

Weak Hyperon Decays from Lattice QCD

Raoul Hodgson

University of Edinburgh

raoul.hodgson@ed.ac.uk

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RBC UKQCD Collaboration

[UC Berkeley/LBNL](#)

Aaron Meyer

[BNL and BNL/RBRC](#)

Yasumichi Aoki (KEK)

Peter Boyle (Edinburgh)

Taku Izubuchi

Yong-Chull Jang

Chulwoo Jung

Christopher Kelly

Meifeng Lin

Hiroshi Ohki

Shigemi Ohta (KEK)

Amarjit Soni

[CERN](#)

Andreas Jüttner (Southampton)

[Columbia University](#)

Norman Christ

Duo Guo

Yikai Huo

Yong-Chull Jang

Joseph Karpie

Bob Mawhinney

Ahmed Sheta

Bigeng Wang

Tianle Wang

Yidi Zhao

[University of Connecticut](#)

Tom Blum

Luchang Jin (RBRC)

Michael Riberdy

Masaaki Tomii

[Edinburgh University](#)

Matteo Di Carlo

Luigi Del Debbio

Felix Erben

Vera Gülpers

Tim Harris

Raoul Hodgson

Nelson Lachini

Michael Marshall

Fionn Ó hÓgáin

Antonin Portelli

James Richings

Azusa Yamaguchi

Andrew Z.N. Yong

[KEK](#)

Julien Frison

[University of Liverpool](#)

Nicolas Garron

[Michigan State University](#)

Dan Hoyal

[Milano Bicocca](#)

Mattia Bruno

[Peking University](#)

Xu Feng

[University of Regensburg](#)

Davide Giusti

Christoph Lehner (BNL)

[University of Siegen](#)

Matthew Black

Oliver Witzel

[University of Southampton](#)

Nils Asmussen

Alessandro Barone

Jonathan Flynn

Ryan Hill

Rajnandini Mukherjee

Chris Sachrajda

[University of Southern Denmark](#)

Tobias Tsang

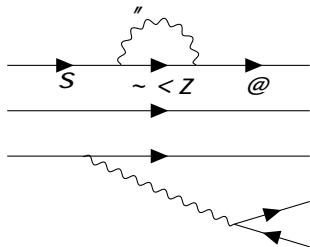
[Stony Brook University](#)

Jun-Sik Yoo

Sergey Syritsyn (RBRC)

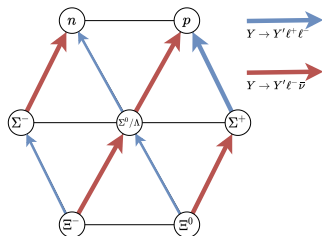
Rare Hyperon Decay

- $S \rightarrow S' \ell^+ \ell^-$ is an $S \rightarrow S' \ell^+ \ell^-$ FCNC process
- Sensitive to new physics
- Baryonic equivalent to the rare kaon decay $V \rightarrow V' \ell^+ \ell^-$
[Felix Erben Fri. 8:30 EDT]

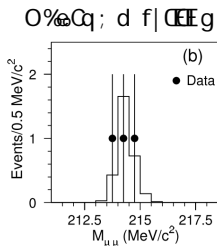


$S \rightarrow S' \ell^+ \ell^-$ Semileptonic

- $\Sigma \rightarrow \Sigma' \ell^+ \ell^-$ et al. are $S \rightarrow S' \ell^+ \ell^-$ processes
- Alternate measurement of $|V_{cs}|$, $|V_{cb}|$

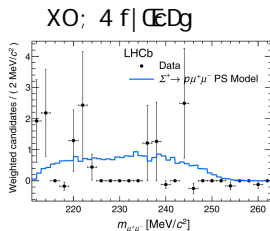


S! @Motivation: Experiment



- Observed 3 events
- $B(\Sigma^+ \rightarrow e^+ \chi)$ _{HCP} = $8.6^{+6.6}_{-5.4} \times 5.5 \times 10^{-8}$
- Originally thought this could be signal for a new particle of mass ~ 214 MeV/c²

[HyperCP 10.1103/PhysRevLett.94.021801]



- $O(10)$ events
- $B(\Sigma^+ \rightarrow e^+ \chi)$ _{LHCb} = $2.2^{+1.8}_{-1.3} \times 10^{-8}$
- No evidence for new particle
- Interest in improving measurement with the latest data, including first measurement of $B(\Sigma^+ \rightarrow e^+ \chi)$

[LHCb 10.1103/PhysRevLett.120.221803]

- SM prediction from combination of experimental input, unitarity cuts, vector meson dominance and Chiral Perturbation Theory

$$1.6 \cdot 10^{-8} < B(\pi^+ \rightarrow e^+ \nu_e) < 9.0 \cdot 10^{-8}$$

- Short distance contribution $O(10^{-12})$
- Dominated by the long distance contribution

$$\pi^+ \rightarrow e^+ \nu_e, \quad \pi^+ \rightarrow \mu^+ \nu_\mu$$

- Large range mainly coming from different types of ChPT

[X. He 10.1103/PhysRevD.72.074003]

[X. He et al. 10.1007/JHEP10(2018)040]

- Long distance part $\mathcal{M}(S \rightarrow P) \sim e^{-iE_{\text{eff}} T}$ given by amplitude

$$A = \langle P | \mathcal{O}_w | S \rangle$$

- T : EM current \mathcal{O}_w : $S \rightarrow P$ @effective weak Hamiltonian

$$\mathcal{O}_w(\not{x}) = \frac{K_F}{2} \sum_{us', ud} [\gamma_1 (k_1^u \gamma_5) + \gamma_2 (k_2^u \gamma_5) + \dots] + \text{c.c.}$$

with the 4-quark operators

$$k_1^q = (s \gamma_L) (\gamma_L l) \quad k_2^q = (s \gamma_L) (\gamma_L l)$$

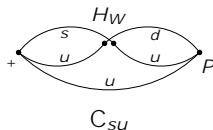
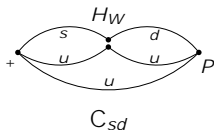
- Form factor decomposition

$$A = \tilde{p} \cdot l^2 \left[H_1 + L_1 \not{p} \right] + \not{l} \left[H_2 + L_2 \not{p} \right] \sim$$

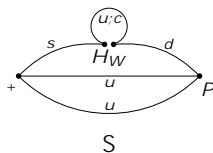
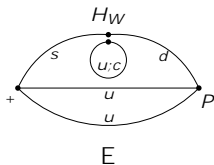
- Can be extracted from the 4pt correlation function

$$;^4 = h_N(z) O_w(z_H) T(z_J) (0)_i$$

- Requires computation of diagrams of the type



[github.com/paboyle/Grid]



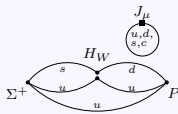
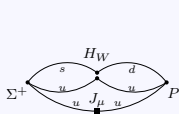
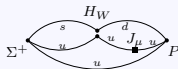
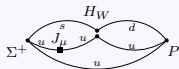
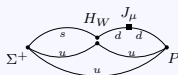
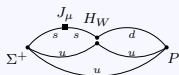
[github.com/aportelli/Hadrons]

- Can be extracted from the 4pt correlation function

$$i^4 = h_N(z) O_w(z_H) T(z_J) (0) i$$

Including vector current insertions e.g.

- Requ



[github.com/
paboyle/Grid]

[github.com/
aportelli/Hadrons]

- Can use method very similar to the rare Kaon decay $V \rightarrow \pi^+ \pi^-$
 [Felix Erben Fri. 8:30 EDT] [N. Christ et al. 10.1103/PhysRevD.92.094512]
 [N. Christ et al. 10.1103/PhysRevD.94.114516]

- Fix EM current position and integrate z_H in a window around z_J

$$R(y_a; y_b) = \int_{t_T - T}^{t_T + T_4} \langle \mathcal{O}_H | h_N(z) \mathcal{O}_w(z_H) T(z_J) | (0) \rangle$$

- Growing exponential contamination in y_a from intermediate states J and J' with $B < B'$
- Can construct and remove J' contribution similarly to J state in $V \rightarrow \pi^+ \pi^-$ decay [N. Christ et al. 10.1103/PhysRevD.100.114506]
- Understanding of finite volume corrections from J' state in progress

Summed method where operator insertion is summed over all time

[L. Maiani et al. 10.1016/0550-3213(87)90078-2]
 [C. Bouchard et al. 10.1103/PhysRevD.96.014504]

$$\begin{aligned}
 \tilde{R}^B(z) &= \sum_{t=0}^{\infty} h_f(z) T(z) i(0) i \\
 &= \sum_{n,m} \tilde{S}_{0n}^i \tilde{S}_{m0}^f T_{nm} \frac{1}{C^{E^\wedge E^\vee}} \frac{C^{(E^\wedge E^\vee)}(t-1)}{1} C^{E^\wedge t} \\
 &+ \sum_n \tilde{S}_{0n}^i \tilde{S}_{n0}^{fJ} + \sum_l \frac{\tilde{S}_{0n}^i \tilde{S}_{nl}^f T_{l0}}{C^{E_Y}} C^{E^\wedge t} \\
 &+ \sum_m \tilde{S}_{0m}^{iJ} \tilde{S}_{m0}^f + \sum_l \frac{T_{0l} \tilde{S}_{lm}^i \tilde{S}_{m0}^f}{C^{E_Y}} C^{E^\wedge t} + \sum_k \frac{\tilde{S}_{0k}^i \tilde{S}_{k0}^f T_{00}}{C^{E_W}} C^{E_W t}
 \end{aligned}$$

- First line gives regular linear z behaviour for $\wedge = \vee$ states
- Second and third lines give combination of contact terms ($z = 0$ and $z = z$) and out of order terms ($z > z$)

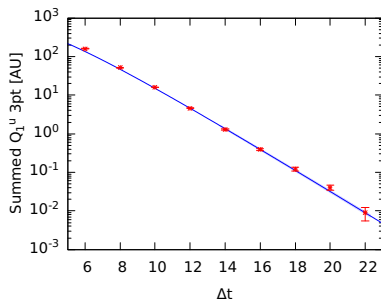
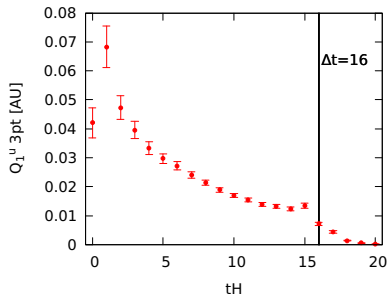
- Can extend the summed method to $\pi^+ \rightarrow e^- \nu_e$ 4pt function

$$R^4(z) = \int_0^1 \int_0^1 \mathcal{O}_H(z_H) \mathcal{O}_J(z_J) \mathcal{O}_w(z_H) T(z_J) (0) i$$

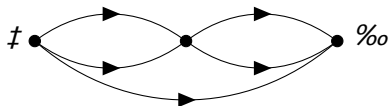
- Get contamination from intermediate states N, J, \dots
Can in theory be constructed and removed
- Full understanding of contact and out of order terms in progress
- Investigating practicality of this method
- Could benefit both the rare Hyperon and rare Kaon decays

S! @Preliminary Results

- Working towards a first exploratory calculation of this matrix element on a larger than physical pion mass ensemble
- RBC UKQCD 2 + 1 flavour $24^3 \times 64$ Iwasaki gauge configurations
 - $\beta = 1.780$ [C] and $\beta = 1.340$ [C] using Shamir domain wall fermions
- 3pt correlator $\langle h_N(z) O_w(z_H) \rangle (0)$ using Gaussian smearing without summing (left) and summed (right)

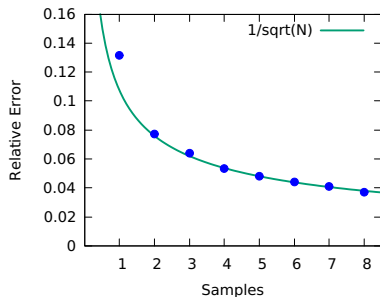
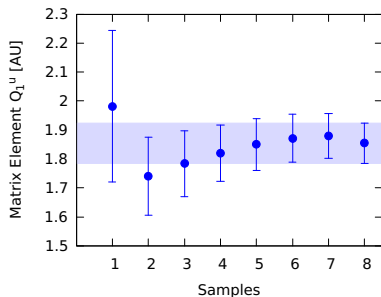


Source-Sink Sampling



- Positions t and $%0$ fixed in solves
- Volume sum for momentum projection requires 14,000 solves on 24^3 64 lattice

- Approximate with sum over N random position samples
- Could get up to $1 = \sqrt{N}$ error scaling [Y. Li et al. 10.1103/PhysRevD.103.014514]
- Only observe approx. $1 = \sqrt{N}$ scaling for this quantity



- Currently best $f_{+}^{K_{us}}$ measurement from (semi)leptonic Kaon decays $K \rightarrow \pi \ell \nu$ and $K \rightarrow \pi \ell \nu$: $f_{+}^{K_{us}} = 0.2243(4)$
 $\int_{\text{CERN}}^{\text{LHC}} \text{p} \rightarrow \text{q} \rightarrow \text{B} \rightarrow \text{q}$: [\[Particle Data Group 10.1093/ptep/ptaa104\]](#)
- Gives a $\sim 2\%$ tension with first row CKM unitarity
- Semileptonic Hyperon octet transitions can give alternative determination of $f_{+}^{K_{us}}$ from the experimental measurements and theory prediction of form factors
- From $\mathbf{r} \in (3)_F$ symmetry : $f_{+}^{K_{us}} = 0.2250(27)(?)_{SU(3)}$
[\[N. Cabibbo 10.1103/PhysRevLett.92.251803\]](#)
- $\mathbf{r} \in (3)_F$ breaking required for improved precision

- $\langle \bar{l} \gamma^\mu \gamma^5 l \rangle$ hadronic amplitude $A = h^{\alpha\beta} T^{\alpha\beta\gamma} \langle u \gamma^\mu \gamma^5 d \rangle$
where $T^{\alpha\beta\gamma} = \int d^4x \langle \bar{l}(x) \gamma^\mu \gamma^5 l(x) \mathcal{S} \rangle$
- Form factor decomposition

$$A = \int d^4x \langle \bar{l}(x) \gamma^\mu \gamma^5 l(x) \mathcal{S} \rangle = H_1 \langle \bar{l} \gamma^\mu \gamma^5 l \rangle + H_2 \frac{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle}{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle} + H_3 \frac{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle}{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle} + L_1 \langle \bar{l} \gamma^\mu \gamma^5 l \rangle + L_2 \frac{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle}{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle} + L_3 \frac{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle}{\langle \bar{l} \gamma^\mu \gamma^5 l \rangle} \sim$$

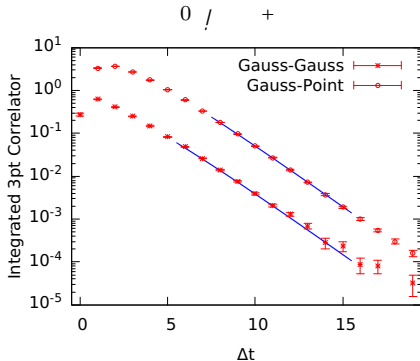
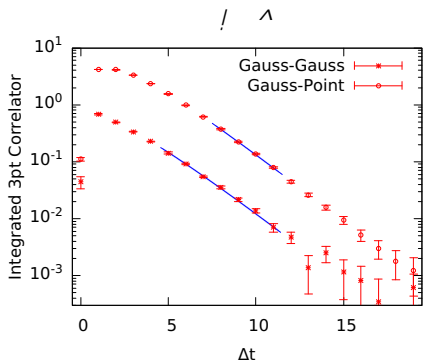
- H_1 subject to at most second order SU(3) breaking
- H_2 and $L_2 = 0$ in the SU(3) limit
- H_3 and L_3 terms suppressed in the full amplitude by the charged lepton mass m_l . Therefore negligible in electronic decays but relevant for muonic decays

- Using RBC UKQCD 2 + 1 flavour $48^3 \times 96$ Iwasaki gauge configurations and Möbius domain wall fermions.
 - $\beta = 1.730$ and $\beta' = 1.40$
- Using generalised summed method to reduce cost and improve signal

$$R^{(3)}(z) = \sum_t h_{Y^t}(z) T^{s^t u}(z) Y(0)^t$$

- Twisted boundary conditions on the strange quark to achieve $I^2 = 0$ point for the Λ and Σ decays

- Example integrated 3pt correlators of the temporal component of the current $R_0^{(3)}(z)$ [arbitrary units]



Summary

- SM prediction of $\pi^+ \rightarrow e^+ \nu_e$ is a quantity of interest
- Have a method to compute on the lattice and working towards a calculation at larger than physical pion mass
- Also working on $S \rightarrow \Lambda$ hyperon transitions at the physical point

Future outlook

- Compute $S \rightarrow \Lambda$ hyperon transitions with second lattice spacing
- Compute $\pi^+ \rightarrow e^+ \nu_e$ decay at the physical point



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