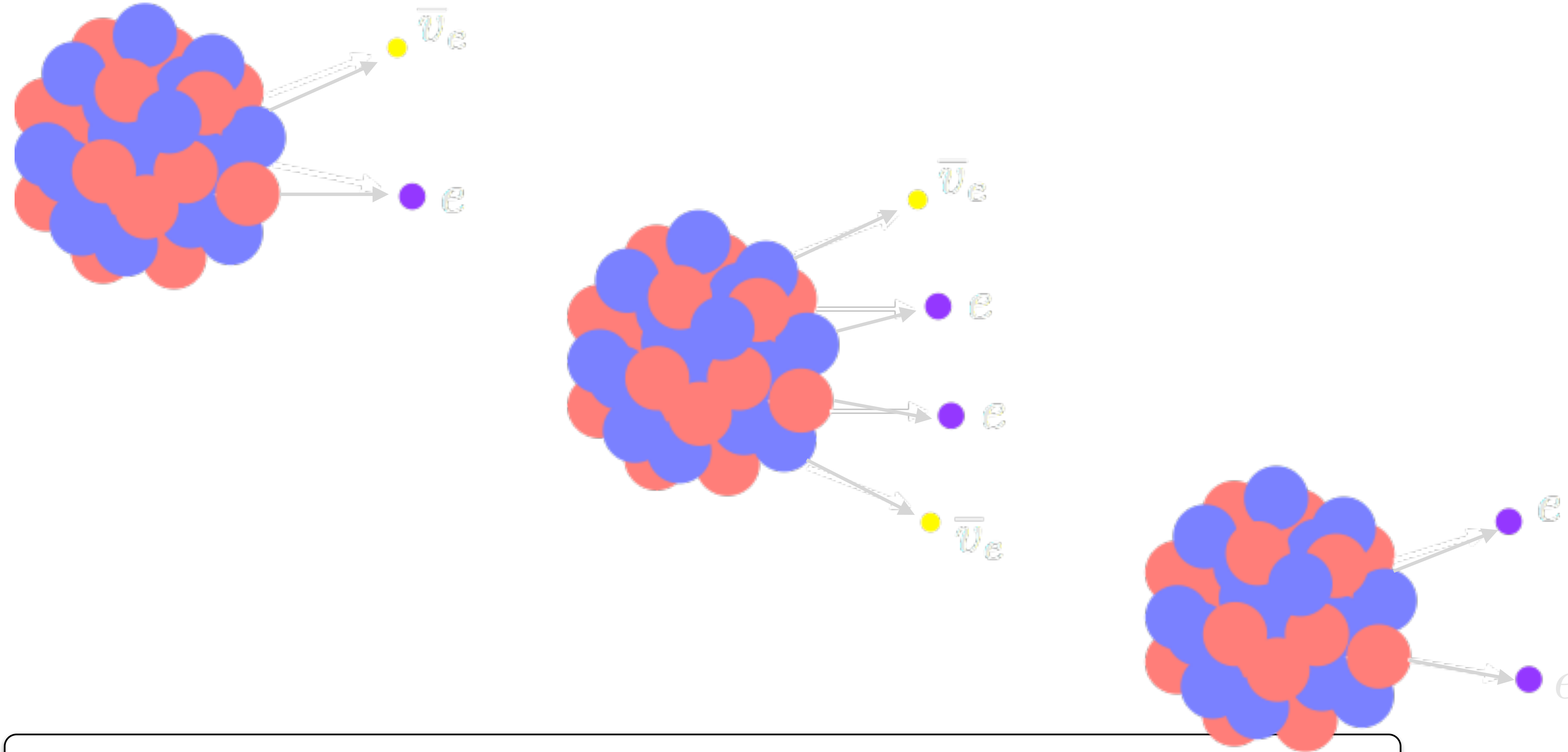
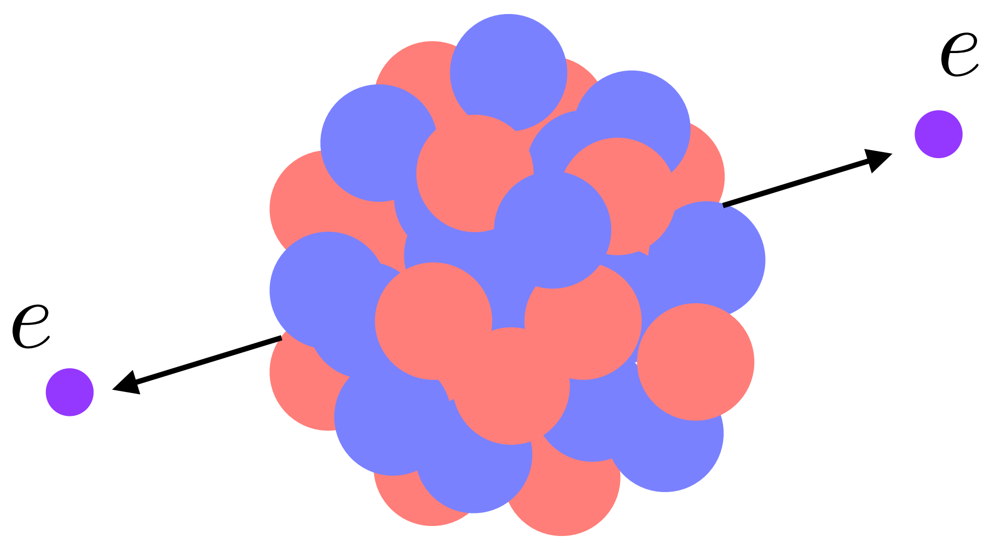


THE 2021 BNL-HET & RBRC Joint Workshop "DWQ@25"

NUCLEAR DOUBLE-β DECAYS FROM LATTICE QCD?

ZOHREH DAVOUDI
UNIVERSITY OF MARYLAND

We cannot study the matrix elements of Germanium or other experimentally relevant isotopes directly from QCD…



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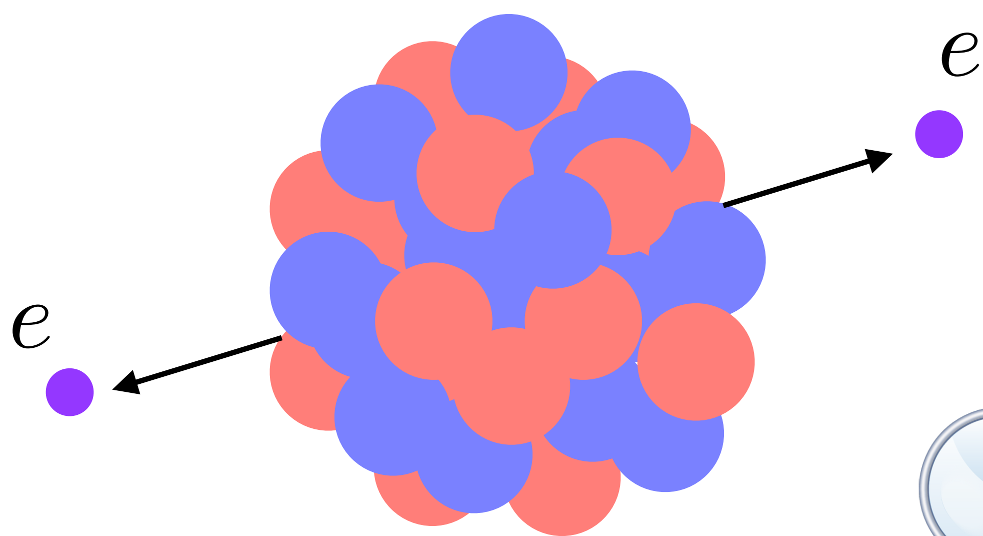
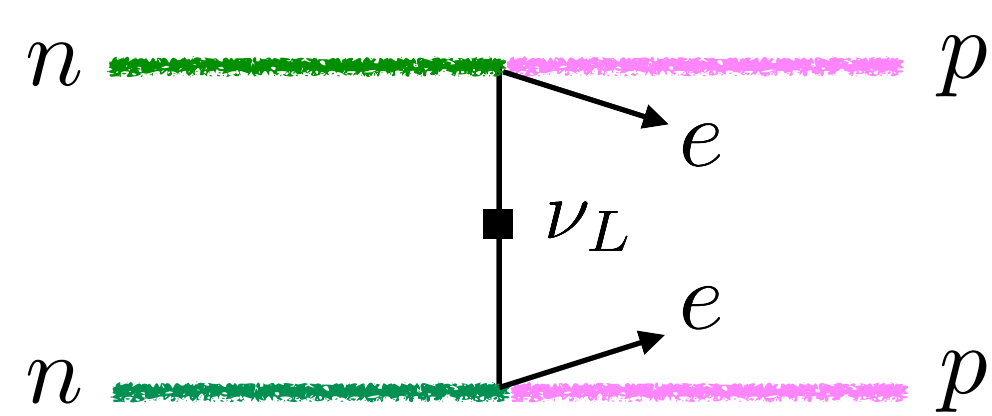


[Caution] Momentum exchanged is ~100 MeV. Three and multi-nucleon effects? Pion contributions?



Lattice QCD combined with EFT can help improve nucelar structure predictions of the rates.

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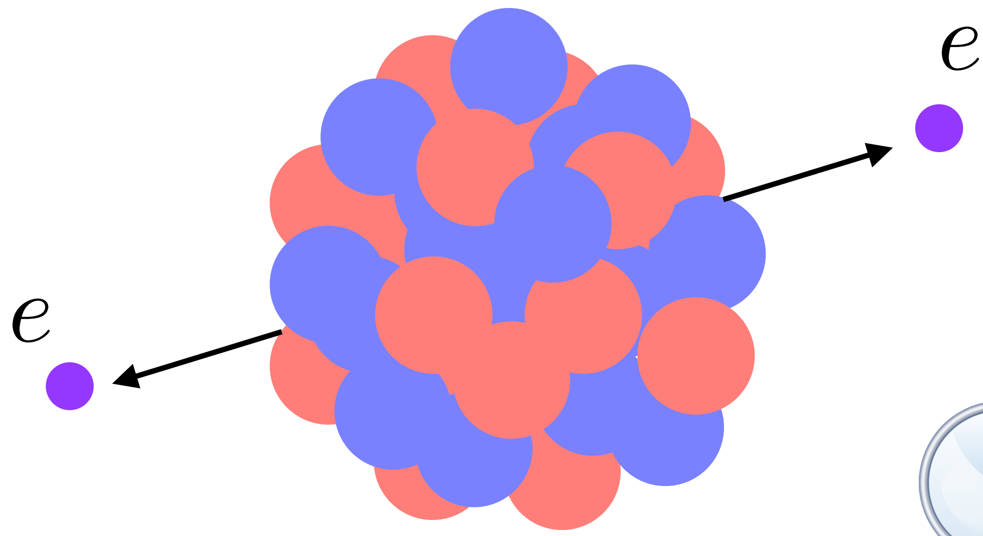
See Cirigliano's talk.



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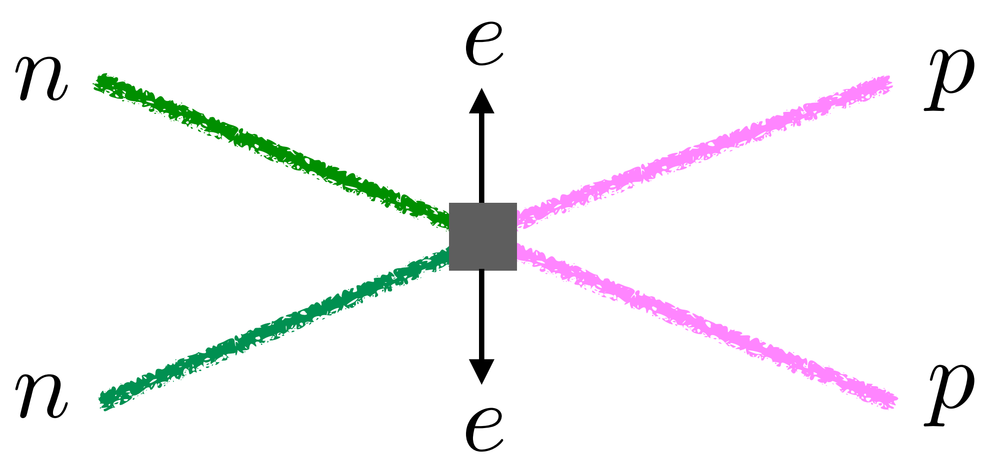
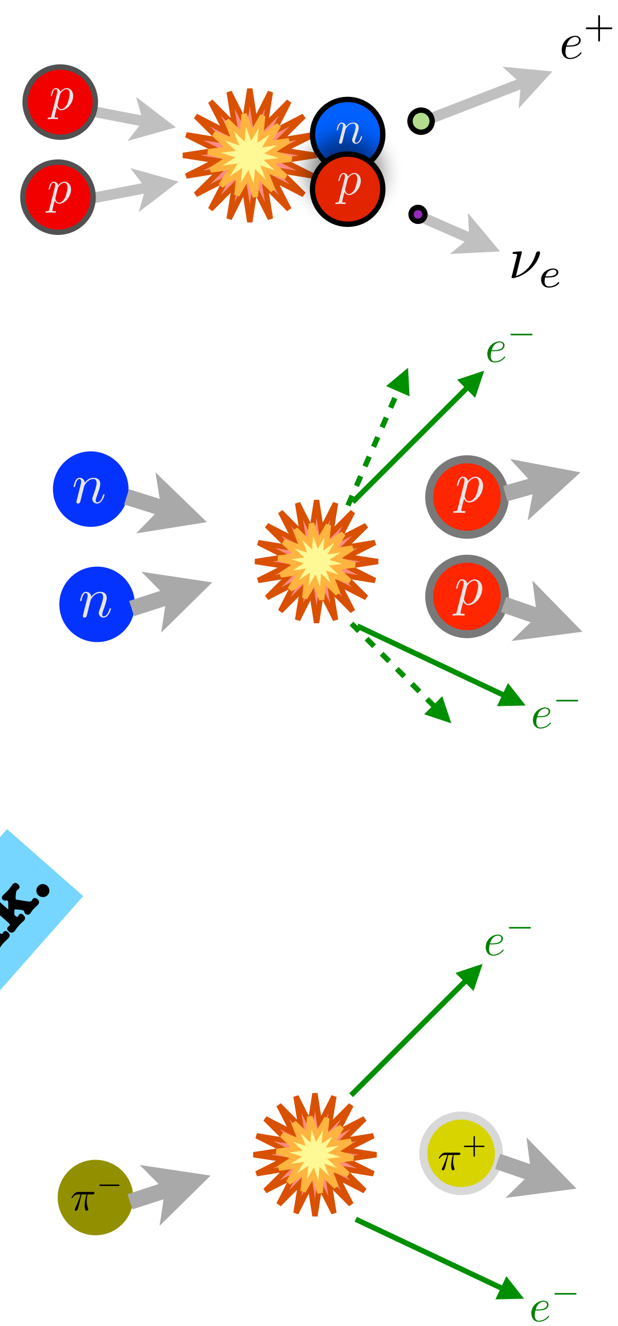


Lattice QCD combined with EFT can help improve nucelar structure predictions of the rates.

Lattice QCD calculations of the β-decay processes have started and more complete calculations will emerge in the upcoming years…



See Nicholson's talk.

Proton-proton fusion and tritium β-decay from lattice QCD

Savage et al [NPLQCD], Phys. Rev. Lett. 119, 062002 (2017).

The isotensor axial polarizability and lattice QCD input for nuclear double-β decay phenomenology

Shanahan et al [NPLQCD], Phys. Rev. Lett. 119, 062003 (2017).

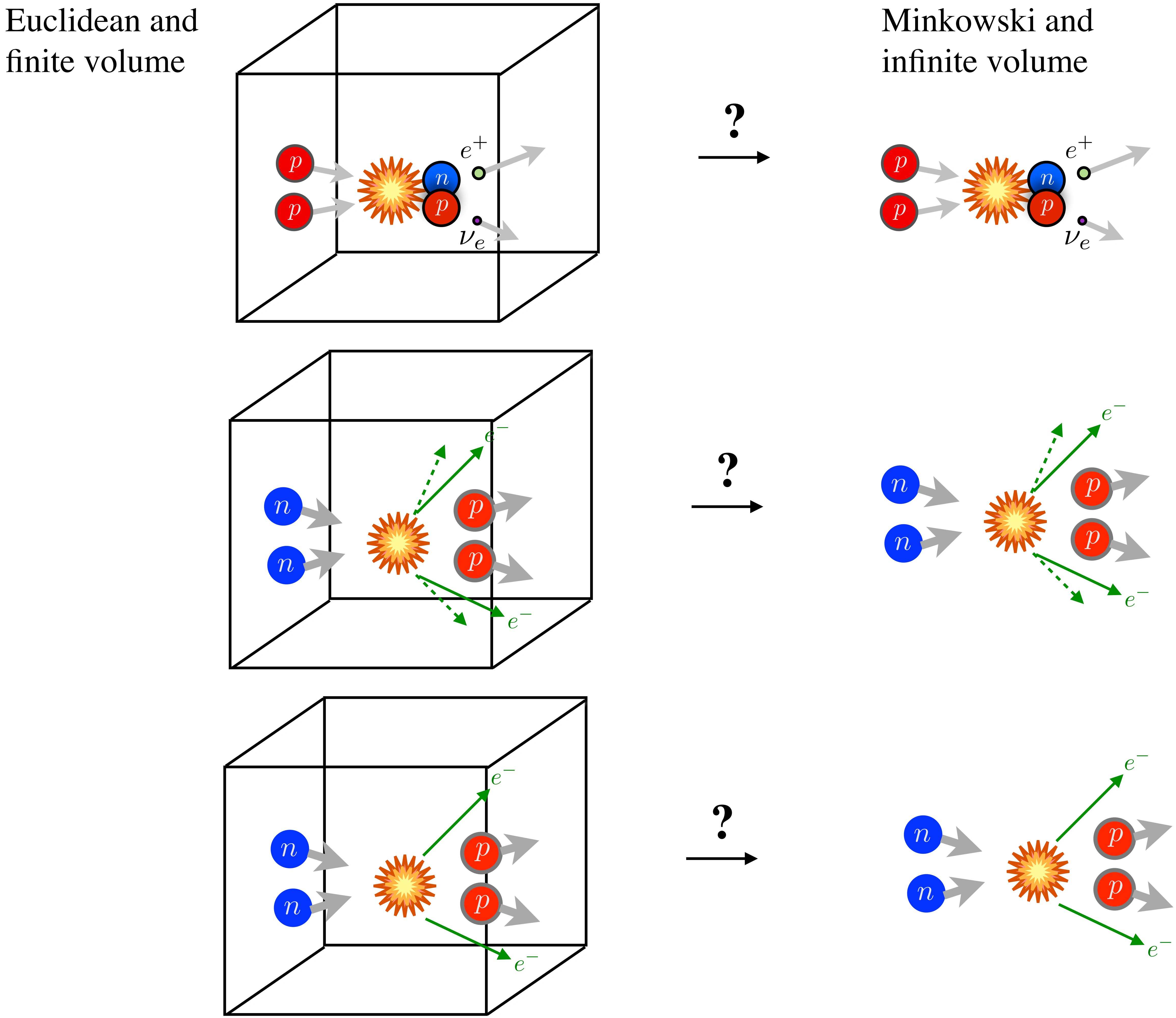
Pionic $0\nu\beta\beta$ decay matrix elements from lattice QCD in the heavy neutrino scenario:

Nicholson at al [CALLATT], Phys. Rev. Lett. 121, 172501 (2018).

Pionic $0\nu\beta\beta$ decay matrix elements from lattice QCD in the light neutrino scenario:

Tuo, Feng, Jin, *phys. Rev. D*, 100 (2019) 9, 094511.

Detmold and Murphy, 2004.07404 [hep-lat].



Euclidean and finite volume
Minkowski and infinite volume
?
?
?
p
p
n
p
e^+
ν_e
n
n
p
p
e^-
e^-

The **finite-volume technology** for electroweak matrix elements is crucial for the success of the program and builds upon many valuable developments of the past, to mention a few…

Weak transition matrix elements from finite volume correlation functions

Lellouch and Luescher, Commun. Math. Phys. 219, 31–44 (2001).

Finite-volume effects for two-hadron states in moving frames

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Matrix elements of unstable states

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Moving Multi-Channel Systems in a Finite Volume with Application to Proton-Proton Fusion

Briceno and Davoudi, Phys. Rev. D 88, 094507 (2013).

Relativistic, model-independent, multichannel 2 → 2 transition amplitudes in a finite volume

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Feng, Jin, Wang, Zhang, Phys. Rev. D 103, 034508 (2021)

Two-neutrino double-β decay in pionless effective field theory from a Euclidean finite-volume correlation function

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The path from LQCD to the short distance cont. to $0\nu\beta\beta$ decay with a light Majorana neutrino

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On the extraction of low-energy constants of single- and double-β decays from lattice QCD: A sensitivity analysis

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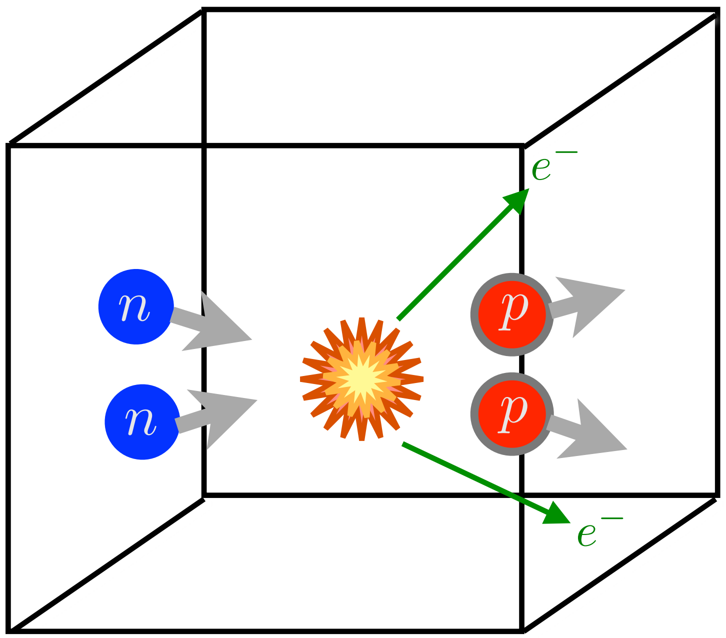
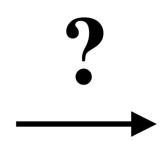
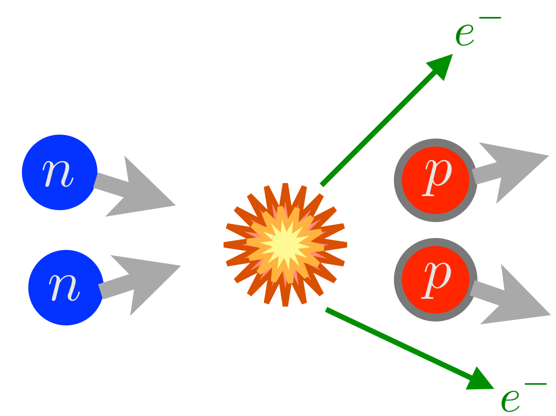
The finite-volume technology for **two-nucleon double-β decays with light neutrinos** is now developed. I will focus only on the neutrinoless case…and will comment on how the neutrinofull case is similar (but not entirely).

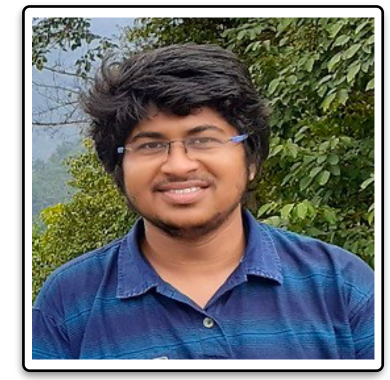
Euclidean and finite volume





Minkowski and infinite volume





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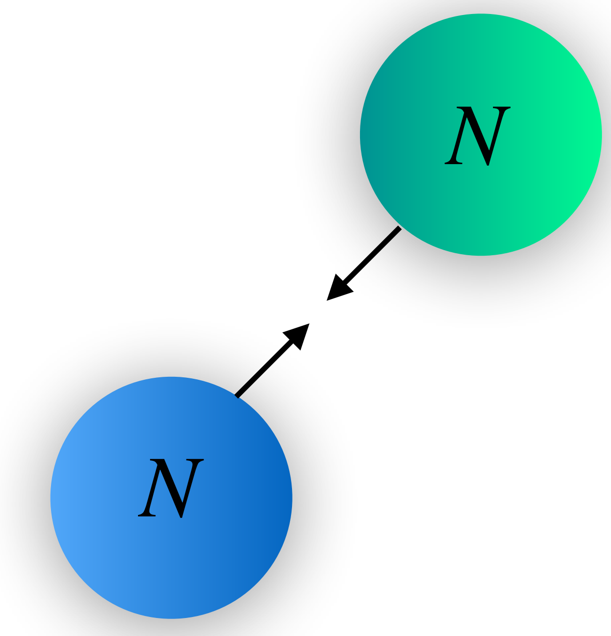
Our theoretical framework is the **pionless nuclear effective field theory**.

Seminal work of Kaplan, Savage, Wise, van Kolck, and of Chen, Kong, Ravndal, Bedaque and many others.



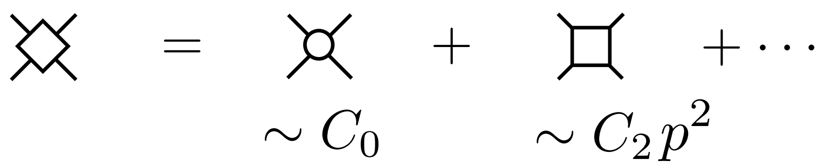
There are two *NN* systems in s-wave in nature, and both are unnatural (c.w. atomic systems near Feshbach resonance)!

$$a^{(^1S_0)} \approx -23\ [\mathtt{fm}] \gg 1/m_\pi$$
$$a^{(^3S_1)} \approx \quad 5\ [\mathtt{fm}] \gg 1/m_\pi$$

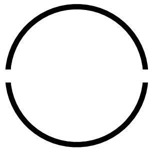
Leading contact interactions must be summed to all orders (expansion near unitarity) and pion exchanges are not leading order. In the pionless theory, pions are integrated out.

Some notation…

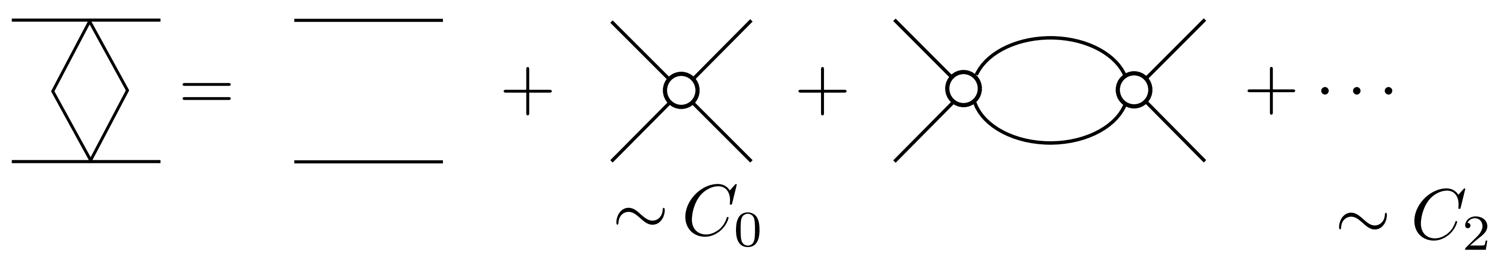
Strong couplings



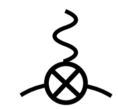
s-channel loop functions



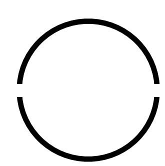
Elastic 2 to 2 S-matrix element:



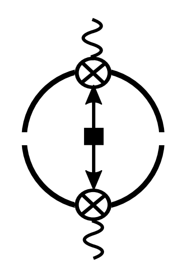
Weak coupling

 $\sim g_A$

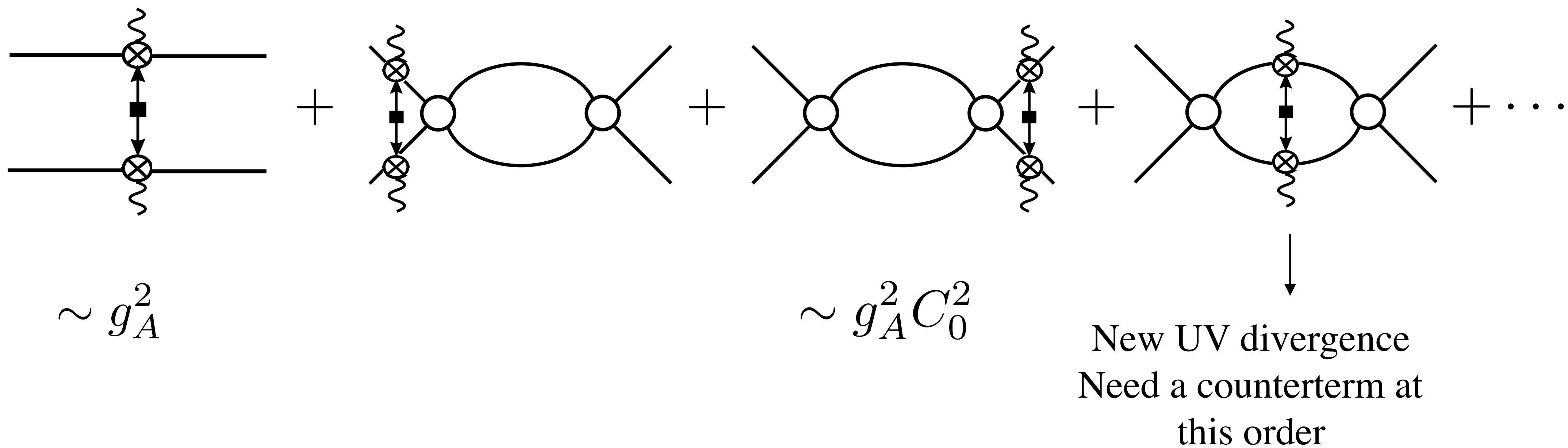
s-channel loop functions



$\sim m_{\beta\beta}$



$0\nu\beta\beta$ decay amplitude in the EFT (assuming low-energy S-wave interactions, isospin limit, and negligible neutrino mass compared to momenta):



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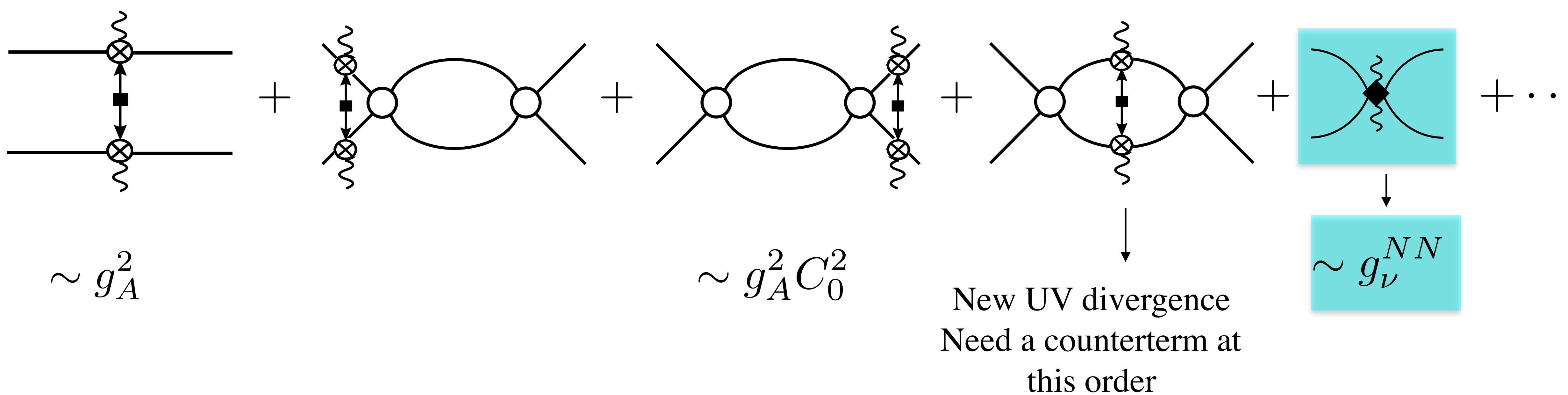
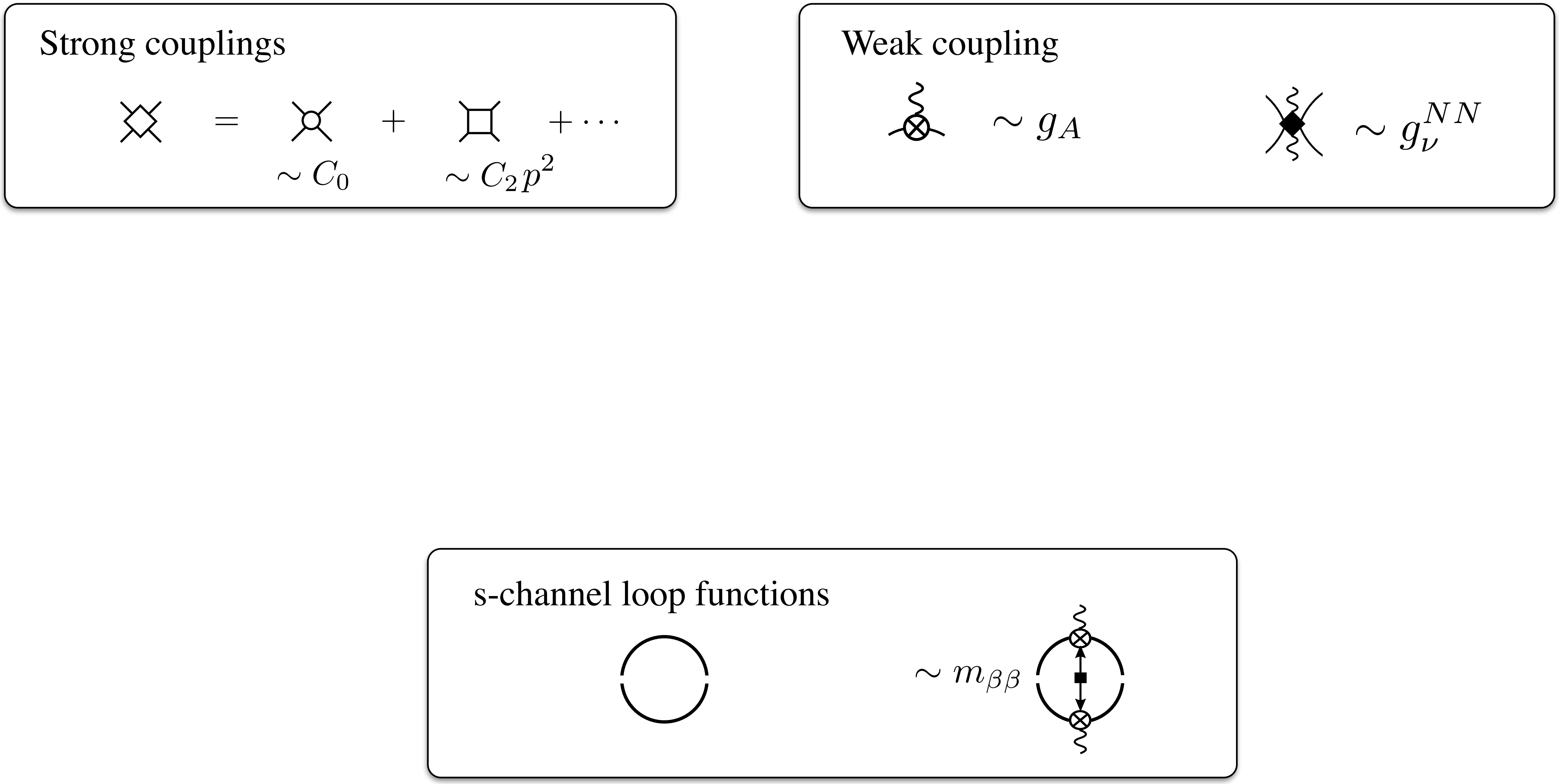


Cirigliano, Dekens, De Vries, Graesser, Mereghetti, Pastore, and Van Kolck, Phys. Rev. Lett. 120, 202001 (2018), Cirigliano, Dekens, Mereghetti, and Walker-Loud, Phys. Rev. C 97, 065501 (2018).


Strong couplings
$= \quad + \quad + \cdots$
$\sim C_0$
$\sim C_2 p^2$
Weak coupling
$\sim g_A$
$\sim g_\nu^{NN}$
s-channel loop functions
$\sim m_{\beta\beta}$

Ingredient i) Finite-volume **2-point** function

$2 \longrightarrow 2$ physical amplitude

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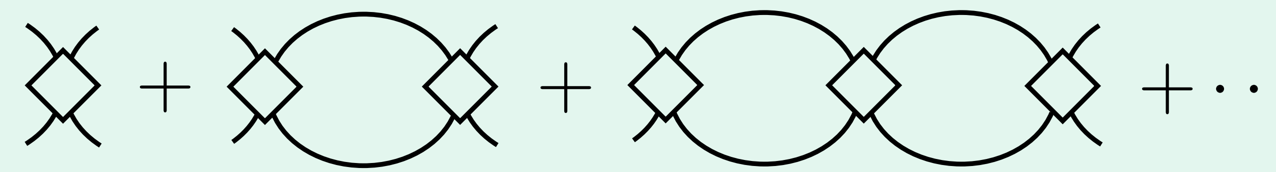
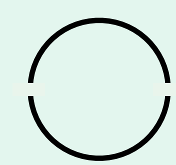
$2 \longrightarrow 2$ physical amplitude

$i\mathcal{M} =$

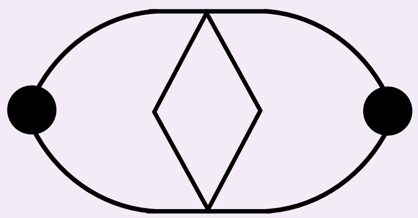


$$\mathcal{M}^{(\mathrm{LO})} = -\frac{C_0}{1 - I_0(E)\, C_0}$$



Ingredient i) Finite-volume
2-point function

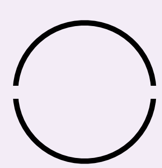
$2 \longrightarrow 2$ physical amplitude

$C_L =$ 

$$C_L(P) = C_\infty(P) + \mathcal{B}(E)\, i\mathcal{F}(E)\, \mathcal{B}^\dagger(E)$$

$$\mathcal{F} \equiv \frac{1}{F_0^{-1} + \mathcal{M}}$$

Related to Luescher's
Z-function



Ingredient i) Finite-volume
2-point function

$2 \longrightarrow 2$ physical amplitude

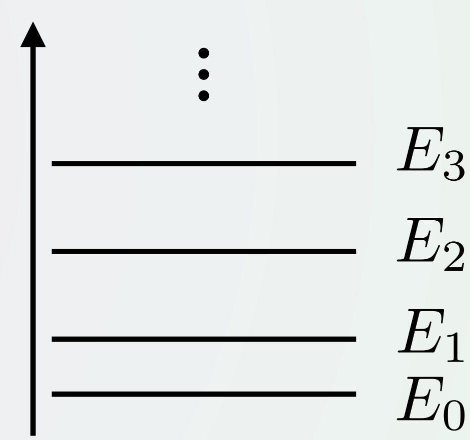
Matching relation:

Luescher (1986, 1991). In the context of NN EFT derived by: Beane, Bedaque, Parreno, and Savage, *Phys.Lett.B* 585 (2004) 106-114.

Luescher's quantization condition

$$F_0^{-1}(E) + \mathcal{M}(E) = 0, \quad \text{for} \quad E = E_n.$$

Tower of finite-volume energy eigenvalues



Ingredient ii) Finite-volume
4-point function

$2 \xrightarrow{\mathcal{J}\mathcal{J}} 2$ physical amplitude

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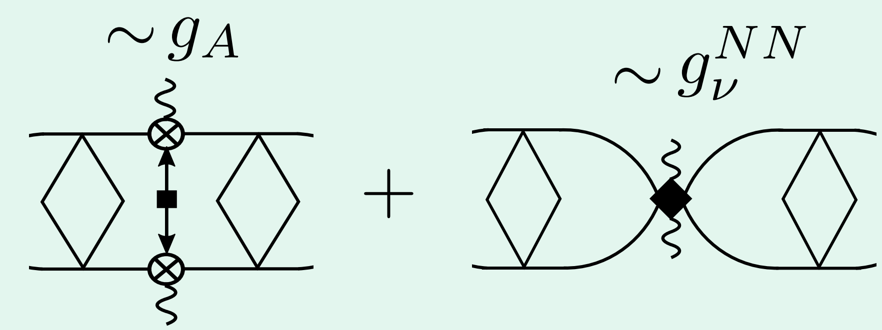
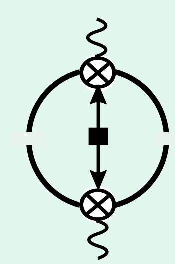
4-point function

$2 \xrightarrow{\mathcal{J}\mathcal{J}} 2$ physical amplitude

$\mathcal{M}^{(\text{Int.})}_{nn\to pp} =$



$$\mathcal{M}^{(\text{Int.})}_{nn\to pp}(E_i, E_f) = m_{\beta\beta}\,\mathcal{M}(E_f)\left[-\left(1+3g_A^2\right) J^{\infty}(E_i, E_f;\mu) + \frac{2g_\nu^{NN}(\mu)}{C_0^2(\mu)}\right]\mathcal{M}(E_i).$$

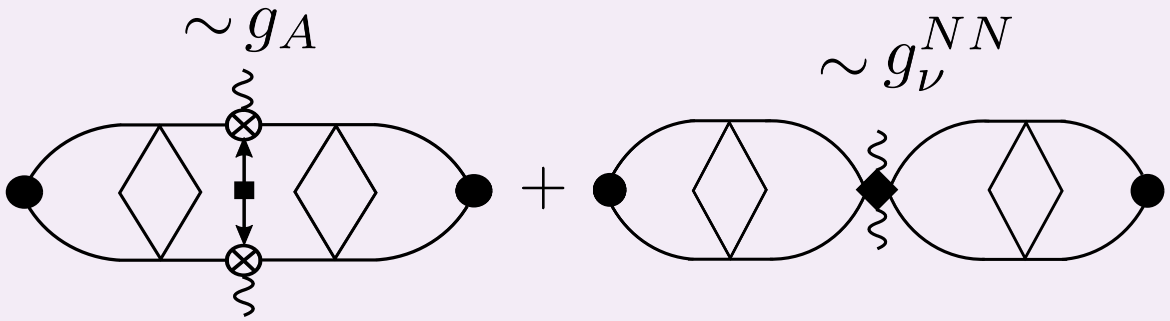


The unknown
short-distance
coupling of EFT

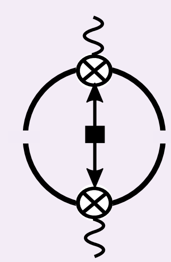
Ingredient ii) Finite-volume
4-point function

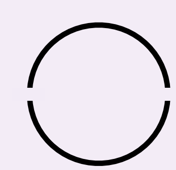
$2 \xrightarrow{\mathcal{J}\mathcal{J}} 2$ physical amplitude

$C_L =$



$$C_L(E_i,E_f) = C_\infty(E_i,E_f) + \mathcal{B}_{pp}(E_f)\, i\mathcal{F}(E_f)\left[i\mathcal{M}^{(\mathrm{Int.})}_{nn\to pp}(E_i,E_f) + m_{\beta\beta}(1+3g_A^2)\times i\mathcal{M}(E_f)\, i\,\delta J^V(E_f,E_i)\, i\mathcal{M}(E_i)\right] i\mathcal{F}(E_i)\mathcal{B}^\dagger_{nn}(E_i) + \cdots$$





Ingredient ii) Finite-volume **4-point** function

$$2 \xrightarrow{\mathcal{J}\mathcal{J}} 2 \quad \text{physical amplitude}$$

Matching relation (not quite):

$$L^6 \left|\mathcal{T}_L^{(\mathrm{M})}\right|^2 = \left|\mathcal{R}(E_{n_f})\right| \left|\mathcal{M}^{(\mathrm{Int.})}_{nn\to pp}(E_{n_i}, E_{n_f}) - m_{\beta\beta}\,(1+3g_A^2)\mathcal{M}(E_{n_f})\delta J^V(E_{n_f}, E_{n_i})\mathcal{M}(E_{n_i})\right| \left|\mathcal{R}(E_{n_i})\right|$$

$$\mathcal{T}_L^{(\mathrm{M})} \equiv \int dz_0\, e^{iE_1 z_0} \int_L d^3z \left[\langle E_{n_f}, L|\, T[\mathcal{J}(z_0,\boldsymbol{z})\, S_\nu(z_0,\boldsymbol{z})\mathcal{J}(0)]\, |E_{n_i}, L\rangle\right]_L .$$

- $\mathcal{J}$: Weak current
- S_ν: Zero-mode removed finite-volume neutrino propagator

$\mathcal{R}(E_{n_i})$: Residue of F-function at the finite-volume energies

$$\mathcal{R}(E_n) = \lim_{E\to E_n} (E - E_n)\, \mathcal{F}(E).$$

Davoudi and Kadam, Phys. Rev. Lett. 126, 152003 (2021).

What we need is:

$$\mathcal{T}_L^{(\mathrm{M})} \equiv \int dz_0\, e^{iE_1 z_0} \int_L d^3 z \left[\langle E_{n_f}, L|\, T[\mathcal{J}(z_0, \boldsymbol{z})\, S_\nu(z_0, \boldsymbol{z}) \mathcal{J}(0)]\, |E_{n_i}, L\rangle \right]_L$$

?

Lattice QCD gives access to:

$$G_L^{(\mathrm{E})}(\tau) = \int_L d^3 z\, \left[\langle E_f, L| T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau, \boldsymbol{z}) S_\nu^{(\mathrm{E})}(\tau, \boldsymbol{z}) \mathcal{J}^{(\mathrm{E})}(0)] |E_i, L\rangle \right]_L$$

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$$i \int d\tau\, e^{E_1 \tau}\, G_L^{(\mathrm{E})}(\tau)$$

?

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No if the total available energy in the process is larger than the possible intermediate-state energies, i.e., that of the neutrino and the NN state.

$$i)\ |\boldsymbol{P}_{*m}| + E_{*m} \leq E_f + E_1 \text{ or } ii)\ |\boldsymbol{P}_{*m}| + E_{*m} \leq E_i - E_1$$

Davoudi and Kadam, Phys. Rev. D 102, 114521 (2020).

See also Christ, Feng, Martinelli, and Sachrajda, Phys. Rev. D 91, 114510 (2015), and Briceno, Davoudi, Hansen, Schindler, and Baroni, Phys. Rev. D101, 14509 (2020).

$$i \int d\tau\, e^{E_1 \tau}\, G_L^{(\mathrm{E})}(\tau)$$

?

Lattice QCD gives access to:

$$G_L^{(\mathrm{E})}(\tau) = \int_L d^3z\, \left[\langle E_f, L| T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau,\boldsymbol{z}) S_\nu^{(\mathrm{E})}(\tau,\boldsymbol{z}) \mathcal{J}^{(\mathrm{E})}(0)] |E_i, L\rangle\right]_L$$

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$$\mathcal{T}_L^{(\mathrm{M})} \equiv \int dz_0\, e^{iE_1 z_0} \int_L d^3z \left[\langle E_{n_f}, L|\, T[\mathcal{J}(z_0,\boldsymbol{z})\, S_\nu(z_0,\boldsymbol{z}) \mathcal{J}(0)]\, |E_{n_i}, L\rangle\right]_L$$

$$i \int d\tau\, e^{E_1 \tau}\, G_L^{(\mathrm{E})}(\tau)$$

?

Davoudi and Kadam, Phys. Rev. Lett. 126, 152003 (2021).

Consider a lattice QCD calculation at *L*=8 fm:

$E_1 = E_2 = 0$ (Electrons' energies)

$E_{n_i} \approx -2.6$ MeV (Initial *nn* FV ground-state energy)

$\widetilde{E}_{*m} \approx \{-5.6, 10.5, \cdots\}$ MeV (Intermediate *np* FV energies)

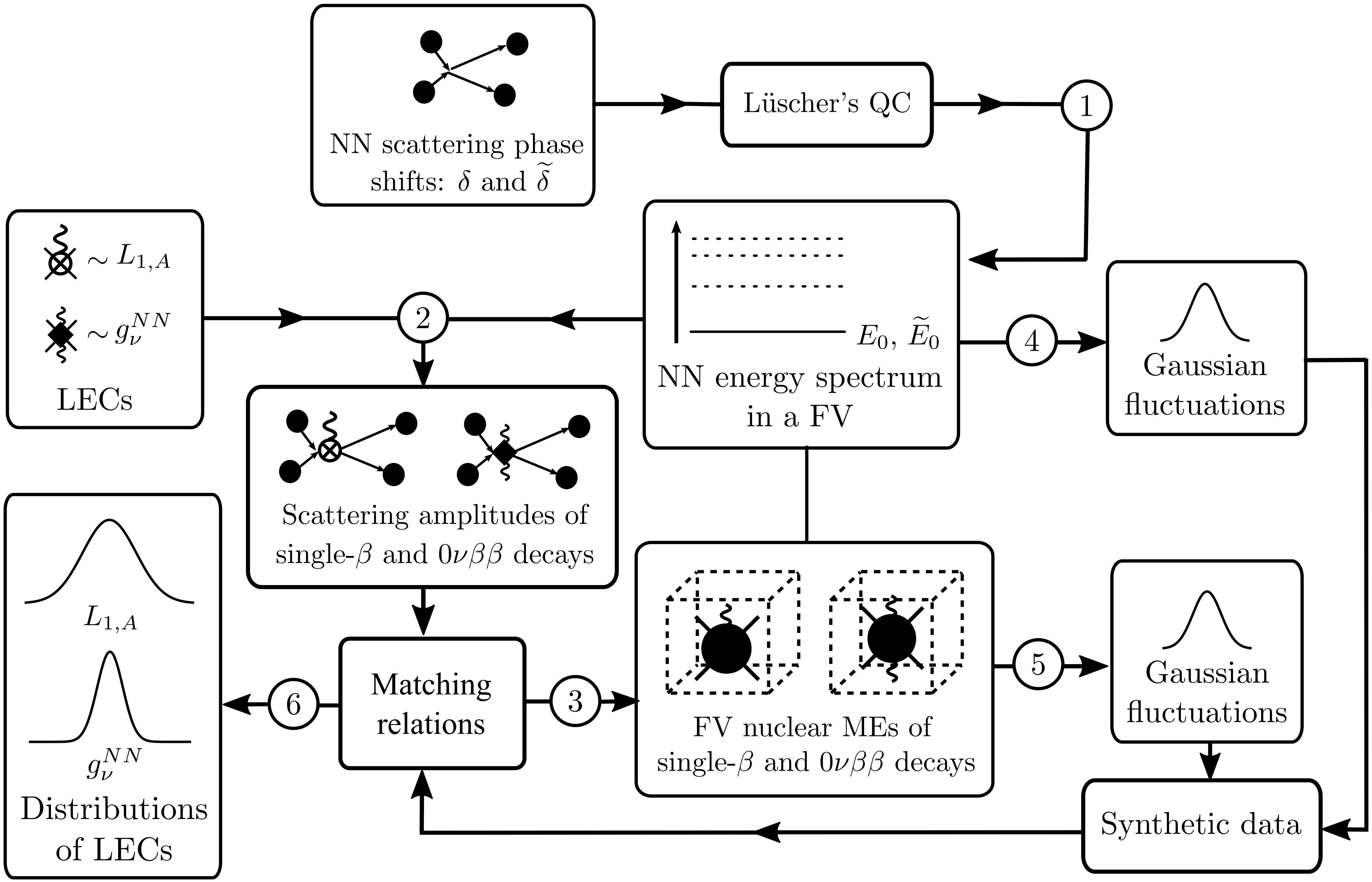
$|\boldsymbol{P}_*| = 2\pi/L \approx 155.0$ MeV (The smallest allowed neutrino energy)

No on-shell intermediate state!!

Lattice QCD gives access to:

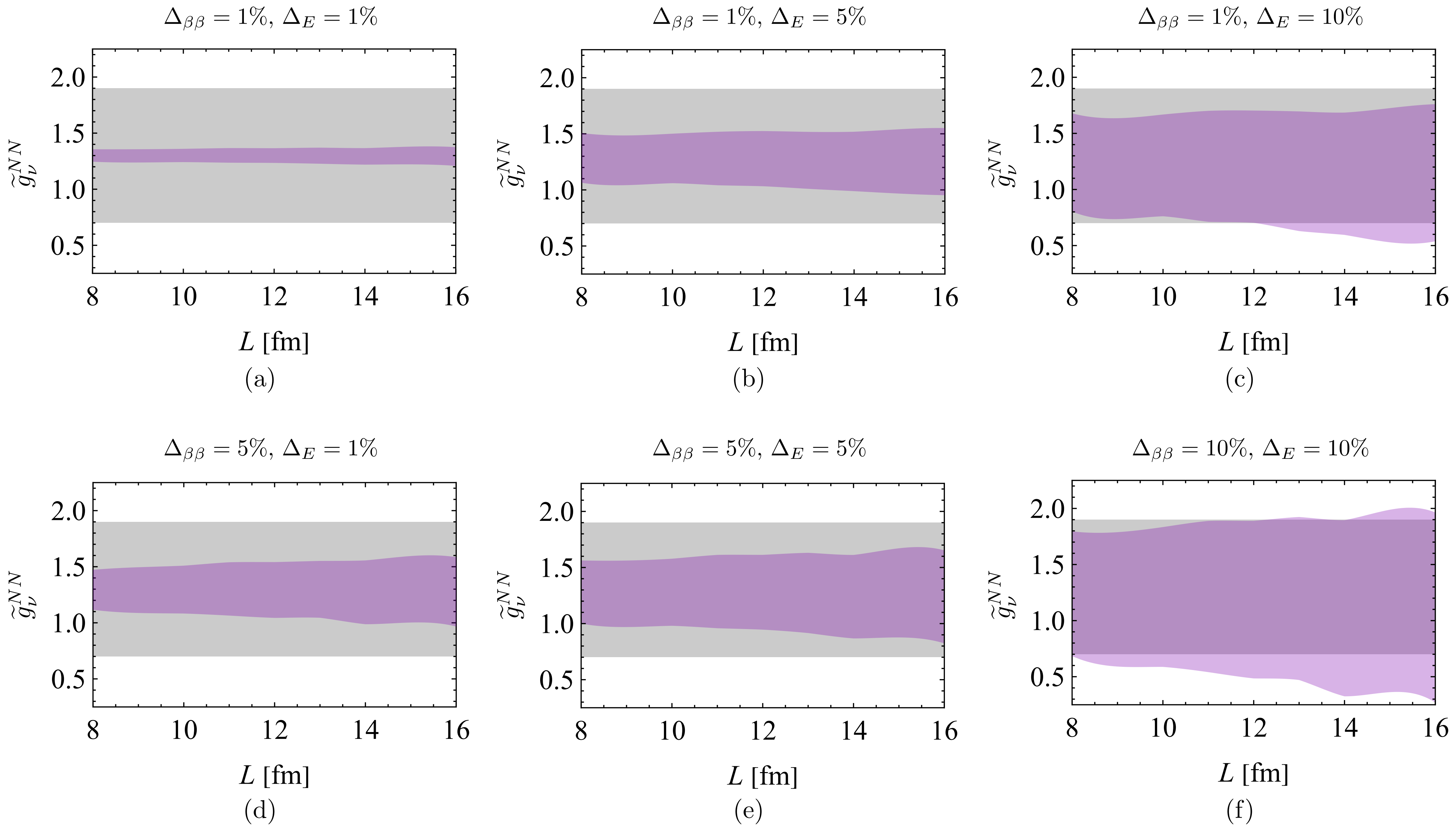
$$G_L^{(\mathrm{E})}(\tau) = \int_L d^3z \left[\langle E_f, L|T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau,\boldsymbol{z}) S_\nu^{(\mathrm{E})}(\tau,\boldsymbol{z}) \mathcal{J}^{(\mathrm{E})}(0)]|E_i, L\rangle\right]_L$$

[A synthetic data analysis] How sensitive is the new short-distance coupling to uncertainties in the lattice QCD energy and matrix-element inputs?



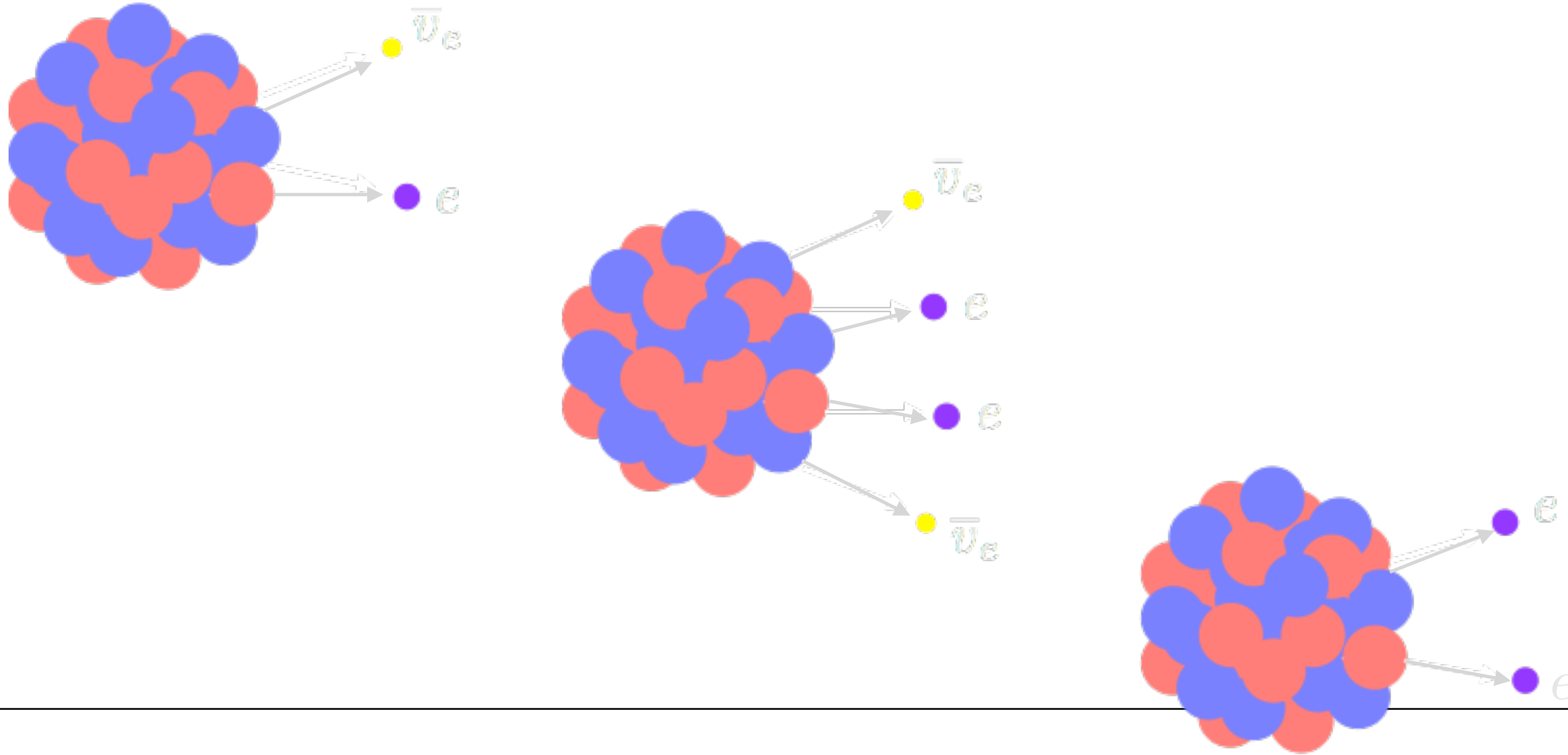
Davoudi and Kadam, arXiv: 2111.11599 [hep-lat] (2021).

The expected uncertainty on the new short-distance coupling for various uncertainty levels in the lattice QCD energy and matrix element inputs:



Davoudi and Kadam, arXiv: 2111.11599 [hep-lat] (2021).

Existing constraint by Cirigliano, Dekens, de Vries, Hoferichter, and Mereghetti, arXiv: 2012.11602 [nucl-th] (2020). Impact on many-body calculations of decay rate by Jokiniemi, Soriano, Menendez, *Phys.Lett.B* 823 (2021) 136720, and by Wirth, Yao, and Hergert, *Phys.Rev.Lett.* 127 (2021) 24, 242502.



[Conclusion] A **new short-distance LEC** at LO in the EFT amplitude of the **$0\nu\beta\beta$ decay with light neutrinos** arises. Its first determination has rather significant uncertainties and is shown to lead to a **significant effect** on the matrix elements of 0vBB decay **in experimentally relevant isotopes**. It would be great if lattice QCD can directly constrain this coupling from first principles. We have built **a path between computable quantities in lattice QCD and the desired LEC of the EFT** and have shown it can be constrained **with percent-level precision** with **moderate uncertainties on the energy and matrix-element** determinations from lattice QCD.

THANK YOU