# Weak Hyperon Decays from Lattice QCD

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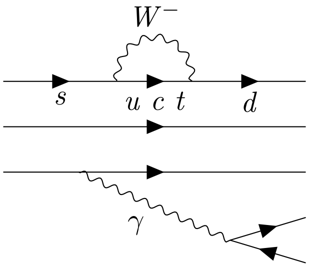
## Introduction

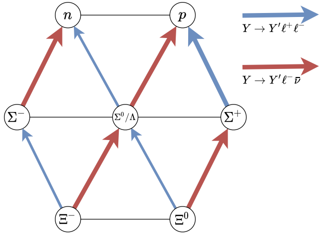
### Rare Hyperon Decay

- $\Sigma^+ \to p\ell^+\ell^-$ is an $s \to d$ FCNC process
- Sensitive to new physics
- Baryonic equivalent to the rare kaon decay $K \to \pi\ell^+\ell^-$ [Felix Erben Fri. 8:30 EDT]



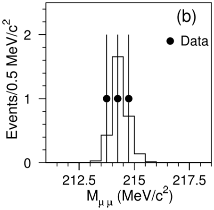

### $s \to u$ Semileptonics

- $\Sigma^- \to n\ell^-\bar{\nu}$ et al. are $s \to u$ processes
- Alternate measurement of $V_{us}$



# $s \to d$ Motivation: Experiment

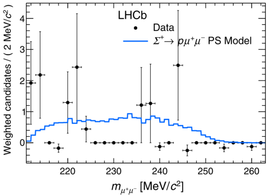
Hyper CP (2005)




- Observed 3 events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{HCP} = 8.6^{+6.6}_{-5.4} \pm 5.5 \times 10^{-8}$
- Originally thought this could be signal for a new particle of mass $\simeq 214 MeV$

[HyperCP 10.1103/PhysRevLett.94.021801]

LHCb (2018)




- $\mathcal{O}(10)$ events
- $\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{LHCb} = 2.2^{+1.8}_{-1.3} \times 10^{-8}$
- No evidence for new particle
- Interest in improving measurement with the latest data, including first measurement of $\mathcal{B}(\Sigma^+ \to pe^+e^-)$

[LHCb 10.1103/PhysRevLett.120.221803]

# $s \to d$ Motivation: Theory

- SM prediction from combination of experimental input, unitarity cuts, vector meson dominance and Chiral Perturbation Theory

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 9.0 \times 10^{-8}$$

- Short distance contribution $\mathcal{O}(10^{-12})$
- Dominated by the long distance contribution

$$\Sigma^+ \to p\gamma^* \, , \, \gamma^* \to \mu^+\mu^-$$

- Large range mainly coming from different types of ChPT

[X. He 10.1103/PhysRevD.72.074003]

[X. He et al. 10.1007/JHEP10(2018)040]

# $s \to d$ Lattice Theory

- Long distance part $\Sigma^+ \to p\gamma^*$ given by amplitude

$$\mathcal{A}_\mu = \langle p|\, T\left\{H_w J_\mu\right\} |\Sigma\rangle$$

- $J_\mu$: EM current $\qquad$ $H_w$: $s \to d$ effective weak Hamiltonian

$$H_w(x) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[C_1(Q_1^u - Q_1^c) + C_2(Q_2^u - Q_2^c) + ...\right] + \text{c.c.}$$

with the 4-quark operators

$$Q_1^q = (\bar{s}\gamma_\mu^L d)(\bar{q}\gamma_\mu^L q) \qquad Q_2^q = (\bar{s}\gamma_\mu^L q)(\bar{q}\gamma_\mu^L d)$$

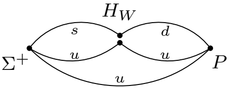
- Form factor decomposition

$$\mathcal{A}_\mu = \bar{u}_p \left[\left(q^2\gamma_\mu - \not{q} q_\mu\right)(f_1 + g_1\gamma_5) + \sigma_{\mu\nu} q^\nu (f_2 + g_2\gamma_5)\right] u_\Sigma$$

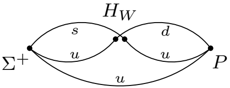
- Can be extracted from the 4pt correlation function

$$C^4 = \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

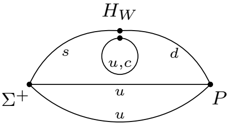
- Requires computation of diagrams of the type



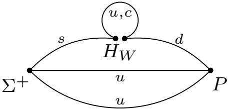

$C_{sd}$




$C_{su}$




E




S

GRID

[github.com/paboyle/Grid]

Hadrons

[github.com/aportelli/Hadrons]

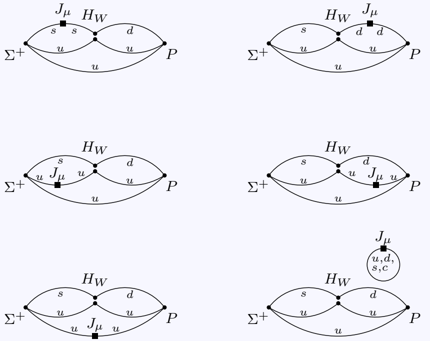
# $s \to d$ Lattice Theory

- Can be extracted from the 4pt correlation function

$$C^4 = \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

- Req

**Including vector current insertions e.g.**




GRID

[github.com/ paboyle/Grid]

Hadrons

[github.com/ aportelli/Hadrons]

# $s \to d$ Computational Approach

- Can use method very similar to the rare Kaon decay $K \to \pi \ell^+ \ell^-$ [Felix Erben Fri. 8:30 EDT] [N. Christ et al. 10.1103/PhysRevD.92.094512] [N. Christ et al. 10.1103/PhysRevD.94.114516]
- Fix EM current position and integrate $t_H$ in a window around $t_J$

$$I(T_a, T_b) = \int_{t_J - T_a}^{t_J + T_b} dt_H \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

- Growing exponential contamination in $T_a$ from intermediate states $N$ and $N\pi$ with $E < E_\Sigma$
- Can construct and remove $N\pi$ contribution similarly to $\pi\pi$ state in $K \to \pi\nu\bar{\nu}$ decay [N. Christ et al. 10.1103/PhysRevD.100.114506]
- Understanding of finite volume corrections from $N\pi$ state in progress

# 3pt Summed Method

Summed method where operator insertion is summed over all time

[L. Maiani et al. 10.1016/0550-3213(87)90078-2]
[C. Bouchard et al. 10.1103/PhysRevD.96.014504]

$$
\begin{aligned}
I^3(\Delta t) &= \sum_{t=0}^{T} \langle \psi_f(\Delta t) J(t) \bar{\psi}_i(0) \rangle \\
&= \sum_{n,m} \left[ Z^i_{0n} Z^f_{m0} J_{nm} \frac{1 - e^{-(E_n - E_m)(\Delta t - 1)}}{e^{E_n - E_m} - 1} e^{-E_m \Delta t} \right] \\
&+ \sum_{n} \left[ Z^i_{0n} Z^{fJ}_{n0} + \sum_{l} \frac{Z^i_{0n} Z^f_{nl} J_{l0}}{e^{E_l} - 1} \right] e^{-E_n \Delta t} \\
&+ \sum_{m} \left[ Z^{iJ}_{0m} Z^f_{m0} + \sum_{l} \frac{J_{0l} Z^i_{lm} Z^f_{m0}}{e^{E_l} - 1} \right] e^{-E_m \Delta t} + \sum_{k} \cancel{Z^i_{0k} Z^f_{k0}} J_{00} e^{-E_k \Delta t}
\end{aligned}
$$

- First line gives regular linear $\Delta t$ behaviour for $n = m$ states
- Second and third lines give combination of contact terms ($t = 0$ and $t = \Delta t$) and out of order terms ($t > \Delta t$)

# 4pt Summed Method

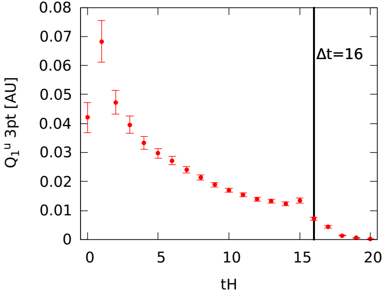
- Can extend the summed method to $\Sigma^+ \rightarrow p\gamma^*$ 4pt function

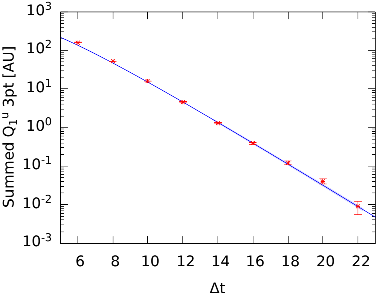
$$I^4_\mu(\Delta t) = \int_0^T dt_H \int_0^T dt_J \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

- Get contamination from intermediate states N, $\Sigma$, $N\pi$, $\Sigma\pi$ ...
  Can in theory be constructed and removed
- Full understanding of contact and out of order terms in progress
- Investigating practicality of this method
- Could benefit both the rare Hyperon and rare Kaon decays

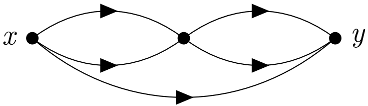
# $s \to d$ Preliminary Results

- Working towards a first exploratory calculation of this matrix element on a larger than physical pion mass ensemble
- RBC UKQCD 2 + 1 flavour $24^3 \times 64$ Iwasaki gauge configurations $a^{-1} = 1780 MeV$ and $m_\pi \simeq 340 MeV$ using Shamir domain wall fermions
- 3pt correlator $\langle \psi_N(\Delta t) H_w(t_H) \bar{\psi}_\Sigma(0) \rangle$ using Gaussian smearing without summing (left) and summed (right)



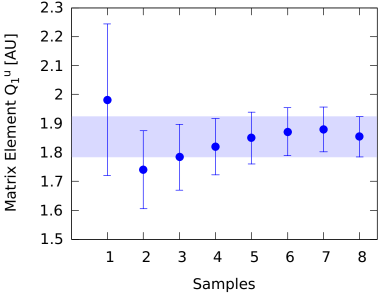



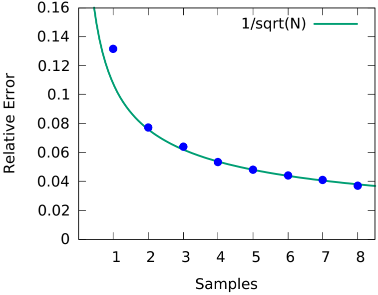
# Source-Sink Sampling




- Positions $x$ and $y$ fixed in solves
- Volume sum for momentum projection requires $\sim 14,000$ solves on $24^3 \times 64$ lattice

- Approximate with sum over N random position samples
- Could get up to $1/N$ error scaling [Y. Li et al. 10.1103/PhysRevD.103.014514]
- Only observe approx. $1/\sqrt{N}$ scaling for this quantity



# $s \to u$ Motivation

- Currently best $V_{us}$ measurement from (semi)leptonic Kaon decays $K \to \ell\bar{\nu}$ and $K \to \pi\ell\bar{\nu}$ : $|V_{us}| = 0.2243(4)$
  [Andrew Yong Thur. 12:20 EDT] [Particle Data Group 10.1093/ptep/ptaa104]
- Gives a $\sim 2\sigma$ tension with first row CKM unitarity

- Semileptonic Hyperon octet transitions can give alternative determination of $|V_{us}|$ from the experimental measurements and theory prediction of form factors
- From $SU(3)_F$ symmetry : $|V_{us}| = 0.2250(27)(?)_{SU(3)}$
  [N. Cabibbo 10.1103/PhysRevLett.92.251803]
- $SU(3)_F$ breaking required for improved precision

# $s \to u$ Theory

- $Y \to Y'\ell\bar{\nu}$ hadronic amplitude $\mathcal{A}_\mu = \langle Y'| J_\mu^{s\to u} |Y\rangle$ where $J_\mu^{s\to u} = \bar{u}\gamma_\mu(1-\gamma_5)s$
- Form factor decomposition

$$\begin{aligned}\mathcal{A}_\mu =&\bar{u}' \left[ f_1\gamma_\mu + f_2\frac{\sigma_{\mu\nu}q^\nu}{M+M'} + f_3\frac{q_\mu}{M+M'} \right.\\ &\left. + g_1\gamma_\mu\gamma_5 + g_2\frac{\sigma_{\mu\nu}q^\nu\gamma_5}{M+M'} + g_3\frac{q_\mu\gamma_5}{M+M'}\right] u\end{aligned}$$

- $f_1$ subject to at most second order SU(3) breaking
- $f_3$ and $g_2 = 0$ in the SU(3) limit
- $f_3$ and $g_3$ terms suppressed in the full amplitude by the charged lepton mass $m_\ell$. Therefore negligible in electronic decays but relevant for muonic decays

# $s \to u$ Lattice Setup

- Using RBC UKQCD $2+1$ flavour $48^3 \times 96$ Iwasaki gauge configurations and Möbius domain wall fermions. $a^{-1} = 1730 MeV$ and $m_\pi \simeq 140 MeV$
- Using generalised summed method to reduce cost and improve signal

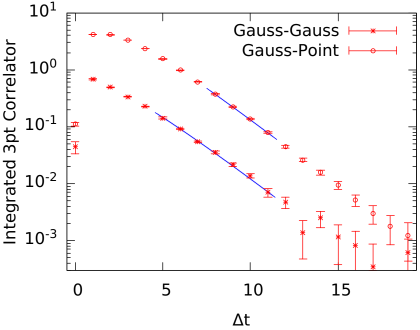
$$I_\mu^{(3)}(\Delta t) = \sum_t \langle \psi_{Y'}(\Delta t)\ J_\mu^{s\to u}(t)\ \bar{\psi}_Y(0)\rangle$$

- Twisted boundary conditions on the strange quark to achieve $q^2 = 0$ point for the $\Sigma \to N$ and $\Xi \to \Sigma$ decays

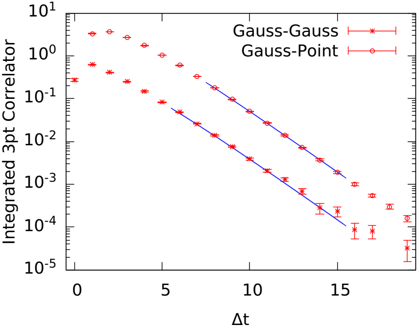
# $s \to u$ Preliminary Results

- Example integrated 3pt correlators of the temporal component of the current $I_0^{(3)}(\Delta t)$ [arbitrary units]

$\Sigma^- \to n$




$\Xi^0 \to \Sigma^+$



# Summary / Outlook

## Summary

- SM prediction of $\Sigma^+ \to p\ell^+\ell^-$ is a quantity of interest
- Have a method to compute on the lattice and working towards a calculation at larger than physical pion mass
- Also working on $s \to u$ hyperon transitions at the physical point

## Future outlook

- Compute $s \to u$ hyperon transitions with second lattice spacing
- Compute $\Sigma^+ \to p\ell^+\ell^-$ decay at the physical point

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