Global Fits of Domain Wall QCD

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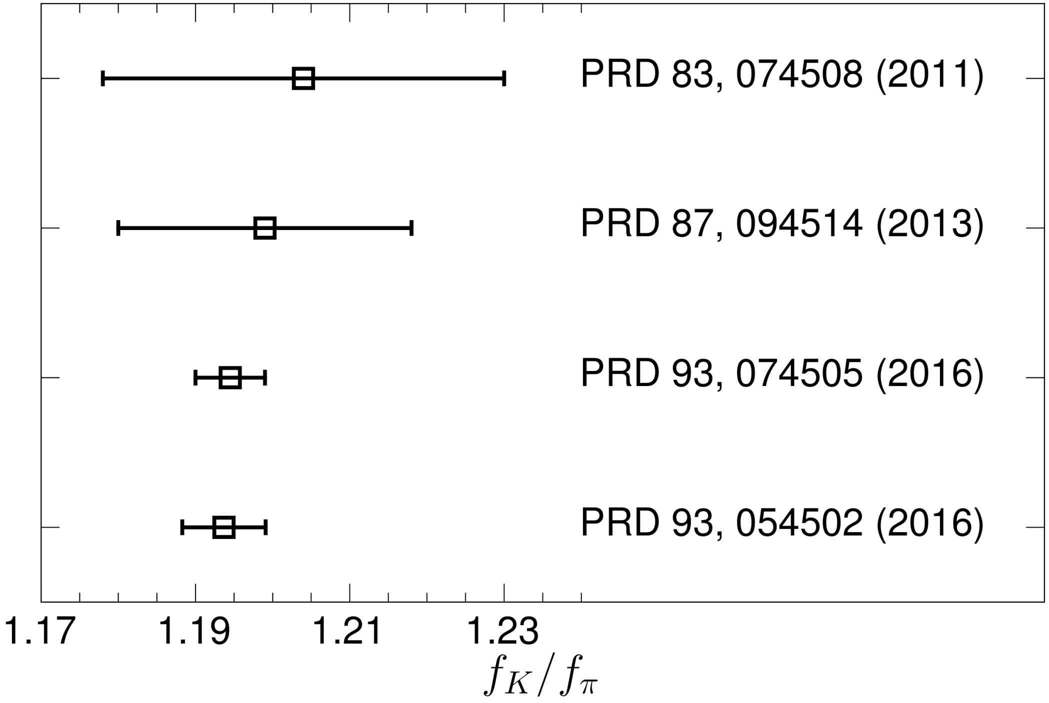
Objectives

- Scale setting – precisions of physics calculations, e.g., g-2, depend on knowing precise lattice spacings
- New physical ensemble 96I is included in the global fit
- Prediction of physical quark masses, f_K/f_π, ...
- Low Energy Constants (LECs) in ChPT can be extracted
- RBC-UKQCD's more than a decade long efforts with the global fits
  - [R. Mawhinney, PoS Lattice 2009, arXiv:0910.3194] - NLO and NNLO global fits
  - [Y. Aoki *et al.*, Phys. Rev. D 83, 074508 (2011)] – continuum limit with 32I, 24I
  - [R. Arthur *et al.*, Phys. Rev. D 87, 094514 (2013)] – with near-physical points and 32ID
  - [T. Blum *et al.*, PRD 93, 074505(2016)] - C. Kelly, NLO global fits incl. 48I_M, 64I_M, 32Ifine
  - [P. A. Boyle *et al.*, PRD 93, 054502 (2016)] - D. Murphy, NNLO ChPT study incl. 48M_ID

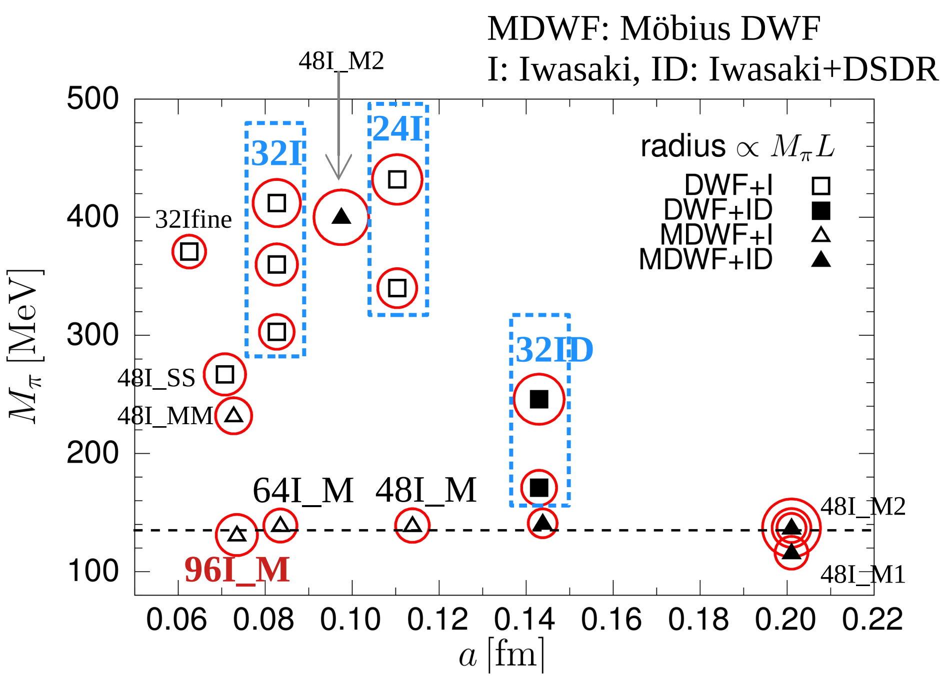
f_K / f_π



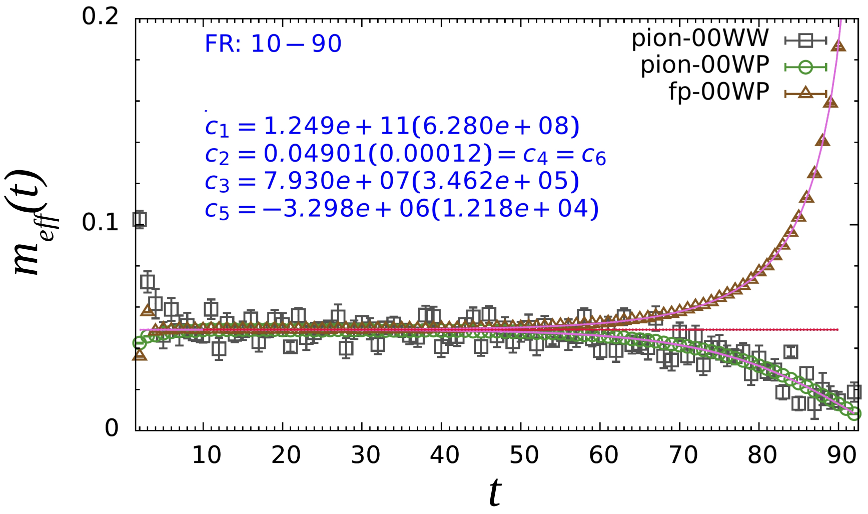
PRD 83, 074508 (2011)
PRD 87, 094514 (2013)
PRD 93, 074505 (2016)
PRD 93, 054502 (2016)
1.17
1.19
1.21
1.23
f_K/f_π

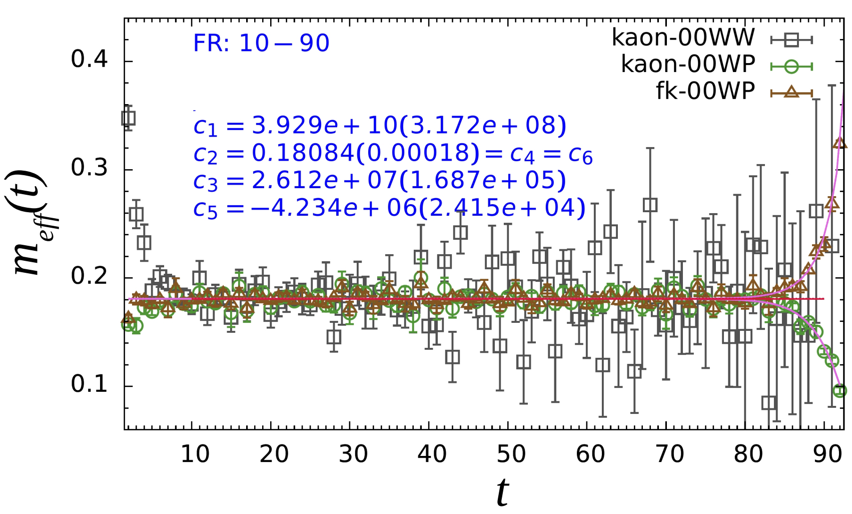
Ensembles

- 96I_M - physical mass ensemble with the smallest lattice spacing
- 24I, 32I, and 32ID lattices have partially quenched valence quark masses combinations (Fig. unitary pion only)
- 3 volumes at a ~ 0.2 fm, M_π ~ 135 MeV
- Heavy pion mass (>500MeV) and G-parity ensembles are not shown



96I_M: M_π, f_π / M_K, f_K





- Correlators are simultaneously fitted with a single state - uncorrelated fits
- Effective masses calculated for both data and fit are compared: $m_{\rm eff}(t) = \ln \frac{C(t)}{C(t+1)}$
- Statistical errors at present ~ 0.2% (0.1%) for M_π (M_K)
- 3x measurements will be completed soon

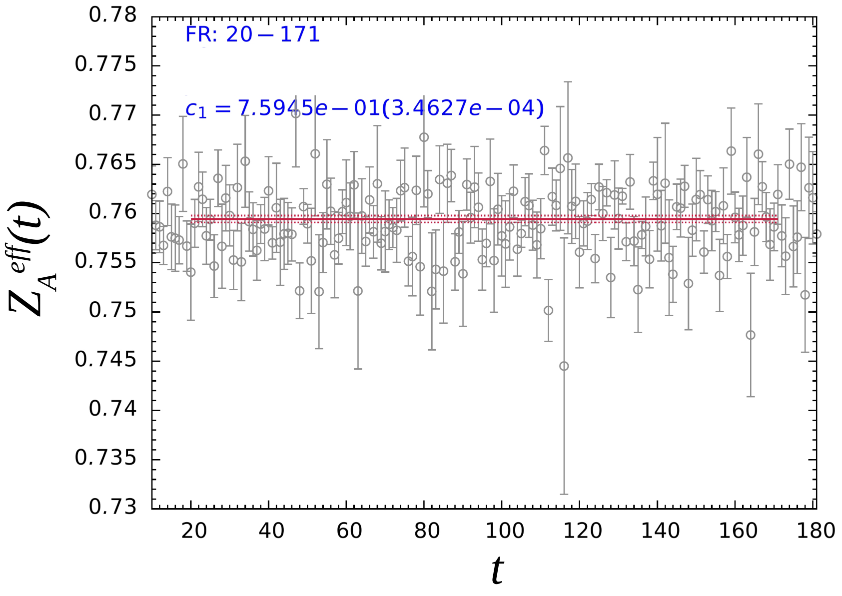
96I_M: Z_A

$$Z_A{}^{\text{eff}}(t) = \frac{1}{2}\left[\frac{C_{\mathcal{A}}(t-1)+C_{\mathcal{A}}(t)}{2C_A(t-1/2)} + \frac{2C_{\mathcal{A}}(t)}{C_A(t+1/2)+C_A(t-1/2)}\right]$$

$$C_{\mathcal{A}}(t) \equiv \langle 0|\sum_x \partial_\mu \mathcal{A}^a_\mu(x,t)|\pi\rangle$$

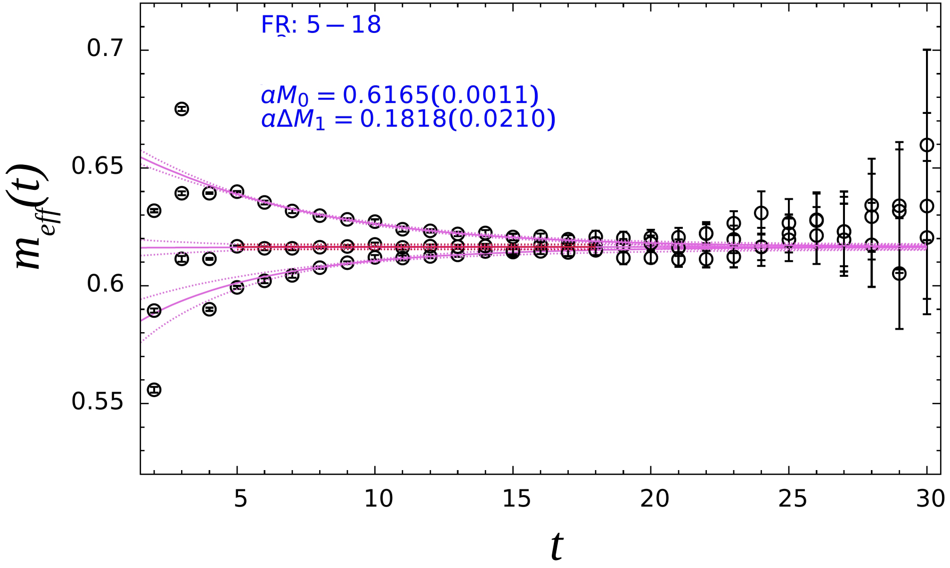
$$C_A\,(t-1/2) \equiv \langle 0|\sum_x \partial_\mu A^a_\mu(x,t)|\pi\rangle$$

- Z_A from the improved ratio of *5-dim.* to 4-dim. axial current divergences
- uncorrelated fits
- statistical errors at present ~ 0.05%
- used for f_π, f_K



96I_M: M_Ω

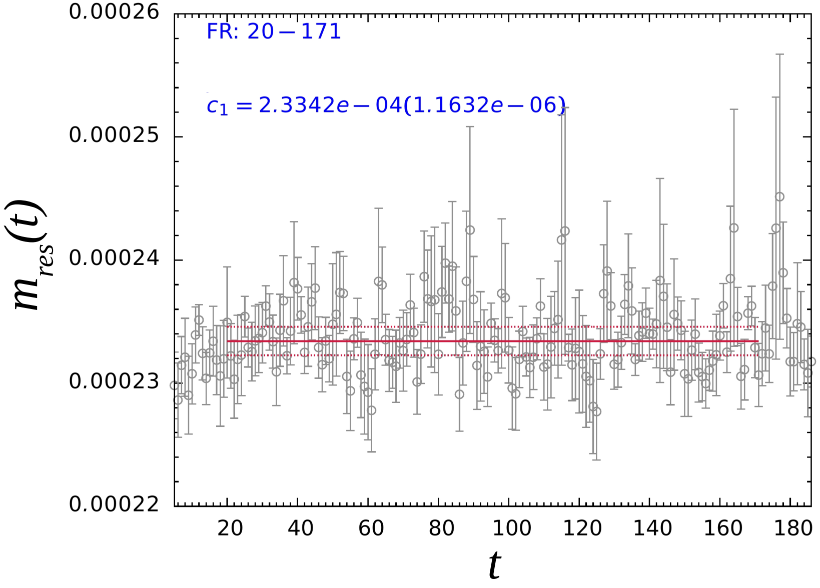
- Three box sources 24^3, 32^3, 48^3
- Correlators are simultaneously fitted with two states M_0 and M_1
- Uncorrelated fit
- Effective mass: $m_{\text{eff}}(t) = \ln \frac{C(t)}{C(t+1)}$
- Statistical error in M_Ω ~ 0.2%



96I_M: residual mass

- uncorrelated fits
- sea quark masses m_l = 0.00054, m_s=0.02132

$$m_{\rm res}(t) = \frac{\langle 0|\sum_x j^a_{5q}(x,t)|\pi\rangle}{\langle 0|\sum_x j^a_5(x,t)|\pi\rangle}$$



Global Fits

- On a lattice with dynamical quark masses am_l (u,d) and am_h (s)
- Q is measured with valence quark masses $am_x \leq am_y$

$$Q = (a^2 m_\pi^2, af_\pi, a^2 m_K^2, af_K, am_\Omega, \sqrt{t_0}/a, w_0/a),$$

$$I = (am_l, am_h, am_x, am_y; a, L)$$

  – deals with *dimensionless* quantities

- The global fit finds the best description $H(I)$ of $Q(I)$

$$H = (h_{m_\pi}, h_{f_\pi}, h_{m_K}, h_{f_K}, h_{m_\Omega}, h_{t_0}, h_{w_0})$$

Global Fits – domain *I*

- Renormalized Trajectory → Common LECs for all lattices
- Intermediate renormalization scheme, i.e., $Z_{l,h} = 1$ on the reference ensemble "r"

$$I \to I' = (aZ_l m_l, aZ_h m_h, aZ_l m_x, aZ_h m_y; a, L)$$

- Further, the fit is performed with relative lattice spacing $R_a = a_r/a$; the reference lattice spacing is absorbed into the LECs

$$I \to \tilde{I} = (R_a Z_l am_l, R_a Z_h am_h, R_a Z_l am_x, R_a Z_h am_y; R_a^{-1} a_r, L)$$

Global Fits – fit function H

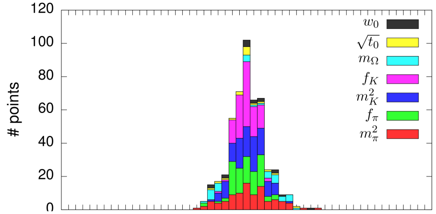
$$H = (h_{m_\pi}, h_{f_\pi}, h_{m_K}, h_{f_K}, h_{m_\Omega}, h_{t_0}, h_{w_0})$$

$$h_X(\tilde{I}) = \text{(light quark) + (heavy quark)}$$

- SU(2) ChPT for the light quark mass dependences for each of hadronic quantity
- Analytic linear expansion for the heavy (strange) degrees of freedom
- e.g., $M_\pi^2 = \frac{\chi_x + \chi_y}{2} \left\{ 1 + L^{m_\pi}(\chi_x, \chi_y, \chi_l) + c[m_\pi, m_h] \frac{1}{2B}(m_h - m_h^{\text{phys}}) \right\}$

  $(\chi_l = 2Bm_l)$
- Or, fully analytic ansatz for the both light and heavy quarks; always for $m_\Omega, w_0, \sqrt{t_0}$

Global Fits – scale setting



- LECs $\{c\}$, Rs, and Zs are determined by minimizing,

$$\chi_1^2(\{c\},\{Z_l,Z_h,R_a\}) = \sum_{\tilde{I}} (Q(\tilde{I}) - H(\tilde{I}))^\intercal \mathrm{Cov}^{-1}(Q(\tilde{I}) - H(\tilde{I}))$$

- Given LECs, Rs, and Zs, the secondary fit solves for physical quark masses

$$\chi_2^2(m_l^{\mathrm{phys}}, m_h^{\mathrm{phys}}) = \left(\frac{\sqrt{h_{m_\pi}}}{h_{m_\Omega}} - \frac{m_\pi^{\mathrm{phys}}}{m_\Omega^{\mathrm{phys}}}\right)^2 + \left(\frac{\sqrt{h_{m_K}}}{h_{m_\Omega}} - \frac{m_K^{\mathrm{phys}}}{m_\Omega^{\mathrm{phys}}}\right)^2$$

- Then, lattice spacings are given by

$$\begin{aligned} a_r =& h_\Omega(m_l^{\mathrm{phys}}, m_h^{\mathrm{phys}}, Z_l = Z_h = R_a = 1)/m_\Omega^{\mathrm{phys}} \\ =& \sqrt{h_\pi}/m_\pi^{\mathrm{phys}} = \sqrt{h_K}/m_K^{\mathrm{phys}} \end{aligned}$$

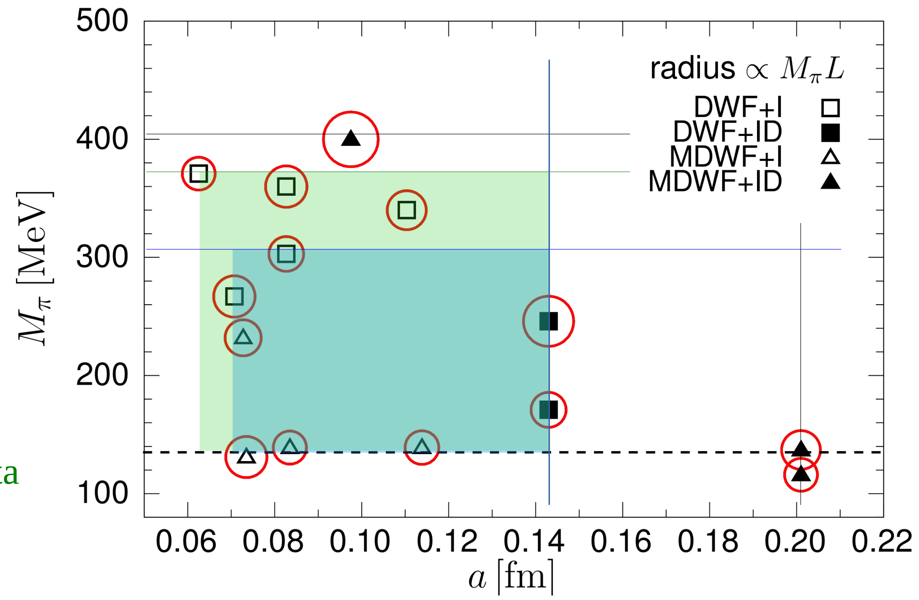
, first on the reference lattice, and subsequently for the rests $a = a_r/R_a$

Global Fit Variations

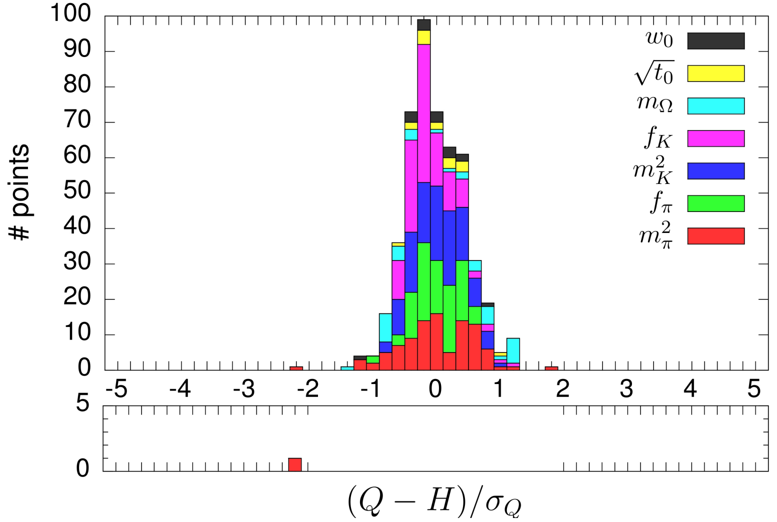
- SU(2) ChPTFV fit: NLO
- Discretization effect: a^2 term only, but different coefficients for I and ID ensembles
- Valence mass variations are in 32I, 24I, 32ID
- 32I is the reference ensemble, i.e., $R_a = 1$

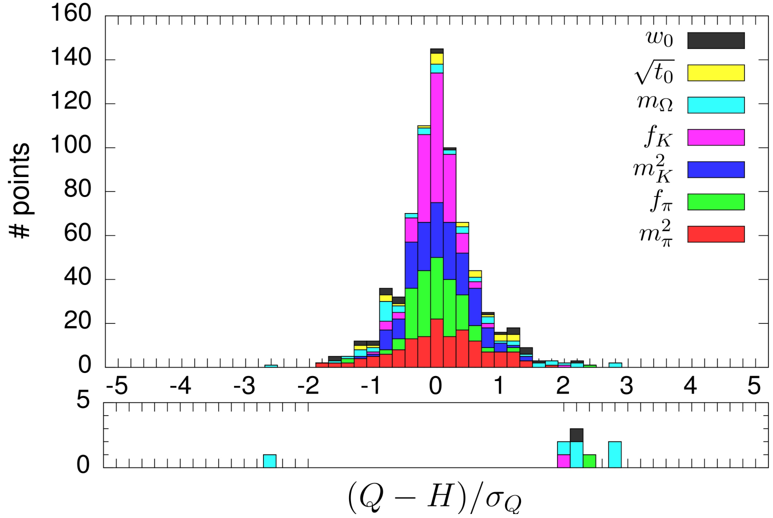
- Varying data included in fits with cuts:
  A) a ~ 0.14fm, $\boldsymbol{M_\pi}$ **~ 400 MeV**: B + 32ID_M2 (M_π ~ 400MeV)
  B) a ~ 0.14fm, $\boldsymbol{M_\pi}$ **~ 370 MeV** ➡ same cuts and methodology as the 2016 analysis with more data
  C) $\boldsymbol{a}$ **~ 0.14fm**, $\boldsymbol{M_\pi}$ **~ 300 MeV**
  D) $\boldsymbol{a}$ **~ 0.20fm**, M_π ~ 300 MeV: C + 32ID_M3 (M_π ~ 135MeV)
  E) a ~ 0.20fm, M_π ~ 300 MeV: D + 32ID_M1 (M_π ~ 116MeV)

ensembles included in this work

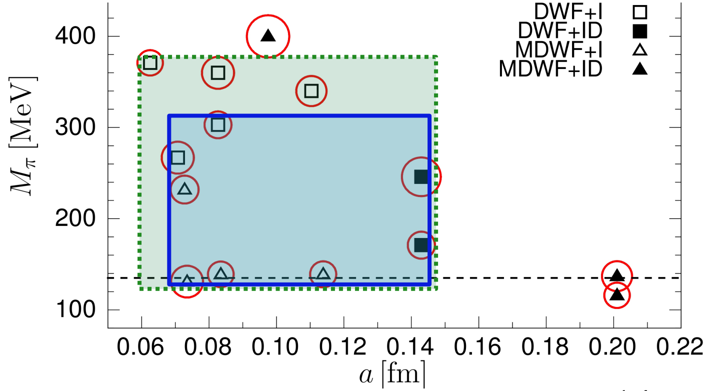


Fits with M_π ~ 300 MeV vs. 370 MeV cuts

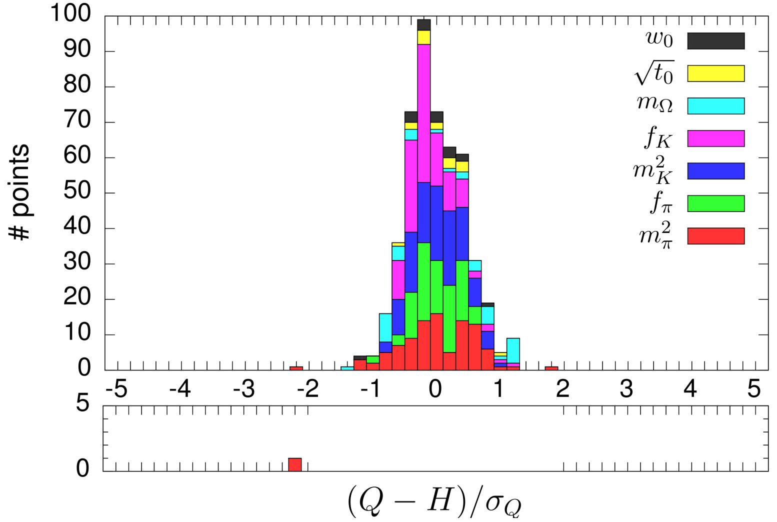


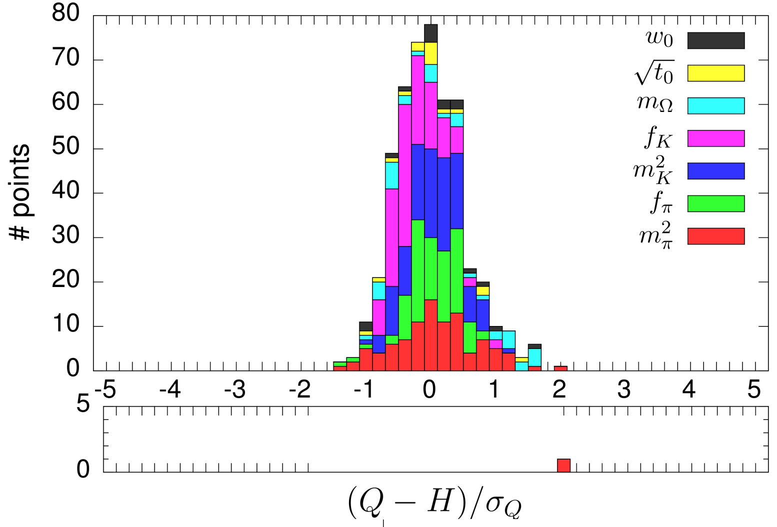


- Lattice spacing cut ~ 0.14 fm
- left: m_π^{2} in 32I
- right: f_π, f_K in 32Ifine / m_Ω in 32Ifine, 32I / w_0 in 32I

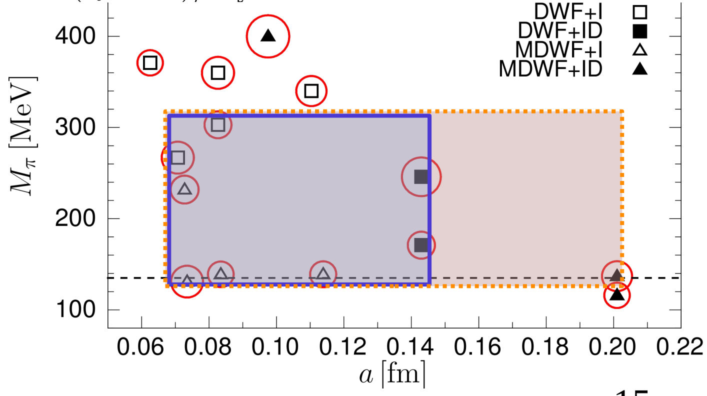


Fits with a ~ 0.14 fm vs. 0.20 fm cuts





- Pion mass cut ~ 300 MeV
- *M_π ~ 135 MeV ID lattice at a ~ 0.2 fm is included*
- left: $m_\pi^{\ 2}$ in 32I
- right: $m_\pi^{\ 2}$ in 32I



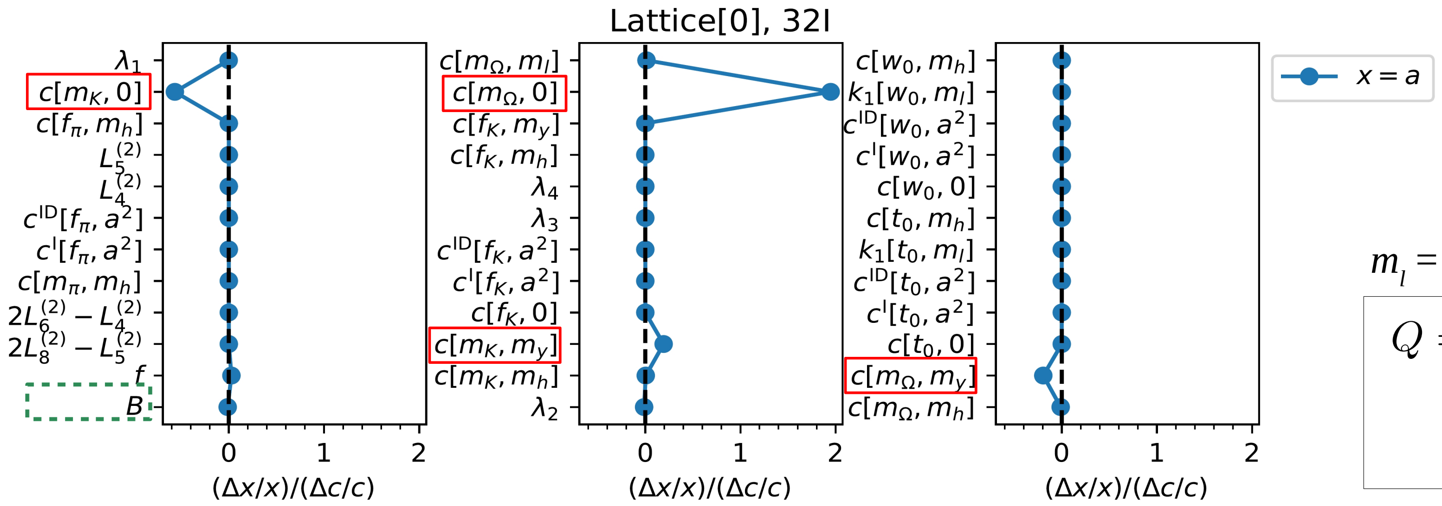
Sensitivity Test I

- Fit with cuts a ~ 0.14fm, M_π ~ 370 MeV is examined
- A controlled LEC is shifted from the central value of a global fit examined by a small amount
- All other LECs and Zs and Rs for non-primary lattices are fixed to the global fit centeral value
- Then, physical quark masses are readjusted:

$$\chi_2^2(m_l^{\mathrm{phys}}, m_h^{\mathrm{phys}}) = \left(\frac{\sqrt{h_{m_\pi}}}{h_{m_\Omega}} - \frac{m_\pi^{\mathrm{phys}}}{m_\Omega^{\mathrm{phys}}} \right)^2 + \left(\frac{\sqrt{h_{m_K}}}{h_{m_\Omega}} - \frac{m_K^{\mathrm{phys}}}{m_\Omega^{\mathrm{phys}}} \right)^2$$

Sensitivity Test I

- Lattice spacing "*a*" (on the primary lattice) is sensitive to four fit parameters
- Physical quark masses for m_l and m_h are responding similarly,
- except an additional sensitivity in m_l with *B*; "a" is insensitive to *B*
- The m_l dependencies in $M_\pi^2, f_\pi, M_K^2, f_K$ appear with $\chi_l = 2Bm_l$; same for m_x, m_y for the pion and m_y for the Kaon



$m_l = m_x = 0$, $m_h = m_y = m_h^{phys}$ at "0"

$$Q = Q_0 + (\partial Q/\partial x)_0 x + \cdots$$
$$c[Q, 0] = Q_0$$
$$c[Q, x] = c'(\partial Q/\partial x)_0$$

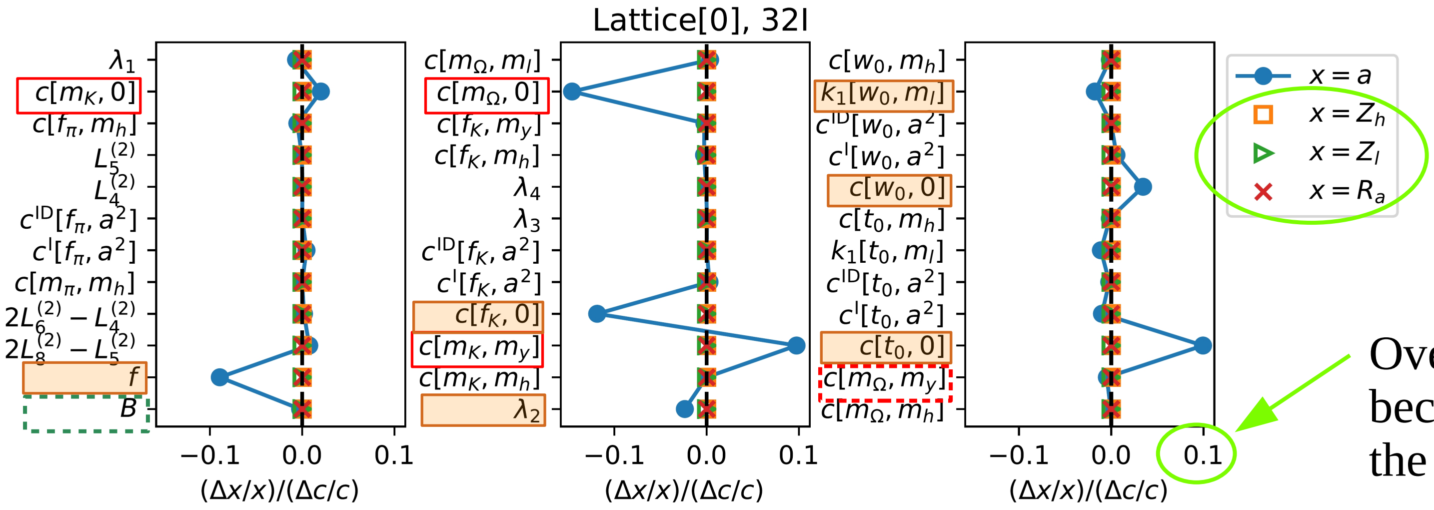
Sensitivity Test II

- The first sensitivity test ignores correlations
- A controlled LEC is shifted and all other LECs, Zs, and Rs are free
  ➡ LECs, Zs, and Rs are redetermined
- Then, physical quark masses are readjusted

Sensitivity Test II: prim. lattice

- Lattice spacing "*a*" of the *primary lattice* is still sensitive to 3 of 4 fit parameters identified with the test I
- Valence heavy mass (m_y) dependence in m_Ω becomes tiny whereas the dependence in m_K is enhanced relative to the change with c[m_Ω, 0]
- "*a*" becomes sensitive to **six additional LECs**

$$Q = Q_0 + (\partial Q/\partial x)_0 x + \cdots$$
$$c[Q,0] = Q_0$$
$$c[Q,x] = c'(\partial Q/\partial x)_0$$



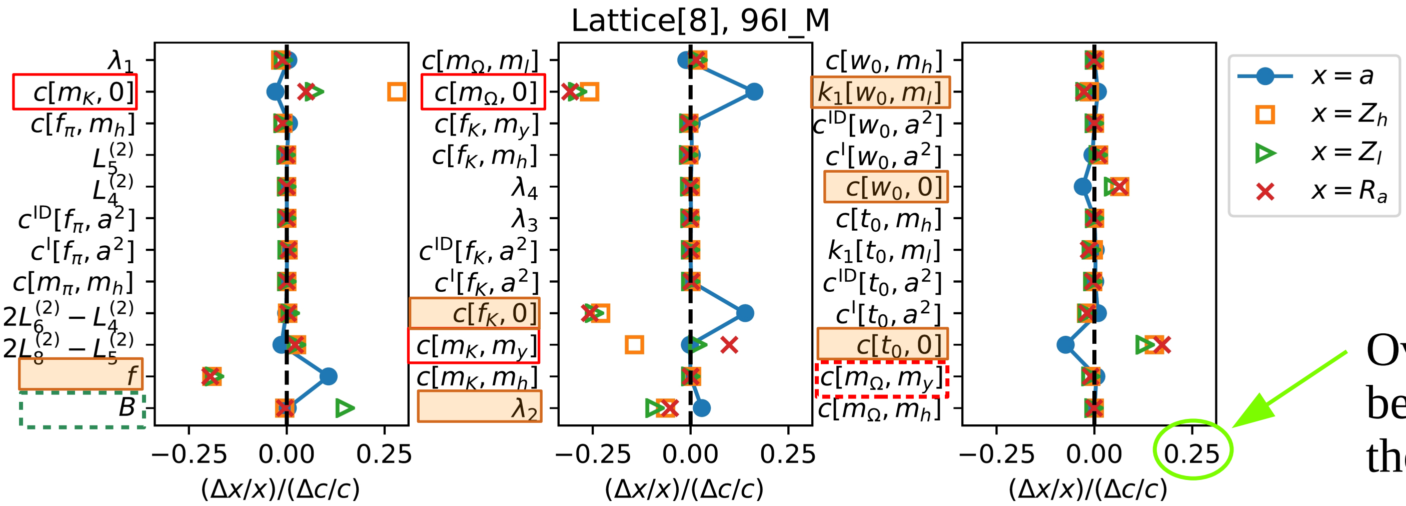
=1 for the primary lattice

Overall, changes become smaller than the test I, c.f., 2

Sensitivity Test II: non-prim. lattice

- the same set of major sensitive parameters for *non-primary lattices*
- Valence heavy mass (m_y) dependence in m_K (c[m_K, m_y]) becomes tiny for non-primary lattices, except for the 64I_M
- $c[f_\pi, 0] = f$, $\frac{\partial M_K^2}{\partial m_x} = \left(c[m_K, 0]\frac{2B}{f^2}\right) \lambda_2$

$$Q = Q_0 + (\partial Q/\partial x)_0 x + \cdots$$
$$c[Q,0] = Q_0$$
$$c[Q,x] = c'(\partial Q/\partial x)_0$$



Overall, changes become smaller than the test I, c.f., 2

Summary & Outlook

- A work in progress with the global fit including 96I_M is reported
- Statistics on 96I_M will be increased by a factor of 3 targeting precision on "*a*" ~ 0.1%
- Systematics will be assessed further, e.g., NNLO ChPT
- Sensitivity test shows that
  - leading coefficients c[Q,0] for $Q = f_\pi, f_K, m_\Omega, t_0$ are the most sensitive parameters, and then for $Q = m_K, w_0$
  - sea (valence) light mass dependence c[w_0, m_l] (c[m_K, m_x]) has smaller effects on "*a*"
  - valence heavy mass dependence c[m_K, m_y] could be either as large as the effect of c[m_Ω,0] or tiny
  - correlations among Z_l, Z_h, R_a, and LECs reduce the net impact on the lattice spacing "*a*"

Thank you for your attention