Calculating the amplitude of $K_L \to \gamma\gamma$ and $K_L \to \mu^+\mu^-$ decay

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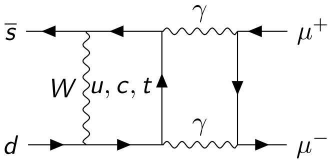
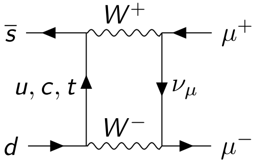
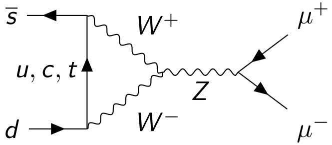
Introduction of the $K_L \to \mu^+\mu^-$ decay

- Long distance contribution



- Short distance contribution





Introduction

- The short distance contribution is interesting in physics, as it involves a flavor changing neutral current process that is highly suppressed in the Standard Model.
- The short-distance contribution to $K_L \rightarrow \mu^+\mu^-$ is well known at NLO. To compare it with experimental value, we have to calculate and subtract the long distance background.

Introduction

- The $K_L \to \mu^+\mu^-$ decay is well established experimentally (Ambrose et al., 2000 and PDG table 2020)..

$$BR(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9} \tag{1}$$

- The decay rate is completely saturated by the absorptive contribution (Nakamura et al., 2010).

$$R = \frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = R_{abs} + R_{disp} \tag{2}$$

$$\text{Experimental result: } R = (1.24 \pm 0.02) \times 10^{-5} \tag{3}$$

$$\text{Absorptive contribution: } R_{abs} = 1.195 \times 10^{-5} \tag{4}$$

- $$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = 2\beta_\mu \left(\frac{\alpha_{\rm EM}}{\pi}\frac{m_\mu}{M_K}\right)^2 \left(|F_{\rm imag}|^2 + |F_{\rm real}|^2\right). \quad (5)$$

- The imaginary part gets contributions from several available on-shell states, but is completely dominated by the $\gamma\gamma$ intermediate state:

$$F_{\rm imag} = \frac{\pi}{2\beta_\mu} \ln\left(\frac{1-\beta_\mu}{1+\beta_\mu}\right), \quad (6)$$

Introduction

- The experimental decay rate and the known imaginary part determine $|F_{\rm real}| = 1.167 \pm 0.094$. Finally we can write

$$F_{\rm real} = (F_{\rm real})_{\rm EM} + (F_{\rm real})_{\rm weak}\,. \qquad (7)$$

- The standard model predicts at one loop: $(F_{\rm real})_{\rm weak} = -1.82 \pm 0.04$.
- Thus, a lattice calculation of $(F_{\rm real})_{\rm EM}$ with 10% accuracy would determine $(F_{\rm real})_{\rm weak}$ to 6% or 17% depending on whether $F_{\rm real}$ and $(F_{\rm real})_{\rm weak}$ have the same or opposite signs, which could serve as an important test of the standard model

Intermediate states for $K_L \to \mu^+\mu^-$

- The major difficulty in the lattice calculation of $K_L \to \mu^+\mu^-$ decay is the presence of intermediate states whose energies are smaller than M_K.
- The most important intermediate states with energy less than that of the kaon is the two-photon intermediate state $|\gamma\gamma\rangle$.
- In a previous work, we proposed an analytic continuation method to tackle the problem of the $|\gamma\gamma\rangle$ intermediate states and applied this method to successfully perform a lattice computation of the $\pi^0 \to e^+e^-$ decay (Norman H. Christ, Yidi Zhao et al., 2020)

Intermediate states for $K_L \to \mu^+\mu^-$

Besides the two-photon intermediate state, depending on the time order of the operator in the matrix element $\langle T\{J_\mu(u)J_\nu(v)H(x)K(t_k)\}\rangle$, it will have the following intermediates states whose energies are lower than or close to that of kaon:

- $\langle J_\mu(u)J_\nu(v)H(x)K(t_k)\rangle$: There can be $|\pi\rangle$ and $|\eta\rangle$ states between $J_\nu(v)$ and $H(x)$.

  Since we ignore CP violation, $|0\rangle$ and $|\pi\pi\rangle$ intermediate states are not allowed between JJ and H.
  We also ignore the $|\pi\pi\pi\rangle$ state because its phase space is much smaller than the $|\pi\rangle$ state.

- $\langle J_\mu(u)H(x)J_\nu(v)K(t_k)\rangle$: There can be $|\gamma\pi\pi\rangle$ state between $J_\mu(u)$ and $H(x)$.

- $\langle H(x)J_\mu(u)J_\nu(v)K(t_k)\rangle$: There are no intermediate states with energies lower than Kaon mass.

Intermediate states for $K_L \to \mu^+\mu^-$

- The $|\pi\pi\gamma\rangle$ intermediate state is difficult to deal with as it carries momentum that is dependent on the final state.
- We calculate a simpler process, the $K_L \to \gamma\gamma$. Since the two photons are onshell, the $|\pi\pi\rangle$ state has to carry momentum $\vec{p}$ that whose magnitude is equal to half of the initial kaon energy$|\vec{p}| = \frac{M_K}{2}$. Therefore, the energy of the $|\pi\pi\rangle$ intermediate state is larger than Kaon mass.

$K_L \to \gamma\gamma$

If we ignore CP violation, the $K_L \to \gamma\gamma$ amplitude has the following symmetry

$$\langle\gamma(p_1)\gamma(p_2)|H_w|K_L(q)\rangle = (2\pi)^4\delta^{(4)}(p_1 + p_2 - q)i\mathcal{M}$$
$$i\mathcal{M} = \epsilon_{\mu\nu\alpha\beta}\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)p_{1\alpha}p_{2\beta}\left[e^2\frac{G_F}{\sqrt{2}}V_{us}V_{ud}F_{K_L\gamma\gamma}\right].$$

where $\mathcal{M}$ is the scattering amplitude, and $F_{K_L\gamma\gamma}$ is the only unknown scalar factor.

Amplitude of $K_L \to \gamma\gamma$ decay

The scalar factor $F_{K_L\gamma\gamma}$ can be separated into an EM part and a hadronic part

$$F_{K_L\gamma\gamma} = \sum_{u,v} E_{\mu\nu}(u,v) H_{\mu\nu}(u,v).$$

where

- The EM factor $E_{\mu\nu}(u,v)$ is analytically known
- The hadronic part $H_{\mu\nu}(u,v) = \langle J_\mu(u) J_\nu(v) H_w(0) | K_L \rangle$

Ensemble

Ensemble	24ID
Lattice size	$24^3 \times 64$
a^{-1}	1GeV
am_π	0.14030(80)
am_K	0.50438(85)
Z_V^π	0.7267(17)
Z_V^K	0.743(14)
Configurations	120

For each configuration, We have:

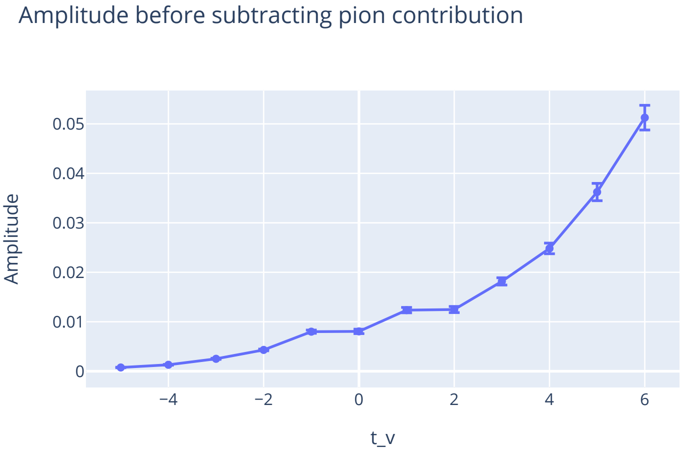
- Coulomb gauge fixed wall source propagator on each time slice
- 512 point source propagators
- A2A propagators with 2000 low modes and 768 high modes (64 hits with spin and color dilution)

Calculating strategy

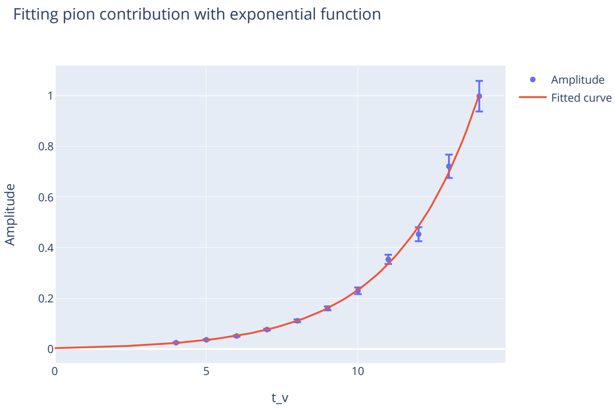
$$H_{\mu\nu}(u,v) = \langle J_\mu(u) J_\nu(v) H_w(0) | K_L \rangle$$

- The weak Hamiltonian is fixed at origin $x = 0$
- We break the symmetry between the positions u and v and assume that $t_u \geq t_v$.
- The distance that the intermediate π^0/η states propagate is $t_v - t_x$ when $t_v > t_x$.
  We plot the amplitude as a function of t_v to demonstrate that the intermediate states have been successfully removed.
- We ignore the disconnected diagrams for now.

Amplitude without removing intermediate stats



Amplitude without removing intermediate stats



We fit the exponential increase for $t_v \in [4, 14]$ with the exponential function $A(t_v) = C \exp(M t_v) + b$. The fitted mass $M = 0.3628(152)$ GeV is close to $M_K - M_\pi = 0.364$ GeV.

Pion intermediate state

We directly calculate the remove the pion intermediate state by subtracting the following product of matrix elements

$$\langle J_\mu(u) J_\nu(v) | \pi \rangle \langle \pi H_w(0) | K_L \rangle \tag{8}$$

Eta intermediate state

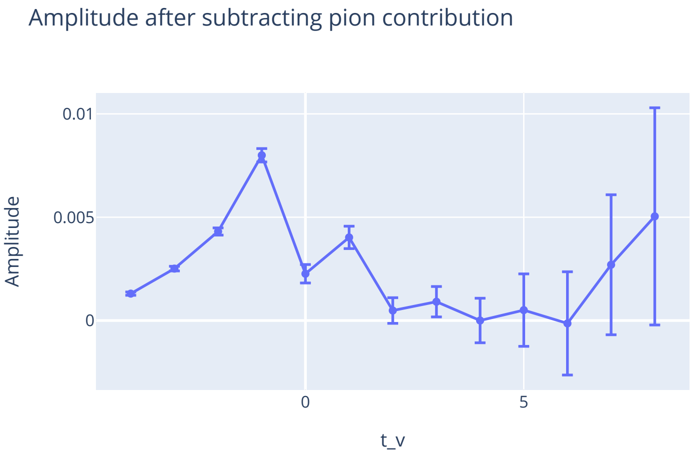
- We plan to use total divergence term $\bar{s}d$ to remove the intermediate $|\eta\rangle$ state in the future

$$H = H_w + c_s \bar{s}d$$
$$c_s = -\frac{\langle \eta | H_w(0)^{\Delta S=1} | K^0 \rangle}{\langle \eta | \bar{s}d(0) | K^0 \rangle}.$$

- The contributions from disconnected diagrams are too noisy to deal with. For now, we ignore the disconnected diagrams so that eta is the basically the same as pion can be removed together.

Amplitude after removing intermediate stats

The exponential increase has been successfully removed:



Comprison with experiment

Our result from only connected diagrams is about 1.5σ away from experimental result

$$\text{Our result: } F_{K_L\gamma\gamma} = 0.0238(20)\text{GeV} \tag{9}$$

$$\text{Experimental result: } F_{K_L\gamma\gamma} = 0.0205(1)\text{GeV}. \tag{10}$$

Conclusion

- We performed a lattice calculation of the $K_L \to \gamma\gamma$ decay amplitude
- Together with our previous work on the calculation of $pi^0 \to e^+e^-$ decay, they lay the foundation of the future calculation of a more interesting and more complicated process, the $K_L \to \mu^+\mu^-$ decay

Thank You!