Variations on inverse problems and Maiani-Testa

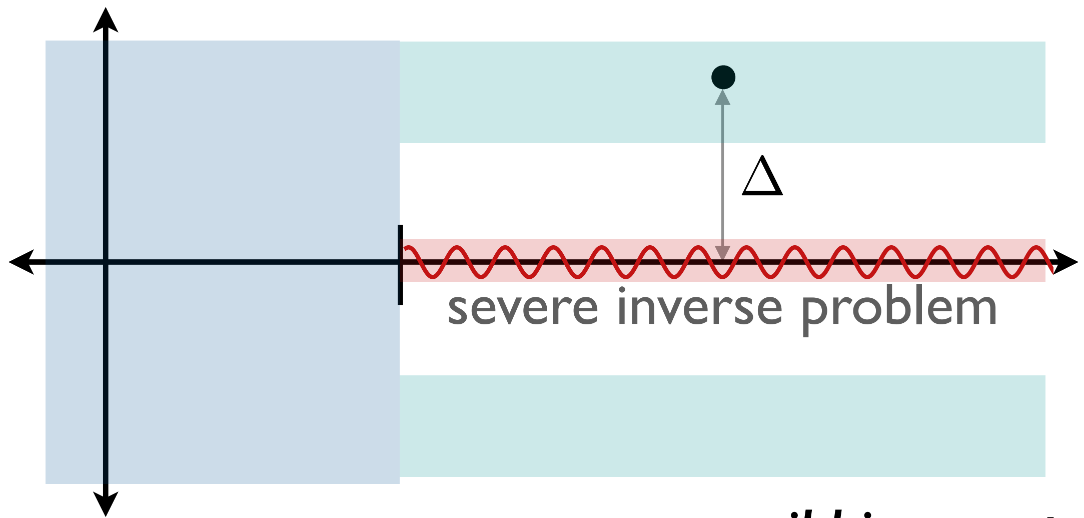
Maxwell T. Hansen

December 17th, 2021

- Based on work with M. Bruno, J. Bulava, M. Hansen, A. Patella, N. Tantalo

THE UNIVERSITY of EDINBURGH

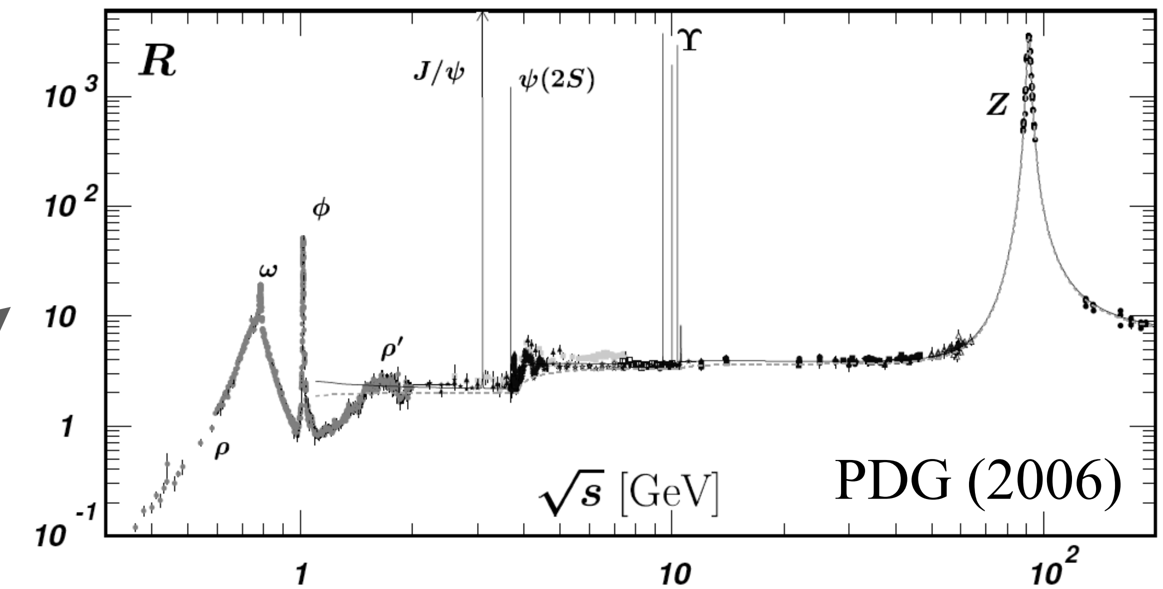
R ratio



no inverse problem

Hashimoto, 2019 $\dfrac{1}{\bar{\omega}-\omega}$

mild inverse problem?



PHYSICAL REVIEW D VOLUME 13, NUMBER 7 1 APRIL 1976

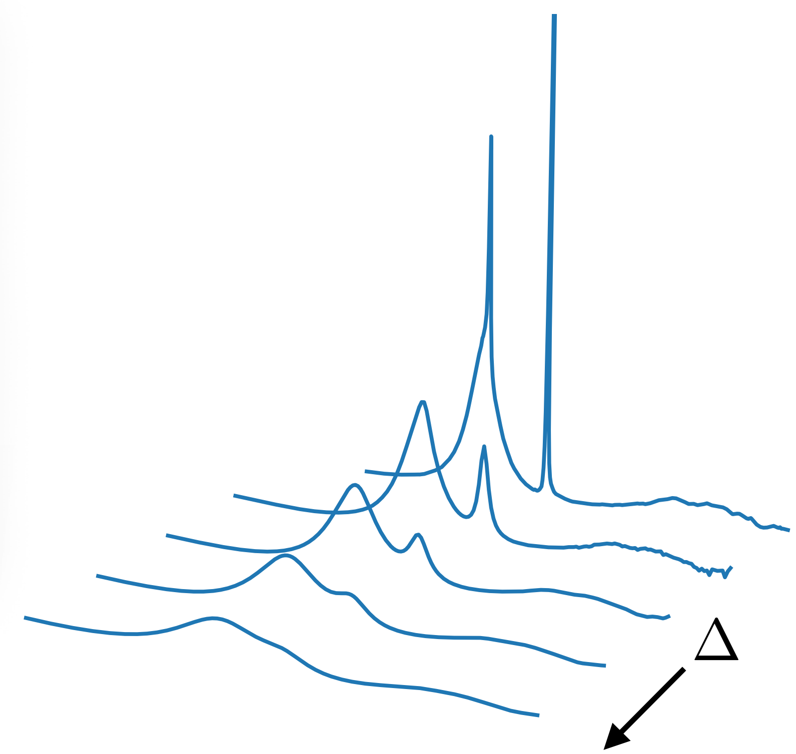
Smearing method in the quark model*

E. C. Poggio, H. R. Quinn,† and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 8 December 1975)

We propose that perturbation theory in a gauge field theory of quarks and gluons can be used to calculate physical mass-shell processes, provided that both the data and the theory are smeared over a suitable energy range. This procedure is explored in detail for the process of electron-positron annihilation into hadrons. We show that a smearing range of 3 GeV^2 in the squared center-of-mass energy should be adequate to allow the use of lowest-order perturbation theory. The smeared data are compared with theory for a variety of models. It appears to be difficult to fit the present data with theory, unless there is a new charged lepton or quark with mass in the region 2 to 3 GeV.

$$\widehat{R}_\Delta(s) = \frac{\Delta}{\pi}\int_0^\infty ds' \, \frac{R(s')}{(s'-s)^2+\Delta^2}$$

into the complex plane!



courtesy of M. Bruno
(using F. Jegerlehner’s ***alphaQED***)

Reconstruction methods

$$\underset{\text{have}}{G(\tau)} = \int d\omega\, e^{-\omega\tau}\, \underset{\text{want}}{\rho(\omega)}$$

- ***Linear, model-independent reconstruction*** (e.g. Backus-Gilbert-like, Chebyshev)

$$\begin{aligned}\sum_\tau \mathcal{K}(\bar{\omega},\tau)G(\tau) &= \sum_\tau \mathcal{K}(\bar{\omega},\tau)\int d\omega\, e^{-\omega\tau}\,\rho(\omega)\\ &= \int d\omega\,\Big[\sum_\tau \mathcal{K}(\bar{\omega},\tau)\, e^{-\omega\tau}\Big]\,\rho(\omega)\\ &= \int d\omega\, \widehat{\delta}_\Delta(\bar{\omega},\omega)\,\rho(\omega)\end{aligned}$$

δ is exactly known

- Maximum Entropy Method (MEM) ⟵ Not discussed here…
- Direct fits
- Neural networks

See multiple ECT and CERN workshops, work by Aarts, Allton, Amato, Brandt, Burnier, Del Debbio, Francis, Giudice, Hands, Harris, Hashimoto, Jäger, Karpie, Liu, Meyer, Monahan, Orginos, Robaina, Rothkopf, Ryan, …*

- Key idea here… we aim only to construct

$$\widehat{\rho}(\bar{\omega}) \equiv \int_{-\infty}^{\infty} d\omega\, \widehat{\delta}_\Delta(\bar{\omega},\omega)\,\rho(\omega)$$

Linear reconstruction

$$\widehat{\rho}(\bar{\omega}) = \sum_{\tau} \mathcal{K}(\bar{\omega},\tau) G(\tau) = \int_{-\infty}^{\infty} d\omega \, \widehat{\delta}_{\Delta}(\bar{\omega},\omega) \rho(\omega)$$

$$\widehat{\delta}_{\Delta}(\bar{\omega},\omega) = \sum_{\tau} \mathcal{K}(\bar{\omega},\tau) e^{-\omega\tau}$$

- Given $\delta^{\text{target}}(\bar{\omega},\omega)$, best $K(\bar{\omega},\tau)$ = whatever minimizes combination of

$$\Delta(\mathcal{K}\,|\,\bar{\omega},\omega) = \left| \delta^{\text{target}}(\bar{\omega},\omega) - \sum_{\tau} \mathcal{K}(\bar{\omega},\tau) e^{-\omega\tau} \right|$$

+

statistical uncertainty on $\widehat{\rho}(\omega)$

- $\Delta(K\,|\,\bar{\omega},\omega)$ is known
- If $\Delta(K_A\,|\,\bar{\omega},\omega) < \Delta(K_B\,|\,\bar{\omega},\omega)$ (for similar uncertainties) … then K_A is just better
- Chebyshev polynomials and Backus-Gilbert-like approaches are some options

Chebyshev idea

- Take $\bar{\omega}$ and Δ and 'target shape' as fixed and write...

$$\delta_{\Delta}^{\text{target}}(\bar{\omega}, \omega) = f(\omega) = \sum_n \mathcal{K}_n (e^{-\omega a_t})^n = \sum_n \mathcal{K}_n x^n$$

- One specific family of choices for K_n is given using Chebyshevs

$$f(\omega) = \sum_n^N c_n T_n(x)$$

can be readily converted to K_n and compared to other methods

- Chebyshev is best if and only if it achieves some version of the desired minimization

$$\boxed{\Delta(\mathcal{K} \,|\, \bar{\omega}, \omega) = \left| \delta^{\text{target}}(\bar{\omega}, \omega) - \sum_\tau \mathcal{K}(\bar{\omega}, \tau) e^{-\omega\tau} \right|} \quad + \quad \boxed{\textit{statistical uncertainty on } \widehat{\rho}(\omega)}$$

Miscellaneous comments

- If correlator is indistinguishable from a single exponential, then

$$G(\tau) = c_0 e^{-E_0 \tau} \quad \Longrightarrow \quad \rho(\omega) = c_0 \delta(\omega - E_0)$$

- Ground-state-dominated time slices do not add much

- Systematic uncertainty on $\widehat{\rho}(\bar{\omega})$ can be challenging

$$\Delta(\mathcal{K}\,|\,\bar{\omega}, \omega) = \left| \delta^{\text{target}}(\bar{\omega}, \omega) - \sum_{\tau} \mathcal{K}(\bar{\omega}, \tau) e^{-\omega \tau} \right|$$

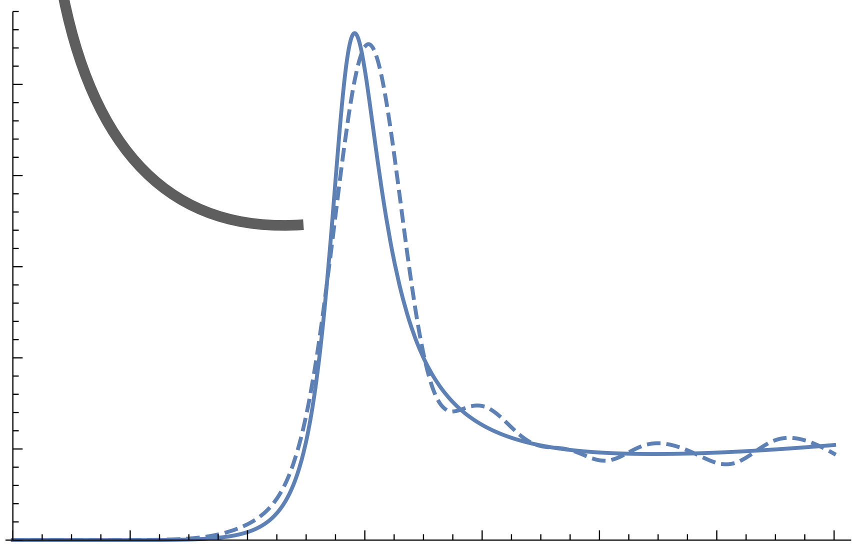
want $\Delta(K\,|\,\bar{\omega}, \omega)$ small when $\rho(\omega)$ is large

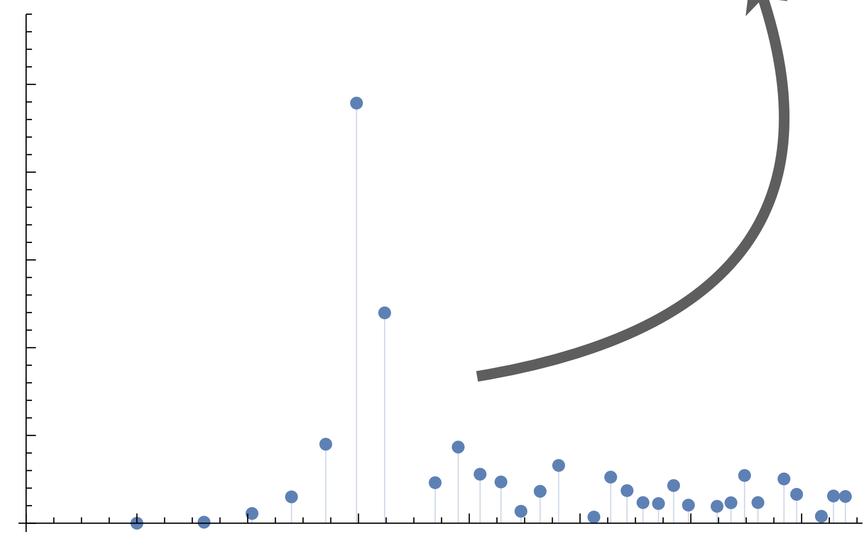
- A GEVP that saturates the correlator contains all relevant information

$$G(\tau) = \sum_n c_n e^{-E_n \tau} \quad \Longrightarrow \quad \rho(\omega) = \sum_n c_n \delta(\omega - E_n)$$

Role of the finite volume

$$\widehat{\rho}_L(\bar{\omega}) \equiv \int_0^{\infty} d\omega\, \widehat{\delta}_{\Delta}(\bar{\omega}, \omega)\, \rho_L(\omega) \qquad G_L(\tau) = \int d\omega\, e^{-\omega\tau}\, \rho_L(\omega)$$





- Any reconstructed spectral function that ≠ forest of deltas…

contains implicit smearing (or else $L \to \infty$)

We require…

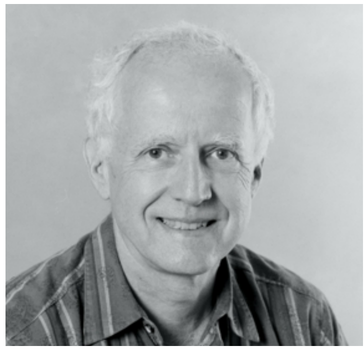
$$1/L \ll \Delta \ll \mu_{\text{physical}}$$

smearing function
covers many delta peaks

smearing does not overly
distort observable

MTH, Meyer, Robaina (2017)

Backus-Gilbert (*Numerical Recipes* version)




- Un-stabilized inverse

Without uncertainties, minimize the functional…

$$A[\mathcal{K}] \equiv \int_0^\infty d\omega\,(\bar{\omega}-\omega)^2\,\widehat{\delta}_\Delta(\bar{\omega},\omega)^2 = \int_0^\infty d\omega\,(\bar{\omega}-\omega)^2\left[\sum_\tau \mathcal{K}_\tau e^{-\omega\tau}\right]^2$$

with unit area constraint on δ

- Stabilized inverse

With uncertainties, instead minimize…

$$W_\lambda[\mathcal{K}] \equiv (1-\lambda)A[\mathcal{K}] + \lambda\frac{\mathcal{K}\cdot\mathrm{COV}\cdot\mathcal{K}}{\mathcal{N}}$$

wants oscillating K

wants well-behaved K

λ parametrizes the tradeoff between *final resolution* and *final* uncertainty

Extended Backus Gilbert

- Important generalization from ***Martin Hansen, Lupo, Tantalo***
- Addressed two limitations…

Shape and width of δ depends on data quality

No clear way to set λ

$$W_\lambda[\mathcal{K}] \equiv (1-\lambda)A[\mathcal{K}] + \lambda \frac{\mathcal{K}\cdot \mathrm{COV}\cdot \mathcal{K}}{\mathcal{N}}$$

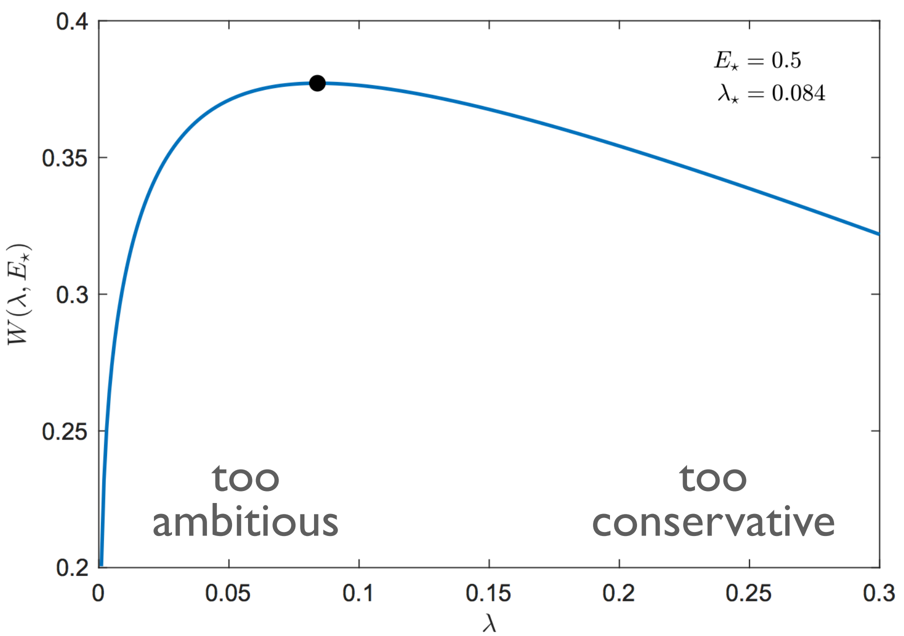
Change minimizing functional

$$A[\mathcal{K}] \equiv \int_0^\infty d\omega\, (\bar\omega - \omega)^2\, \widehat{\delta}_\Delta(\bar\omega,\omega)^2$$

$$\widetilde{A}[\mathcal{K}] \equiv \int_0^\infty d\omega \left[\widehat{\delta}_\Delta^{\,\mathrm{target}}(\bar\omega,\omega) - \widehat{\delta}_\Delta(\bar\omega,\omega)\right]^2$$

Extremize *W w.r.t.* λ

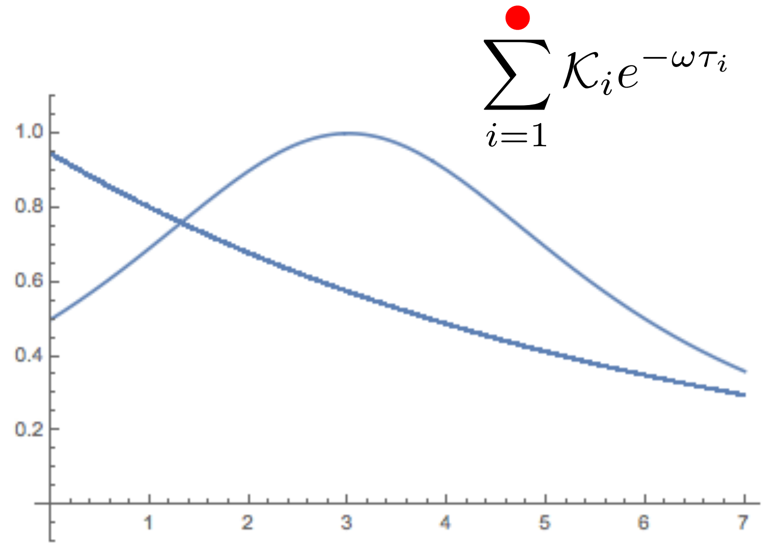
$$W_\lambda[\mathcal{K}_{\min}(\lambda)]$$

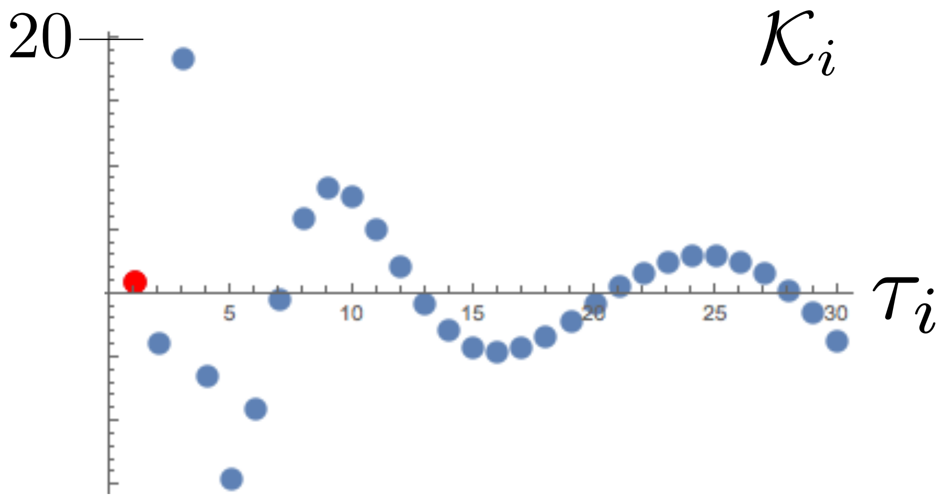


Hansen, Lupo, Tantalo (2019)

Extended Backus Gilbert example

- e.g. target a Breit-Wigner





Hansen, Lupo, Tantalo (2019) method

Extended Backus Gilbert example

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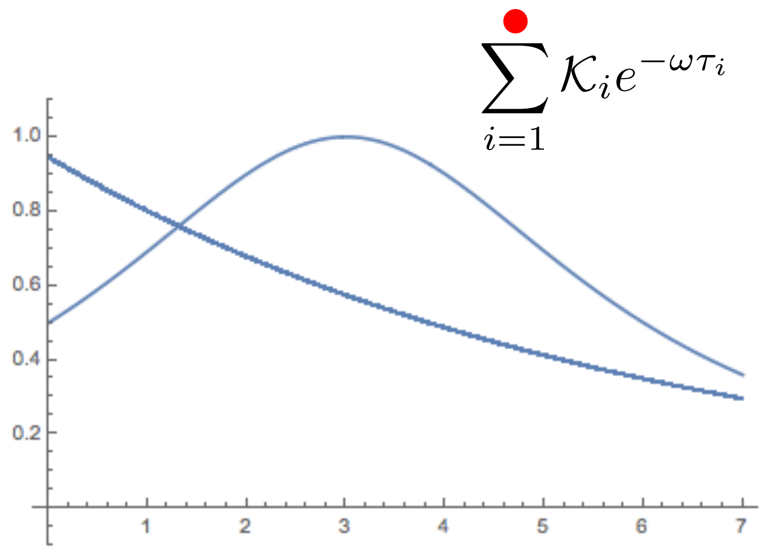

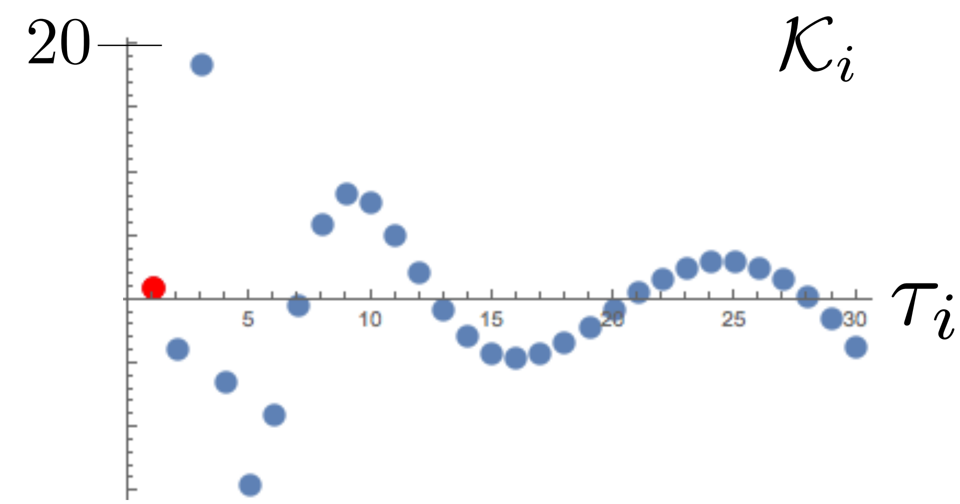

Hansen, Lupo, Tantalo (2019) method

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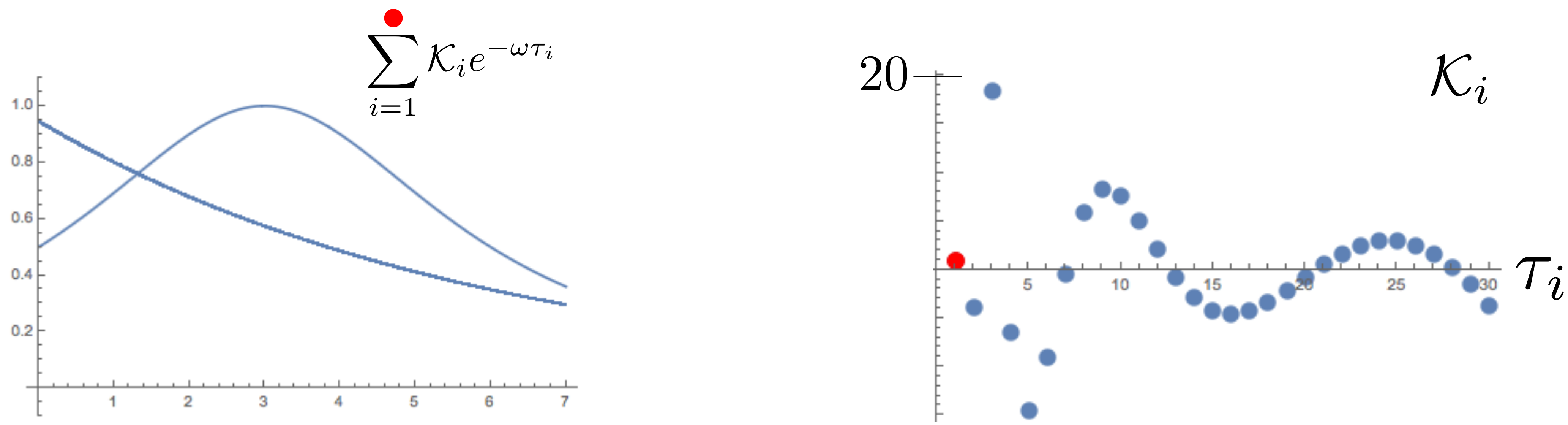

Hansen, Lupo, Tantalo (2019) method

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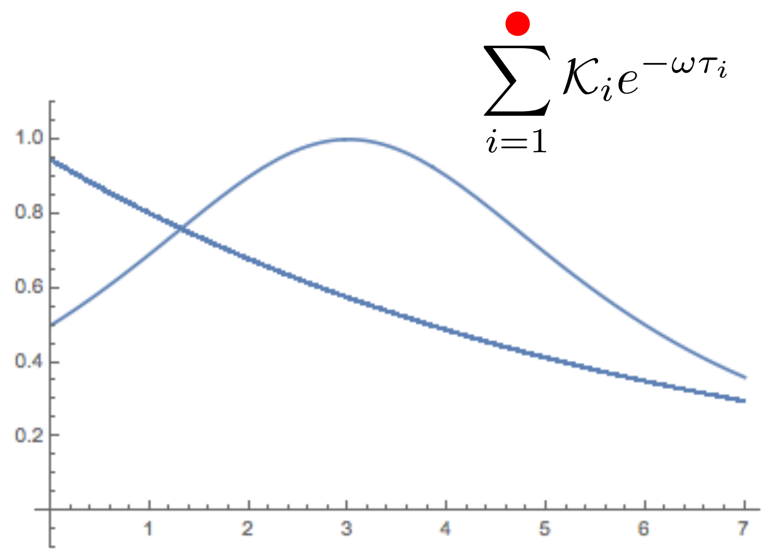
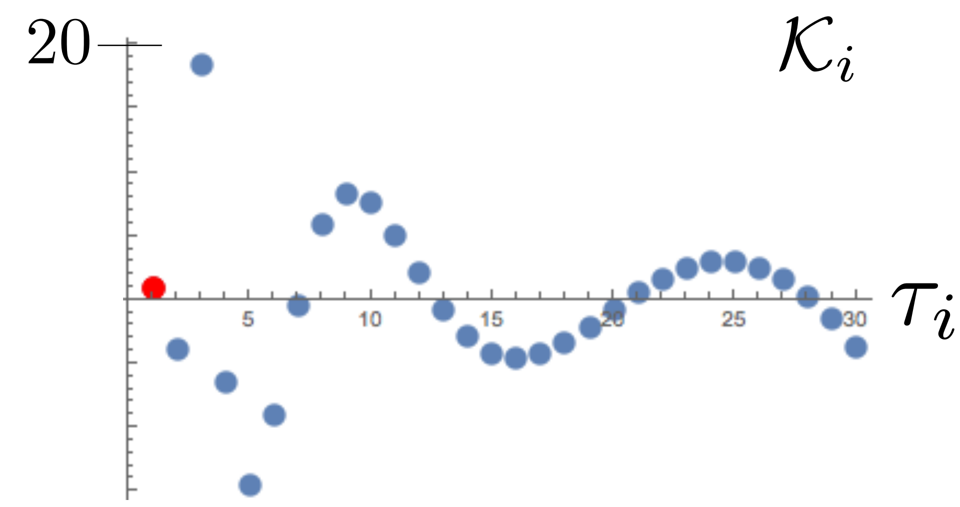
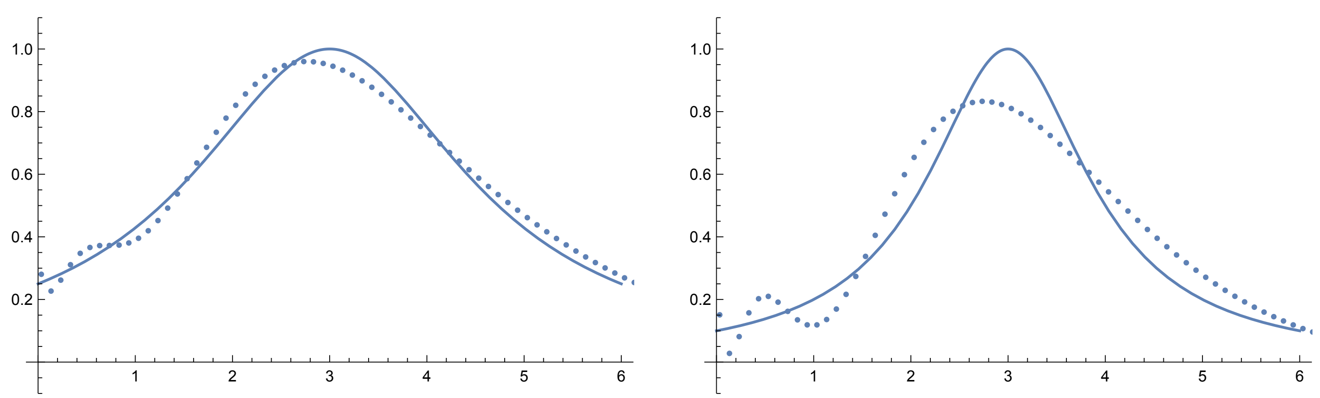


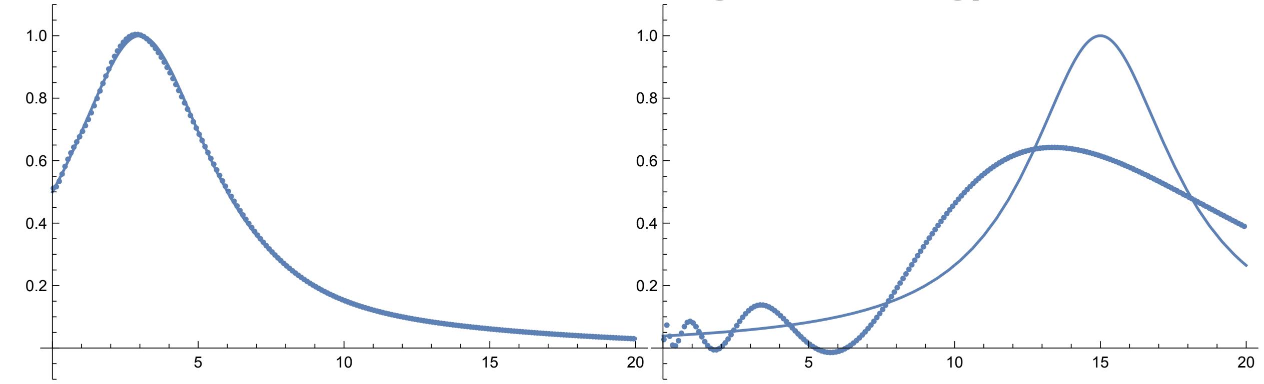


- Method will fail if one tries to…

make the resolution function too narrow



move the resolution too high in energy



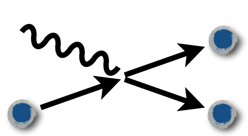
Hansen, Lupo, Tantalo (2019) method

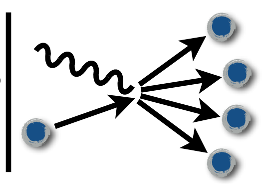
Total rate based applications

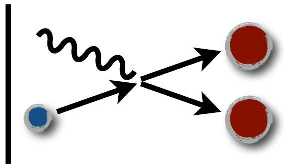
- Hadronic tensor, heavy flavor lifetimes

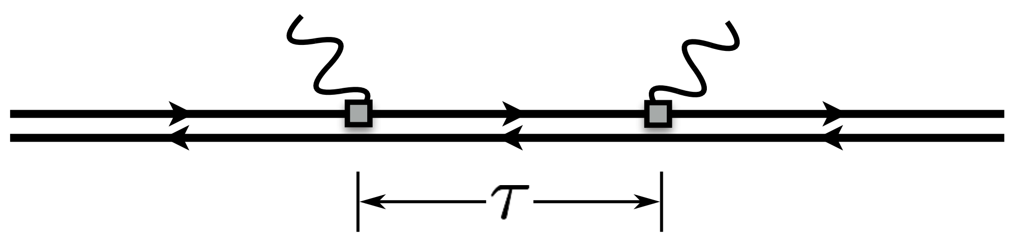
$$W_{\mu\nu}(p,q) \equiv \int d^4x \; e^{iqx} \langle \pi, p | \mathcal{J}_\mu^\dagger(x) \mathcal{J}_\nu(0) | \pi, p \rangle \qquad \propto \langle \pi, p | \widetilde{\mathcal{J}}^\dagger_{\boldsymbol{q},\mu}(0) \delta(\hat{H} - \omega) \mathcal{J}_\nu(0) | \pi, p \rangle$$

$$\propto \int d\Phi \Big| \quad \Big|^2 + \int d\Phi \Big| \quad \Big|^2 + \int d\Phi \Big| \quad \Big|^2 + \cdots$$









$$= \int_0^\infty d\omega \, e^{-\omega\tau} \, W_{\mu\nu}(p,q)_{\omega = p^0 + q^0}$$

$$W_{\mu\nu} = \lim_{\Delta \to 0} \lim_{L \to \infty} \widehat{W}_{\mu\nu;\Delta,L}$$

- Inclusive and semi-inclusive, importance of smearing/volume/kinematics
- What about ***scattering*** and ***transition amplitudes?***

MTH, Meyer, Robaina (2017), see also K.F.-Liu (2016), Hashimoto (2019-2021)

Amplitudes from spectral functions

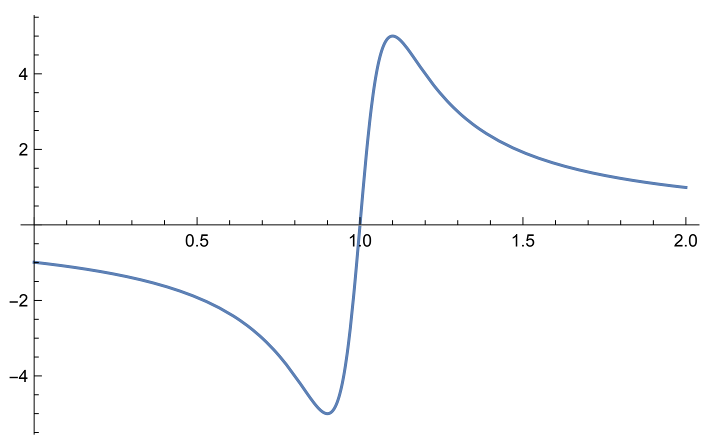
- First define the Euclidean correlator

$$G_L(\tau) = \langle \pi_{\boldsymbol{p}_4} | \pi(\tau_3, \boldsymbol{p}_3) \pi(0) | \pi_{\boldsymbol{p}_1} \rangle_L = \int_0^\infty dE_3 \, \rho(E_3) e^{-E_3 \tau_3}$$

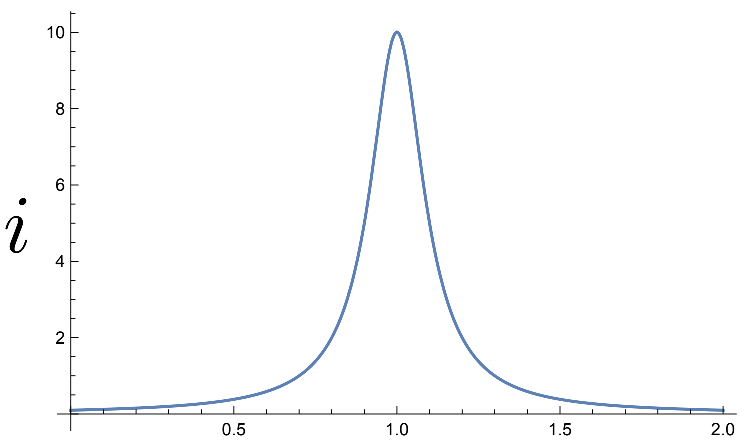
and the smeared spectral function

$$\widehat{\rho}^{L,\epsilon}_{\boldsymbol{p}_4 \boldsymbol{p}_1}(q_3) = \int_0^\infty dE_3 \, \frac{1}{q_3^0 - E_3 + i\epsilon} \, \rho(E_3) = \int_0^\infty dE_3 \, \widehat{\delta}_\epsilon(q_3^0, E_2) \, \rho(E_3)$$

$$\frac{1}{q_3^0 - E_3 + i\epsilon} =$$



$$-i$$



Bulava, MTH (2019)

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- Next project *on shell* at finite ϵ

$$\mathcal{M}_{\mathrm{c}}^{L,\epsilon}(p_4 p_3 | p_2 p_1) \equiv \frac{2E(\boldsymbol{p}_3)}{Z^{1/2}(\boldsymbol{p}_3)} \frac{2E(\boldsymbol{p}_2)}{Z^{1/2}(\boldsymbol{p}_2)} \epsilon^2 \, \widehat{\rho}^{L,\epsilon}_{\boldsymbol{p}_4 \boldsymbol{p}_1}(E(\boldsymbol{p}_3), \boldsymbol{p}_3)$$

- Finally project out the ***scattering amplitude***

$$\mathcal{M}_{\mathrm{c}}(p_4 p_3 | p_2 p_1) = \lim_{\epsilon \to 0^+} \lim_{L \to \infty} \mathcal{M}_{\mathrm{c}}^{L,\epsilon}(p_4 p_3 | p_2 p_1)$$

Bulava, MTH (2019)

Some comments

- Derivation based in modified **LSZ** + *signature-independence* of ρ(E)
- Holds when LSZ holds

$$\langle m, \mathrm{out} | n, \mathrm{in} \rangle \qquad \langle m, \mathrm{out} | \mathcal{J}(0) | n, \mathrm{in} \rangle$$

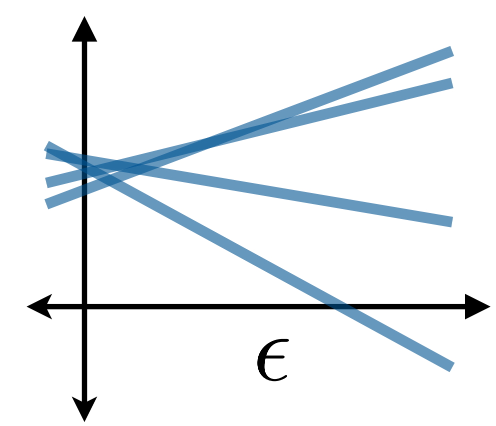
- Very challenging… but systematic

  for some (unknown) volume + correlator quality, we can get $D \to \pi\pi, K\overline{K}$

- Some nice features…

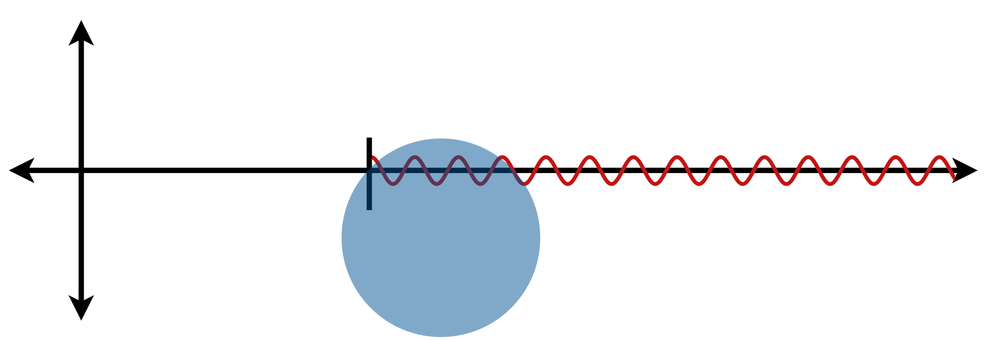
  GEVP-like operator freedom

$$G_L^{[ab]}(\tau) = \langle \pi_{\boldsymbol{p}_4} | \pi^a(\tau_3, \boldsymbol{p}_3) \pi^b(0) | \pi_{\boldsymbol{p}_1} \rangle_L \longrightarrow \mathcal{M}^{[ab],\epsilon,L}$$



Finite range of analyticity in ϵ

Uses overlaps



Bulava, MTH (2019)

Perturbative study…

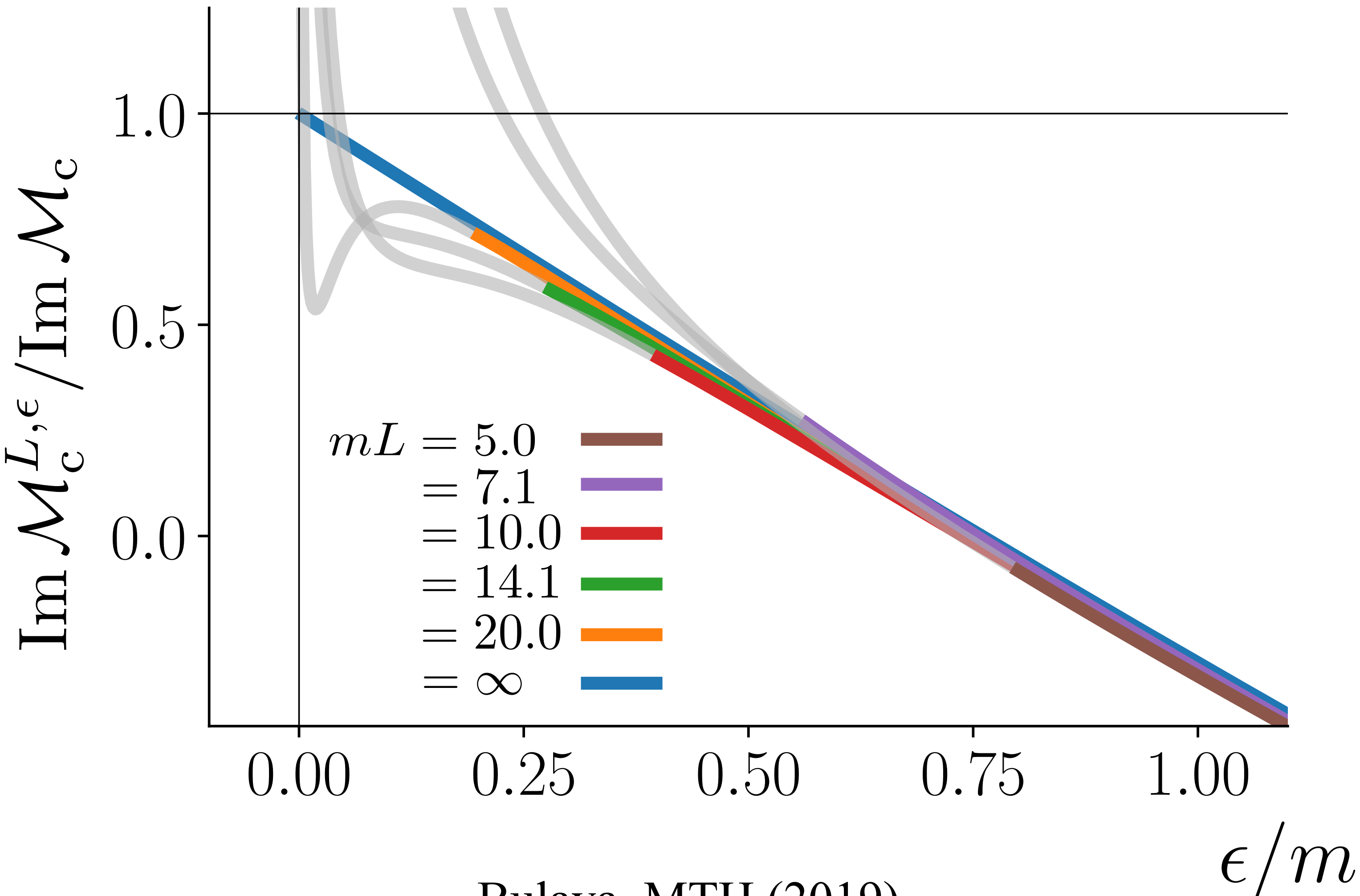
Calculate in PT

$$G_L(\tau) = \langle \pi_{\boldsymbol{p}_4} | \pi(\tau_3, \boldsymbol{p}_3) \pi(0) | \pi_{\boldsymbol{p}_1} \rangle_L$$

Convert to this

$$\mathcal{M}_{\mathrm{c}}^{L,\epsilon}(p_4 p_3 | p_2 p_1)$$

$$\mathrm{Im}\, \mathcal{M}_{\mathrm{c}}^{L,\epsilon}(p_4 p_3 | p_2 p_1) = \frac{\lambda^2}{2} \frac{1}{L^3} \sum_{\boldsymbol{k}'}^{\Lambda} \frac{1}{(2E(\boldsymbol{k}'))^2}\, \mathrm{Im} \left\{ \frac{1}{(E_{\mathrm{cm}} - 2E(\boldsymbol{k}') + i\epsilon)} \left[1 - \frac{\epsilon^2}{4E(\boldsymbol{k}')^2} - \frac{\epsilon(\epsilon + 2iE(\boldsymbol{k}'))}{E_{\mathrm{cm}} E(\boldsymbol{k}')} \right] \right\}$$



Bulava, MTH (2019)

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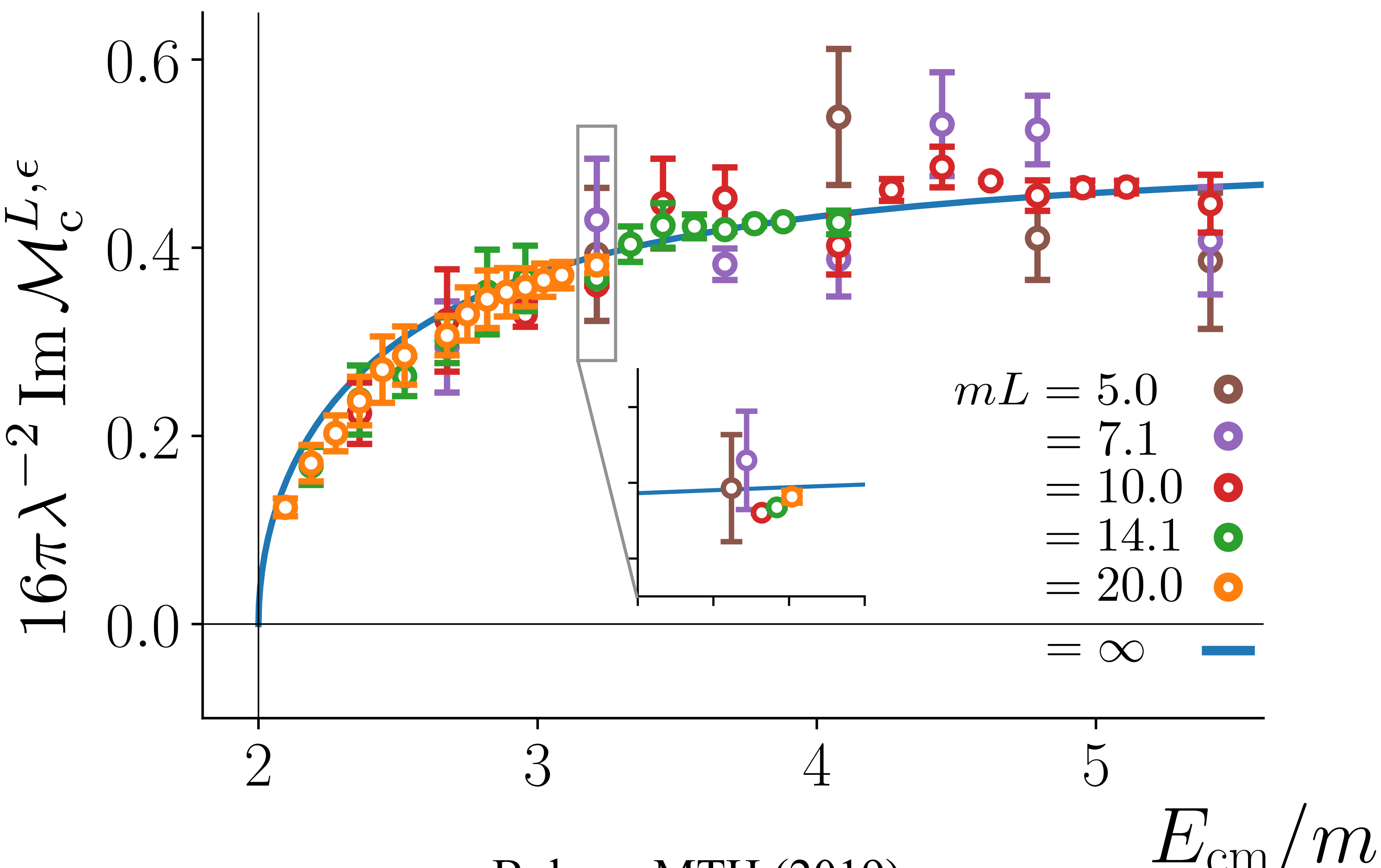


Bulava, MTH (2019)

Connection to Maiani-Testa

- Maiani and Testa considered correlators of the form

$$G_{[\boldsymbol{p}]}(\tau) = \langle \pi_{-\boldsymbol{p}} | \pi_{\boldsymbol{p}}(\tau) J(0) | 0 \rangle$$

- And showed two key points

$$G_{[\mathbf{0}]}(\tau) = \frac{\sqrt{Z_\pi}}{2M_\pi} e^{-M_\pi \tau} f(4M_\pi^2) \left[1 - a_{\pi\pi} \sqrt{\frac{m_\pi}{\pi \tau}} + O(\tau^{-3/2}) \right]$$

$G_{[\boldsymbol{p} \neq \mathbf{0}]}(\tau)$ is plagued by un-physical (operator dependent) contributions

- Recent work with M. Bruno connects this story to spectral function amplitudes

Variations on the Maiani-Testa approach and the inverse problem

M. Bruno[a] and M. T. Hansen[b]

arXiv: 2012.11488 (JHEP)

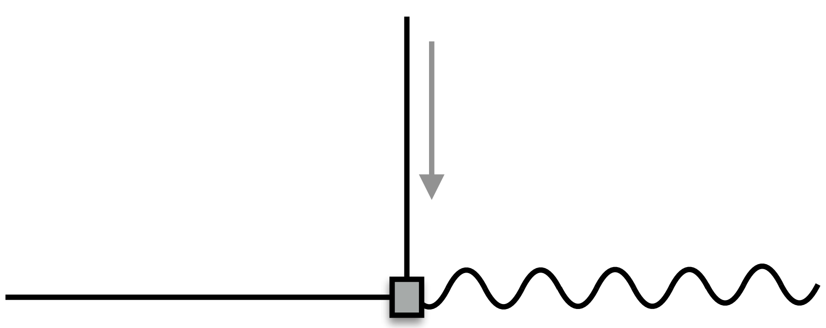
G is a smeared spectral function

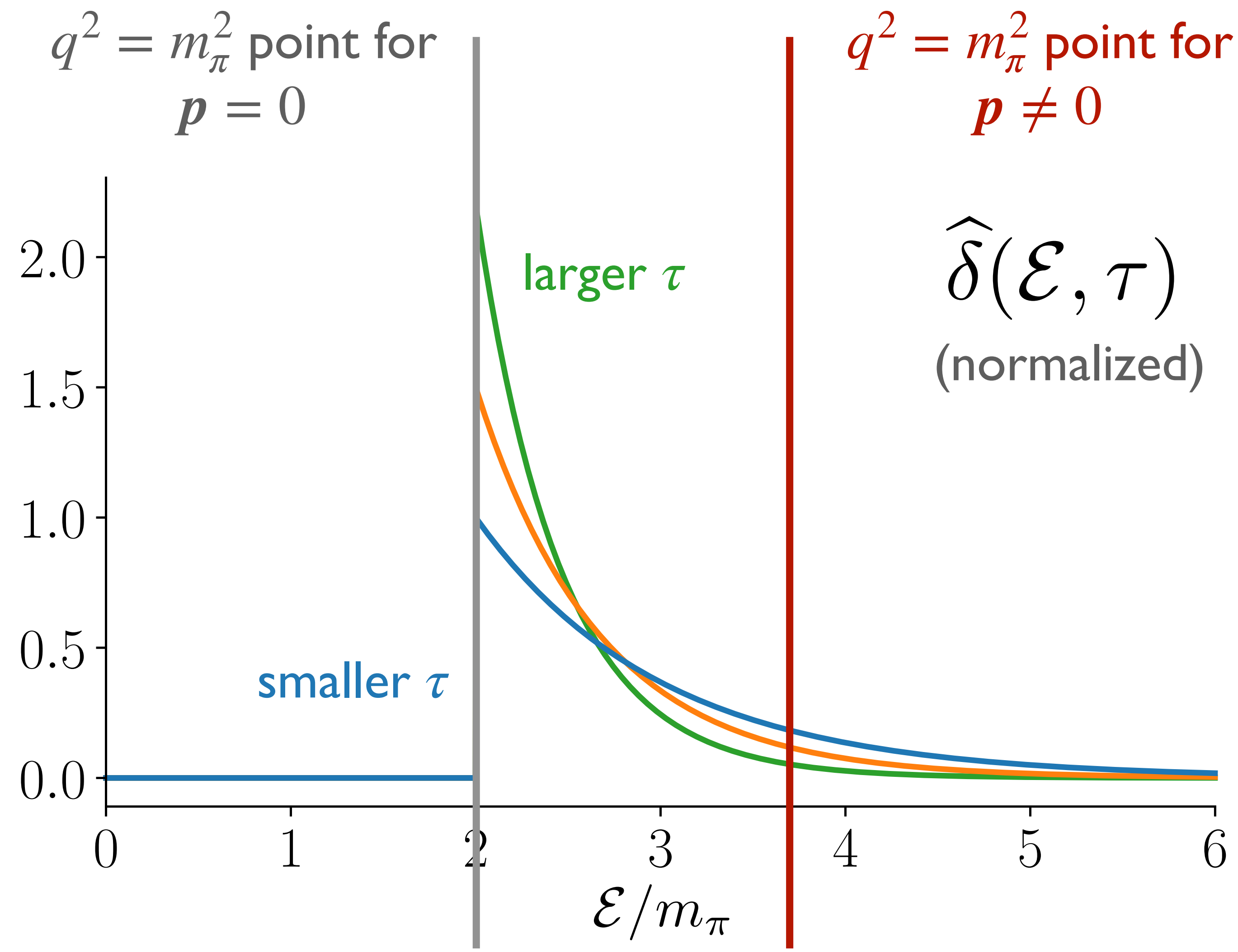
- Correlator can be viewed as a smeared spectral function

$$G_{[\boldsymbol{p}]}(\tau) = \int_0^\infty d\mathcal{E}\, \rho(\mathcal{E})\, \widehat{\delta}(\mathcal{E}, \tau) \qquad \widehat{\delta}(\mathcal{E}, \tau) = \theta(\mathcal{E} - 2m_\pi)\exp[-\mathcal{E}\tau]$$

$$\rho(\mathcal{E})$$

$$q^2 = (\mathcal{E} - \omega_{\boldsymbol{p}})^2 - \boldsymbol{p}^2$$

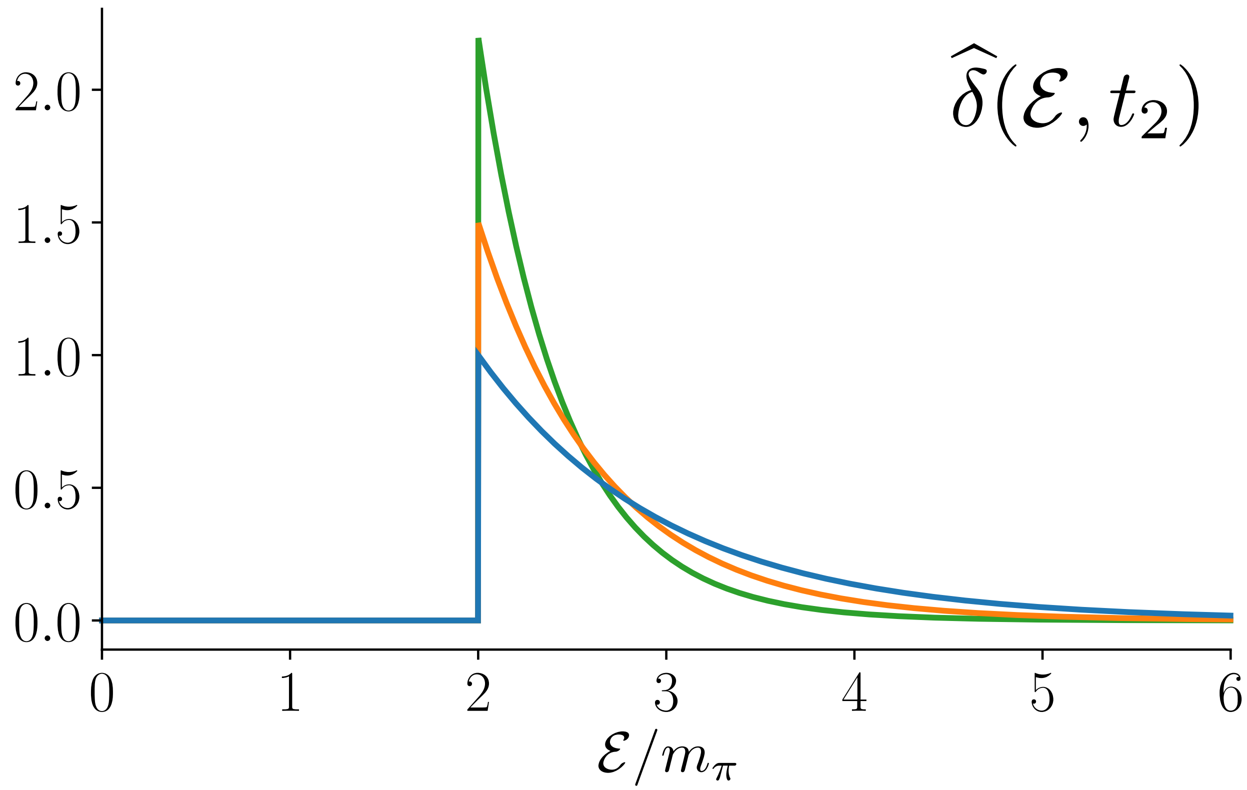




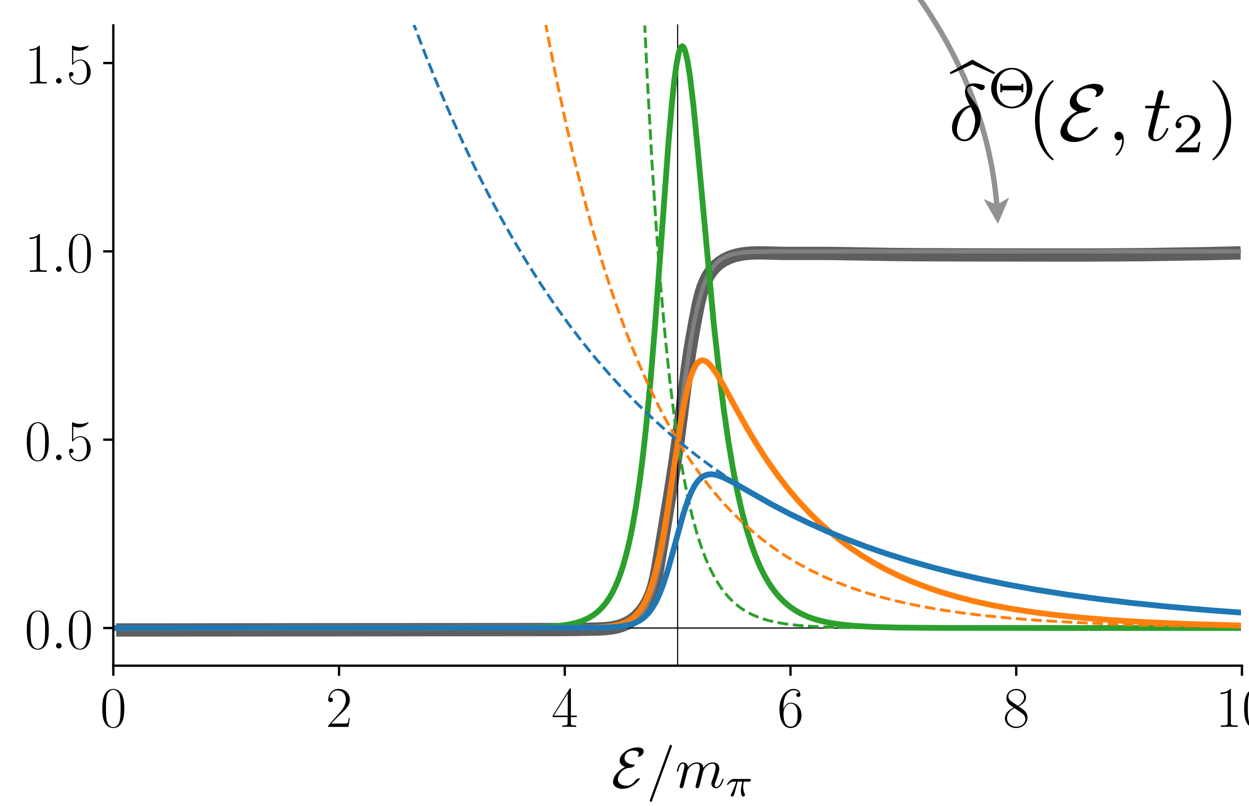
Bruno, MTH (2020)

Shift the peak!

$$\widehat{\delta}(\mathcal{E},\tau) = \theta(\mathcal{E} - 2m_\pi)\exp[-\mathcal{E}\tau]$$



$$\widehat{\delta}^{\Theta}(\mathcal{E},\tau) = \Theta(\mathcal{E} - 2m_\pi, \Delta)\exp[-\mathcal{E}\tau]$$



Bruno, MTH (2020)

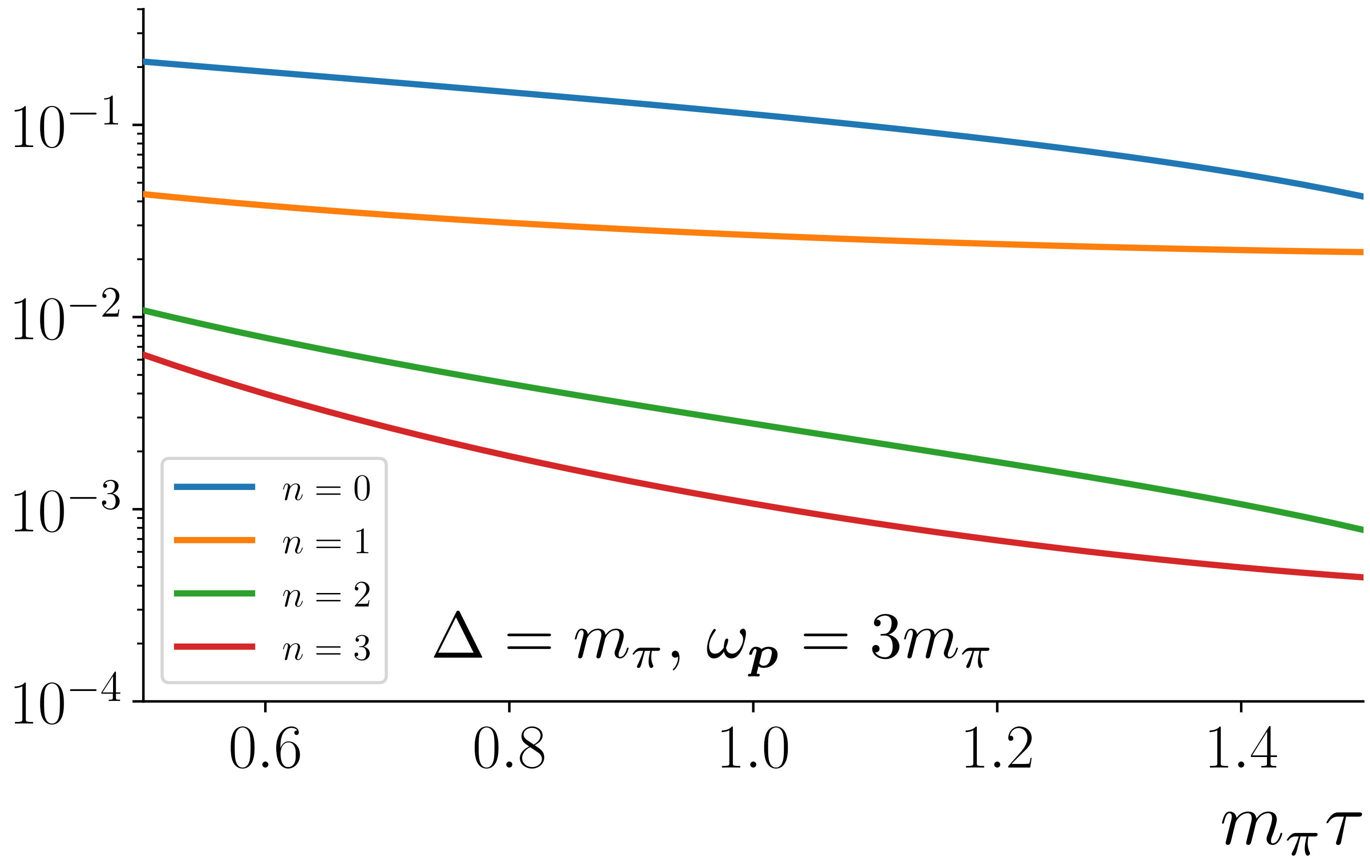
Reaching above threshold

- So the Maiani and Testa correlator becomes

$$G^{\Theta}_{[\boldsymbol{p}]}(\tau) = \langle \pi_{-\boldsymbol{p}} | \pi_{\boldsymbol{p}}(\tau)\, \Theta(\hat{H} - 2\omega_{\boldsymbol{p}}, \Delta)\, J(0) | 0 \rangle$$

- Now separating the fields gives something useful above threshold

$$G^{\Theta}_{[\boldsymbol{p}]}(\tau) = \frac{\sqrt{Z_\pi}}{2\omega_{\boldsymbol{p}}} e^{-\omega_{\boldsymbol{p}}\tau} \Big[\Theta(0,\Delta)\, \mathrm{Re} f(4\omega_{\boldsymbol{p}}^2) - 2\mathcal{J}^{(0)}(\tau,\Delta)\, \mathrm{Im} f(4\omega_{\boldsymbol{p}}^2) + \cdots \Big]$$



Hierarchy of $\mathcal{J}^{(n)}$ provides a useful fit function

Constructing the Θ correlator

$$G^{\Theta}_{[\boldsymbol{p}]}(\tau) = \langle \pi_{-\boldsymbol{p}} | \pi_{\boldsymbol{p}}(\tau)\, \Theta(\hat{H} - 2\omega_{\boldsymbol{p}}, \Delta)\, J(0) | 0 \rangle$$

- Two main methods:
  - Backus-Gilbert and HLT method
  - GEVP

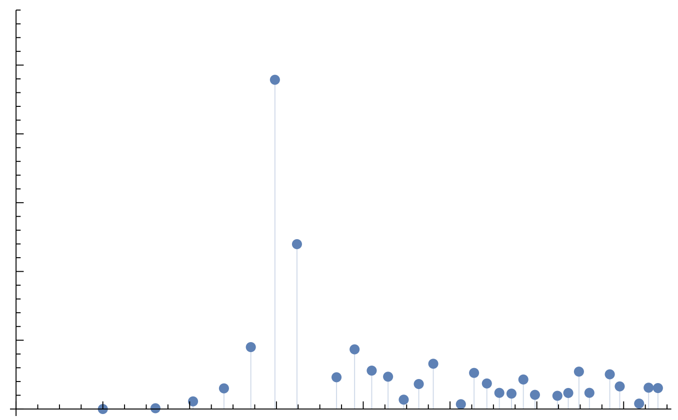
$$G^{\Theta}_{[\boldsymbol{p}]}(\tau) = \sum_n \Theta(E_n - 2\omega_{\boldsymbol{p}}, \Delta) e^{-(E_n - \omega_{\boldsymbol{p}})\tau} \langle \pi_{-\boldsymbol{p}} | \pi_{\boldsymbol{p}}(0) | n \rangle \langle n | J(0) | 0 \rangle$$

combine finite-volume energies and matrix elements

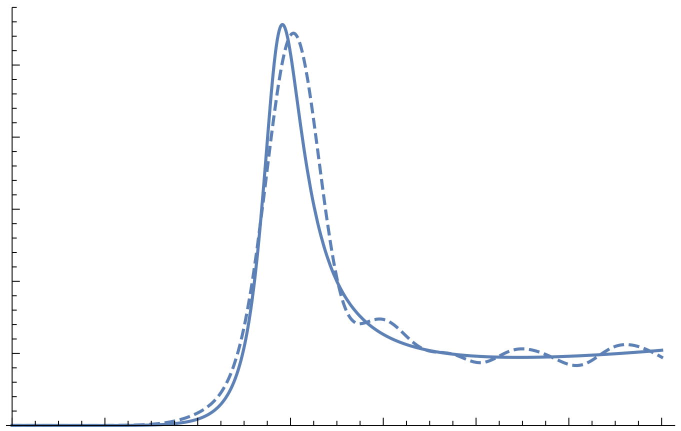
- Volume effects?… Suppressed as $e^{-\Delta L}$, but more investigation is needed

Summary

- Cannot solve the inverse problem, we can get $\widehat{\rho}_{L,\Delta}(\bar{\omega}) \equiv \int_0^\infty d\omega \, \widehat{\delta}_\Delta(\bar{\omega}, \omega) \, \rho_L(\omega)$
- Smearing is needed anyway to ***suppress volume effects***

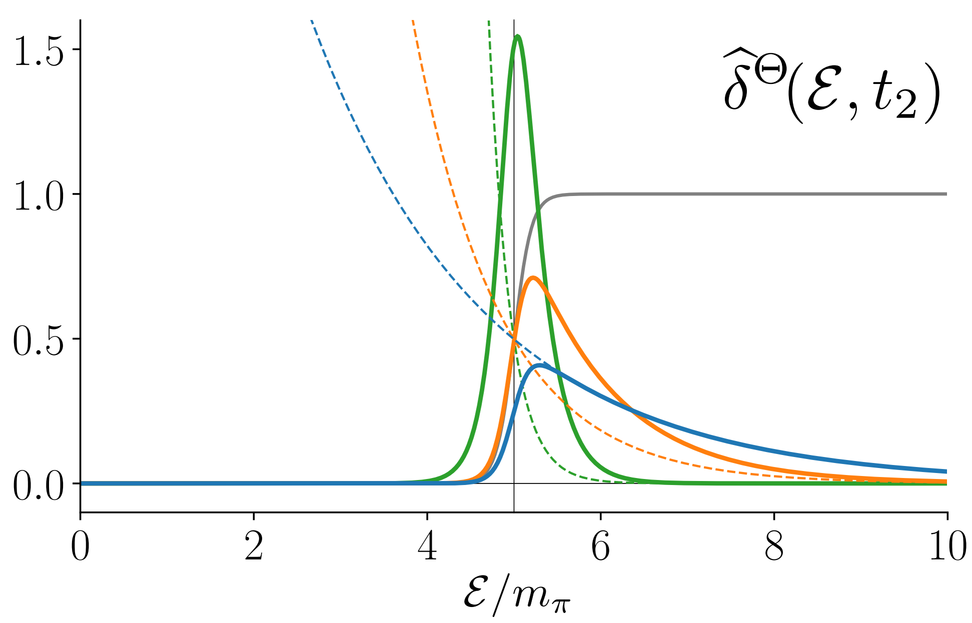


$$1/L \ll \Delta \ll \mu_{\text{physical}}$$



- Generalized Backus-Gilbert takes $\widehat{\delta}_\Delta^{\text{target}}(\bar{\omega}, \omega)$ as input

- Can extract smeared versions of
  — Inclusive and semi-inclusive rates
  — Scattering amplitudes



- Closely related to the Maiani-Testa result

- *Can reconstruction this actually work?*

Monte-Carlo test

- Full lattice calculation in two-dimensional O(3) non-linear sigma model

- Demonstrating the modified Backus-Gilbert (HLT) method for the "R-ratio"

$$C(t) \equiv \int d\boldsymbol{x}\, \langle\Omega|\, \hat{j}_1^a(0,\boldsymbol{x})\, e^{-\hat{H}t}\, \hat{j}_1^a(0)\, |\Omega\rangle = \int_0^\infty d\omega\, e^{-\omega t}\, \rho(\omega)$$

- Data + theory driven analysis of finite-L and -T effects and discretization

ID	$(L/a) \times (T/a)$	β	$am_\star$	$m_\star L$	$m_\star T$
A1	640×320	1.63	0.0447989(62)	29	14
A2	1280×640	1.72	0.0257695(31)	33	17
A3	1920×960	1.78	0.0176104(31)	34	17
A4	2880×1440	1.85	0.0112608(29)	32	16
B1	5760×1440	1.85	0.0112607(73)	65	16
B2	2880×2880	1.85	0.0112462(72)	32	32

Bulava, MTH, Hansen, Patella, Tantalo (2021), arXiv:*2111.12774*

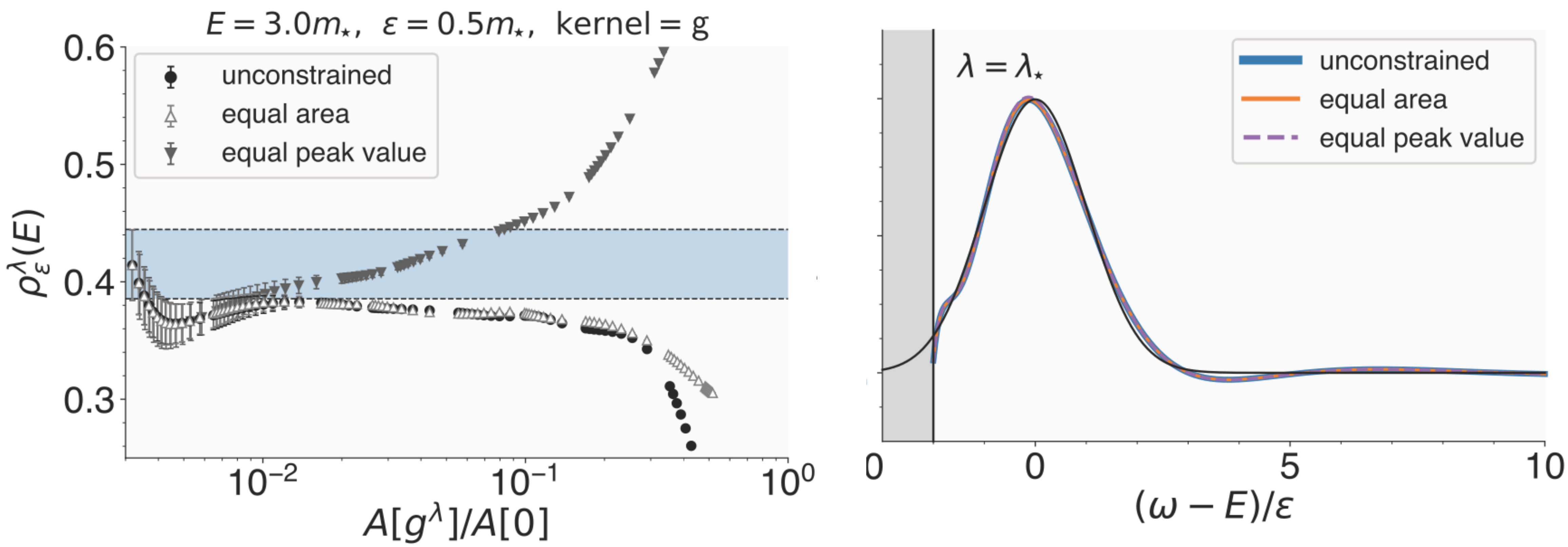
Monte-Carlo test

- Construct different smearings of $\rho(\omega)$

$$\rho_\epsilon^\lambda(E) = \int_0^\infty d\omega\, \delta_\epsilon^\lambda(E,\omega)\, \rho(\omega)$$

$$\delta_\epsilon^{\mathrm{g}}(x) = \frac{1}{\sqrt{2\pi}\epsilon} \exp\left[-\frac{x^2}{2\epsilon^2}\right], \qquad \delta_\epsilon^{\mathrm{c0}}(x) = \frac{1}{\pi}\frac{\epsilon}{x^2+\epsilon^2},$$

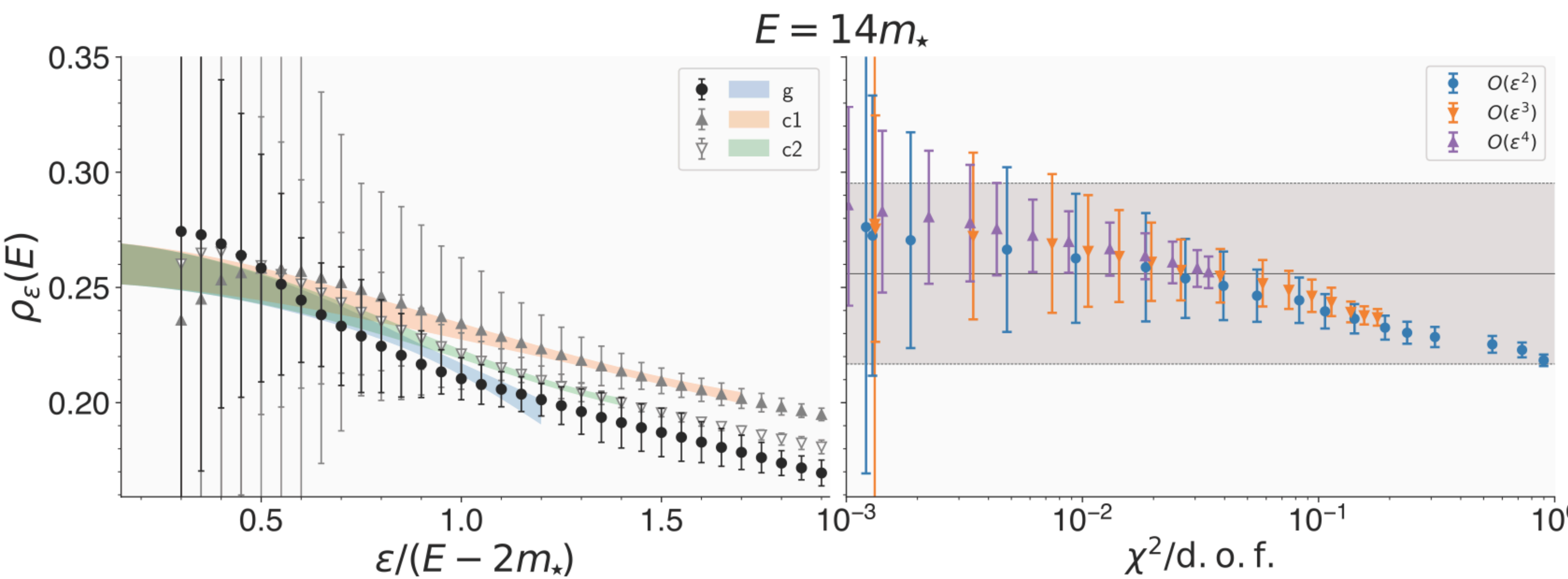
$$\delta_\epsilon^{\mathrm{c1}}(x) = \frac{2}{\pi}\frac{\epsilon^3}{(x^2+\epsilon^2)^2}, \qquad \delta_\epsilon^{\mathrm{c2}}(x) = \frac{8}{3\pi}\frac{\epsilon^5}{(x^2+\epsilon^2)^3}.$$



Bulava, MTH, Hansen, Patella, Tantalo (2021), arXiv:*2111.12774*

Extrapolation

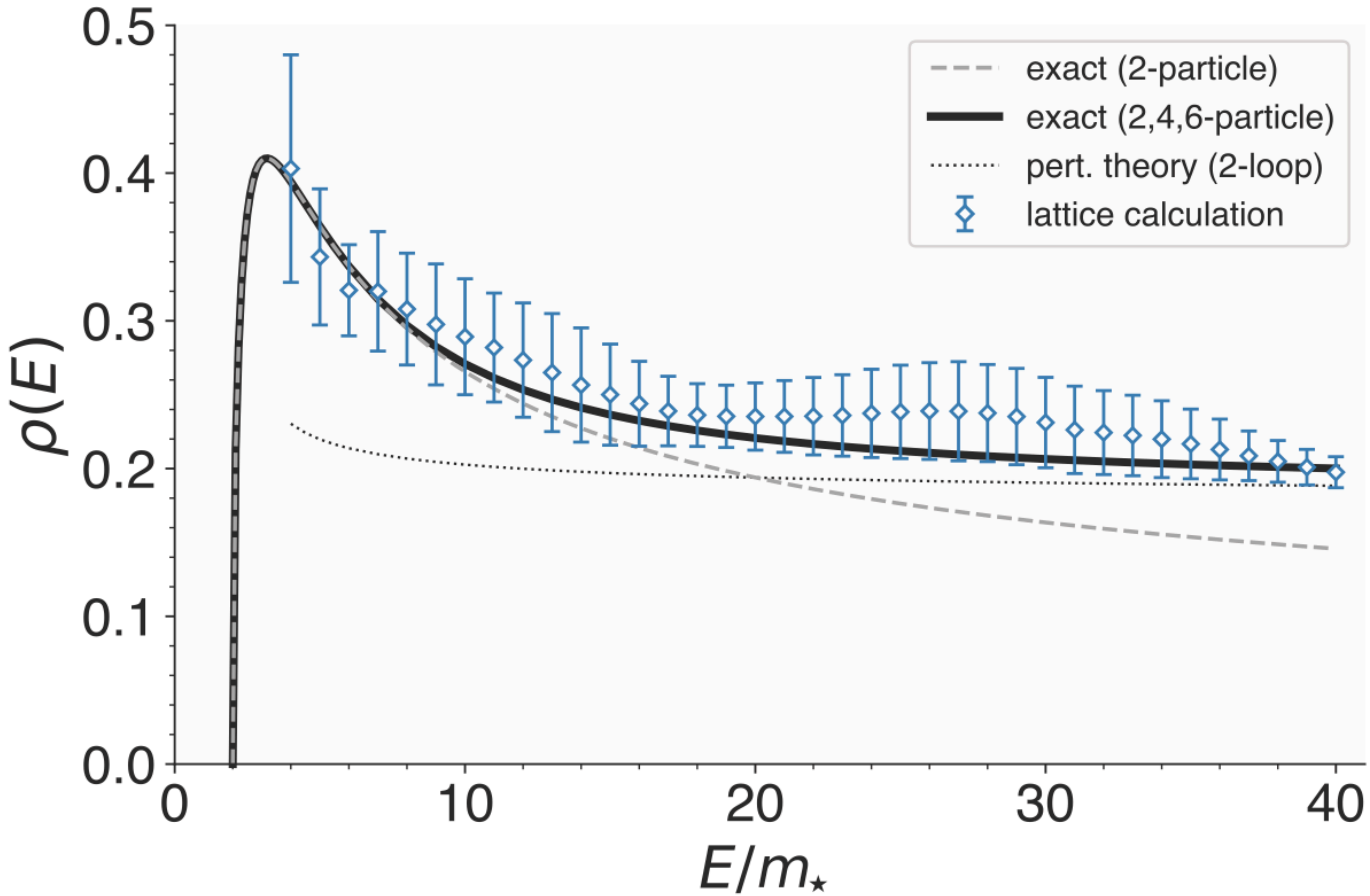
- Targeting $\rho(E)$ for $E = 14m$ here



- Use known relations between different smearing kernels

Bulava, MTH, Hansen, Patella, Tantalo (2021), arXiv:*2111.12774*

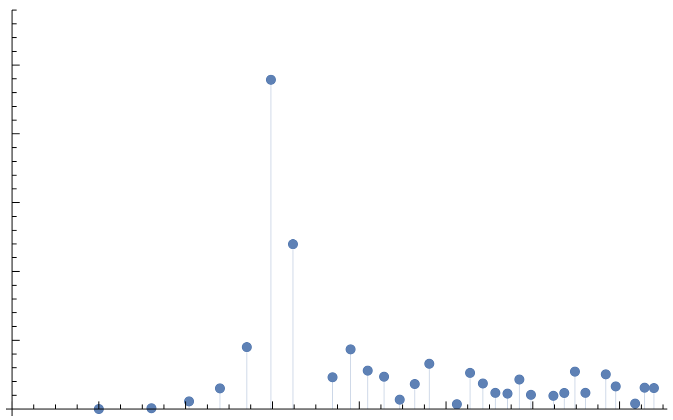
Result



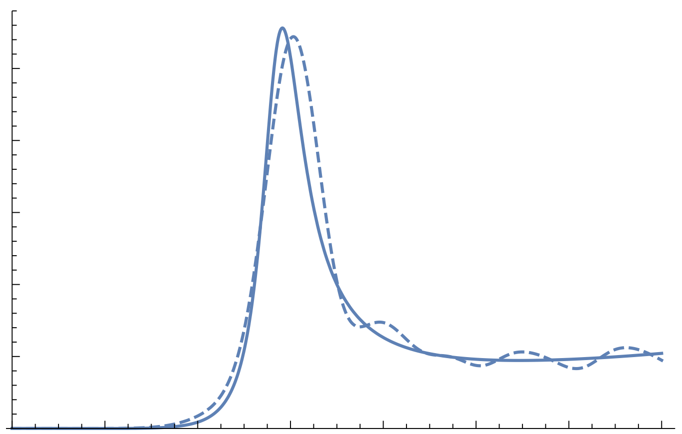
Bulava, MTH, Hansen, Patella, Tantalo (2021), arXiv:*2111.12774*

Conclusions

- Cannot solve the inverse problem, we can get $\widehat{\rho}_{L,\Delta}(\bar{\omega}) \equiv \int_0^\infty d\omega\, \widehat{\delta}_\Delta(\bar{\omega},\omega)\, \rho_L(\omega)$
- Smearing is needed anyway to ***suppress volume effects***



$$1/L \ll \Delta \ll \mu_{\text{physical}}$$



- Generalized Backus-Gilbert (HLT) takes $\widehat{\delta}_\Delta^{\text{target}}(\bar{\omega},\omega)$ as input
- Unlocks many observables

This has unlocked a ***playground*** of calculations that we ***just beginning*** to explore

Thanks!