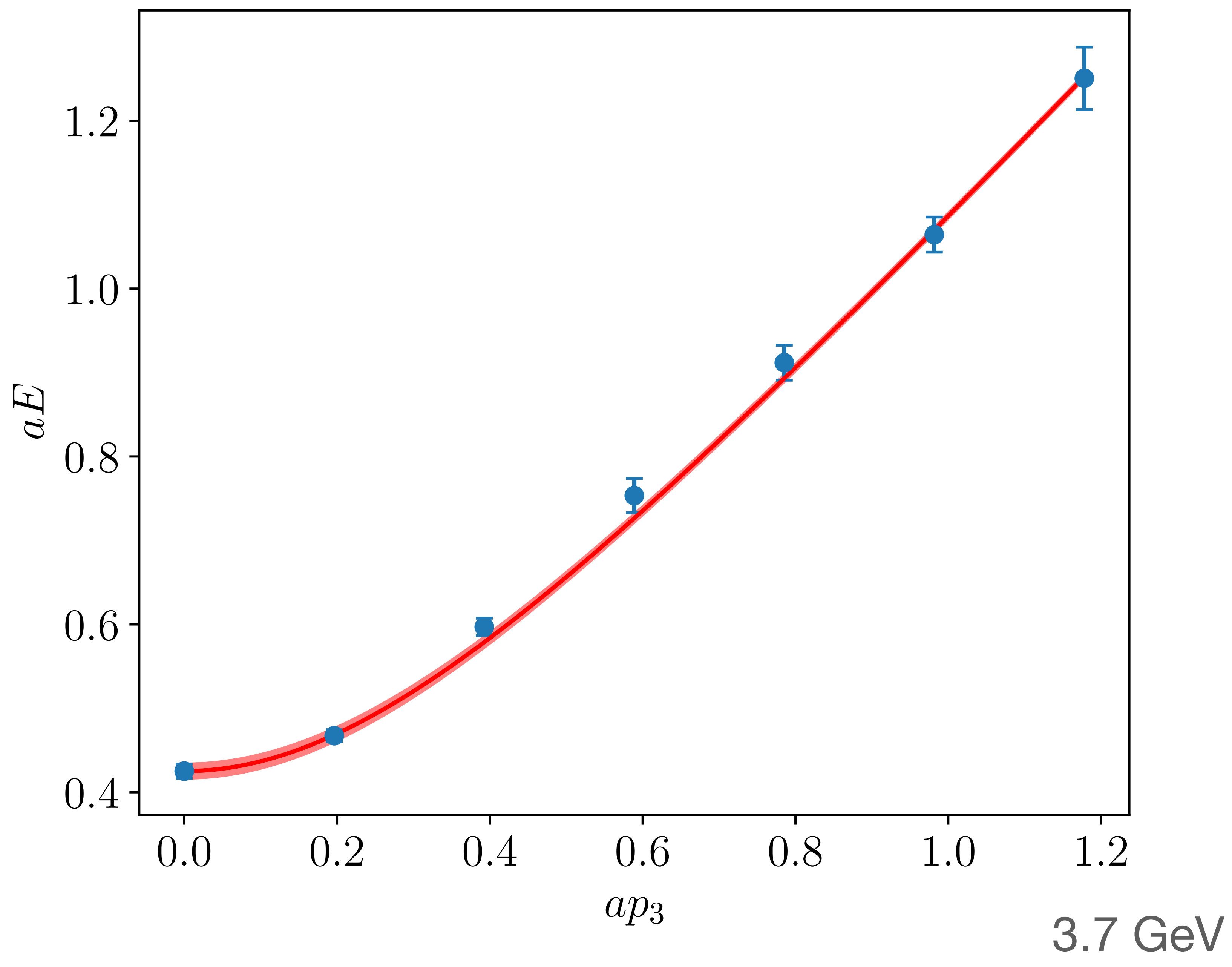
Nucleon Momentum scan

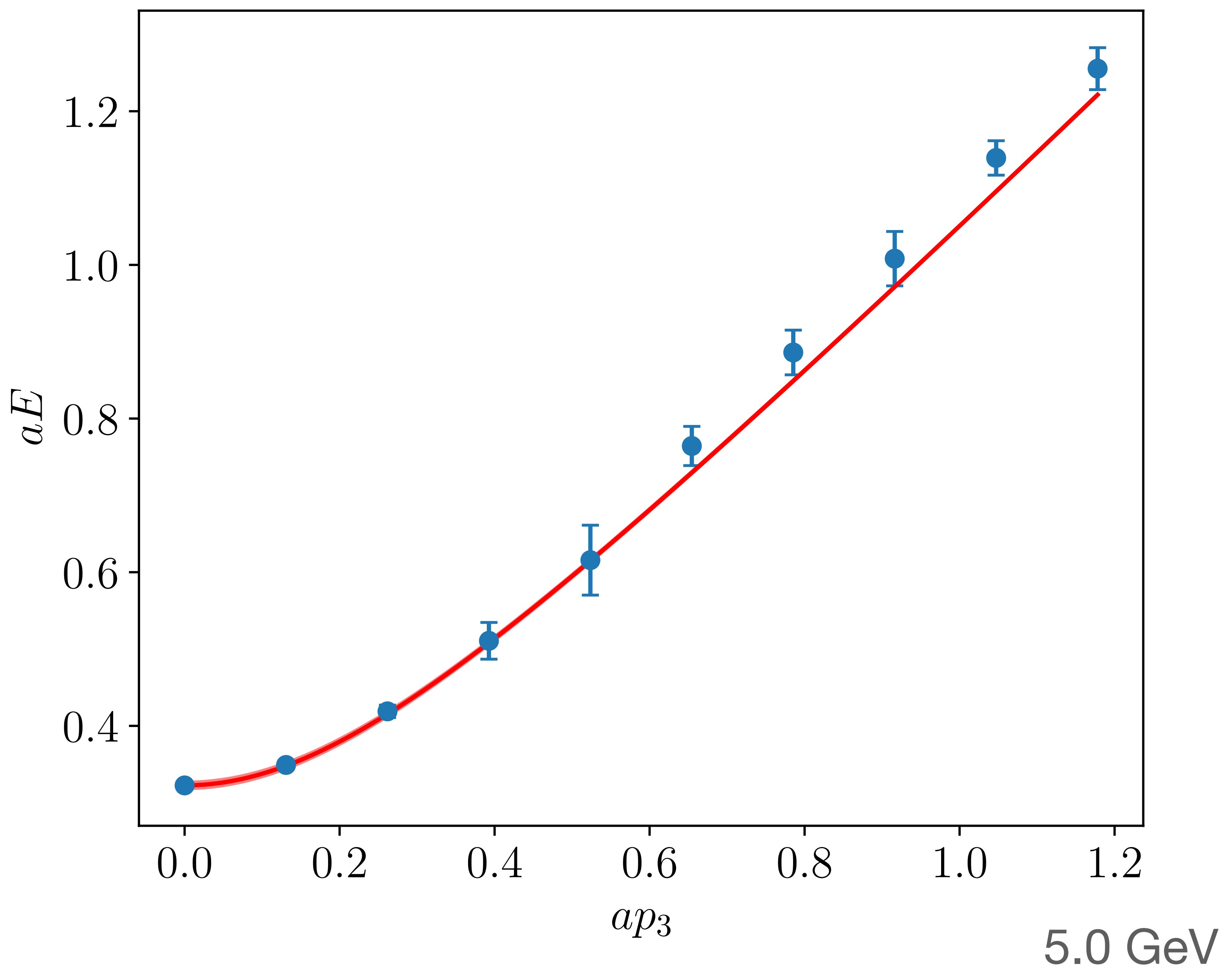
Energy *vs* momentum



a=0.075 fm



a=0.065 fm



a=0.048 fm

Maximum attainable momentum in lattice units can be up to $\mathcal{O}(1)$

Smaller lattice spacing allows for physically larger momentum

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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Jacobi Polynomials
Inverse problem

$q_+(x) = q(x) + \bar{q}(x)$

$q_-(x) = q(x) - \bar{q}(x)$

PDF parametrization

$$q_\pm(x) = x^\alpha (1-x)^\beta \sum_{n=0}^{\infty} {}_\pm d_n^{(\alpha,\beta)} J_n^{(\alpha,\beta)}(x)$$

$J_n^{(\alpha,\beta)}(x)$ Jacobi Polynomials: Orthogonal and complete in the interval [0,1]

$$\int_0^1 dx\, x^\alpha (1-x)^\beta J_n^{(\alpha,\beta)}(x) J_m^{(\alpha,\beta)}(x) = N_n^{(\alpha,\beta)} \delta_{n,m}$$

Complete basis of functions in the interval [0,1] for any α and β

$$J_n^{(\alpha,\beta)}(x) = \sum_{j=0}^{n} \omega_{n,j}^{(\alpha,\beta)} x^j \ ,$$

$$\omega_{n,j}^{(\alpha,\beta)} = \binom{n}{j} \frac{(-1)^j}{n!} \frac{\Gamma(\alpha+n+1)\Gamma(\alpha+\beta+n+j+1)}{\Gamma(\alpha+\beta+n+1)\Gamma(\alpha+j+1)} \ .$$

$$\mathrm{Re}\,\mathfrak{M}(\nu,z^2)=\int_0^1 dx\,\mathcal{K}_R(x\nu,\mu^2 z^2)q_-(x,\mu^2) \qquad \mathrm{Im}\,\mathfrak{M}(\nu,z^2)=\int_0^1 dx\,\mathcal{K}_I(x\nu,\mu^2 z^2)q_+(x,\mu^2)$$

$$\mathcal{K}_R(x\nu,\mu^2 z^2)=\sum_{n=0}^{\infty}\frac{\sigma_n^{(\alpha,\beta)}(\nu,\mu^2 z^2)}{N_n^{(\alpha,\beta)}}J_n^{(\alpha,\beta)}(x)$$

$$\mathcal{K}_I(x\nu,\mu^2 z^2)=\sum_{n=0}^{\infty}\frac{\eta_n^{(\alpha,\beta)}(\nu,\mu^2 z^2)}{N_n^{(\alpha,\beta)}}J_n^{(\alpha,\beta)}(x)\,,$$

$$\sigma_n^{(\alpha,\beta)}(\nu,z^2\mu^2)=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}c_{2k}(z^2\mu^2)\omega_{n,j}^{(\alpha,\beta)}\,B(\alpha+2k+j+1,\beta+1)\,\nu^{2k}$$

$$\eta_n^{(\alpha,\beta)}(\nu,z^2\mu^2)=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}c_{2k+1}(z^2\mu^2)\omega_{n,j}^{(\alpha,\beta)}B(\alpha+2k+j+2,\beta+1)\nu^{2k+1}($$

$$\mathrm{Re}\,\mathfrak{M}_{\mathrm{lt}}(\nu,z^2)=1+\sum_{n=1}^{N_-}\sigma_n^{(\alpha,\beta)}(\nu,z^2\mu^2)_-d_n^{(\alpha,\beta)}$$

$$\mathrm{Im}\,\mathfrak{M}_{\mathrm{lt}}(\nu,z^2)=\sum_{n=0}^{N_+-1}\eta_n^{(\alpha,\beta)}(\nu,z^2\mu^2)_+d_n^{(\alpha,\beta)}\,.$$

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\text{cont}}(\nu,z^2) + \sum_{n=1}\left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\text{QCD}})^n R_n(\nu)\,.$$

$$\mathfrak{M}(\nu,z^2) = \int_0^1 dx\, q_v(x,\mu)\mathcal{K}(x\nu,z^2\mu^2) + \sum_{k=1}^{\infty}\mathcal{B}_k(\nu)(z^2)^k\,.$$

Parametrization of correction terms - Only use one of each kind

Higher Twist

$$\text{Re}\,B_1(\nu) = \sum_{n=1}^{N_{R,b}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) b_{R,n}^{(\alpha,\beta)}\,, \qquad \text{Im}\,B_1(\nu) = \sum_{n=1}^{N_{I,b}} \eta_{0,n}^{(\alpha,\beta)}(\nu) b_{I,n}^{(\alpha,\beta)}$$

z-dependent lattice spacing

$$\text{Re}\,P_1(\nu) = \sum_{n=1}^{N_{R,p}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) p_{R,n}^{(\alpha,\beta)}\,, \qquad \text{Im}\,P_1(\nu) = \sum_{n=1}^{N_{I,p}} \eta_{0,n}^{(\alpha,\beta)}(\nu) p_{I,n}^{(\alpha,\beta)}$$

z-independent lattice spacing

$$\text{Re}\,R_1(\nu) = \sum_{n=1}^{N_{R,r}} \sigma_{0,n}^{(\alpha,\beta)}(\nu) r_{R,n}^{(\alpha,\beta)}\,, \qquad \text{Im}\,R_1(\nu) = \sum_{n=1}^{N_{I,r}} \eta_{0,n}^{(\alpha,\beta)}(\nu) r_{I,n}^{(\alpha,\beta)}\,,$$

$$\sigma_{0,n}^{(\alpha,\beta)}(\nu) = \int_0^1 dx\, \cos(\nu x) x^{\alpha}(1-x)^{\beta} J_n^{(\alpha,\beta)}(x)$$

$$\eta_{0,n}^{(\alpha,\beta)}(\nu) = \int_0^1 dx\, \sin(\nu x) x^{\alpha}(1-x)^{\beta} J_n^{(\alpha,\beta)}(x)\,,$$

Bayesian Inference

Optimize model parameters

- Fix the expansion order in the Jacobi polynomial expansion
- Optimize α,β and the expansion of coefficients by maximizing the posterior probability
- Average over models using AICc
- Note that one could fix α,β at a reasonable value and the vary the order of trancation in the Jacobi polynomial expansion

$$P\left[\theta|\mathfrak{M}^L, I\right] = \frac{P\left[\mathfrak{M}^L|\theta\right] P\left[\theta|I\right]}{P\left[\mathfrak{M}^L|I\right]}\,.$$

$$P\left[\theta|\mathfrak{M}^L, I\right] = \frac{P\left[\mathfrak{M}^L|\theta\right] P\left[\theta|I\right]}{P\left[\mathfrak{M}^L|I\right]}\,.$$

Probability distribution of the data given the parameters

$$P[\mathfrak{M}^L|\theta] \propto \exp[-\frac{\chi^2}{2}]$$

$$\chi^2 = \sum_{k,l} (\mathfrak{M}_k^L - \mathfrak{M}_k) C_{kl}^{-1} (\mathfrak{M}_l^L - \mathfrak{M}_l),$$

Prior distributions

Shifted lognormal for α,β so that α>-1 and b>-1

Normal distribution for all linear parameters (expansion coefficients)

Optimize parameters using non-linear optimizer for α,β only

VarPro (Variable projection method) allows for exact optimization of all expansion coefficients given α,β

model	Real L^2/d.o.f.	Real χ^2/d.o.f.	Imag L^2/d.o.f.	Imag χ^2/d.o.f.
Q only	3.173	3.094	3.146	3.095
Q and B_1	2.721	2.479	3.054	2.969
Q and R_1	3.028	2.748	3.068	2.871
Q and P_1	0.876	0.809	1.186	1.088
Q, B_1, and R_1	2.610	2.057	2.917	2.619
Q, B_1, and P_1	0.852	0.723	1.020	0.888
Q, R_1, and P_1	0.881	0.763	1.289	1.063
All terms	0.857	0.727	1.026	0.893

the z-dependent lattice spacing effect seems the most important systematic error

arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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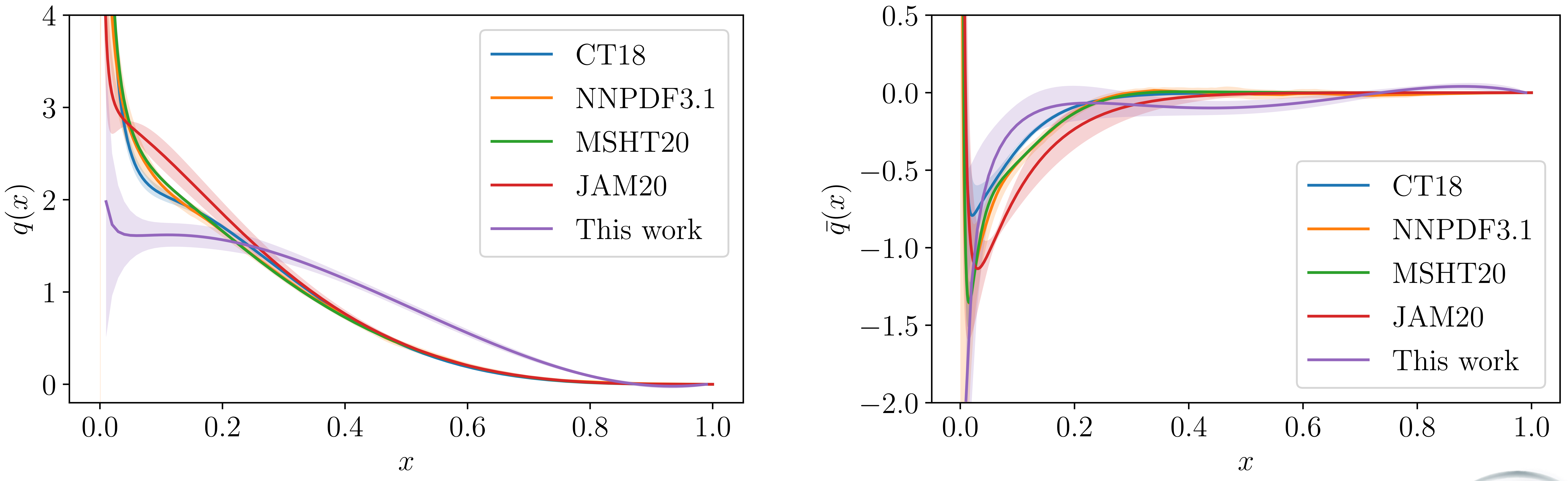
Isovector quark and anti-quark distributions

Comparison with phenomenology



arXiv:2105.13313 **[hep-lat] J. Karpie** *et. al.*

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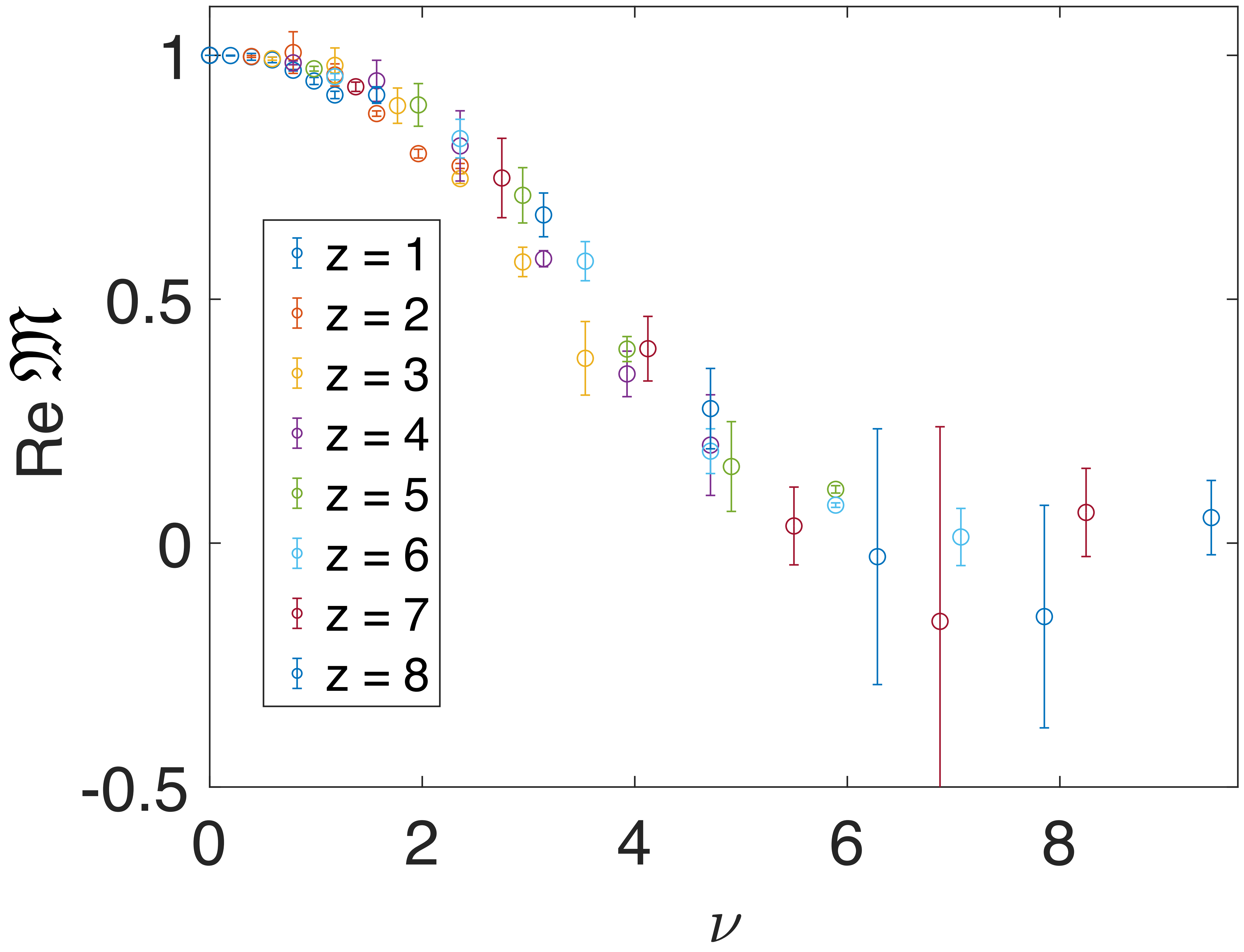
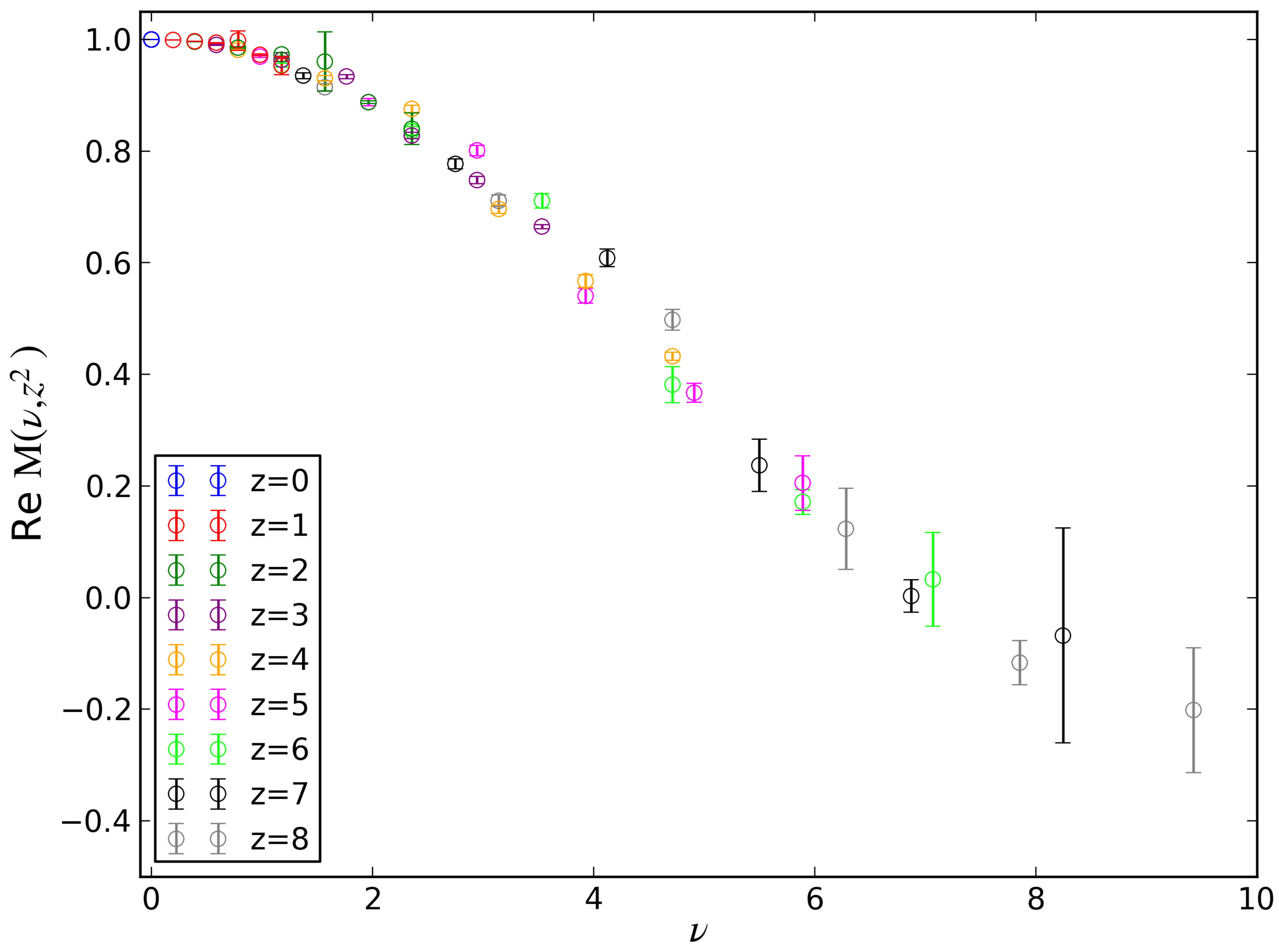
Sequential operator approach



plot: J. Karpie

distillation

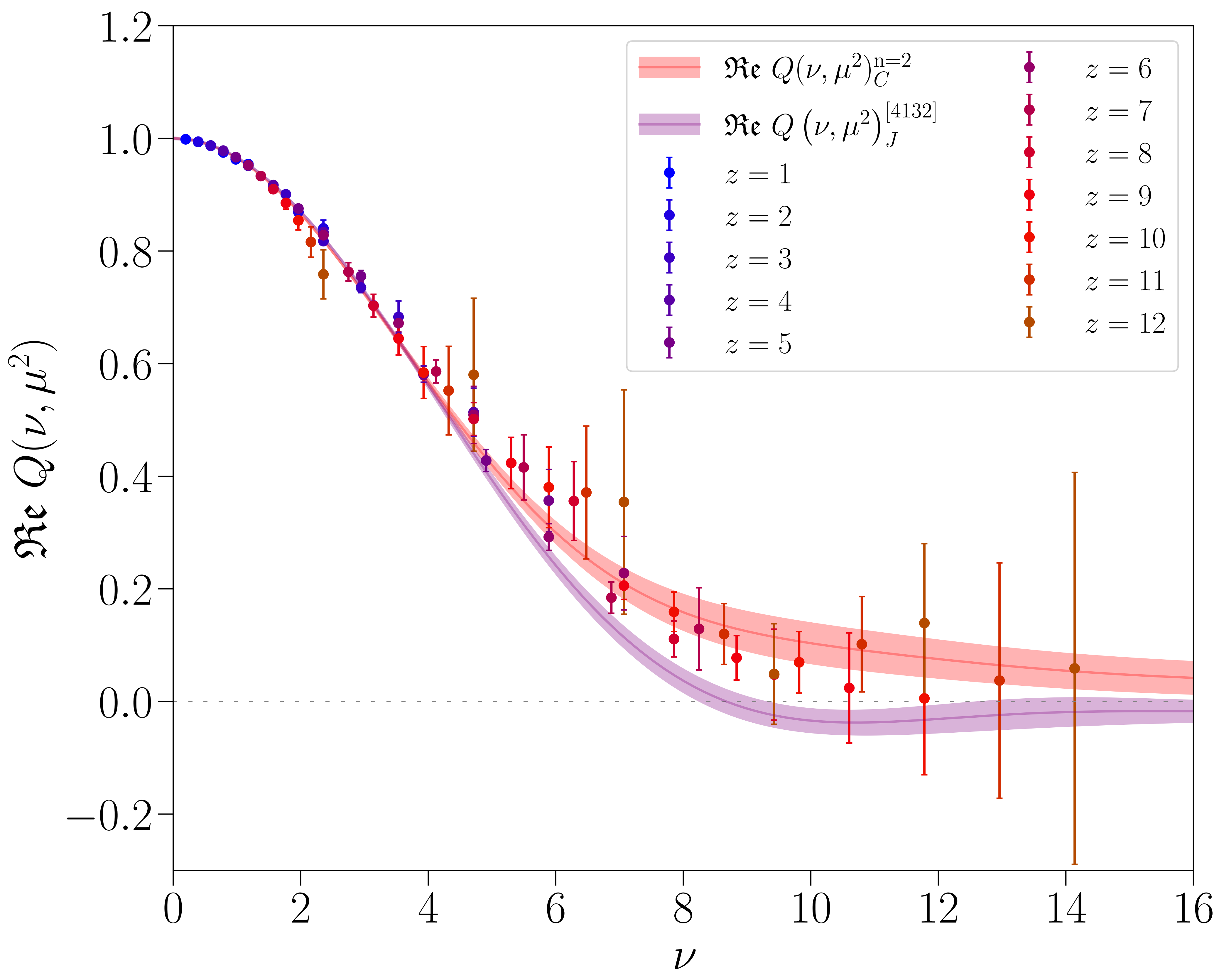


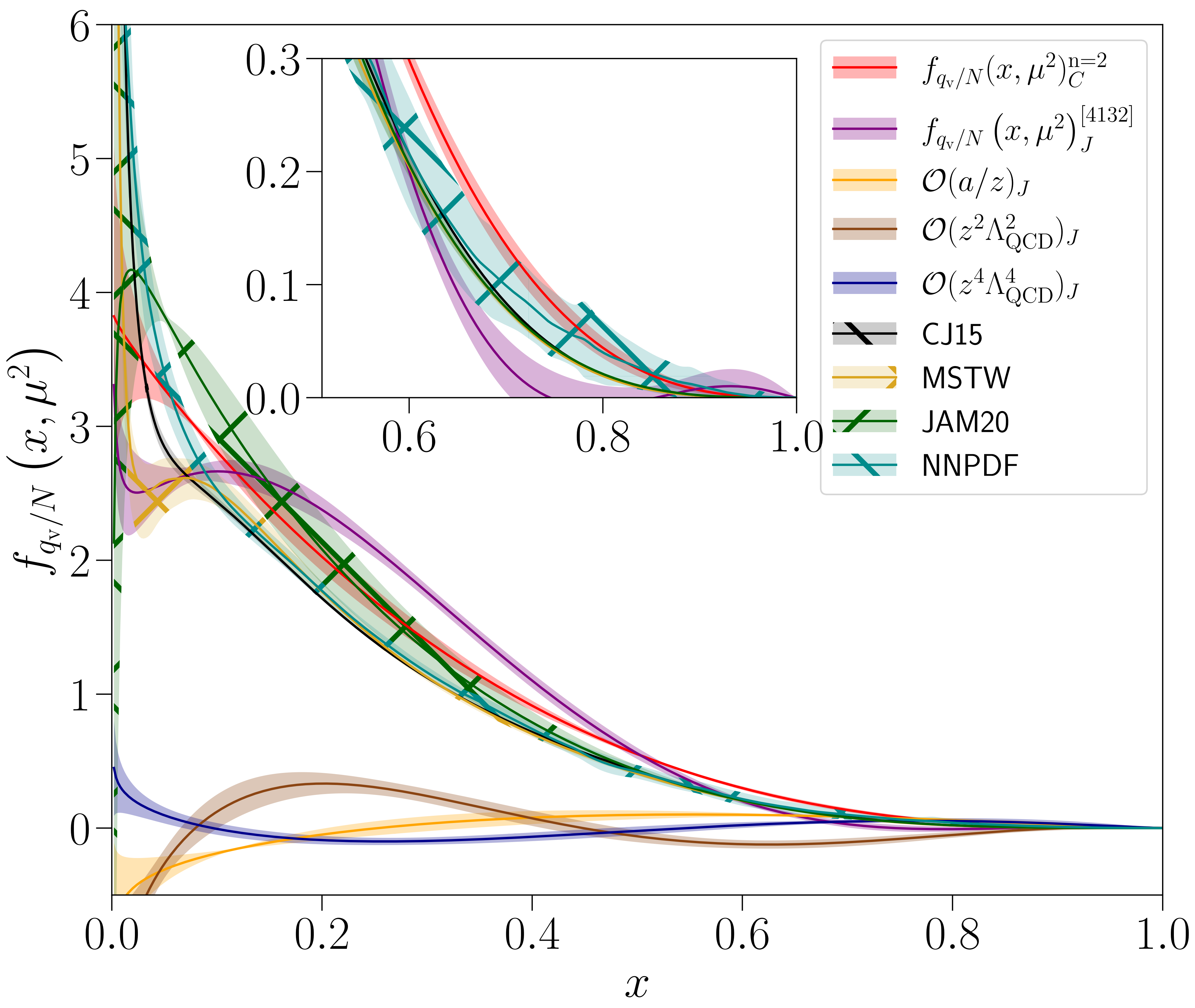
plot: C. Egerer

a=0.093fm $m_\pi \sim 350$ MeV $32^3 \times 64$

First distillation results

2+1 flavors single lattice spacing





arXiv:2107.05199 [hep-lat] C. Egerer *et. al.*

HadStruc

Conclusions

Outlook

- Understanding hadronic structure is a major goal in nuclear physics
  - Large experimental effort: JLab 12 GeV and future EIC
- Lattice QCD calculations can in principle compute (Generalized) parton distribution functions from first principles
- Controlling all systematics of the calculation is important and that complicates the solution of the inverse problem at hand
  - Both lattice spacing and higher twist effects need to be controlled
- New ideas are needed for pushing to higher momentum and improved sampling of the Ioffe time
  - The range of Ioffe time is essential for obtaining the x-dependence of distribution functions
- Synergy between lattice and experiment may be proven essential in providing precision estimates of (Generalized) parton distribution functions

END