News from the lattice and the Unitarity Triangle Fit

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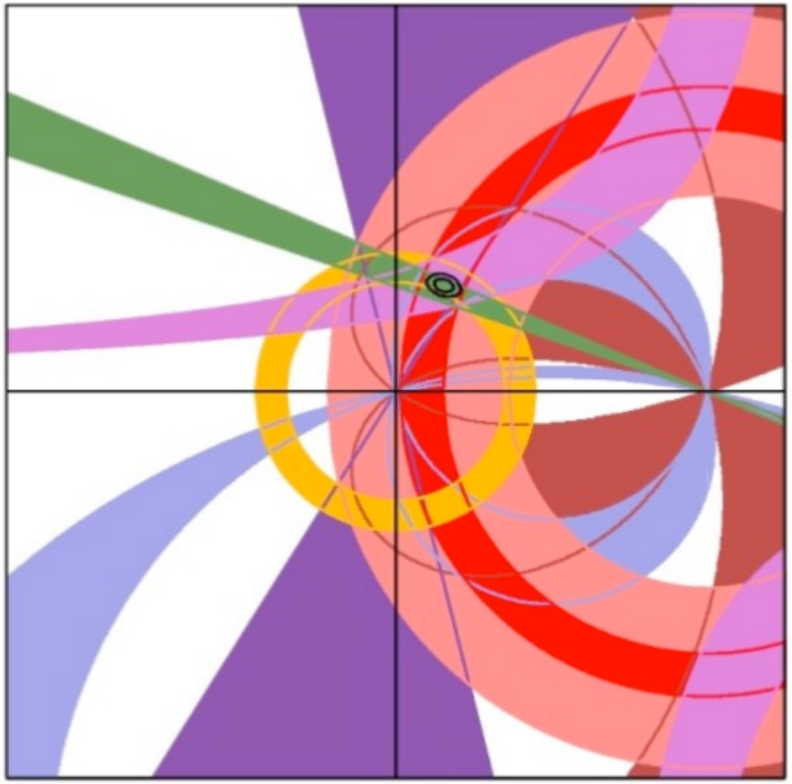
UTfit

PRACE

INFN
Istituto Nazionale di Fisica Nucleare

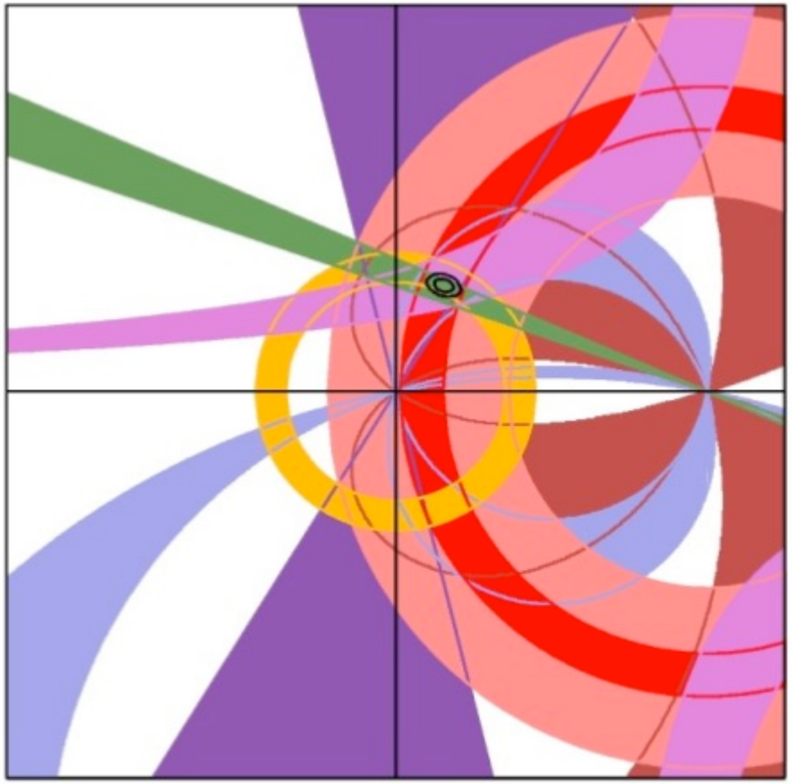
PLAN OF THE TALK

- *General introduction to the Unitary Triangle Fit*
- *SM Analysis*
- *Tensions and unknown*
- *Some news in lattice calculations;*
- *Future directions, new/old ideas*
- *Conclusion*



Thanks to
Bona, Lubicz,
Silvestrini, Simula,
Vittorio,

STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (Flavor Physics)



- *Provides the best determination of the CKM parameters;*
- *Tests the consistency of the SM (``direct" vs ``indirect" determinations) @ the quantum level;*
- *Provides predictions for SM observables (in the past for example sin 2β and Δm_s)*
- *It could lead to new discoveries (CP violation, Charm, !?)*

The fundamental issue is ***to find signatures of new physics*** *and to unravel the underlying theoretical structure;*

Precision Flavor physics is a key tool*, complementary to the large energy searches at the LHC;*

If the LHC discovers new elementary particles BSM, then precision flavor physics will be necessary to constrain the underlying framework;

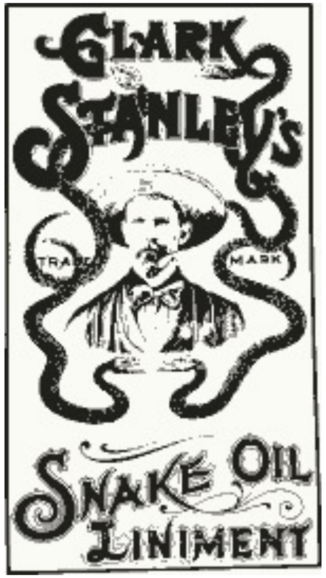
The discovery potential of precision flavor physics should not be underestimated.

The extraordinary progress of the experimental measurements requires accurate theoretical predictions

Precision flavour physics requires the control of hadronic effects for which lattice QCD simulations are essential.

$$Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle$$

$$Q^{EXP} = \sum_i C^i_{SM}(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C^{i'}_{Beyond}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle$$



UTfit

RELOADED

www.utfit.org

M.Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco, V. Lubicz, G. Martinelli, D. Morgante, M. Pierini, L. Silvestrini, S. Simula, C. Tarantino, V. Vagnoni, M. Valli, and L.Vittorio

Plots and numbers in this talk are hot-off-the-press for this workshop

Measure	V_{CKM}	Other NP parameters
$\Gamma(b\to u)/\Gamma(b\to c)$	$\bar\rho^2+\bar\eta^2$	$\bar\Lambda, \lambda_1, F(1), \ldots$
ε_K	$\eta[(1-\bar\rho)+\ldots]$	B_K
Δm_d	$(1-\bar\rho)^2+\bar\eta^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d/\Delta m_1$	$(1-\bar\rho)^2+\bar\eta^2$	ξ
$A_{CP}(B_d\to J/\psi K_s)$	$\sin 2\beta$	

M. Bona et al. (UTfit Collaboration) JHEP 0507:028,2005 hep-ph/0501199

M. Bona et al. (UTfit Collaboration) JHEP 0603:080,2006 hep-ph/0509219

classical UT Bayes analysis

$$f(\bar\rho,\bar\eta,X|c_1,\ldots,c_m)\sim\prod_{j=1,m} f_j(\mathcal{C}|\bar\rho,\bar\eta,X)*\prod_{i=1,N} f_i(x_i)f_0(\bar\rho,\bar\eta)$$

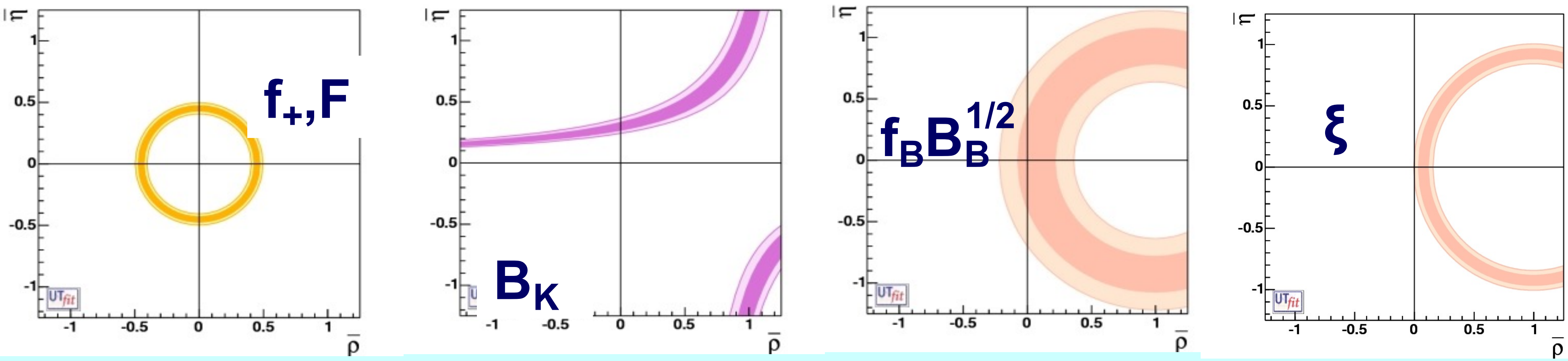
$$X\equiv x_1,\ldots,x_n=m_t,B_K,F_B,\ldots$$

$$\mathcal{C}\equiv c_1,\ldots,c_m=\epsilon,\Delta m_d/\Delta m_s,A_{CP}(J/\psi K_S),\ldots$$

Quantities used in the Standard UT Analysis

levels @
68% (95%) CL

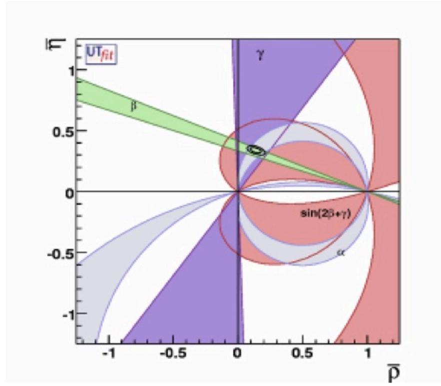
V_{ub}/V_{cb} | ε_K | Δm_d | $\Delta m_d/\Delta m_s$



Inclusive vs Exclusive
Opportunity for lattice QCD

UT-LATTICE

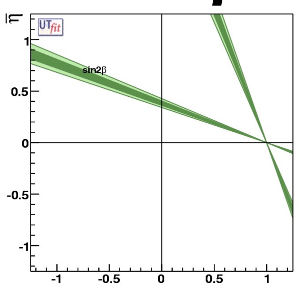
Other Quantities used in the UT Analysis



UT-ANGLES

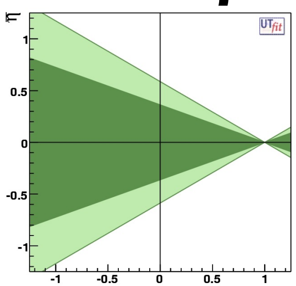
Several new determinations of UT angles are now available, thanks to the results coming from the B-Factory experiments

$\sin 2\beta$



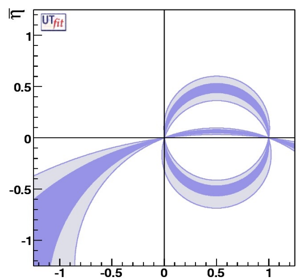
$B \to J/\Psi K^0$

$\cos 2\beta$



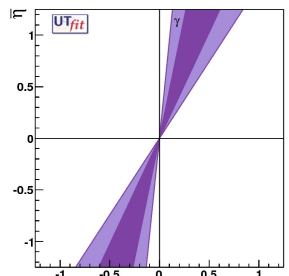
$B \to J/\Psi K^{*0}$

α



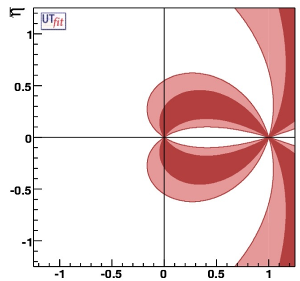
$B \to \pi\pi, \rho\rho$

γ



$B \to D^{(*)}K$

$\sin(2\beta+\gamma)$

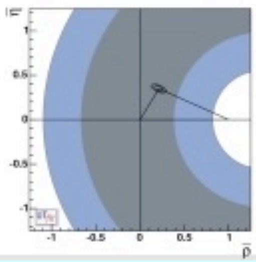


$B \to D^{(*)}\pi, D\rho$

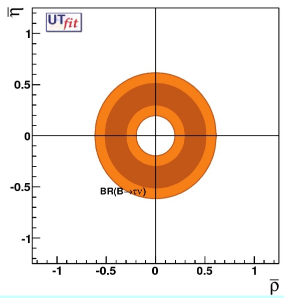
New Constraints from B and K rare decays (not used yet)

New bounds are available from rare B and K decays. They do not still have a strong impact on the global fit and they are not used at present.

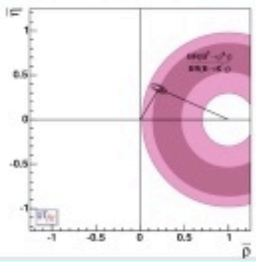
$K \to \pi \nu \bar{\nu}$



$B \to \tau \nu$

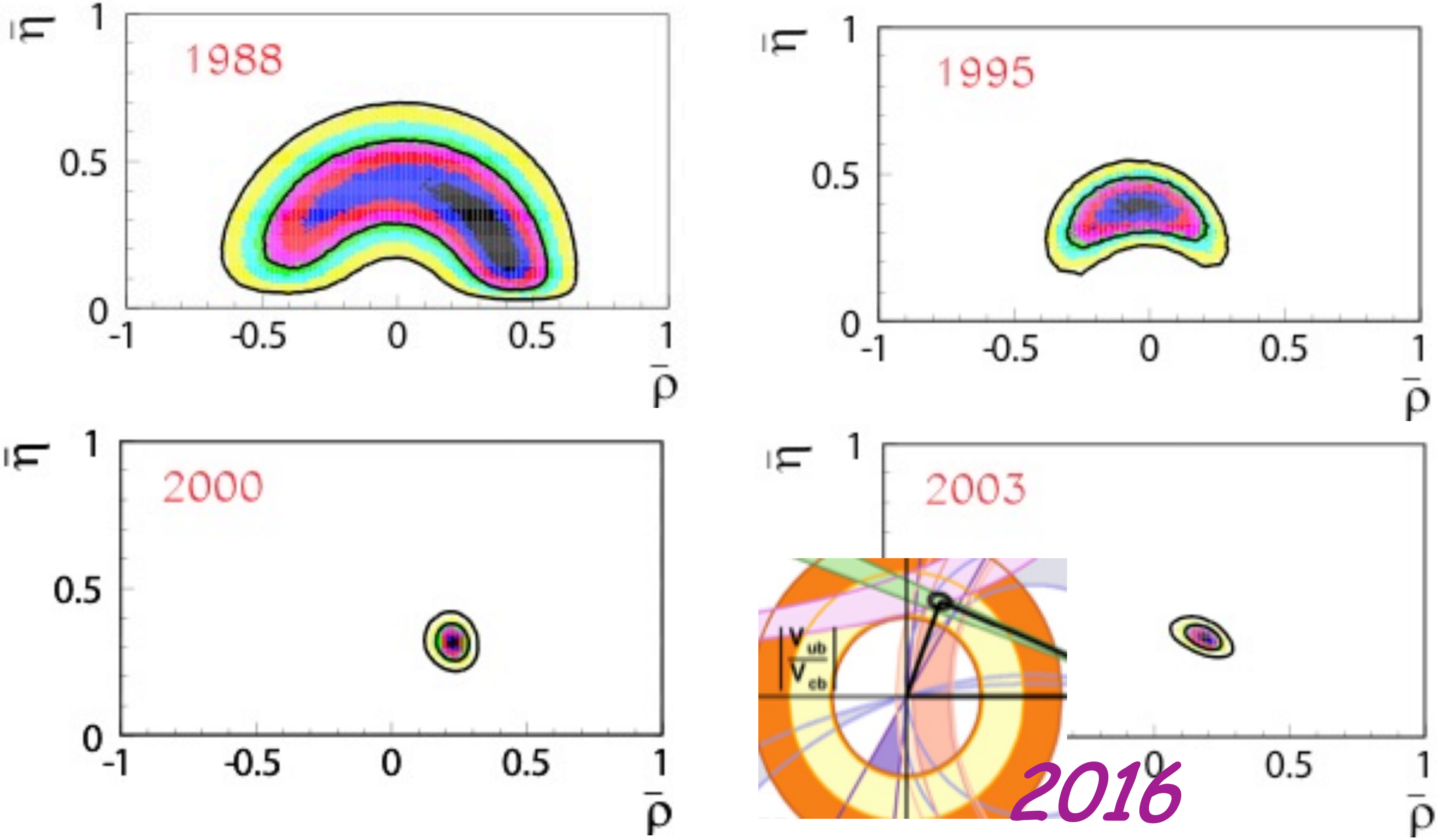


$(B \to \rho/\omega\,\gamma)/(B \to K^*\gamma)$



PROGRESS SINCE 1988

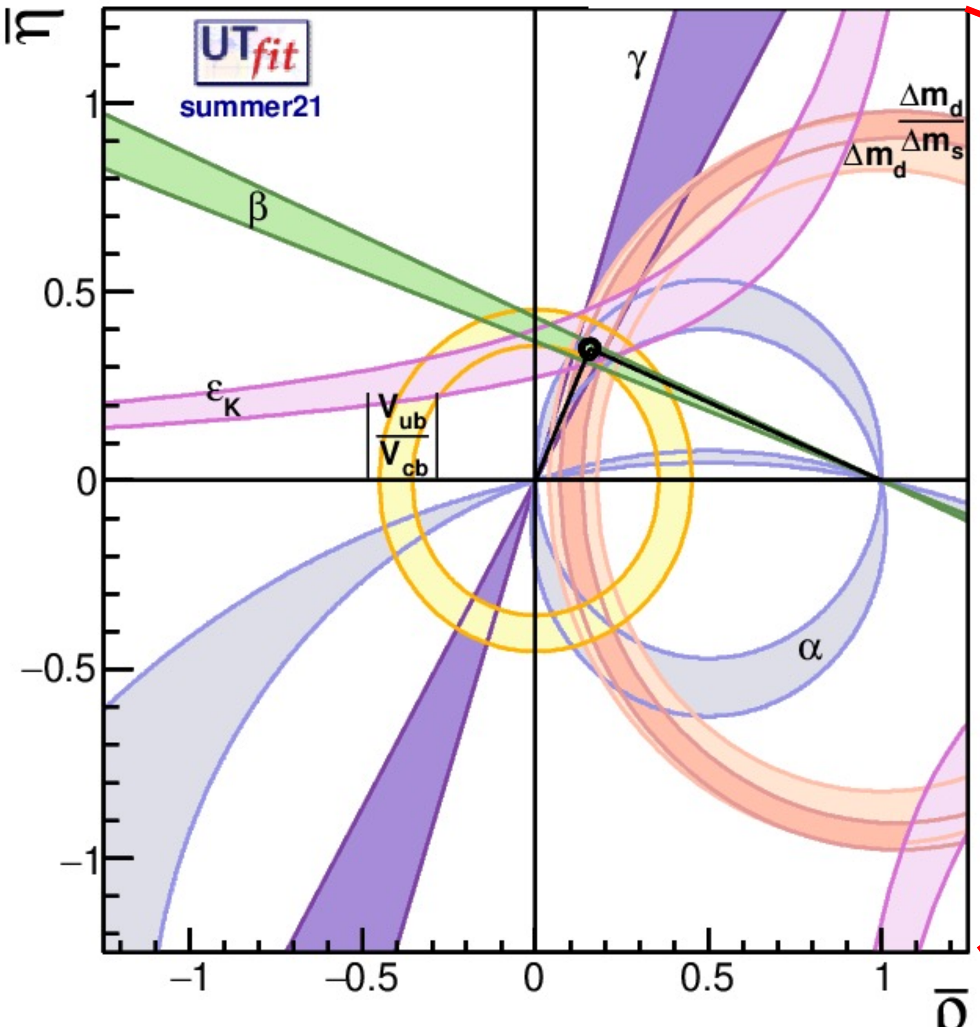
Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)

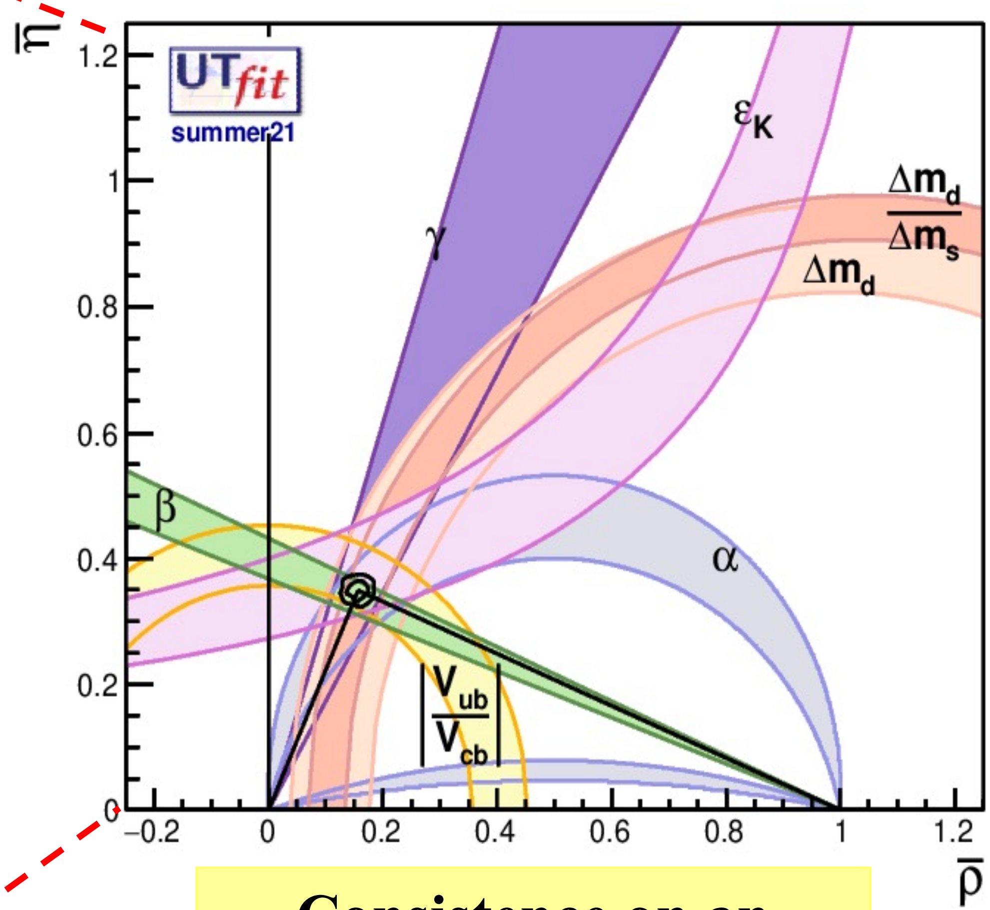


Unitarity Triangle analysis in the SM   Summer 2021

PLOTS AND NUMBERS TO BE UPDATED

levels @ 95% Prob





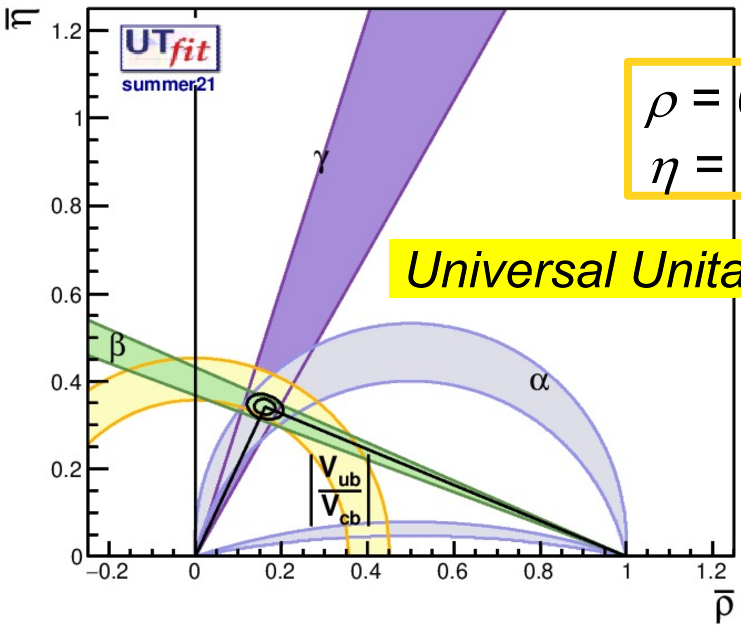
Consistence on an over constrained fit of the CKM parameters

$\rho = 0.156 \pm 0.012$ ~8%

$\eta = 0.350 \pm 0.010$ ~3%

The CKM matrix is the dominant source of flavour mixing and CP violation

Some interesting configurations

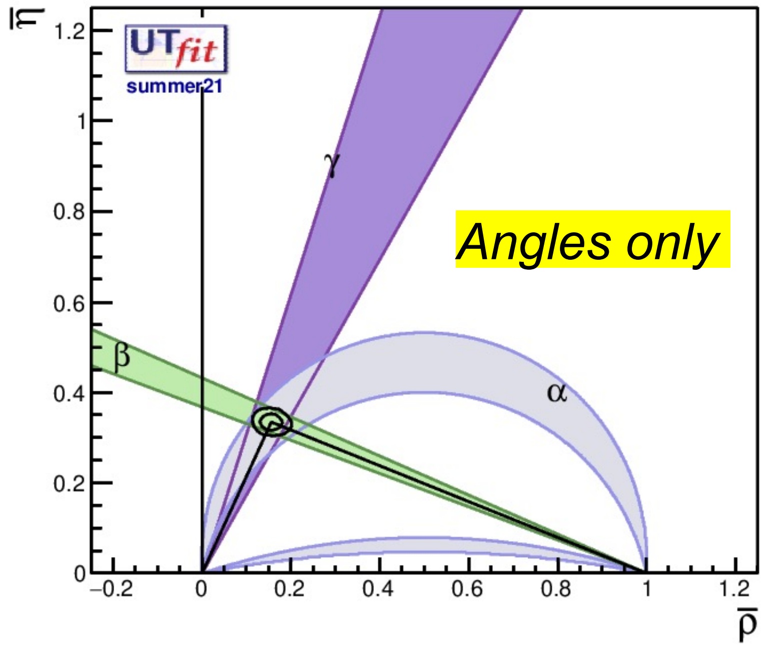


~10%

ρ = 0.162 ± 0.017
η = 0.341 ± 0.011

~3%

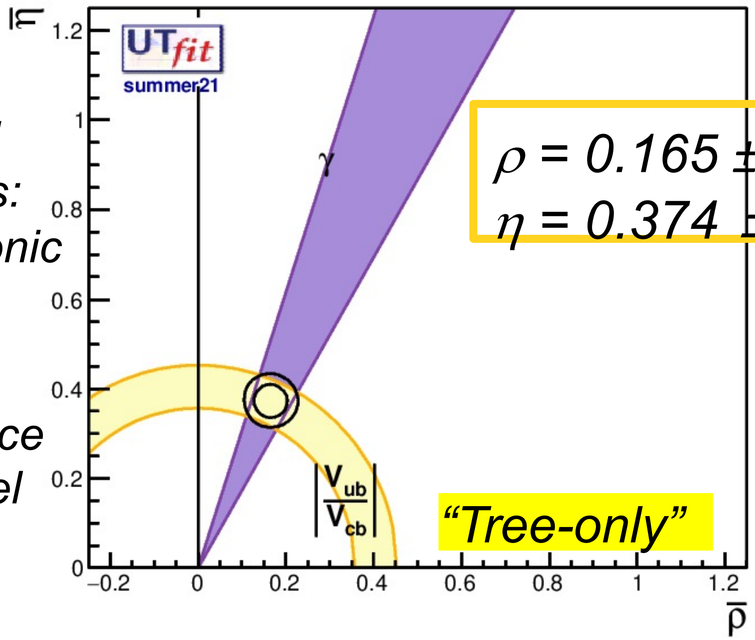
Universal Unitary Triangle



Angles only

Tree-level processes: Semileptonic and DK B decays

→ reference for model building

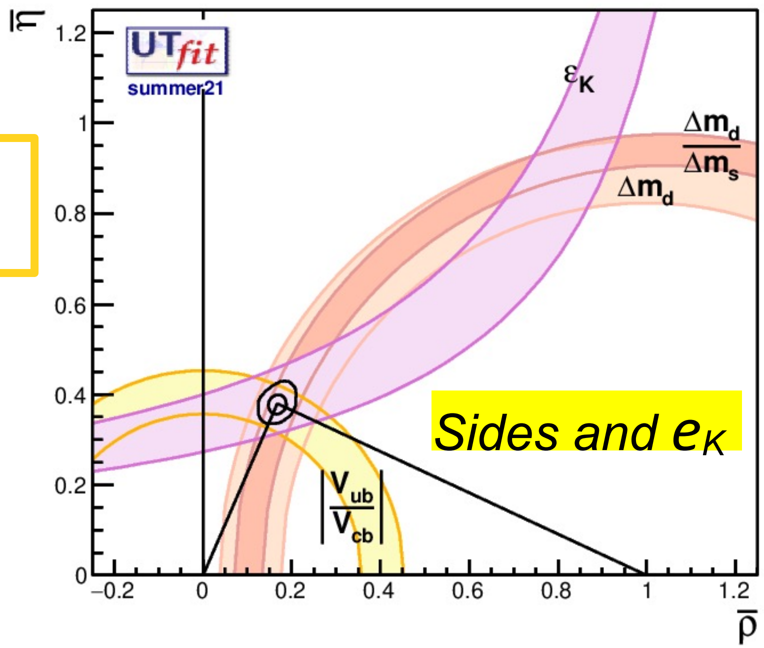


~15%

ρ = 0.165 ± 0.025
η = 0.374 ± 0.024

~7%

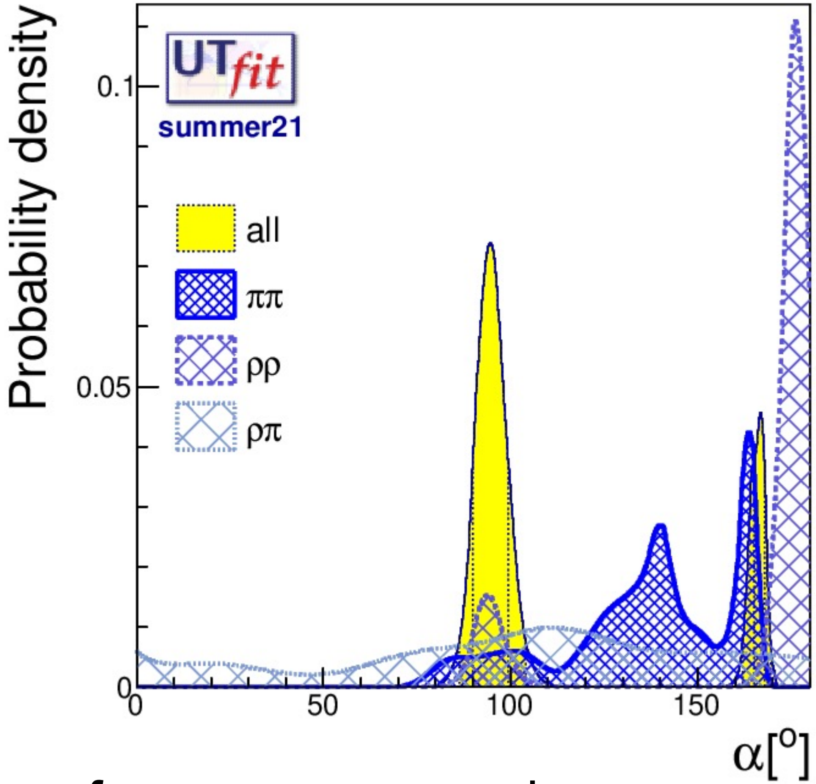
"Tree-only"



Sides and ε_K

$\sin 2\alpha$ (ϕ_2) and γ (ϕ_3)

α updated with latest $\pi\pi/\rho\rho$ BR and C/S results



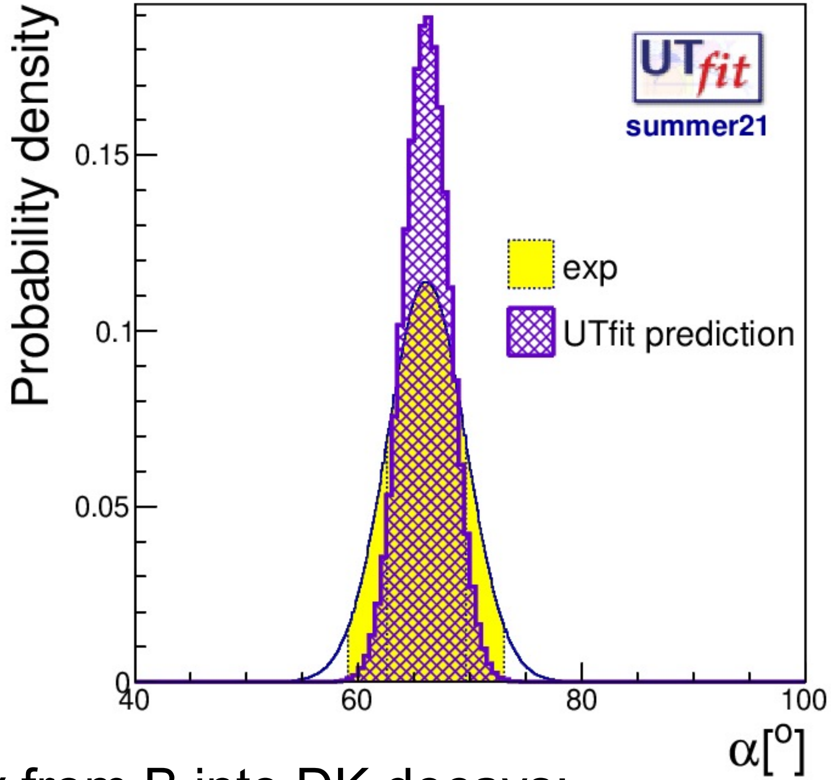
α from $\pi\pi$, $\rho\rho$, $\pi\rho$ decays:

combined SM: $(93.6 \pm 4.2)^\circ$

UTfit prediction: $(90.8 \pm 2.0)^\circ$

α from HFLAV: 85.5 ± 4.6

γ updated with all the latest results (LHCb)



γ from B into DK decays: γ

HFLAV: $(66.2 \pm 3.5)^\circ$

UTfit prediction: $(66.2 \pm 2.1)^\circ$

LHCb just released an update of the γ combination + charm mixing parameters $\gamma = (65.4 + 3.8 - 4.2)^\circ$

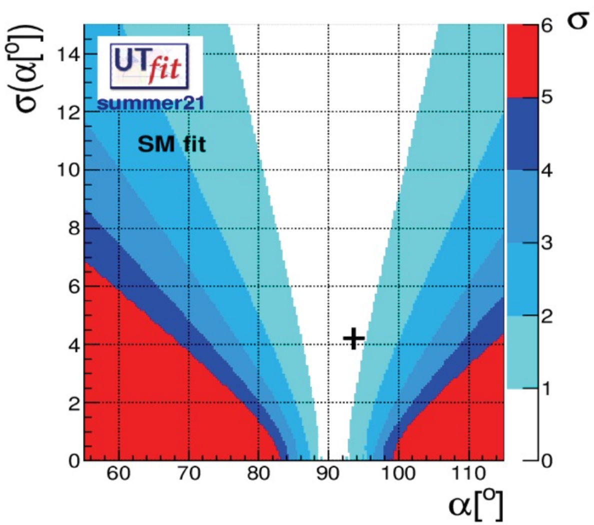
We can perform the same combination in the context of UTFit

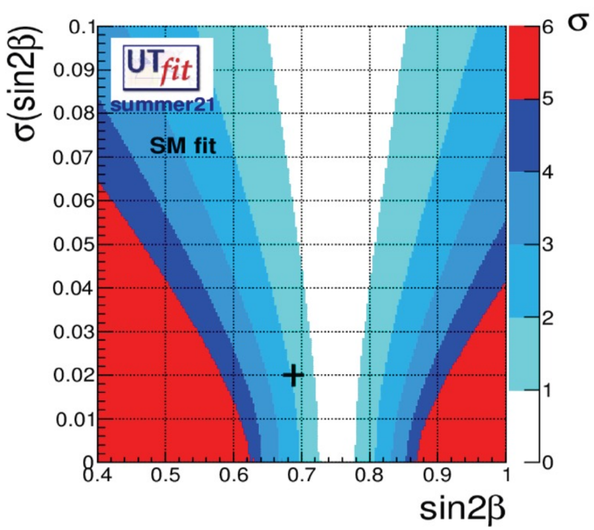
Compatibility plots – CKM angles

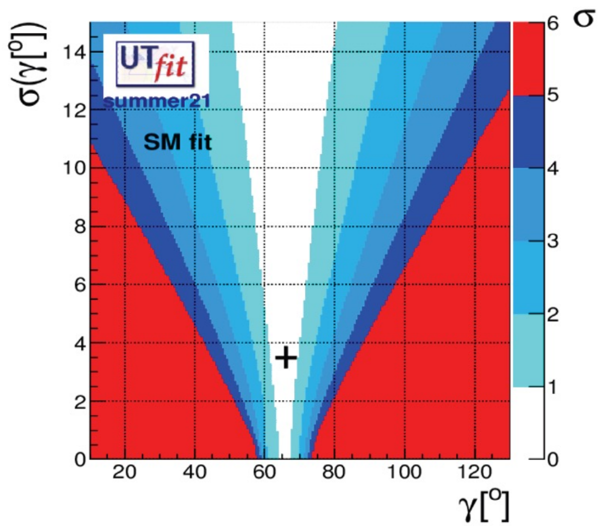
- "Measure" the agreement of a single measurement with the indirect determination from the fit using all the other inputs → test for the SM description of flavour physics

Coloured scale: level of agreement between measured value and indirect determination at better than $n\sigma$

Black cross: value (x) and uncertainty (y) of the experimental determination







$\alpha_{exp} = (93.6 \pm 4.2)°$

$\alpha_{UTfit} = (90.8 \pm 2.0)°$

$\sin 2\beta_{exp} = 0.688 \pm 0.020$

$\sin 2\beta_{UTfit} = 0.752 \pm 0.028$

~1.9σ

$\gamma_{exp} = (66.2 \pm 3.5)°$

$\gamma_{UTfit} = (66.2 \pm 2.1)°$

The V_{cb} vs V_{ub} saga

$|V_{cb,excl}| = (39.48 \pm 0.68)\times10^{-3}$ FLAG 2021 review

$|V_{cb,incl}| = (42.16 \pm 0.50)\times10^{-3}$ from Bordone et al. arXiv:2107.00604

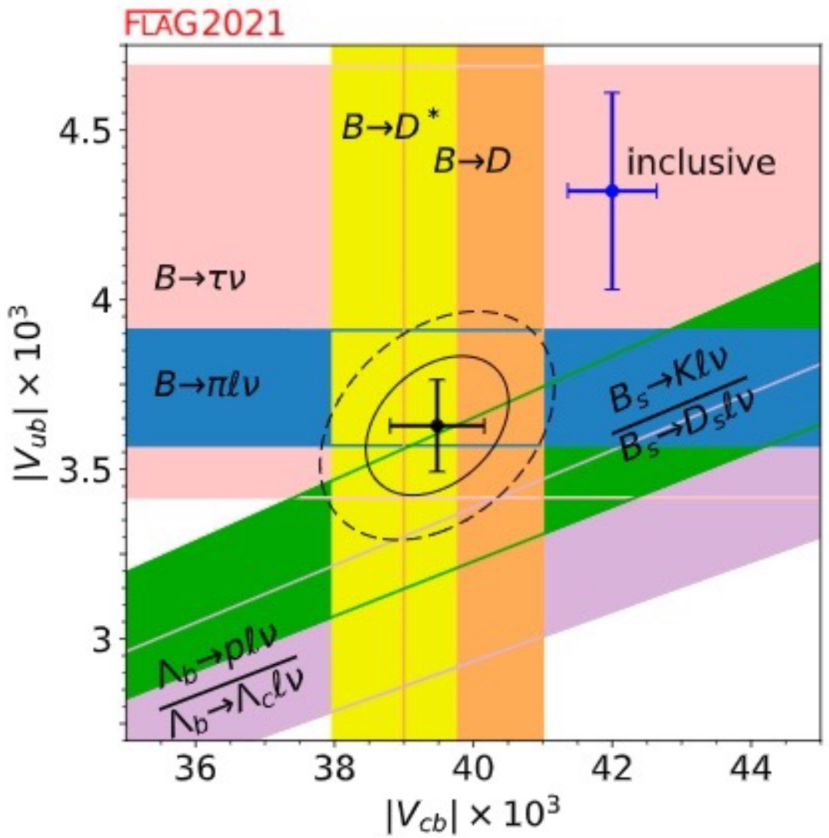
$\sim 3.2\sigma$ discrepancy

$|V_{ub,excl}| = (3.63 \pm 0.14)\times10^{-3}$ FLAG 2021 review

$|V_{ub,incl}| = (4.19 \pm 0.17 \pm 0.18\,[flat])\times10^{-3}$

from GGOU HFLAV 2021 adding a flat uncertainty covering the spread of central values

$\sim 1.6\sigma$ discrepancy



Other players in the game:

$|V_{ub}/V_{cb}|(LHCb) = (9.46 \pm 0.79)\times10^{-2}$ From B_s to K at high q^2

$|V_{ub}/V_{cb}|(LHCb) = (7.9 \pm 0.6)\times10^{-2}$ From Λ_b, excluded following FLAG guidelines

V_{cb} and V_{ub}

from FLAG 2021

$|V_{cb}|$ *(excl)* = $(39.48 \pm 0.68)\ 10^{-3}$

$|V_{cb}|$ *(incl)* = $(42.16 \pm 0.50)\ 10^{-3}$

from Bordone et al.
arXiv:2107.00604

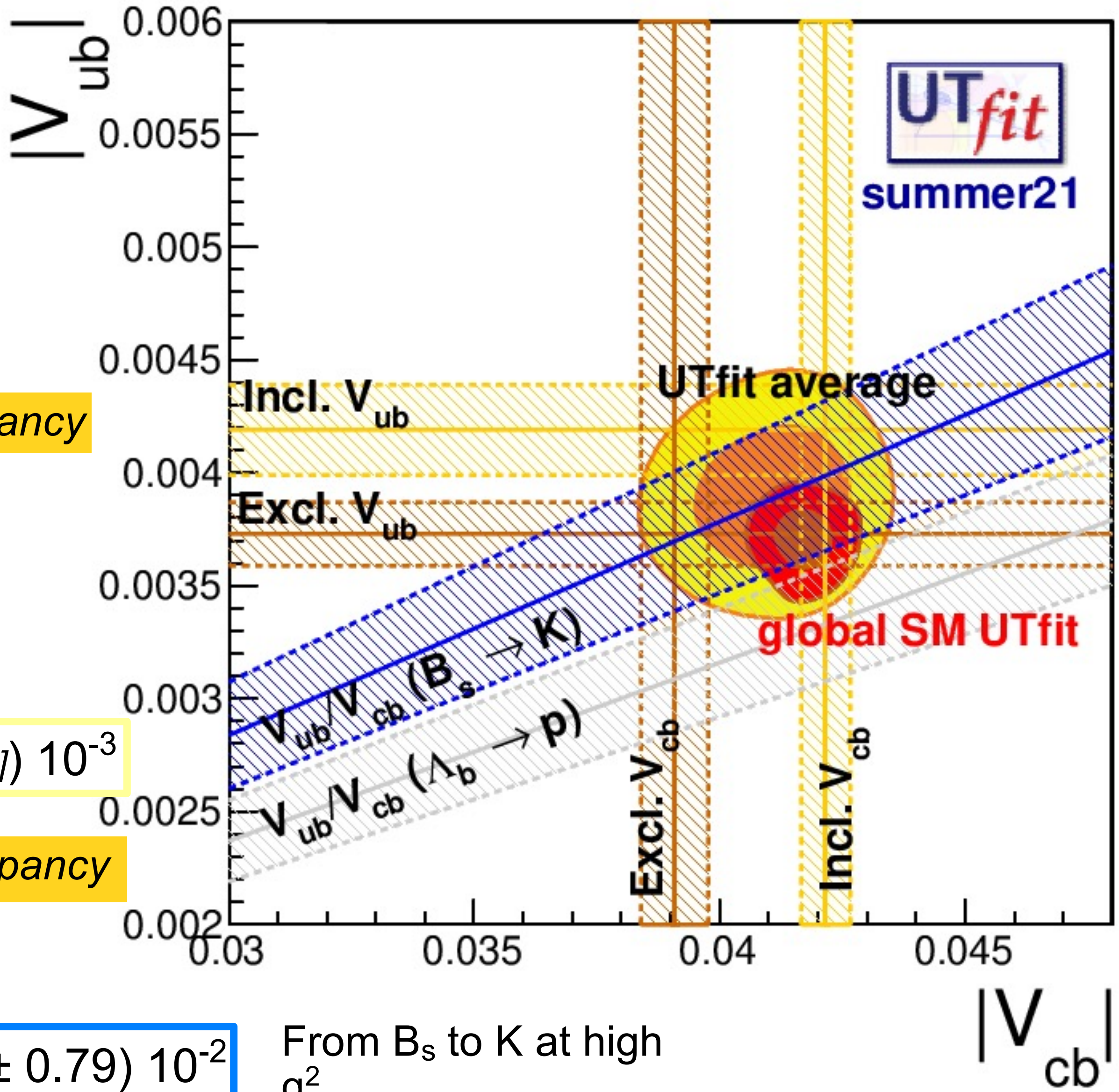
~ 3.2σ discrepancy

$|V_{ub}|$ *(excl)* = $(3.63 \pm 0.14)\ 10^{-3}$

$|V_{ub}|$ *(incl)* = $(4.19 \pm 0.17 \pm 0.18$ *[flat]*$)\ 10^{-3}$

from GGOU HFLAV 2021
adding a flat uncertainty
covering the spread
of central values

~1.6σ discrepancy



$|V_{ub} / V_{cb}|$ *(LHCb)* = $(9.46 \pm 0.79)\ 10^{-2}$

From B_s to K at high q^2

$|V_{ub} / V_{cb}|$ *(LHCb)* = $(7.9 \pm 0.6)\ 10^{-2}$

From Λ_b, excluded following FLAG guidelines

Utfit average gives Vcb= $(39.45 \pm 0.59)\ 10^{-3}$ Vub= $(3.75 \pm 0.13)\ 10^{-3}$

V_{cb} and V_{ub}

A-la-D'Agostini two-dimensional average procedure:

$|V_{cb}| = (41.1 \pm 1.0)\ 10^{-3}$

uncertainty ~ 2.4%

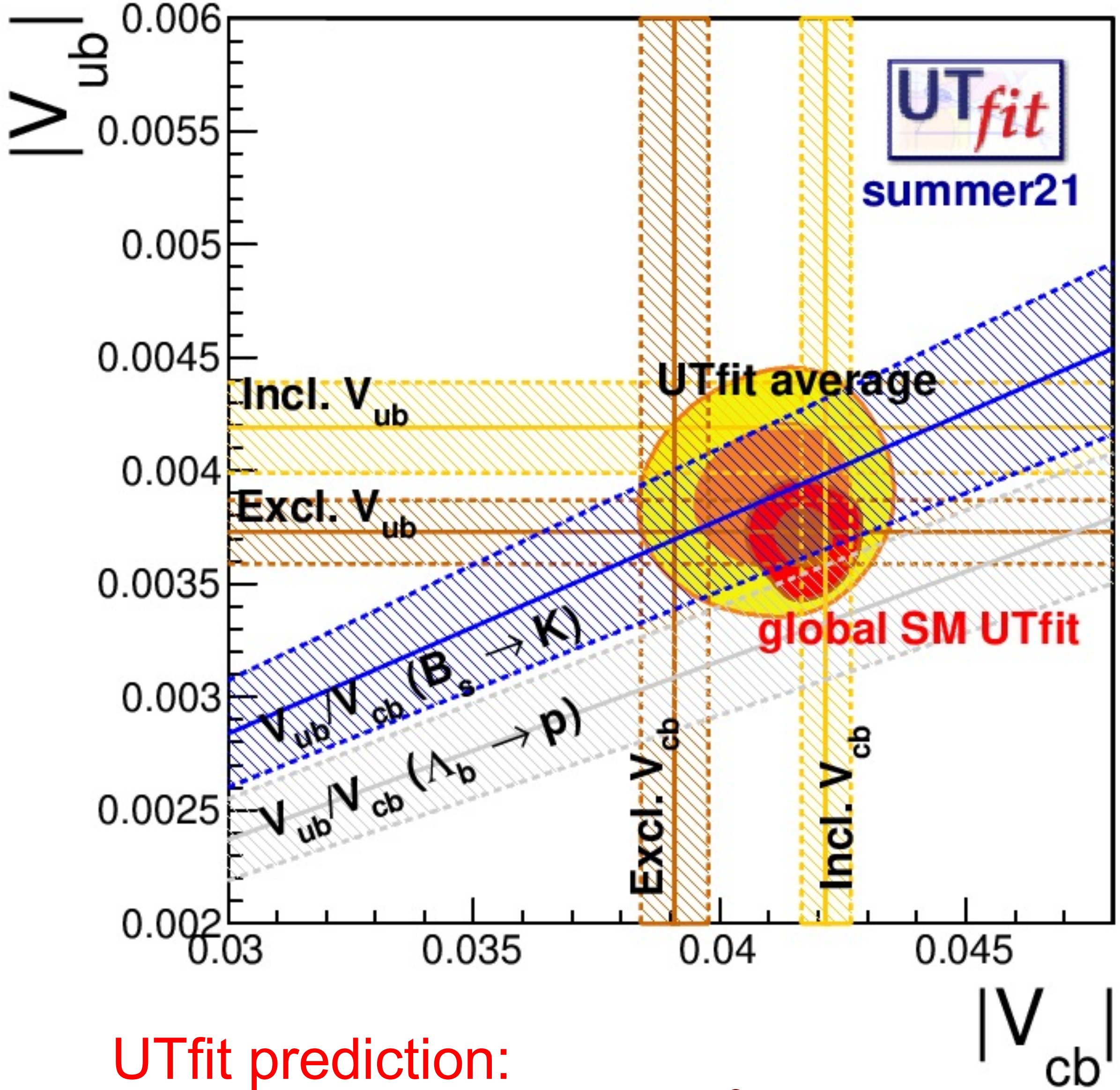
$|V_{ub}| = (3.89 \pm 0.21)\ 10^{-3}$

uncertainty ~ 5.4%

From global SM fit

$|V_{cb}| = (42.0 \pm 0.4)\ 10^{-3}$

$|V_{ub}| = (3.72 \pm 0.09)\ 10^{-3}$



UTfit prediction:

$|V_{cb}| = (41.9 \pm 0.5)\ 10^{-3}$

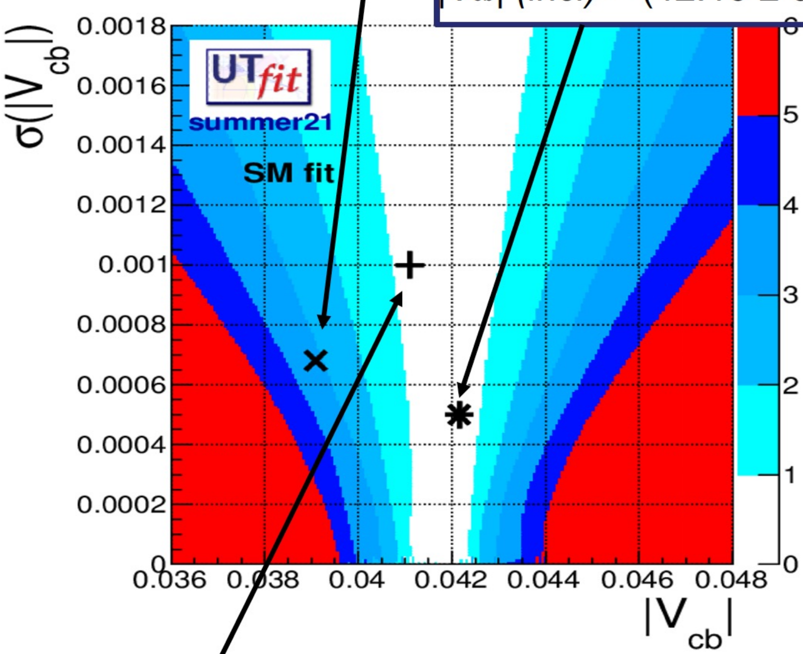
$|V_{ub}| = (3.68 \pm 0.10)\ 10^{-3}$

PLOTS AND NUMBERS TO BE UPDATED

Compatibility plots – $|V_{cb}|$ and $|V_{ub}|$

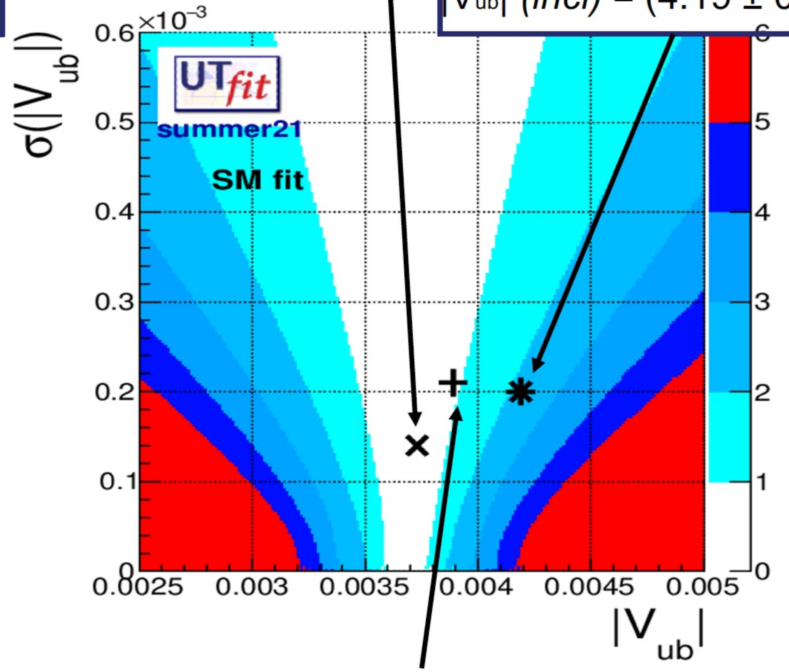
$|V_{cb}|$ *(excl)* = $(39.09 \pm 0.68)\ 10^{-3}$

$|V_{cb}|$ *(incl)* = $(42.16 \pm 0.50)\ 10^{-3}$



$|V_{ub}|$ *(excl)* = $(3.73 \pm 0.14)\ 10^{-3}$

$|V_{ub}|$ *(incl)* = $(4.19 \pm 0.20)\ 10^{-3}$



$V_{cb_{exp}} = (41.1 \pm 1.0) \cdot 10^{-3}$

$V_{cb_{UTfit}} = (41.9 \pm 0.5) \cdot 10^{-3}$

$V_{ub_{exp}} = (3.89 \pm 0.21) \cdot 10^{-3}$

$V_{ub_{UTfit}} = (3.68 \pm 0.10) \cdot 10^{-3}$

Experimental progress so impressive that we can fit the hadronic matrix elements (in the SM)

Updates from UTfit

FLAG21 0.756 ± 0.016

obtained excluding the given constraint from the fit

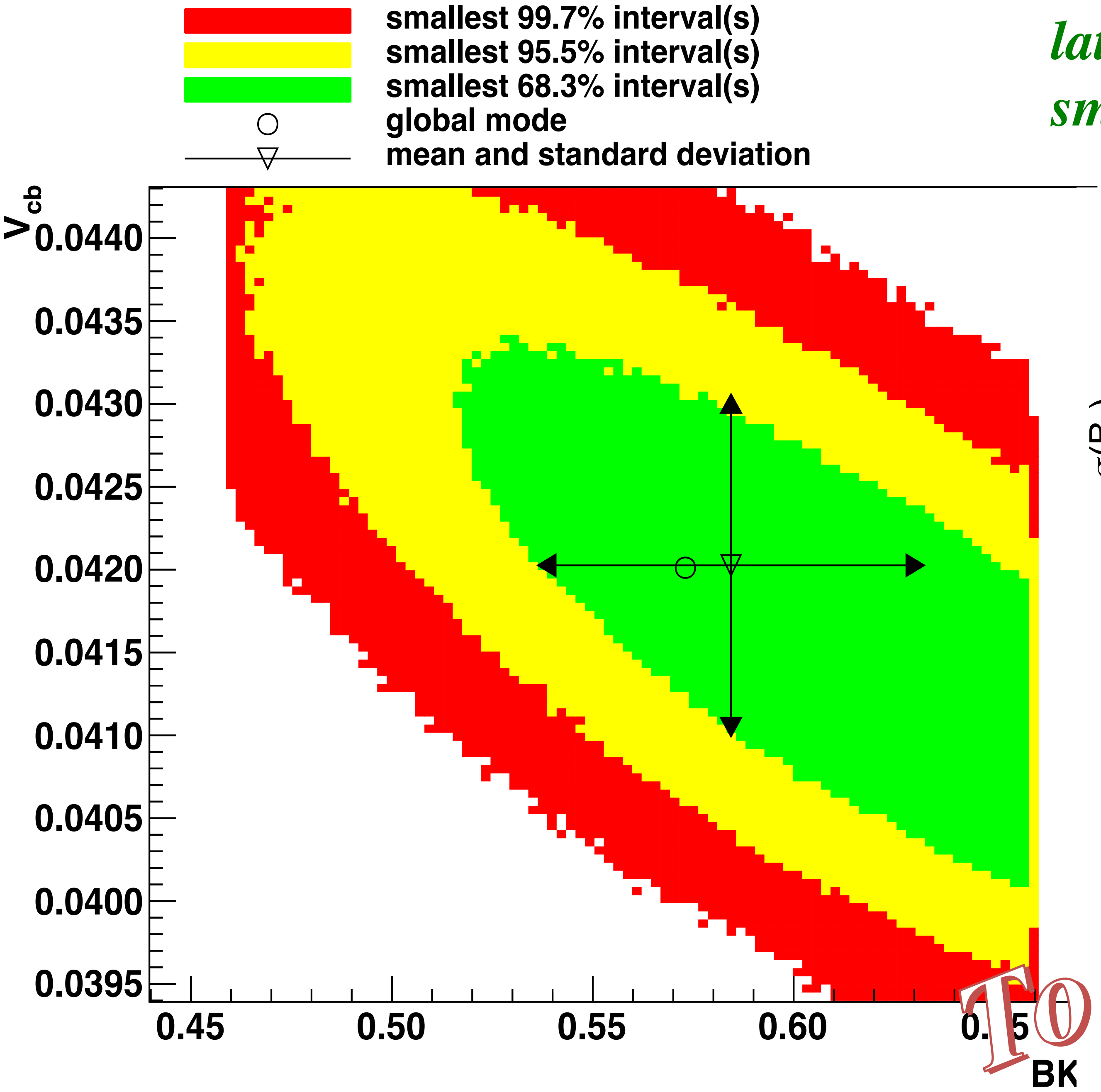
Observables	Measurement	Prediction	Pull (#σ)
B_K	0.740 ± 0.029	0.81 ± 0.07	< 1
f_{Bs}	0.226 ± 0.005	0.220 ± 0.007	< 1
f_{Bs}/f_{Bd}	1.203 ± 0.013	1.210 ± 0.030	[illegible]
B_{Bs}/B_{Bd}	1.032 ± 0.036	1.07 ± 0.0[illegible]	< 1
B_{Bs}	1.35 ± 0.08	1.3[illegible] ± [illegible].07	< 1

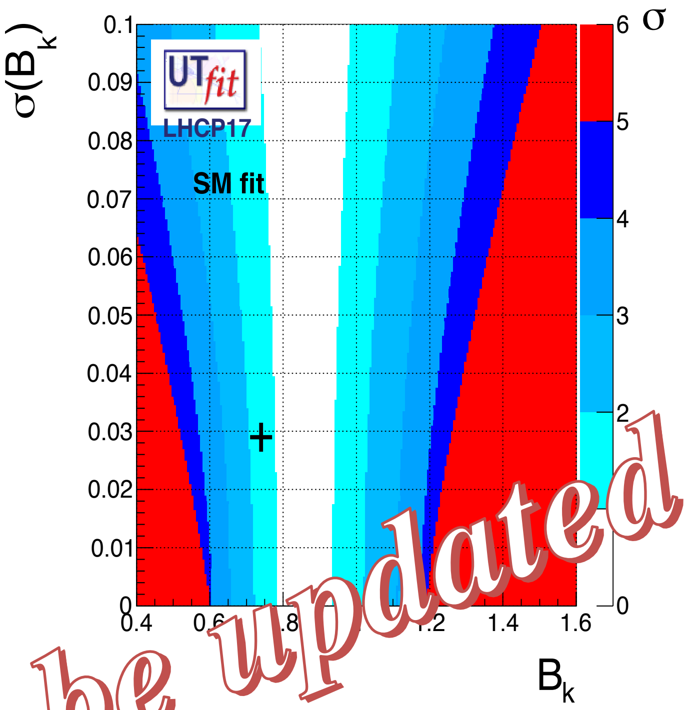
To be updated

It does not make sense to improve the precision on B_K if we do not control long distance effects; Similarly for f_π or f_K without radiative corrections

UT-fit Preliminary

- ε_K large Vcb
-B mixing with large lattice matrix elements smaller Vcb





To be updated

Power corrections to the CP-violation parameter ε_K

M. Ciuchini[(a)], E. Franco[(b)], V. Lubicz[(c,a)],
G. Martinelli[(d,b)], L. Silvestrini[(b)], C. Tarantino[(c,a)]

2021: an estimate from the 1/mc expansion of the effective Hamiltonian

$$\varepsilon_K = (1.99 \pm 0.14) \times 10^{-3}\,.$$

$$\varepsilon_K^{exp} = (2.228 \pm 0.011) \cdot 10^{-3}$$

Computing the long-distance contributions to ε_K

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Norman Christ[*†]
Columbia University, USA
E-mail: nhc@phys.columbia.edu

RBC and UKQCD Collaborations

2015: a real exploratory calculation

no physical masses, no extrapolation to the continuum etc.

$$|\varepsilon| = (1.806(41) + 0.891(11) + 0.209(6) + 0.112(13)) \times 10^{-3} = 3.019(45) \times 10^{-3}$$

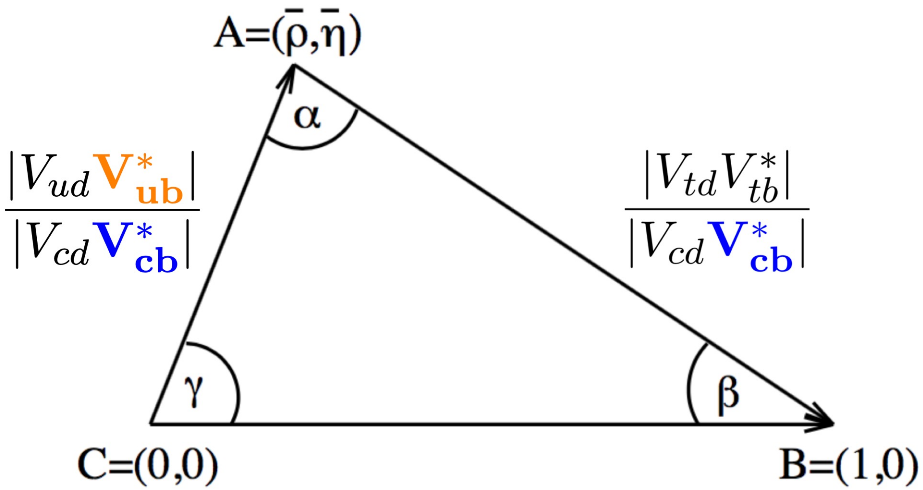
tt $\quad ut_{SD}$ $\quad ut_{LD}$ $\quad \mathrm{Im}(A_0)$,

e'/e from RBC now in UTfit

Exclusive semileptonic B → {D(*),π} decays through unitarity

Work in collaboration with M. Naviglio. S. Simula and L. Vittorio

(PRD ’21 (2105.02497), PRD ’21 (2105.07851), 2105.08674, 2109.15248 + in prep.)



Mr. Nosferatu
from Transylvania

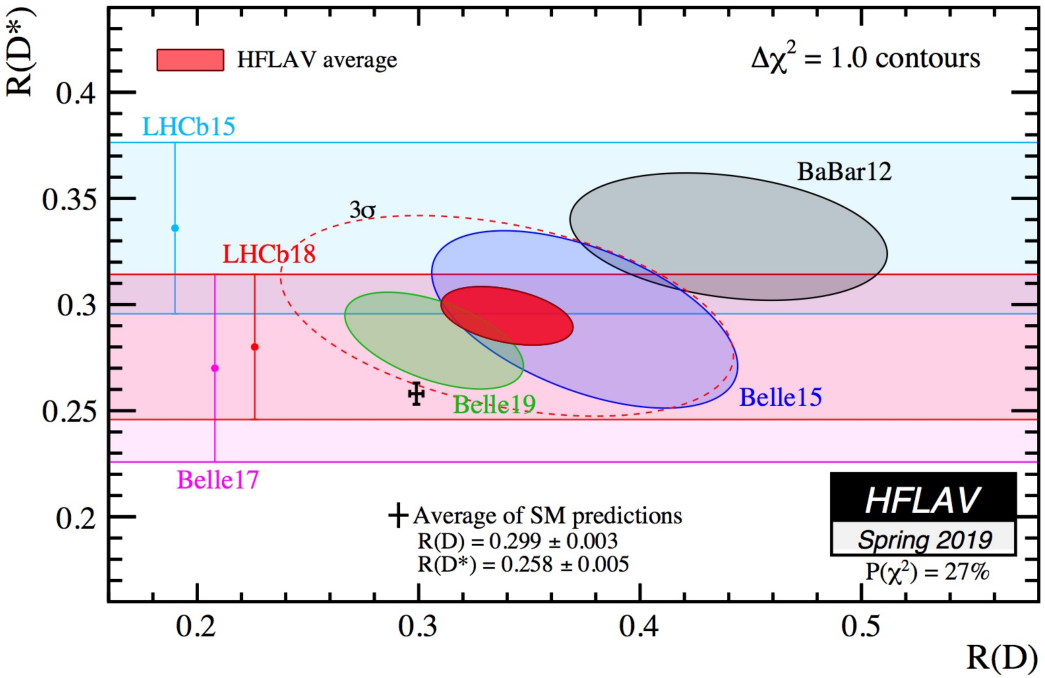
State-of-the-art of the semileptonic B → {D(*),π} decays

Three critical issues:

- $R_{D^{(*)}}$ anomalies:

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)},$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)}$$



HFLAV Coll., see https://hflav-eos.web.cern.ch/hflaveos/semi/spring19/html/RDsDsstar/RDRDs.html

3.08σ discrepancy

Form Factors (FFs) in exclusive semileptonic *B* decays

- Production of a pseudoscalar meson (*i.e. D,π*):

$$\frac{d\Gamma}{dw} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{48\pi^3} \frac{4r\, m_D^3\, (m_B + m_D)^2\, (w^2-1)^{3/2}}{(1+r)^2} \boxed{f_+(w)|^2}$$

- Production of a vector meson (*i.e. D**):

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$

$$\times B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$
$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$
$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$
$$- 4\sin\theta_\ell (1-\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0$$
$$+ 4\sin\theta_\ell (1+\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_- H_0\},$$

$$H_\pm(w) = \boxed{f(w)} \mp m_B m_{D^*} \sqrt{w^2-1}\, \boxed{g(w)}$$

$$H_0(w) = \frac{\boxed{\mathcal{F}_1(w)}}{\sqrt{m_B^2 + m_D^2 - 2 m_B m_D w}}$$

$$\boxed{q^2 = m_B^2 + m_P^2 - 2 m_B m_P\, w}$$

relation between the momentum transfer and the recoil

If the lepton is not massless two other FFs

$$f_0(w)\ \text{(pseudoscalar)},\ P_1(w)\ \text{(vector)}$$

The Dispersive Matrix (DM) method

A novel, non-perturbative and model independent approach: starting from the available LQCD computations of the FFs in the high-q^2 (or low-w) regime, we extract the FFs behaviour in the low-q^2 (or high-w) region!

E. de Rafael and J. Taron, PRD, 50 (1994), p. 373-380
C. Bourrely, B. Machet, and E. de RafaelNPB, 189 (1981), pp. 157 – 18

For LQCD original idea from L. Lellouch: NPB, 479 (1996)
New developments in PRD '21 (2105.02497)

The resulting description of the FFs will be:

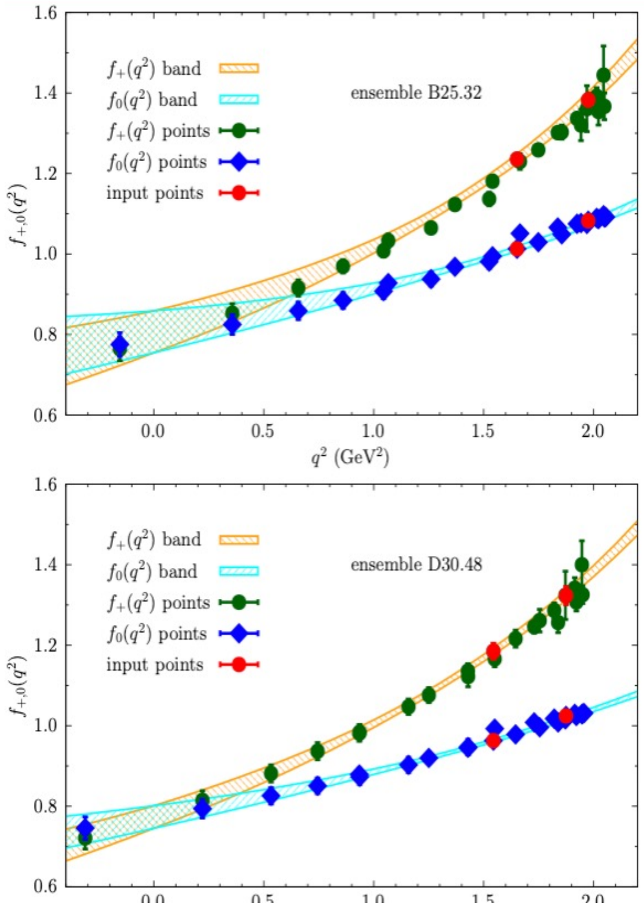
- entirely based on first principles (LQCD non-perturbative evaluation of 2- and 3-point Euclidean correlators)
- independent of any assumption on the functional dependence of the FFs on the momentum transfer
- applicable to theoretical calculations of the FFs, but also to experimental data
- independent of any mixing among theoretical calculations and experimental data
- universal: it can be applied to any exclusive semileptonic decays of mesons and baryons

→ No HQET, no series expansion, no perturbative bounds with respect to the other popular parametrizations

The DM method works

example from D -> K decays

The great advantage of studying the $D \to K$ decay is that we can compare our results obtained with the unitarity procedure to the ones obtained from a direct calculation of the form factors that cover indeed all the kinematical region in q^2.



The red points are the only data used as input for the DM method!!

The figures show the bands obtained by using as inputs only the red points and the rest of the lattice points that are not used as input in our analysis in the case of the ETMC ensembles B25.32 and D30.48.

The agreement is excellent!

These results suggest that it will be possible to obtain quite precise determinations of the form factors for B decays by combining form factors at large q^2 with the non perturbative calculation of the susceptibilities.

The Dispersive Matrix (DM) method

$$B \to D$$

$$t \equiv q^2$$

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

$$\langle h_1 | h_2 \rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}(t) z}$$

The conformal variable z is related to the momentum transfer as:

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}$$

[0, t_{max}=t_-] ⇨ [z_{max}, 0]

$$t_\pm \equiv (m_B \pm m_D)^2$$

$$\det \mathbf{M} \geq 0$$

The DM method

$$\mathbf{M} = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle \end{pmatrix}$$

We also have to define the kinematical functions

$$\phi_0(z,Q^2) = \sqrt{\frac{2n_I}{3}}\sqrt{\frac{3t_+t_-}{4\pi}}\frac{1}{t_+-t_-}\frac{1+z}{(1-z)^{5/2}}\left(\beta(0)+\frac{1+z}{1-z}\right)^{-2}\left(\beta(-Q^2)+\frac{1+z}{1-z}\right)^{-2},$$

$$\phi_+(z,Q^2) = \sqrt{\frac{2n_I}{3}}\sqrt{\frac{1}{\pi(t_+-t_-)}}\frac{(1+z)^2}{(1-z)^{9/2}}\left(\beta(0)+\frac{1+z}{1-z}\right)^{-2}\left(\beta(-Q^2)+\frac{1+z}{1-z}\right)^{-3}, \quad \beta(t) \equiv \sqrt{\frac{t_+-t}{t_+-t_-}}$$

Thus, we need **these external inputs** to implement our method:

- estimates of the FFs, computed on the lattice, *@ {$t_1,\ldots,t_n$}*: from Cauchy's theorem (for generic *m*)

$$\langle g_{t_m}|\phi f\rangle = \phi(t_m,Q^2)f(t_m)$$

LQCD data!

$$\langle g_{t_m}|g_{t_l}\rangle = \frac{1}{1-\bar{z}(t_l)z(t_m)}$$

Q^2

- non-perturbative values of the susceptibilities, since from the dispersion relations (calling the Euclidean quadratic momentum)

$$\chi(Q^2) \geq \langle\phi f|\phi f\rangle$$

Since the susceptibilities are computed on the lattice, we can in principle use whatever value of $Q^2 \equiv -q^2$

The DM method

The positivity of the original inner products guarantees that $\det \mathbf{M} \geq 0$ namely

LOWER bound

$$\boxed{\beta - \sqrt{\gamma}} \leq f(z) \leq \boxed{\beta + \sqrt{\gamma}}$$

UPPER bound

$$\beta = \frac{1}{d(z)\phi(z)} \sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_f} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)} \frac{1}{1 - z^2} \left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} \right]$$

UNITARITY FILTER: unitarity is satisfied if γ is semipositive definite, namely if

$$\chi \geq \sum_{i,j=1} N f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}$$

This is a parametrization-independent unitarity test of the LQCD input data

A detailed discussion of the treatment of statistical errors and constraints was also presented (simplified with respect to L. Lellouch NPB, 479 (1996))

Non-perturbative computation of the susceptibilities

The possibility to compute the χs on the lattice allows us to choose *whatever value of Q²* (i.e. near the region of production of the resonances)

→

NOT POSSIBLE IN PERTURBATION THEORY!!!

$$(m_b + m_c)\Lambda_{QCD} << (m_b + m_c)^2 - q^2$$

POSSIBLE IMPROVEMENT IN THE STUDY OF THE FFs through our method

Work in progress…

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^+}(t)\ , \xrightarrow{W.\ I.} \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t')\right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^-}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^-}(t)\ , \xrightarrow{W.\ I.} \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t')\right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^+}(t)$$

Non-perturbative computation of the susceptibilities

Let us choose for the moment zero Q^2:

$$\boxed{\chi_{0^+}(Q^2=0)} = \int_0^\infty dt\ t^2\ C_{0^+}(t)\ ,$$

$$\boxed{\chi_{1^-}(Q^2=0)} = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^+}(t)\ .$$

$$\chi_{0^+}(Q^2=0) = \frac{1}{12}(m_b-m_c)^2\int_0^\infty dt\ t^4\ C_S(t)$$

$$\chi_{0^-}(Q^2=0) = \frac{1}{12}(m_b+m_c)^2\int_0^\infty dt\ t^4\ C_P(t)$$

$$C_{0^+}(t) = \boxed{\widetilde{Z}_V^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \boxed{\widetilde{Z}_V^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \boxed{\widetilde{Z}_A^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \boxed{\widetilde{Z}_A^2}\frac{1}{3}\sum_{j=1}^{3}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_S(t) = \boxed{\widetilde{Z}_S^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)q_2(x)\ \bar{q}_2(0)q_1(0)\right]|0\rangle\ ,$$

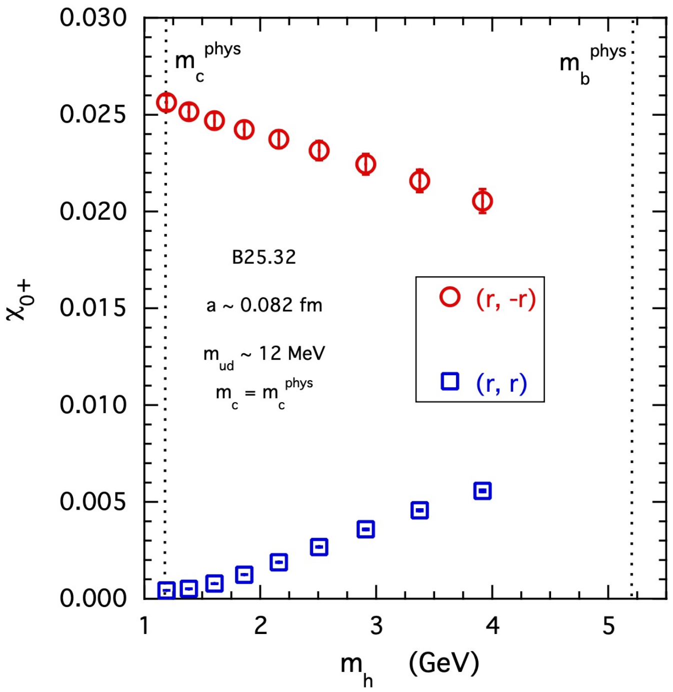
$$C_P(t) = \boxed{\widetilde{Z}_P^2}\int d^3x\langle 0|T\left[\bar{q}_1(x)\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_5 q_1(0)\right]|0\rangle\ ,$$

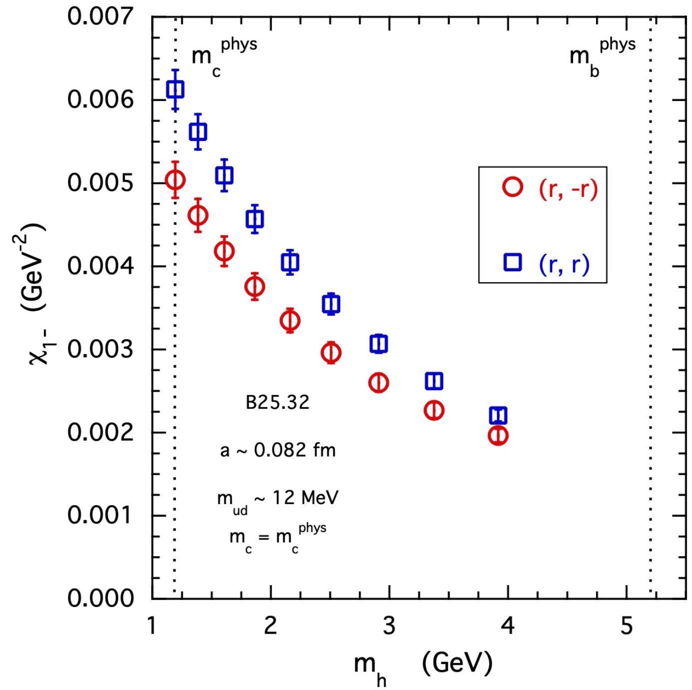
Z: appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

Non-perturbative computation of the susceptibilities

r: (unphysical) Wilson parameter





Following set of nine quark masses:

$$m_h(n) = \lambda^{n-1}\, m_c^{phys} \qquad \text{for } n = 1, 2, \ldots$$

$$m_h(1) = m_c^{phys}$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602$$

$$m_h(9) \simeq 3.9\ \text{GeV} \simeq 0.75\ m_b^{phys}$$

$$m_h = a\mu_h/(Z_P a)$$

Contact terms &

Large discretisation effects

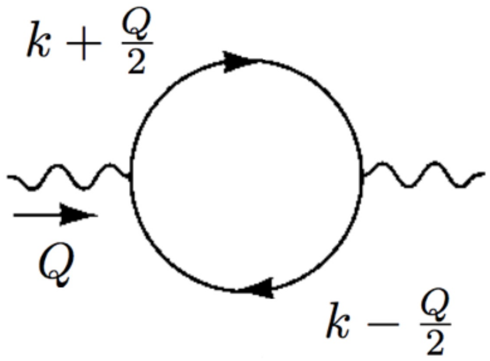
Contact terms & perturbative subtraction

In **twisted mass LQCD** (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i\mu_{q,i}\gamma_5\tau^3}{\mathring{p}^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}, \qquad i = 1,2$$

$$\mathring{p}_\mu \equiv \frac{1}{a}\sin(ap_\mu), \qquad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2}a\hat{p}_\mu^2, \qquad \hat{p} \equiv \frac{2}{a}\sin\left(\frac{ap_\mu}{2}\right).$$



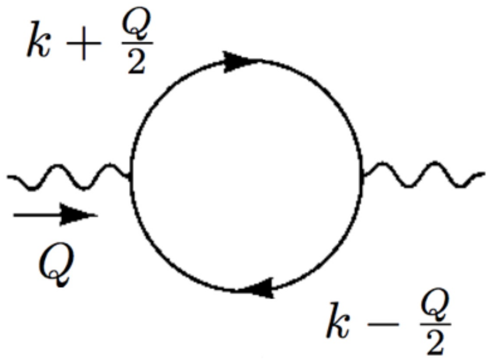
$$\begin{aligned}\Pi_V^{\alpha\beta} = {} & a^{-2}(Z_1^I + (r_1^2 - r_2^2)Z_2^I + (r_1^2 - r_2^2)(r_1^2 + r_2^2)Z_3^I)g^{\alpha\beta} \\ & + (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1\mu_2 Z^{\mu_1\mu_2})g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2 - r_2^2)Z_2^{Q^2})Q\cdot Q g^{\alpha\beta} \\ & + (Z_1^{Q^\alpha Q^\beta} + (r_1^2 - r_2^2)Z_2^{Q^\alpha Q^\beta})Q^\alpha Q^\beta + r_1 r_2(a^{-2}Z_1^{r_1r_2}g^{\alpha\beta} + (Z_2^{r_1r_2} + (r_1^2 + r_2^2)Z_3^{r_1r_2} \\ & + (r_1^4 + r_2^4)Z_4^{r_1r_2})Q\cdot Q g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1r_2} + \mu_2^2 Z_6^{r_1r_2})g^{\alpha\beta}) + O(a^2),\end{aligned}$$

CONTACT TERMS!!!

Contact terms & perturbative subtraction

In **twisted mass LQCD** (tmLQCD):

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr}\Big[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\Big],$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

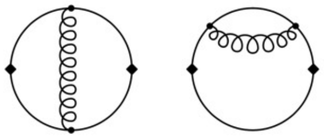
$$\chi_j^{free} = \boxed{\chi_j^{LO}} + \boxed{\chi_j^{discr}}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

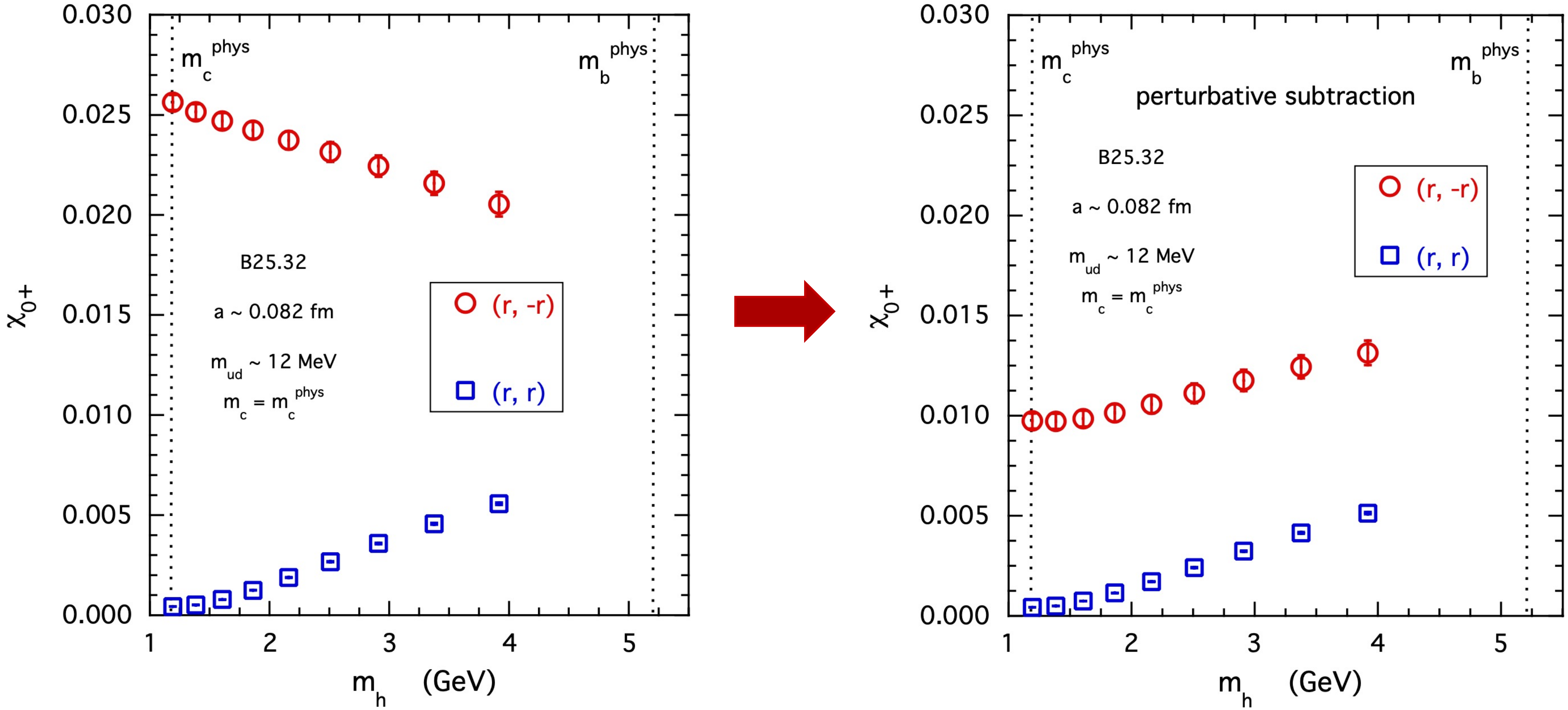
$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

Higher order corrections?



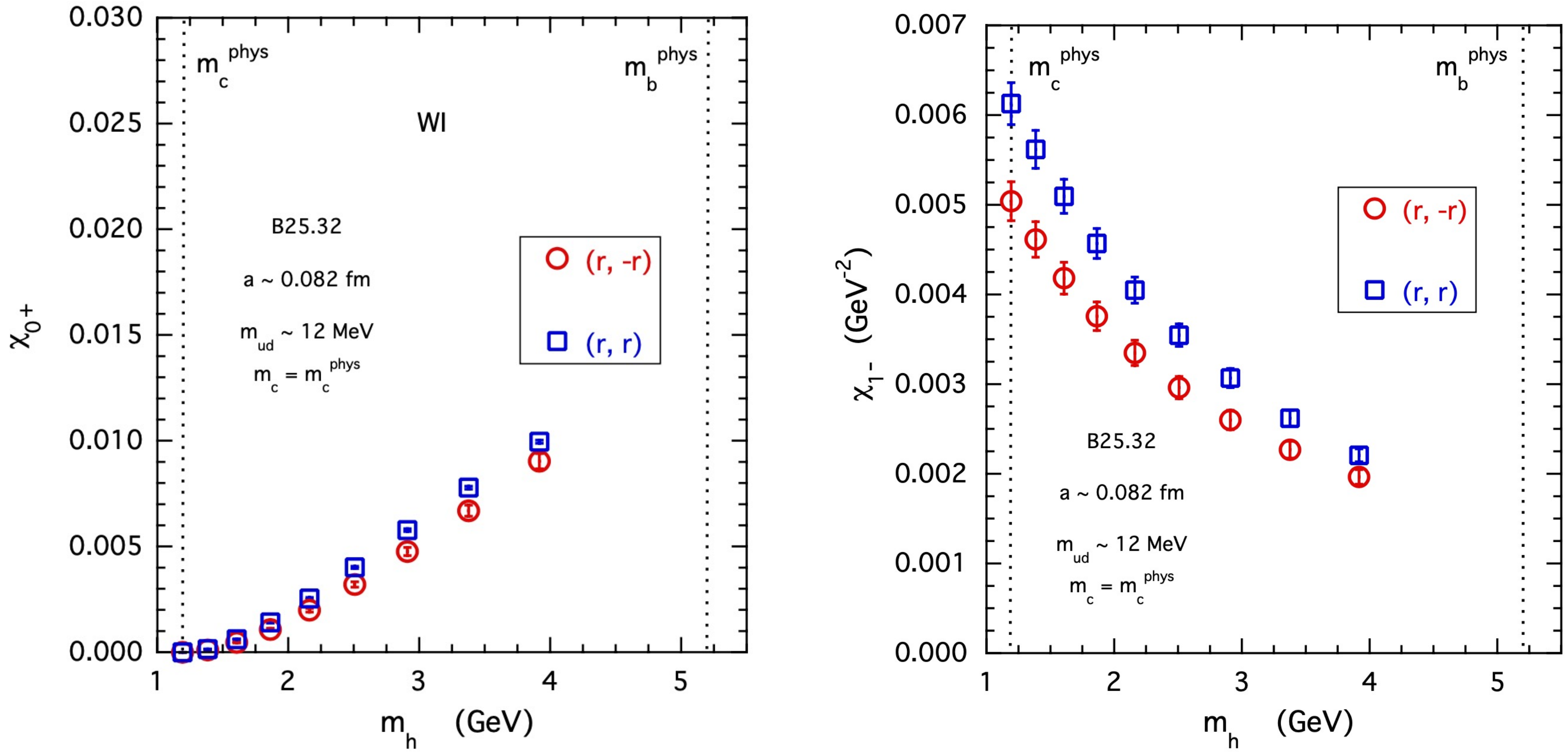
Work in progress…

Contact terms & perturbative subtraction



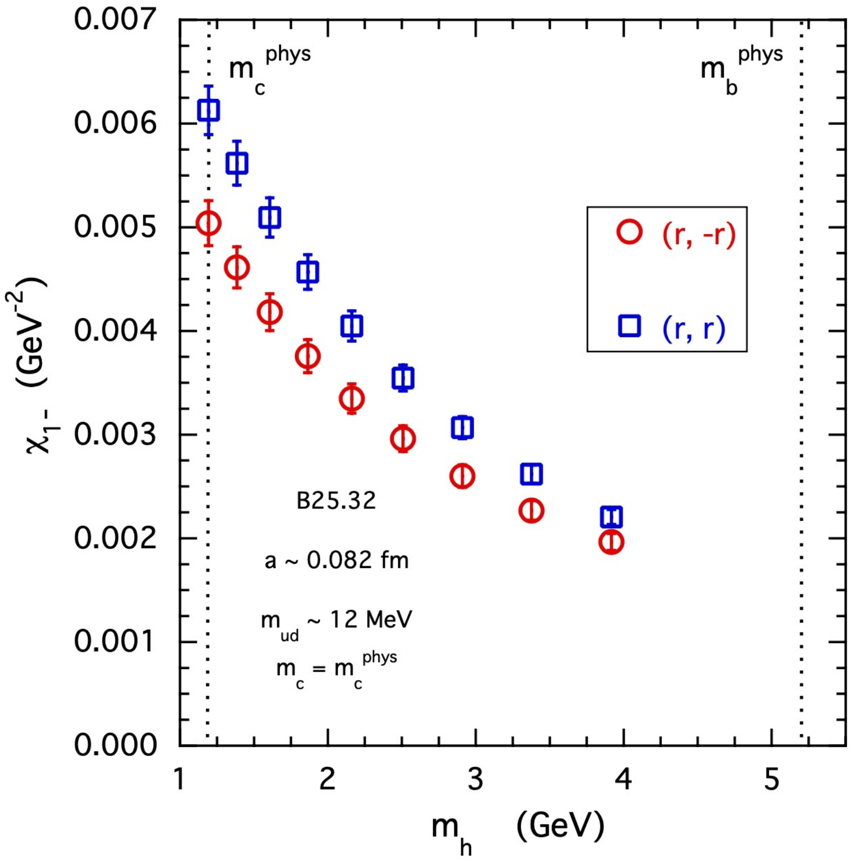
NOT ENOUGH…

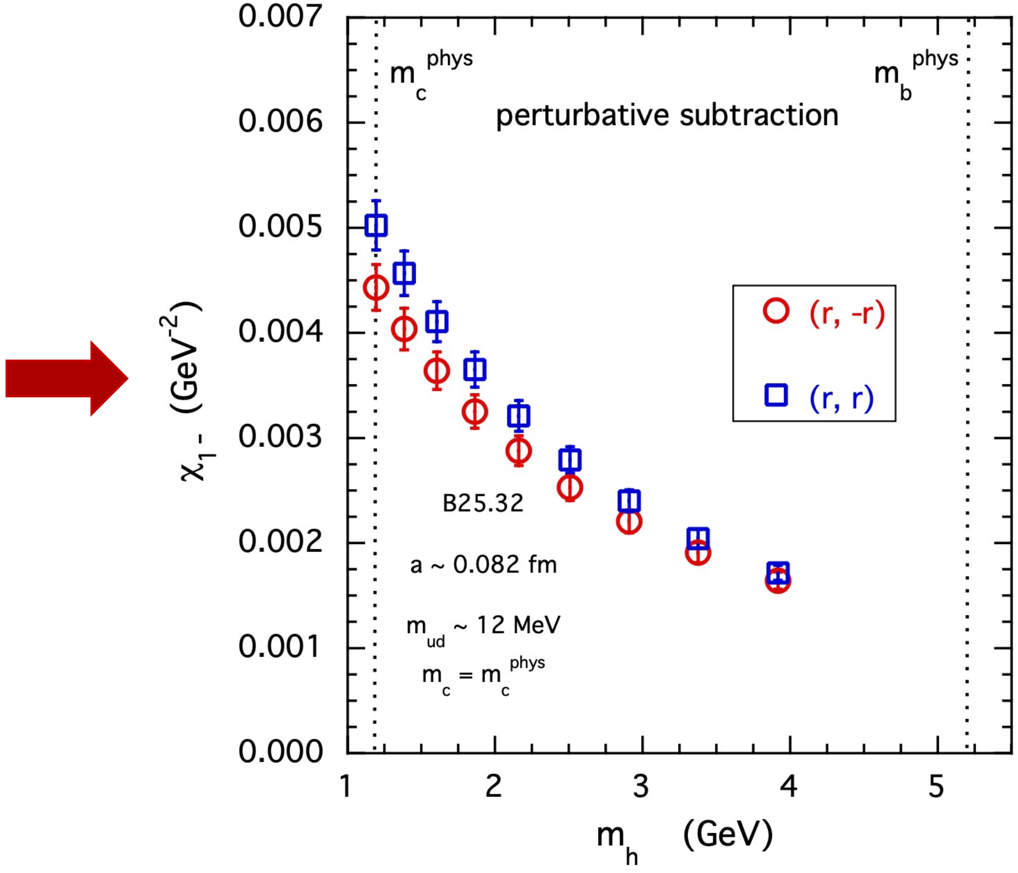
Non-perturbative computation of the susceptibilities



Much better using the Ward Identity

Contact terms & perturbative subtraction





OK

An extrapolation to the continuum limit was implemented

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}}$$

to ensure that

$$\lim_{n\to\infty} R_j(n) = 1$$

$$\Rightarrow \boxed{\begin{aligned} &\rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1 \ , \\ &\rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2 \end{aligned}}$$

All the details are deeply discussed in *arXiv:2105.07851*. In this way, we have obtained **the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, *in prep.*) transition current densities:**

b → c

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}[10^{-4}\ \text{GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}[10^{-4}\ \text{GeV}^{-2}]$	3.894	—	4.69(30)	—

b → u

Non-perturbative	With subtraction
2.04(20)	—
2.34(13)	—
4.88(1.16)	4.45(1.16)
4.65(1.02)	—

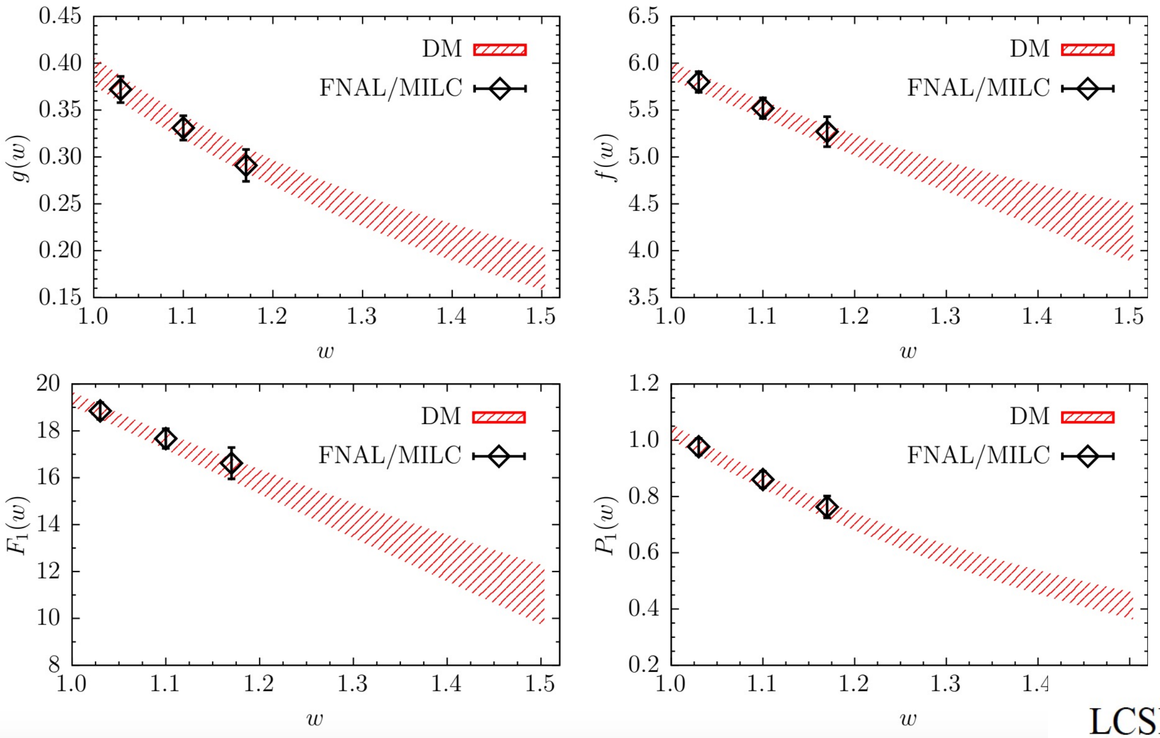
Differences with PT? **~4% for 1⁻, ~7% for 0⁻, ~20 % for 0⁺ and 1⁺**

Bigi, Gambino PRD '16
Bigi, Gambino, Schacht PLB '17
Bigi, Gambino, Schacht JHEP '17

form factors for $B \to D^* \ell \nu_\ell$ decays

results from FNAL/MILC computations of the FFs arXiv:2105.14019
non-perturbative susceptibilities from arXiv:2105:07851

arXiv:2109.15248



Two kinematical constraints (KCs):

$$\mathcal{F}_1(1) = (m_B - m_{D^*}) f(1)$$

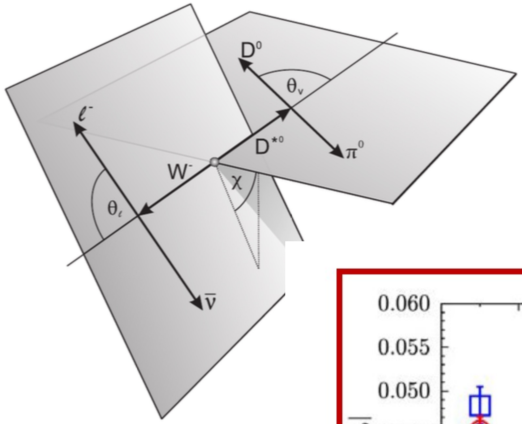
$$P_1(w_{max}) = \frac{\mathcal{F}_1(w_{max})}{(1 + w_{max})(m_B - m_{D^*})\sqrt{m_B m_{D^*}}}$$

$$f(w_{max}) = 4.19 \pm 0.31 \,,$$
$$g(w_{max}) = 0.180 \pm 0.023 \,,$$
$$\mathcal{F}_1(w_{max}) = 11.0 \pm 1.3 \,,$$
$$P_1(w_{max}) = 0.411 \pm 0.048 \,.$$

LCSR: $\mathcal{F}_1(w_{max}) = 16.0 \pm 2.1$ (arXiv:1811.00983)

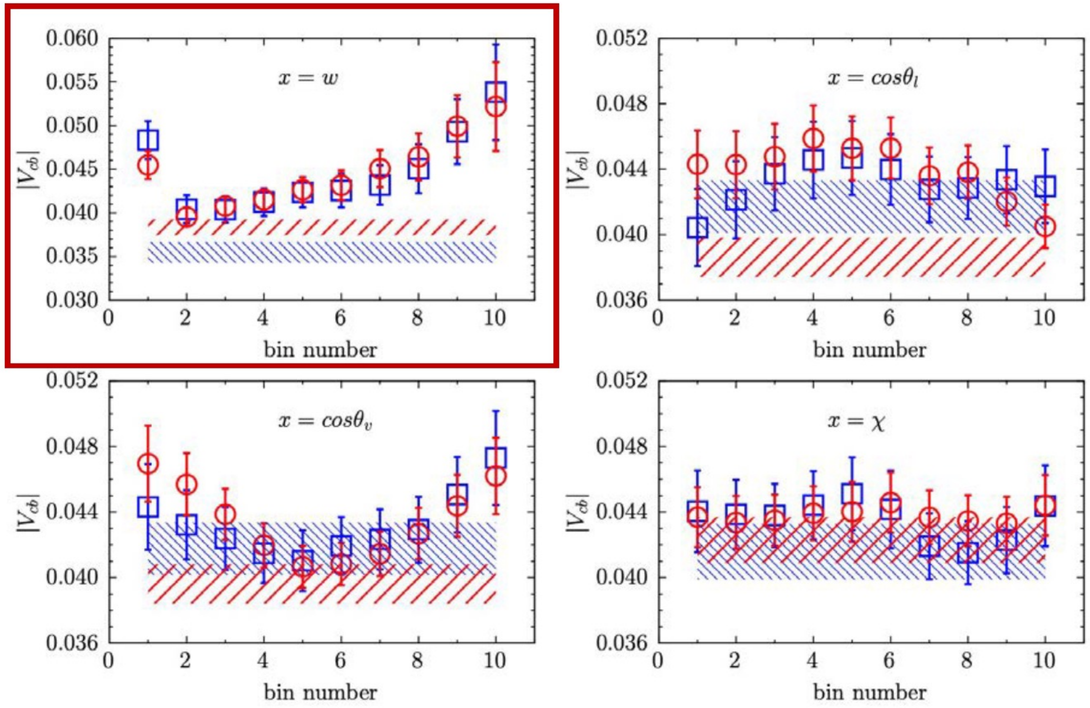
$$R(D^*) = 0.269 \pm 0.008, \quad P_\tau = -0.52 \pm 0.01, \quad F_L = 0.42 \pm 0.01,$$

$$R(D^*)|_{\rm exp} = 0.295 \pm 0.011 \pm 0.008, \quad P_\tau(D^*)|_{\rm exp} = -0.38 \pm 0.51^{+0.21}_{-0.16}, \quad F_L(D^*)|_{\rm exp} = 0.60 \pm 0.08 \pm 0.04.$$



Exclusive Vcb determination from B -> D*

$d\Gamma/dx$, $x = w, \cos\theta_l, \cos\theta_v, \chi$



Blue squares: arXiv:1702.01521

Red points: arXiv:1809.03290

In the w differential decay rate data systematically above the result of the fit

This problem is known and has been studied, for example, in Nucl. Instrum. Meth. A346 (1994) 306-311

Our interpretation is that there is a problem related to the Experimental calibration and to the covariance matrix

experimental data for $B \to D^* \ell \nu_\ell$ decays

- two sets of data from Belle collaboration arXiv:1702.01521 and arXiv:1809.03290
- four different differential decay rates $d\Gamma/dx$ where $x = \{w, \cos\theta_v, \cos\theta_\ell, \chi\}$: 10 bins for each variable

total of 80 data points

*** we do not mix theoretical calculations with experimental data to describe the shape of the FFs ***

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}} \qquad i = 1, \ldots, N_{bins}$$

* issue with the covariance matrix $C_{ij}^{exp.}$ of the Belle data: $\Gamma^{exp.} \equiv \sum_{i=1}^{10} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$ should be the same for all the variables x

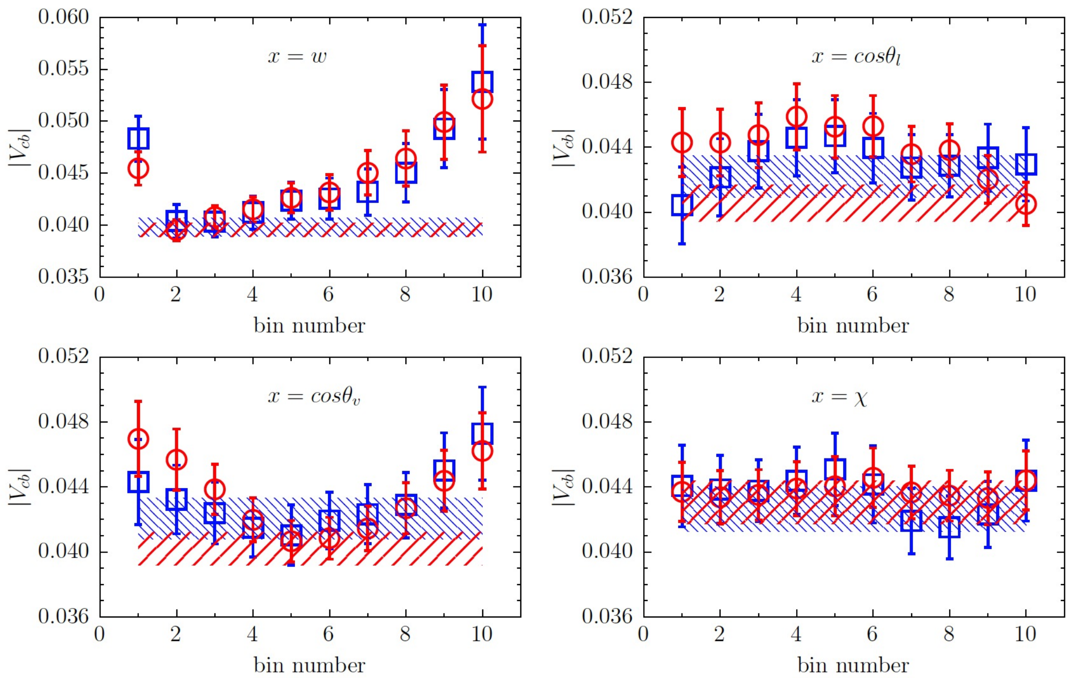
(see D'Agostini, arXiv: 2001.07562)

- we recover the above property by evaluating the correlation matrix of the experimental ratios

$$\frac{1}{\Gamma^{exp.}} \left(\frac{d\Gamma}{dx}\right)_i^{exp.}$$

and by considering the new covariance matrix of the experimental data given by (see arXiv:2105.08674)

$$\widetilde{C}_{ij}^{exp.} \to \rho_{ij}^{ratios} \sqrt{C_{ii}^{exp.} C_{jj}^{exp.}}$$



blue data: Belle 1702.01521

red data: Belle 1809.03290

bands are (correlated) weighted averages

$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} \,,$$

$$\sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}} \,,$$

Exclusive Vcb determination from B → D*

	experiment	$\vert V_{cb}\vert(x=w)$	$\vert V_{cb}\vert(x=\cos\theta_l)$	$\vert V_{cb}\vert(x=\cos\theta_v)$	$\vert V_{cb}\vert(x=\chi)$
Belle 1702.01521	Ref. [11]	0.0398 (9)	0.0422 (13)	0.0421 (13)	0.0426 (14)
Belle 1809.03290	Ref. [12]	0.0395 (7)	0.0405 (11)	0.0402 (10)	0.0430 (13)

averaging procedure

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k \ ,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu_x)^2,$$

→ $|V_{cb}|_{excl.} \cdot 10^3 = 41.3 \pm 1.7$

with the original covariance of Belle data

$|V_{cb}|_{excl.} \cdot 10^3 = 40.0 \pm 2.6$

$|V_{cb}|_{excl.} \cdot 10^3 = 39.6^{+1.1}_{-1.0}$ Gambino et al., arXiv:1905.08209

$|V_{cb}|_{excl.} \cdot 10^3 = 39.56^{+1.04}_{-1.06}$ Jaiswal et al., arXiv:2002.05726

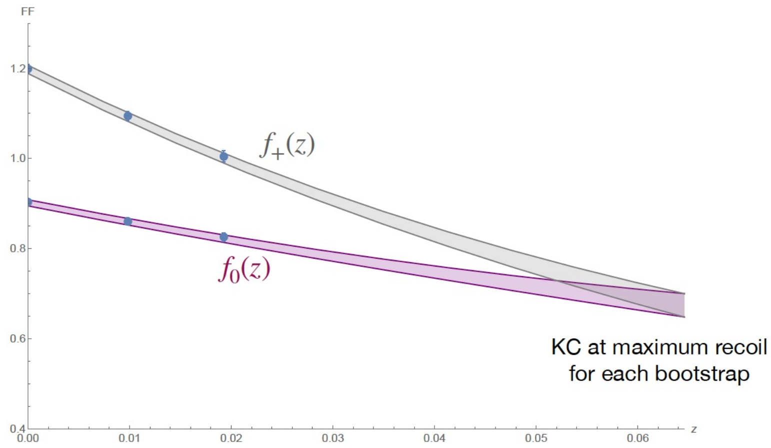
$|V_{cb}|_{excl.} \cdot 10^3 = 38.86 \pm 0.88$ FLAG '21, arXiv:2111.09849

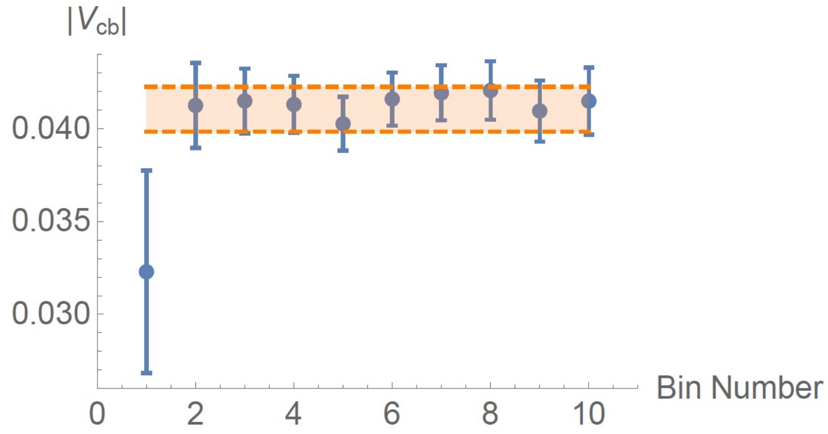
← the use of exp. data to describe the shape of the FFs leads to smaller errors, but the use of truncated BGL fits does not guarantee that the final error is not underestimated

$|V_{cb}|_{incl.} \cdot 10^3 = 42.16 \pm 0.50$ (Bordone et al: arXiv:2107.00604)

extraction of $|V_{cb}|$ from $B \to D \ell \nu_\ell$ decays

* lattice QCD form factors from FNAL/MILC (arXiv:1503.07237): synthetic data points at 3 (small) values of the recoil
* experimental data from Belle collaboration in 10 bins (arXiv:1510.03657)





$|V_{cb}|_{excl.} \cdot 10^3 = 41.0 \pm 1.2$ nice consistency with $B \to D^*$

$R(D)$, $R(D^*)$ and polarization observables

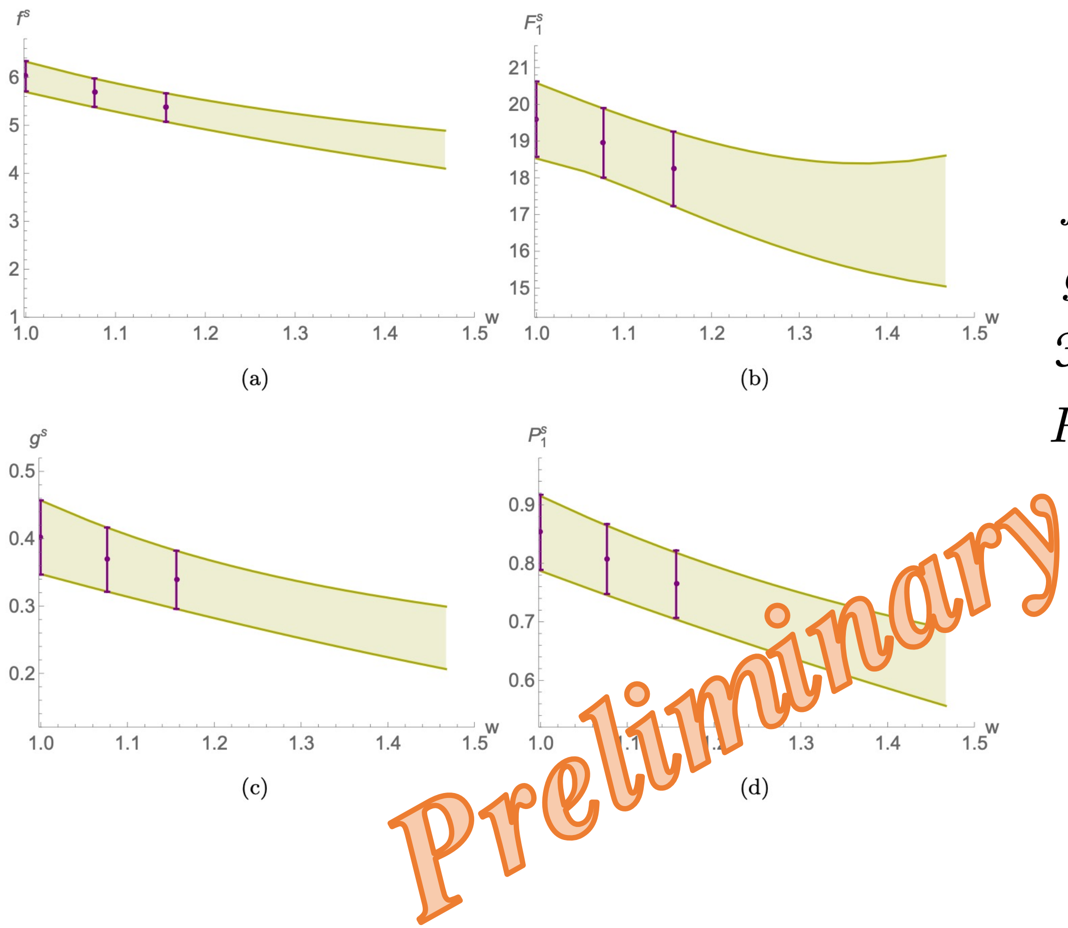
observable	DM	experiment	difference
$R(D)$	0.289 (8)	0.340 (27) (13)	$\simeq 1.6\,\sigma$
$R(D^*)$	0.269 (8)	0.295 (11) (8)	$\simeq 1.6\,\sigma$
$P_\tau(D^*)$	−0.52 (1)	$-0.38\,(51)\,(^{+21}_{-16})$	
$F_L(D^*)$	0.42 (1)	0.60 (8) (4)	$\simeq 2.0\,\sigma$

*** pure theoretical and parameterization-independent determinations within the DM approach ***

Semileptonic $B_s \to D_s^*$ decays

arXiv:2105.11433, From the results of the HPQCD Collaboration , we have determined the values of the FFs @ three values of small recoil, namely *w*={1.00, 1.08, 1.16}, and then applied the DM method to describe them in the whole kinematical range.

Results?

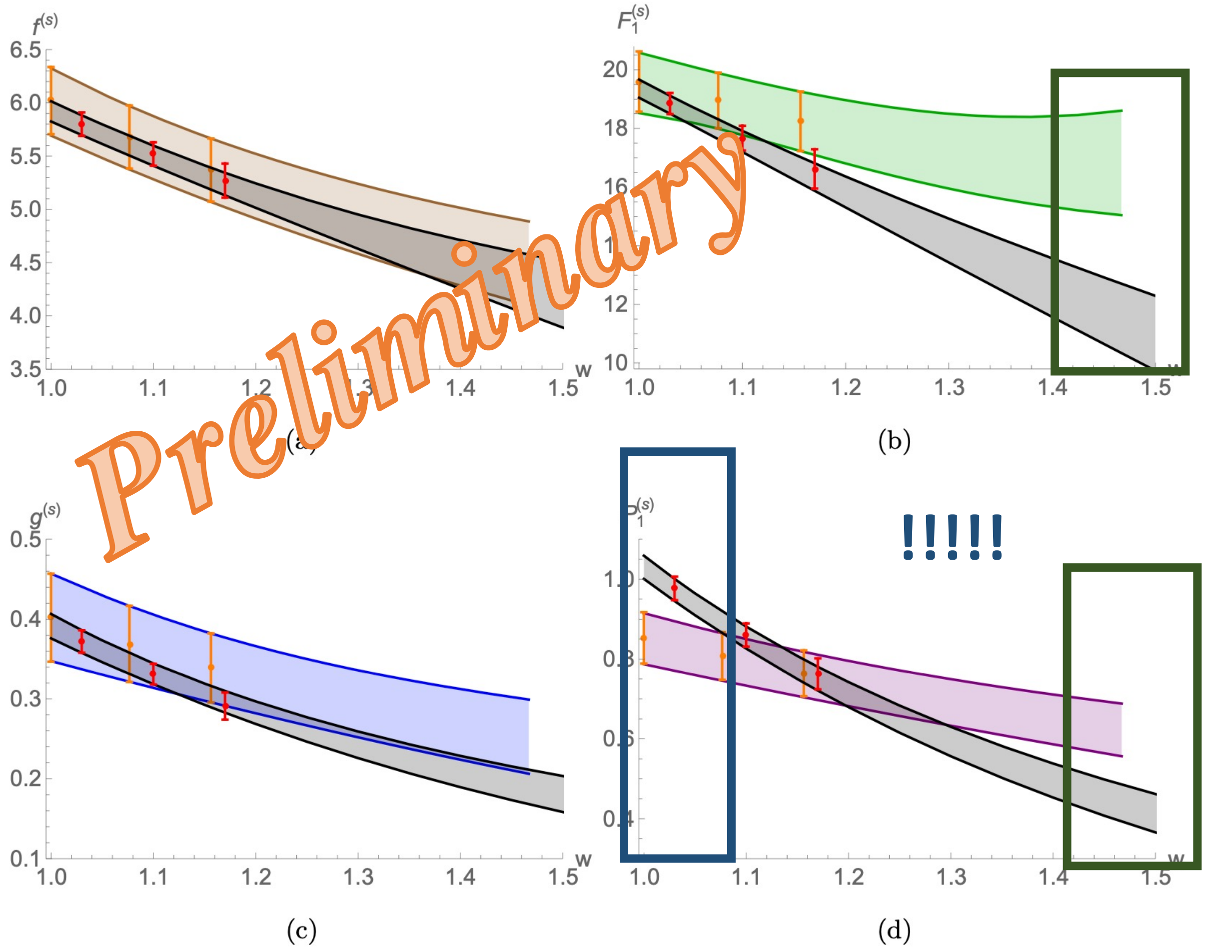


(a) (b) (c) (d)

$$\begin{aligned} f^s(w_{\max}) &= 4.49 \pm 0.39, \\ g^s(w_{\max}) &= 0.253 \pm 0.046, \\ \mathcal{F}_1^s(w_{\max}) &= 16.8 \pm 1.8, \\ P_1^s(w_{\max}) &= 0.622 \pm 0.066. \end{aligned}$$

Semileptonic $B_s \to D_s^*$ decays

Possible large (huge!) $SU(3)_F$ symmetry breaking effects
Hard to believe



$$f(w_{max}) = 4.19 \pm 0.31,$$
$$g(w_{max}) = 0.180 \pm 0.023,$$
$$\mathcal{F}_1(w_{max}) = 11.0 \pm 1.3,$$
$$P_1(w_{max}) = 0.411 \pm 0.048.$$

!!!!!

$$f^s(w_{\rm max}) = 4.49 \pm 0.39,$$
$$g^s(w_{\rm max}) = 0.253 \pm 0.046,$$
$$\mathcal{F}_1^s(w_{\rm max}) = 16.8 \pm 1.8,$$
$$P_1^s(w_{\rm max}) = 0.622 \pm 0.066.$$

DM confronts BGL

two important differences in the DM method with respect to BGL parametrization

- No series expansion to describe the FFs → ***NO TRUNCATION ERRORS***

particularly relevant for semileptonic decays characterized by a very large q^2 range

$B \to \pi \ell \nu$ Maximum q^2 = 26.46 GeV^2

$\Lambda_b \to p \ell \nu$ Maximum q^2 = 21.9 GeV^2

- Unitarity check of FFs data completely independent of the parameterization

The DM approach

i) reproduces exactly the known data

ii) allows to extrapolate the form factor in the whole kinematical range

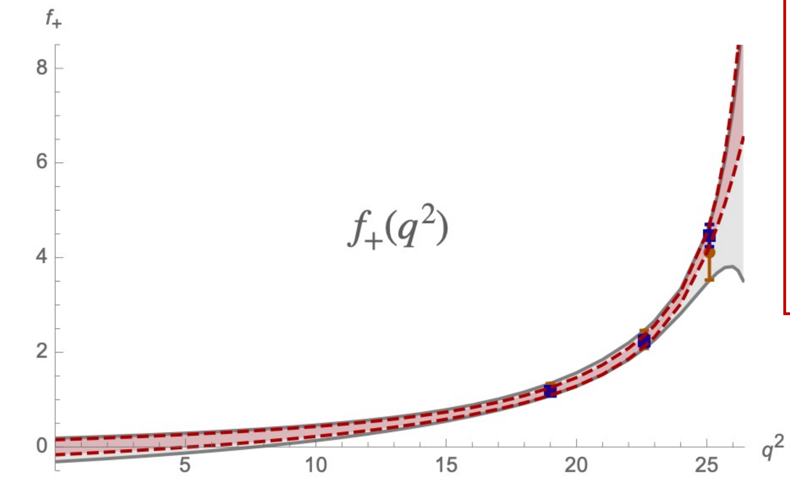
iii) in a parameterization-independent way

iv) providing a band of values representing the results of all possible BGL fits satisfying unitarity and passing through the known points (important for estimating uncertainties)

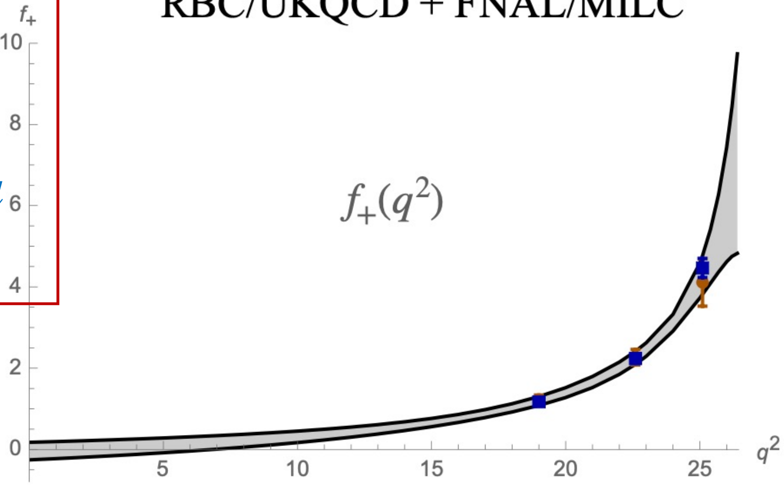
Semileptonic B → π decays *(in prep.)*

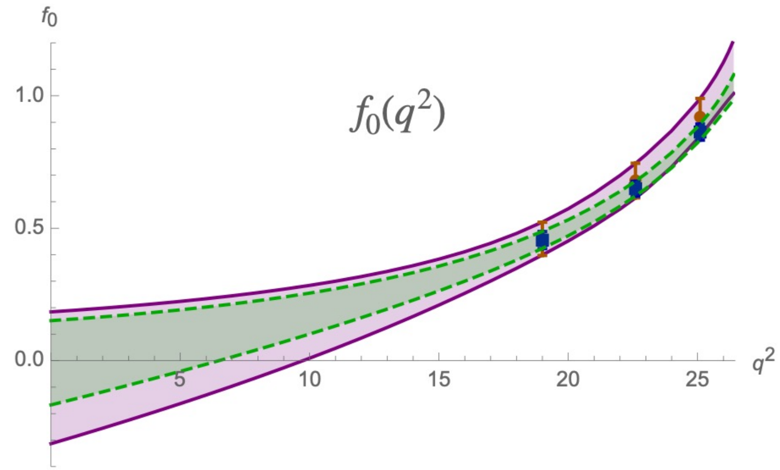
Solid: RBC/UKQCD
Dashed: FNAL/MILC

Non-perturbative susceptibility for the $b \to u$ current

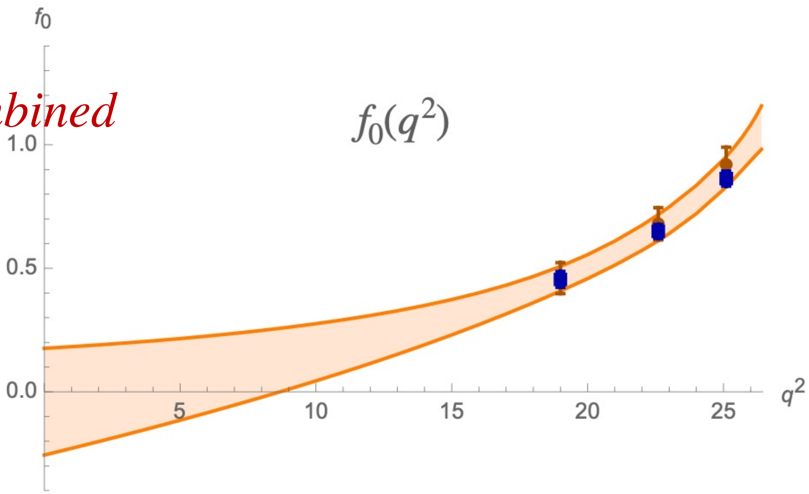


RBC/UKQCD + FNAL/MILC





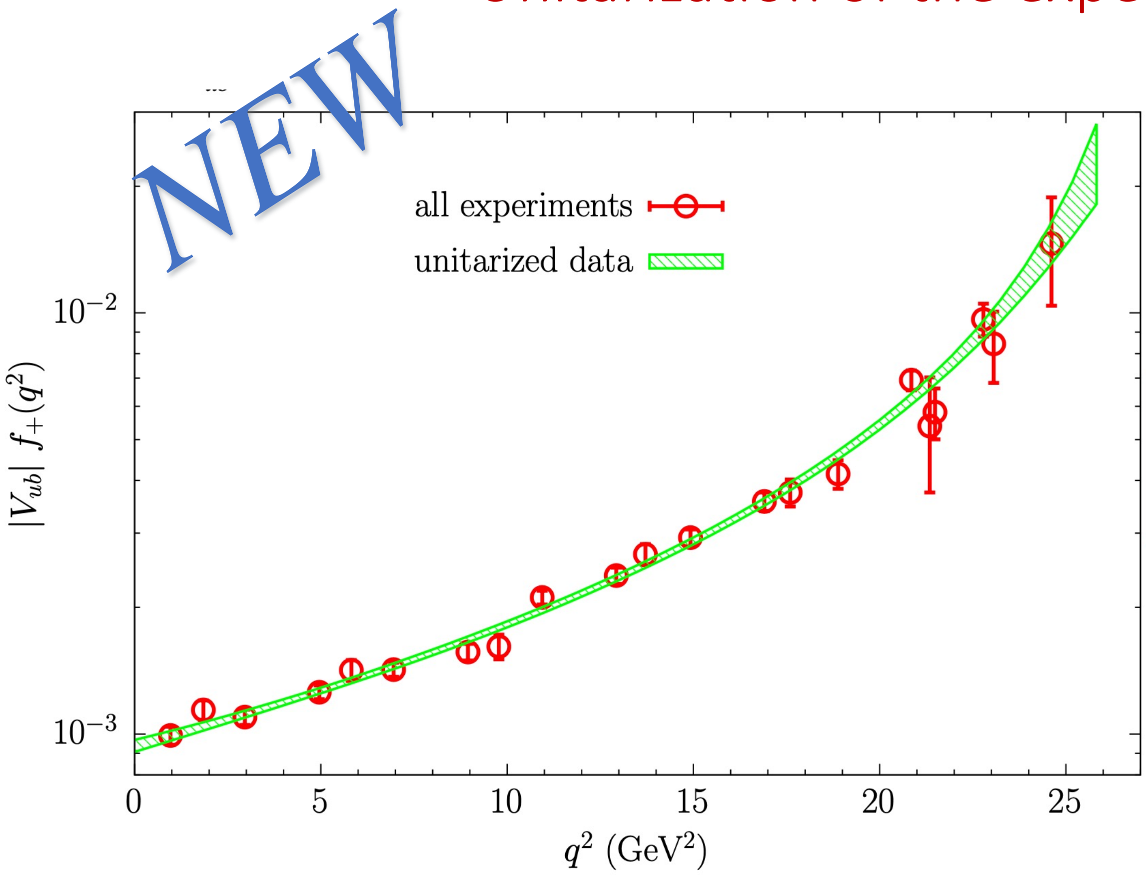
combined



	$f_+(0) = f_0(0)$
RBC/UKQCD	-0.06 ± 0.25
FNAL/MILC	-0.01 ± 0.16
Combined	-0.04 ± 0.22
LCSR	0.28 ± 0.03

- **3 RBC/UKQCD data (points) for each FF [arXiv:1501.05363]**
- **3 FNAL/MILC data (squares) for each FF [arXiv:1503.07839]**

Unitarization of the experimental data



$$|\mathbf{V_{ub}}|_{\mathbf{DM}} \times \mathbf{10^3} = \mathbf{3.88 \pm 0.32}$$

Reference	$\lvert V_{ub}\rvert \times 10^3$
FLAG '21	3.74 ± 0.17
HFLAV '18 & PDG '20	4.32 ± 0.29
Belle Coll. '21	4.10 ± 0.28

Exclusive and the inclusive values are compatible at the 1σ level

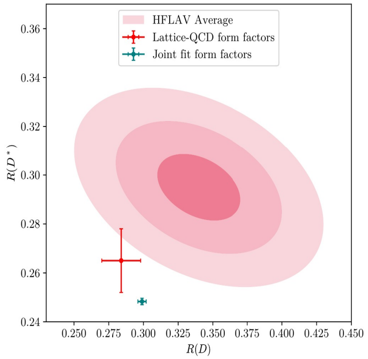
* construct the experimental values of $|V_{ub} f_+(q_i^2)| = \sqrt{\Delta\Gamma_i / z_i}$ (z_i = kinematical coefficient in the i-th bin)

* apply the DM method on the data points $|V_{ub} f_+(q_i^2)|$ using the unitarity bound $|V_{ub}|^2 \chi_{1^-}(0)$ with an initial guess for $|V_{ub}|$

* determine $|V_{ub}|$ using the theoretical DM bands and iterate the procedure until consistency for $|V_{ub}|$ is reached

S. Meinel CKM21

$R(D^{(*)})$



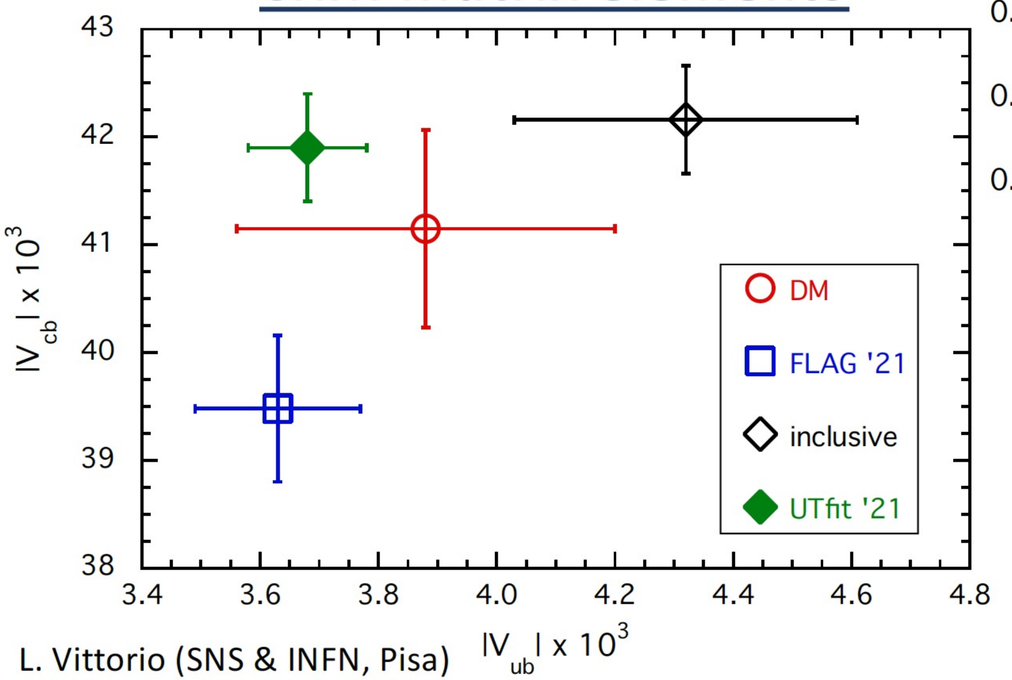
Lattice-QCD-only SM predictions:

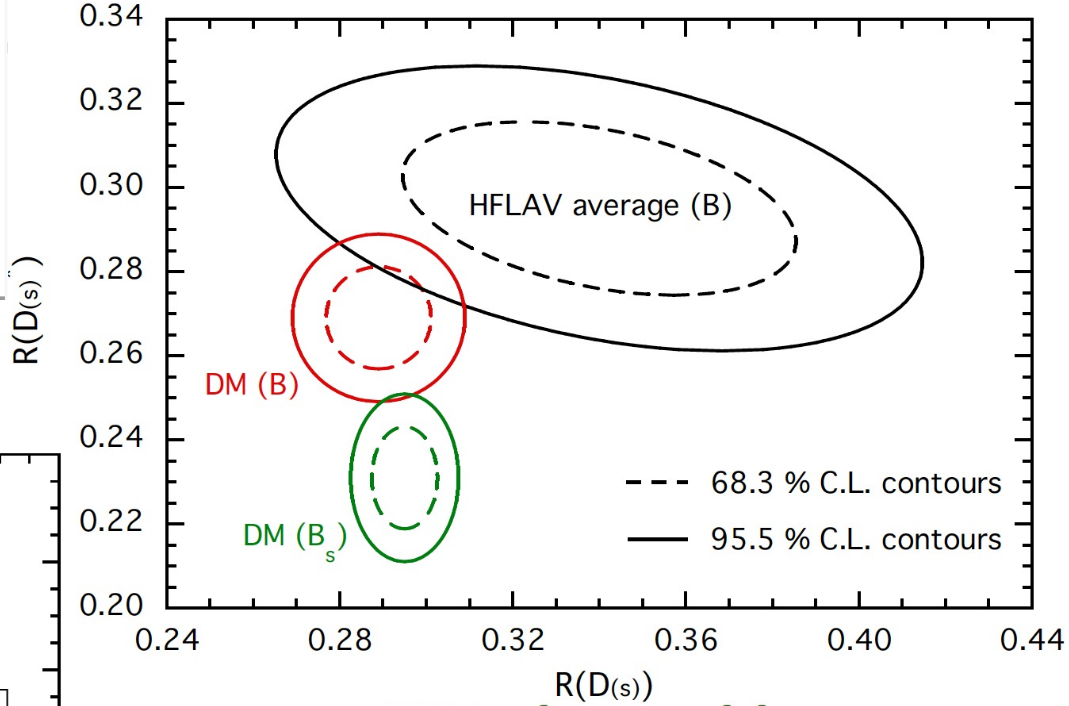
$$R(D^*)_{\text{FNAL/MILC 2021}} = 0.265 \pm 0.013$$
$$R(D)_{\text{FNAL/MILC 2015}} = 0.284 \pm 0.014$$
$$R(D)_{\text{FLAG 2021}} = 0.2934 \pm 0.0053$$

[FNAL/MILC, arXiv:2105.14019; HFLAV, arXiv:1909.12524/EPJC 2021]

See the talks by Judd Harrison, Shoji Hashimoto, Takashi Kaneko, Gumaro Rendon, and Silvano Simula for more lattice results on heavy→heavy decays!

CKM matrix elements





LFU observables

IMPORTANT: *the difference between* the red and the green area *comes from the difference in the LQCD computations by FNAL/MILC and HPQCD Collaborations*

Conclusions

The ***Dispersion Matrix approach*** *is a powerful tool to implement unitarity in the analysis of exclusive semileptonic decays of mesons and baryons*

- it does not rely on any assumption about the momentum dependence of the hadronic form factors

- it can be based entirely on first principles (*i.e*. unitarity and analiticity) using non-perturbative lattice determinations of both the relevant form factors and the dispersive bounds (the susceptibilities) from appropriate 2-point and 3-point (Euclidean) correlation functions

- it predicts band of values that are equivalent to all possible BGL fits satisfying unitarity and reproducing exactly a given set of data points. Larger but more reliable uncertainties

- It is not biased by the fit of the experimental data

- it is universal, namely it can be applied to any exclusive semileptonic decay e.g. baryon decays

Conclusions 2

New insight on both:

- *the |Vcb|, |Vub| puzzles (exclusive and inclusive determinations compatible @ the 1σ level)*

We found problems with the Belle covariance matrix

- *the R(D(*)) anomalies (theoretical values and measurements compatible @ the 1.6σ level)*

Is there really a problem with Lepton Flavor Universality in $B \rightarrow D^{(*)}$ *decays ?*

or

Much ado about nothing

absence says more than presence

FRANK HERBERT

(Dune)

THANKS FOR YOUR ATTENTION

Istituto Nazionale di Fisica Nucleare