$K \to \pi\pi$ decay matrix elements at physical points with periodic BCs

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K → ππ & Direct CPV

CP odd CP even

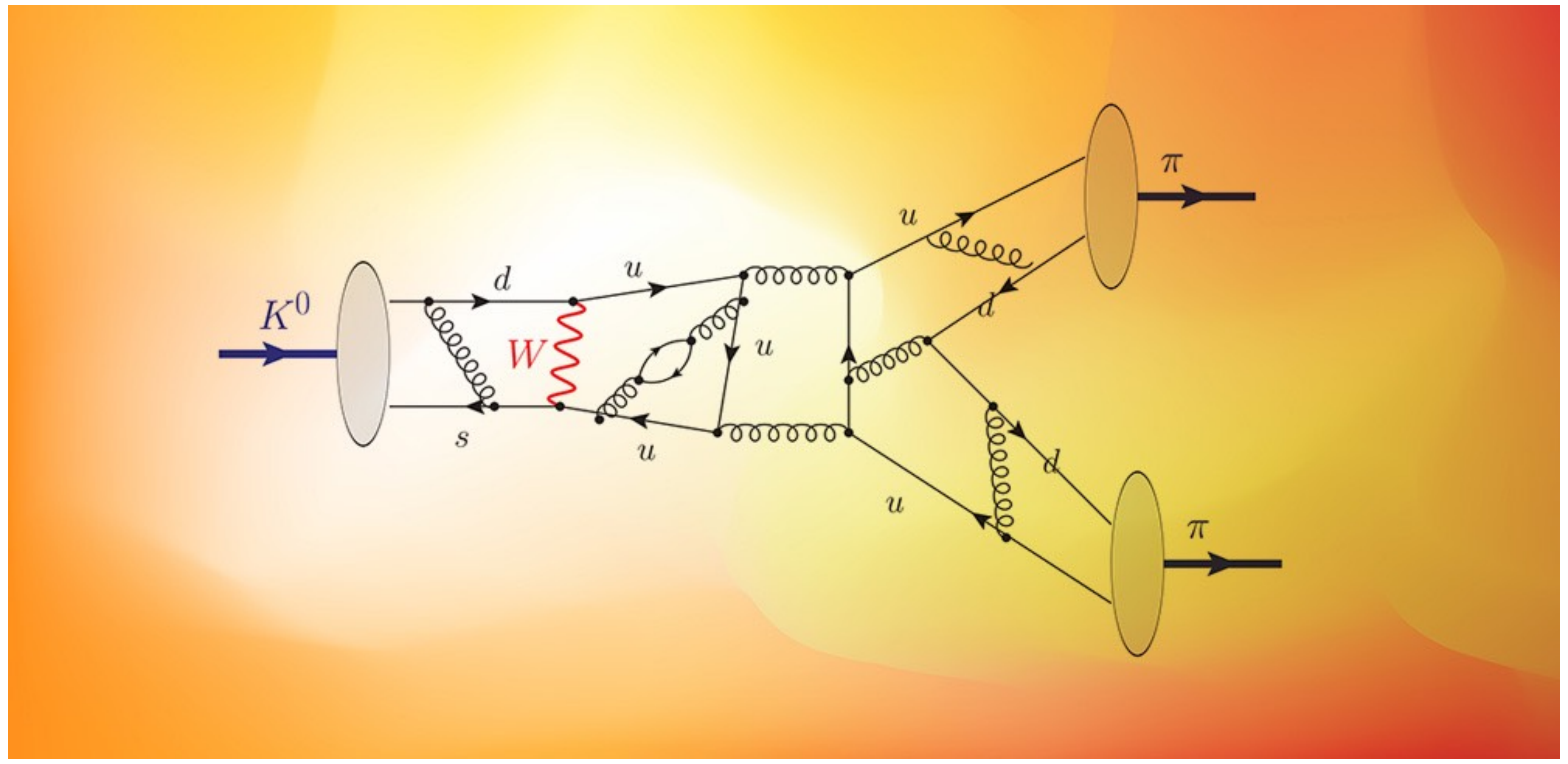
$$|K_L\rangle = |K_2\rangle + \varepsilon\,|K_1\rangle$$

ε′ direct CPV

ε indirect CPV

$|\pi\pi\rangle$

CP even

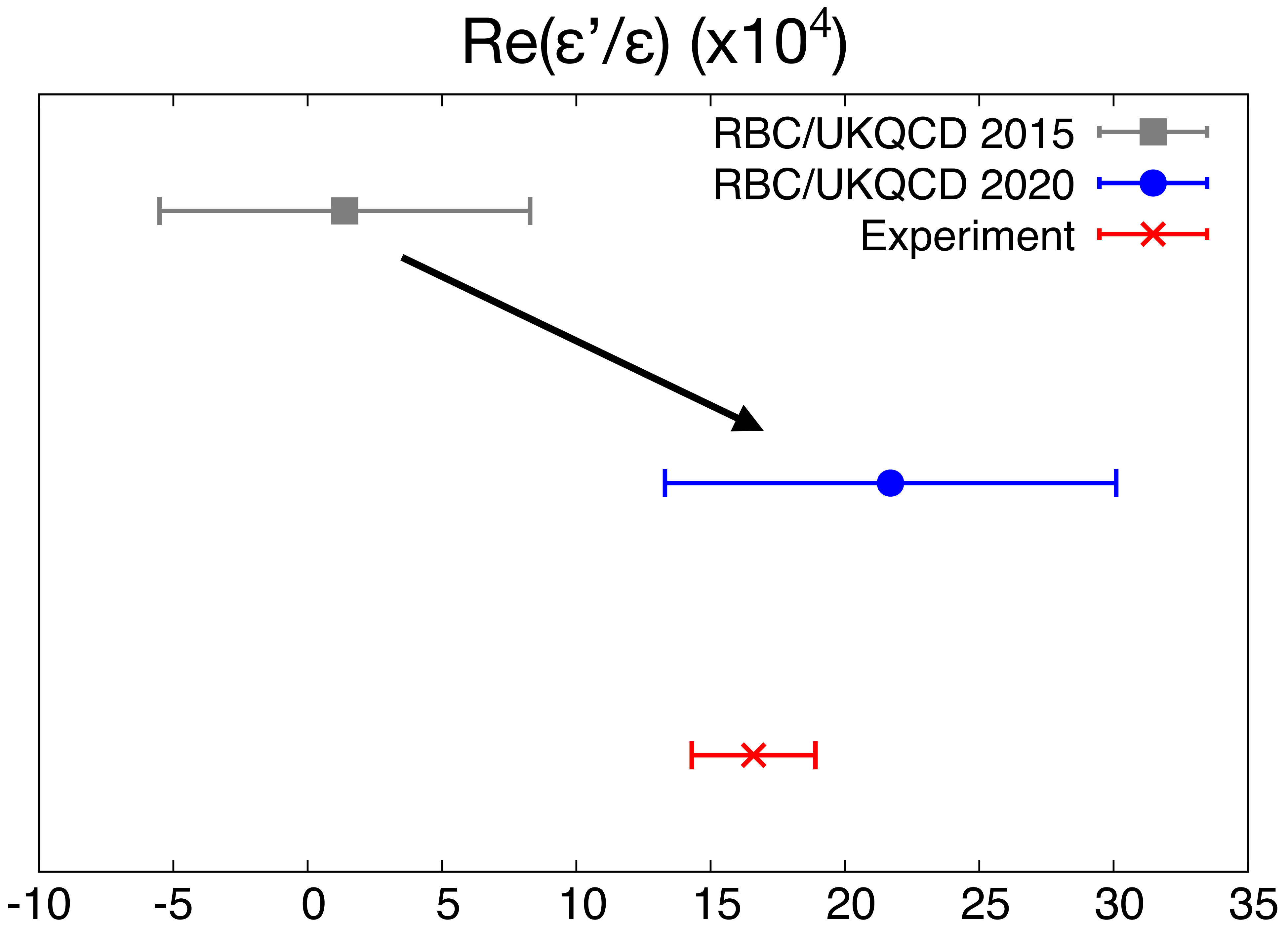


- ε’ vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (KTeV, NA48)
  - Explained by SM?
- Key to understanding the nature of matter/anti-matter asymmetry

G-parity BC calculation done

- $E_{\pi\pi} = 2m_\pi \approx 280$ MeV state in Euclidean correlators forbidden
- Useful to extract $E_{\pi\pi} = m_K$ state at large time separations

- C. Kelly just talked
- PRD 102,054509



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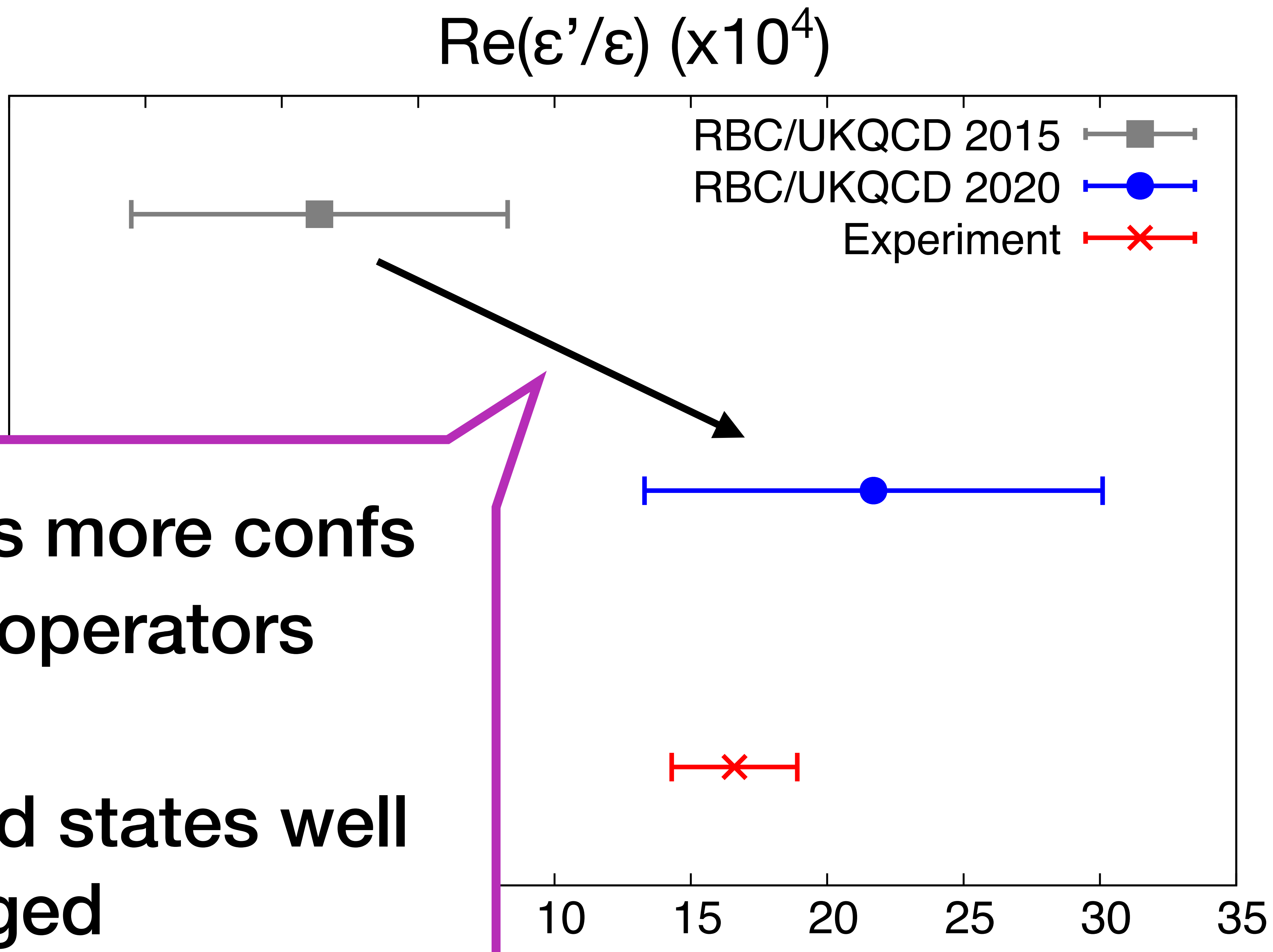

- 3+ times more confs
- # of ππ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

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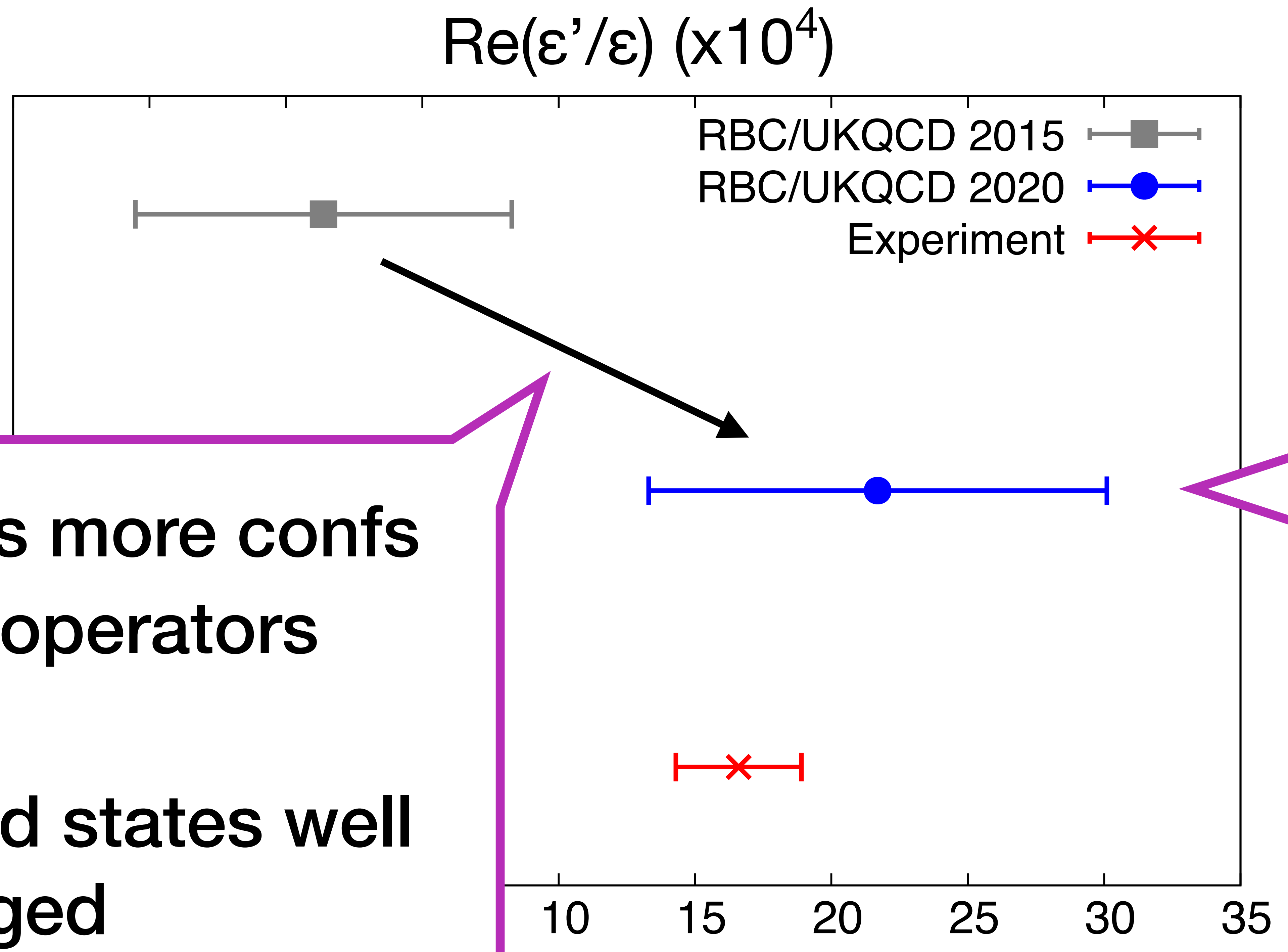

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- Step scaling in NPR

$$21.7(2.6)_{\mathrm{stat}}(6.2)_{\mathrm{sys}}(5.0)_{\mathrm{EM/IB}} \times 10^{-4}$$

- Independent calculations desired
- Systematic errors
  - Isospin breaking effects
  - Finite lattice cutoff
  - Wilson coefficients

Why periodic BCs?

- Already have lattice ensembles with physical pion mass
  - 1 GeV, 24^3 x 64, 1.4 GeV, 32^3x64 and …
  - Continuum limit possible
- Hope to introduce QED/IB effects near future
  - Difficult with G-parity BCs
  - Straightforward with periodic BCs (J. Karpie's talk at 11AM ET)
- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as in G-parity BCs: $\sim e^{-(m_K - 2m_\pi)t}$
  - interesting to see if it's possible to extract signal of excited state

Lattice setup

- RBC/UKQCD's 2+1-flavor ensembles with MDWFs at physical pion & kaon masses
  - 24^3 x 64, a^{-1} = 1.0 GeV, ~250 confs
  - 32^3 x 64, a^{-1} = 1.4 GeV, ~200 confs (measurement on going)
- Chiral symmetry of DWFs → strong constraints on operator mixings
  - with lower-dimensional operators
  - among different representations WRT chiral symmetry (8,1), (8,8) & (27,1)

What to calculate

ππ phase shifts

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction

Renormalization matrix

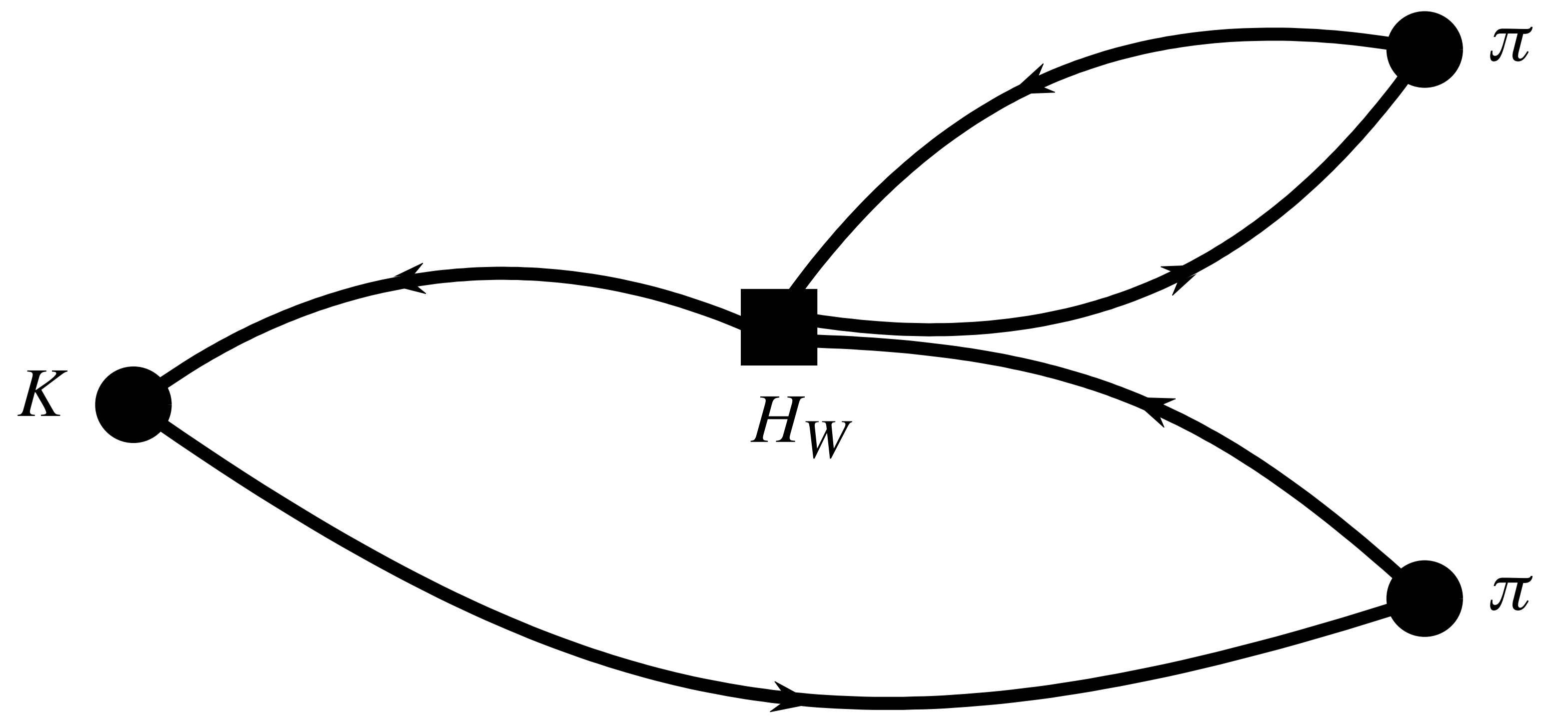
$$A_I = F\frac{G_F}{2}V_{us}^* V_{ud}\sum_{i,j}\underbrace{[z_i(\mu)+\tau y_i(\mu)]}_{\text{Wilson coefs. pQCD}}\underbrace{Z_{ij}(\mu)}_{\text{LQCD (+pQCD)}}\underbrace{\langle(\pi\pi)_I|Q_j^{\mathrm{lat}}|K\rangle}_{\text{LQCD}}$$

- δ_I, F being determined via π-π scattering work w/ GEVP & Lüscher formalism
- A_I = < $(\pi\pi)_I$ | H_W | K > from 3pt correlation functions
- I = 0 challenging — disconnected diagrams, power divergences

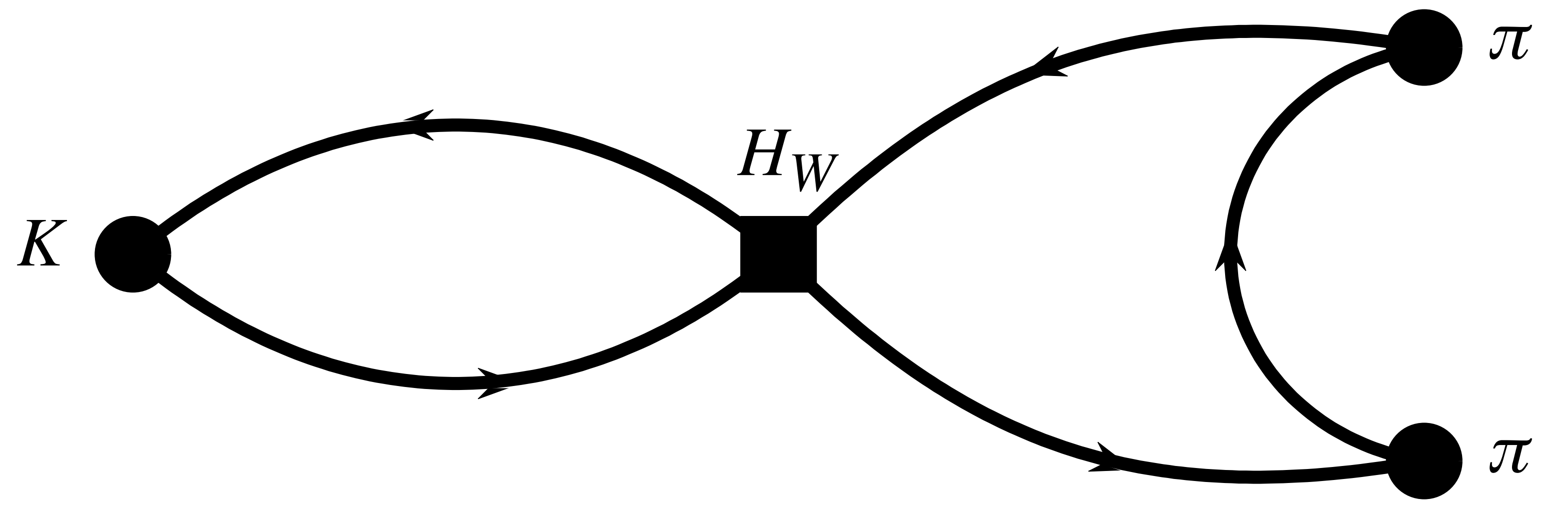
K → ππ calculation

- 258 configurations (on 24^3x64), physical pion & kaon masses
- All-to-all quark propagators
  - 2,000 low modes for light quarks (no low mode for strange)
  - high-mode part: spin, color and time dilutions => 768 inversions
- 28 (5 independent) interpolation ππ operators
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$, $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$, $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$, $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$ & σ
  - to control effects from various states

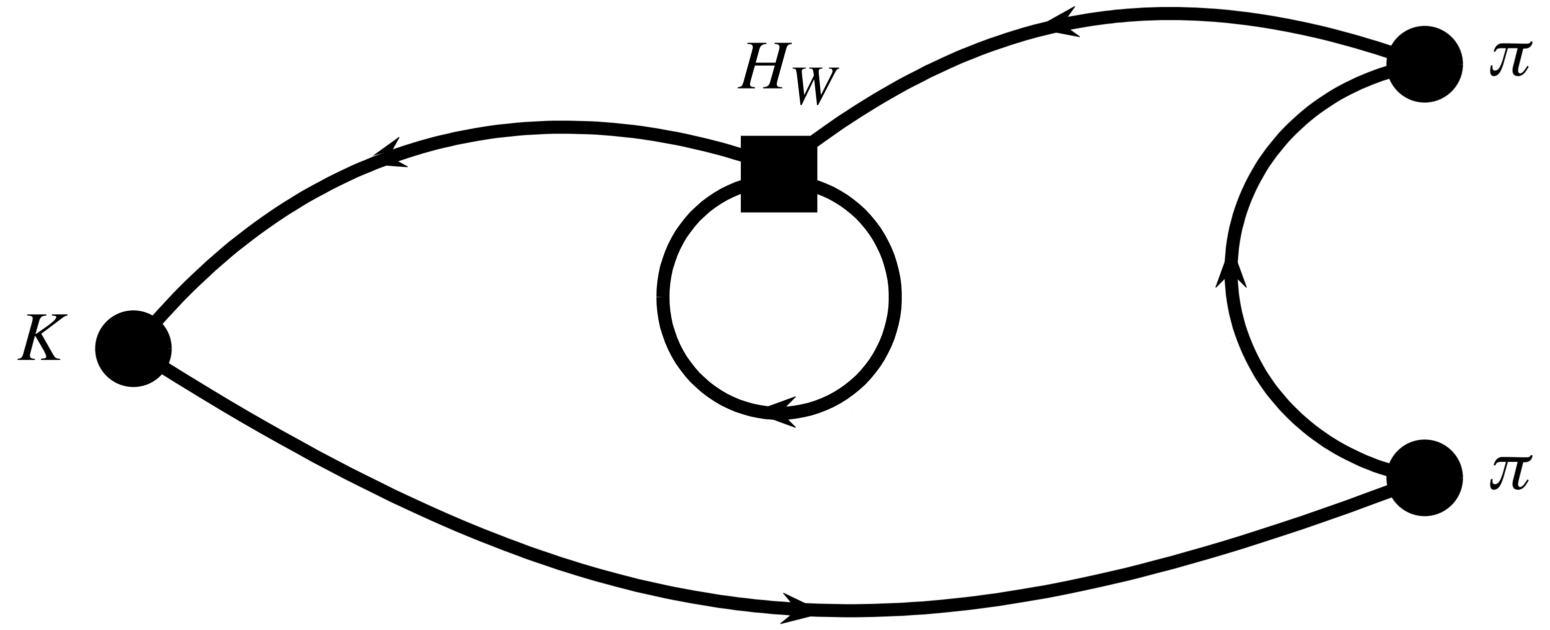
4 types of diagrams



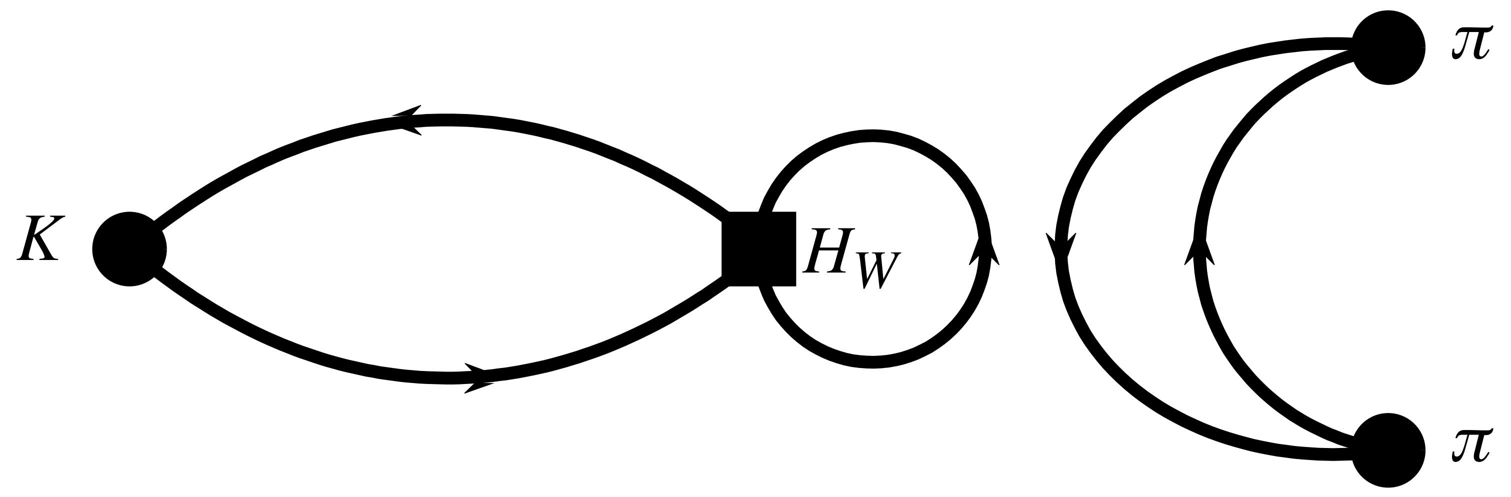
type 1



type 2



type 3



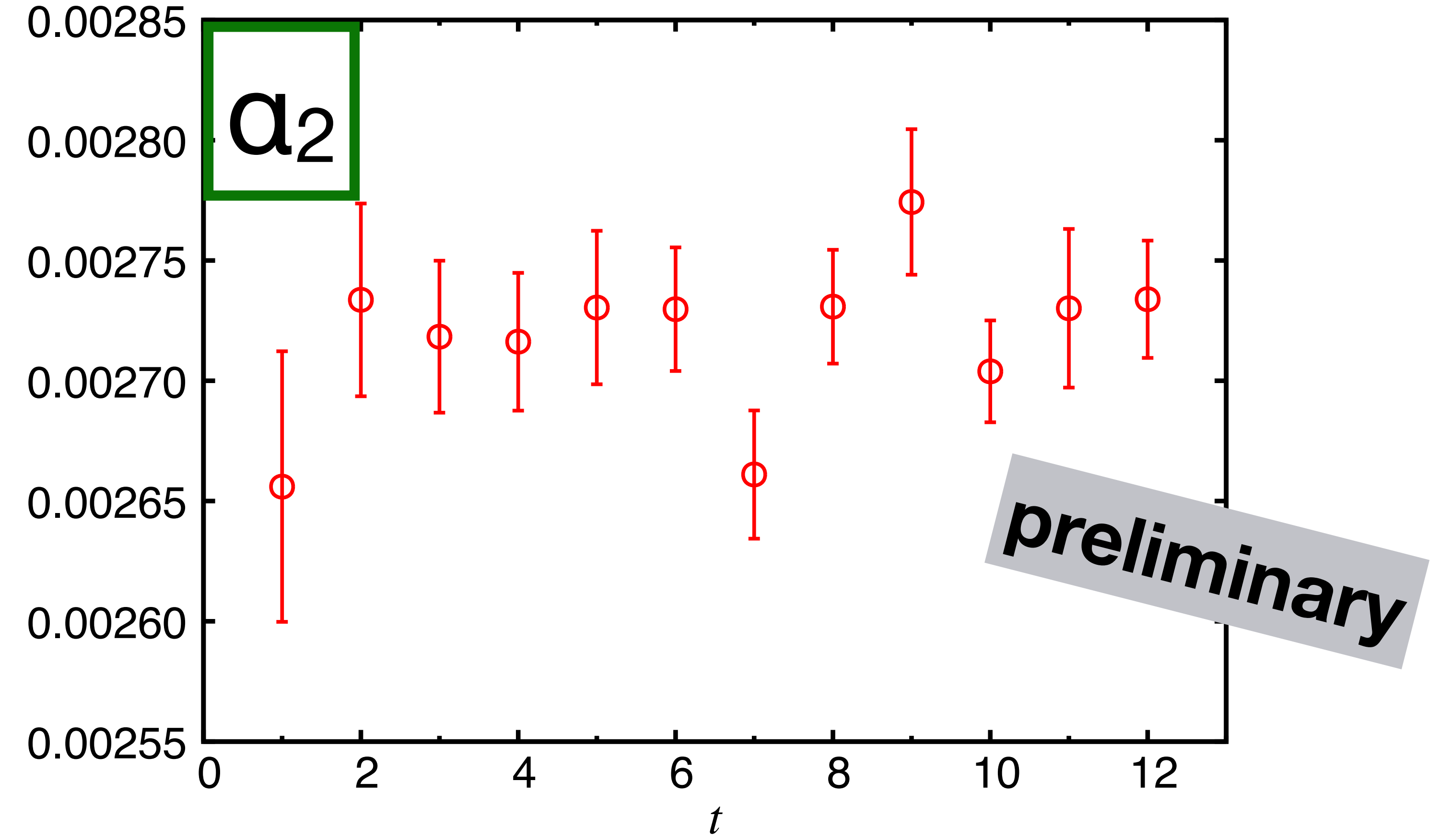
type 4

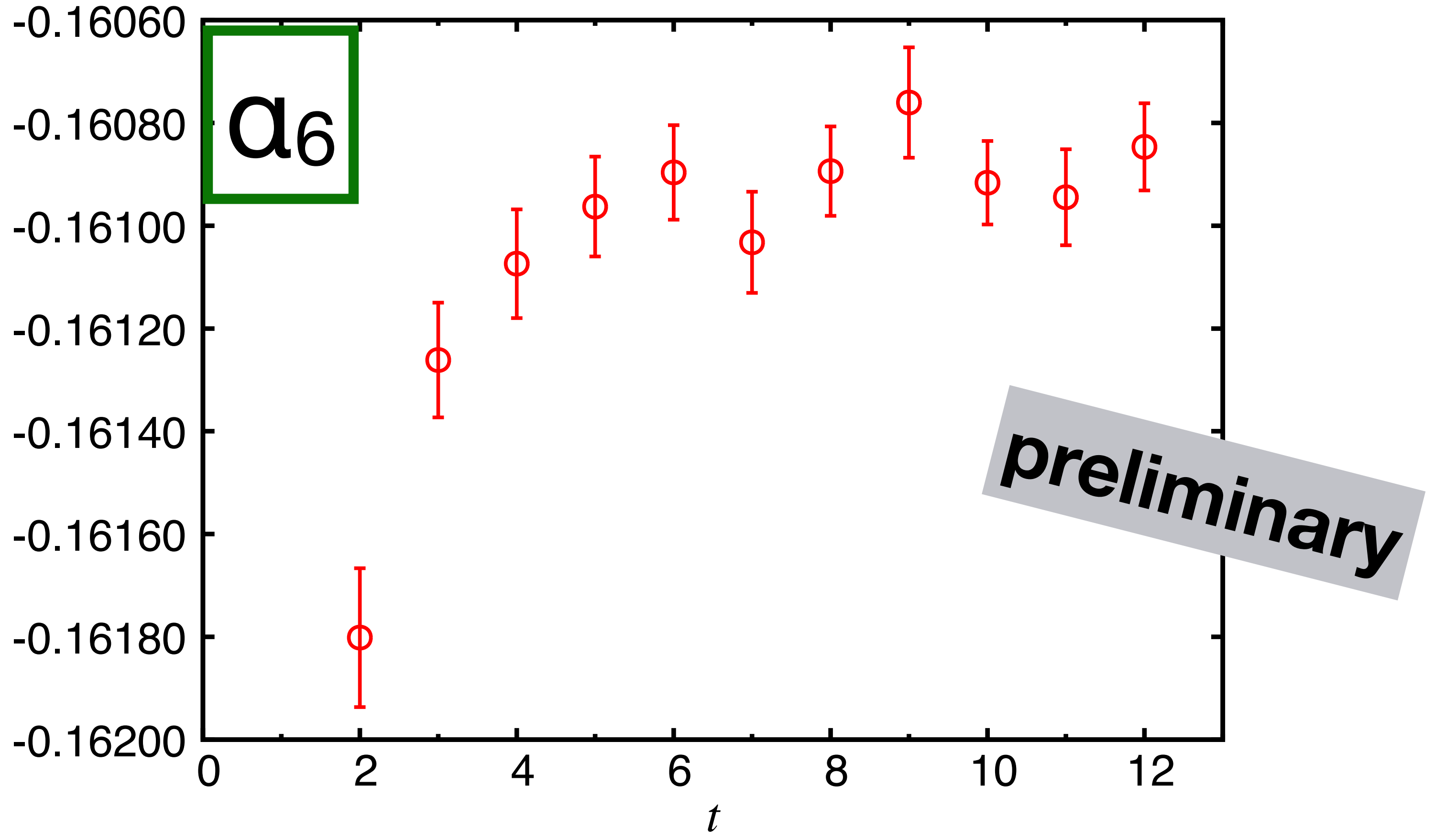
Subtraction of quadratic divergence

- Loop diagrams (types3,4) have a^{-2} divergence due to mixing with bilinear operators

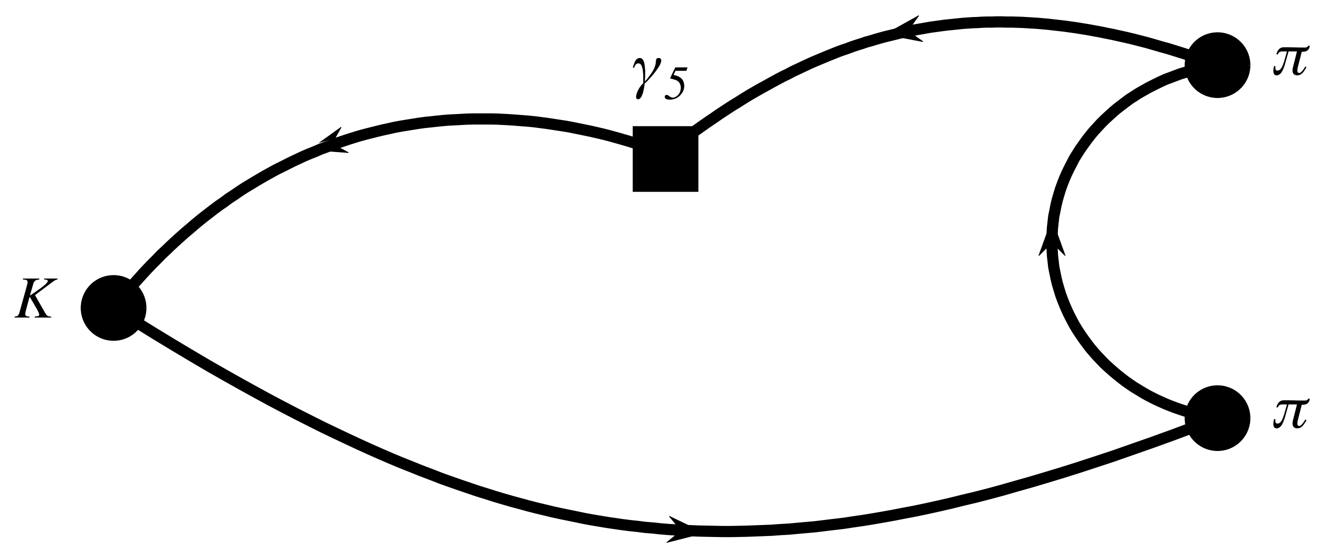
- Subtraction
  - $Q'_i = Q_i - \alpha_i \, \bar{s}\gamma_5 d$
  - Condition: $\langle Q'_i(t) K(0)^\dagger \rangle = 0$

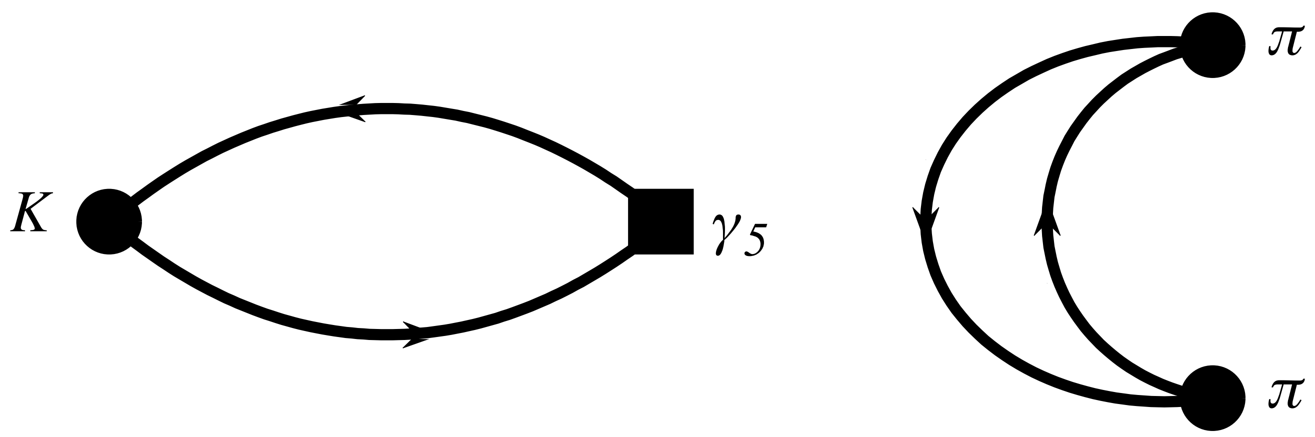
$$\boxed{\alpha_i = \frac{\langle Q_i(t) K(0)^\dagger \rangle}{\langle \bar{s}\gamma_5 d(t) K(0)^\dagger \rangle}}$$





- Additional contractions



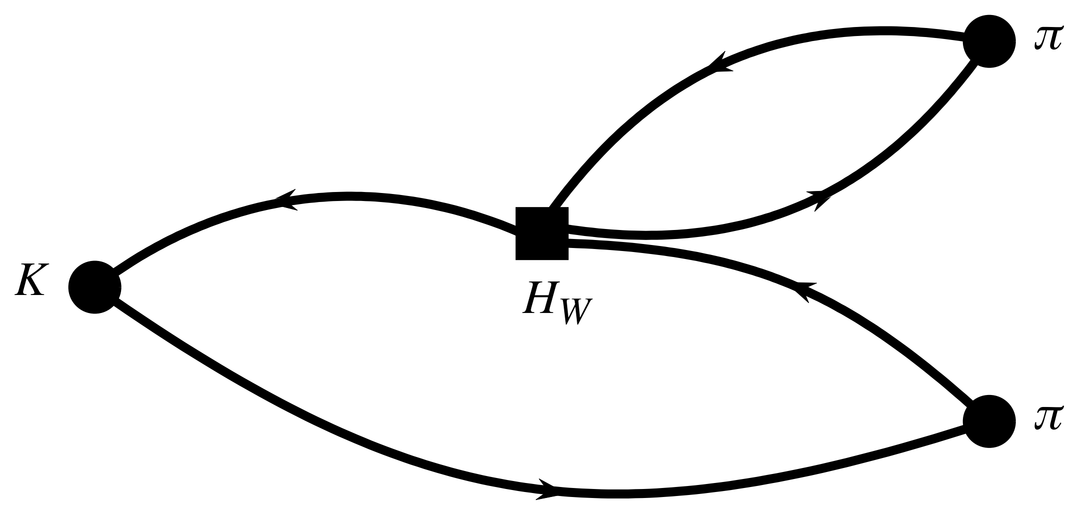


type 4 dominates stat. error

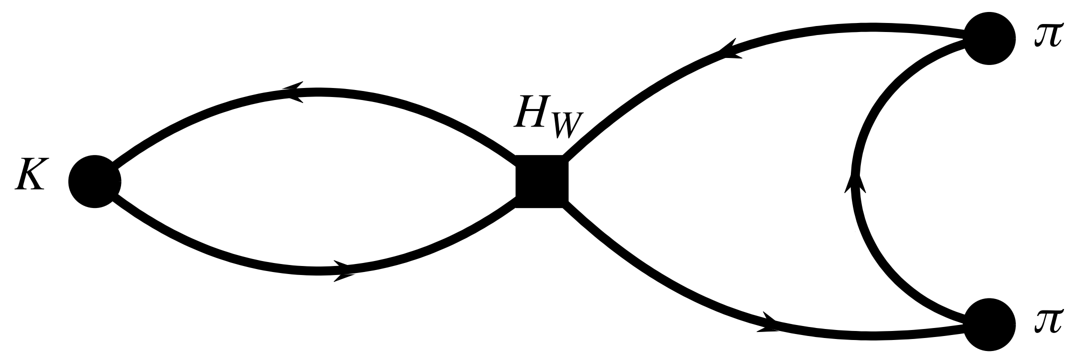
- Previous G-parity calculation
  - types 1,2: averaged over every 8 time translations
  - types 3,4: averaged over every time translation

- types 1,2 still expensive but no need of such precision

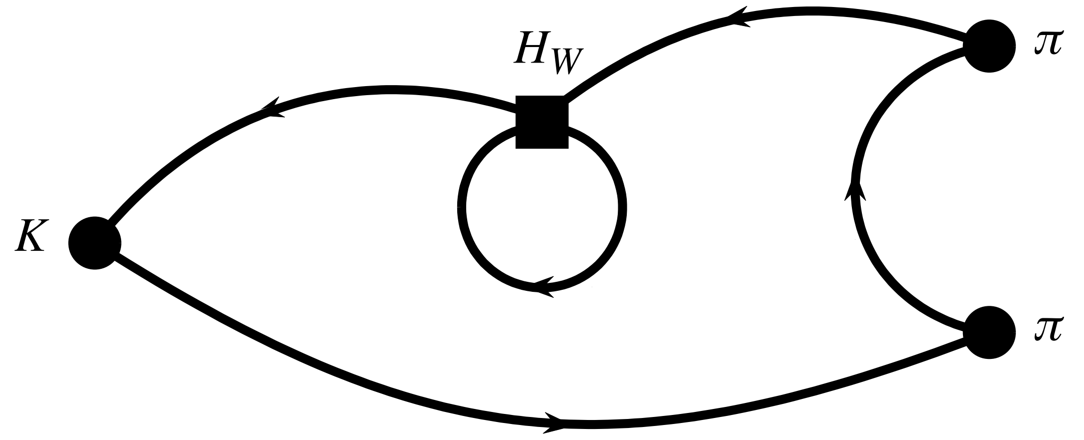
  → cost reduction?



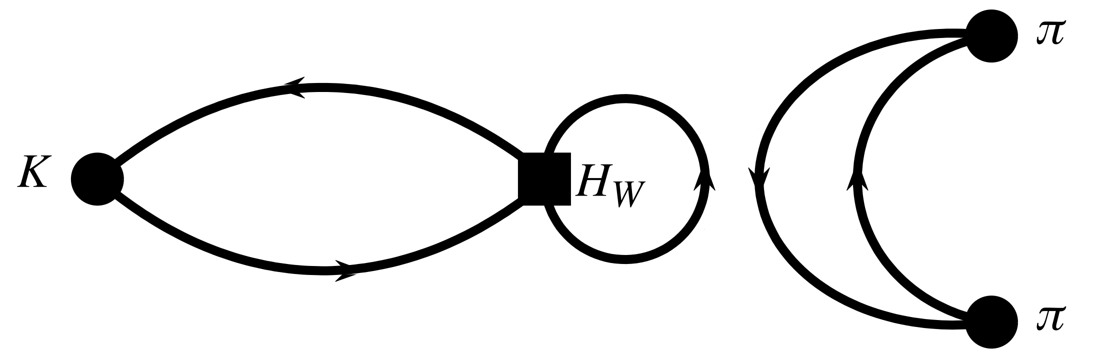
type 1



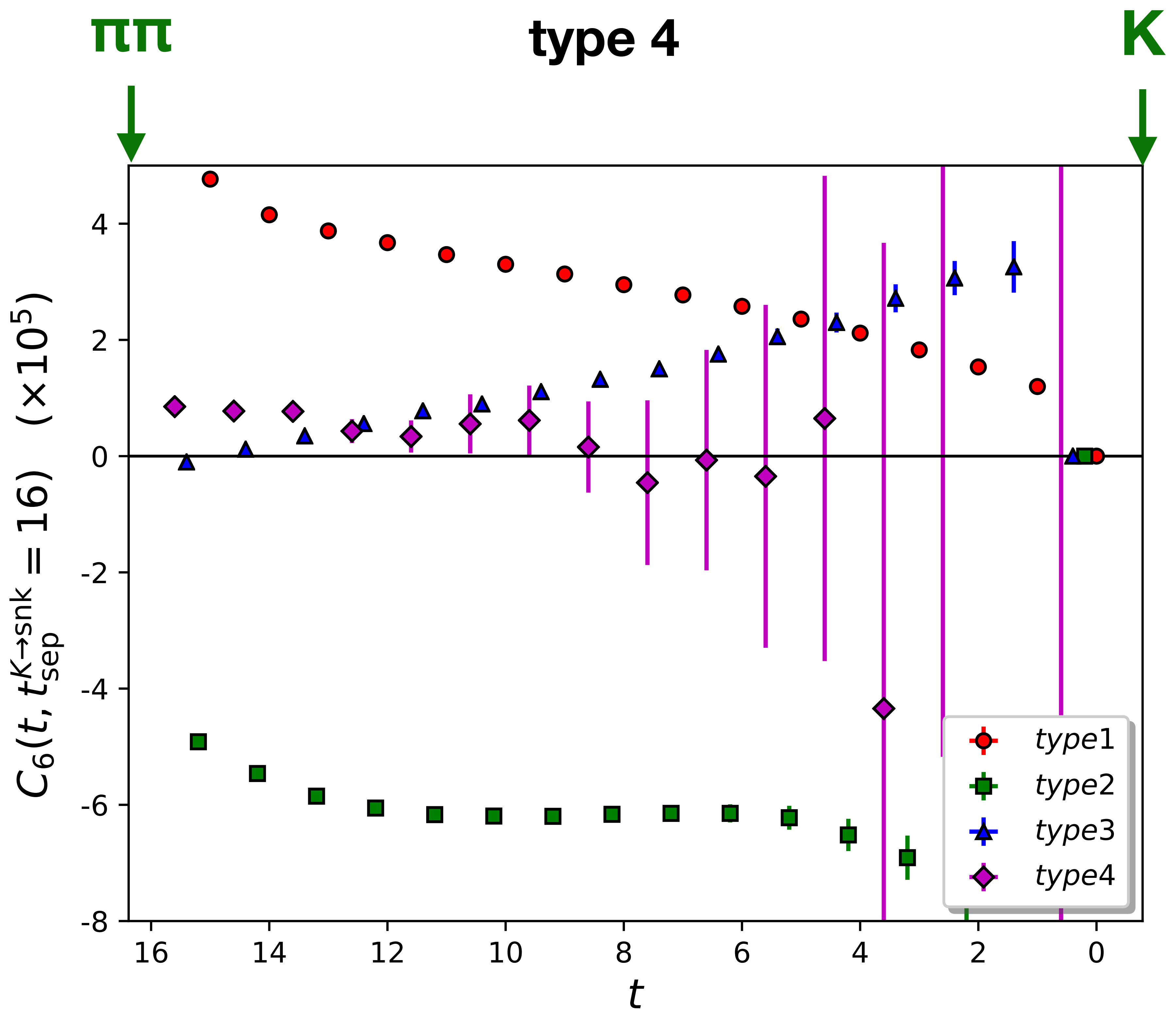
type 2



type 3



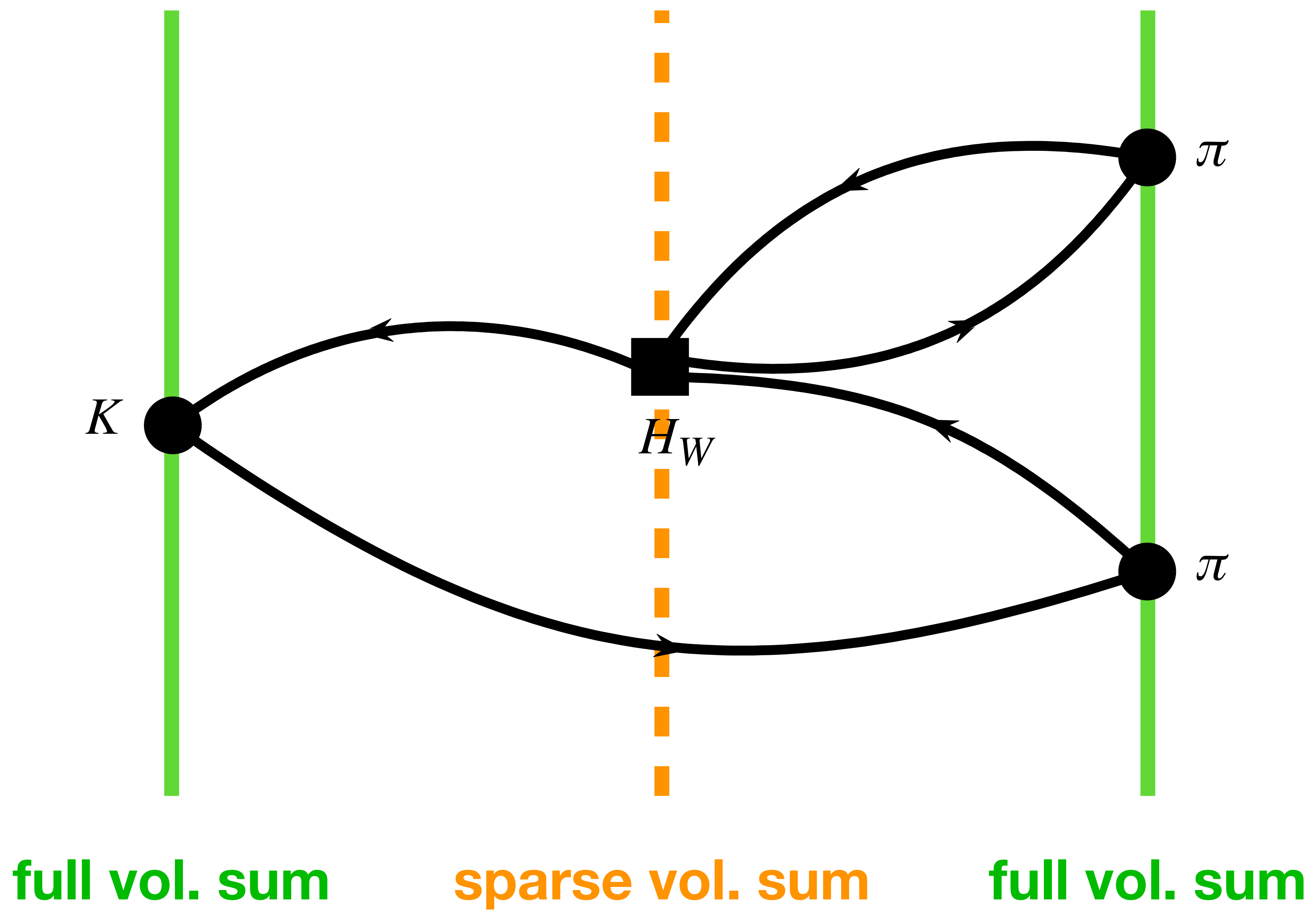
type 4



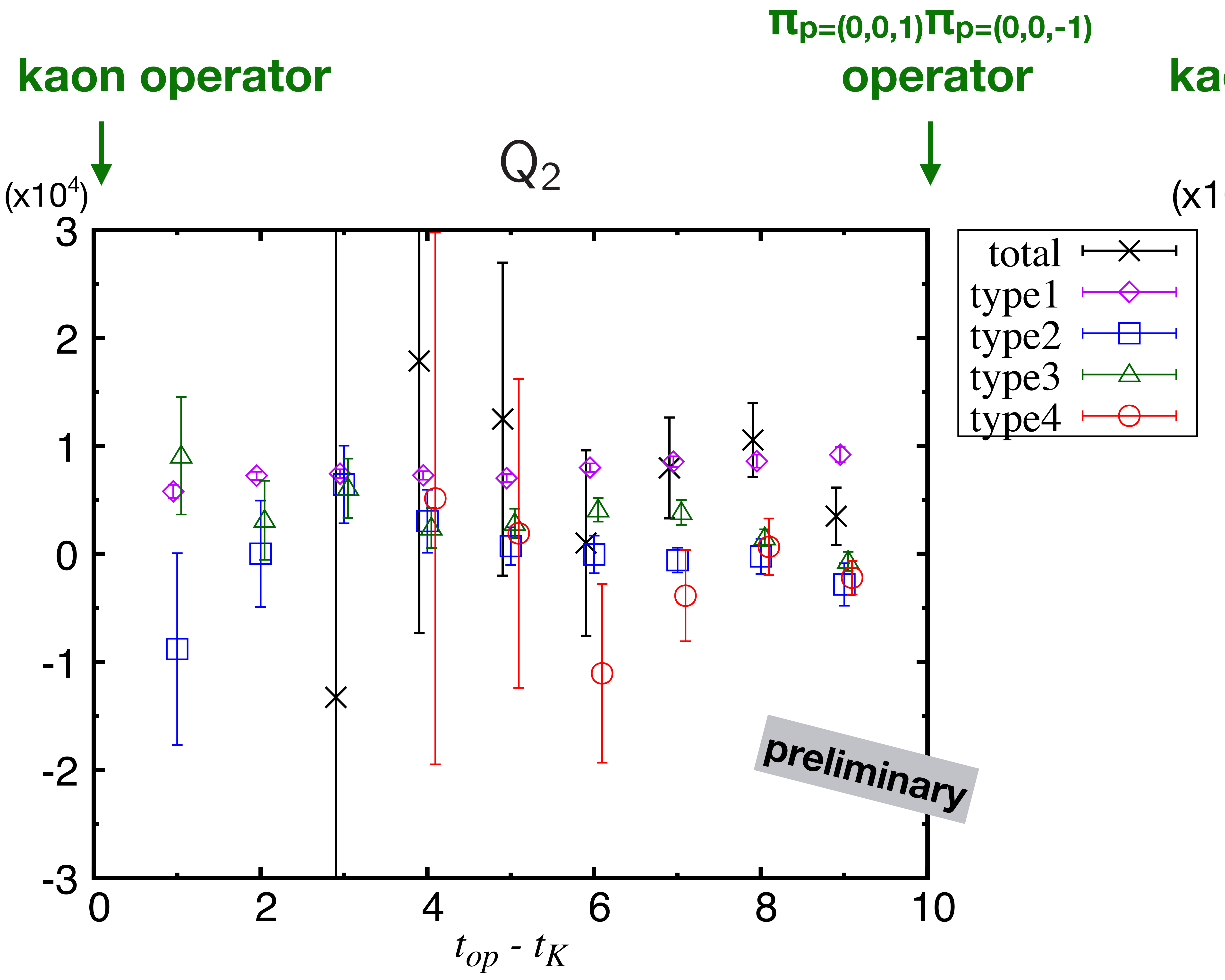
RBC/UKQCD, PRD 102,054509

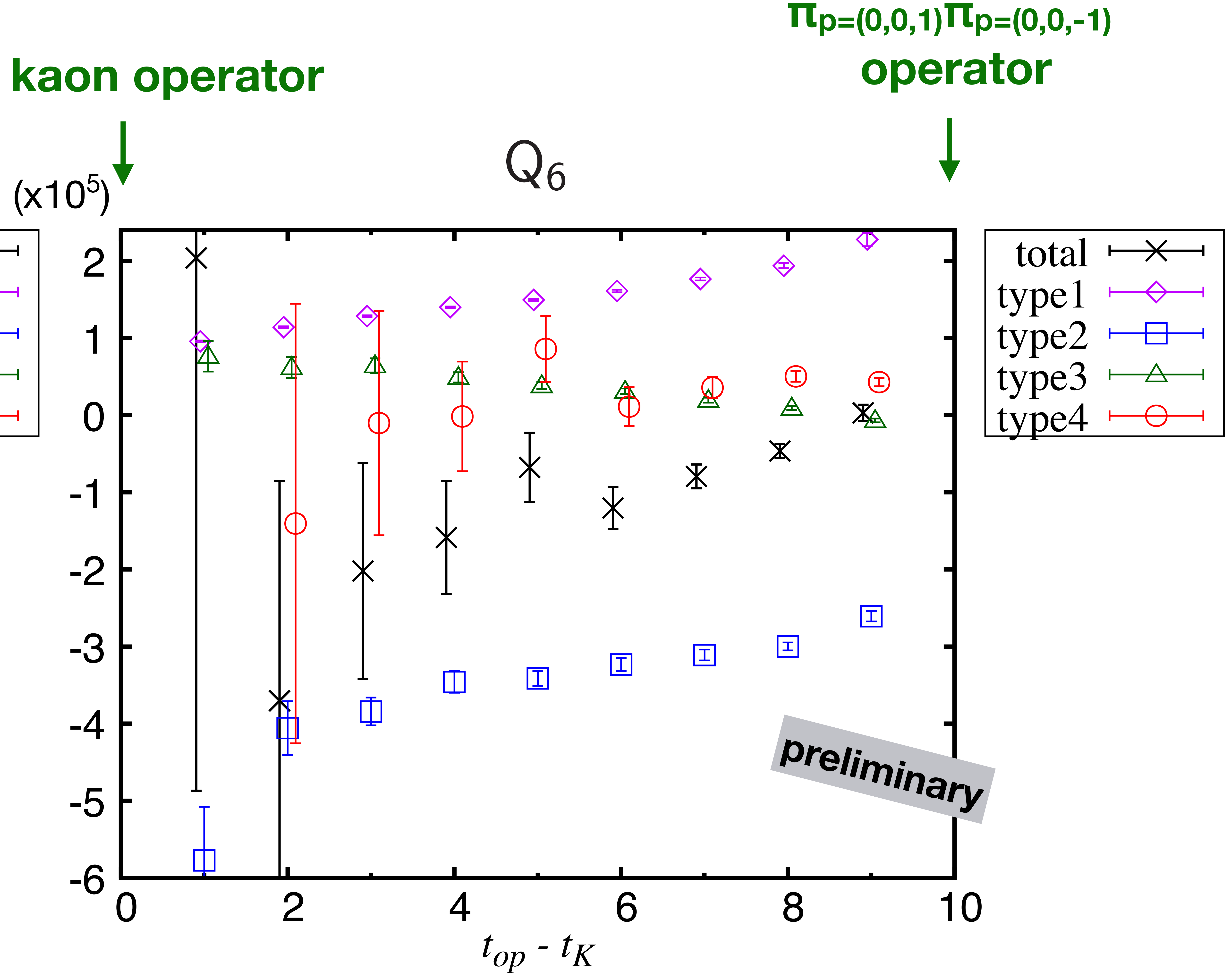
Sparsening H_W

- Contraction cost mostly promotional to volume of H_W
- G-parity calculation: summed H_W over whole 3D volume
- This calculation: volume of H_W ($24^3 \rightarrow 8^3$: 27x speed up) for types 1 & 2



I = 0 correlation functions





- Sparsen H_W for types1,2 – still much more precise than type4

State decompositions

- 2pt functions of interpolation operators:

$$C_{ab}(t) = \langle O_a(t) O_b(0)^\dagger \rangle = \sum_n A_{n,a} A^*_{n,b} e^{-E_n t}$$

- Good combinations of interpolation operators:

$$O_a \to O'_n = \sum_a v^*_{n,a} O_a$$

$$C'_n(t) = \sum_{a,b} v^*_{n,a} C_{ab}(t) v_{n,b} = A'_n A'^*_n e^{-E_n t} + O(e^{-(E_{N+1} - E_n)t})$$

- $v_{n,a}$ well determined by solving GEVP (Generalized Eigenvalue Problem)

$$C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0)$$

Before solving GEVP...

- More precise evaluation of 2pt functions on the lattice

$$C_{ab}(t) = \sum_n A_{n,a} A^*_{n,b} e^{-E_n t}$$ – assumed in the previous slide

$$+ \sum_n A_{n,a} A^*_{n,b} e^{-E_n (T-t)}$$ – negligible for $E_0(T - 2t) \gtrsim 3$

$$+ \langle O_a \rangle \langle O_b \rangle$$ – needs to be subtracted for I = 0

$$+ \langle \pi | O_a | \pi \rangle \langle \pi | O_b | \pi \rangle e^{-E_\pi T} + \dots$$ – also needs to be subtracted

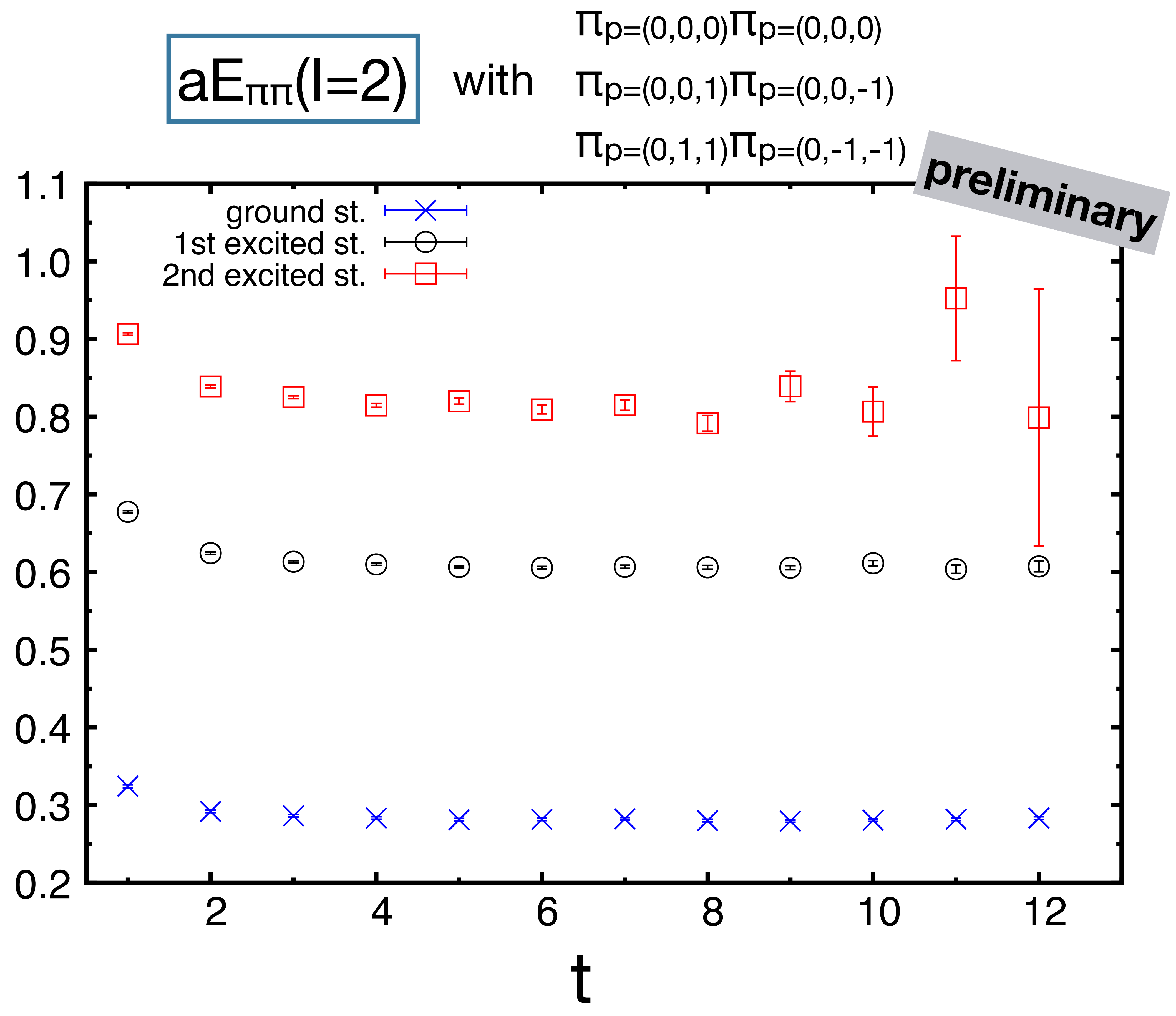
- Subtraction of thermal effects

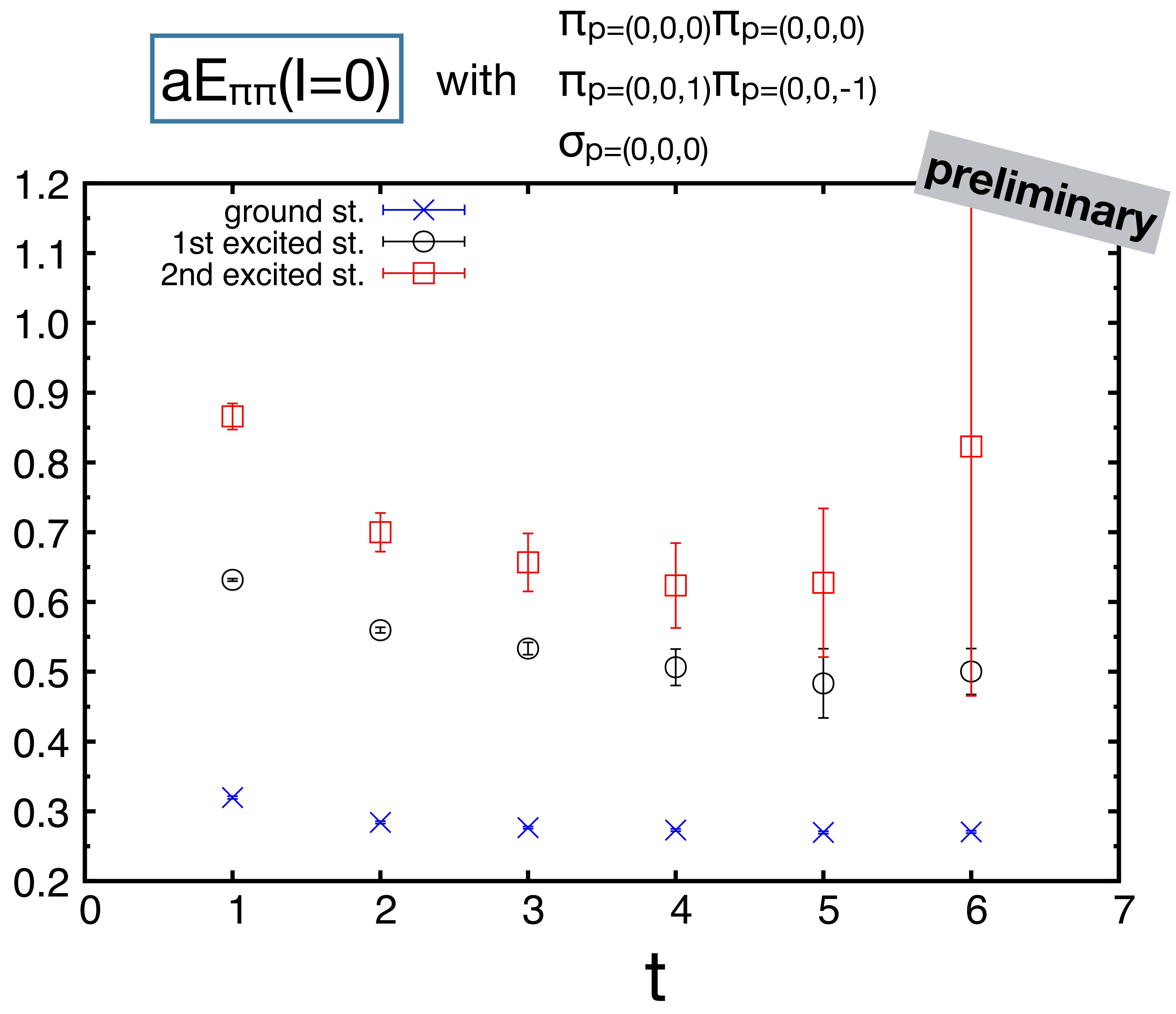
$$C_{ab}(t) \to C_{ab}(t) - C_{ab}(t + \delta t) = \sum_n (A_{n,a} - e^{-E_n \delta t/2})(A_{n,b} - e^{-E_n \delta t/2})^* e^{-E_n t}$$

ππ states

- Effective ππ energies from 2pt functions
- $t_0 = t - 1$

Led by D. Hoying





Effective matrix elements

- 3pt functions

$$C'^{(3)}_{i,mn}(\underbrace{t_{snk} - t_{op}}_{t_2}, \underbrace{t_{op} - t_{src}}_{t_1}) = \langle O'_{snk,m}(t_{snk}) Q_i(t_{op}) O'_{src,n}(t_{src})^\dagger \rangle$$

$$= A'_{snk,m} M_{i,mn} A'^{*}_{src,n} e^{-E_{snk,m} t_2} e^{-E_{src,n} t_1} + (\mathrm{N+1th\ state})\text{'s effect}$$

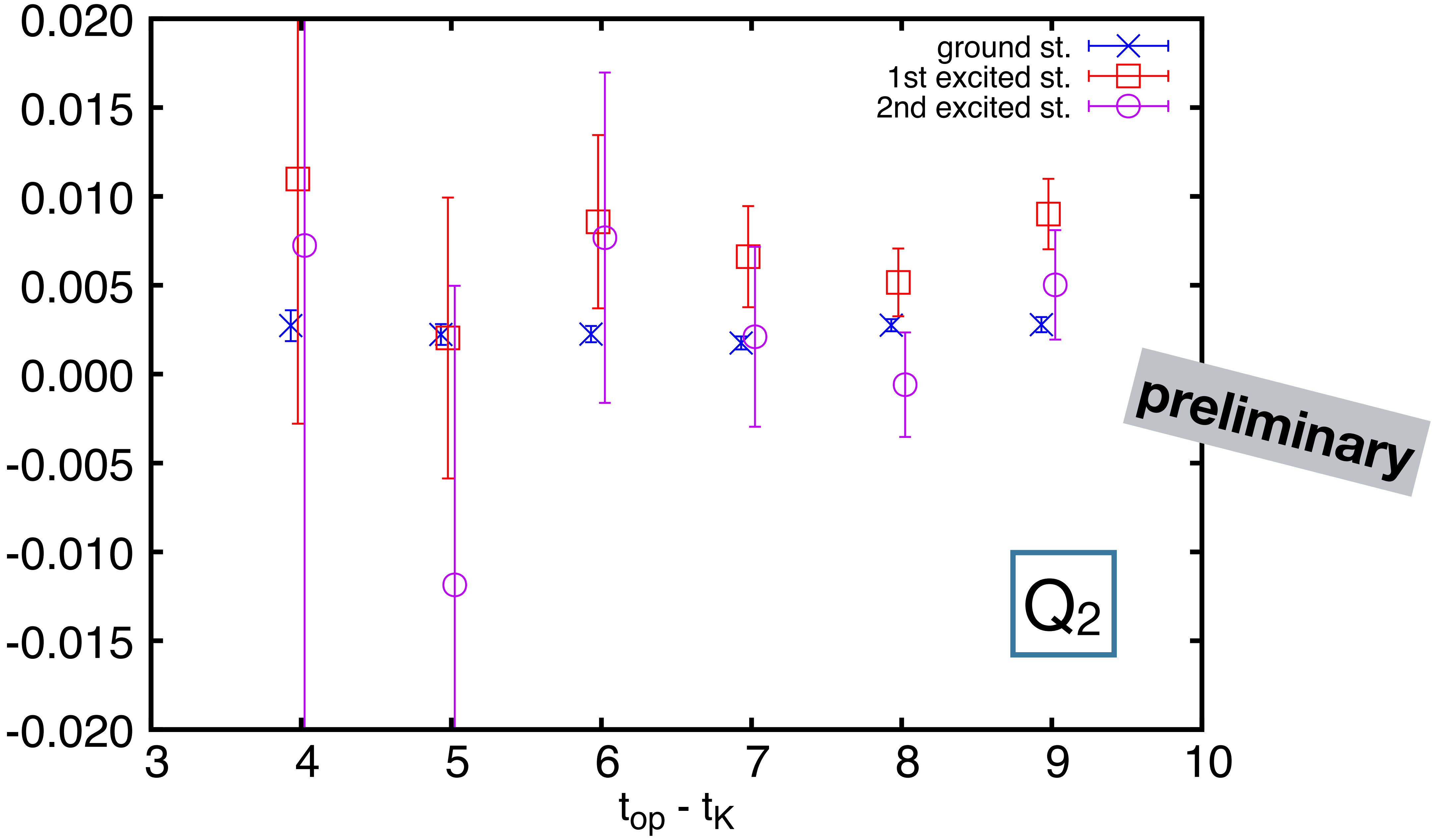
- Effective matrix elements

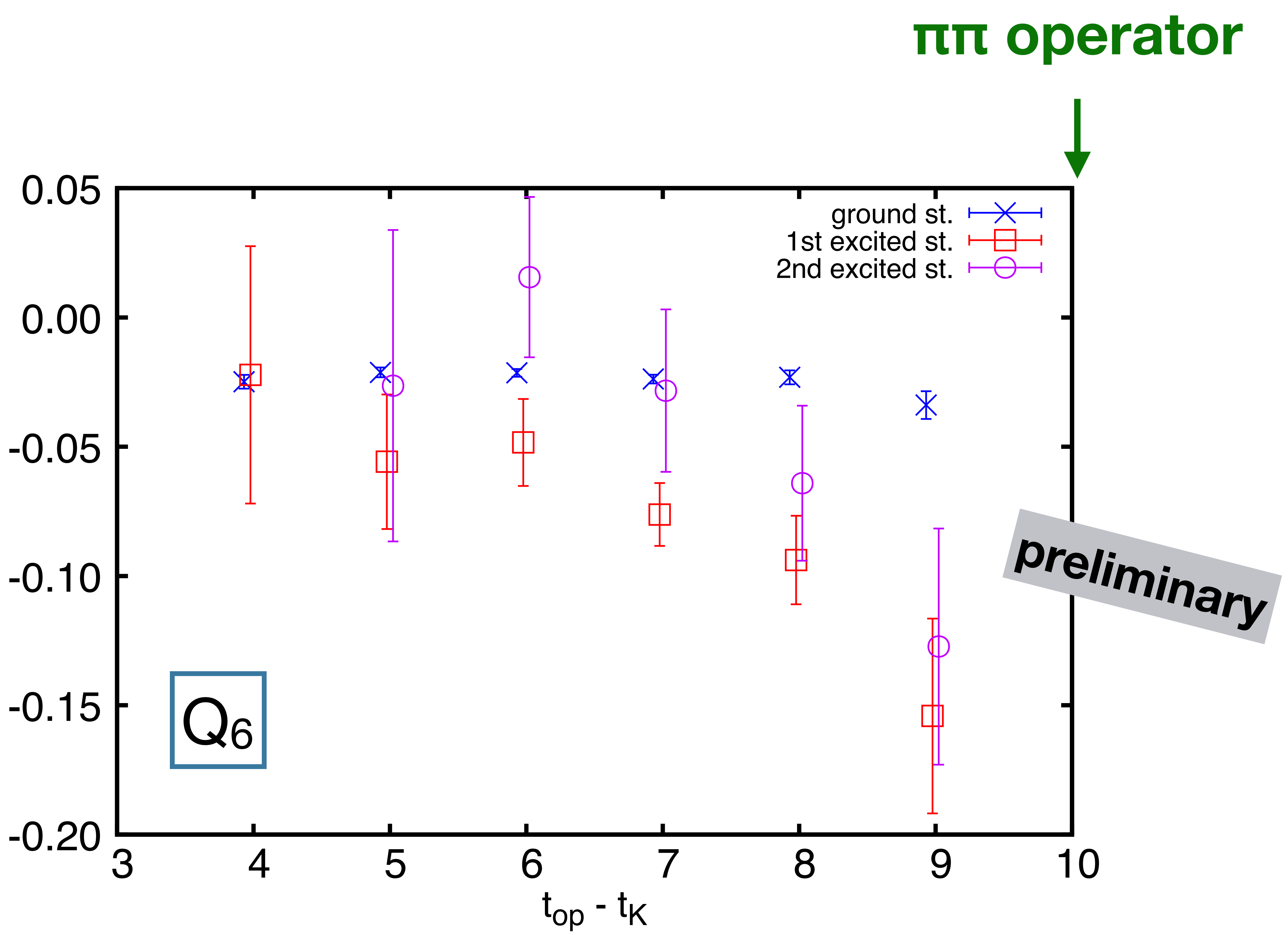
$$M^{eff}_{i,mn}(t_2, t_1) = C'^{(3)}_{i,mn}(t_2, t_1) \left[\frac{e^{E_{snk,m} t_2} e^{E_{src,n} t_1}}{C'_{snk,m}(t_2) C'_{src,n}(t_1)} \right]^{1/2}$$

- We employ 1 kaon operator & multiple ππ operators

Effective matrix elements ($I = 0$)

- Plateau appears
- Further improvement being considered
  - sGEVP (Alpha Collab. *JHEP* 01 (2012) 140)
  - fit to all data (6 numbers of $t_{\pi\pi}$-t_K)





Summary

- Purpose
  - New independent calculation of $K \to \pi\pi$ decays
  - Periodic-BC study gives prospect of introducing QED/IB effects
- On-shell final state = 1st excited state
  - Interesting challenge to extract signal of excited state
  - GEVP works
- Various analyses being considered for better signal