Scaling approach to nuclear structure in isobar collisions

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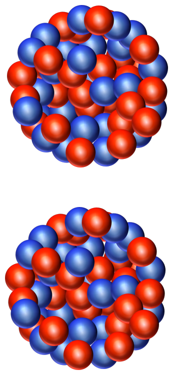
Stony Brook University

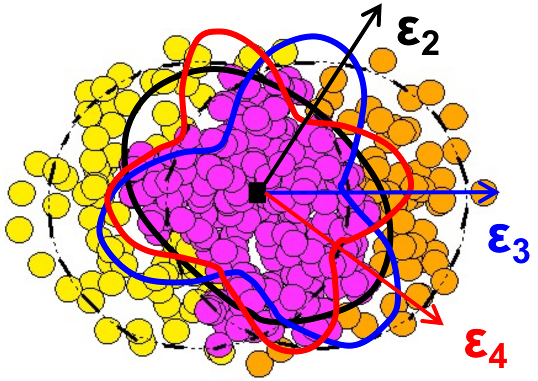
Connecting the initial state to the final state: hydrodynamic response

Nuclear Structure



Imaging?

Initial State

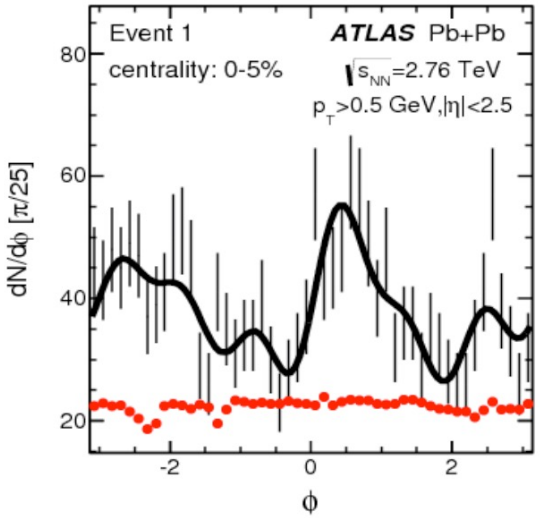


Hydrodynamic response

Approximate linear response
in each event:

D. Teaney and L. Yan, PRC86, 044908(2012) (Editors' Suggestions)

Produced Particle Flow



$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi)))/a_0}}$$

Initial Size

$$R_\perp^2 \propto \langle r_\perp^2 \rangle$$

Initial Shape

$$\mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$$

?

$$R_0 \qquad a_0 \qquad \beta_n$$

Radial Flow

Anisotropic Flow

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n e^{-in\phi}\right)$$

$\beta_2 \rightarrow$ Quadrupole deformation

$\beta_3 \rightarrow$ Octupole deformation

$a_0 \rightarrow$ Surface diffuseness

$R_0 \rightarrow$ Nuclear size

High energy: approximate linear
Response in each event

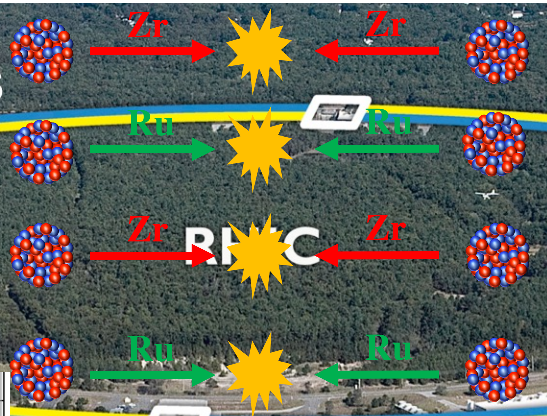
$$\boxed{\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n}$$

The unique isobar run in heavy-ion collisions

1) ${}^{96}_{44}$Ru+${}^{96}_{44}$Ru, ${}^{96}_{40}$Zr+${}^{96}_{40}$Zr with same mass number (A)

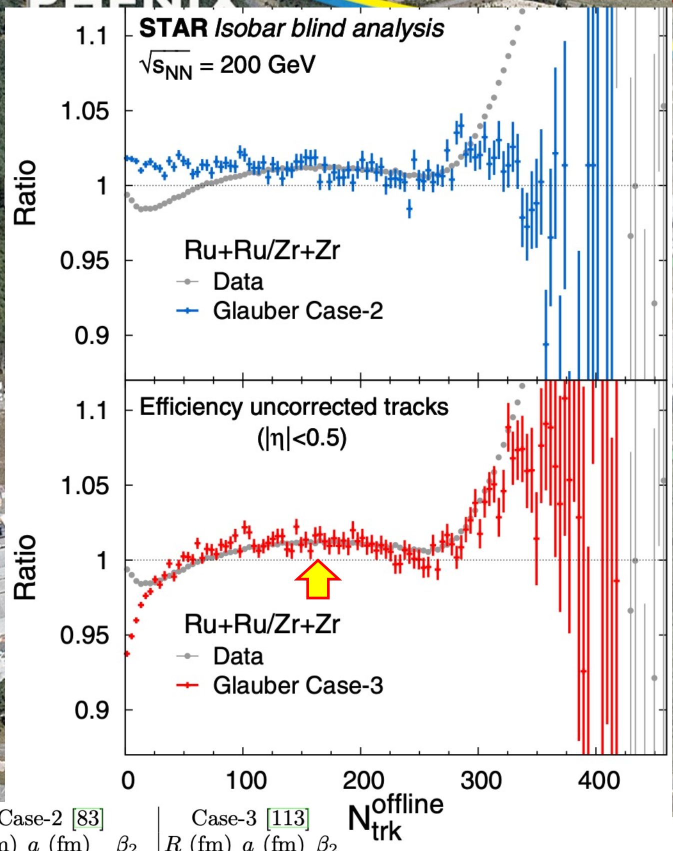
2) Special operation mode:

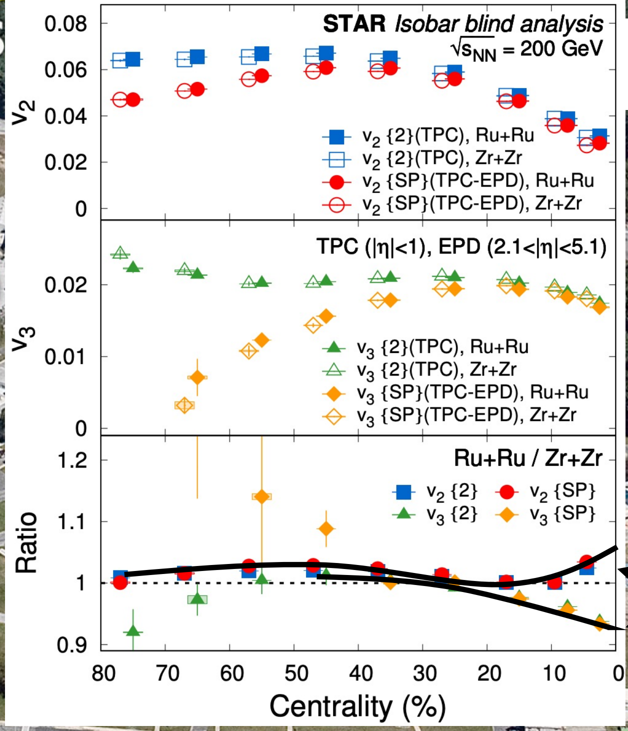
- Fill-by-fill switching between Ru+Ru and Zr+Zr
- Similar run conditions at STAR (minimize the systematics)



3) Ideal system to study nuclear structure:

$$\frac{\mathcal{O}_{\mathrm{Ru+Ru}}}{\mathcal{O}_{\mathrm{Zr+Zr}}} \stackrel{?}{=} 1$$





Isobar run originally dedicated to CME search

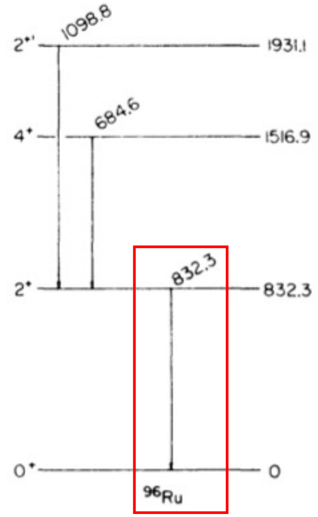
M. S. Abdallah et al. (STAR), PRC105, 014901(2022)

Significant departure from one

Nucleus	Case-1 [83] R (fm)	a (fm)	β_2	Case-2 [83] R (fm)	a (fm)	β_2	Case-3 [113] R (fm)	a (fm)	β_2
${}^{96}_{44}$Ru	5.085	0.46	0.158	5.085	0.46	0.053	5.067	0.500	0
${}^{96}_{40}$Zr	5.02	0.46	0.08	5.02	0.46	0.217	4.965	0.556	0

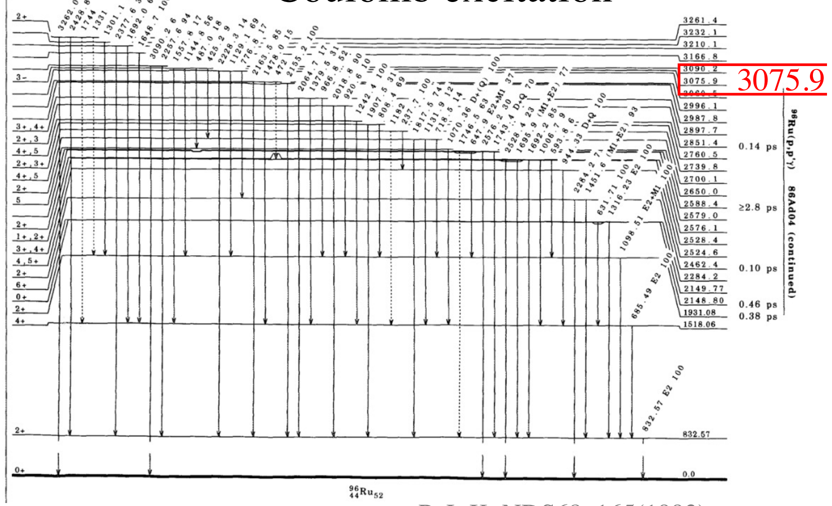
Nuclear deformation experimental data on ^{96}Ru and ^{96}Zr

^{96}Ru



S. Landsberger et al. PRC21, 588(1980)

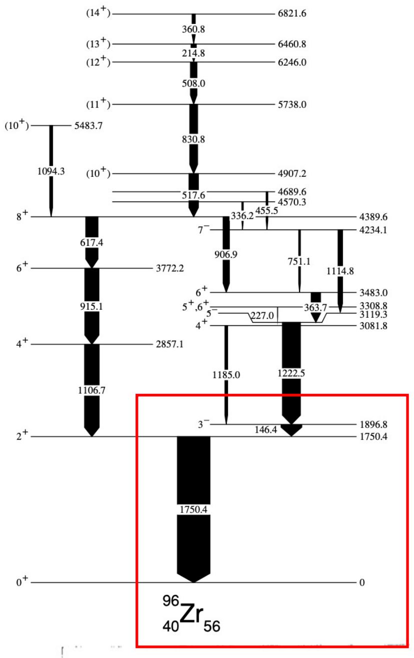
Coulomb excitation

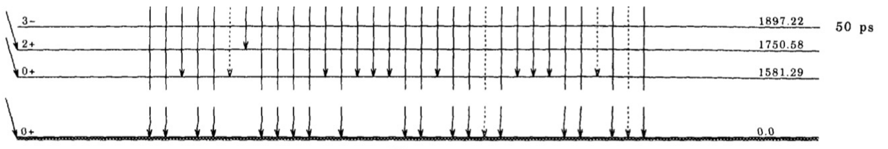


P. L.K, NDS68, 165(1993)

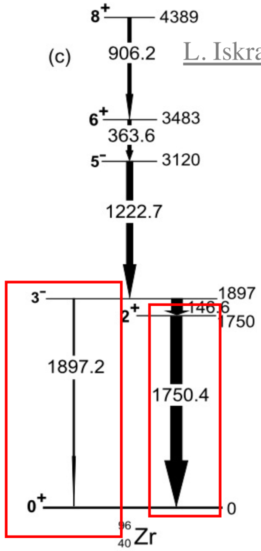
^{96}Zr

D. Pantelica et al., PRC72, 024304(2005)

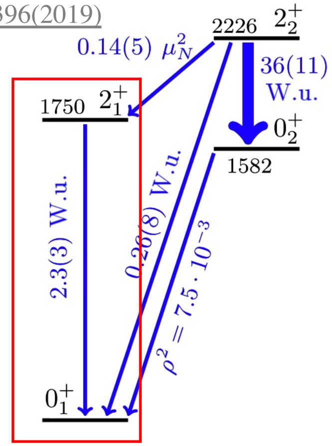




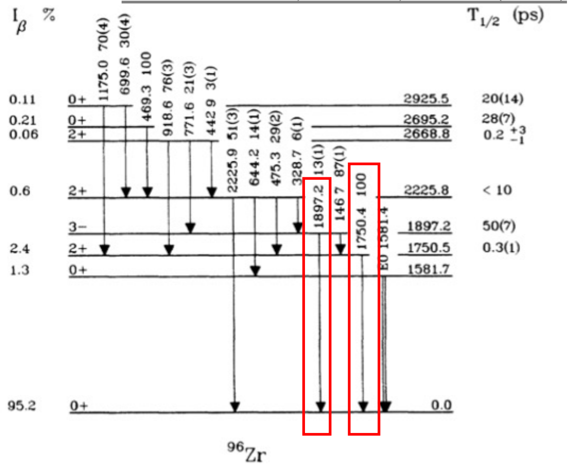
P. L.K, NDS68, 165(1993)



L. Iskra et al., PLB788, 396(2019)

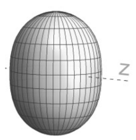


C. Kremer et al., PRL117, 172503 (2016)

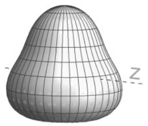


H. Mach et. al., PRC42R, 811(1990)

^{96}Ru



^{96}Zr



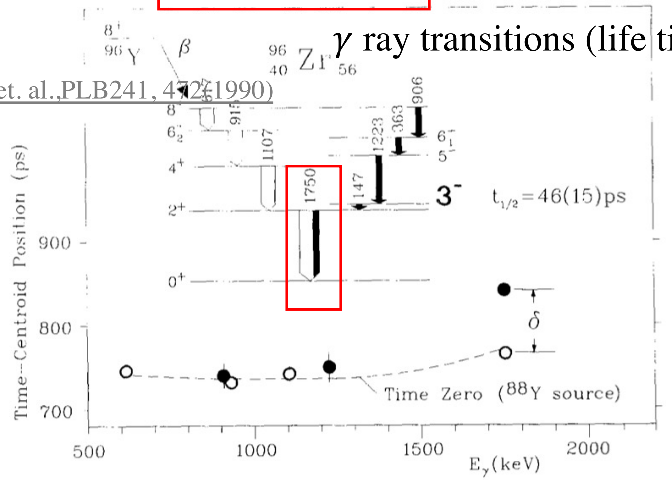
Nuclear structure experimental data on Ru/Zr β_n:

	β_2	$E_{2_1^+}$ (MeV)	β_3	$E_{3_1^-}$ (MeV)
^{96}Ru	0.154	0.83	-	3.08
^{96}Zr	0.062	1.75	0.202,0.235,0.27	1.90

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\sqrt{\frac{B(E2)\uparrow}{e^2}} \qquad \beta_3 = \frac{4\pi}{3ZR_0^3}\sqrt{\frac{B(E3)\uparrow}{e^2}}$$

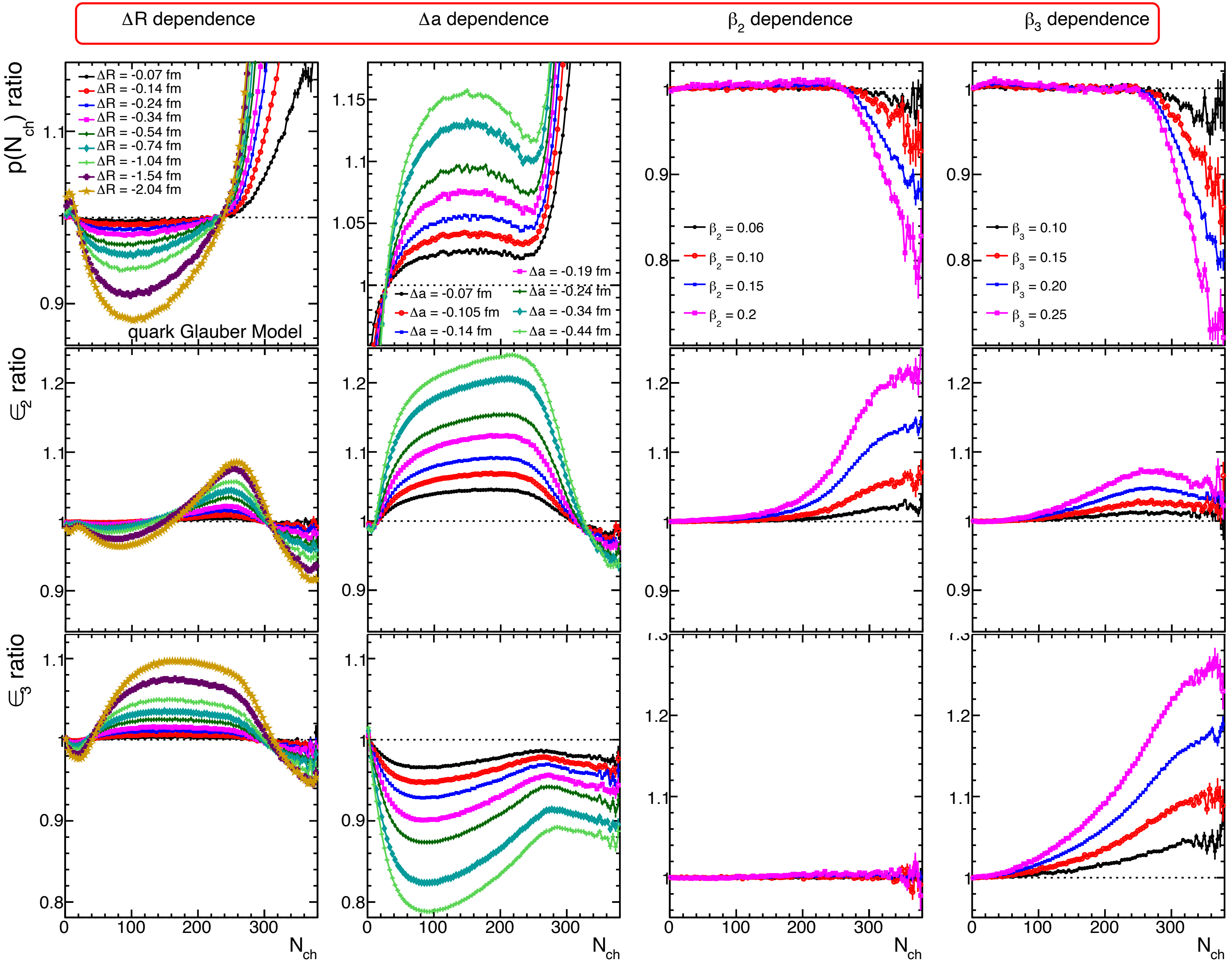
γ ray transitions (life time)

H. Ohm et. al.,PLB241, 472(1990)



Lower energy experiments show large $\beta_{2,Ru}$ and strong $\beta_{3,Zr}$

Nuclear structure effect on the initial state



Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm

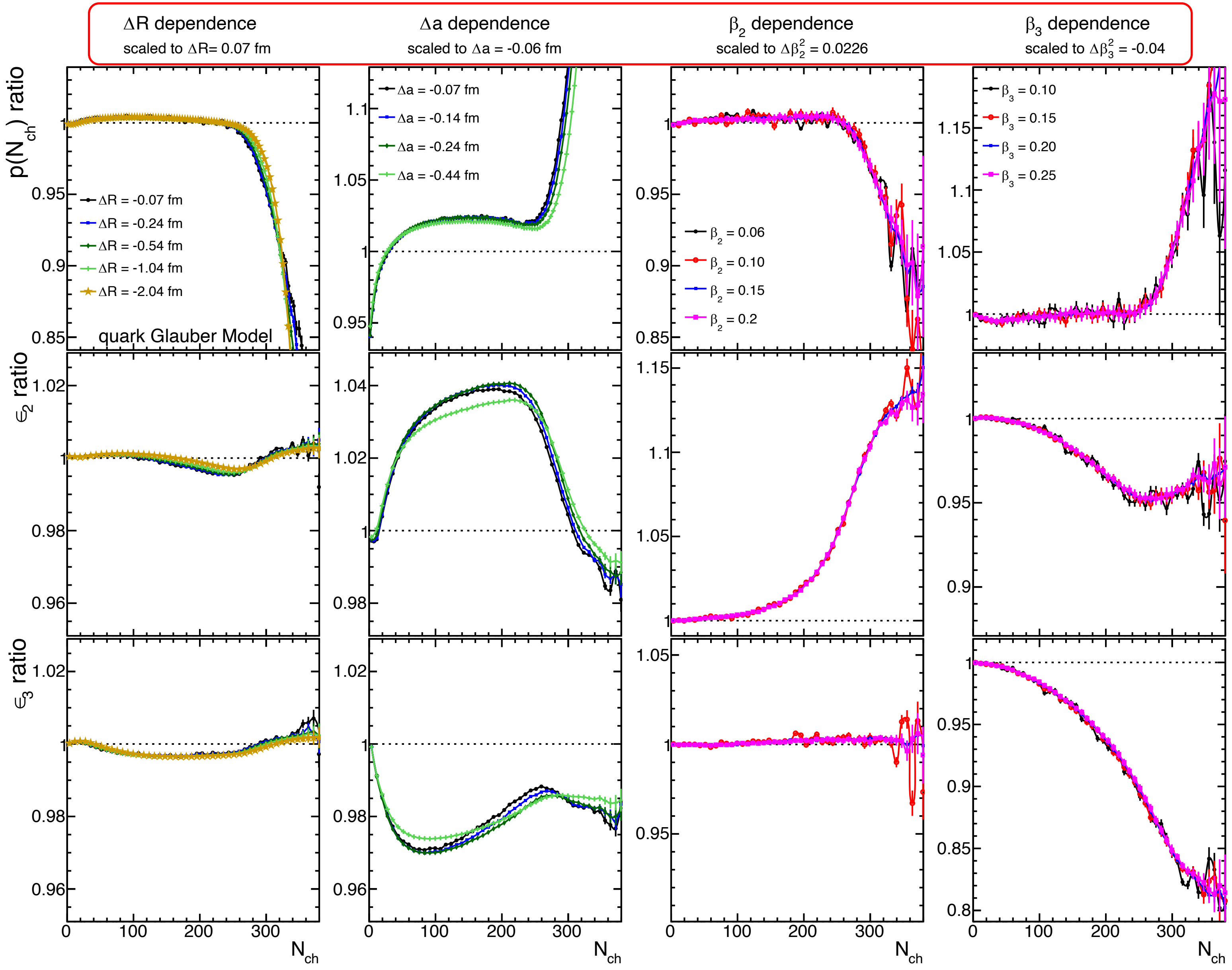
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

R_0 has some effect with unexpected big change
a_0 enhance it in mid-central
β_2 decrease it in central
β_3 decrease it in central

R_0 has some effect with unexpected big change
a_0 enhance ϵ_2 in mid-central
β_2 enhance ϵ_2 in from mid-central to central
β_3 enhance ϵ_2 in mid-central

R_0 has some effect with unexpected big change
a_0 decrease ϵ_2 in mid-central
β_2 has no effect on ϵ_3
β_3 enhance ϵ_3 in mid-central

Scaling approach to nuclear structure on the initial state



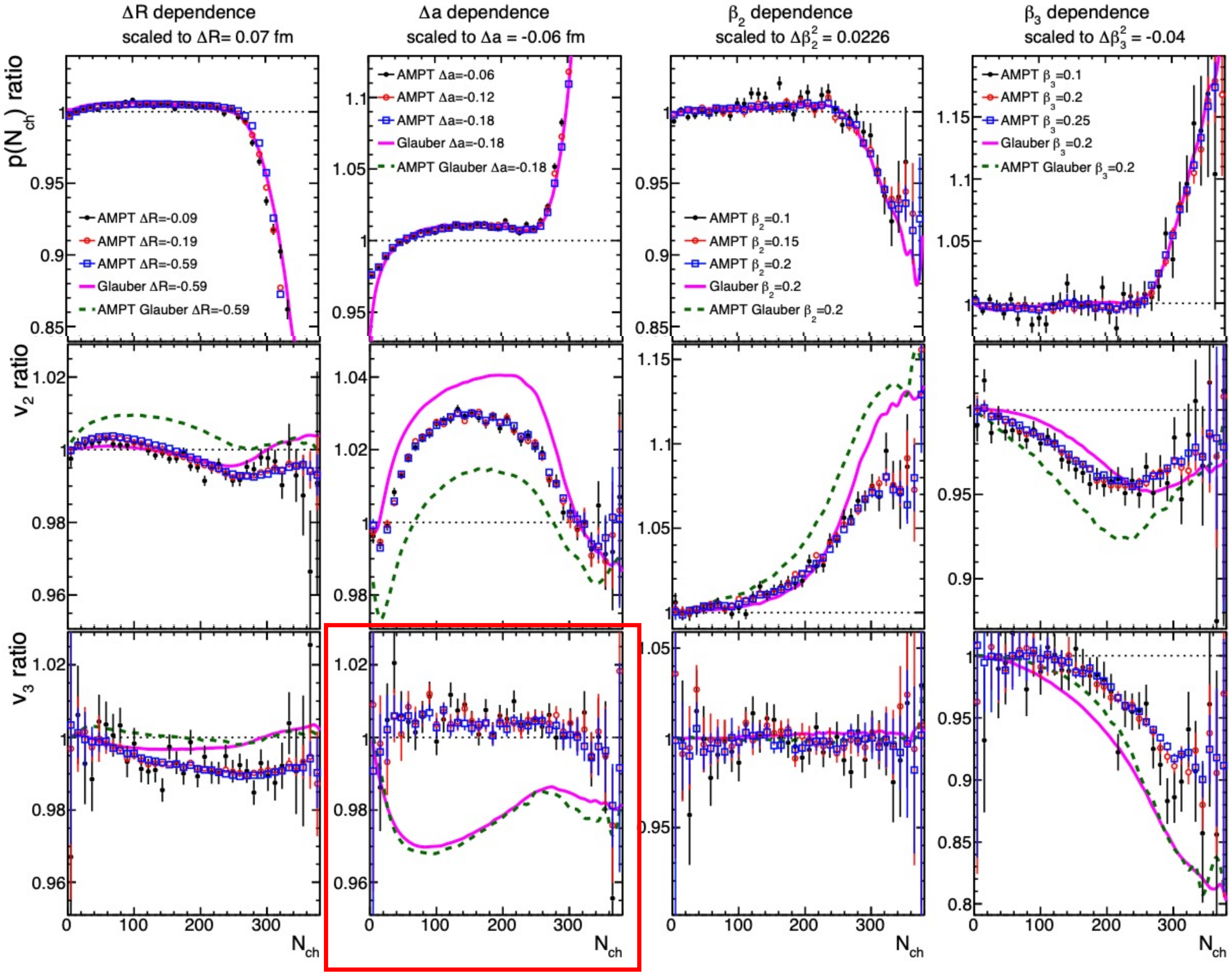
Is it linear?

Yes, only scale a constant value

nearly perfect scaling over the wide range of parameter values

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

Scaling approach to nuclear structure on the final state



Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

nearly perfect scaling over the wide range of parameter values

c_n can be determined more precisely by using a larger change of these parameters

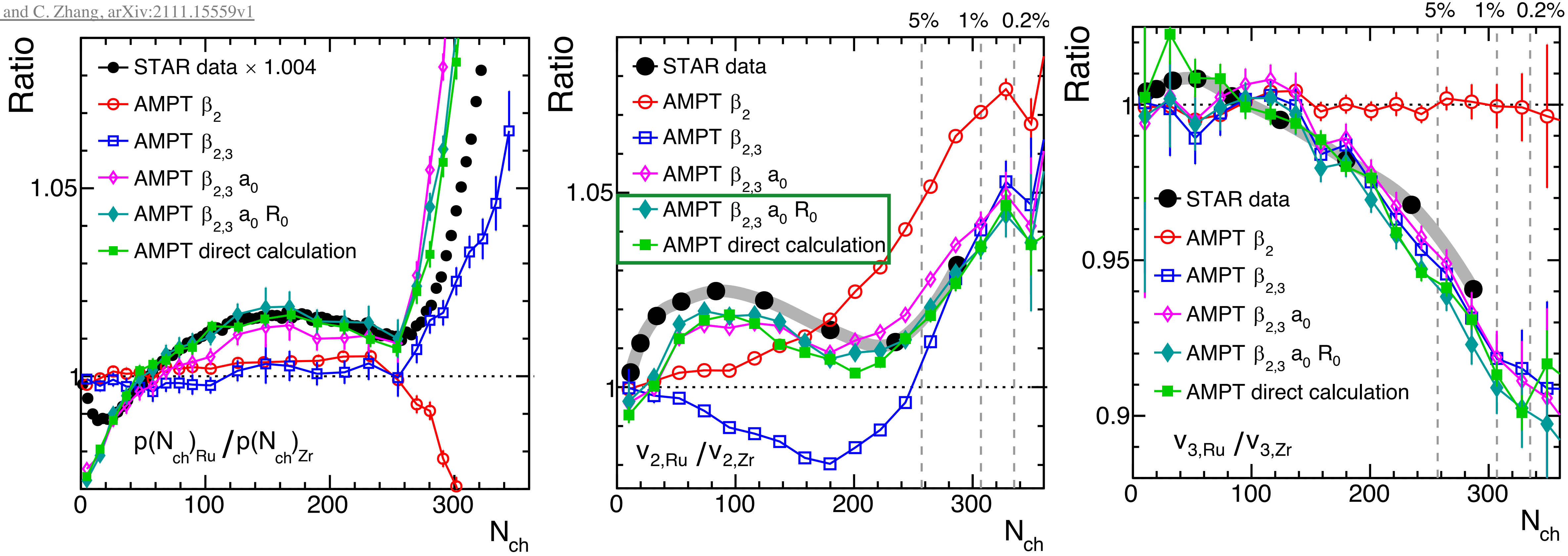
Verifies the relation:

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Nuclear structure via multiplicity ratio and flow ratio

J. Jia and C. Zhang, arXiv:2111.15559v1



Heavy-ion expectation:

$$v_2^2 = a_2 + b_2\beta_2^2 + b_{2,3}\beta_3^2, \quad v_3^2 = a_3 + b_3\beta_3^2$$

$$\frac{v_{2,\mathrm{Ru}}^2}{v_{2,\mathrm{Zr}}^2} \approx 1 + \frac{b_2}{a_2}\left(\beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2\right) - \underline{\frac{b_{2,3}}{a_2}\beta_{3,\mathrm{Zr}}^2}$$

$$\frac{v_{3,\mathrm{Ru}}^2}{v_{3,\mathrm{Zr}}^2} \approx 1 - \frac{b_3}{a_3}\beta_{3,\mathrm{Zr}}^2 < 1$$

Cancelation expected in non-central collisions

1) v_2 ratio: large $\beta_{2,\mathrm{Ru}}$, negative contribution from $\beta_{3,\mathrm{Zr}}$ ⇒ Sharper increase in central

2) v_3 ratio: strong decrease from $\beta_{3,\mathrm{Zr}}$ with negligible $\beta_{2,\mathrm{Ru}}$ distortion

3) Residual effect due to radial structure, e.g., neutron skin in Zr

4) No significant effect due to nuclear size

✓ The large differences of v_2 and v_3 suggest $\boldsymbol{\beta_{2,\mathrm{Ru}} \gg \beta_{2,\mathrm{Zr}}}$ **and** $\boldsymbol{\beta_{3,\mathrm{Ru}} \ll \beta_{3,\mathrm{Zr}}}$ **.**

C. Zhang and J. Jia, PRL128, 022301(2022); J. Jia, PRC105, 014905(2022) (Editors' Suggestions)

A direct algebra linked to neutron skin

Using relation for WS: $R^2 \equiv \langle r^2 \rangle \approx \left(\frac{3}{5}R_0^2 + \frac{7}{5}\pi^2 a^2\right) \Big/ \left(1 + \frac{5}{4\pi^2}\sum_n \beta_n^2\right)$

Neutron skin expressed by R and a parameters for nucleons and protons:

$$\Delta r_{np} \approx \frac{R^2 - R_p^2}{R(\delta+1)} \approx \frac{3\left(R_0^2 - R_{0,p}^2\right) + 7\pi^2\left(a^2 - a_p^2\right)}{\sqrt{15}R_0\sqrt{1 + \frac{7\pi^2}{3}\frac{a^2}{R_0^2}}\left(1 + \delta + \frac{5}{8\pi^2}\sum_n \beta_n^2\right)}$$

$$\delta = (N-Z)/A$$

The difference between two isobar systems can be expressed as:

$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{\bar{R}_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0\left(1 + \bar{\delta} + \frac{5}{8\pi^2}\sum_n \overline{\beta_n^2}\right)}$$

where $Y \equiv 3\left(R_0^2 - R_{0,p}^2\right) + 7\pi^2\left(a^2 - a_p^2\right)$ $\quad \Delta x = x_1 - x_2 \quad \bar{x} = (x_1 + x_2)/2$

Can obtain skin diff. from ΔR_0 Δa for nucleons and known ΔR_0 Δa for protons

Example: H.J. Xu et. al., PLB819, 1136453(2021)

	^{96}Ru		^{96}Zr	
	R	a	R	a
p	5.060	0.493	4.915	0.521
n	5.075	0.505	5.015	0.574
p+n	5.067	0.500	4.965	0.556

Direct calc.: $\Delta(\Delta r_{np})$=0.0296 fm – 0.1606 fm = -0.1310 fm

Formula: $\Delta(\Delta r_{np})$= -0.1319 fm

<1% difference

Conclusions and Outlooks

1) Demonstration: the nuclear structure effect on bulk observables.

2) AMPT could describe the STAR published data quantitatively in a broad centrality.

3) A new approach to constrain the collective nuclear structure parameters:

✓ The final state bulk observables $v_{2,}$ v_3 and $p(N_{ch})$ follow a simple dependences on the variation of parameters:

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}}) \qquad R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

✓ The c_n can be determined precisely in a given model.

✓ The data-model comparison can precisely constrain the dependence of nuclear parameters:

$$\Delta\beta_2^2 = \beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2 \qquad \Delta\beta_3^2 = \beta_{3,\mathrm{Ru}}^2 - \beta_{3,\mathrm{Zr}}^2 \qquad \Delta R_0 = R_{0,\mathrm{Ru}} - R_{0,\mathrm{Zr}} \qquad \Delta a = a_{\mathrm{Ru}} - a_{\mathrm{Zr}}$$

✓ Achieve to obtain the difference of neutron skin between two isobar systems:

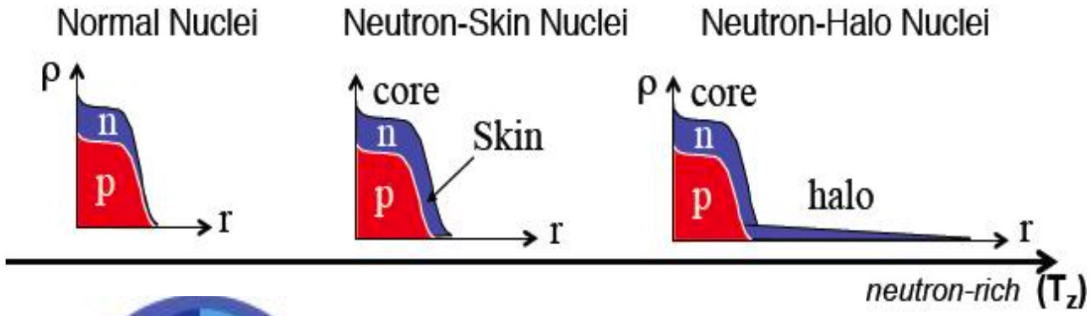
$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{R_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0\left(1 + \bar{\delta} + \frac{5}{8\pi^2}\sum_n \overline{\beta_n^2}\right)}$$

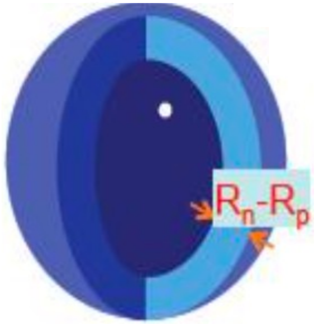
4) Unique opportunities by relativistic collisions of isobars as a tool to study nuclear structure.

Thank you for listening and also many thanks to RBRC

Neutron skin or halo in ^{96}Zr

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + e^{(r - R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi)))/a_0}}$$





Neutron skin

$$\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

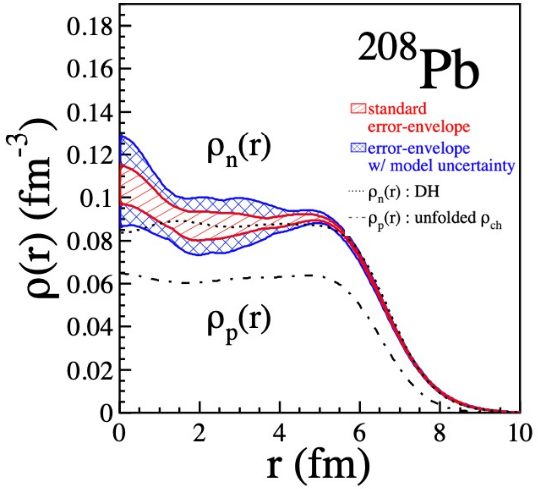
B. S. Hameed, Ph. D thesis

Fundamental importance in nuclear and astrophysics

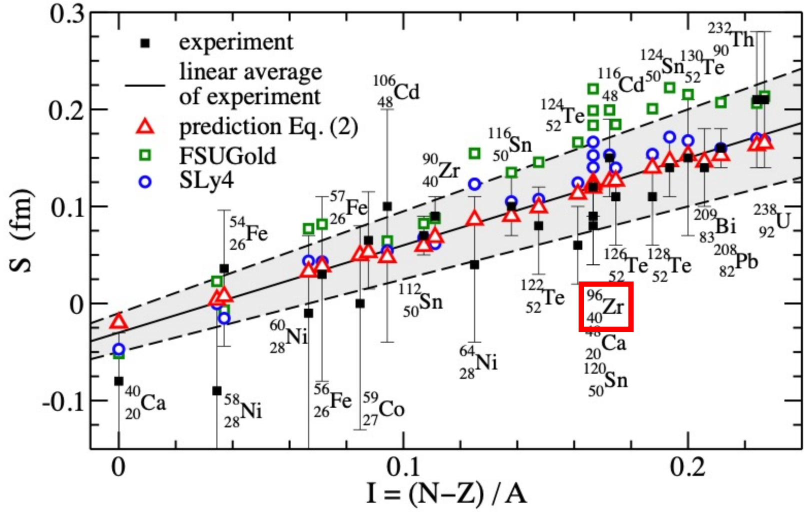
Halo or Skin in Zr-Isotopes?

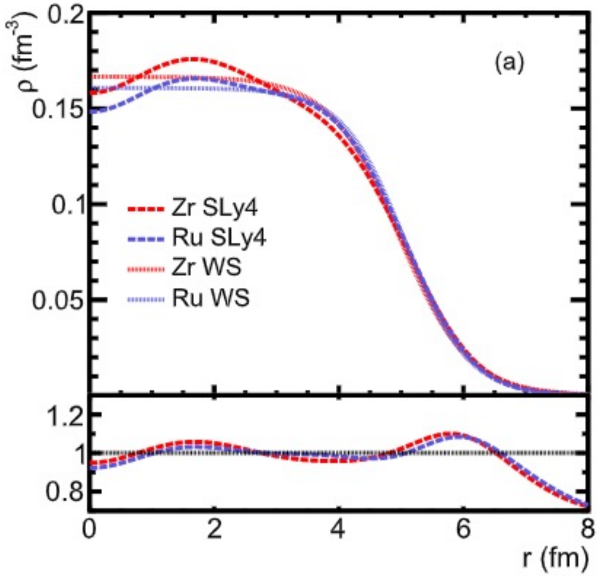
H. Kaur et al., Nucl. Phys.62, 350 (2017)

B. T. Reed et al., PRL126, 172503(2021)
A. Trzcinska et al., PRL87, 082501(2001)



D. Adhikari et al., (PREX) PRL126, 172502(2021)
M. Centelles et al., PRL102, 122502(2009)





Distribution has more sophisticated form, but typically parameterized with WS

^{96}Zr have neutron skin

H.J. Xu et al., PLB819, 1146453(2021)