[Phys.Rev. D91 (2015) no.7, 074502]
[Phys.Rev. D102 (2020) 5, 054509]
[arXiv:2103.15131, accepted to PRD]

Direct CP Violation & the $\Delta I=1/2$ Rule in $K\to\pi\pi$ Decay in the Standard Model

Christopher Kelly
Brookhaven National Laboratory

(RBC & UKQCD collaborations)

Friday December 17th 2021,
DWQ@25,
Brookhaven National Laboratory, USA
(Online)

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner (Southampton)

Columbia University
Norman Christ
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda

University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Motivation

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \rightarrow \pi\pi$:

$$\eta_{00} = \frac{A(K_{\rm L} \to \pi^0\pi^0)}{A(K_{\rm S} \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_{\rm L} \to \pi^+\pi^-)}{A(K_{\rm S} \to \pi^+\pi^-)}.$$

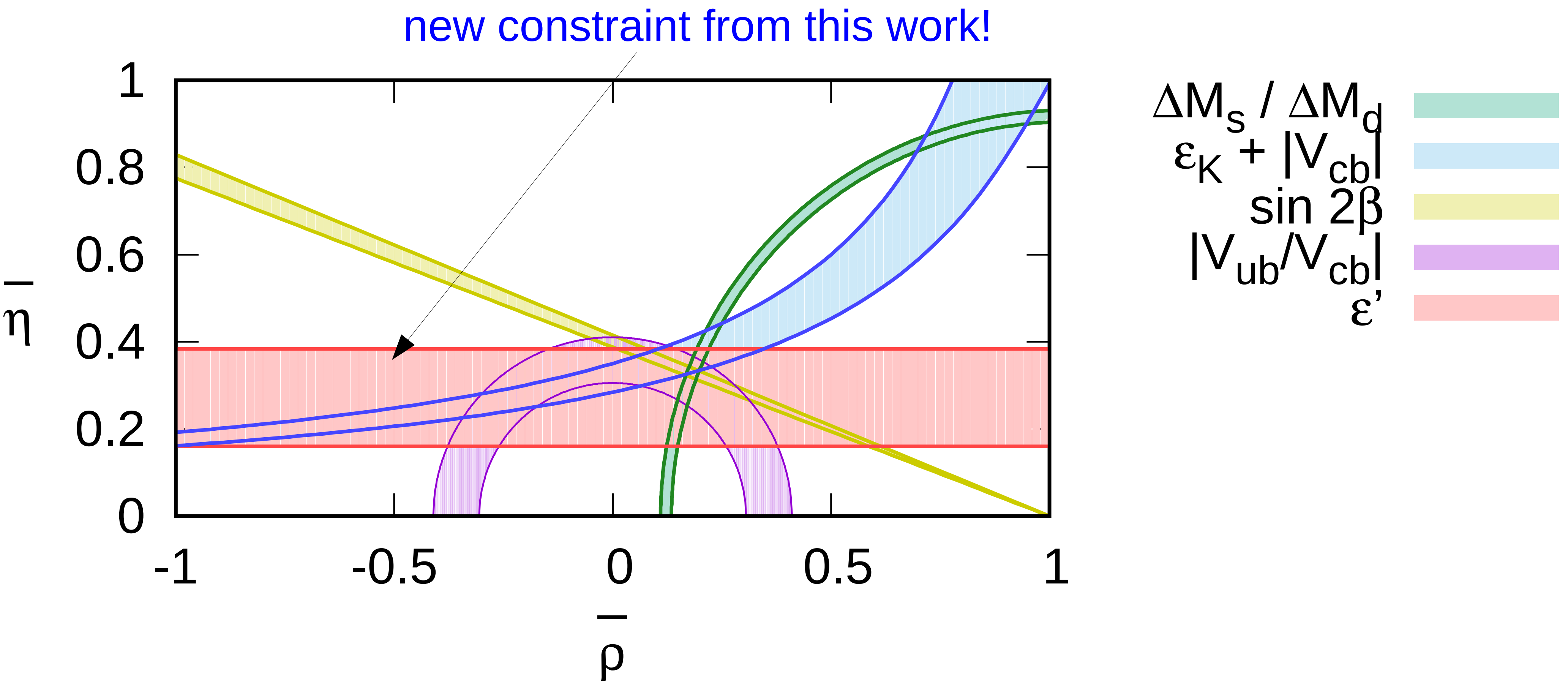
$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \quad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by many BSM models.
- Looking for deviations from experiment may help shed light on origin of M/AM asymmetry.

- A Standard Model prediction of ε' also provides a new horizontal band constraint on CKM matrix in ρ-η plane:



- While underlying weak process occurs at high energies ~M_W=80 GeV, K→ππ decays receive large corrections from low-energy hadronic physics O(Λ_{QCD})~250 MeV.
- Lattice QCD is the only known *ab initio*, **systematically improvable** technique for studying non-perturbative QCD.

Overview of calculation

ππ phase shifts

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

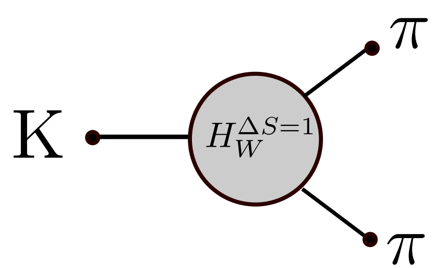
$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2} \,,$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2} \,.$$

$$\longrightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

I=2 decay

I=0 decay

- **Hadronic energy scale << M_W – use weak effective theory (3 flavors)**



$$H_W = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\left(z_i(\mu) + \tau y_i(\mu)\right)^{\overline{\mathrm{MS}}} Q_i^{\overline{\mathrm{MS}}(\mu)} \right]$$

perturbative Wilson coeffs.

10 effective four-quark operators

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

Lattice QCD for K→ππ

$$\mathcal{M}_i^{\overline{\rm MS}} = F Z_{ij}^{{\rm lat}\to\overline{\rm MS}} \underbrace{\langle(\pi\pi)_I|Q_j|K\rangle}_{\mathcal{M}_j^{\rm lat}}$$

Use intermediate non-perturbative scheme and convert to MSbar perturbatively at high energy

Lellouch-Luscher finite volume correction

operator with *ππ* q. numbers.

operator with kaon q. numbers.

Measured Green's function

$$\langle 0|\mathcal{O}_{\rm snk}(t_{\rm snk})Q_i(t)\mathcal{O}_{\rm src}(t_{\rm src})|0\rangle$$

$$= \sum_{n,m} A^n_{\rm snk} \underbrace{\langle n|Q_i(t)|m\rangle}_{\mathcal{M}_i^{\rm lat}} A^m_K \times e^{-E_n(t_{\rm snk}-t)} e^{-E_m(t-t_{\rm src})}$$

many states contribute at source and sink

- Extract matrix elements by fitting time dependence in limit of large (t_{snk}-t), (t_{src}- t) at which lower-energy states dominate.

Stationary ππ state

- Spectrum of *ππ* operator includes state with both pions at rest with energy $E_{\pi\pi}$~270 MeV << m_K
- Thus 3-point correlator dominated by *unphysical* matrix element.

(see next talk by M. Tomii)

- Consider multi-exponential fits : difficult, especially for noisy I=0
- Instead exploit freedom to modify boundary conditions while incurring only exponentially suppressed corrections.
- <u>I=2 calculation:</u>

➢ Use isospin rotation to recast $K^0 \to (\pi\pi)^{I=2}_{I_3=0} \to K^+ \to \pi^+\pi^+$ involving only **charged** pions

➢ Antiperiodic spatial BCs on *down quark*: charged pions become antiperiodic

$$p_i^{\pi^\pm} = \pm(2n+1)\pi/L$$

➢ Tune L to match *ππ* and kaon energy such that physical decay becomes **ground-state martrix element.**

➢ Use of $\pi^+\pi^+$ final state avoids mixing resulting from isospin breaking induced by applying BCs only to down quark as this is the only doubly-charged state in the system.

Stationary ππ state II

- <u>I=0 calculation</u>

- I=0, I_3=0 final state cannot be isospin rotated
- No way to avoid breaking of isospin symmetry and applicability only to charged pions of APBC on down quarks.
- Use G-parity BCs: $\hat{G}\pi^{\pm,0}\hat{G}^{-1} = -\pi^{\pm,0}$: charged and neutral pions antiperiodic
- Commutes with isospin!
- Tricky application at quark level:

$$\hat{G}d\hat{G}^{-1} = C\bar{u}^T \qquad \hat{G}u\hat{G}^{-1} = -C\bar{d}^T$$

- Various complications make calculation more expensive:
    - Factor of 2 more expensive due to explicit flavors
    - New gauge configurations required as gauge field satisfies charge conjugation BCs
    - Additional Wick contractions across boundaries complicates Green's functions
    - Introducing fictional strange quark partner to obtain stationary kaon state
    - ......

I=2 calculation

[Phys.Rev. D91 (2015) no.7, 074502]

- Other than unusual BCs, precise I=2 calculation can be performed relatively easily using traditional lattice methods
- Most recent result (2015):
  - Computed with large, ~ (5.5 fm)^3 volumes
  - Physical quark masses
  - Two lattice spacings (2.36 GeV and 1.73 GeV) → Continuum limit taken.

experimental result

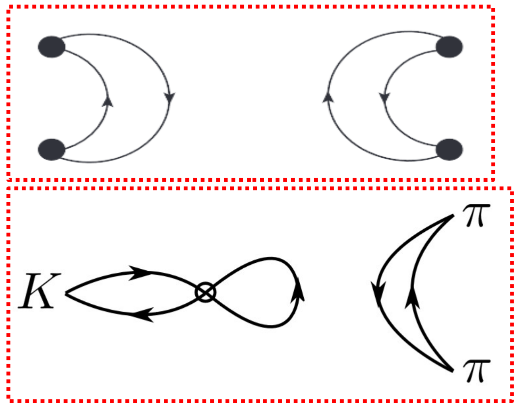
$$\boxed{\begin{aligned} \mathrm{Re}A_2 &= 1.50(4)(14)\times 10^{-8}\ \mathrm{GeV} \\ \mathrm{Im}A_2 &= -6.99(20)(84)\times 10^{-14}\ \mathrm{GeV} \end{aligned}} \longrightarrow 1.479(3)\times 10^{-8}\ \mathrm{GeV}$$

- <1% statistical error!
- 10% and 12% total errors on Re(A_2) and Im(A_2) resp.
- Dominant sys. errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

I=0 challenges

- I=0 ππ state has **vacuum quantum numbers**

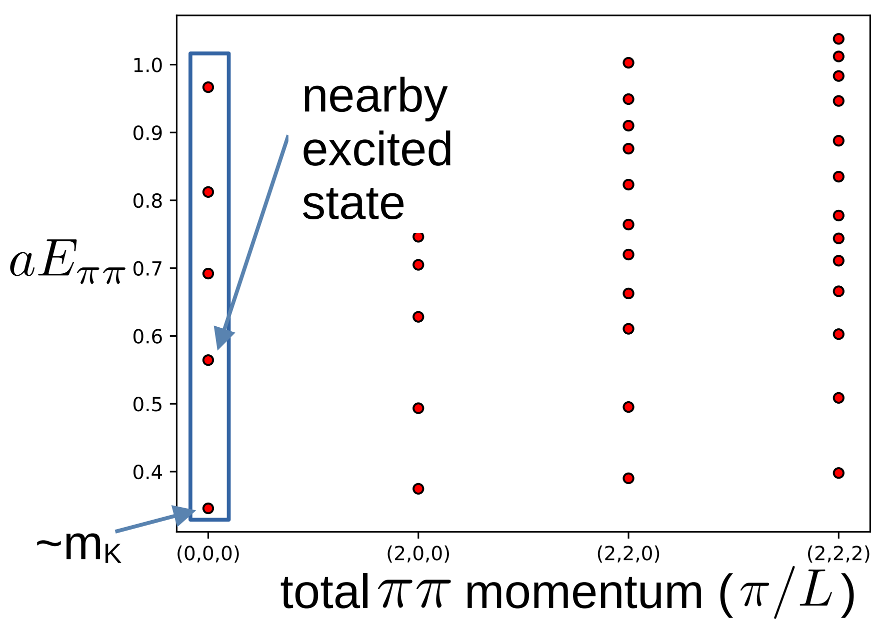
“Disconnected diagram”

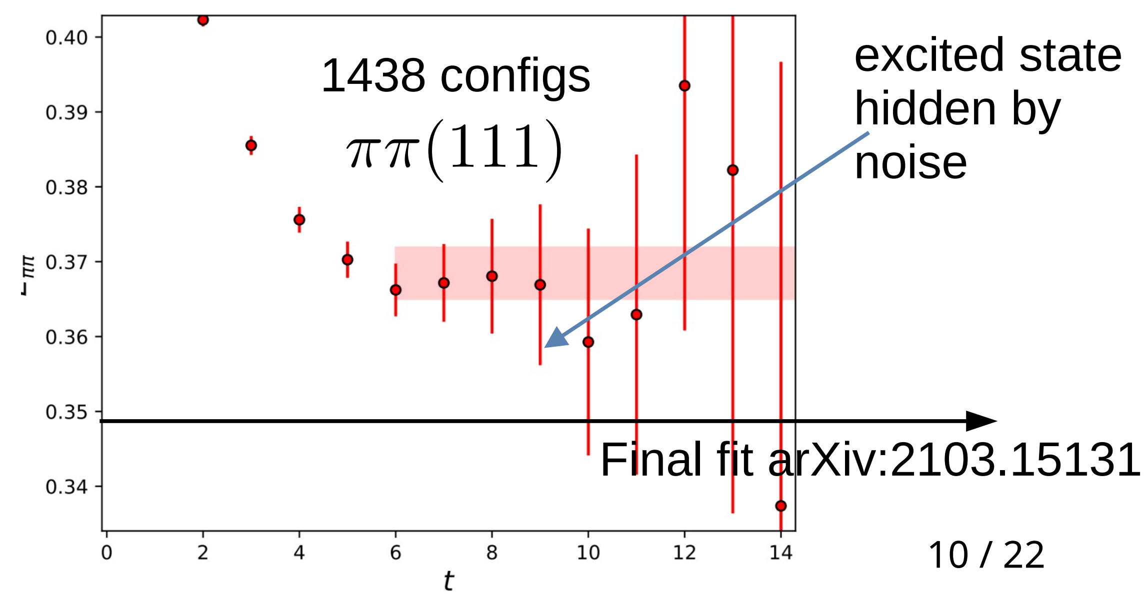


Very noisy, requiring large statistics and advanced methods

- Rapid degeneration of signal/noise increases susceptibility to excited state contamination

Predicted lattice energies

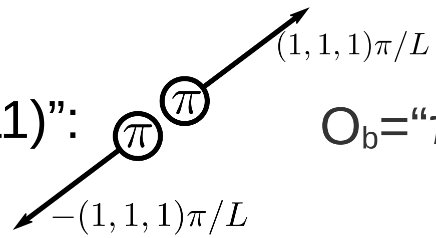




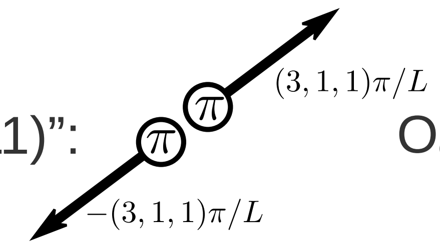
Controlling excited states

- Introduction of multiple operators (with same quantum numbers) very powerful means of resolving excited states.

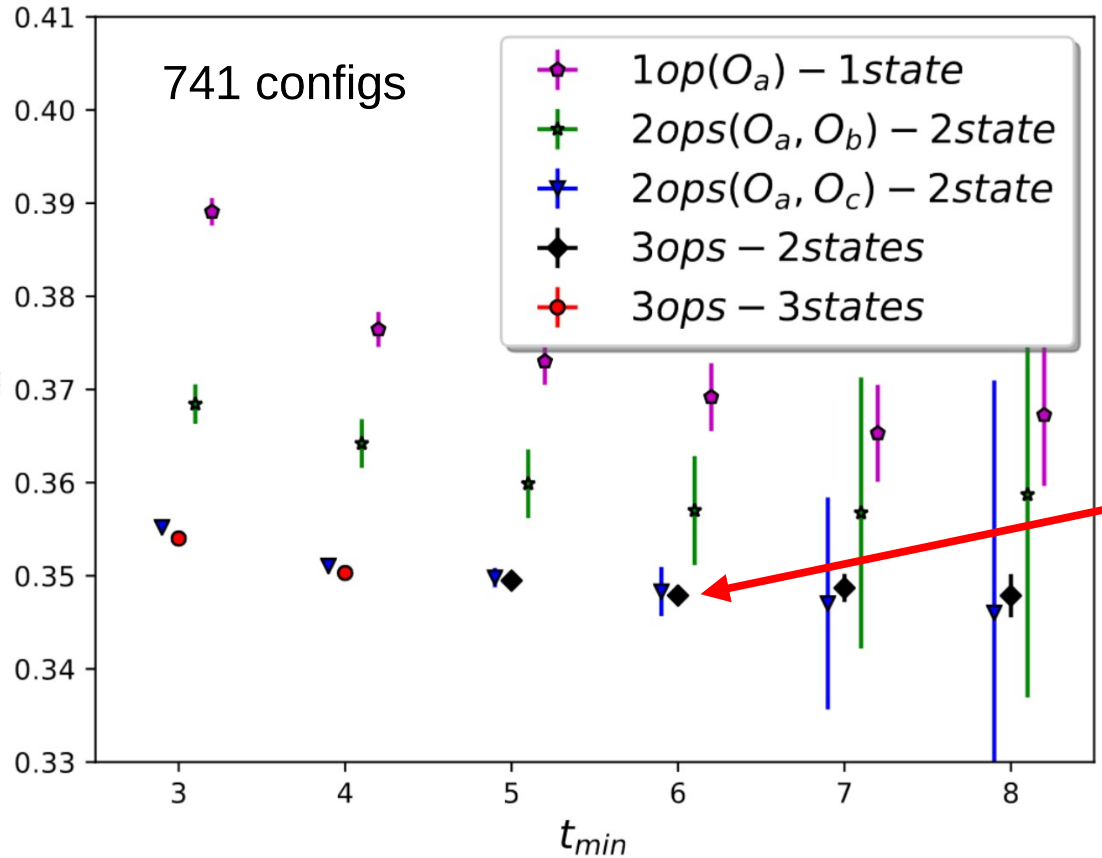
O_a="ππ(111)":



O_b="ππ(311)":



$O_c = \sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$



fitted energy (lattice units)

[arXiv:2103.15131]

Result at kaon mass scale

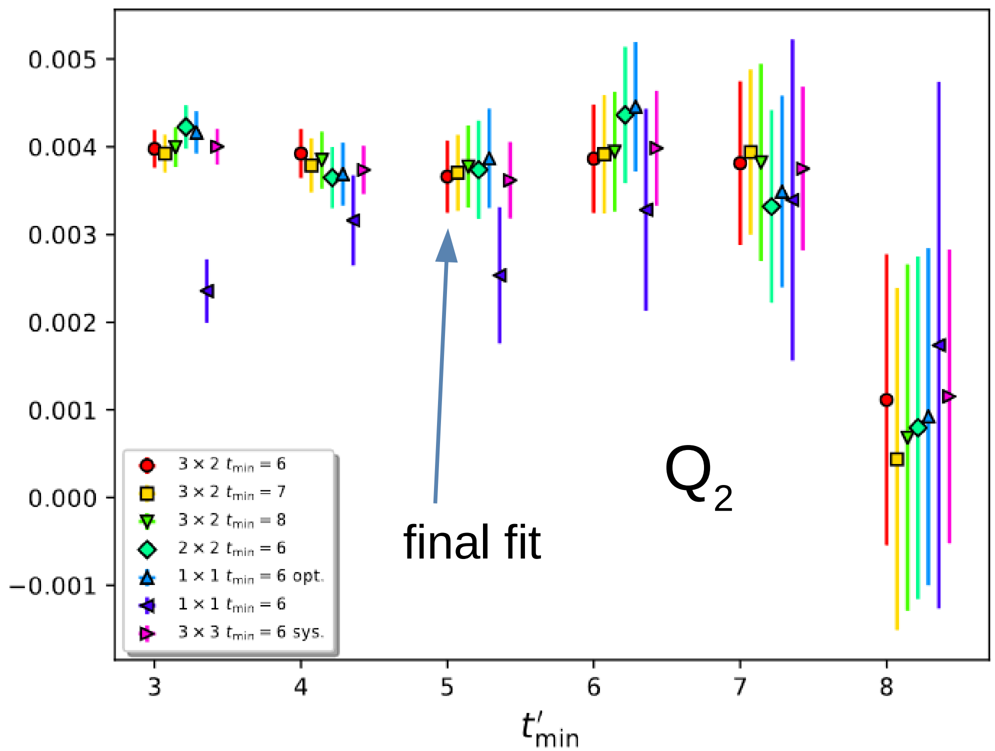
$$\delta_0(479.5\ \mathrm{MeV}) = 32.3(1.0)(1.4)^\circ$$

agrees well with dispersive result 35.9°

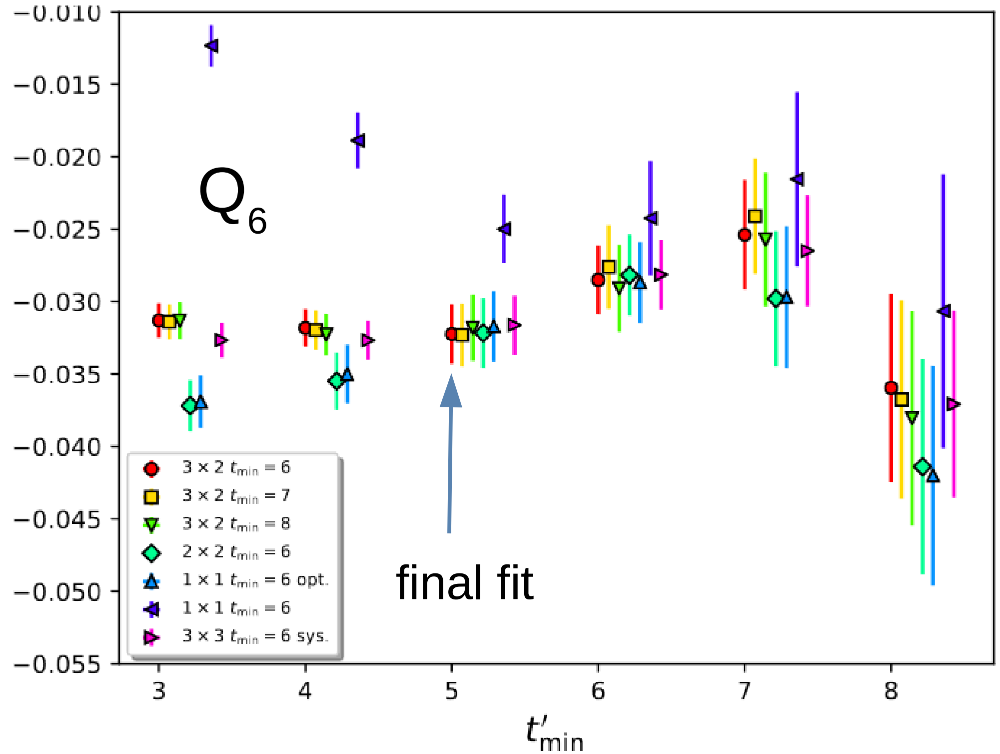
Effect of multiple operators on I=0 K→ππ fit results

- Examine many fit ranges, #states and #operators

Dominant contribution to ReA0



Dominant contribution to ImA0



- Again see significant improvement in quality of result and stability with additional operators.

I=0 calculation

[Phys.Rev.D 102 (2020) 5, 054509]

- Most recent result (2020):
  - Computed with large, ~ (4.6 fm)^3 volume
  - Large statistics (741 configurations)
  - Physical quark mass
  - Single, somewhat coarse lattice spacing (1.378(7) GeV)
  - Step-scaling RI-SMOM to 4.0 GeV to minimize PT error in matching.
  - 3 two-pion operators to control excited states.

$$\boxed{\begin{aligned} \mathrm{Re}A_0 &= 2.99(0.32)(0.59)\times 10^{-7}\ \mathrm{GeV} \\ \mathrm{Im}A_0 &= -6.98(0.62)(1.44)\times 10^{-11}\ \mathrm{GeV} \end{aligned}}$$

experimental result

$$3.320(2)\times 10^{-7}\ \mathrm{GeV}$$

- 11% and 9% statistical error
- 20% and 21% systematic errors
- Dominant sys. errors due to discretization and truncation of PT series in computation of Wilson coefficients.

A_0 systematic error budget

- Use of multiple operators results in negligible excited state systematic
- Step-scaling to 4.0 GeV results in low, 4% renormalization error.

- Single, coarse lattice spacing gives large, 12% discretization error.
- Evidence that Wilson coefficient systematics are driven by using PT for 3-4f matching, not improved by evaluating at μ=4.0 GeV in 3f theory.

Error source	Value
Excited state	-
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

Error source	Value	
	$\mathrm{Re}(A_0)$	$\mathrm{Im}(A_0)$
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

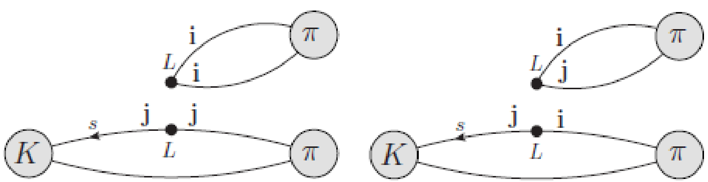
ΔI=1/2 rule

- In experiment kaons ~450x (!) more likely to decay into I=0 pi-pi states than I=2.

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 22.45(6) \quad \text{(the ΔI=1/2 rule)}$$

- Perturbative running to charm scale accounts for about a factor of 2. Where does the remaining 10x come from? New Physics?
- The answer is **low-energy *QCD*!** [arXiv:1212.1474, arXiv:1502.00263]

Strong cancellation between the two dominant contractions not predicted by naive factorization:

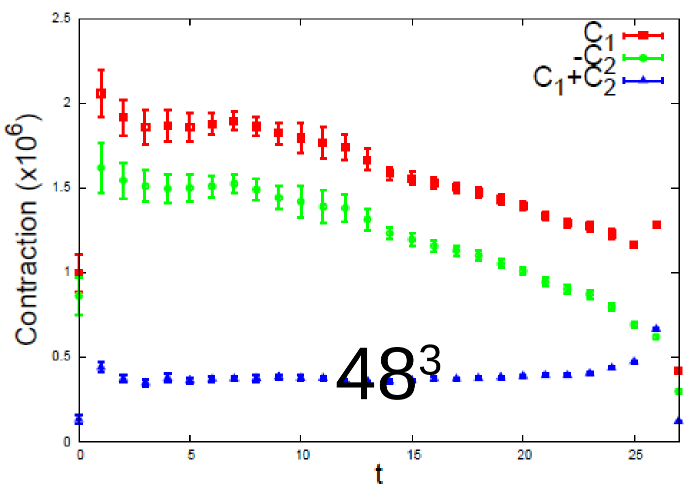


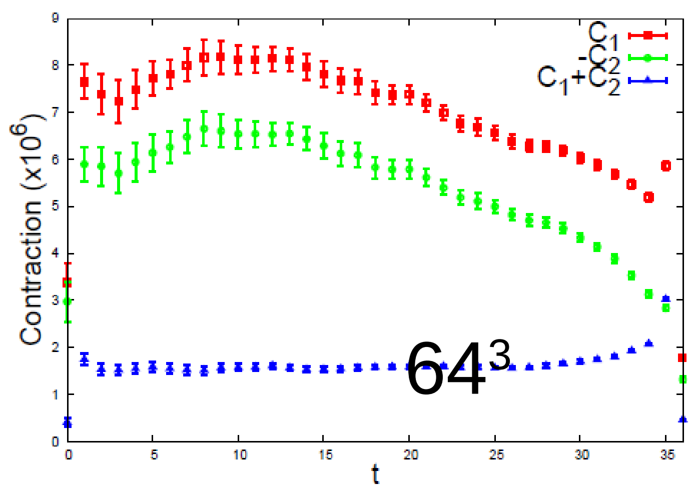
Contraction ①. Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

find $② \approx -0.7①$ heavily suppressing $\mathrm{Re}(A_2)$.

[Phys.Rev. D91 (2015) no.7, 074502]





Pure-lattice calculation

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} = 19.9(5.0)$$

[$\mathrm{Re}(A_0)$ agrees with expt.]

[Phys.Rev.D 102 (2020) 5, 054509]

Isospin breaking + EM effects

- Our simulation does not include effects of isospin breaking or EM effects.
- While these effects are typically small O(1%), heavy suppression of A_2 ($\Delta I=1/2$ rule) means relative effect and ε' could be O(20%).
- Current best determination of effect uses NLO χPT and $1/N_c$ expansion predicts 23% correction to our result: [Cirigliano *et al*, JHEP 02 (2020) 032]

Include as separate systematic error on ε'.

Final result for ε'

- Combining our results for $\mathrm{Im}(A_0)$ and $\mathrm{Im}(A_2)$, and using expt. for the real parts, we find

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 0.00217(\underbrace{26}_{\text{stat}})(\underbrace{62}_{\text{sys}})(\underbrace{50}_{\text{IB + EM}})$$

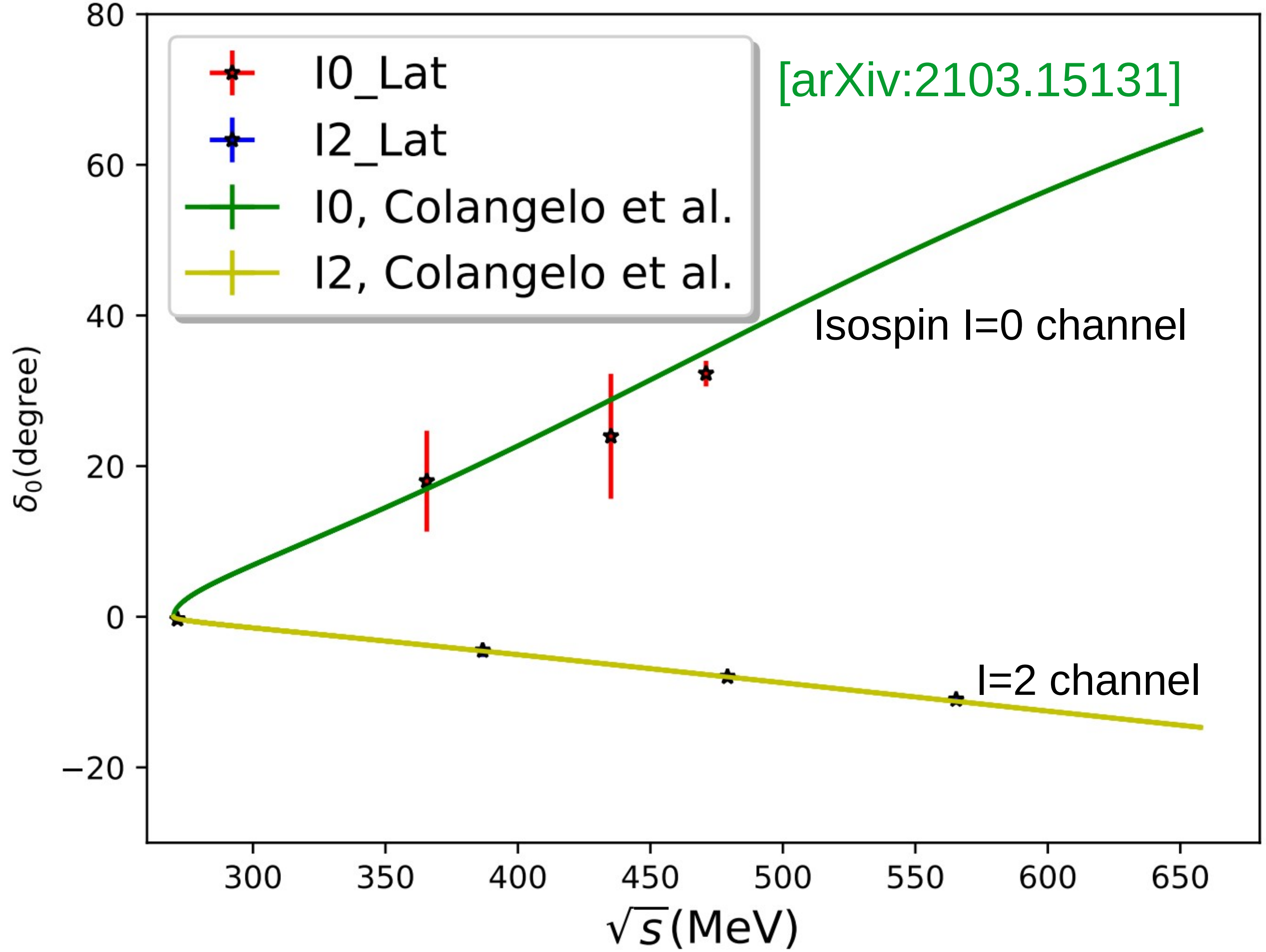
Consistent with experimental result:

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$$

Total error ~3.6x that of experiment.

Results: Phase shift energy dep.

- We need the total momentum (0,0,0) pi-pi calculation for $K^0 \to \pi\pi$
- With only a slight amount more work we can compute also with non-zero total momentum and map out the energy dependence of the phase shift:



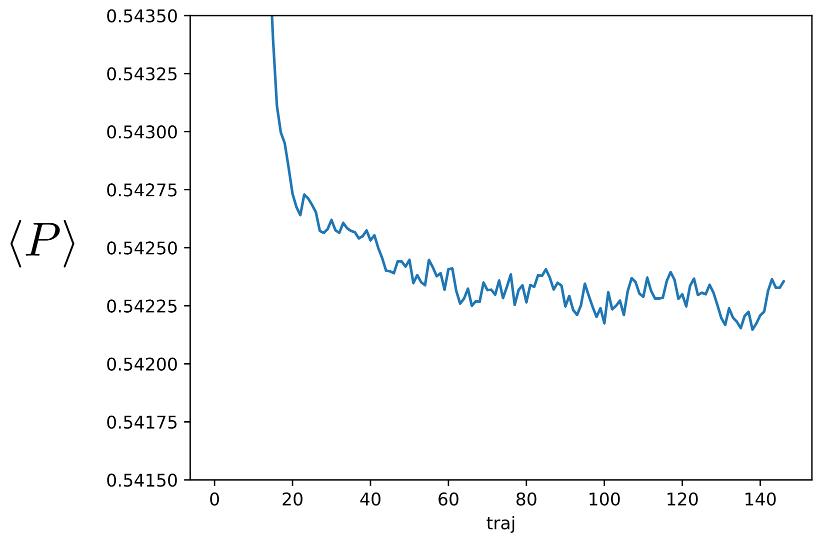
The Next Big Thing

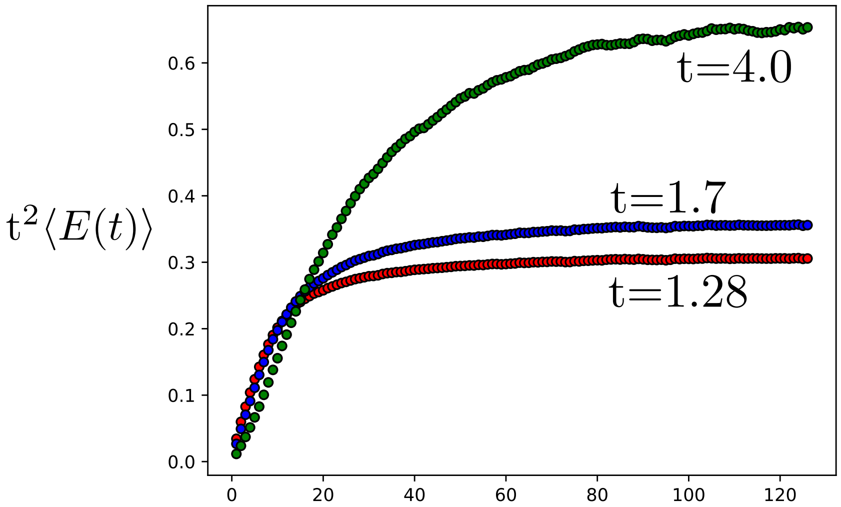
- ε’ remains a promising avenue to search for new physics, but greater precision is required.
- Primary pure-lattice systematic is discretization error (12%). Currently estimated using scaling of I=2 operators but there may be significant “error on the error”.
- Near-term availability of next-gen supercomputers (Perlmutter, Aurora) opens up opportunity to perform a full continuum extrapolation.
- Generate two additional ensembles with following properties:
  - Physical pion and kaon masses.
  - Same gauge action allowing continuum extrapolation with 3 points.
  - Same physical volume and G-parity BCs such that ππ energy remains the same and the interaction remains physical.
- 40^3x64 , a^{-1}=1.7 GeV and 48^3x64, a^{-1}=2.1 GeV
  are computationally feasible while providing a good lever arm (a^2 scaling).

The Next Big Thing pt.2

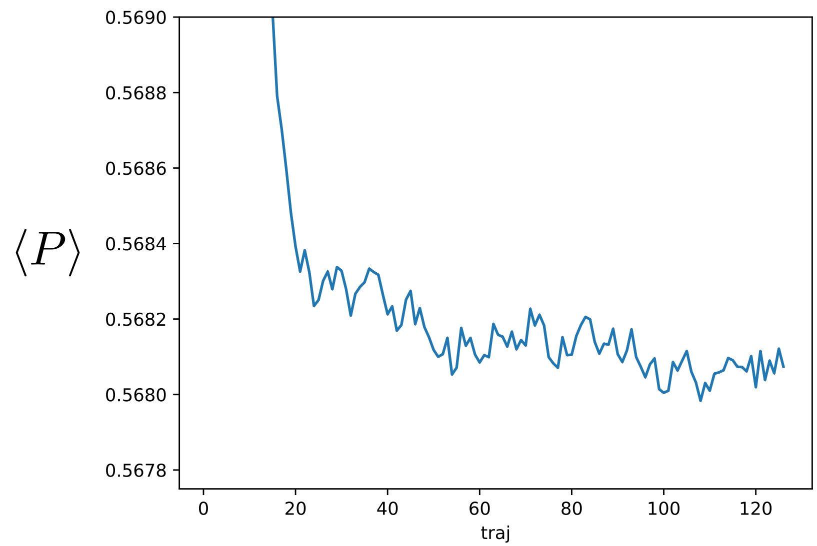
- Ensemble generation has already commenced on Perlmutter and ensembles are nearly thermalized.

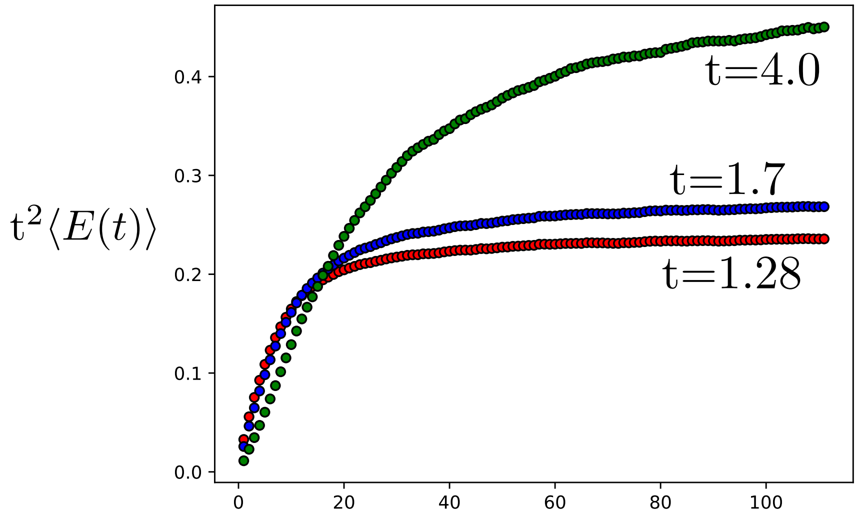
40^3x64





48^3x64





Other work in progress

- Independent calculation of ϵ' using multiple operators to extract on-shell matrix elements as excited-state contributions in a periodic lattice is well under way. [cf. next talk by M.Tomii]

  We are developing techniques to perform 3-4f matching in the Wilson coefficients non-perturbatively in order to avoid relying on PT at the charm scale. [PoS LATTICE2018, 216][PoS LATTICE2019, 174]

- Also working on laying the groundwork for the lattice calculation of EM contributions. [PoS LATTICE2021, 312]

Conclusions

- The RBC & UKQCD collaborations have performed a precise *ab initio* Standard Model calculation of the $K^0 \to \pi\pi$ decay amplitudes
- Reproduce experimental value for *ΔI*=1/2 rule, demonstrating that QCD sufficient to solve this decades-old puzzle.
- Subsidiary calculation of I=0 and I=2 s-wave ππ scattering phase shifts with physical pion masses agrees well with dispersive predictions.
- Result for ε' consistent with experimental value and total error is ~3.6x that of experiment.
- Continued work is being performed to further reduce error for a more precise test of the Standard Model.