Proton Decay Amplitudes
with Physical Chirally-Symmetric Quarks

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[arXiv:2111.01608]

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- Proton decay basics
  - *Experimental lifetime limits & outlook*
  - *Motivation and theory status*
  - *Effective nucleon decay operators and matrix elements*
- Need for lattice calculations
  - *Past calculations and model uncertainty*
  - *Summary of the present calculation*
- Lattice calculation and analysis
  - *Hadron masses and energies*
  - *Extraction of matrix elements*
  - *Operator renormalization*
  - *Momentum & continuum extrapolations*
- Results
  - *Comparison to earlier calculations*
  - *Nucleon annihilation amplitudes*
  - *Conclusions*

Baryogenesis and Broken Symmetries

Why More Matter > Antimatter in the Universe?

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \cdot 10^{-10}$$

Three necessary components [A.Sakharov (1967)] :

proton decay, neutron oscillations

Baryon number-changing interactions

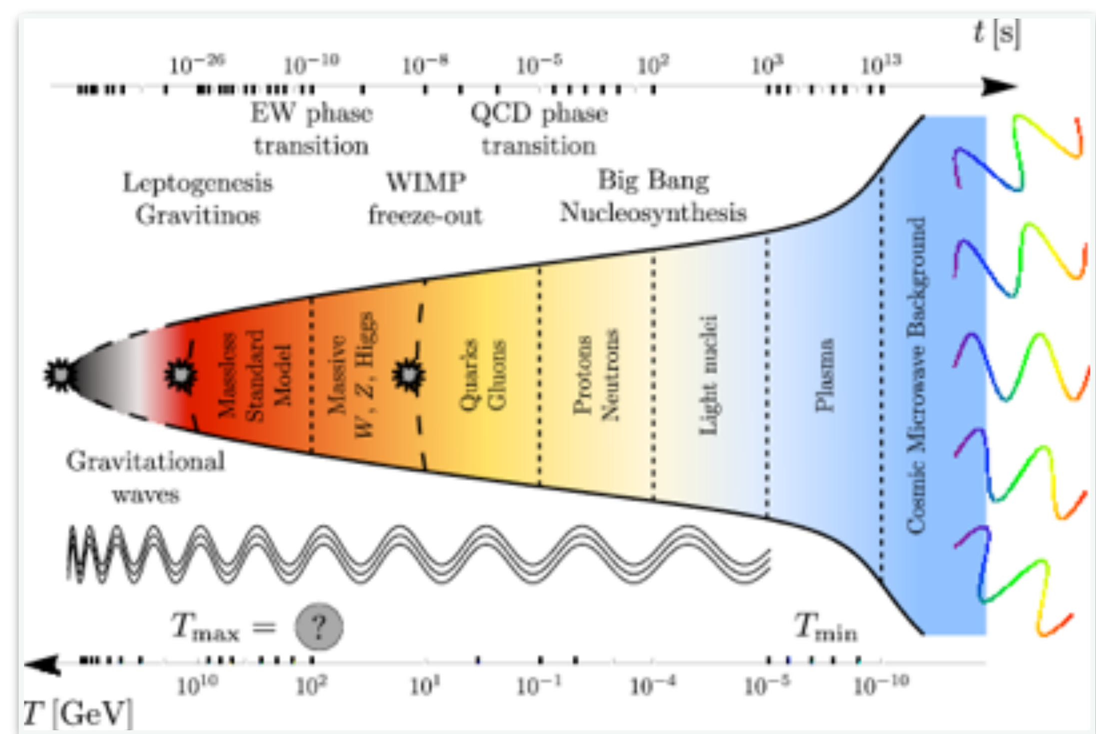
(alternatively, leptogenesis + sphalerons)

neutrinoless beta-decays

Violations of C- and CP-symmetries

(electric dipole moments of p, n, e⁻, nuclei, atoms)

Interactions out of equlibrium



Proton Decays and Grand Unification

Proton decay rate in a Grand Unified Theory

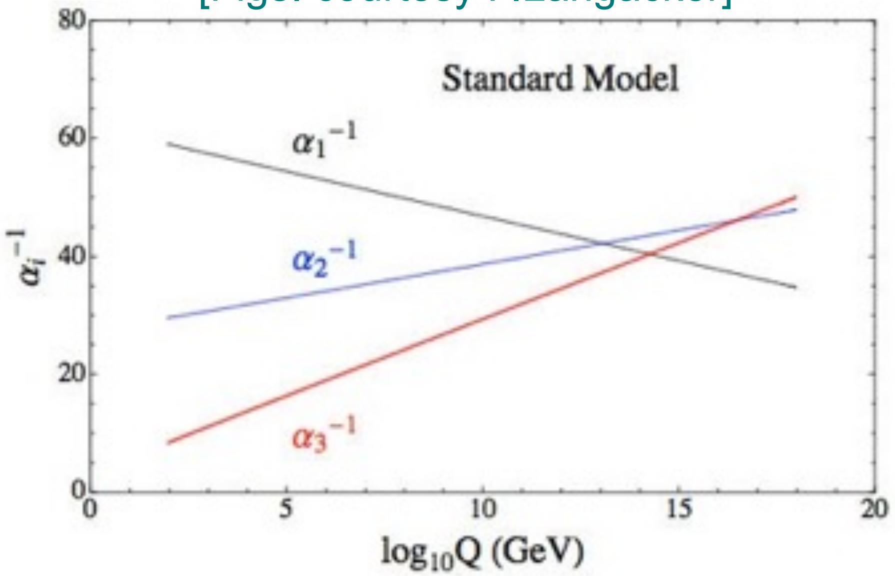
$$\frac{\tau_p}{Br(p \to \pi\bar{\ell})} \approx \frac{1.4 \cdot 10^{33}\ \text{years}}{|c_I|^2 \cdot \left|\langle\pi|\mathcal{O}_{\text{decay}}|p\rangle\right|^2} \cdot \left(\frac{\Lambda_{\text{GUT}}}{10^{15}\ \text{GeV}}\right)^4$$

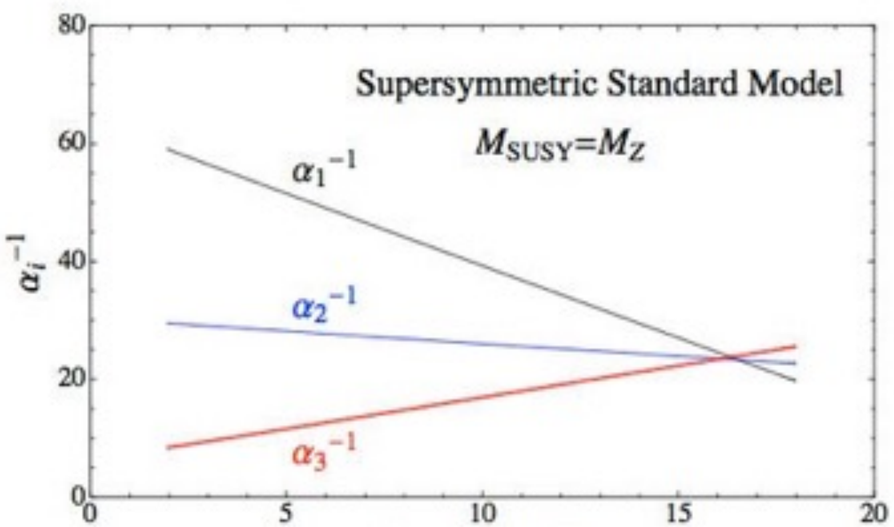
$\approx$ GUT boson mass

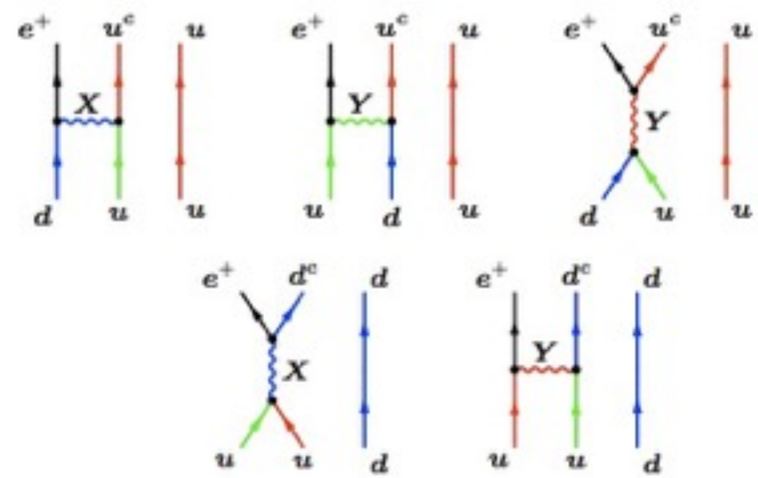
O(1) coupling at Λ_{GUT}

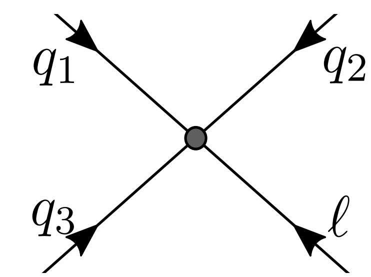
decay amplitude at nuclear scale

Coupling constants of the Standard Model
[Figs. courtesy P.Langacker]

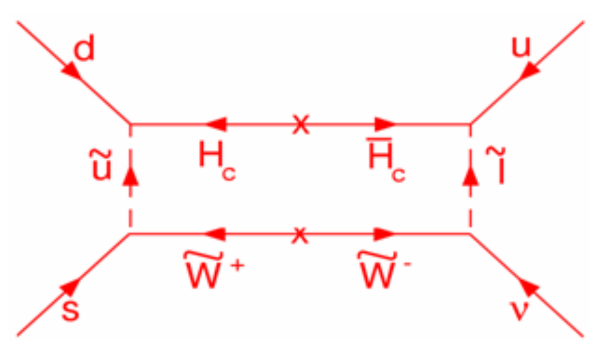






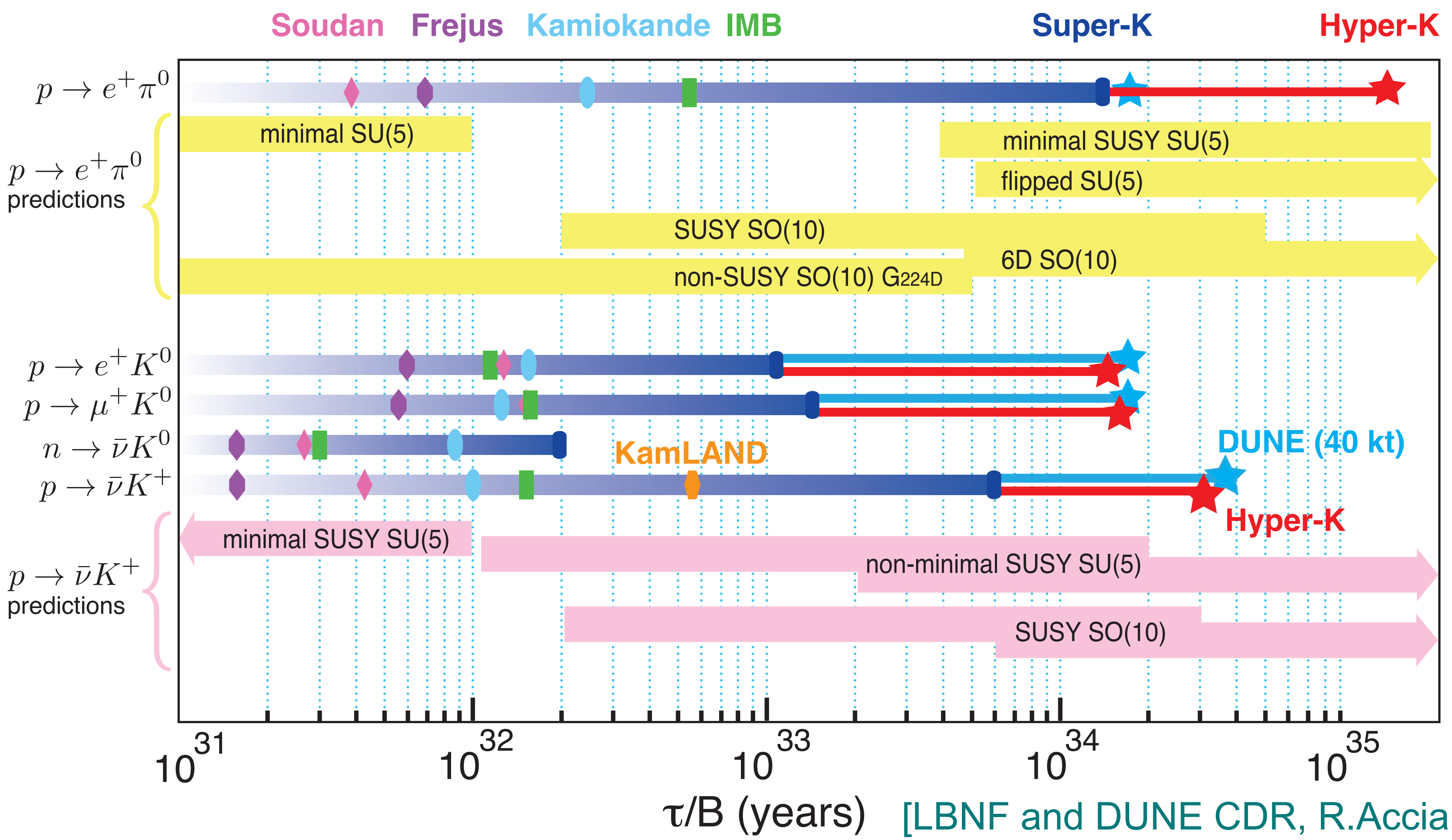


(qqql) + h.c.
effective interaction at hadron scale



SUSY $\Lambda_{\text{GUT}} \approx 10^{16}$ GeV, but $p \to \bar{\nu}K^+$ faster due to colored Higgs

Proton Stability: Status and Outlook



[LBNF and DUNE CDR, R.Acciarri et al (2015)]

- Current limits $\tau(p \to e^+\pi^0) \gtrsim 1.6\cdot10^{34}$, $\tau(p \to \bar{\nu}K^+) \gtrsim 5.9\cdot10^{33}$ [Super-K]
- Expect x10 improvement on lifetime limit from Hyper-K and DUNE
- Better sensitivity to $p \to \bar{\nu}K^+$ that affects supersymmetric GUT models

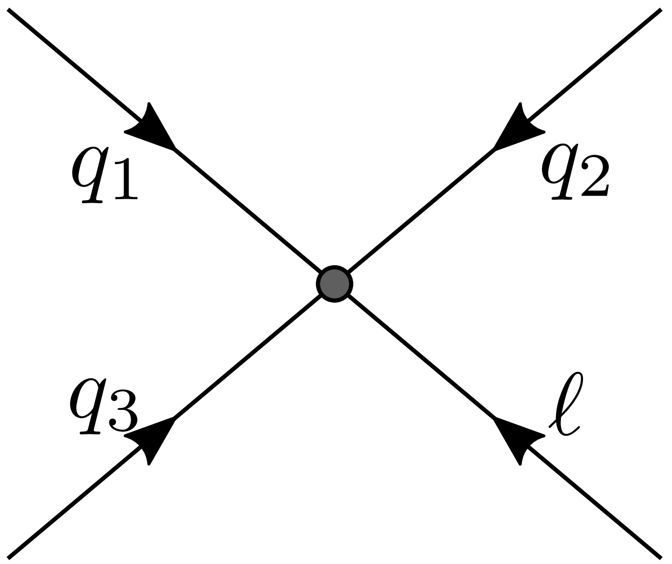
How Nucleon Structure Affects GUT Limits

Effective interaction

$$\mathcal{L}_{\text{eff}} = \sum_I C_I \mathcal{O}_I + \text{h.c.}$$

$$\mathcal{O}_I = \epsilon^{abc}(\bar{q}_1^{aC} P_{\chi_I} q_2^b)(\bar{\ell}^C P_{\chi'_I} q_3^c) = \bar{\ell}^C_\alpha\, \mathcal{O}^{3q}_{I,\alpha}$$

$$q_{1,2,3} \in \{u, d, s\}\,, \quad P_{\chi_I^{(\prime)}} = \frac{1 \pm \gamma_5}{2}$$



Decay matrix elements $(W_{0,1})_I$

$$\langle \bar{\ell}(q)\Pi(p)|\mathcal{O}^{\chi'}|N(k)\rangle = \bar{v}^C_{\ell\alpha}(q)\, P_{\chi'} \left[W_0(-q^2) - \frac{i\not{q}}{m_N} W_1(-q^2)\right] u_N(k)$$

and $W_{\bar{\ell}} = \left[W_0 + W_1 \cdot O(m_{\bar{\ell}}/m_N)\right]_{q^2=m^2_{\bar{\ell}}}$

nonperturbative QCD

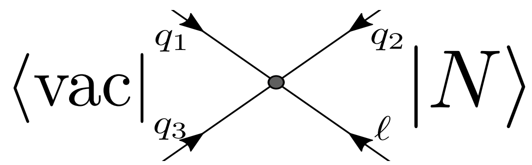
negligible for e^+;
≈10% for μ^+

[S.Aoki et al, PRD62:014506 (2000)]

Protons Stable due to Topology?

Why these experiments found nothing ?

- ***more complicated*** GUT scenario,
- **no BV interactions** at GUT scale ,
- OR protons&neutrons are ***just not likely*** to decay ?

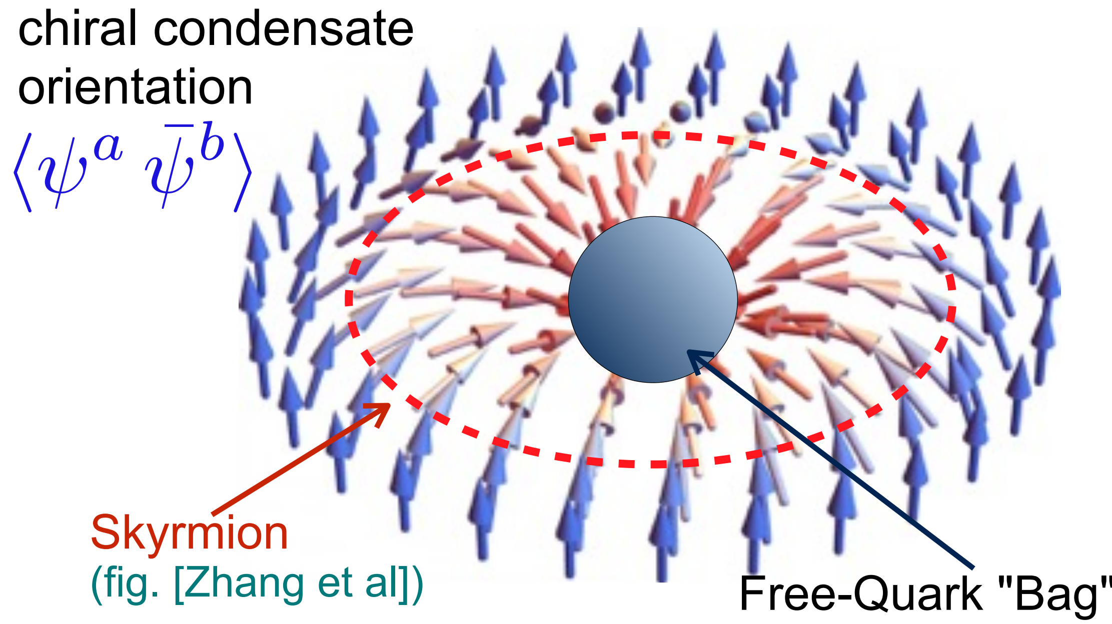


small?

"quark pudding" estimate:

$$\langle \mathrm{vac}|\mathcal{O}^{3q}|N\rangle \sim \rho_q^{3/2}\sqrt{V_N} \sim \frac{1}{V_N} \approx 0.004\,\mathrm{GeV}^3$$

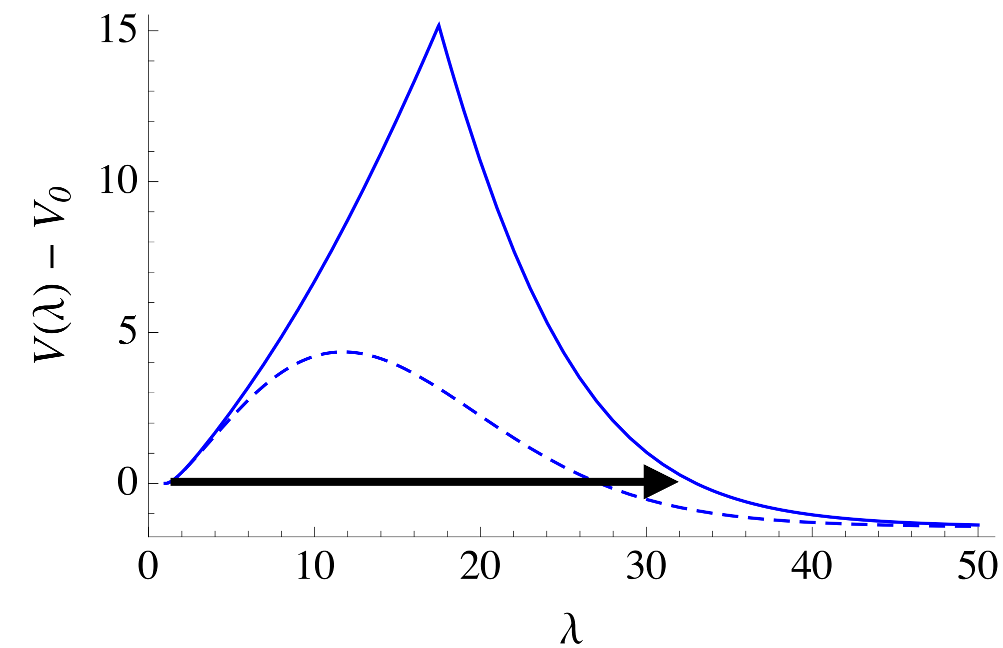
$$\langle \Pi|\mathcal{O}^{3q}|N\rangle \sim \langle \mathrm{vac}|\mathcal{O}^{3q}|N\rangle/f_\pi \approx 0.03\,\mathrm{GeV}^2$$



"Chiral Bag" Model

Topological stability of proton as a "Chiral Bag" [A.Martin, G.Stavenga '12]

- proton decay as quantum tunneling over topological barrier



Depending on R_{Bag}, decay rate may be suppressed ~ $O(10^{-4}) - O(10^{-12})$

Uncertainty can be addressed only by an ab initio QCD calculation

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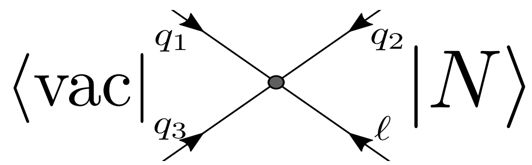

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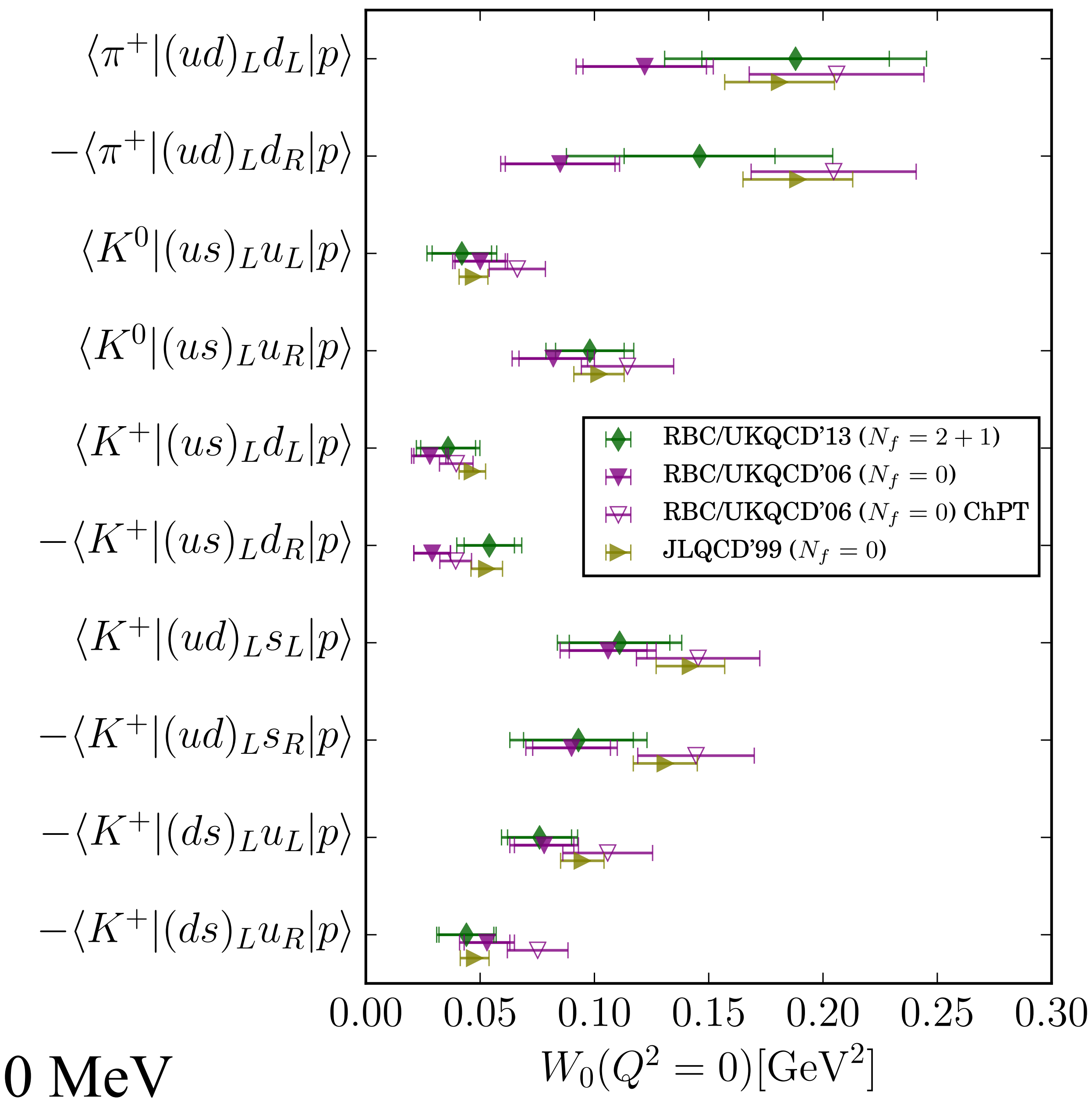
$$\langle \Pi|\mathcal{O}^{3q}|N\rangle \sim \langle \mathrm{vac}|\mathcal{O}^{3q}|N\rangle / f_\pi \approx 0.03\,\mathrm{GeV}^2$$

lattice calculations with $m_\pi \gtrsim 330$ MeV



Lattice QCD

- [S.Aoki et al (2000)] Wilson fermions
- [Y.Aoki et al (2006)] DW quenched
- [Y.Aoki et al (2013)] DW Nf=2+1 $m_\pi \gtrsim 330$ MeV

Any surprises in chiral limit? at physical light quarks?

This Work: Lattice Setup

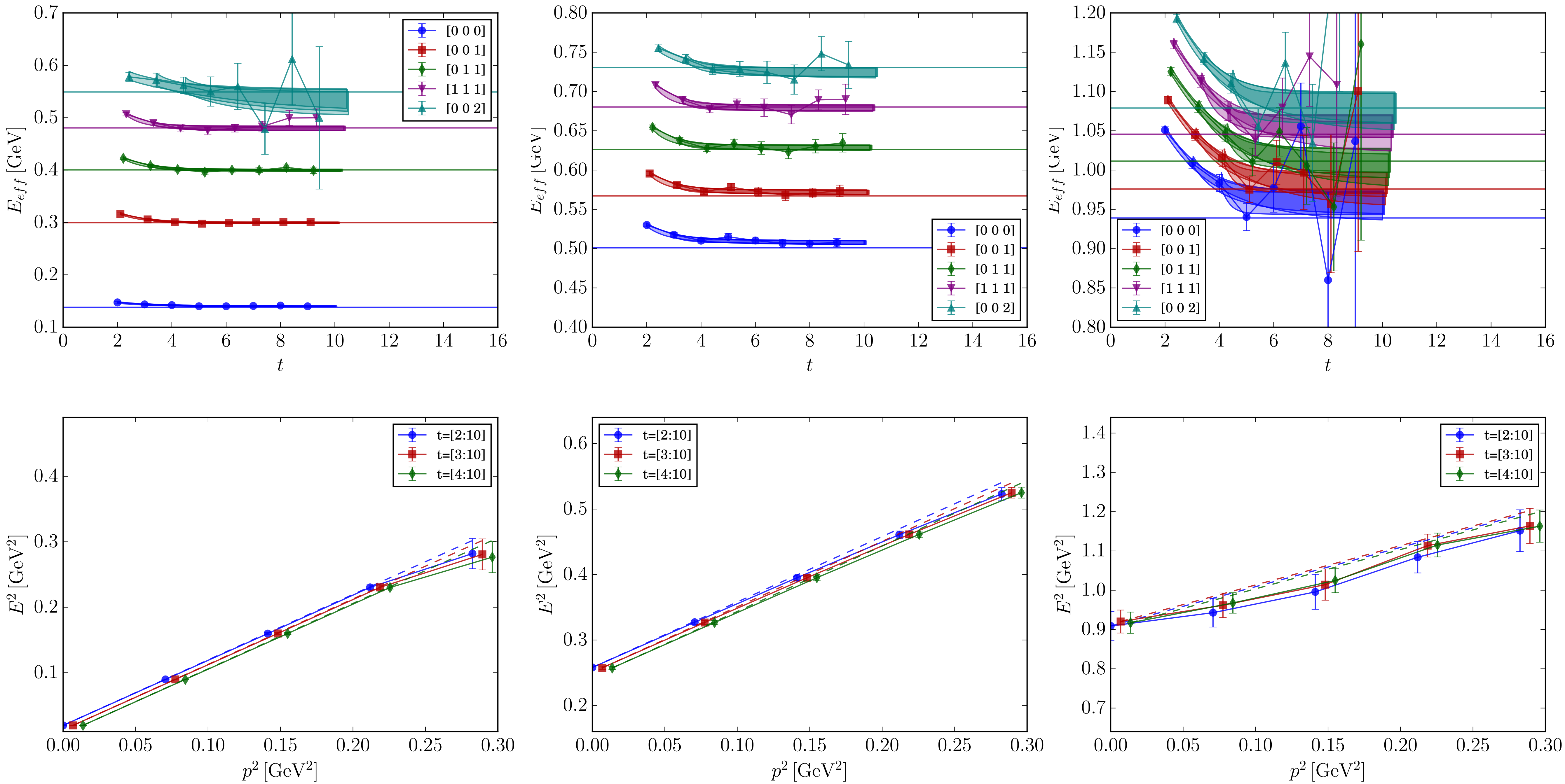
- Two ensembles: [32ID] $32^3\times64$(a=0.14 fm) and [24ID] $24^3\times64$(a=0.20 fm)
- Iwasaki gauge action+ Dislocation-supp. det.ratio (DSDR)
- $N_f = 2+1$ Chirally-symmetric (Mobius-)Domain Wall fermion action with physical light and strange quark masses
- Multigrid deflation of z-Mobius operator + AMA
- "Direct" ($p \to \pi,K$ matrix elements) and "Indirect" ($p \to$vacuum + ChPT)
- Nonperturbative renormalization
- Two state-fit analysis of π,K,N spectrum and $p \to \pi,K$ matrix elements
- a^2 Continuum extrapolation

	24ID $24^3 \times 64$	32ID $32^3 \times 64$
β	1.633	1.75
a, fm	0.20	0.14
a^{-1}, GeV	1.02	1.37
$m_\pi L$	3.4	3.3
N_{conf}	134	94
N_{samp}	4288	3008

- three kinematic (Q^2) points to interpolate matrix elements to decay kinematic $Q^2 = -(m_{\bar{\ell}})^2$

Π	$\vec{n}_\Pi$	$\vec{n}_N$	Q^2(GeV^2) (24c)	Q^2(GeV^2) (32c)
π	[1 1 1]	[0 0 0]	0.010	−0.012
	[1 1 1]	[0 1 0]	0.113	0.095
	[0 0 2]	[0 0 0]	−0.116	−0.140
K	[0 1 1]	[0 0 0]	−0.034	−0.042
	[0 1 1]	[0 1 0]	0.058	0.056
	[0 0 1]	[0 0 0]	0.075	0.074

Proton and Meson Spectrum



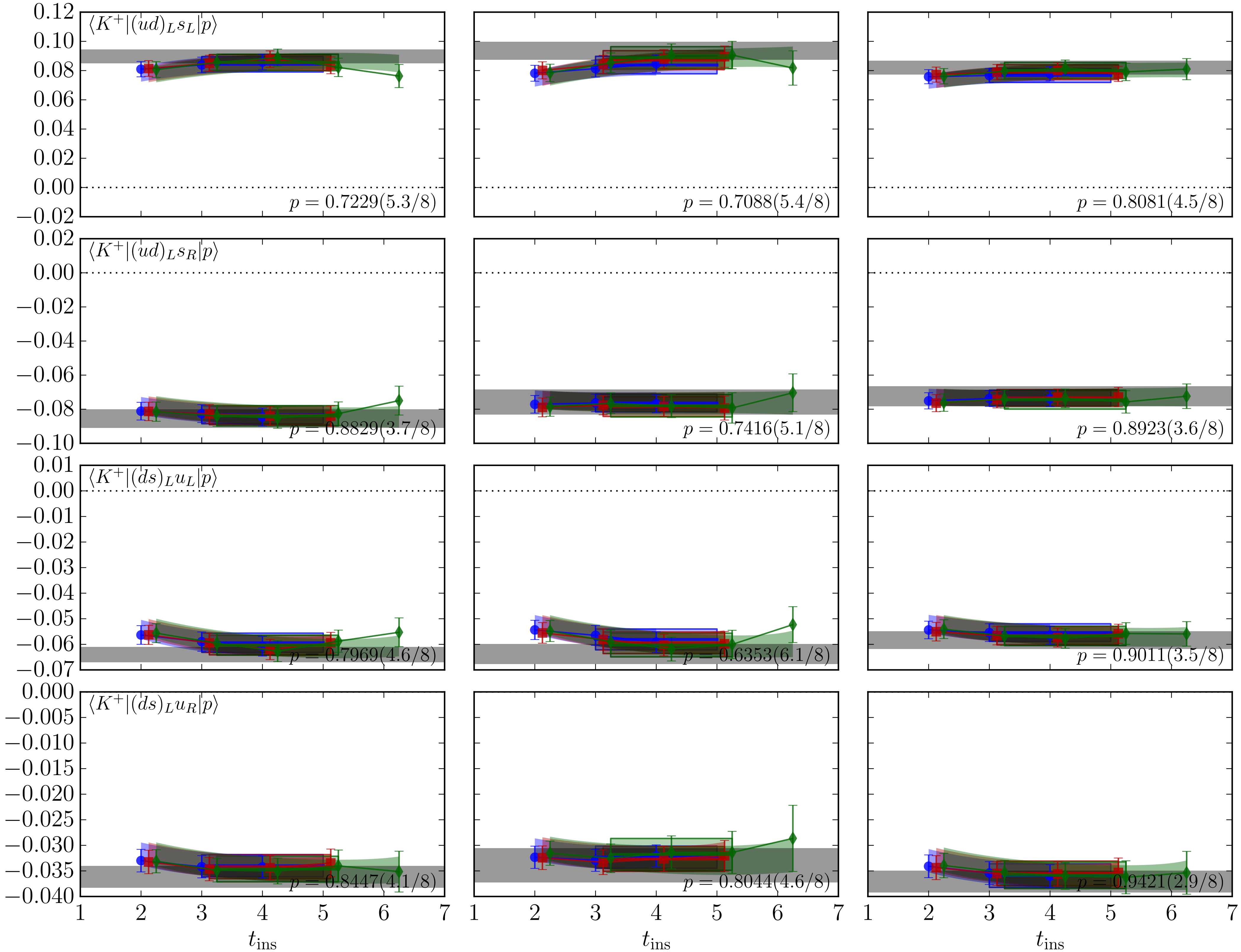
pion **kaon** **nucleon**

- 24ID ensemble (a=0.20 fm)
- Two-state fits + priors from large-t_{min} one-state fits

Extraction of Matrix Elements

32ID

W_0



- Two-state fits with energies fixed from spectrum fits: *no signs of excited-state sys.errors*

Nonperturbative Renormalization

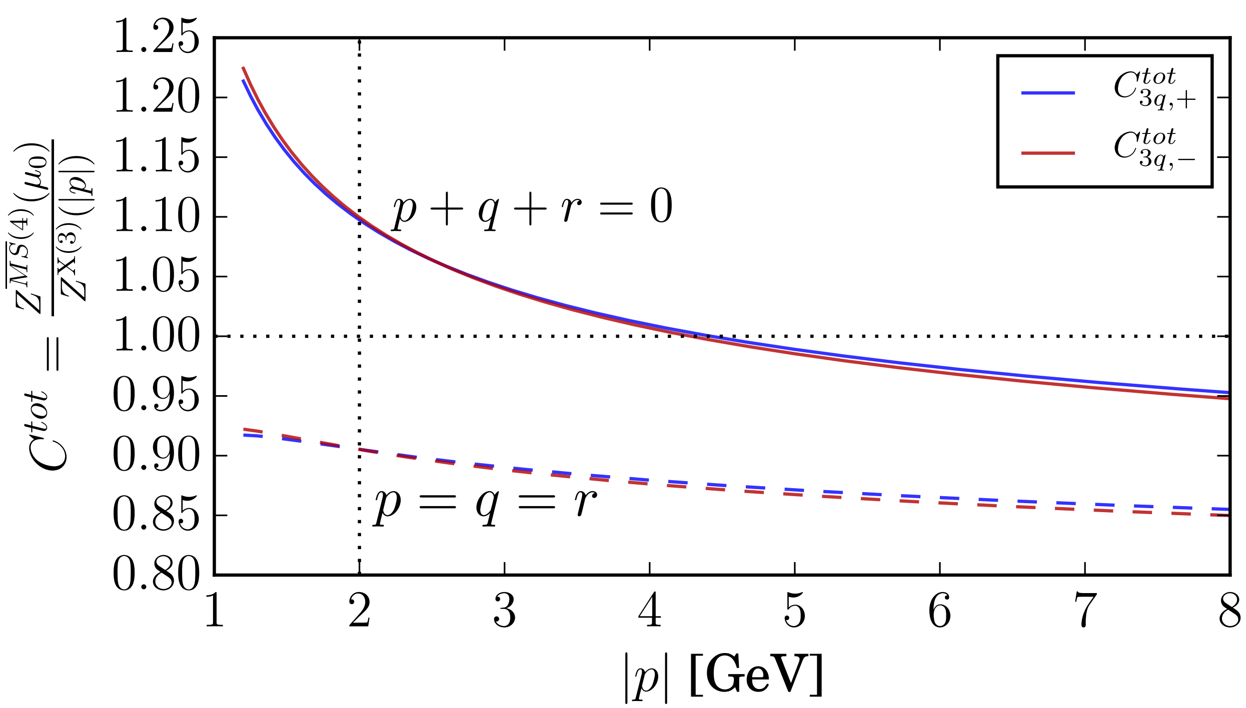
- symmetry-allowed mixing

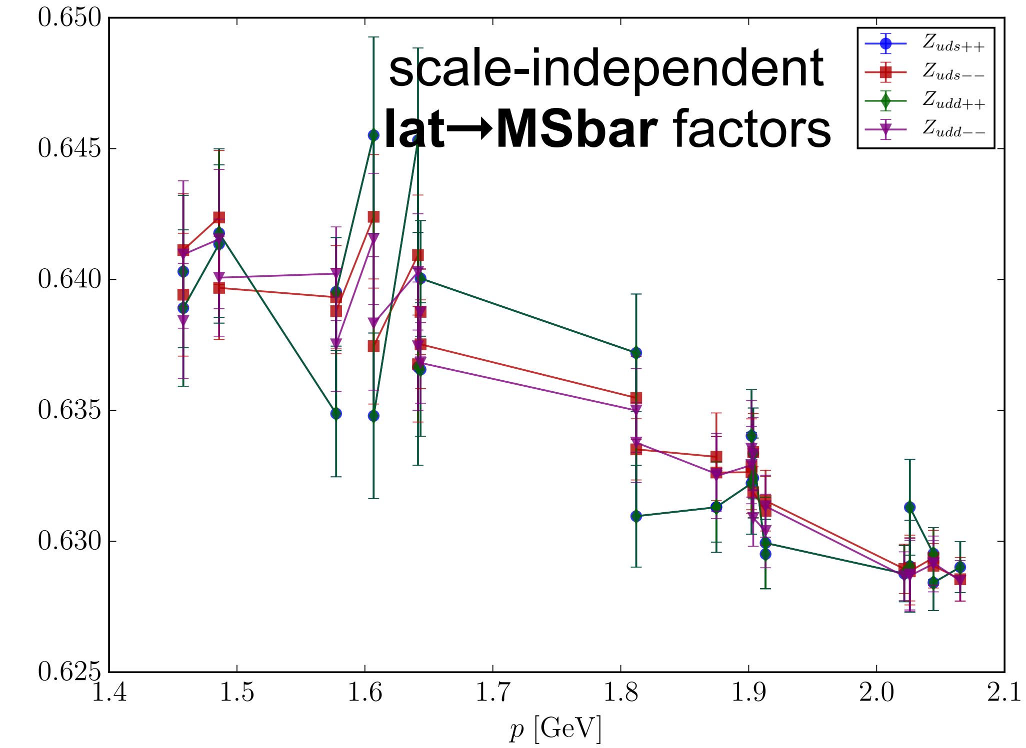
	$\mathcal{S}=-1$	$\mathcal{S}=+1$
$\mathcal{P}=-1$	$SS,\ PP,\ AA$	$VV,\ TT$
$\mathcal{P}=+1$	$SP,\ PS,\ AV$	$VA,\ TQ$

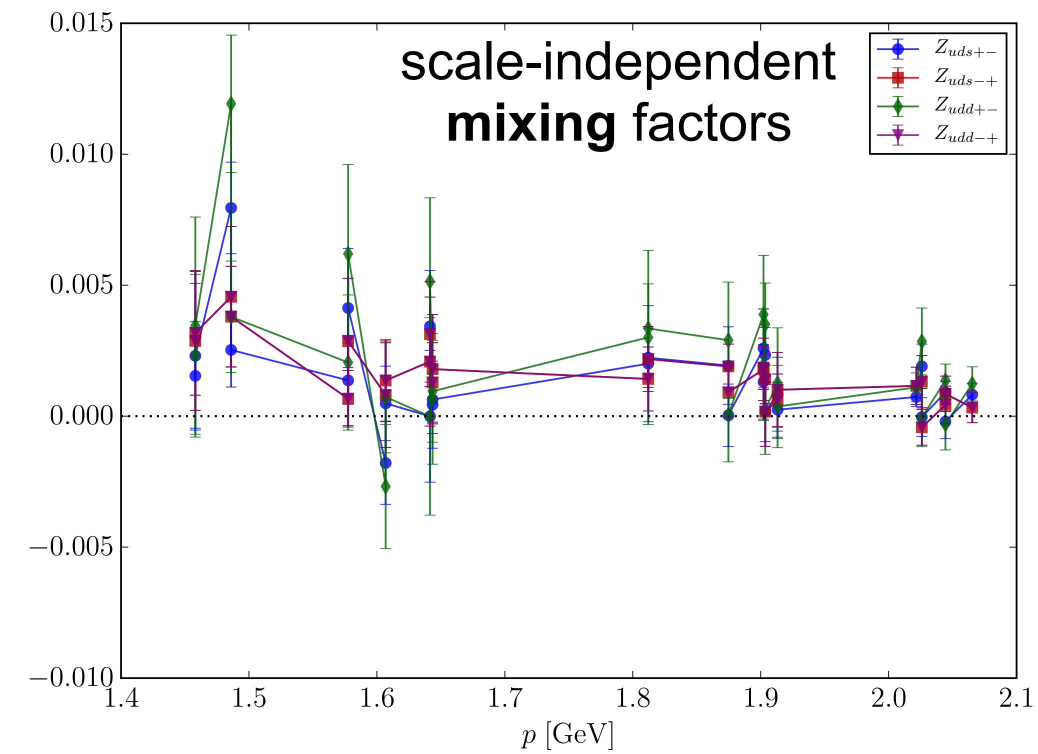
- *symmMOM* scheme : $p+q+r=0,\ \ p^2=q^2=r^2=\mu^2$

$$Z^{3q}_{IK}(\mu)\ \mathrm{Proj}_J\left[\langle \bar{q}_1(p)\bar{q}_2(q)\bar{q}_3(r)\,\mathcal{O}^{3q}_K\rangle_{\mathrm{amp}}\right]=\delta_{IJ}$$

- symmMOM(p)→MSbar(2 GeV) perturbative conversion at $O(\alpha^3)$ [J.Gracey, JHEP09:052 (2012)]

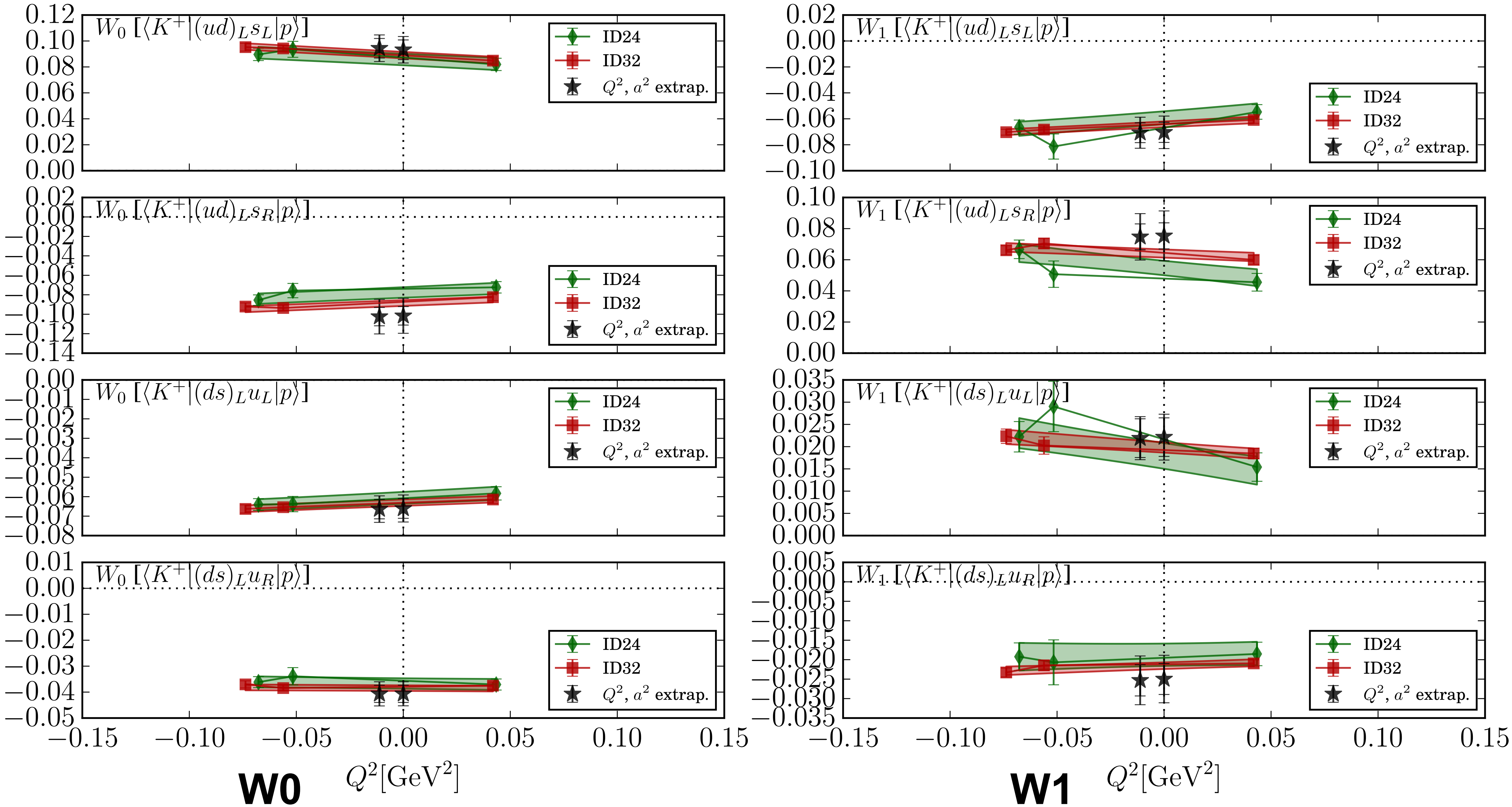






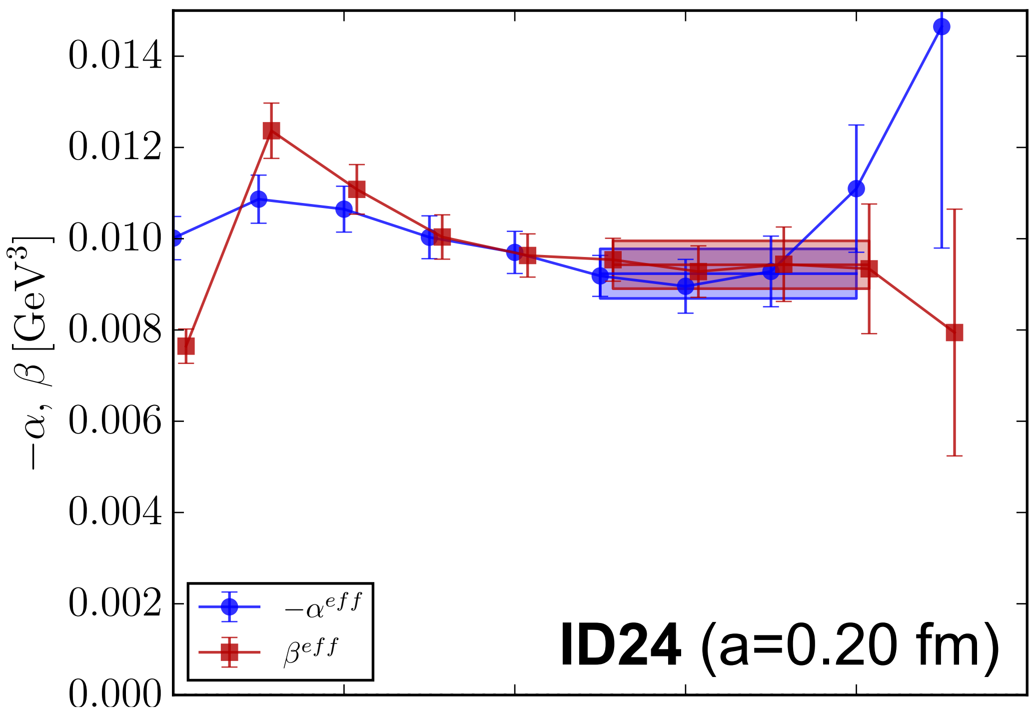
- *chiral symmetry suppresses mixing of L⟺R fields & operators*

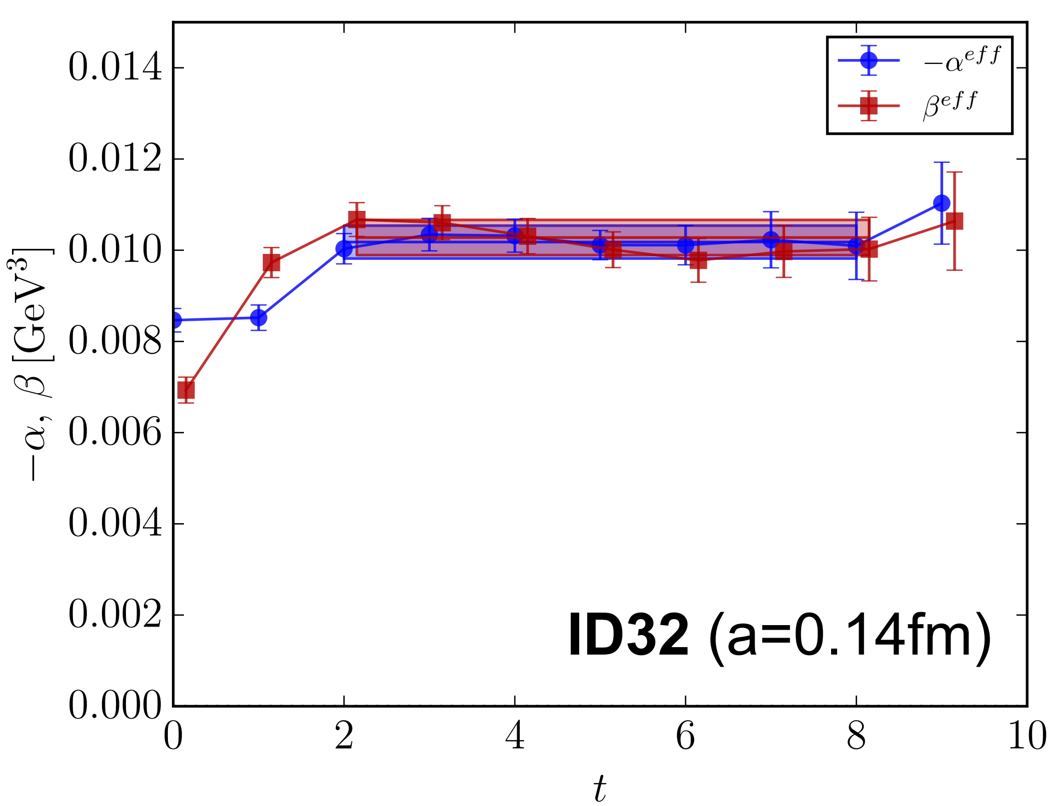
Momentum and Continuum Extrapolation



- linear momentum extrapolation $Q^2 \rightarrow m_e^2, m_\mu^2$ to the decay kinematics
- Continuum extrapolation $A(a^2) \sim (A_0 + A_2\, a^2)$; *conservative* sys.error = $|A_0 - A_{[a=0.14\mathrm{fm}]}|$

Proton Decay constants



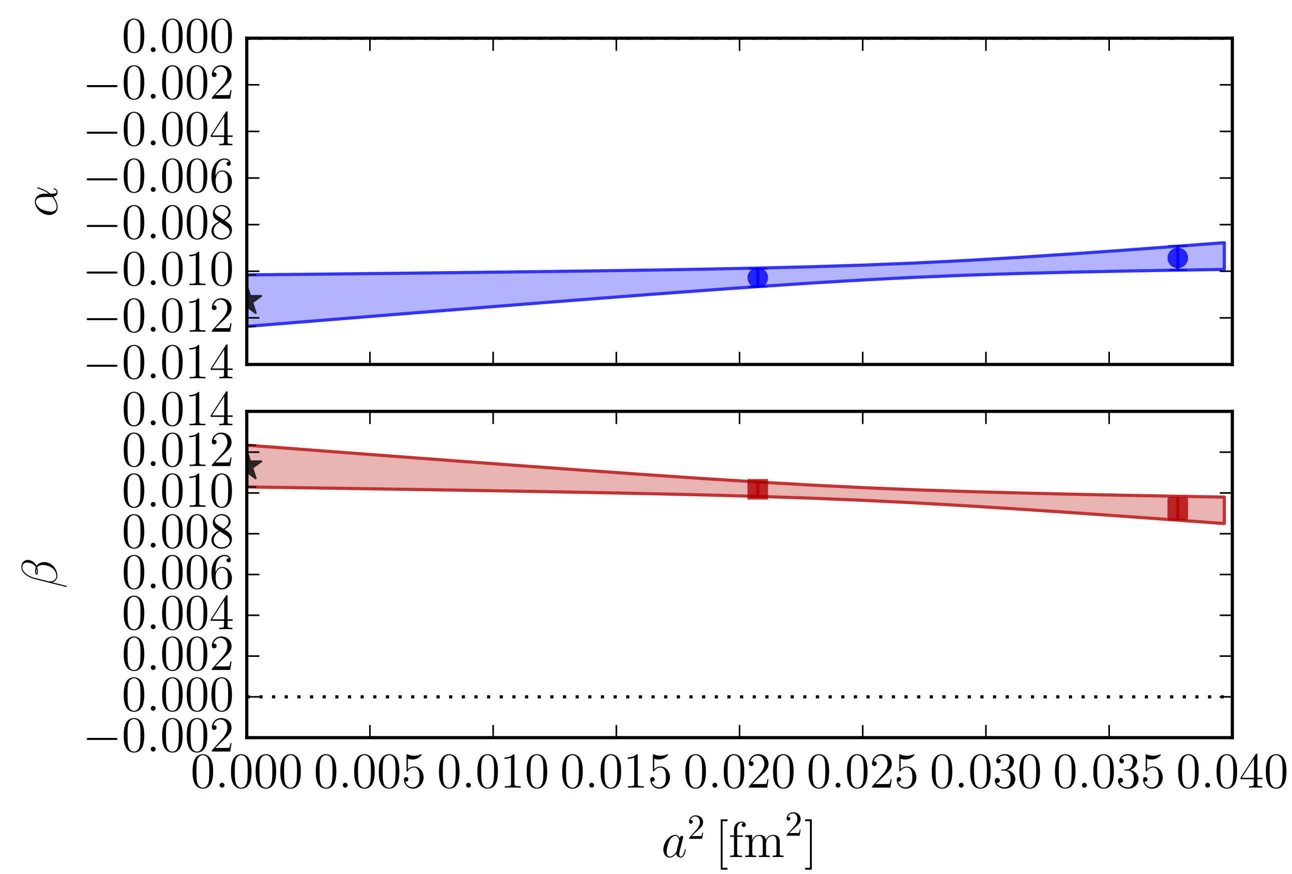


$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC} d^b)_R u^c_L|N\rangle = \alpha\, P_L U_N$$
$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC} d^b)_L u^c_L|N\rangle = \beta\, P_L U_N$$

- connected to $\langle \pi/K|O^{3q}|N\rangle$ by soft-pion theorem
- $(\alpha+\beta) = 0$ [within errorbars] implying

$$\epsilon^{abc}(\bar{u}^{aC} d^b)\gamma_5 u^c\,|N\rangle \overset{?}{\approx} 0$$

parity (–) (–) (+)



Proton Annihilation Amplitudes

LO ChPT proton decay amplitudes

$$\langle\pi^+|(ud)_L u_L|p\rangle = \frac{\beta}{f}(1+D+F),$$
$$\langle\pi^+|(ud)_L u_R|p\rangle = \frac{\alpha}{f}(1+D+F),$$
$$\langle K^0|(us)_L u_L|p\rangle = \frac{\beta}{f}\left(1-(D-F)\frac{m_N}{m_B}\right),$$
$$\langle K^0|(us)_L u_R|p\rangle = -\frac{\alpha}{f}\left(1+(D-F)\frac{m_N}{m_B}\right),$$
$$\langle K^+|(us)_L d_L|p\rangle = \frac{\beta}{f}\left(\frac{2D}{3}\frac{m_N}{m_B}\right)$$
$$\langle K^+|(us)_L d_R|p\rangle = \frac{\alpha}{f}\left(\frac{2D}{3}\frac{m_N}{m_B}\right)$$
$$\langle K^+|(ud)_L s_L|p\rangle = \frac{\beta}{f}\left(1+\left(\frac{D}{3}+F\right)\frac{m_N}{m_B}\right),$$
$$\langle K^+|(ud)_L s_R|p\rangle = \frac{\alpha}{f}\left(1+\left(\frac{D}{3}+F\right)\frac{m_N}{m_B}\right),$$
$$\langle K^+|(ds)_L u_L|p\rangle = -\frac{\beta}{f}\left(1-\left(\frac{D}{3}-F\right)\frac{m_N}{m_B}\right),$$
$$\langle K^+|(ds)_L u_R|p\rangle = \frac{\alpha}{f}\left(1+\left(\frac{D}{3}-F\right)\frac{m_N}{m_B}\right),$$

$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC}d^b)_R u_L^c|N\rangle = \alpha\, P_L U_N$$
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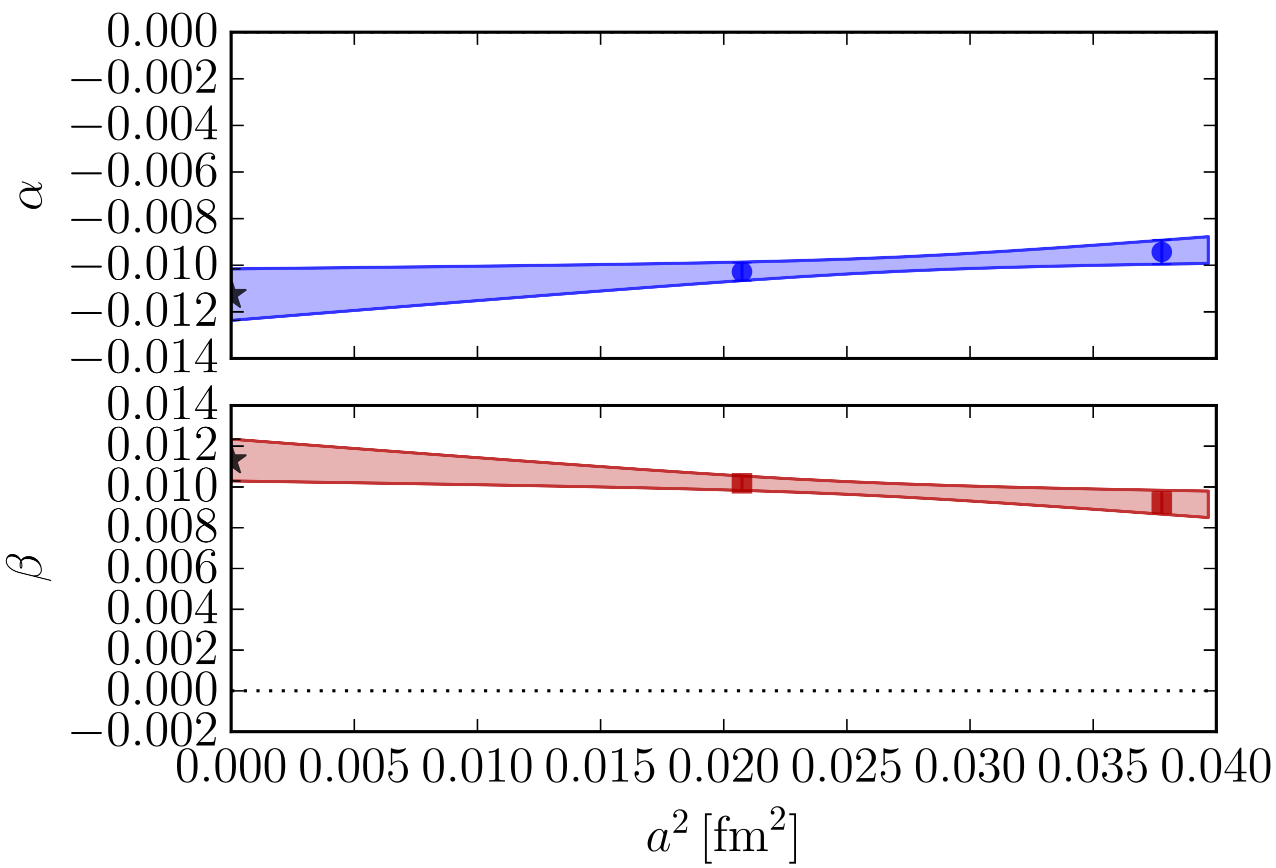
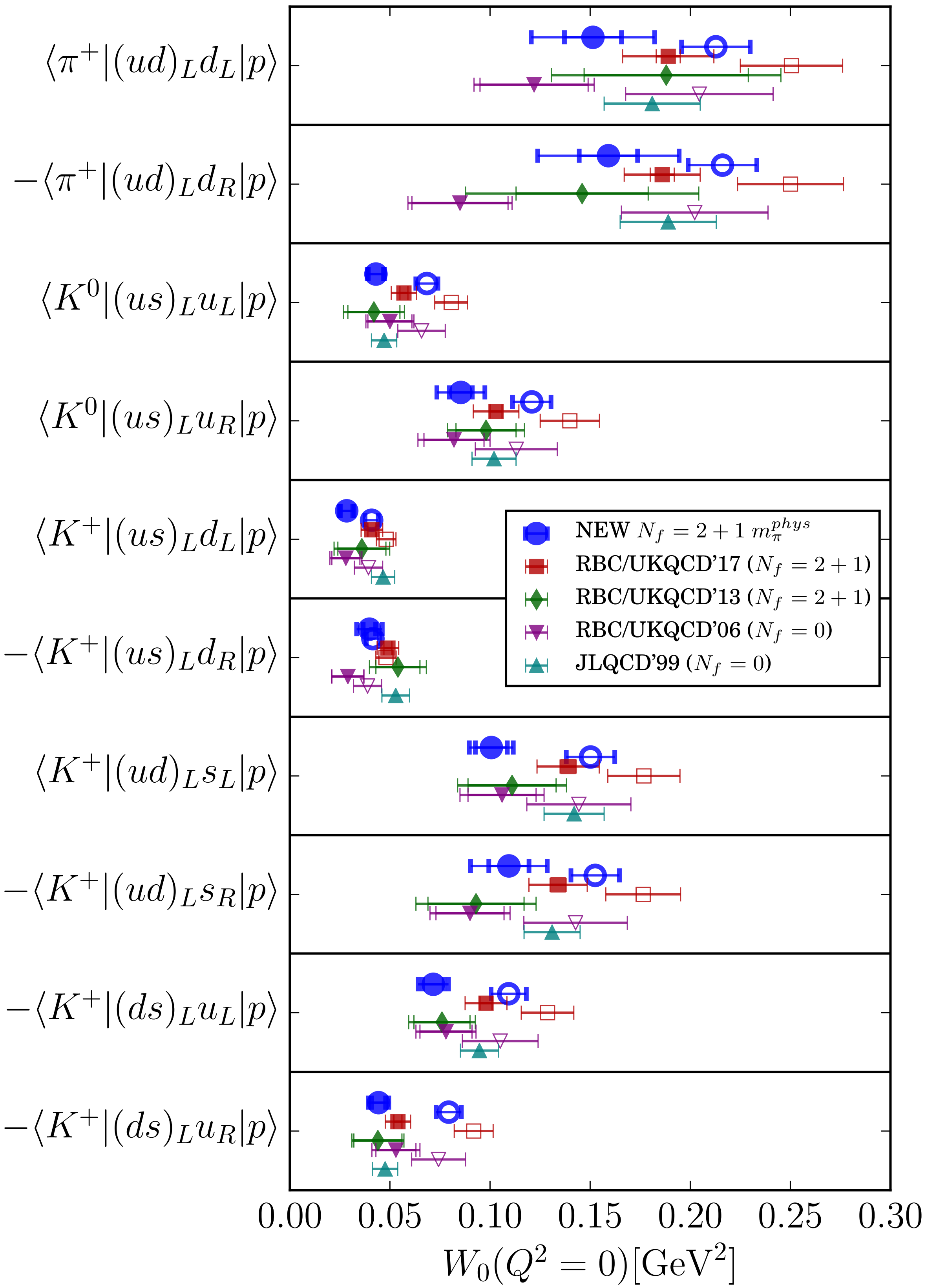


Comparison to Previous Work



- New results with **physical quarks**: stat.+sys. uncertainty ~ 10-20% [J.Yoo, S.S., et al, arXiv:2111.01608]
- (No finite-volume check: mpi*L~3.4)
- (Coarse lattices a=0.14 ... 0.20 fm)
- physical-point results agree with prev. calculations at mπ ≳340 MeV [Y.Aoki et al (2000–2013)]

***NO SUPPRESSION** of nucleon decay due to chiral skyrmion topology*

*Protons are indeed **sensitive probes** of Grand-Unified Theories*

Summary & Conclusions

- *Proton decay amplitudes at the physical point with chiral symmetric quarks and continuum extrapolation*
- *No topological suppression of nucleon decay found; limits on Grand-Unified Theories* ***stand***
- *Sys+Stat. precision O(10-20%) ; may be improved with more statistics, finer lattice spacing, finite-volume study*

BACKUP