Calculating Nature Naturally: Quantum Simulating Quantum Fields

Natalie Klco

Brookhaven Quantum Journal Club

November 22, 2021

Predicament:

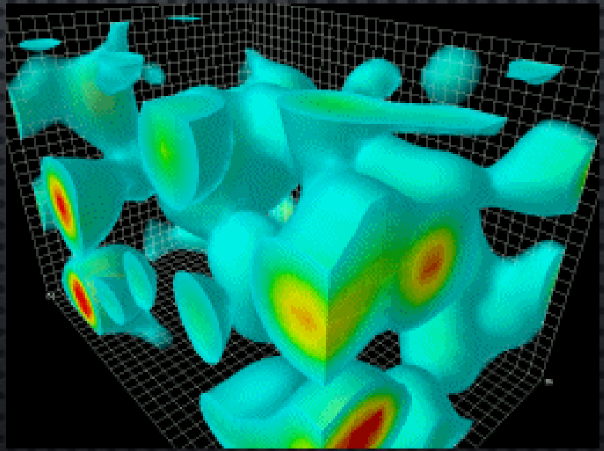
- Quantum mechanics successfully describes every experiment we have performed in the quantum world

- In all known formulations of quantum mechanics, classical calculations require exponential compute resources with the number of particles

ORNL Summit

~340 tons
5,600 sq. ft.
185 miles of fiber optics

HPCwire

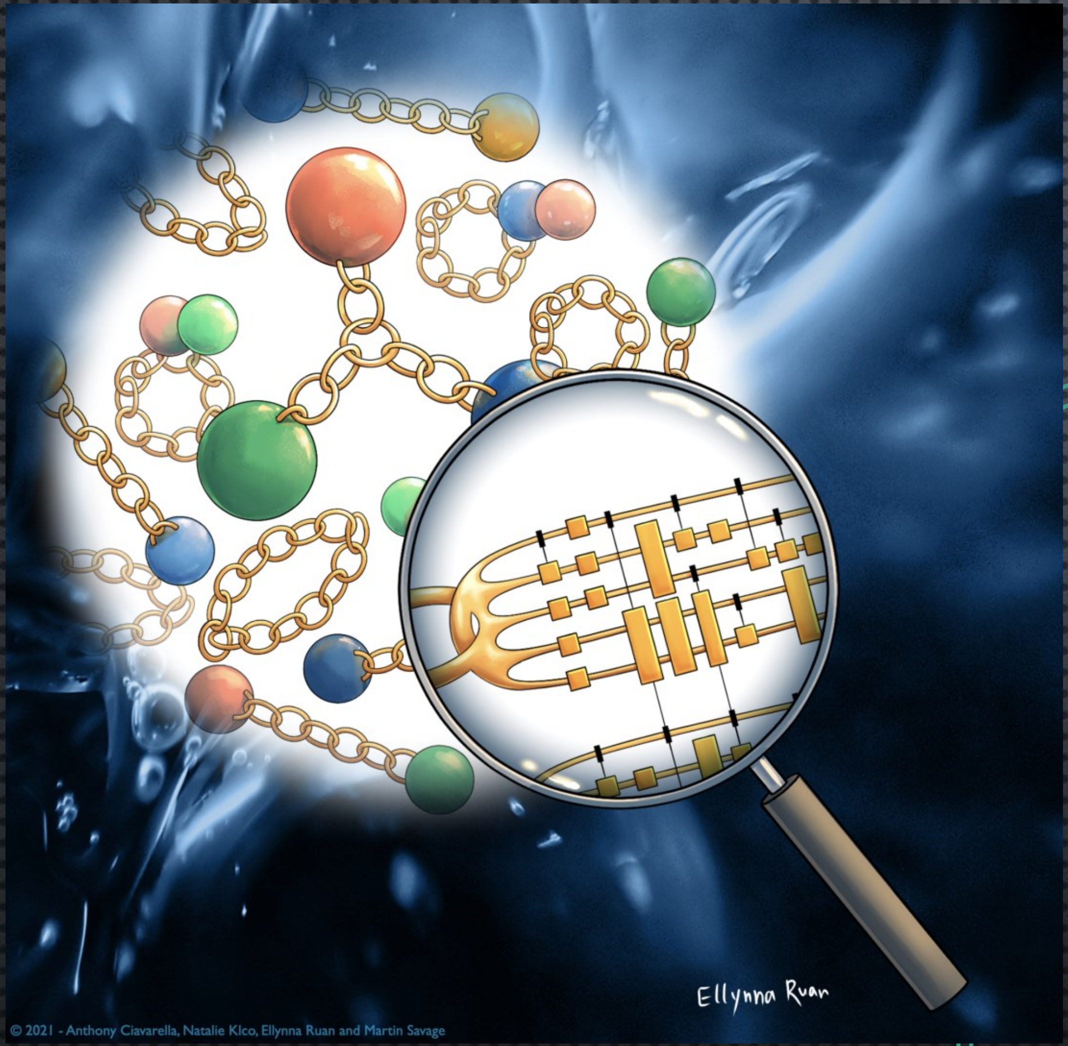


Derek Leinweber

Gluon field configurations in QCD vacuum
(action density)

2.4 x 2.4 x 3.6 fm

Forced to use large portions of spacetime to calculate properties of infinitesimal pieces of themselves.



Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis

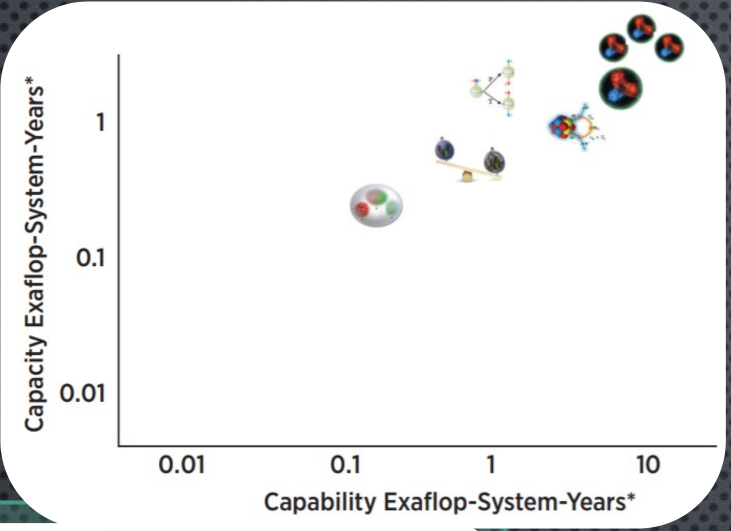
Anthony Ciavarella, Natalie Klco, and Martin J. Savage
Phys. Rev. D **103**, 094501 – Published 4 May 2021

Question: If quantum mechanical degrees of freedom are incorporated into computational frameworks, is that enough to create a computationally efficient description of nature?

Beyond theoretically, how can we do it?

DOE EXASCALE REQUIREMENTS REVIEW — NP/ASCR

2016 estimates for 2025 computing requirements



Exotic decays · Gluonic structure · EDM · 0νββ

Precision g_A, and charge radii and electromagnetic form factors · Three-nucleon forces

* Exaflop-system-year refers to the total amount of computation produced by an exascale computer in 1 year.

IBM

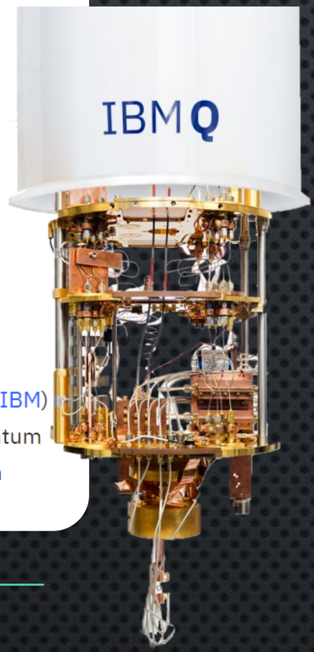
IBM Makes Quantum Computing Available on IBM Cloud to Accelerate Innovation

Users can run experiments on an IBM quantum processor

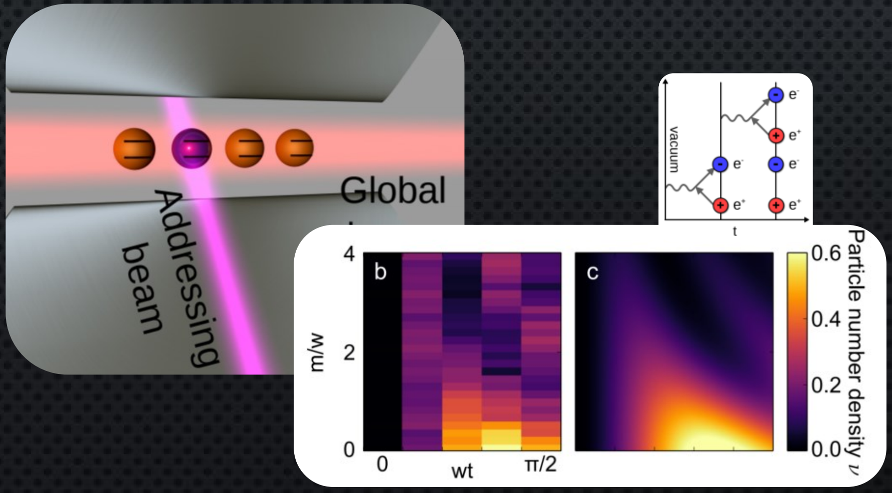
May 3, 2016

PR Newswire
YORKTOWN HEIGHTS, N.Y.

YORKTOWN HEIGHTS, N.Y., May 4, 2016 /PRNewswire/ -- IBM (NYSE: IBM) Research announced today that for the first time ever it is making quantum computing available to members of the public, who can access and run experiments on IBM's quantum processor.



Summer 2016



Simulation of Small U(1) Lattice Gauge Theory with Trapped Ions

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

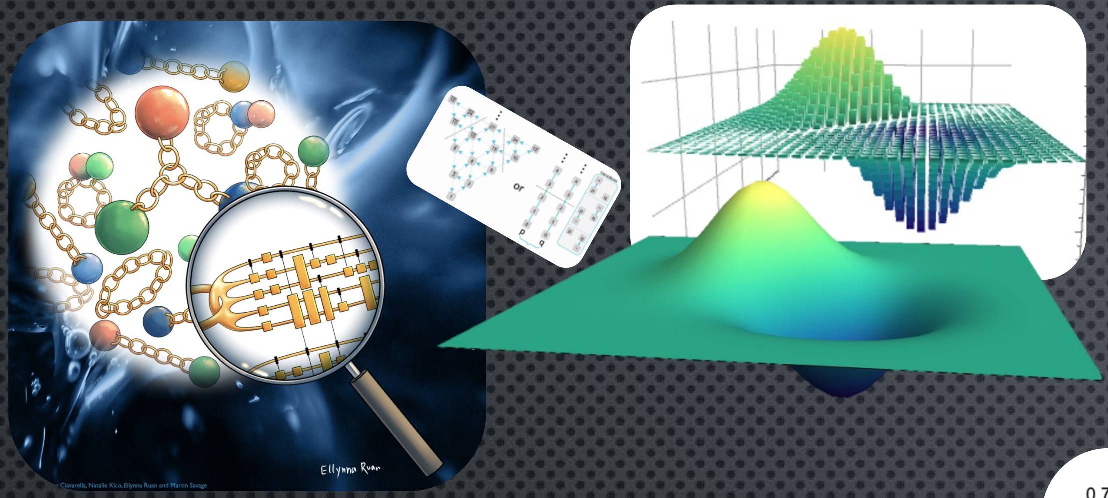
Esteban A. Martinez,[1,*] Christine Muschik,[2,3,*] Philipp Schindler,[1] Daniel Nigg,[1] Alexander Erhard,[1] Markus Heyl,[2,4] Philipp Hauke,[2,3] Marcello Dalmonte,[2,3] Thomas Monz,[1] Peter Zoller,[2,3] and Rainer Blatt[1,2]

The Vision:

Exploring dynamical, sub-atomic quantum phenomena through precision control of atomic-scale quantum devices

Dynamically Evolving Parameter Space

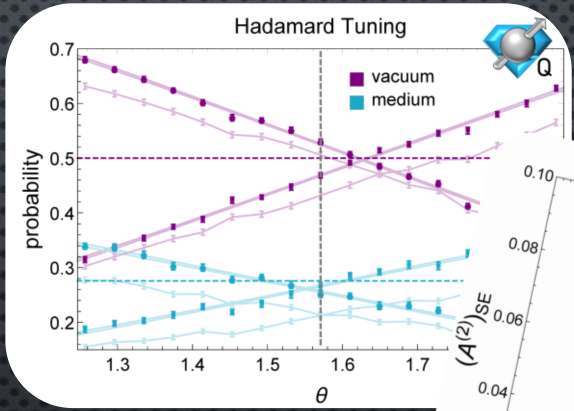
High Dimensional Optimization of Hilbert Space Design

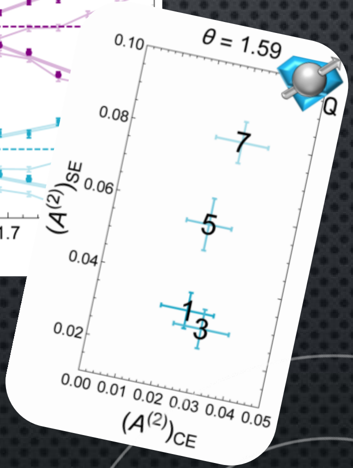


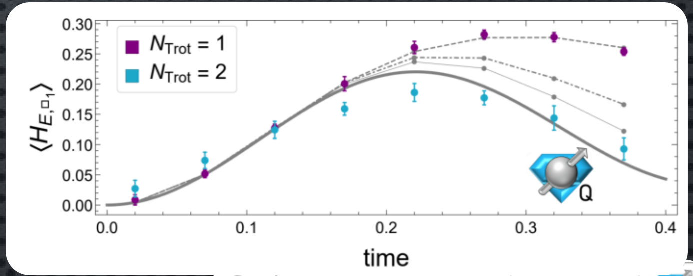
Fundamental Physics or Large-Scale Error-Corrected Computation

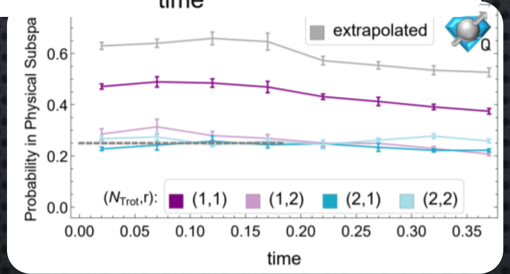
Representing fields on quantum devices

- Build intuition for the high-dimensional optimization
  - Qubits, coherence time, gate fidelities, ...
- "Spending" quantum resources on physics
  - i.e., not field truncations, boundary conditions etc.
- Naturally Mapping + Enforcing Physical Subspaces
  - Hierarchical Qubit Maps (e.g., UV/IR)







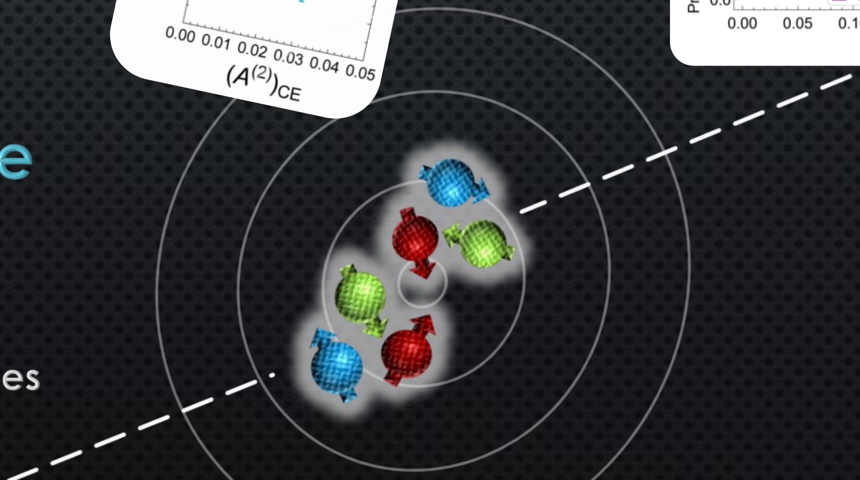


Small-scale simulations

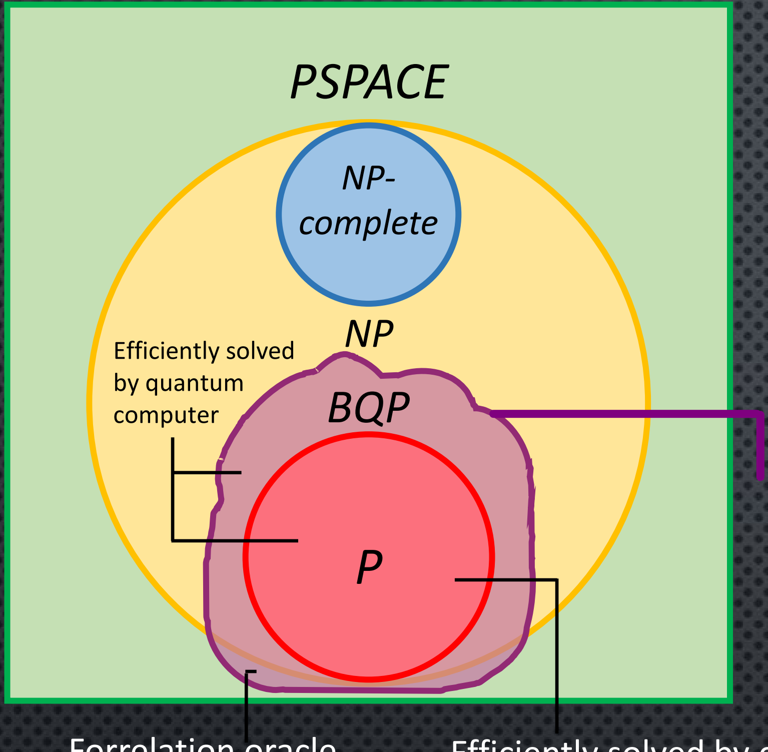
- Natural Hardware Interactions
- Codesign with developing hardware
- Error mitigation strategies
  - Vacuum/medium operations
  - Stochastic error exacerbation + extrap
  - Workflow: temporally correlated in-vivo calibrations
  - Gauge invariant subspace survival
- Maintain strong connection to reality

Entanglement structure of nature

- Guides quantum simulation design
- Informs EFT construction
  - Symmetries affect Ent. Fluctuations
  - Distillable: sensitive to Λ_{UV} at long distances
- Natural quantum architecture



Aaronson (Sci. Am.)



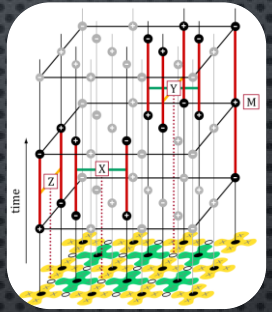
Role of Quantum Fields

computation ~ quantum field

nature ~ quantum field

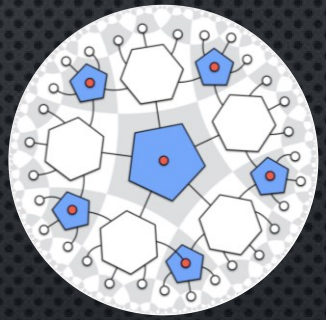
qubit array ~ quantum field

Vac-vac $\lambda\phi^4$+ classical sources (Jordan, Krovi, Lee, Preskill) 2018



Surface Codes
Kitaev (1997)

Holographic Codes
Pastawski, Yoshida, Harlow, Preskill (2015)



Analog Simulators

- Q Sim. efficient for local Hamiltonians (Feynman, Lloyd)
- Scattering efficient--massive $\lambda\phi^4$, Gross Neveu--precision, energy, particle #, coupling strength (Jordan, Lee, Preskill)
- BQP Hard: Vacuum-to-Vacuum in massive $\lambda\phi^4$ with classical sources. Map all of BQP. (Jordan, Krovi, Lee, Preskill)
- BQP Complete: universal for QC (Jordan, Krovi, Lee, Preskill)

"BQP Leading Order" and Entanglement Hierarchies

arXiv:2107.04769

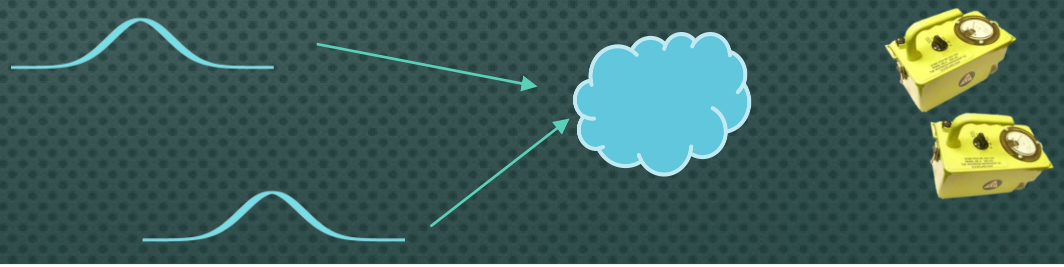
Where is the line?

Many Complementary Trajectories for Incorporation of Quantum Devices

Quantum Computation of Scattering in Scalar Quantum Field Theories

Stephen P. Jordan (NIST, Wash., D.C. and Caltech), Keith S.M. Lee (Pittsburgh U. and Caltech), John Preskill (Caltech)

Published in: Quantum Information and Computation 14, 1014-1080 (2014), *Quant.Inf.Comput.* 14 (2014) 1014-1080



Simulating collider physics on quantum computers using effective field theories

Christian W. Bauer* and Benjamin Nachman†
Marat Freytsis‡

PhysRevLett.127.212001

$$\sigma = H \otimes J_1 \otimes \cdots \otimes J_n \otimes S$$

$$\langle X | \mathrm{T}[Y_n \, Y_{\bar{n}}^{\dagger}] | \Omega \rangle$$

Wilson Line in Gauge Field

Soft Function

Leveraging similar dynamical quantum computational techniques

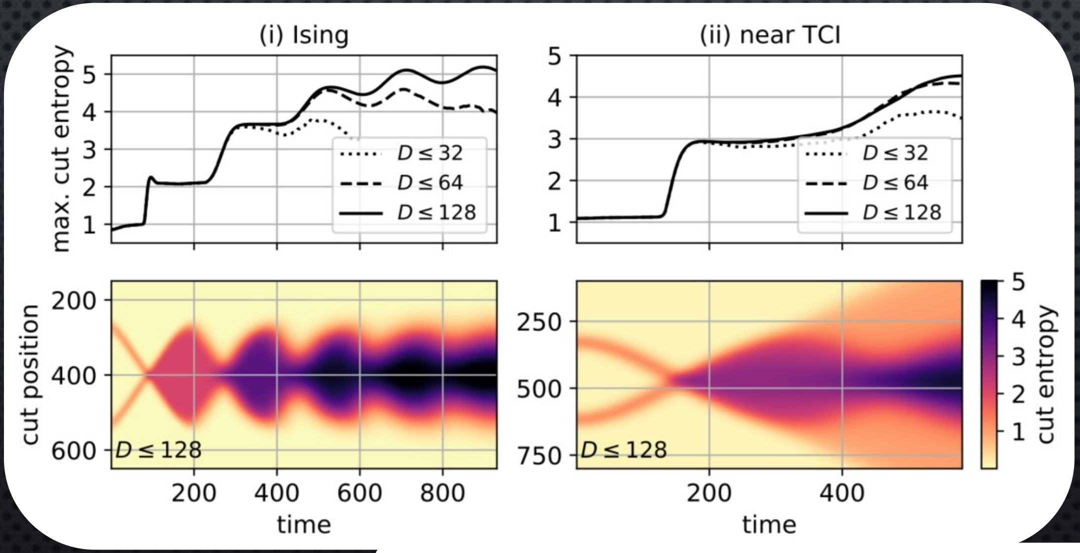
Entanglement

~~Perplexing Burden~~

Valuable Resource

Guiding Principle for Simulation Design

Dynamics of entanglement across bipartitions



growing MPS
bond dimension

arXiv:2012.07243v1

Collisions of false-vacuum bubble walls in a quantum spin chain

Ashley Milsted,[1,2,3,4,*] Junyu Liu,[1,2,†] John Preskill,[1,2,4] and Guifre Vidal[3,5]

Vacuum Field Entanglement

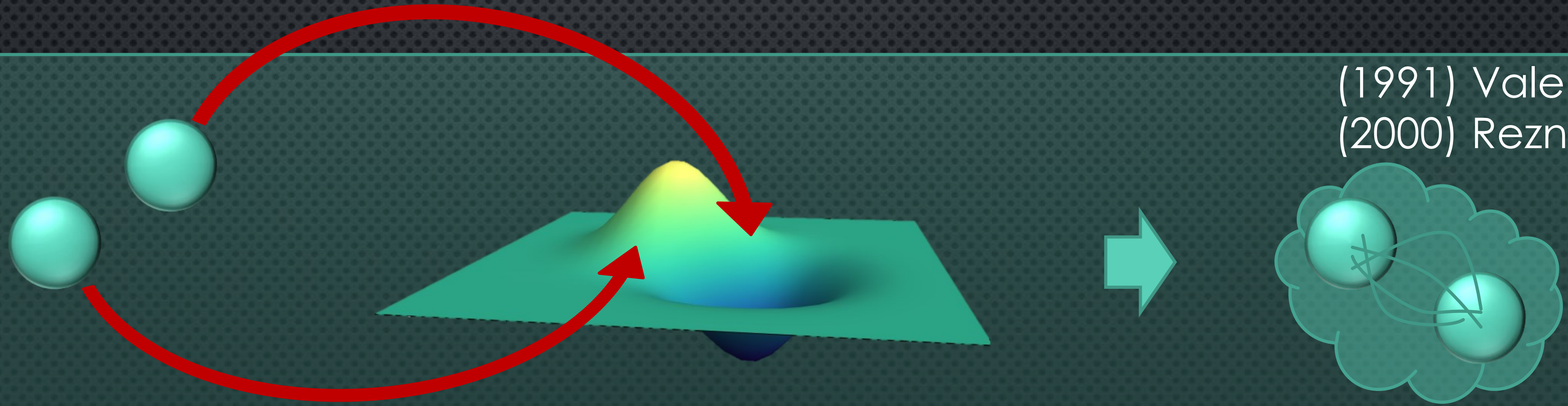
(1961) Reeh-Schlieder

only if $|\chi\rangle = 0$

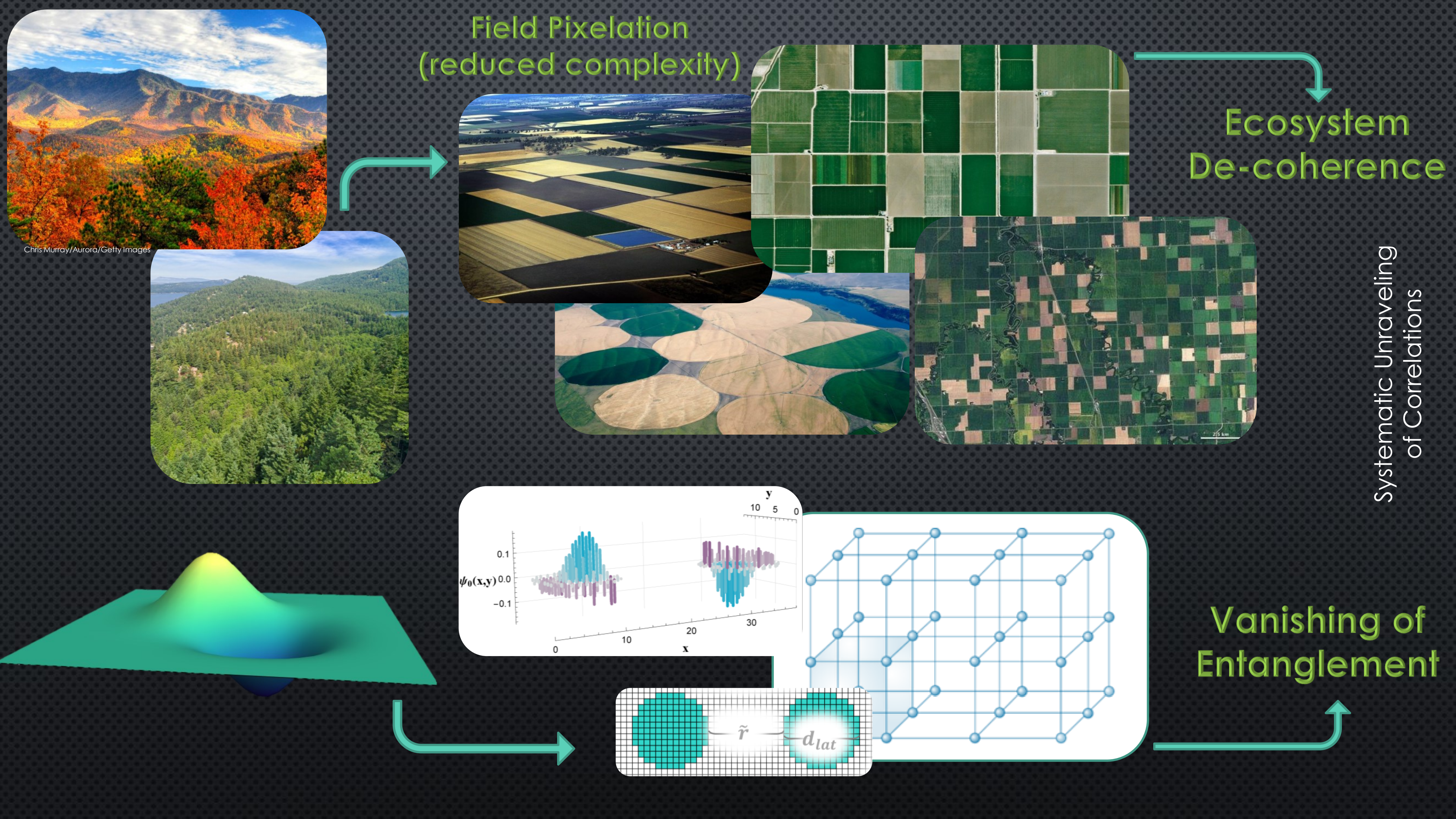
$$\langle \chi | \phi(x_1)\phi(x_2)\cdots\phi(x_n)|\Omega\rangle \neq 0, \quad x_1,\ldots,x_n \in \mathcal{U}_{\mathcal{V}}$$

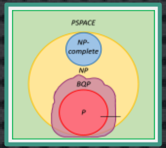
Dense exploration of the field vacuum sector through local operators on the vacuum

(1991) Valentini
(2000) Reznik

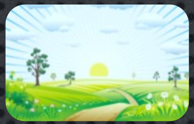


Distillation of vacuum entanglement to EPR pairs


Field Pixelation
(reduced complexity)
Chris Murray/Aurora/Getty Images
Ecosystem
De-coherence
Systematic Unraveling
of Correlations
y
10
5
0
0.1
$\psi_0(x,y)$ 0.0
−0.1
0
10
20
30
x
$\tilde{r}$
d_{lat}
Vanishing of
Entanglement



Motivation: Free scalar a simple toy model



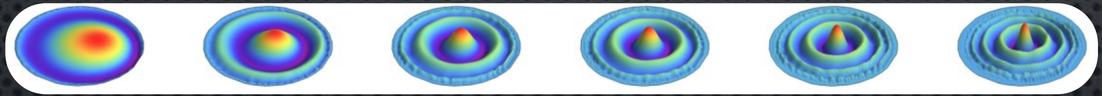
1D: Marcovitch + (2008)
2D: NK, Savage (2020)
3D: NK, Savage (2021)

$\frac{\tilde{r}}{d} > 1$

Regime of experimental interest is outside of CFT analytic control

Distillable Entanglement ⬆ $\mathcal{N} \sim e^{-\beta\frac{\tilde{r}}{d}}$ ⬆ Decay Rate

Exp. decay from poly/log
2-point correlation functions
(sign problem)



Long distance entanglement sensitive to UV physics

Conjecture: $\mathcal{N}_{\mathcal{N}} \sim e^{-\Lambda d}$

➡ Effective field theory design for coherent observables

State Preparation
in "reverse"

"Bound Entangled"
⬆
Distribute entangled pairs + separable resource
➡ Local unitaries ➡ vacuum state

Consolidate entanglement into pairs through local operations

Peres (1996)
Horodecki^3 (1996)
Eisert-Plenio (1999),
Vidal-Werner (2002)

Negativity

Entanglement Monotone
Sufficient (~~necessary~~) for indicating entanglement
Upper bound to distillable entanglement
UV, IR finite

~~Entanglement Entropy~~ : mixed states
~~Mutual Information~~ : Quantum Correlations

Density Matrices Positive Definite:

$$\rho > 0 \quad \rho^T > 0$$

$$\rho_A \otimes \rho_B > 0$$

Partial Transpose

$$\rho_A \otimes (\rho_B)^T = (\rho_A \otimes \rho_B)^{\Gamma_B} > 0$$

$$(\rho_{AB})^{\Gamma_B} < 0$$

Detects inseparability

Physically: local $p \to -p$ transformation

Simon (2000)

One of an infinite family of positive maps/symmetry transformations that a separable state would be required to satisfy

Negativity Tasseography

Upper bound for distillable entanglement

PPT $\mathcal{N} = 0$

Is it distillable? **No**

Is it entangled? **No** ~~?~~

NPT $\mathcal{N} \neq 0$

Is it entangled? **Yes**

Is it distillable? **Yes** ~~Maybe~~

free field vacuua are not generic states

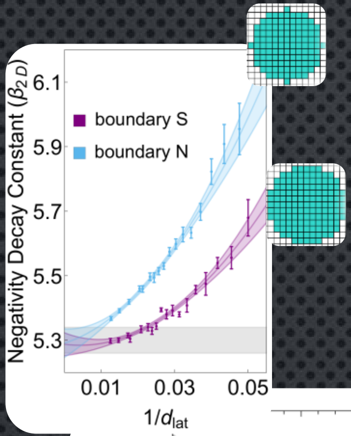
…is it all distillable? **Yes, Yes**

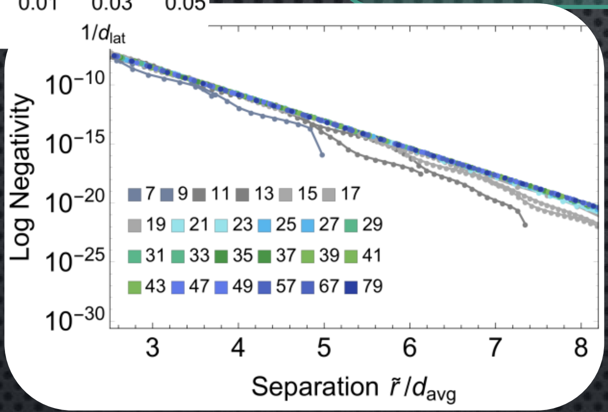
…is there more?

arXiv:2110.10736

Exponential Decay

Entanglement Sphere





Marcovitch+ (2008)

$\beta_{1D} = 2.82(3)$

$\beta_{2D} = 5.29(4)$

$\beta_{3D} = 7.6(1)$

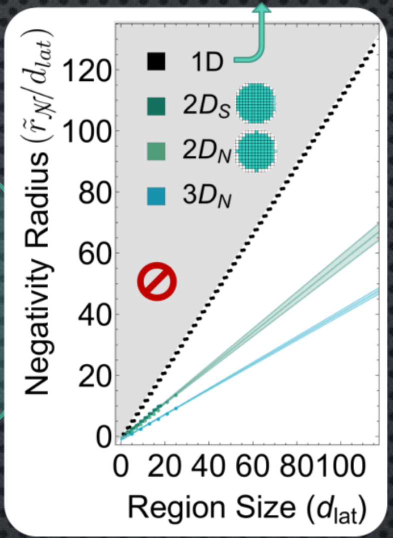
$\beta_D \sim D$

Rapid Decay in Higher Dimension

D-dependent

$$\mathcal{N} \sim e^{-\beta \frac{\tilde{r}}{d}}$$

$$\frac{\tilde{r}_{\mathcal{N}}}{d} \sim \left(\frac{\gamma}{a}\right) d$$



$\gamma_{1D} = 1.114(2)$

$\gamma_{2D} = 0.60(2)$

$\gamma_{3D} = 0.43(2)$

$\gamma_D \sim 1/D$

Reduced Radius in Higher Dimension

$$\mathcal{N}_{\mathcal{N}} \sim e^{-\beta\gamma \frac{d}{a}}$$

$\beta\gamma$	
$1D$:	3.14(3)
$2D$:	3.2(1)
$3D$:	3.2(1)

Numerical conjecture

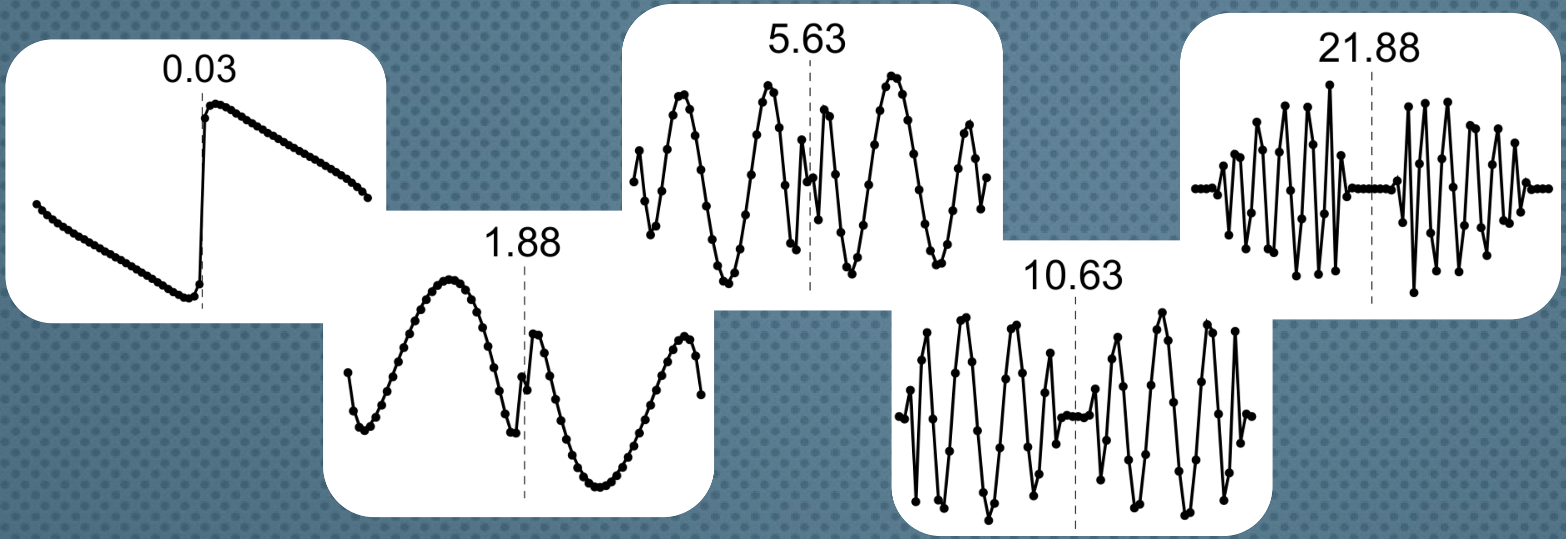
Suggests a saturation of information in the field description for supporting non-locality

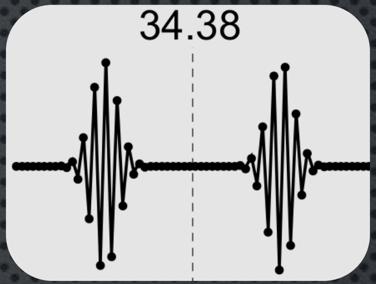
UV trunc.

$$\Lambda = \frac{\pi}{a}$$

$$\mathcal{N}_{\mathcal{N}} \sim e^{-\beta\gamma \frac{d}{a}} \sim e^{-\pi \frac{d}{a}} \sim e^{-\Lambda d}$$

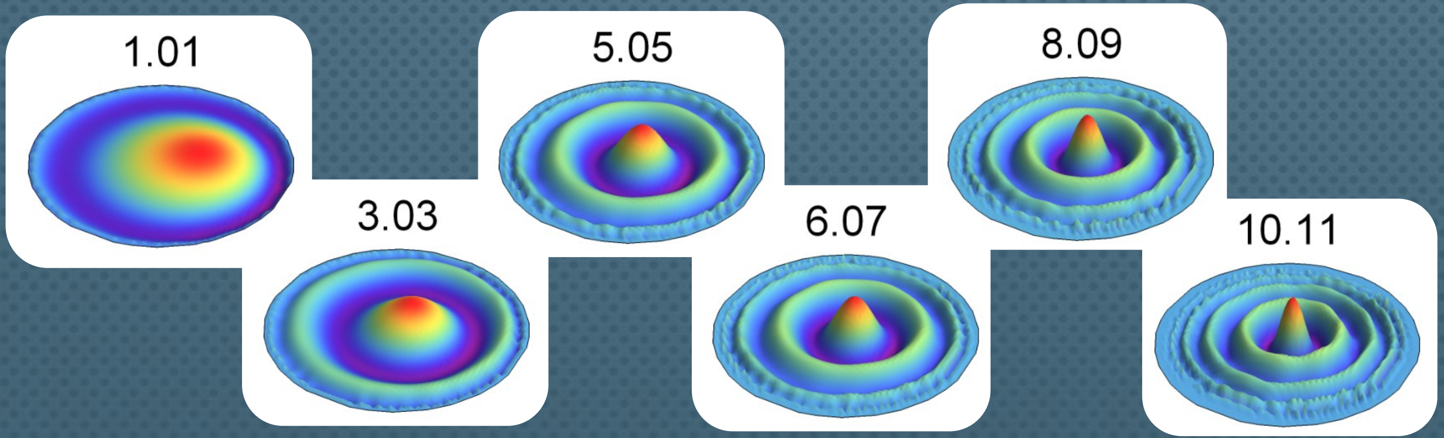
1D:





Separable

2D:



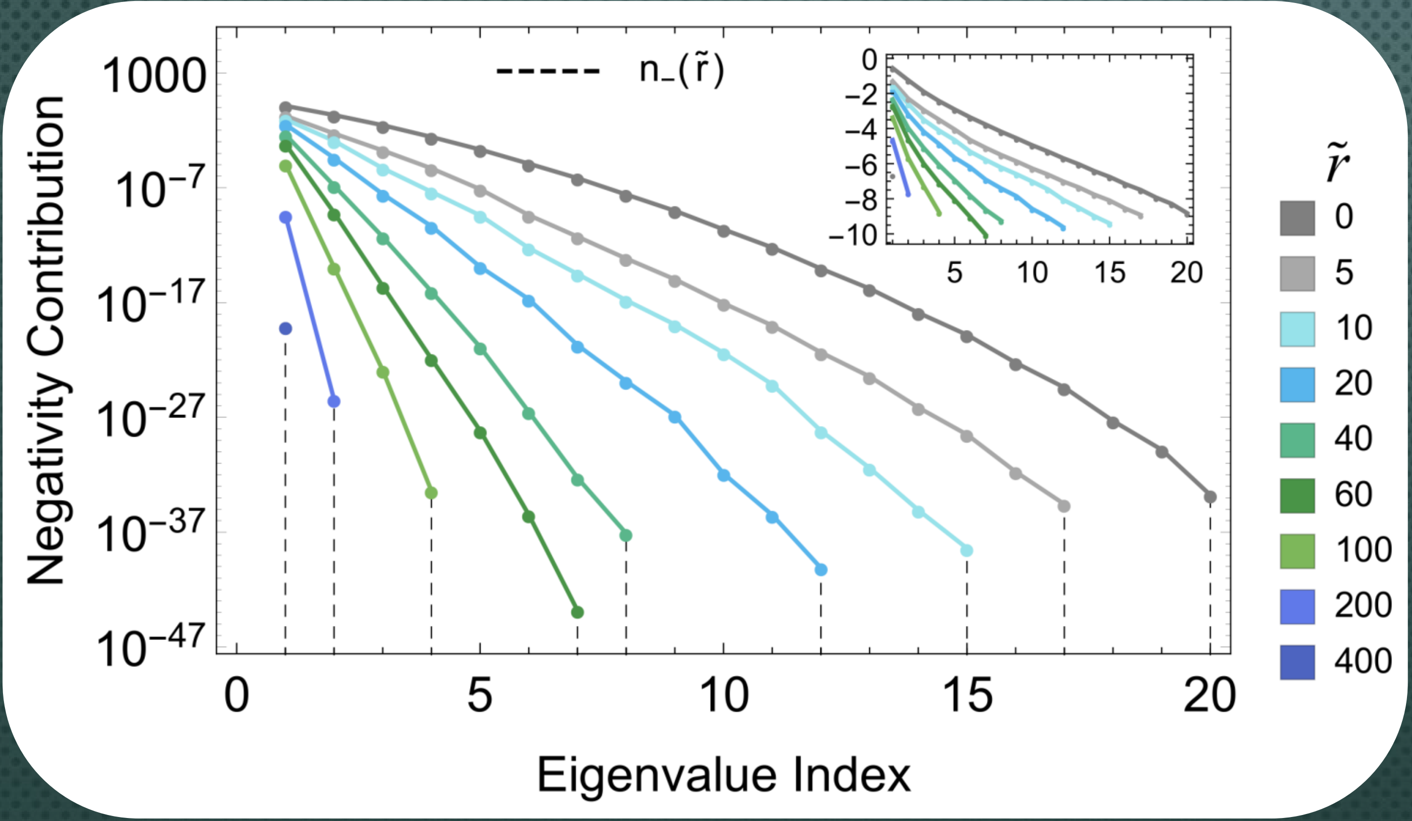
Fluctuations at the lattice scale outside of entanglement sphere

long-distance quantum correlations probe short-distance physics

Inside the Entanglement Sphere

(Symplectic Spectrum)

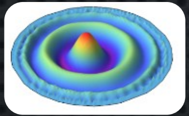
1D, m = 0.003, d = 30



$$\mathcal{N} = -\sum_{j=1}^{n} \log_2 \min(\nu_i^\Gamma, 1)$$

$$\boldsymbol{\nu}^\Gamma = \text{spec}|i\Omega\sigma^\Gamma|$$

- Number of negative eigenvalues reduces with increasing separation
- Single-mode (ground state) precision is high, suggesting systematic truncation viable



--Localize dominant mode through local unitaries?

Consolidating Entanglement

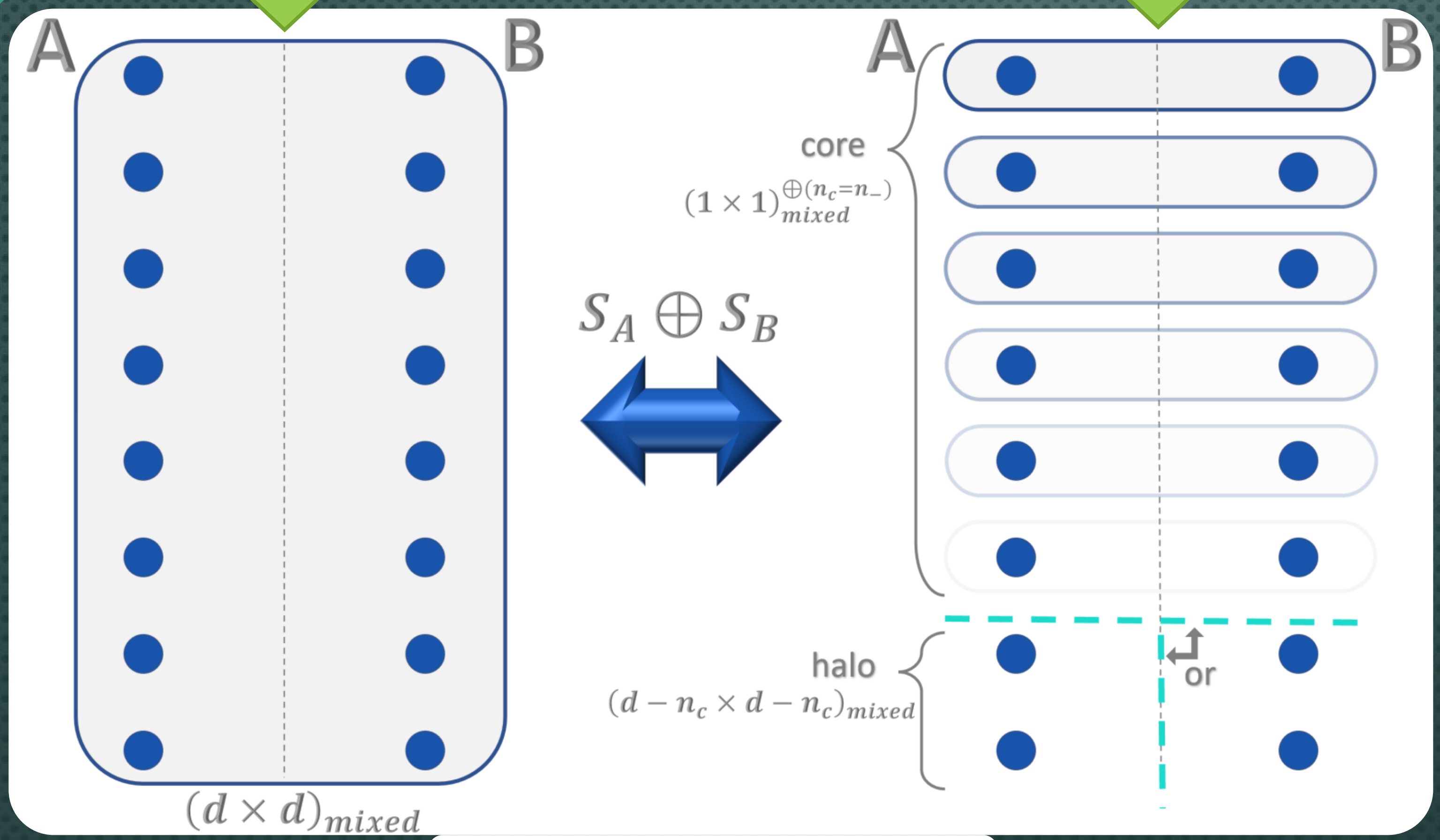
Simplifies Entanglement Structure

$$\mathcal{N} = -\sum_{j=1}^{n} \log_2 \min(\nu_i^\Gamma, 1)$$

$$\boldsymbol{\nu}^\Gamma = \text{spec}|i\Omega\sigma^\Gamma|$$

Genuine high-body entangled

Exponential hierarchy



Entanglement Structures in Quantum Field Theories:
Negativity Cores and Bound Entanglement in the Vacuum
arXiv:2110.10736v1
Natalie Klco,[1,*] D. H. Beck,[2,†] and Martin J. Savage[3,‡]

Locally Transform

$$\sigma' = (S_A \oplus S_B)\sigma(S_A \oplus S_B)^T$$

Create core of Entangled Pairs

$$\sigma' = \sigma_{c,1} \oplus \sigma_{c,2} \oplus \cdots \oplus \sigma_{c,n_c} \oplus \sigma_h + Y$$

Explicitly distillable entanglement

In reverse, provides technique for digital quantum state preparation of vacuum

JLP Identified

$$$$$

…is it all distillable? **Yes,**

…is there more? **Yes**

arXiv:2110.10736

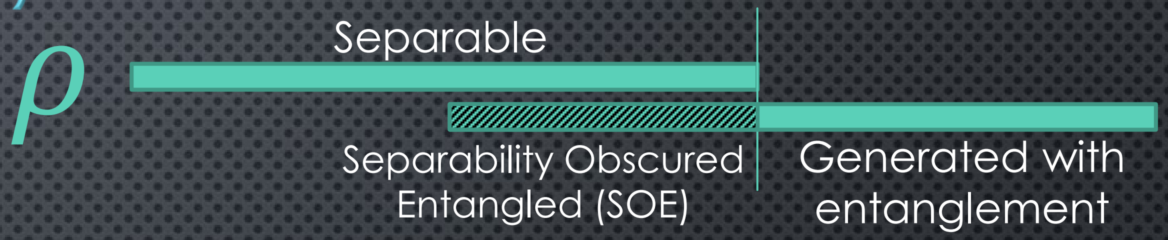
Definition:

$$\rho_{\text{separable}} = \sum_i p_i \ \rho_i^A \otimes \rho_i^B$$

(From a perspective of entanglement avoidance)

(Exasperated) Do I have to?

Separability: No, but you perhaps can!



Classical Noise

$$\rho(\eta) = \eta \ \rho\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) + (1-\eta)\frac{\mathbb{I}}{4}$$

$$= \frac{\eta}{2}\left[\rho\left(|++\rangle_x\right) + \rho\left(|--\rangle_x\right) + \rho\left(|+-\rangle_y\right) + \rho\left(|-+\rangle_y\right)\right]$$
$$+ \frac{1-\eta}{4}\left[\rho\left(|00\rangle\right) + \rho\left(|11\rangle\right)\right] + \frac{1-3\eta}{4}\left[\rho\left(|01\rangle\right) + \rho\left(|10\rangle\right)\right]$$

(Convex $\eta \leq \frac{1}{3}$)

Many physical procedures can lead to the same density matrix (decomps. not unique)

Terminology Quirk: leads to confusing possibilities as

$$separable + classical\ comms. \rightarrow entanglement$$

Separability classifications can obscure entanglement (SOE)

Of little consequence for bipartite systems…

…can cause entanglement to become “bound” in many-body systems

What is bound entanglement?

Clearest Example: PPT inseparable state

Indistillable entangled

Horodecki Phys.Lett. A232 (1997) 333

More Broadly: Operational Difference Between Formation and Distillable Entanglement

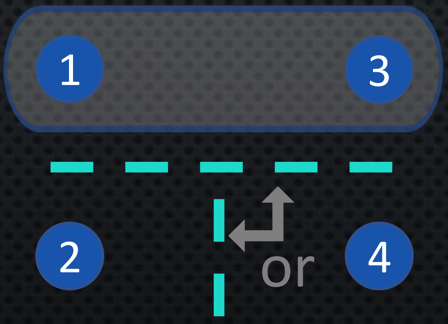
challenging   Upper bound "computable"

"Irreversibility in Quantum Correlations"

Method of Deduction:

Subsystem
- connected separably
- is itself separable in isolation

but requires entanglement to comprise the full QMB state



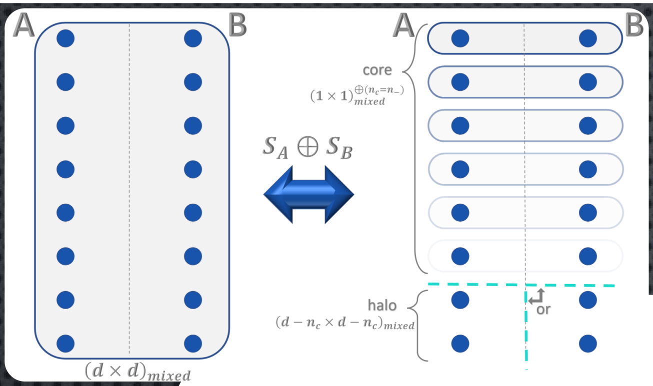
ρ separable: 13 | 24

ρ_{24} separable: 2 | 4

$$\cancel{\rho_{13} \otimes \rho_2 \otimes \rho_4}$$

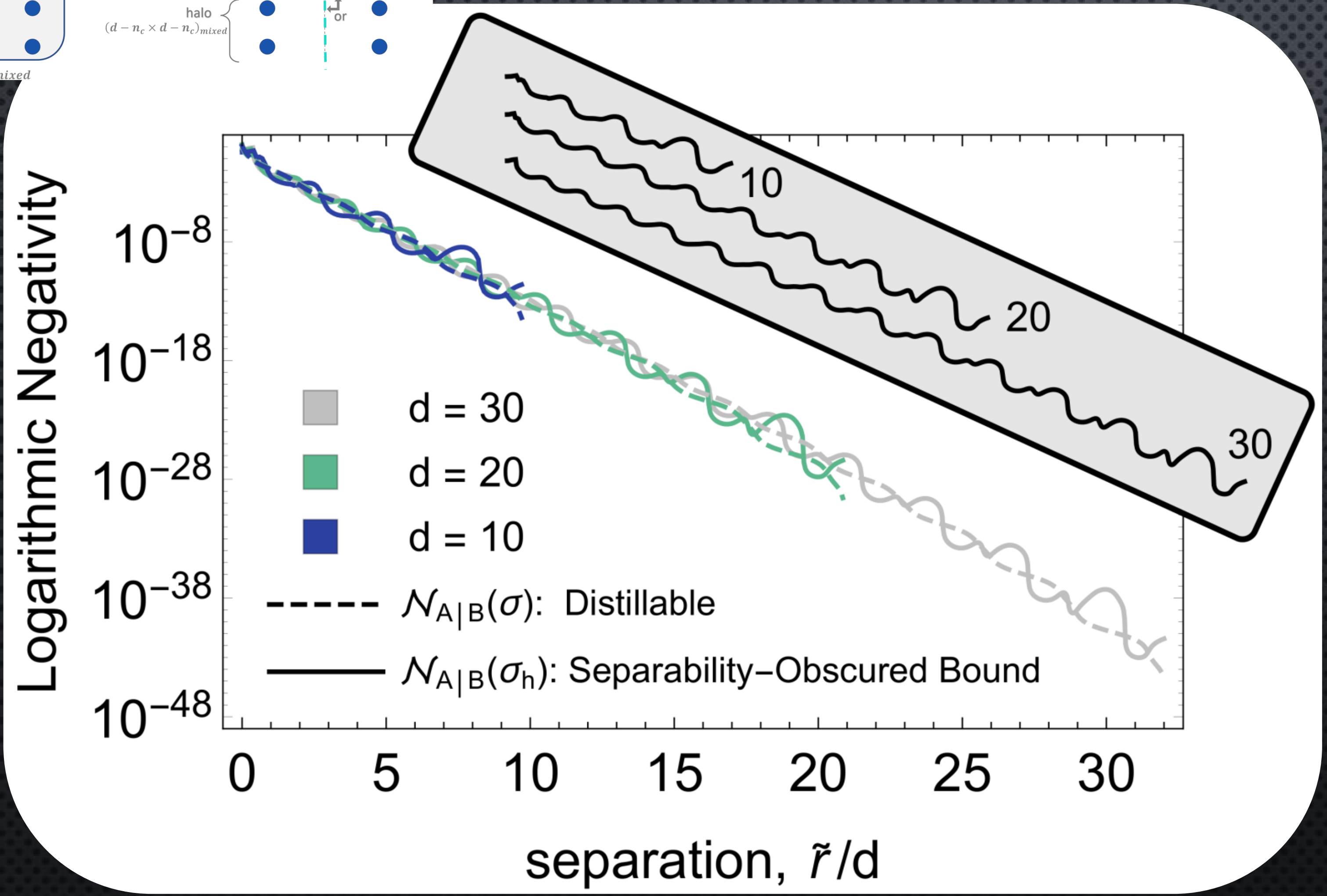
$$\mathcal{N}_{2|134} > 0$$

$$\sigma' = \sigma_{c,1} \oplus \sigma_{c,2} \oplus \cdots \oplus \sigma_{c,n_c} \oplus \sigma_h + Y$$



Free scalar field exhibits bound entanglement

Or more broadly, subsystem entanglement obscured by separability through classical correlations



- Stable toward continuum
- Transition through plateaus
- Magnitude comparable to distillable entanglement

Big Picture

Formulating a calculation for QC requires new perspectives on the role of measurements, entanglement generation, superposition, and interference as computational resources.

The NISQ era provides small devices to begin developing intuition and hardware-specific informed decisions for physics application design

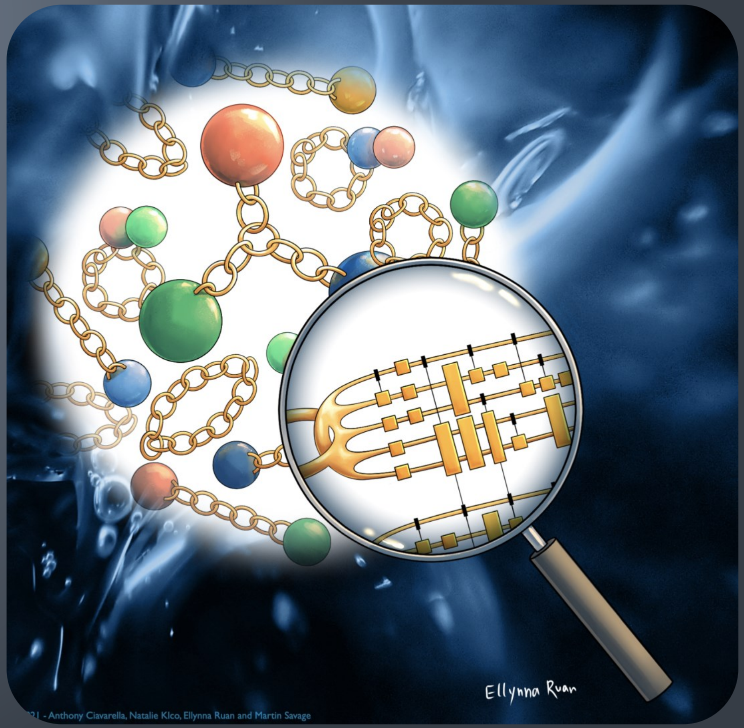
Technical Points:

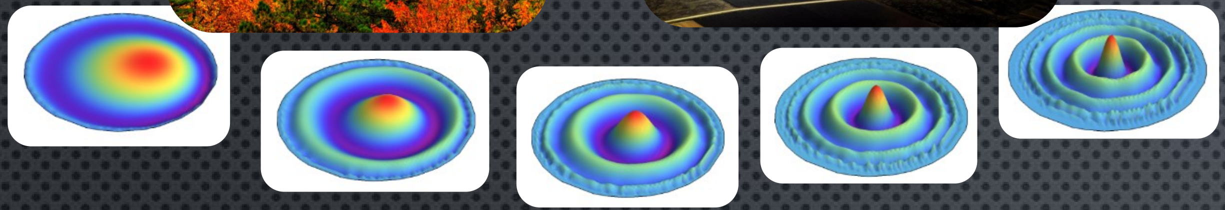
Entanglement

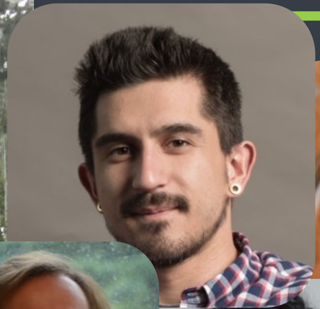
- limits classical dynamical simulations
- guides design of quantum implementations

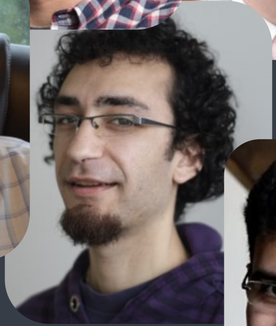
long-distance quantum correlations probe short-distance physics

Entanglement consolidation *identifies bound entanglement* and path for state preparation









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Fin