S-wave $\pi\pi$ I=0 and I=2 scattering with physical pion mass

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December 19, 2021

Why shall we perform this calculation?

- $K \to \pi\pi$ calculation
  - Need $E_{\pi\pi}$ and $A_{\pi\pi}$ to compute lattice matrix element M_{lat}
  - Need δ_0 and $\frac{d\delta_0}{dk}$ to compute LL factor
  - Understand $\pi\pi$ final state
- First calculation with physical pion mass and disconnected diagram
- A calculation on GPBC lattice
- 2015 results gives $\pi\pi$ energy which is 3σ(7σ with more statistics) higher than the phenomenological prediction(Pi-Pi puzzle)

$$\begin{aligned}\delta_0 &= 23.8(4.9)(1.2)^\circ(PRL, 2015)\\ &= 19.1(2.5)^\circ(1386 confs)\\ &\approx 36^\circ(Dispersion)\end{aligned}$$

- Possible reason: Excited state contamination for $\pi\pi$ state
- Solution: Introducing more operators

Calculation details and techniques

Operators list:

- Original $\pi\pi$ interpolating operator (O_a or $\pi\pi(111, 111)$):
  two single π operator with momentum $(\pm 1, \pm 1, \pm 1)\frac{\pi}{L}$ $[\pi(111)]$
- σ operator (O_c or σ):
  $\frac{i}{\sqrt{2}}(\bar{u}u + \bar{d}d)$. This operator has the same quantum number as $\pi\pi_{I=0}$
- Extra $\pi\pi$ operator (O_b or $\pi\pi(311, 311)$):
  two new π operator with one of their component replaced by $\pm\frac{3\pi}{L}$ $[\pi(311)]$

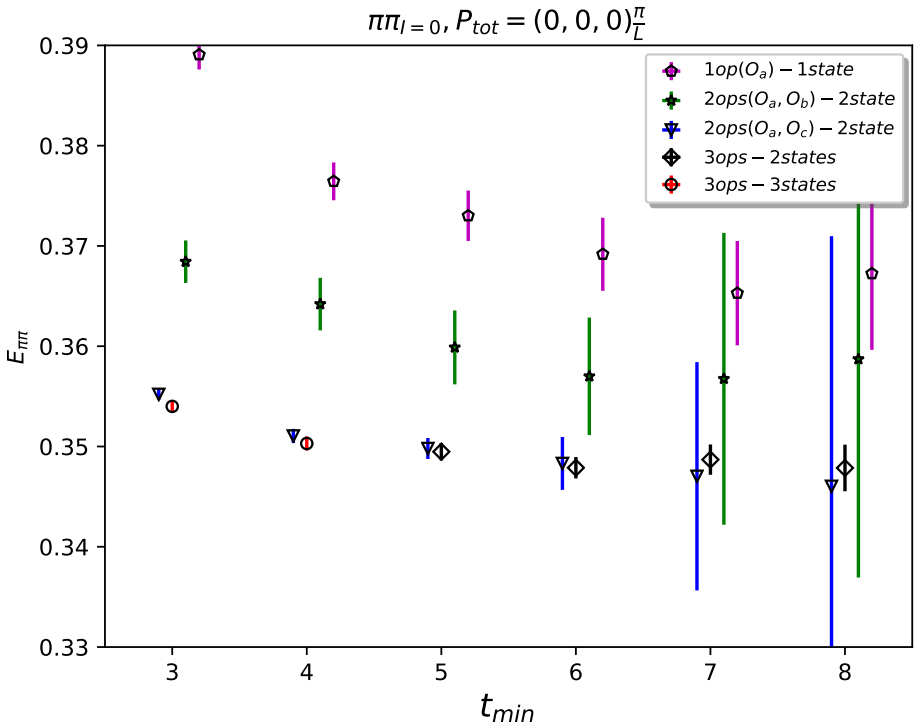
General techniques:

- G-parity boundary condition
- All to all propagator
- Non-overlapping blocked bootstrap [cf. 1911.04582]

Fitting strategies and results

$$C_{ij}(t_{snk}, t = t_{snk} - t_{src}) = \langle O_i^\dagger(t_{snk}) O_j(t_{src}) \rangle - \langle 0|O_i(t_{snk})|0\rangle \langle 0|O_j(t_{src})|0\rangle \times \delta_{I,0} \quad (1)$$

$$C_{ij}(t) = \sum_{x=1}^{N} A_{ix} A_{jx} \left(e^{-E_x t} + e^{-E_x (T-t)} \right) + B_{ij} \times \delta_{I,2} \quad (2)$$



- Non-negligible excited state contamination in $\pi\pi(111, 111)$ operator
- σ significantly suppresses both excited state error and statistical error
- $\pi\pi(311, 311)$ improves statistical error

Pi-Pi puzzle: Solution

- Normalized overlap matrix also supports this argument.

	state_0	state_1	state_2
$\pi\pi(111,111)$	1.0(0.0)	0.47(2)	0.31(7)
σ	1.0(0.0)	-0.83(3)	-0.87(22)
$\pi\pi(311,311)$	0.053(9)	-0.84(12)	1.0(0.0)

- We decide to include all operators

$$\begin{aligned}\delta_0 &= 23.8(4.9)(1.2)^\circ(PRL, 2015, 1op)\\ &= 19.1(2.5)^\circ(1386confs, 1op)\\ &\approx 36^\circ(Dispersion)\\ &= \mathbf{32.3(1.0)^\circ}(741confs, 3op)\end{aligned}$$

- Better control over excited state error
- Better statistical error with smaller number of confs

Moving frame (non-zero total momentum)

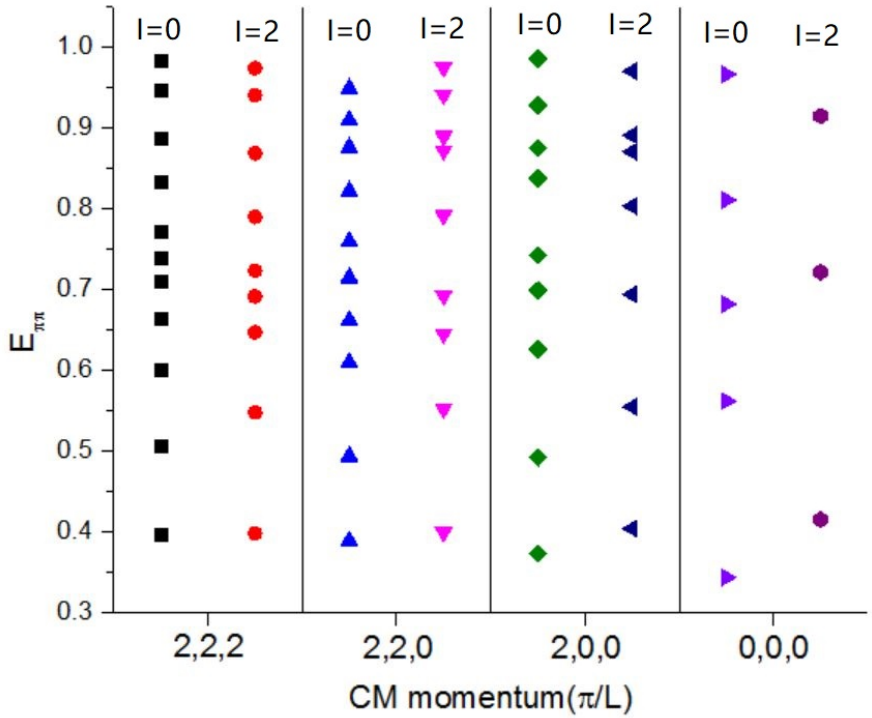
- Two sets of π operator ($\pi(111)$ and $\pi(311)$) allow us to construct $\pi\pi$ operator with non-zero total momentum $\rightarrow$ Moving frame calculation
- Phase shift at multiple $\sqrt{s}$
- LL factor on lattice from finite difference
- Three different magnitudes of total momentum:

$$P_{tot} = (2,0,0)\frac{\pi}{L}, (2,2,0)\frac{\pi}{L}, (2,2,2)\frac{\pi}{L}$$

- For each total momentum, three $\pi\pi$ operators:
  - $\pi\pi(111,111)$
  - $\pi\pi(311,111)$ (Not present in stationary frame calculation)
  - $\pi\pi(311,311)$
- Only compute extra contractions and no need to generate operators in quark level
- Use symmetry to reduce the number of contractions from 7848 to 1037 by removing statistically redundant contractions

Moving frame

- Moving frame calculation is more vulnerable to excited state contamination error due to the denser spectrum of states

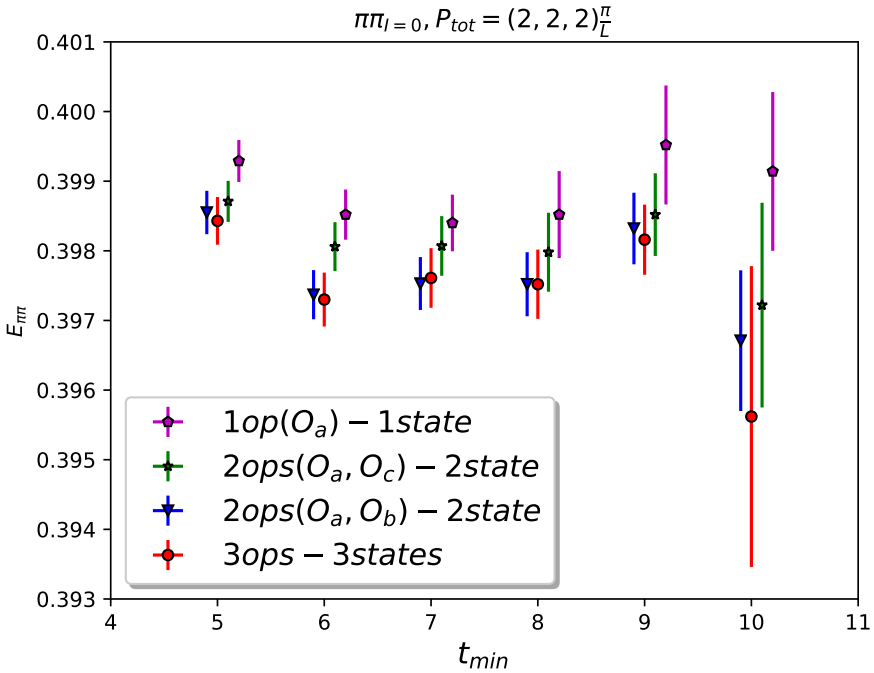


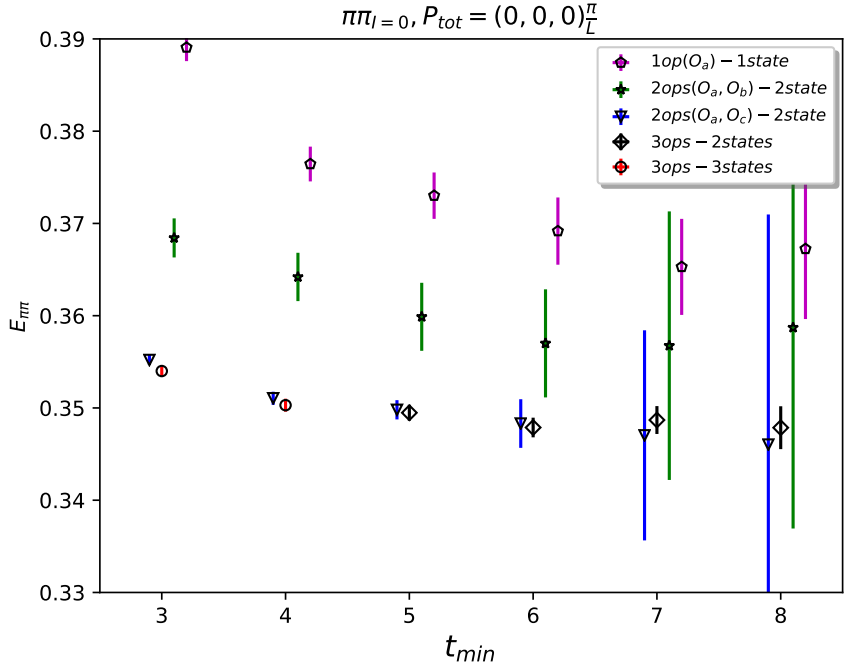
- As total momentum increases ($\sqrt{s}$ decreases), the interaction between two pions decreases, making the inter-coupling between different states and operators decreases (will see that later)

  $\rightarrow$

  Multiple operators become less powerful

Moving frame





- Effect of additional operators is less significant
- No improvement in statistical error

Excited state error

- Simply look at plateau region is less reliable due to rapid increase of error as a function of $t_{\min}$ (fake plateau)
- Including additional state in fitting function makes the result much noisier, even if we freeze the energy of that state
- Different patterns in data: The overlap matrix in moving frames are highly diagonal, while most elements have similar size in stationary frame $I = 0$

$P_{CM} = (2,2,2)$	state_0	state_1	state_2
$\pi\pi(111,111)$	1.0(0.0)	-0.07(1)	-0.035(8)
$\pi\pi(111,311)$	-0.013(6)	1.0(0.0)	-0.19(5)
$\pi\pi(311,311)$	-0.015(2)	0.05(2)	1.0(0.0)

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- Use different error estimation methods on data with different patterns

Excited state error

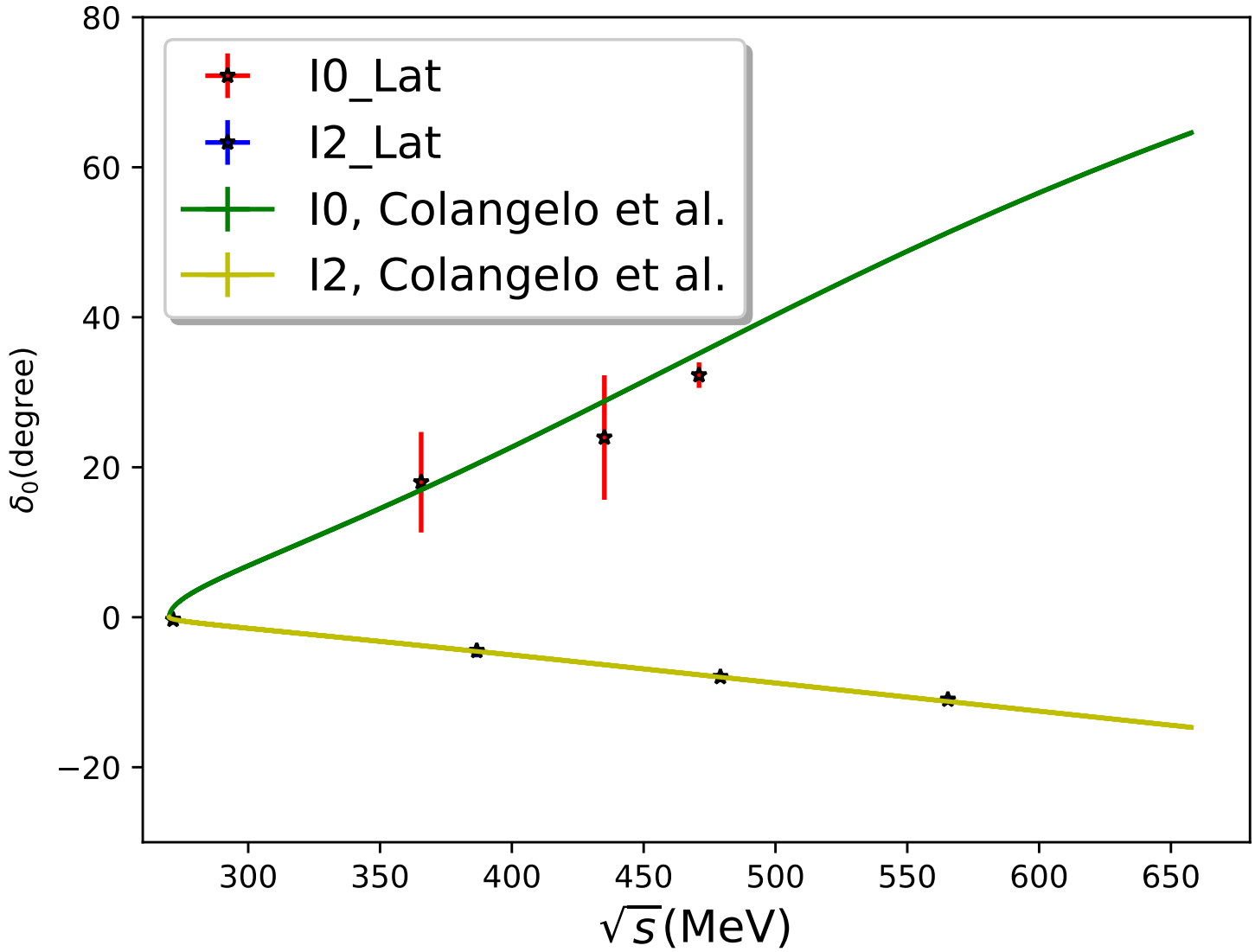
- Include one extra state in the optimal fit
- Energy is obtained from dispersive model
- Nearly diagonal overlap matrix $\Rightarrow$ obtain overlap factors from **single operator fit**
- Operators couple to all states strongly $\Rightarrow$ obtain overlap factors from **multiple operator fit with smaller $t_{\mathbf{min}}$**
- Calculate the maximum energy difference between optimal fit and extra state fit

Final result

P_{tot}	I	$\sqrt{s}$(MeV)	δ	$\Delta\delta_{\mathrm{dis}}$	$\Delta\delta_{\mathrm{FV}}$	$\Delta\delta_{\mathrm{unphy}}$	$\Delta\delta_{\mathrm{exc}}$
$(0,0,0)\frac{\pi}{L}$	0	471.0	32.3**(1.0)**(1.4)	0.64	0.32	0.83	**0.90**
$(2,0,0)\frac{\pi}{L}$	0	435.1	24.0**(3.4)**(7.6)	0.46	0.23	0.71	**7.6**
$(2,2,0)\frac{\pi}{L}$	0	365.6	18.0**(4.5)**(4.9)	0.36	0.18	0.47	**4.9**
$(0,0,0)\frac{\pi}{L}$	2	565.4	-10.98(22)(44)	0.20	0.10	0.18	**0.34**
$(2,0,0)\frac{\pi}{L}$	2	479.1	-7.96(23)(29)	0.16	0.08	0.03	**0.23**
$(2,2,0)\frac{\pi}{L}$	2	386.7	-4.48(40)(77)	0.09	0.04	0.06	**0.76**
$(2,2,2)\frac{\pi}{L}$	2	271.5	-0.32(20)(63)	0.01	0.00	0.02	**0.63**

- Large statistical error with moving $I = 0$
- Small excited state error with stationary $I = 0$ and all $I = 2$ cases
- Huge excited state error with moving $I = 0$
  σ operator might be critical

Final plot


I0_Lat
I2_Lat
I0, Colangelo et al.
I2, Colangelo et al.
80
60
40
20
0
−20
δ_0(degree)
300
350
400
450
500
550
600
650
$\sqrt{s}$(MeV)

Conclusion and future steps

Conclusions:

- Understand and solve Pi-Pi puzzle
- Calculate $\pi\pi$ scattering with $I = 0$ and $I = 2$ at seven different energies
- Estimate excited state error carefully

Future steps:

- Add moving σ operator in moving frame $I = 0$ calculation
- Using ensembles with different lattice spacing to extrapolate to the continuum limit
- Combine this result with other lattice results to improve the dispersive prediction