Proton Mass Decomposition and Hadron Cosmological Constant

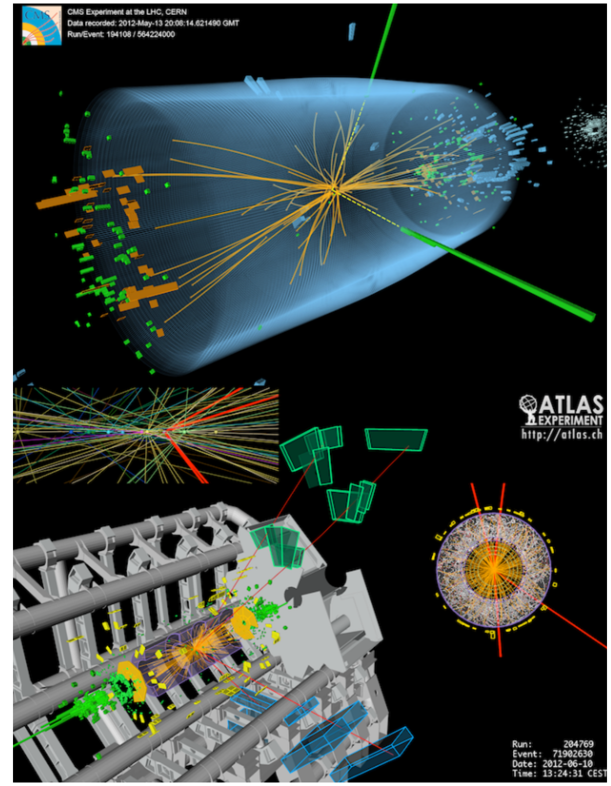
PRD104,076010 (2021)
[arXiv:2103.15768]

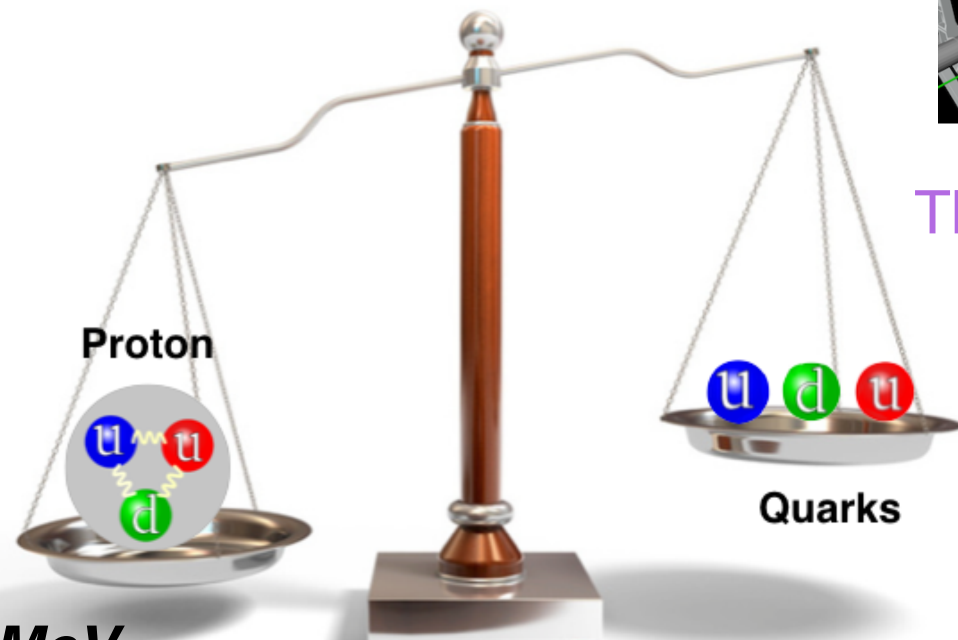
- Mass Decomposition from Hamiltonian
- Gravitational Form Factors of the Energy Momentum Tensor
- Pressure balance of hadrons
- Role of Trace anomaly in pressure equation

DWQ@25
Dec. 16, 2021

Motivation

Where does the proton mass come from, and how ?





The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ MeV

$m_d = 4.73(12)$ MeV

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

But the mass of the proton is ***938.272046(21) MeV.***

~100 times of the sum of the quark masses!

Mass (Rest Energy) from Hamiltonian

- Energy momentum tensor (EMT)

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

X. Ji (1995)

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

- With equation of motion (scale dependent)

$$H_m = \int d^3\vec{x} \sum_f m_f \bar{\psi}_f \psi_f,$$

$$H_E(\mu) = \int d^3\vec{x} \sum_f (\psi_f^\dagger i\vec{\alpha}\cdot\vec{D}\psi_f)_M,$$

Quark kinetic and potential energy

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2)_M,$$

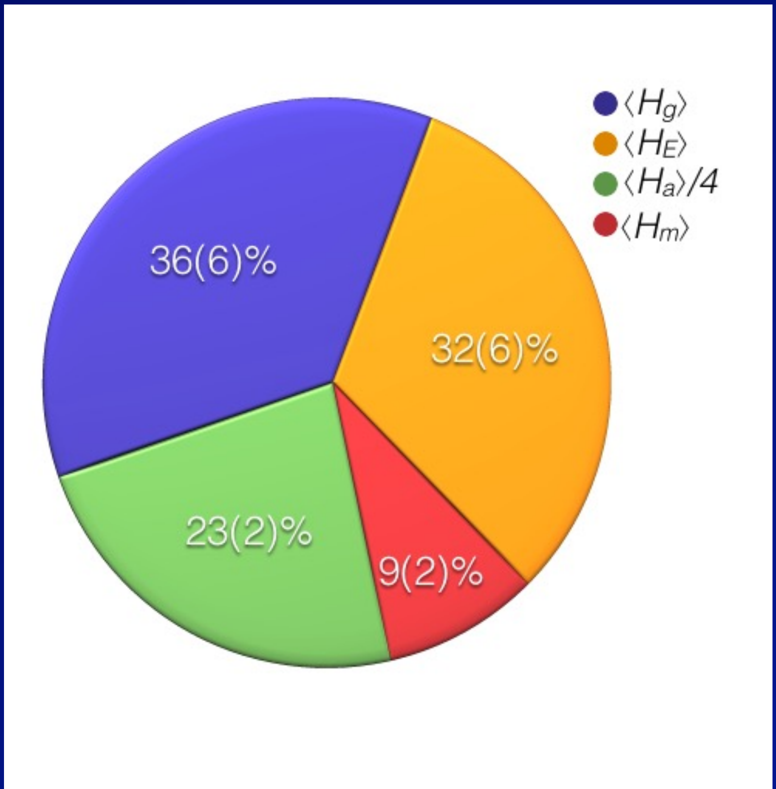
Glue field energy

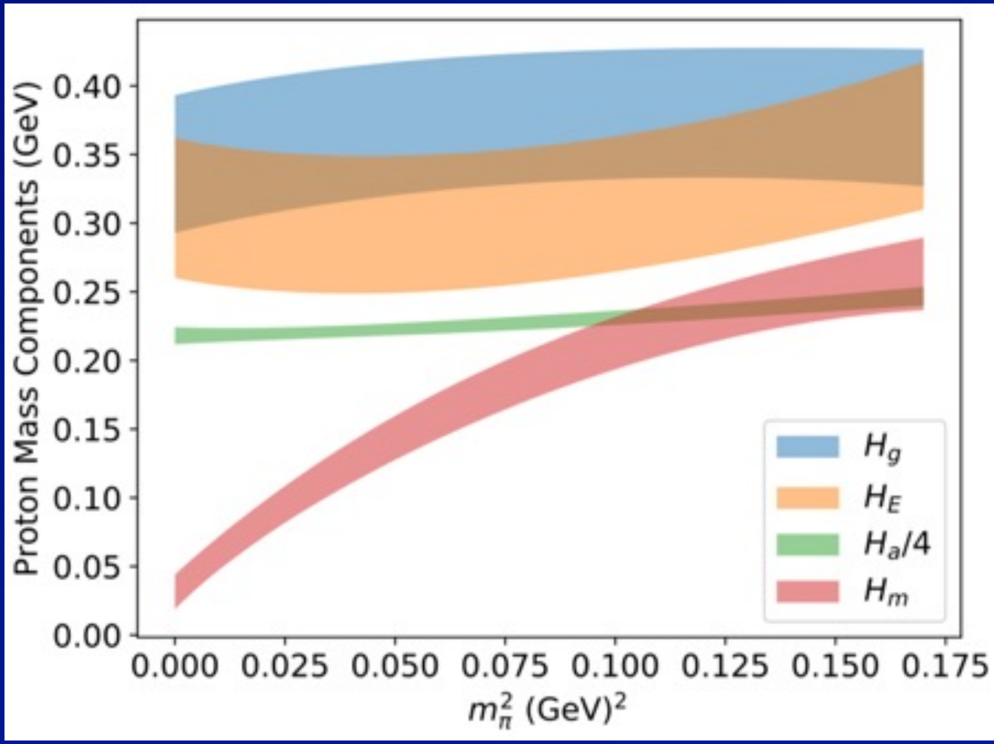
$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T_\mu^\mu)_R.$$

$$E_0 = M = \langle H_M\rangle + \langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$$

Proton Mass Decomposition

Lattice calculation with systematics (physical pion mass, continuum, infinite volume extrapolations, renormalization)





Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)
Physic 11, 118 (2018); ScienceNews, Nov. 16 (2018)

Experimental Measurable Decomposition

- Separate the EMT into traceless part and trace part

$$T_R^{\mu\nu} = \overline{T}_R^{\mu\nu} + \frac{1}{4}\eta^{\mu\nu}(T_\rho^\rho)_R$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x}\,(\frac{i}{4}\sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^{\mu}_{q\,\mu})_M,$$

Quark momentum fraction (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\,\frac{1}{2}(B^2 + E^2)_M,$$

Glue momentum fraction (scale dependent)

$$H_{tr} = \int d^3\vec{x}\,\frac{1}{4}(T_\mu^\mu)_R.$$

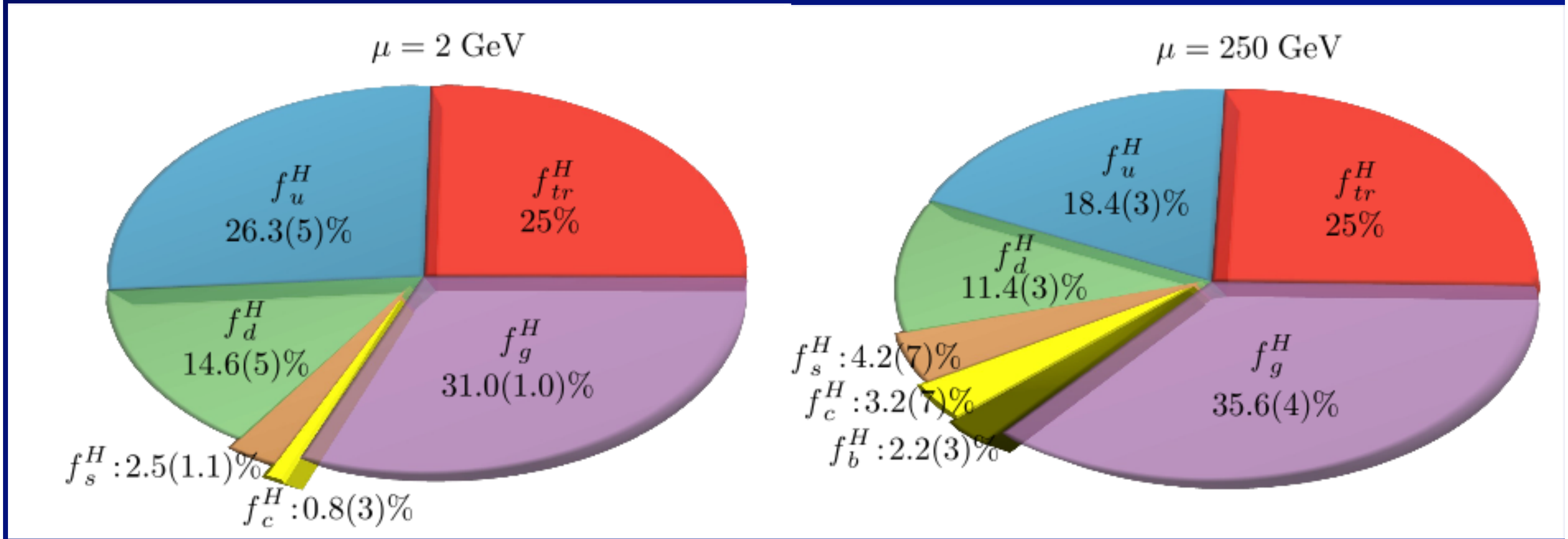
- Rest energy -- $E_0 = M = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle + \langle H_{tr}\rangle,$

$$\langle H_{q_f}(\mu)\rangle = \frac{3}{4}\sum_f \langle x\rangle_f(\mu) M, \qquad \langle H_g(\mu)\rangle = \frac{3}{4}\langle x\rangle_g(\mu) M,$$

$$\langle H_{tr}\rangle = \frac{1}{4}M.$$

<x> - momentum fraction

Mass Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4} \langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4} \langle x \rangle_g(\mu),$$

$$f_{tr}^H = \langle H_{tr} \rangle / M = \frac{1}{4}$$

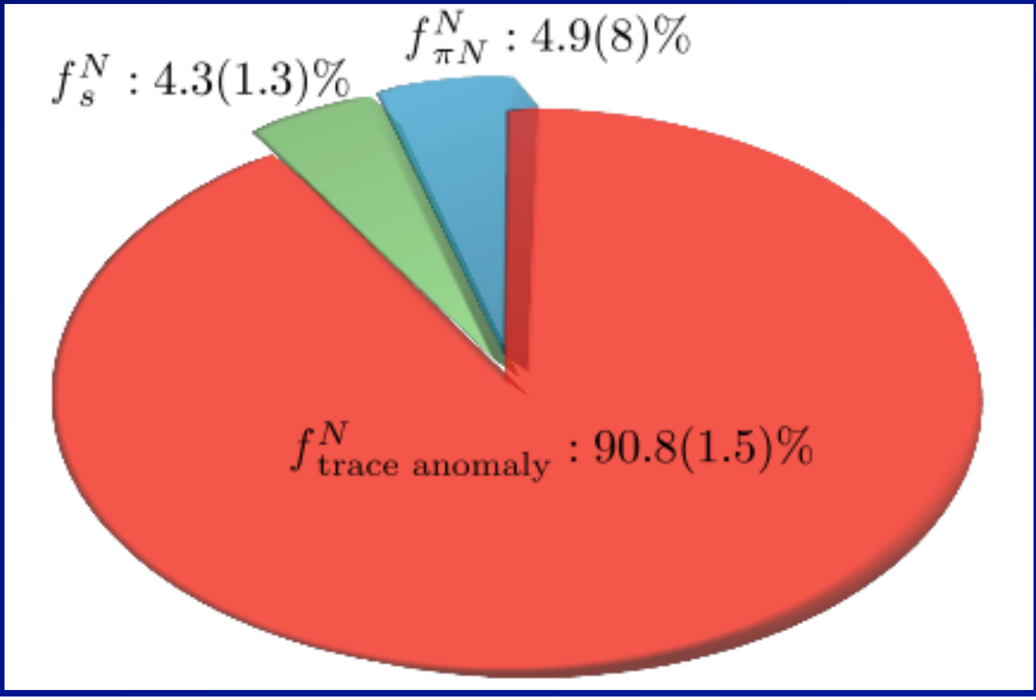
Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

Trace of EMT (¼ of Hadron Mass)

- Trace of EMT – scalar, frame independent, RG invariant

$$T^{\mu}_{\mu} = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta} \right]$$

- Lattice calculation of of quark condensate
  - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]
  - Overlap fermion (Z_m Z_s =1)
  - 3 lattices (one at physical m_π), systematics (volume, continuum)



πN sigma term

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle P | \bar{u}u + \bar{d}d | P \rangle$$

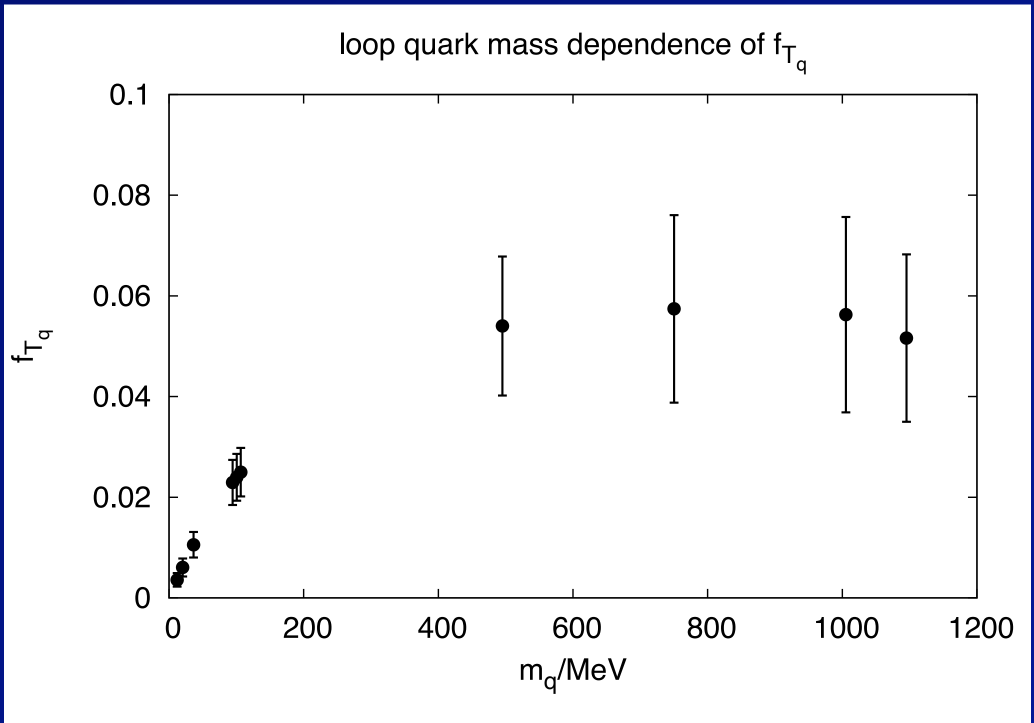
Strangeness sigma term

$$\sigma_s = m_s \langle P | \bar{s}s | P \rangle$$

$$f^N_{\pi N} = \frac{\sigma_{\pi N}}{M_N}, \qquad f^N_s = \frac{\sigma_s}{M_N}$$

Heavy Quarks

- At electroweak scale, the standard model includes Higgs, t, b, c quarks in external states
  - M. Gong et al (χ QCD) [arXiv: 1304.1191]



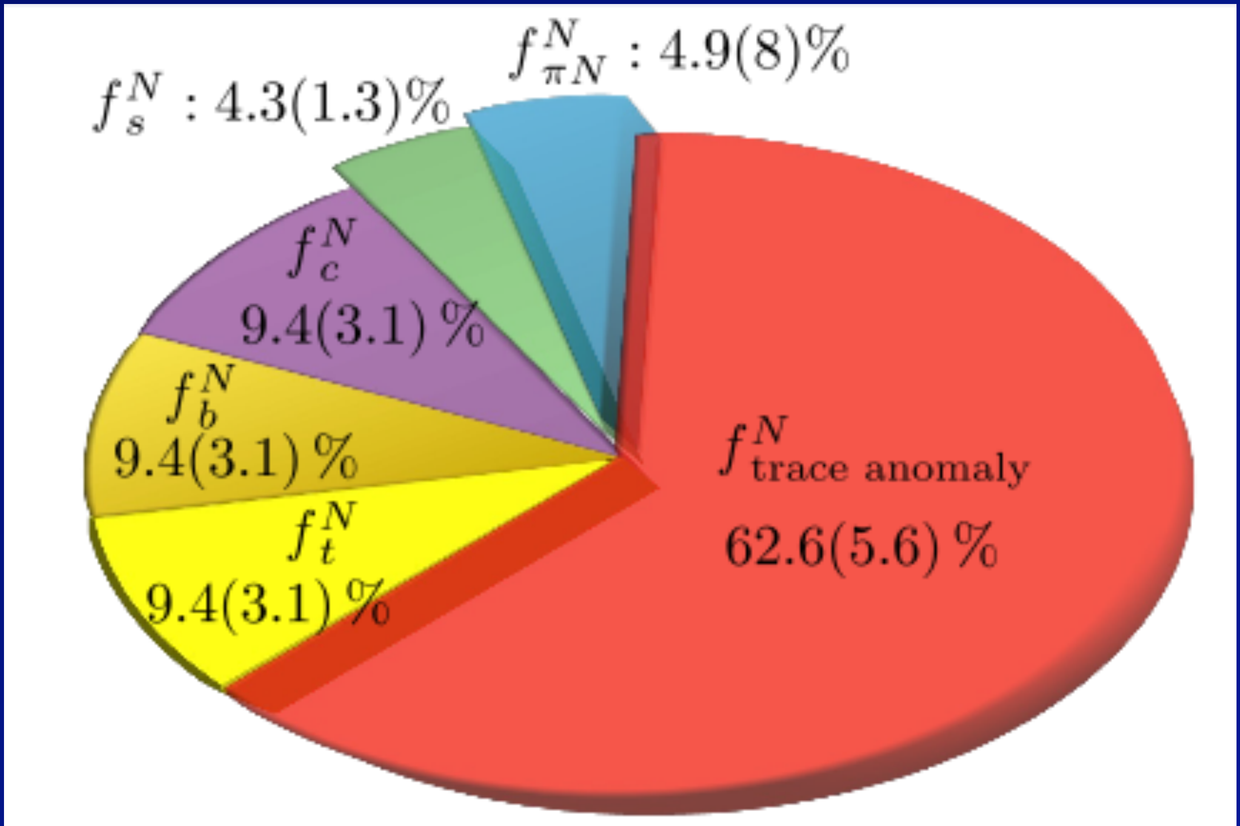
For m_q > ~ 500 MeV, $m_f \langle N|\bar{\psi}_f \psi_f|N\rangle \sim \text{constant}$

Heavy Quark Sigma Terms

- M. A. Shifman, A. Vainshtein, and V. I. Zakharov, Phys.Lett. B 78, 443 (1978) -- heavy quark expansion

$$m_h \langle N|\bar{\psi}_h \psi_h|N\rangle \sim -\frac{n_f}{3}\frac{\alpha_s}{4\pi}\langle N|G^2|N\rangle + \mathcal{O}(1/m_h)$$

$$\frac{\beta(g)}{2g} = -\frac{\beta_0}{2}\left(\frac{\alpha_s}{4\pi}\right) - \frac{\beta_1}{2}\left(\frac{\alpha_s}{4\pi}\right)^2 - \frac{\beta_2}{2}\left(\frac{\alpha_s}{4\pi}\right)^3 + \ldots \qquad \beta_0 = 11 - \frac{2}{3}n_f$$



Higgs coupling in dark matter search

$$f_{f=c,b,t}^N = \frac{m_f \langle P|\bar{\psi}_f \psi_f|P\rangle}{M_N}$$

Decoupling theorem:

$$f_c^N + f_b^N + f_t^N + f_a^N \sim \sum_H \mathcal{O}_H(1/m_H)$$

Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})_R(\mu)|P\rangle/2M_N = \bar{u}(P')[T_{1_{q,g}}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + T_{2_{q,g}}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N}$$
$$+ D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\eta^{\mu\nu}]u(P)$$

  – T_1 and T_2

$$T_{1_{q,g}}(0) = \langle x\rangle_{q,g}(\mu); \qquad \langle x\rangle_q(\mu) + \langle x\rangle_g(\mu) = 1$$

[Ji]

$$T_{1_{q,g}}(0) + T_{2_{q,g}}(0) = 2J_{g,g}(\mu); \qquad 2J_q(\mu) + 2J_g(\mu) = 1$$

  – D term: deformation of space = elastic property - [Polyakov]

  – C-bar term: pressure - [Lorce, Liu]

$$\bar{C}_q + \bar{C}_g = 0, \qquad \partial_\nu T^{\mu\nu} = 0$$

Mass and Pressure from Gravitational FF

- Mass $M_N = M_N(q) + M_N(g)$

$$M_N(q,g) = \langle P|(T^{00}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = \langle x\rangle_{q,g}(\mu)M_N + \bar{C}_{q,g}(0,\mu)M_N$$

Note: Being scale dependent, separate quark and glue T^{00} are renormalized and mixed.

- What are $\bar{C}_q$ and $\bar{C}_g$?

$$\langle P|(T^{ii}_{q,g})_{\rm M}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = 3\bar{C}_{q,g}(0,\mu)M_N$$

$$3\bar{C}_{q,g}(0,\mu)M_N = [\langle P|\eta_{\mu\nu}(T^{\mu\nu}_{q,g})_{RM}|P\rangle - \langle P|(T^{00}_{q,g})_{RM}(\mu)|P\rangle]/2M_N$$

$$\bar{C}_q(0,\mu) = \frac{1}{4}\sum_f (f^N_f - \langle x\rangle_f(\mu)),\ \ \bar{C}_g(0,\mu) = \frac{1}{4}(f^N_a - \langle x\rangle_g(\mu))$$

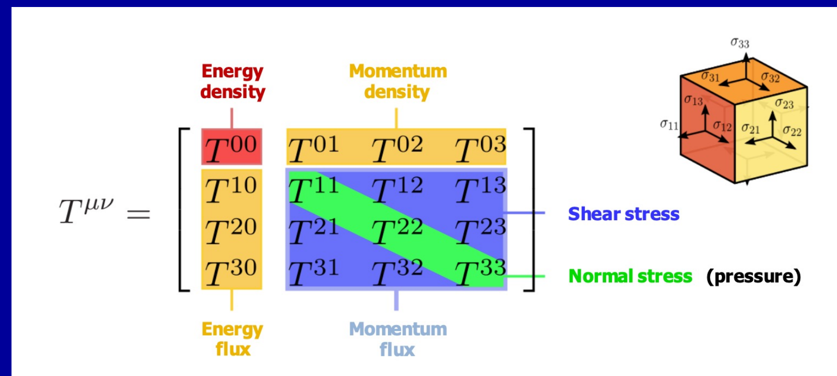
- Therefore,

$$M_N = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f^N_f + f^N_a)M_N$$

the same as from the Hamiltonian.

Trace Anomaly and Cosmological Constant

- What is trace anomaly? What dynamical role does it play, if any?
  - Note $\langle P|(T^{ii}_{q,g})_{\mathrm{M}}(\mu)|P\rangle|_{\vec{P}=0}/2M_N = 3\bar{C}_{q,g}(0,\mu)M_N$
  - $\bar{C}$ is pressure



  - The pressure balance equation

$$\bar{C}_q + \bar{C}_g = \frac{1}{4}\left(\sum_f f_f^N + f_a^N\right) - \frac{1}{4}\left(\langle x\rangle_q + \langle x\rangle_q\right) = 0$$

- Nucleon is a bubble in the sea of gluon condensate

$$\langle H_a\rangle = -\epsilon_{vac}V,$$

where,

$$\epsilon_{vac} = \frac{\beta(g)}{2g}\langle 0|F^{\alpha\beta}F_{\alpha\beta}|0\rangle < 0$$

Trace Anomaly and Cosmological Constant

- Pressure of anomaly:

$$d\langle H_a\rangle = -P_{vac}\, dV\,(dQ = T\,dS = 0), \qquad P_{vac} = -|\epsilon_{vac}| < 0$$

- Quark and glue energy

$$\langle H_E(\mu)\rangle + \langle H_g(\mu)\rangle \propto V^P$$

- Volume dependence of total rest energy

$$E_0 = |\epsilon_{vac}|V + \epsilon_{mat}V^p$$

$$\frac{dE_0}{dV} = -P_{vac} - P_k = |\epsilon_{vac}| + p\,\epsilon_{mat}V^{p-1} = 0$$

- E_0 = d E_S (d=4) ⇒ p = -1/3 (MIT bag model, $E_0 = BV + \Sigma_{q,g}/R$)
- Rest energy as the sum of scalar trace and tensor traceless parts

$$E_0 = E_T + E_S,$$

$$E_T = \langle H_{q_f}(\mu)\rangle + \langle H_g(\mu)\rangle = \frac{3}{4}[\sum_f \langle x\rangle_f(\mu) + \langle x\rangle_g(\mu)]M,$$

$$E_S = \frac{1}{4}[\langle H_m\rangle + \langle H_a\rangle]$$

Trace Anomaly and Cosmological Constant

- Stress-pressure equation

$$\bar{C}_q + \bar{C}_g = 0 \quad \Rightarrow \quad P_{\text{total}} = -\frac{d\,E_0}{d\,V} = -\frac{E_S}{V} + \frac{1}{3}\frac{E_T}{V} = 0$$

- $E_S \propto V$, $E_T \propto V^{-1/3}$

$$\downarrow$$

$$-\epsilon_{\text{vac}}$$

- Vacuum energy density is indeed a constant which is analogous to the cosmological constant in the $g^{\mu\nu}$ term as Einstein introduced for a static universe.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G\rho$$

- Freidman equation for the accelerating expansion of the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

String tension in charmonium

- Heavy quarkonium is confined by a linear potential.
- Constant vacuum energy density and flux tube

$$V(r) = |\epsilon_{vac}|\, A\, r = \sigma\, r$$

- Infinitely heavy quark with Wilson loop

$$V(r) + r\frac{\partial V(r)}{\partial r} = \frac{\langle \frac{\beta}{2g}(\int d^3\vec{x}\, F^2)\, W_L(r,T)\rangle}{\langle W_L(r,T)\rangle}.$$

Dosch, Nachtmann, Rueter - 9503386; Rothe - 9504102

- For charmonium

$$2\sigma\langle r\rangle = \langle H_\beta\rangle_{\bar{c}c} = \frac{\langle \bar{c}c|\frac{\beta}{2g}\int d^3\vec{x}\, F^2|\bar{c}c\rangle}{\langle \bar{c}c|\bar{c}c\rangle}$$

$$\langle H_\beta\rangle_{\bar{c}c} = M_{\bar{c}c} - (1+\gamma_m)\langle H_m\rangle_{\bar{c}c}.$$

- Lattice calculation of charmonium (W. Sun et al.,2012.06228)
- $\langle H_\beta\rangle_{\bar{c}c} = 199\,\mathrm{MeV}$ → σ = 0.153 GeV^2
- Cornell potential fit of charmonium → σ = 0.164(11) GeV^2

Mateu:2018zym

Mass-Pressure Correspondence

- Mass $M_N = \frac{3}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N + \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$
- Pressure $PV = \frac{1}{4}(\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N - \frac{1}{4}(\sum_f f_f^N + f_a^N)M_N$
- Other mass decomposition formulae
  - Trace $\frac{\langle P|\int d^3\vec{x}\, T^\mu_\mu(x)|P\rangle}{\langle P|P\rangle}|_{\vec{P}=0} = M_N$

$$T^\mu_\mu = \sum_f m_f \bar{\psi}_f \psi_f + \left[\sum_f m_f \gamma_m(g) \bar{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^{\alpha\beta} F_{\alpha\beta}\right]$$

No kinetic energy, no connection to pressure

  - Gravitational FF

$$M_N = (\langle x\rangle_q(\mu) + \langle x\rangle_g(\mu))M_N \qquad (\bar{C}_q + \bar{C}_g = 0)$$

No potential energy, no relation to pressure

Summary and Challenges

- Origin of proton mass and pressure in terms of quark and gluon momentum fractions and trace of EMT are closely related.
- m_q ← Higgs mechanism
- Quark condensate ← chiral symmetry breaking (restoration at T and μ)
- Trace anomaly (confinement) ← conformal symmetry breaking (conformal phases with multi-flavors and SU(N); finite $T > T_c$)
- String theory invented in hadron physic finds its home in quantum gravity.
- Cosmological constant introduced in general relativity applies naturally to hadron physics.
- Challenges for EIC to detect $T_{1_{q,g}}(t)$ from GPD and $f_a^N(t)$ from threshold production of heavy quarkonium.