Quantum-number projection technique for nuclear structure

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RBRC workshop - 25/01/2022

Universidad Autónoma de Madrid

Marie Skłodowska-Curie Actions
TAURUS (ID:839847)

Symmetry group of nuclear Hamiltonian $\hat{H}$

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Definition

Let $G \equiv \{g\}$ be a group with a unitary representation $\hat{R}(g)$

If $\forall\, g \in G,\ \hat{R}(g)\hat{H}\hat{R}^{-1}(g) = \hat{H} \quad \Rightarrow$ G is a symmetry group of $\hat{H}$

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- *Irreps* of G can be used to charaterize the eigenstates of $\hat{H}$

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Important consequences

- *Irreps* of G can be used to charaterize the eigenstates of $\hat{H}$
  λ: irrep of G with dimension d_λ ($k \in [\![1, d_\lambda]\!]$)
- Good quantum numbers: $|\Theta_\epsilon^{\lambda k}\rangle$

Symmetry group of nuclear Hamiltonian $\hat{H}$

- In low-energy nuclear physics:

Physical symmetry	Group	Quant. numb.	Comment
Particle-number inv.	$U(1)_Z \times U(1)_N$	N, Z	
Rotational inv.	$SU(2)_A$	J, M_J	
Parity inv.	Z_{2A}	π	
Translational inv.	T_A^3	$\vec{P}$	
Exchange of particles	$S_Z \times S_N$	-1, -1	Pauli principle
Isospin	$SU(2)_A$	T, M_T	*Only approximate*

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- Nuclear wave function: $|\Theta_{\epsilon}^{NZJM\pi}\rangle$

- Quantum numbers can be determined in low-energy experiments

Symmetry-breaking solutions

- Mean-field calculations: $\delta\langle\Phi|\hat{H}|\Phi\rangle = 0$

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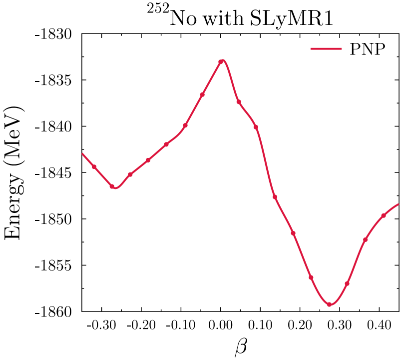
- Symmetry-unrestricted MF calculations favor "deformed" solutions

- Examples: pairing, quadrupole and octupole deformations, ...



Symmetry-breaking solutions

- Mean-field calculations: $\delta\langle\Phi|\hat{H}|\Phi\rangle = 0$
  $|\Phi\rangle \equiv$ Product states (simple wave functions)
- Symmetry-unrestricted MF calculations favor "deformed" solutions
- Examples: pairing, quadrupole and octupole deformations, ...
- Problem: deformed solutions break the symmetries of $\hat{H}$

$$|\Phi\rangle = \sum_{NZJM\pi} \sum_{\epsilon} c_{\epsilon}^{NZJM\pi} |\Psi_{\epsilon}^{NZJM\pi}\rangle$$

$\Rightarrow$ unphysical in nuclei

Symmetry dilemma

- "Symmetry dilemma" of Löwdin

  P. Lykos and G. W. Pratt, Rev. Mod. Phys. 35 496 (1963)

  - MF ansatz respects the symmetries of $\hat{H}$ but is variationally limited
  - MF ansatz is variationally general but breaks the symmetries of $\hat{H}$

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- Examples:

Physical symmetry	Group	Quant. numb.	Correlations
Particle-number inv.	$U(1)_Z \times U(1)_N$	N, Z	Pairing, Finite temp.
Rotational inv.	$SU(2)_A$	J, M_J	Deformation (any)
Parity inv.	Z_{2A}	π	Deformation (odd)
Translational inv.	T_A^3	$\vec{P}$	Localization
Isospin	$SU(2)_A$	T, M_T	Pairing n-p

Solution: restoring the symmetries

- Symmetry-breaking MF $\xrightarrow{\text{reference states}}$ Symmetry-restored BMF

  (BMF ≡ beyond mean field)

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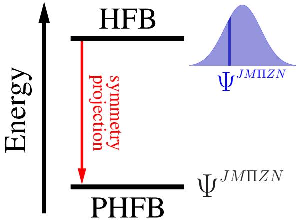
- Quantum-number projection

  B. Bally and M. Bender, PRC 103, 024315 (2021)

  - Obtain symmetry-adapted states (≡ with good quantum numbers)
  - Gain correlation energy (usually)



Symmetry projection: method

- Projection operators

$$\hat{P}^{J}_{MK} = \frac{2J+1}{16\pi^2} \int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \sin(\beta) \int_0^{4\pi} d\gamma\, D^{J}_{MK}{}^{*}(\alpha,\beta,\gamma)\, \hat{R}(\alpha,\beta,\gamma)$$

$$\hat{P}^{NZ} = \frac{1}{4\pi^2} \int_0^{2\pi} d\phi_N \int_0^{2\pi} d\phi_Z\, e^{i\phi_N(\hat{N}-N)} e^{i\phi_Z(\hat{Z}-Z)}$$

$$\hat{P}^{\pi} = \frac{1}{2}(1 + \pi\hat{\Pi})$$

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$$\hat{P}^{\pi} = \frac{1}{2}(1+\pi\hat{\Pi})$$

- Extraction of the components

$$\underbrace{\hat{P}^J_{MK}\hat{P}^{\pi}\hat{P}^{ZN}}_{\text{projection operators}}|\Phi\rangle \xrightarrow{\text{projects}} \left\{ \sum_{\varepsilon} c^{NZJK\pi} |\Psi^{NZJM\pi}_{\varepsilon}\rangle, K \right\} \xrightarrow{\text{diag. } \hat{H}} \left\{ |\Psi^{NZJM\pi}_{\varepsilon}\rangle, \varepsilon \right\}$$

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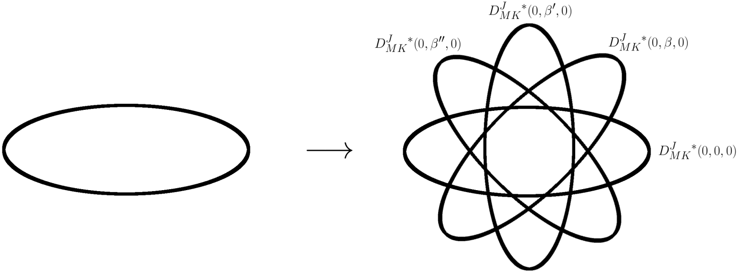
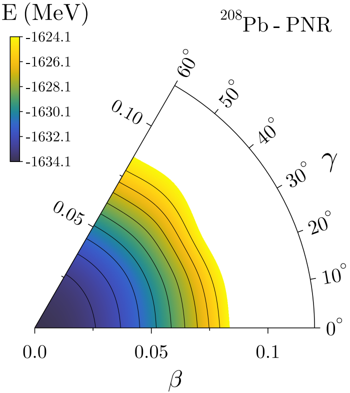
- Projection operator (angular momentum)

$$\hat{P}^J_{MK} = \frac{2J+1}{16\pi^2} \int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \sin(\beta) \int_0^{4\pi} d\gamma \, D^{J}_{MK}{}^*(\alpha,\beta,\gamma) \, \hat{R}(\alpha,\beta,\gamma)$$



Energy surface of ^{208}Pb



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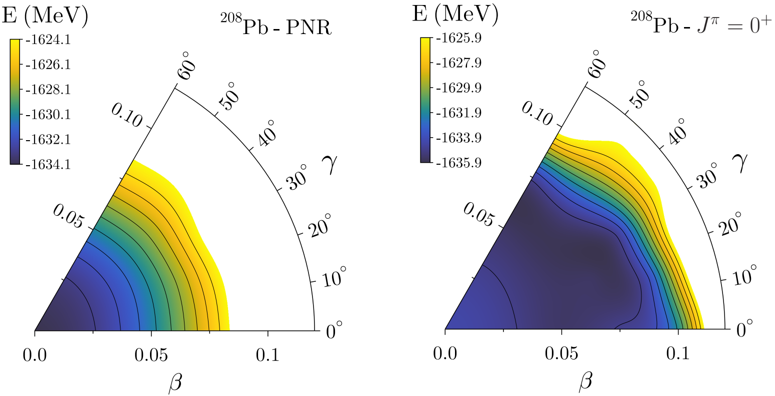
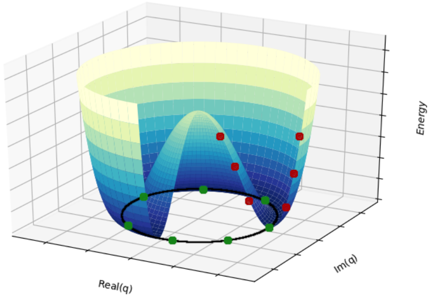


- Order parameter: $q = |q| e^{i \arg(q)}$

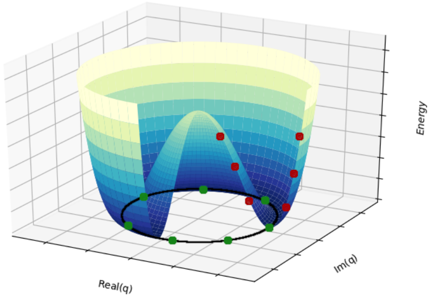


Symmetry breaking and restoration: next level

- Order parameter: $q = |q| e^{i \arg(q)}$
- Example: quadrupole deformations

  $|q| \equiv$ average deformation $\langle \Phi | \hat{Q} | \Phi \rangle$

  $\arg(q) \equiv$ Euler angles (α, β, γ)



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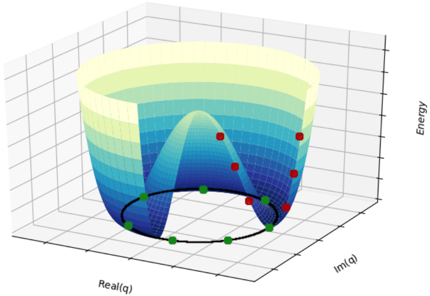

- General ansatz: $|\Theta^{NZJM\pi}\rangle \equiv \int d|q| \, f(|q|) \, P^{NZJM\pi} |\Phi(|q|)\rangle$

  (≡ Projected Generator Coordinate Method (PGCM))

Shape fluctuations for ^{208}Pb

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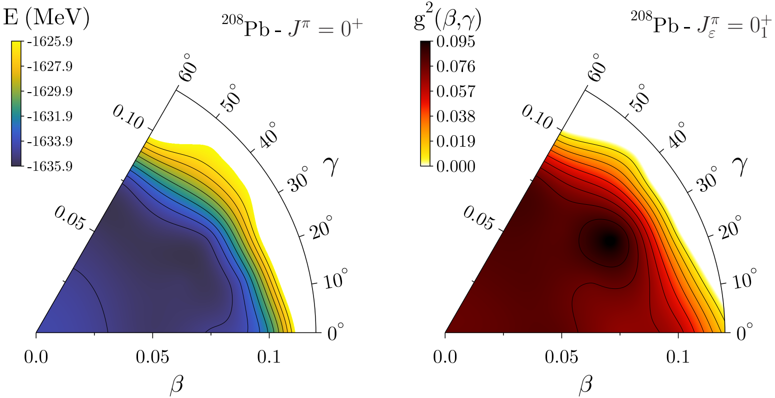
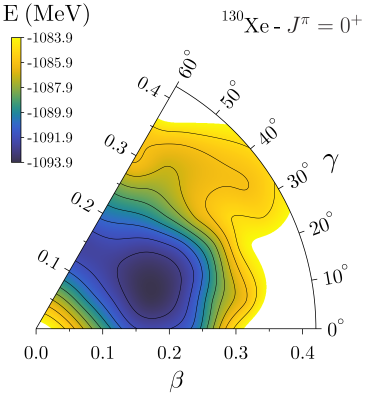
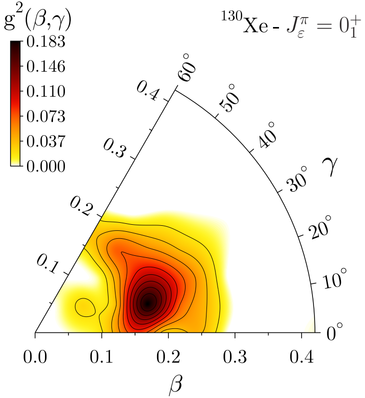
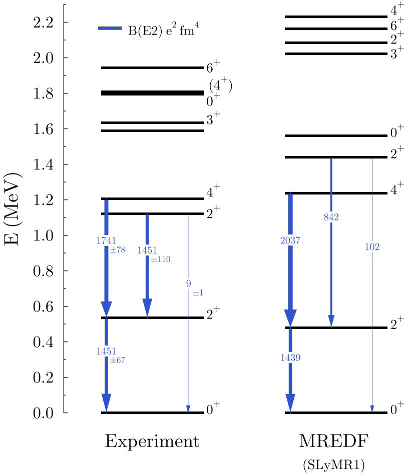


Shape fluctuations for ^{130}Xe





Spectrum for ^{130}Xe (preliminary)



	$Q_s(2_1^+)$ $(e^2\text{fm})$	$\mu(2_1^+)$ (μ_N)
Experiment	-38(17)	+0.67(2)
Theory	-64	+0.62

Conclusion

- Quantum-number projection is powerful tool to study nuclear structure

- It impacts the treatment of deformation
  - Minimum of the energy surface may change
  - There are fluctuations around the minimum

- Recent application in the context of heavy-ion collisions!
  B. Bally, M. Bender, G. Giacalone and V. Somà, arXiv:2108.09578 (2021)