DWQ@25

BNL-HET & RBRC Joint Workshop

Near-Physical Point Isospin-Breaking Corrections to $K_{\ell 2}/\pi_{\ell 2}$

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OVERVIEW

In experiments, inclusive rates are measured

$\mathcal{O}(\alpha)$ and $\mathcal{O}(m_u \neq m_d)$ correction

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) \equiv \Gamma^{\text{tree}} + \delta\Gamma = \Gamma^{\text{tree}}(1 + \boxed{\delta R_P})$$ where $P = K, \pi$

where…

$$\Gamma^{\text{tree}} = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_{\ell^+}^2 \left(1 - \frac{m_{\ell^+}^2}{M_{P^+}^2}\right)^2$$

f_P defined in isospin-symmetric QCD theory ($\alpha = 0, m_u = m_d$)…

Interested in the following:

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \underbrace{\boxed{\frac{\Gamma(K^+ \to \mu^+ \nu_{\mu^+}[\gamma])}{\Gamma(\pi^+ \to \mu^+ \nu_{\mu^+}[\gamma])} \frac{M_{K^+}^3}{M_{\pi^+}^3} \frac{(M_{\pi^+}^2 - m_{\mu^+}^2)^2}{(M_{K^+}^2 - m_{\mu^+}^2)^2}}}_{\text{Experiment}} \underbrace{\boxed{\frac{(f_\pi / f_K)^2}{1 + (\delta R_K - \delta R_\pi)}}}_{\text{Theory}}$$

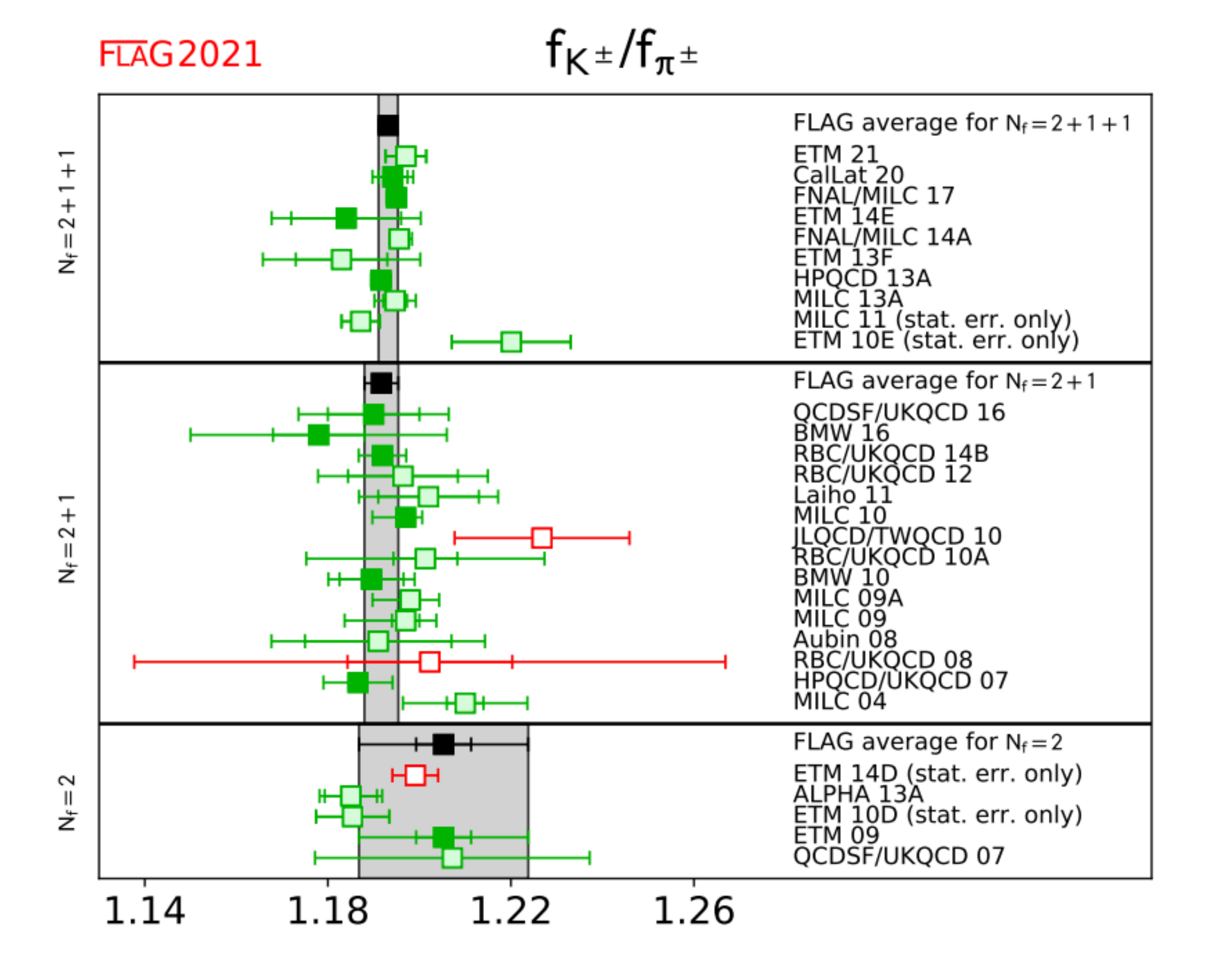
OVERVIEW

Goal: determine $\frac{V_{us}}{V_{ud}}$ using lattice QCD+QED inputs

- Testing unitarity of CKM → search for BSM physics
- From PDG2020…

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

- Theory input (eg. $f_{K^\pm}/f_{\pi^\pm}$) reached percent-level precision or better!
- Need to include isospin-breaking (IB) contributions ($\mathcal{O}(\alpha)$ and $\mathcal{O}(m_u \neq m_d)$) to improve precision on V_{ud} and V_{us}



ISOSPIN-BREAKING CORRECTIONS

Four-fermion operator for this weak decay: $O_W = (\bar{\nu}\gamma_L^{\mu}\ell)(\bar{q}_1\gamma_L^{\mu}q_2)$

$$\mathcal{A}_P^{r,s} \equiv \langle \ell^+, r; \nu_\ell, s | O_W | P^+ \rangle \,, \; \mathcal{A}_P^{0\,r,s} = (\bar{u}^s \gamma_L^{\tau} v^r)\, p_P^{\tau} f_P$$

$$\Gamma^{\text{tree}} = K \sum_{r,s} |\mathcal{A}_P^{0\,r,s}|^2$$

$$\delta\Gamma = 2K \sum_{r,s} \mathrm{Re}\left[\mathcal{A}_P^{0\,r,s*} \boxed{\delta\mathcal{A}_P^{r,s}}\right]$$

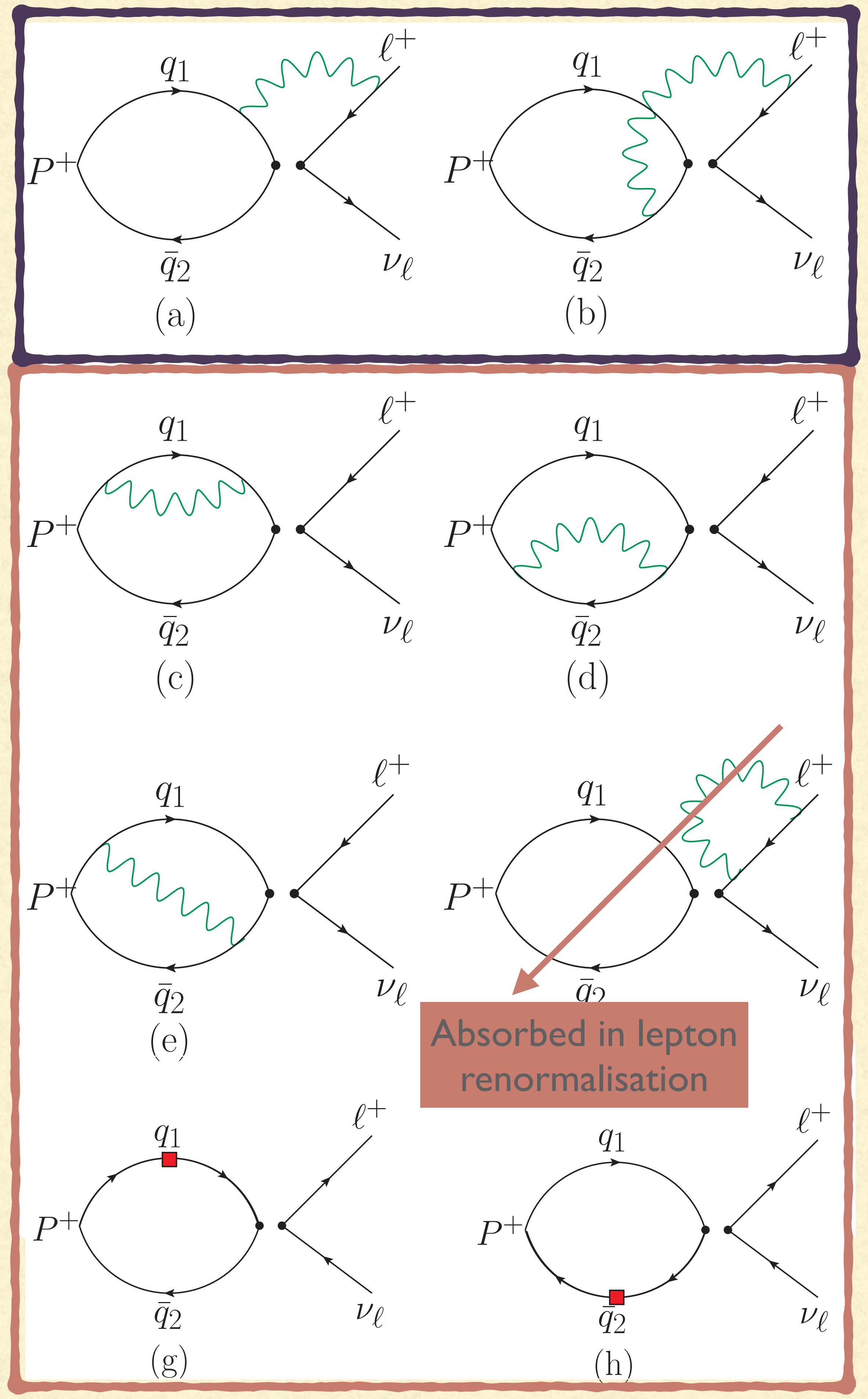
IB correction to matrix element

Perturbative expansion in α and bare quark mass shift $\Delta m = m - m^0$ [1]:

$$\langle O \rangle = \langle O \rangle_0 + \alpha \frac{\partial}{\partial \alpha} \langle O \rangle \Big|_{\alpha=0} + \sum_f \Delta m_f \frac{\partial}{\partial m_f} \langle O \rangle \Big|_{m_f = m_f^0} + \ldots$$

Non-factorisable correction

Factorisable correction



[1] G. M. De Divitis *et* al. Phys. Rev. D 87, 114505

STRATEGY

New challenge: Compute $\delta R_{K\pi}$ in (near) physical point simulation

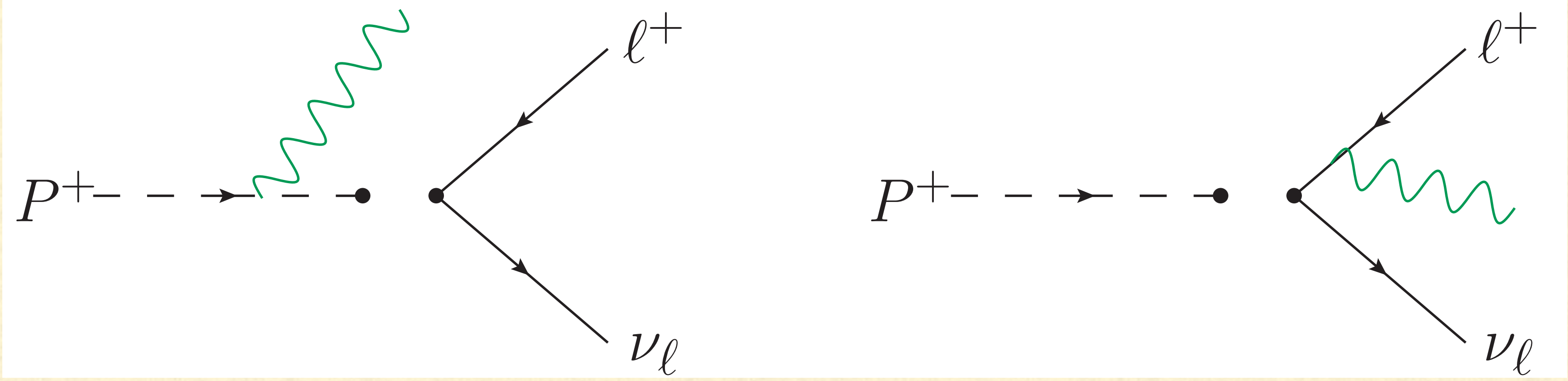
- Virtual photon correction to amplitude → Infrared divergence
- Pioneering work done by the RM123-Southampton collaboration, see…
  - PoS, CD15:023, June-July 2016
  - PhysRevLett., 120.072001, Feb 2018
  - Phys. Rev. D, 100:034514, Aug 2019

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) = \Gamma_0 + \Gamma_1,$$
$$= \lim_{L\to\infty} \left(\boxed{\Gamma_0(L) - \Gamma_0^{\rm pt}(L)}\right) + \boxed{\lim_{m_\gamma \to 0} \left(\Gamma_1(m_\gamma, \Delta E) + \Gamma_0^{\rm pt}(m_\gamma)\right)}$$

$\mathcal{O}(1/L)$ universal FVE removed †

Analytic calculation when $E_{res} < E_\gamma < \Delta E$

$$\Gamma(P^+ \to \ell^+ \nu_\ell[\gamma]) = \Gamma^{\rm tree}(1 + \delta R_P)$$



† For structure-dependent FVE, see 2109.05002.

SIMULATION DETAILS

- Physical point Möbius Domain-Wall Fermions (DWF) [2]
- $N_f = 2 + 1, \quad 48^3 \times 96, \quad L_S = 24, \quad a^{-1} \simeq 1730\mathrm{MeV} \qquad \Rightarrow M_\pi = 139.15(36)\,\mathrm{MeV}$
- Valence light: physical mass ZMöbius, $L_s = 10$
- 60 configurations (20 trajectory-spacing)

[2] RBC/UKQCD Collaboration PhysRevD.93.074505 (2014)

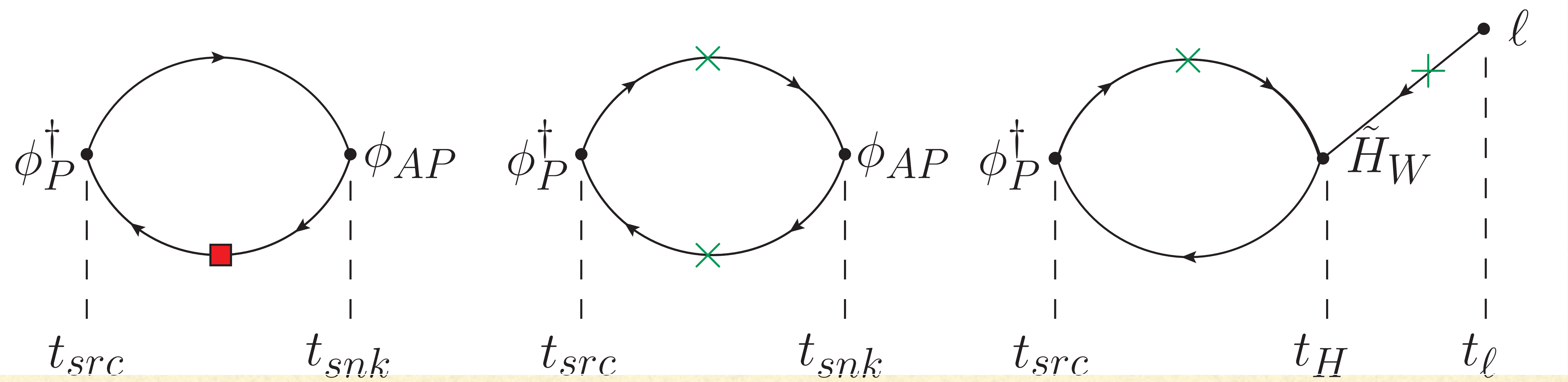
IMPLEMENTATION

Interpolators from standard 2pt analysis:

$$\phi_P = \bar{q}_2\gamma^5 q_1, \quad \phi_{AP} = \bar{q}_2\gamma^0\gamma^5 q_1$$

Four-fermion operator with neutrino leg amputated:

$$\tilde{O}_{W,\beta} = (\gamma^\mu_L \ell)_\beta(\bar{q}_2\gamma^\mu_L q_1), \quad \gamma^\mu_L = \gamma^\mu(\mathbf{1} - \gamma^5)$$



Examples of sequential insertions for (non-)factorisable correlation functions

- $\mathrm{QED_L}$ theory
- Electro-quenched calculation
- Sequential insertion of scalar (S) current and local QED current (renormalised by Z_V) in Feynman gauge on quark propagators
- 96 gauge-fixed wall sources per config.
- Muon propagator generated using free DWF, with twisted B.C. for 4-mom conservation
- 8 $t_\ell - t_H$ separations
- Neutrino propagator included during data processing

FROM SIMULATION TO NATURE

- Linear expansion to physical/isosymmetric QCD point using derivatives w.r.t simulated bare parameters
- For an observable, X (eg hadronic mass),

$$X = X^0 + \alpha\partial_\alpha X|_{\alpha=0} + \sum_f \Delta m_f \partial_{m_f} X|_{m_f=m_f^0}, \quad \text{where} \quad \partial_y X = \frac{\partial X}{\partial y}$$

Tuning to physical point $(\alpha, m_u \neq m_d)$

- Using $\alpha = \alpha_{\mathrm{EM}}$, tune $\{\Delta m_f\}$ such that $M^2_{\pi^+}, M^2_{K^+}, M^2_{K^0}$ match their experimental masses
- Scale setting with Ω^- baryon

Tuning to QCD point $(\alpha = 0)$

- Introduce unphysical mesons as proxy [3,4]:

$$M^2_{ud} = \frac{1}{2}(M^2_{\bar{u}u} + M^2_{\bar{d}d}) \approx 2Bm_{ud} + \dots,$$
$$\Delta M^2 = M^2_{\bar{u}u} - M^2_{\bar{d}d} \approx 2B(m_u - m_d) + \dots,$$
$$M^2_{K\chi} = \frac{1}{2}\left(M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}\right) \approx 2Bm_s + \dots$$

- Set $\alpha = 0$, tune $\{\Delta m'_f\}$ to the following values:

$$M^2_{ud} = (M^{exp}_{\pi^0})^2, \quad M^2_{K\chi} = \frac{1}{2}\left((M^{exp}_{K^+})^2 + (M^{exp}_{K^0})^2 - (M^{exp}_{\pi^+})^2\right),$$
$$\Delta M^2 = \begin{cases} 0 & \text{if} \quad m_u = m_d \\ (\Delta M^2)^{phys} & \text{if} \quad m_u \neq m_d \end{cases}$$

[3] BMW Collaboration PhysRevLett.111.252001 (2013), [4] BMW Collaboration Nature 593 (2021)

FROM SIMULATION TO NATURE

- $(\Delta M^2)^{phys} \approx 2B(m_u - m_d) \rightarrow$ check $\dfrac{m_u}{m_d}$

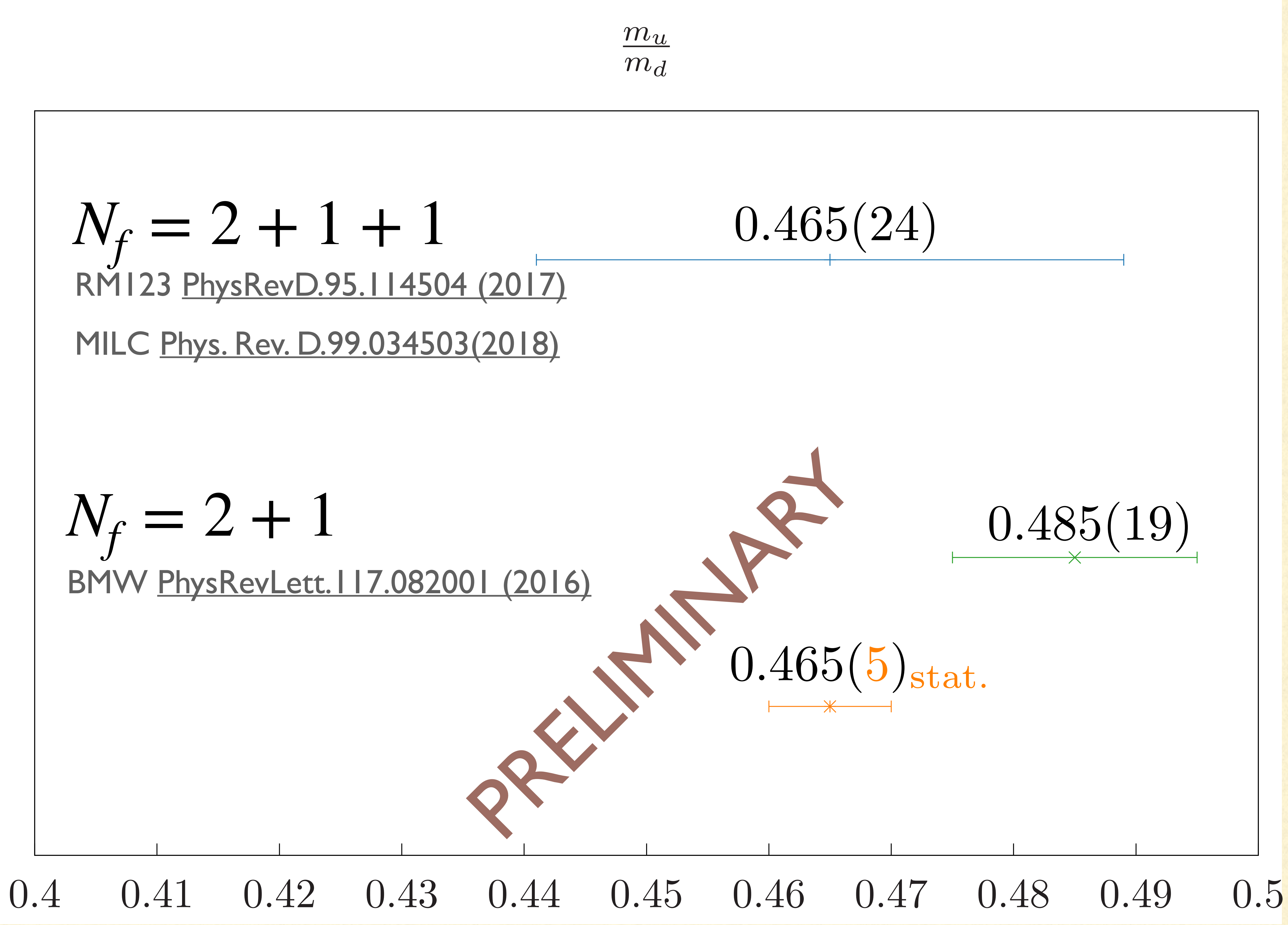
$$(\Delta M^2)^{phys} = -13570(96)_{\text{stat.}}\ \text{MeV}^2$$

- Using inputs in $\overline{\text{MS}} = 2\text{GeV}\ldots$

$$B = 2804(34)\text{MeV}, \quad [5]$$
$$m_{ud} = 3.315(40)\text{MeV} \quad [2]$$

(!) Single lattice spacing; no cont. extrapolation result:

$$\frac{m_u}{m_d} = 0.465(5)_{\text{stat.}}$$ PRELIMINARY



- Comparison with FLAG results (see fig.)…

[2] RBC/UKQCD Collaboration PhysRevD.93.074505 (2014) [5] RBC/UKQCD Collaboration Phys.Rev.D93.054502 (2016)

FROM SIMULATION TO NATURE

- Procedure to separate QCD and QED
  $\rightarrow$ Correction to Dashen's Theorem

$$\epsilon = \frac{\left(M^2_{K^+} - M^2_{K^0}\right)^{\gamma}}{M^2_{\pi^+} - M^2_{\pi^0}} - 1$$

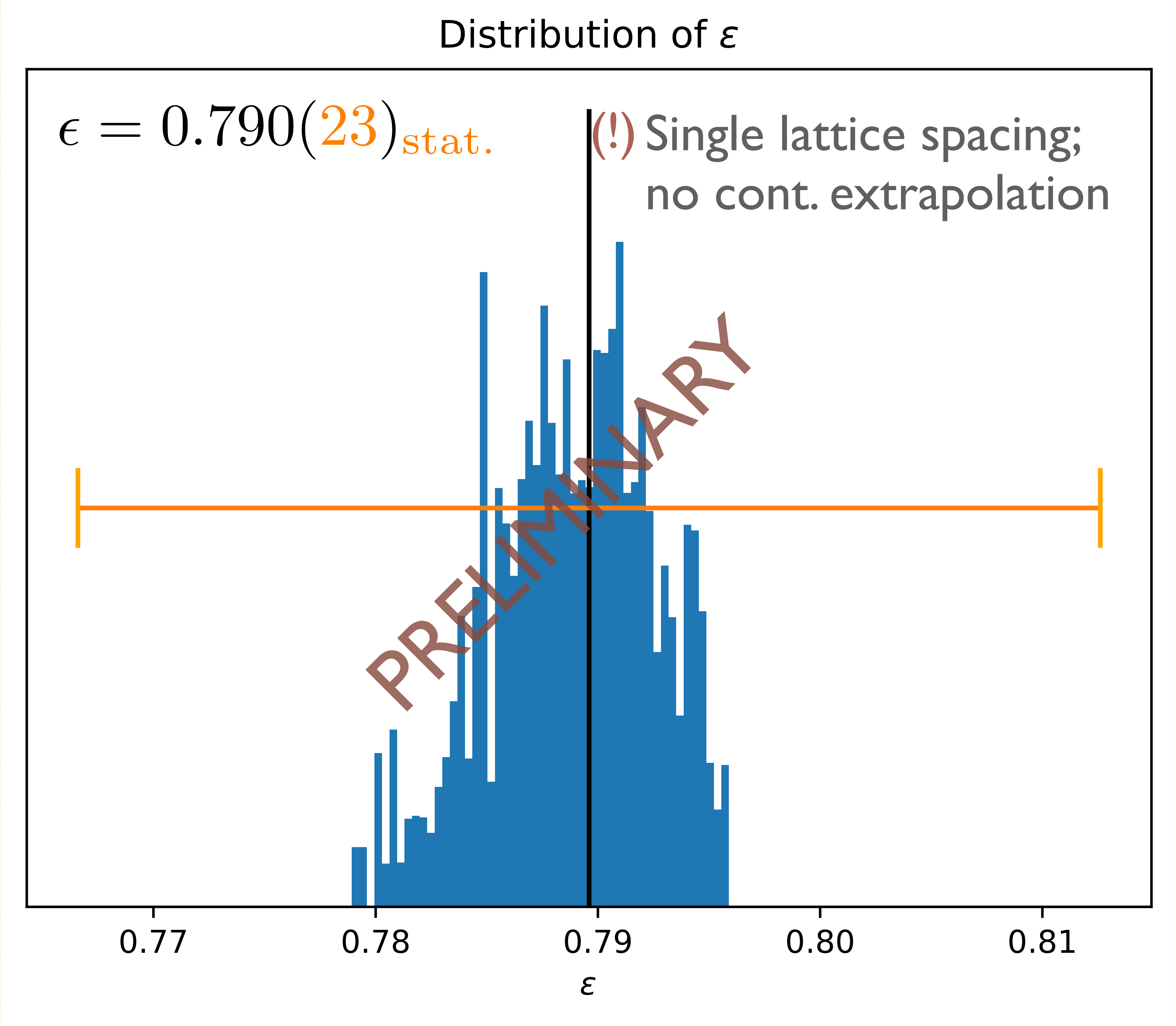
- Comparison with…

$(N_f = 2 + 1 + 1) : \ \epsilon = 0.79(6)$ MILC(2018) RM123 (2017) BMW (2014)

$(N_f = 2 + 1) : \ \epsilon = 0.73(17)$ BMW (2016)

$(N_f = 2 + 1) : \ \epsilon = 0.79(1)_{\mathrm{stat.}}(^{+8}_{-11})_{\mathrm{sys.}}$ MILC (2019)

- Fig: a distribution of ϵ generated by varying fit range of correlation functions



FACTORISABLE AMPLITUDE CORRECTION

- Obtain decay const. and amplitude correction from PP (γ^5, γ^5) and PA $(\gamma^5, \gamma^0\gamma^5)$ correlation functions

- Eg. pion tree-level correlator (excl. backward prop):

$$\hat{C}^i_\pi(t) = A^{i,0}_\pi e^{-M^0_\pi t} \qquad i = PP, PA$$

where $$A^{PP/PA}_\pi = \frac{\langle 0|\phi_{P/AP}|\pi\rangle \langle \pi|\phi^\dagger_P|0\rangle}{2M_\pi}$$

- The QED/scalar insertion correlators are:

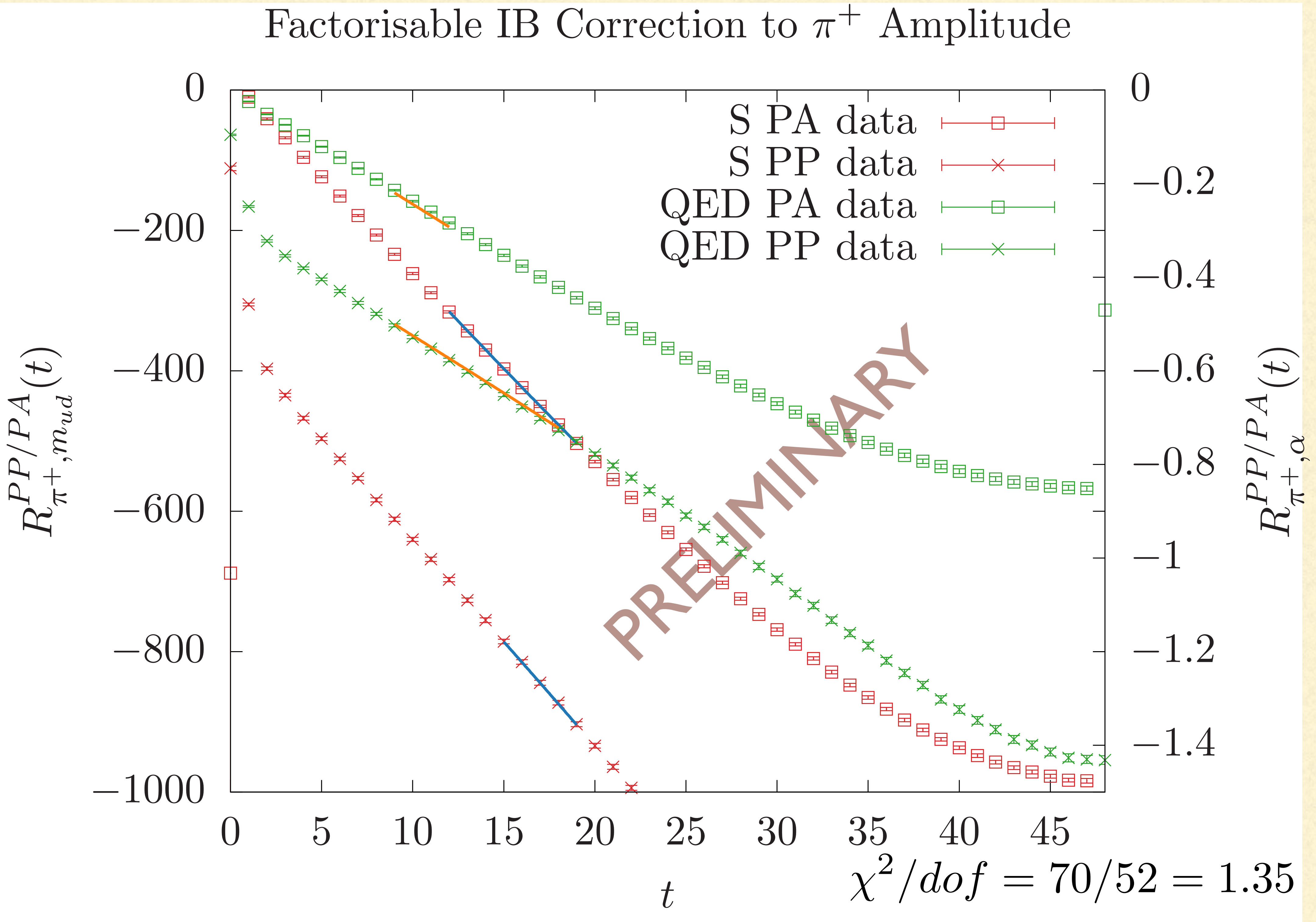
$$\partial_{g_j} C^i_\pi(t) = (\partial_{g_j} A^i_\pi + A^{i,0}_\pi \partial_{g_j} M_\pi t) e^{-(M^0_\pi + \Delta g_j \partial_{g_j} M_\pi)t}$$

$$g_j = \{\alpha, m_u, m_d, m_s\}$$

- Then, the ratios give:

$$R^i_{\pi,g_j}(t) \equiv \frac{\partial_{g_j} C^i_\pi(t)}{\hat{C}^i_\pi(t)} = \boxed{\frac{\partial_{g_j} A^i_\pi}{A^{i,0}_\pi}} - \boxed{\partial_{g_j} M_\pi} t$$

Simultaneous fit → amplitude corrections



$\chi^2/dof = 70/52 = 1.35$

p−value = 0.099

NON-FACTORISABLE AMPLITUDE CORRECTION

- Spectral representation of new correlation function (excl. backward prop and excited states):

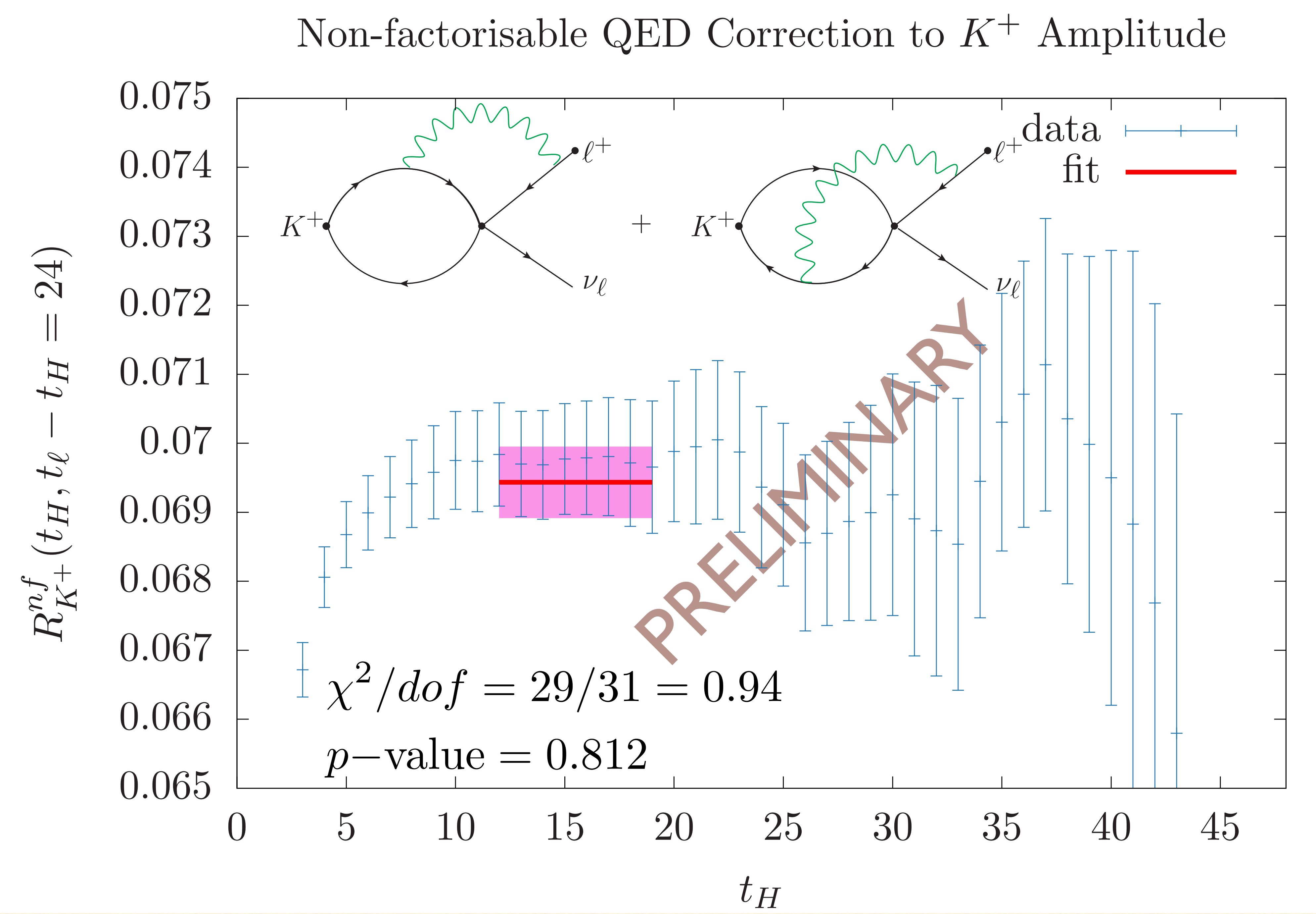
$$\partial_\alpha C_P^{nf}(t_H, t_\ell - t_H)_{\varepsilon\beta} = \frac{\langle P|\phi_P^\dagger|0\rangle}{4E_\ell M_P^0}\left[\partial_\alpha \mathcal{A}_P^{nf}(-\not{p}_\ell - im_\ell)\right]_{\varepsilon\beta} e^{-M_P^0 t_H} e^{-E_\ell(t_\ell - t_H)}$$

- The ratio of correlator traces give:

$$R_P^{nf}(t_H, t_\ell - t_H) = \frac{\mathrm{Tr}\left[\not{p}_\nu \partial_\alpha C_P^{nf}(t_H, t_\ell - t_H)\gamma_L^0\right]}{\mathrm{Tr}\left[\not{p}_\nu C_{P,0}^{nf}(t_H, t_\ell - t_H)\gamma_L^0\right]},$$

$$\overset{t_\ell \gg t_H \gg t_{src}}{=} \boxed{\frac{\mathrm{Tr}\left[\not{p}_\nu \partial_\alpha \mathcal{A}_P^{nf}(-\not{p}_\ell + im_\ell)\gamma_L^0\right]}{M_P^0 f_P\,\mathrm{Tr}\left[\not{p}_\nu \gamma_L^0(-\not{p}_\ell + im_\ell)\gamma_L^0\right]}}$$

Non-factorisable amplitude correction



$\frac{V_{us}}{V_{ud}}$ FROM PHYSICAL POINT CALCULATION

Experiment   Theory

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \boxed{\frac{\Gamma(K^+ \to \mu^+ \nu_{\mu^+}[\gamma])}{\Gamma(\pi^+ \to \mu^+ \nu_{\mu^+}[\gamma])} \frac{M_{K^+}^3}{M_{\pi^+}^3} \frac{(M_{\pi^+}^2 - m_{\mu^+}^2)^2}{(M_{K^+}^2 - m_{\mu^+}^2)^2}} \boxed{\frac{(f_\pi / f_K)^2}{1 + (\delta R_K - \delta R_\pi)}}$$

- PDG → inputs for inclusive rates and masses
- Simultaneous fits of lattice correlators → for calculating $\left(1 + (\delta R_K - \delta R_\pi)\right) f_K^2 / f_\pi^2$
- All lattice inputs available to calculate V_{us}/V_{ud} at physical point
- Full systematics error budget on $\delta R_K - \delta R_\pi$, includes…

1. EM quenching
2. $\mathcal{O}(a^2)$ cut-off effects
3. Fit systematics
4. $\mathcal{O}(L^{-2})$ FVE   See 2109.05002.

- Progress: 2nd independent analysis under way

CONCLUSION & OUTLOOK

- IB effects are necessary to improve precision on light CKM matrix elements
- Lattice calculation setup for physical point calculation
- All ingredients available for calculating $\delta R_{K\pi}$ and $\frac{V_{us}}{V_{ud}}$ - 2nd independent analysis under way

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- Renormalise O_W to obtain V_{us} and V_{ud} individually
- Unquenched calculation
- Lattice calculation of real photon emission diagram
- CKM matrix element from semi-leptonic decays, e.g $K^\pm \to \pi^0 \ell^\pm \nu_\ell$

THANK YOU FOR YOUR ATTENTION