Rare kaon $K \to \pi\ell^+\ell^-$ decay amplitude

DWQ@25

Felix Erben

17 December 2021

THE UNIVERSITY of EDINBURGH

European Research Council
Established by the European Commission

Collaborators

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner (Southampton)

Columbia University
Norman Christ
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda

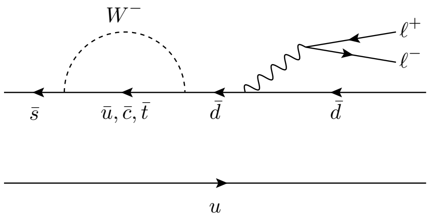
University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

$K \to \pi\ell^+\ell^-$

- flavour-changing neutral current (FCNC)
- $s \to d$ transition, forbidden at tree level
- potential for new physics
- dominated by $K \to \pi\gamma^*$ virtual photon exchange

$\Rightarrow$ dominated by long-distance effects



  - measured at NA48, NA62 experiments
  - NA62 measured $28,000$ samples $K \to \pi\mu^+\mu^-$ in 2016-2018 Run1

    [Bician,PoS(ICHEP2020), 390(364)]
  - more $K \to \pi\mu^+\mu^-$ and $K \to \pi e^+e^-$ in 2021-2024 Run2

    [Lazeroni,CERN SPS, Mar. 2021]

Related talk by Raoul Hodgson, Thu 16 Dec, 12:00: Weak Hyperon Decays from Lattice QCD

RELATED WORK

- theoretical description of form factor via two-loop low-energy expansion

  [D'Ambrosio et al., JHEP 2019 10.1007/jhep02(2019)049], [D'Ambrosio et al., Physics Letters B 797, 134891 (2019)]

- lattice methodology for long-distance contribution first proposed in

  [Isidori et al., Physics Letters B 633, 75 (2006)]

- additional details of strategy, including control over UV divergences

  [Christ et al., Physical Review D 92, (2015)]

- exploratory work at $M_\pi = 430$ MeV

  [Christ et al., Phys. Rev. D 94, 114516 (2016)]

- this work repeats the exploratory study at M_π^{phys}

$K \to \pi\gamma^* \to \pi\ell^+\ell^-$

Long-distance part can be parametrised via form factor V_+: [D'Ambrosio et al., hep-ph/9808289]

$$A_\mu = \frac{-iG_F}{(4\pi)^2} V_+(z)\Big(q^2(k+p)_\mu - (M_K^2 - M_\pi^2)q_\mu\Big)$$

$$z = q^2/M_K^2, \quad q = k - p, \quad V_+ = a_+ + b_+ z + V^{\pi\pi}(z)$$

Predictions from fitting NA48/NA62 data:

$$K \to \pi e^+ e^- : \quad a_+ = -0.578(16), \quad b_+ = -0.779(66)$$

$$K \to \pi\mu^+\mu^- : \quad a_+ = -0.592(15), \quad b_+ = -0.699(58)$$

theory, matching long-distance to short-distance from perturbation theory: [D'Ambrosio et al., 1906.03046]

$$K \to \pi\ell^+\ell^- : \quad a_+ = -1.59(8), \quad b_+ = -0.82(6)$$

LATTICE STRATEGY

Long-distance amplitude:

$$A_\mu(q^2) = \int d^4x \, \langle \pi(\vec{p}) | T[J_\mu(0) \mathcal{H}_W(x)] | K(\vec{k}) \rangle$$

J_μ : electromagnetic current, $\quad \mathcal{H}_W : s \to d$ weak current

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} [C_1 (Q_1^u - Q_1^c) + C_2 (Q_2^u - Q_2^c)]$$

$$Q_1^q = (\bar{s}_i \gamma_\mu^L d_i)(\bar{q}_j \gamma_\mu^L q_j), \quad Q_2^q = (\bar{s}_i \gamma_\mu^L q_i)(\bar{q}_j \gamma_\mu^L d_j), \quad q \in \{u, c, t\}$$

on the lattice, four-point correlation function

$$\Gamma^{(4)}(t_J, t_H) = \langle \phi_\pi(t_\pi) T[J_\mu(t_J) H_W(t_H)] \phi_K^\dagger(t_K) \rangle$$

integrate t_H around fixed t_J:

$$I(T_a, T_b) = \int_{t_J - T_a}^{t_J + T_b} dt_H \, \tilde{\Gamma}^{(4)}(t_J, t_H)$$

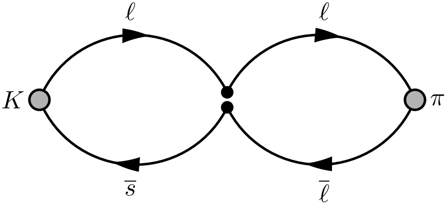
with the reduced correlator

$$\tilde{\Gamma}^{(4)}(t_J, t_H) = \Gamma^{(4)}(t_J, t_H) / Z_{\pi K}$$

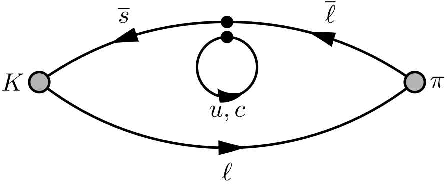
$Z_{\pi K}$ from 2pt correlators. Amplitude via:

$$A_\mu(q^2) = \lim_{T_a, T_b \to \infty} I(T_a, T_b)$$

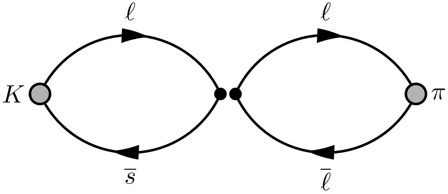
Types of diagrams, H_W 3pt function:



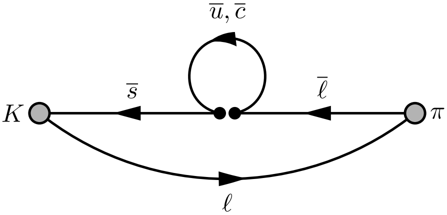
Connected



Eye



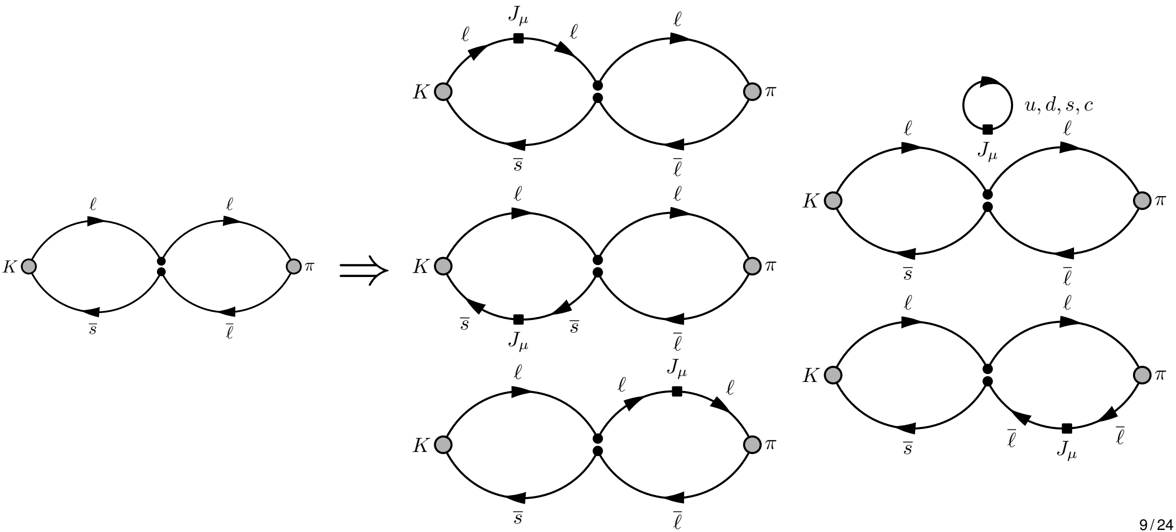
Wing



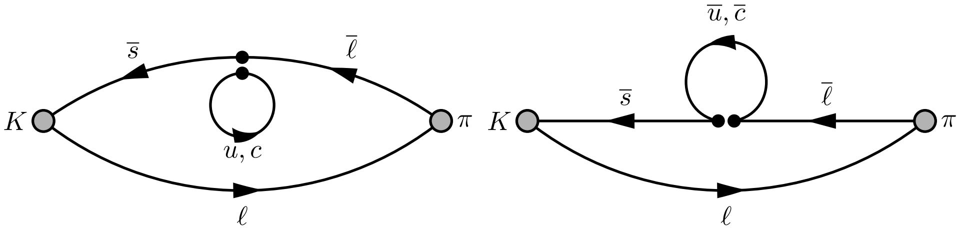
Saucer

LATTICE STRATEGY

4pt function including J_μ, for the C diagram



- J_μ in loop of S, E diagrams
  - ⇒ quadratic divergences as $\mathrm{J}_\mu, \mathrm{H}_W$ approach each other
- conserved current guarantees a logarithmic at most
- remaining divergence dealt with by GIM subtraction [Glashow et al. Phys. Rev. D 2, 1285 (1970)]
  - ⇒ needs charm-quark loops
  - ⇒ difference leads to cancellation of divergence



Ensemble

RBC/UKQCD $N_f = 2+1$ Möbius DWF ensemble [Blum et al. 1411.7017]

- $48^3 \times 96$
- $a^{-1} = 1.73$ GeV
- $m_\pi L = 3.86$
- $m_\pi = 139$ MeV
- $m_K = 499$ MeV

measurements:

- 87 configurations, separated by 20 MC steps
- 6 time translations per configuration
- on this ensemble:
  - $\vec{p}_K = (0,0,0), \vec{p}_\pi = \frac{2\pi}{L}(1,0,0)$

  $\Rightarrow \ z = \frac{q^2}{M_K^2} = 0.013(2)$

Setup of Computation

light quarks:

- Coulomb-gauge fixed wall sources
- zMöbius action
  - approximation of Möbius using complex parameters
  - 5^{th} dimension extent reduced from $L_s = 24$ to $L_s = 10$
  - all-mode-averaging style bias correction needed for Möbius action

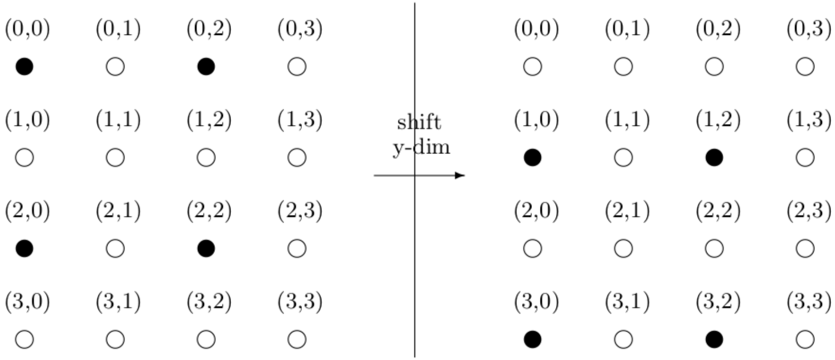
charm quarks:

- GIM demands same action as for l quark to work
- zMöbius action parameter choice makes simulation at m_c^{phys} impossible
  - simulation at unphysical $am_c \in \{0.25, 0.30, 0.35\}$

loops:

- spin-colour diluted sparse sources
- all-mode averaging
  - 1 hit exact ($10^{-8}, 10^{-10}, 10^{-12}, 10^{-14}$ for l, c_1, c_2, c_3)
  - 10 hits inexact (10^{-4} for all)

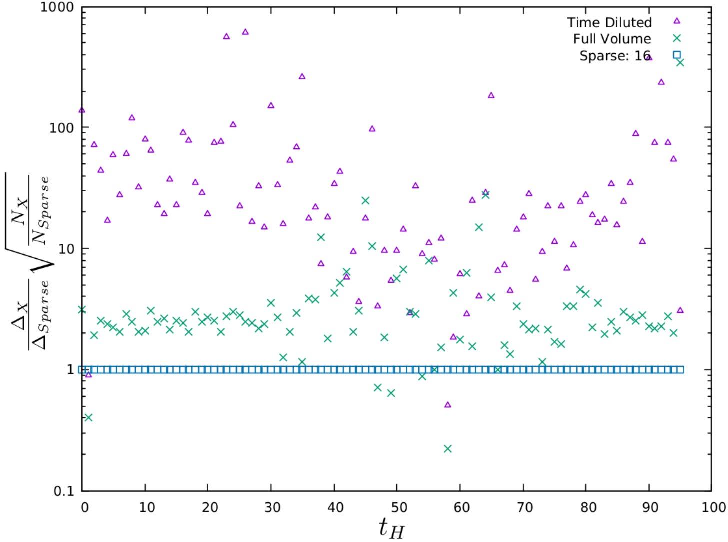
SPARSE SOURCES (LOOPS)



$$\kappa^{a}_{\text{sparse}}(x) = \begin{cases} \kappa_{Z_2}(x), & x = a,\ a \in (\mathbb{Z}/2\mathbb{Z})^4 \\ 0, & \text{else} \end{cases}$$

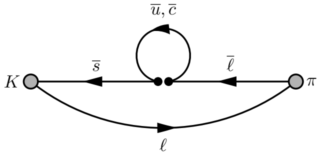
$$\mathbb{Z}/2\mathbb{Z} = \{0 \pmod 2, 1 \pmod 2\}$$

Noise comparison, Saucer diagram

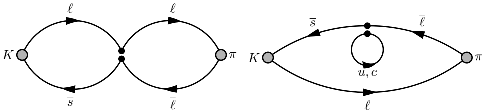


Cost comparison of 3 noise strategies

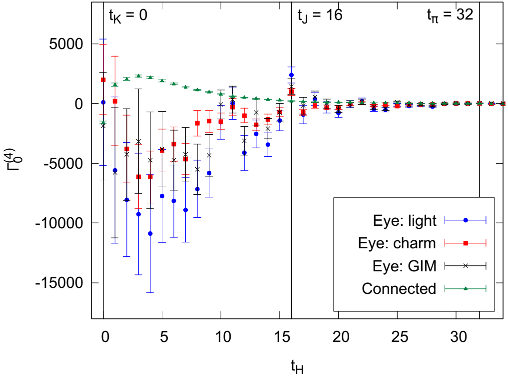
- full-volume Z_2 noise sources
- sparse Z_2 noise sources
- time-diluted A2A vectors

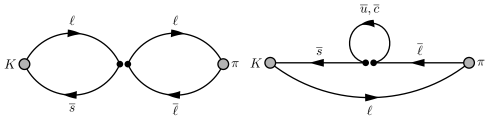


INDIVIDUAL RESULTS AT $m_c = 0.25$

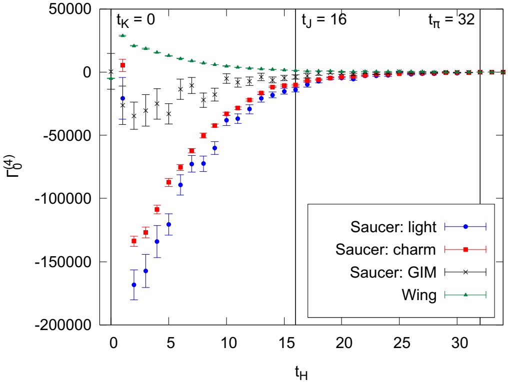


Q1





Q2



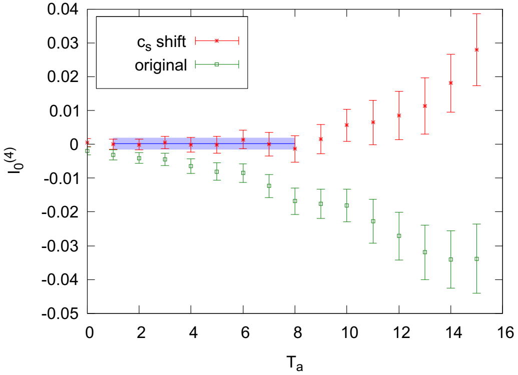
Integrating $\Gamma^{(4)}$

- any intermediate state with $E < E_K$ leads to exponentially growing contribution
  - $\Rightarrow$ needs to be subtracted [Christ et al., 1507.03094]
- Single-pion intermediate state:
- method 1: construct π state directly from correlation functions, subtract explicitly
- method 2: shift H_W by scalar density

$$H_W' = H_W - c_s \bar{s}d\,, \quad \langle \pi(\vec{k})|H_W'|K(\vec{k})\rangle = 0$$

- we compared both, settled for method 2
- Other states ($\pi\pi$, $\pi\pi\pi$) are expected to be insignificant until calculations reach sub-percent precision

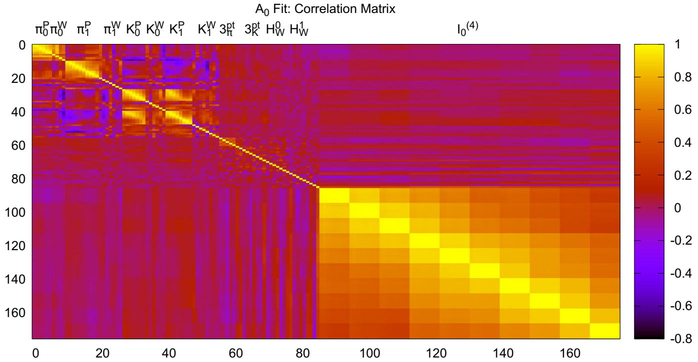
T_a DEPENDENCY AT $m_c = 0.25$



$$I(T_a, T_b) = \int_{t_J - T_a}^{t_J + T_b} dt_H \, \Gamma^{(4)}(t_J, t_H)$$

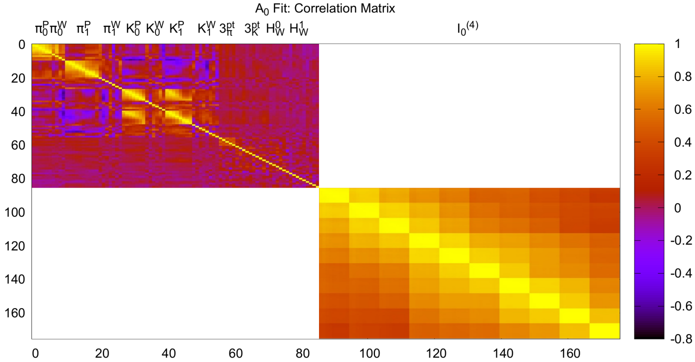
- $T_b = 8$ fixed
- green data has exponentially growing terms from intermediate π states
- red: π states manually removed
- fit result: $A_0 = 0.00022(172)$

Combined Fit



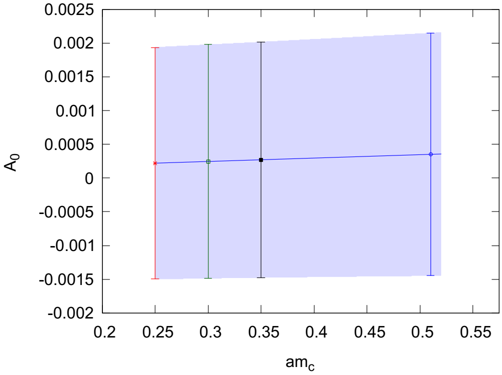
A_0 via combined fit to

- 2pt functions for π, K
  - point (P) and wall (W) sinks
  - zero momentum or $\frac{2\pi}{L}(1, 0, 0)$
- vector currents 3_π^{pt} and (3_K^{pt})
- weak current without (H_W^0) and with momentum (H_W^1)
- 4pt correlator $I_0^{(4)}(T_a, T_b)$



- Highly correlated
- we treat $I_0^{(4)}(T_a, T_b)$ as de-correlated from all other fit variables

AMPLITUDE



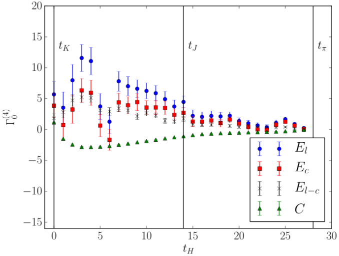
- all 3 charm masses m_c give compatible results
- ... but also compatible with 0
- extrapolated result $A_0 = 0.00035(180)$
- due to high statistical error, we ignored systematic effects

COMPARISON TO EARLIER WORK ($Q1$ DIAGRAMS)

First exploratory calculation of the long-distance contributions to the rare kaon decays $K \to \pi\ell^+\ell^-$

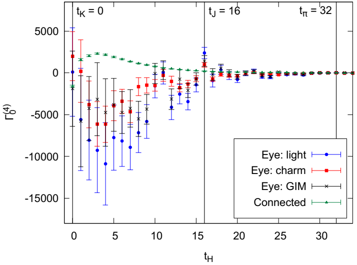
Norman H. Christ,[1] Xu Feng,[1,2,3,4] Andreas Jüttner,[5]
Andrew Lawson,[5] Antonin Portelli,[5,6] and Christopher T. Sachrajda[5]

$M_\pi = 430\text{MeV}$



this work

$M_\pi = 139\text{MeV}$



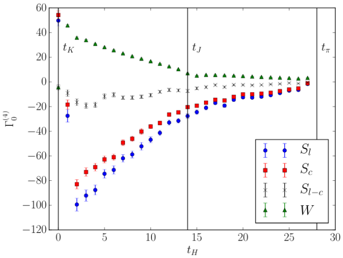
- at M_π^{phys}, u, c loops statistically decorrelate
- even at light m_c!
- GIM subtraction noisier than at unphysical M_π

COMPARISON TO EARLIER WORK (Q2 DIAGRAMS)

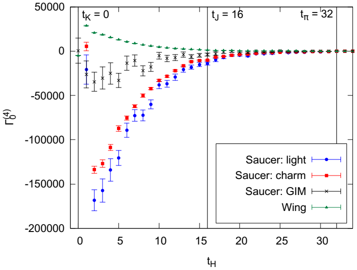
First exploratory calculation of the long-distance contributions to the rare kaon decays $K \to \pi \ell^+ \ell^-$

Norman H. Christ,[1] Xu Feng,[1,2,3,4] Andreas Jüttner,[5]
Andrew Lawson,[5] Antonin Portelli,[5,6] and Christopher T. Sachrajda[5]

$M_\pi = 430\text{MeV}$



this work
$M_\pi = 139\text{MeV}$



- at M_π^{phys}, u, c loops statistically decorrelate
- even at light m_c!
- GIM subtraction noisier than at unphysical M_π

Conclusions

- first lattice QCD calculation of $K \to \pi \ell^+ \ell^-$ and m_π^{phys}
- form factor of the long-distance amplitude: $V(0.013(2)) = -0.87(4.44)$

$\Rightarrow$ $\sim$ 10 times experimental error

  - $V_\mu^{\text{exp}}(0) = a_\mu^{\text{exp}} = -0.578(16)$
  - $V_e^{\text{exp}}(0) = a_e^{\text{exp}} = -0.592(15)$

- Naive $1/\sqrt{N}$ scaling suggests factor $\sim$ 100 in statistics to match experimental error, prohibitively expensive
- manuscript of this study to be submitted for publication within next month

erc
European Research Council
Established by the European Commission

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.

OUTLOOK

- To go forward, new methodology is needed.
- main source of noise / error comes from loop-type diagrams (E, S)

$\Rightarrow$ improvement in estimation of disconnected diagrams is crucial

- Further potential improvement: summed method for $I^{(4)}$ (see Raoul Hodgson's talk):

$$\int_{t_J - T_a}^{t_J + T_b} dt_H \to \int_0^T dt_J \int_0^T dt_H$$

- in 3-flavour QCD, c loop would not have to be computed
  - $\Rightarrow$ requires new renormalisation procedure
- on RBC/UKQCD $64^3 \times 96$ M_π^{phys} ensemble ($a^{-1} = 2.38$GeV), physical m_c could be simulated

European Research Council
Established by the European Commission

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.