Pseudoscalars and Theta equals pi

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Neutral pseudoscalars in the three flavor theory:

$$\pi_0 \propto \frac{1}{2}(\overline{u}_L u_R - \overline{u}_R u_L - \overline{d}_L d_R + \overline{d}_R d_L)$$

$$\eta \propto \frac{1}{2\sqrt{3}}(2\overline{s}_L s_R - 2\overline{s}_R s_L - \overline{u}_L u_R + \overline{u}_R u_L - \overline{d}_L d_R + \overline{d}_R d_L)$$

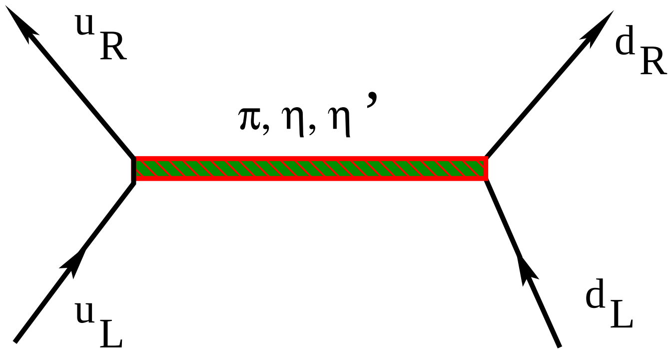
$$\eta' \propto \frac{1}{\sqrt{6}}(\overline{s}_L s_R - \overline{s}_R s_L + \overline{u}_L u_R - \overline{u}_R u_L + \overline{d}_L d_R - \overline{d}_R d_l)$$

$g_p \propto \tilde{F}F$ pseudoscalar glueball

$$M_{\pi_0}^2, M_\eta^2 \propto m_q$$

$$M_{\eta'}, M_{g_p} \propto \Lambda_{qcd}$$

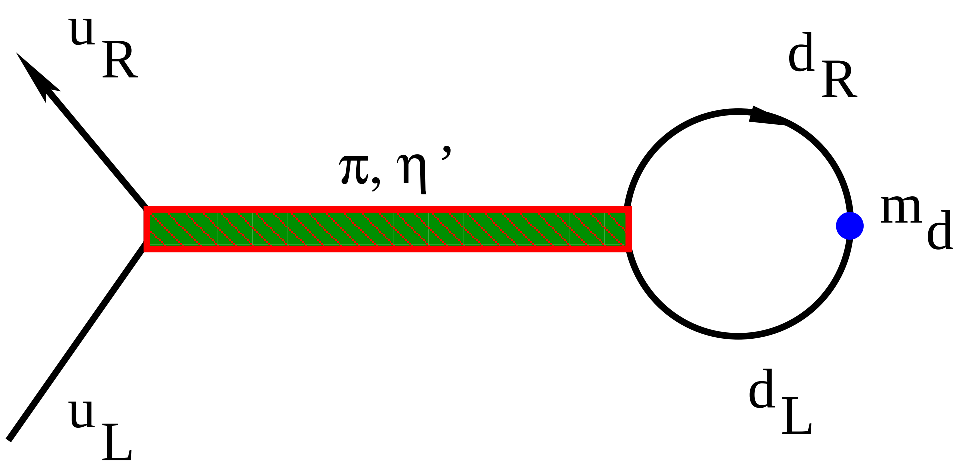
$\overline{u}_L u_R$ or $\overline{d}_L d_R$ can create any of these mesons



- does not vanish for massless quarks
- spin flip process via chiral anomaly
  - “’t Hooft vertex”

Now turn on a small d quark mass

- closing d loop induces $u_L\ u_R$ mixing



Non-zero d quark mass shifts u quark mass

Effect automatically included in lattice simulations

Old point

- Georgi, McArthur, 1981 (unpublished)
- Choi, Kim, Sze, 1988 (PRL)
- Banks, Nir, Seiberg, 1994 (conference proceedings)
- MC, 2004 (PRL)

This is a non-perturbative effect

- mass renormalization is not flavor blind
- mass independent regularization is tricky
- inherent ambiguities defining $m_u = 0$

When $m_u \neq m_d$

- matching perturbative to non-perturbative results
  - problematic

Non-perturbative vs perturbative effects?

- both have perturbative corrections
- topological effects scale dependent
- “instantons” fall through the lattice
- need a non-perturbative regulator: lattice

Wilson fermions

- additive renormalization of Kappa critical
- depends on coefficient of Wilson therm

Domain wall and Overlap fermions

- depend on kernel
- Wilson parameter and Kappa

Staggered fermions

- inherent taste degeneracy cancels effect
    - cannot do non-degenerate quarks

Chiral symmetry

- degenerate light quarks
- $M_\pi^2 \propto m_q \Lambda$

Massless quarks imply massless pions

- for degenerate quarks $m_q = 0$ is well defined

What if we make isospin breaking large?

- $M_\pi^2 \propto \frac{m_u+m_d}{2}\Lambda$
- mass gap persists at $m_u = 0$ if $m_d > 0$

Sensible physics for small negative m_u!

- perturbation theory: sign of mass is a convention

Negative quark mass equivalent to $\Theta = \pi$

With degenerate quarks

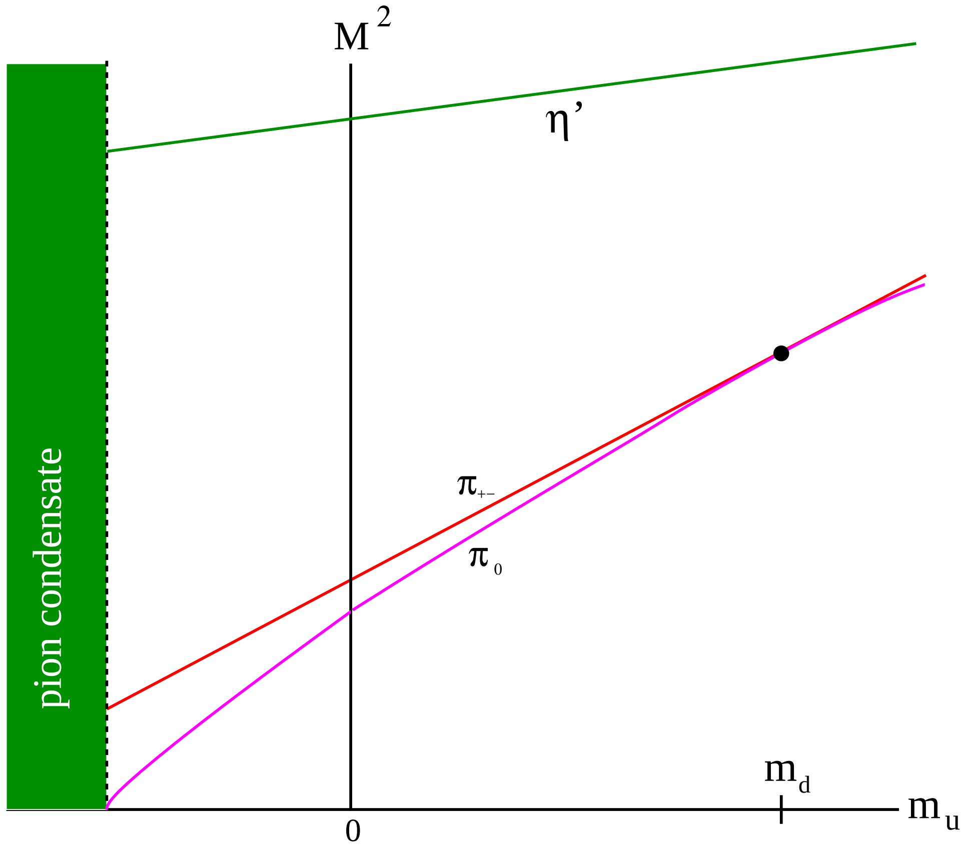
- $\Theta = \pi$ must have spontaneous parity breaking
- many demonstrations of this
  - MC, Annals of Physics 324 (2009), 1573

As the up quark mass varies from $+m_d$ to $-m_d$

  - something interesting must happen!

The Dashen phenomenon

Isospin breaking reduces π_0 mass



- $M^2_{\pi_0}$ vanishes before $m_u = -m_d$

At $M_{\pi_0}^2 < 0$ a pion condensate will form

- $\langle \pi_0 \rangle \neq 0$
- CP broken

Formally at $\Theta = \pi$ $\prod_q m_q < 0$

Dashen 1971

Direct consequence of $(\pi_0,\ \eta,\ \eta')$ mixing

Explicit in sigma models including $(\pi_0,\ \eta)$ mixing

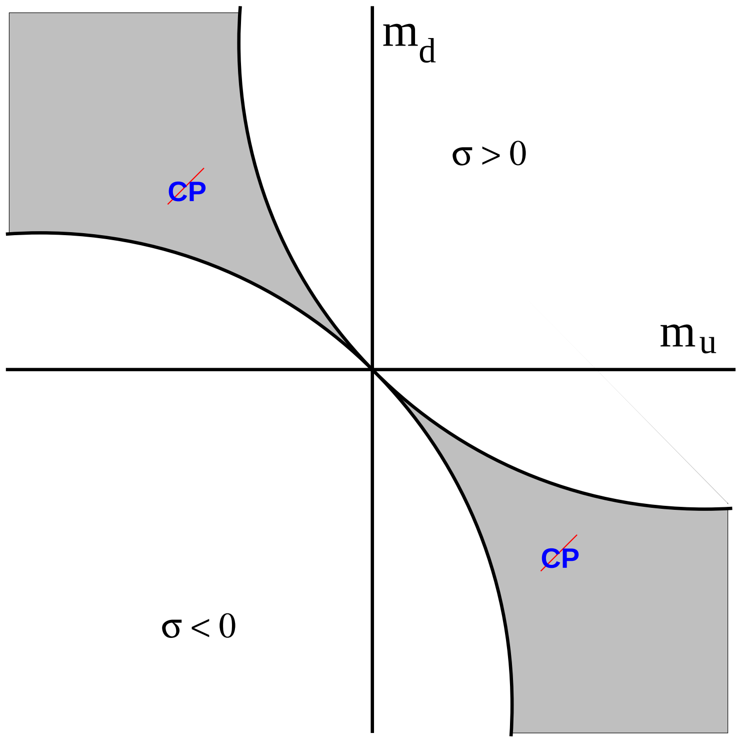
- $$m_{\pi_0}^2 \propto \frac{2}{3}\Big(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\Big)$$

- $$m_\eta^2 \propto \frac{2}{3}\Big(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\Big)$$

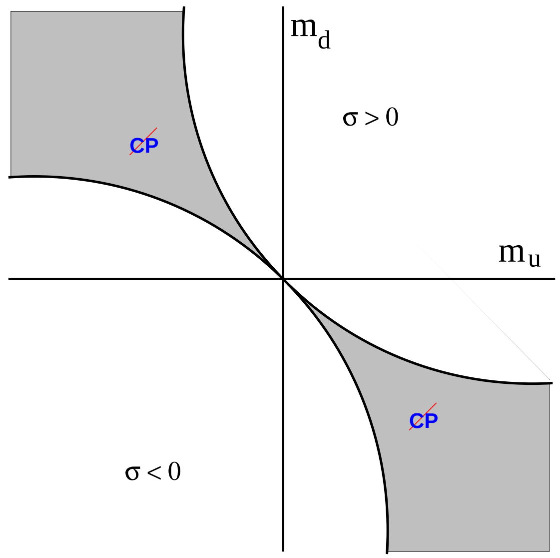
π_0 becomes massless at $m_u = \frac{-m_d m_s}{m_d + m_s} > -m_d$

Ising-like transition at $m_u < 0$

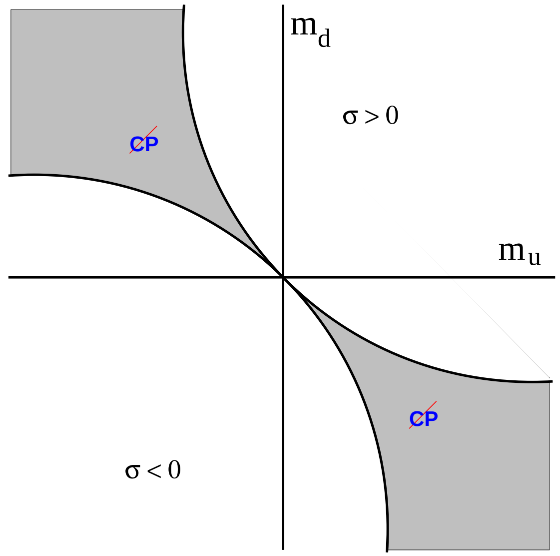
- order parameter $\langle \pi_0 \rangle \neq 0$
- breaks CP spontaneously



Symmetries of the (m_u, m_d) phase diagram



- $m_u \leftrightarrow m_d$ — Isospin
- $(m_u, m_d) \to (-m_u, -m_d)$ — Flavored chiral symmetry
  - $\psi \to e^{i\pi\tau_3\gamma_5}\psi$



NO symmetry under $(m_u, m_d) \to (-m_u, m_d)$

- a strictly non-perturbative effect
- a non-degenerate massless quark
  - not protected from renormalization

Symmetries imply multiplicative mass renormalization

- quark mass difference $m_d - m_u$
- quark mass average $m_d + m_u$

Renormalization factors are not in general equal!!

- m_u can acquire an additive shift
- perturbative/nonperturbative ambiguity
  - depends on scheme

Should we care if quark masses fuzzy?

- not directly measured in scattering
- related to fuzziness in gauge field topology
  - non-differentiable gauge fields

Only particle masses and scatterings are physical

Summary

Non-perturbative physics is crucial for QCD

Different theories with identical perturbative expansions

- $\Theta \neq 0$

Nondegenerate quark masses scheme dependent

- $m_u = 0$ ill defined if $m_d \neq 0$

Gauge field topology inherently ambiguous

Backup

Connection with topology

- fermion determinant $\sim m^\nu$ suppresses topology
- susceptibility $\langle \nu^2 \rangle$
  - vanishes if $m = 0$
  - negative if $m < 0$ (negative weight configurations)

$\langle \nu^2 \rangle = 0$ and $m = 0$ are equivalent

Topology inherently fuzzy in the quantum theory

- fields non-differentiable
- use index theorem with a fermion probe

Fermion eigenvalues depend on lattice scheme

- Wilson fermions: modes small but not exactly zero
- overlap fermions: not unique, "domain wall height"