Microcanonical particlization of relativistic hydrodynamics

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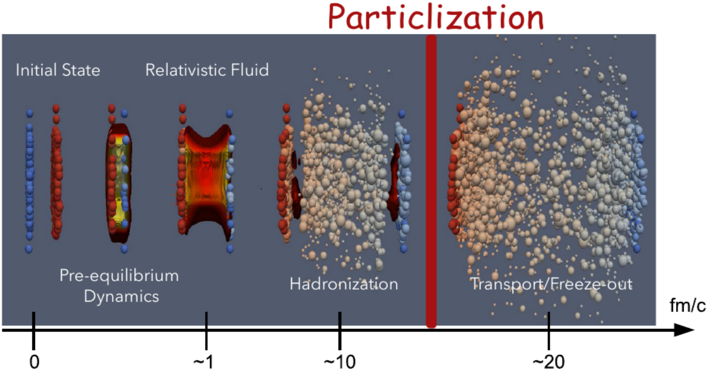
Hybrid approaches: hydrodynamics + transport



- MUSIC + UrQMD, MUSIC + SMASH, JETSCAPE, UrQMD (in hybrid mode), Trajectum, and many others
- Studying correlations and fluctuations
  How not to lose (some) correlations and fluctuations at particlization?

Standard particlization in hydro + transport hybrids

On average by events:
How many particles cross a moving surface $\equiv$
are produced from a hypersurface element with a normal $d\sigma_\mu$?
Cooper-Frye formula:

$$dN = \frac{g}{(2\pi\hbar c)^3} \frac{p^\mu}{p^0} f(p^\alpha u_\alpha, T, \mu) d^3p \, d\sigma_\mu = j^\mu d\sigma_\mu$$

- Cooper-Frye formula does not specify multiplicity distribution
- Standard choice $P(N) = Poisson(\bar{N})$
  motivated by grand-canonical ensemble + classical statistics
- Particles in different cells sampled independently
- Conservation laws fulfilled on average, but not event by event

Conservation laws at particlization: state of the art

Cooper-Frye formula tells nothing about

- correlations between charges, momenta, energies, ...
- fluctuations of B, S, Q, $< p_T >$, ...

They are determined by a sampling algorithm:

- Standard choice: all particles independent
- UrQMD hybrid: attempt to account for conservation laws "mode sampling" Huovinen, Petersen Eur.Phys.J. A48 (2012) 171
- Bozek/Broniowski: always sample particle and antiparticle
  Reproduces charge correlations at $\sqrt{s_{NN}} = 200$ GeV
  Phys.Rev.Lett. 109 (2012) 062301

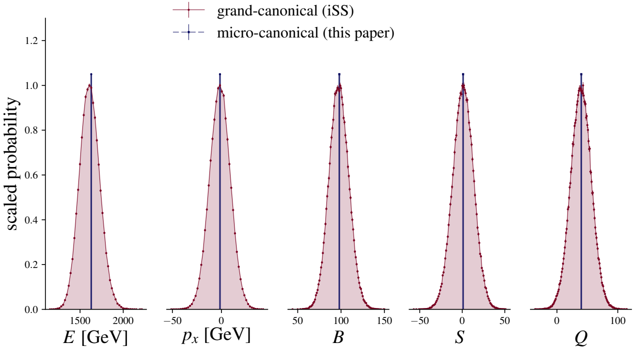
How much algorithm dependence is there?

Bozek's idea relies on $\mu = 0$.

Can one do something similar at lower energies, where $\mu \neq 0$?

Grand-canonical versus microcanonical sampling

Usual grand-canonical particlization assumes independent particles
But particles should be correlated due to conservation laws



AuAu, 19.6 GeV, 30-40% central collisions

E-by-e conservation laws are necessary to study fluctuations

Systematically taking conservation laws into account

Quantities to conserve:

$$\begin{pmatrix} P^{\mu}_{tot} \\ B_{tot} \\ S_{tot} \\ Q_{tot} \end{pmatrix} = \sum_{\substack{\text{cells} \\ i}} \int \begin{pmatrix} p^{\mu}_i \\ B_i \\ S_i \\ Q_i \end{pmatrix} \frac{p^{\nu} d\sigma_{\nu}}{p^0} f_i(p^{\alpha} u_{\alpha}, T, \mu_i) \frac{g_i d^3 p}{(2\pi\hbar)^3}$$

Conservation laws applied independently to parts of the hypersurface:

patches

Plan of the talk:

- Microcanonical particlization in a single patch
- Splitting into patches

Systematically taking conservation laws into account

Distribution to sample:

$$P(N, \{N_s\}^{\text{species}}, \{x_i\}_{i=1}^{N}, \{p_i\}_{i=1}^{N}) = \mathcal{N}$$
$$\left(\prod_s \frac{1}{N_s!}\right) \prod_{i=1}^{N} \frac{g_i}{(2\pi\hbar)^3} \frac{d^3 p_i}{p_i^0} p_i^\mu d\sigma_\mu \, f_i(p_i^\nu u_\nu, T, \mu_i) \times$$
$$\delta^{(4)}(\sum_i p^\mu - P^\mu_{tot}) \, \delta^{B_{tot}}_{\sum_i B_i} \, \delta^{S_{tot}}_{\sum_i S_i} \, \delta^{Q_{tot}}_{\sum_i Q_i}$$

- Total quantities conserved
- Variations of T, μ, u within patch taken into account
- Turns into standard microcanonical sampling in case of one cell
- Sampled with Metropolis algorithm

Metropolis algorithm: general

- Random walk (Markov chain) with many steps
- One step t:
  - in state ξ propose new state ξ', probability $T(\xi \to \xi')$
  - Accept this proposal with probability $A(\xi \to \xi')$
  - $w(\xi \to \xi') = T(\xi \to \xi') A(\xi \to \xi')$
- After many steps reach stationary distribution $P(\xi)$
- $P(\xi)$ should be the desired distribution

$$P^{t+1}(\xi) - P^t(\xi) = \sum_{\xi'} [w(\xi' \to \xi) P^t(\xi') - w(\xi \to \xi') P^t(\xi)]$$

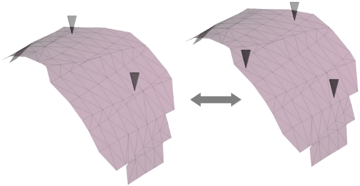
Sufficient condition for stationary distribution (detailed balance):

$$\frac{P(\xi')}{P(\xi)} = \frac{w(\xi \to \xi')}{w(\xi' \to \xi)} \implies \frac{A(\xi \to \xi')}{A(\xi' \to \xi)} = \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')}$$

Common choice:

$$a \equiv A(\xi \to \xi') = \min\left(1, \frac{P(\xi')\, T(\xi' \to \xi)}{P(\xi)\, T(\xi \to \xi')}\right)$$

Proposal function



1. With 50% probability choose a $2 \to 3$ or $3 \to 2$ transition.
2. Select the "incoming" particles by uniformly picking one of all possible pairs or triples.
3. Select the outgoing channel democratically with probability $1/N^{ch}$, N^{ch} – number of possible channels, satisfying quantum number and energy-momentum conservation.
4. For the selected channel sample the "collision" kinematics uniformly from the available phase space with probability $\frac{dR_n}{R_n}$, $n = 2$ or 3.
$$dR_n(\sqrt{s}, m_1, m_2, \dots, m_n) = \frac{(2\pi)^4}{(2\pi)^{3n}} \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \dots \frac{d^3p_n}{2E_n} \delta^{(4)}(P^{\mu}_{tot} - \sum P^{\mu}_i)$$
5. Choose a cell for each of the outgoing particles uniformly from all cells in the patch.

Properties of proposal function

- Never changes total energy, momentum, or quantum numbers
- Generates proposal probabilities:

$$T(2 \to 3) = \frac{1}{2} \frac{G_2^{ch}}{G_2} \frac{1}{N_3^{ch}} \frac{dR_3^{ch}}{R_3^{ch}} \frac{1}{N_{cells}^3}$$

$$T(3 \to 2) = \frac{1}{2} \frac{G_3^{ch}}{G_3} \frac{1}{N_2^{ch}} \frac{dR_2^{ch}}{R_2^{ch}} \frac{1}{N_{cells}^2}$$

$G_2 = \frac{N(N-1)}{2!}$, $G_3 = \frac{N(N-1)(N-2)}{3!}$
total numbers of incoming pairs/triplets of any species
G_2^{ch}, G_3^{ch} – numbers of ways to select given incoming species
N_2^{ch}, N_3^{ch} – numbers of channels with necessary quantum numbers

Acceptance probability

$$a_{n\to m} = \frac{N^{ch}_m R_m}{N^{ch}_n R_n} \frac{N!}{(N+m-n)!} \frac{m!}{n!} \frac{k^{id}_m!}{k^{id}_n!} \times$$

$$\left(\frac{2N_{cells}}{\hbar^3}\right)^{m-n} \frac{\prod_{i=1}^{m} g_i\, f_i(\mu_i - p_i^\alpha u_\alpha, T)\, p_i^\mu d\sigma_\mu}{\prod_{j=1}^{n} g_j\, f_j(\mu_j - p_j^\alpha u_\alpha, T)\, p_j^\mu d\sigma_\mu}$$

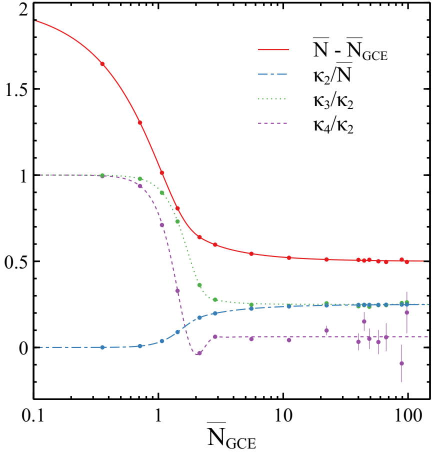
T, μ, u are taken at positions of the incoming/outgoing particles

Testing the sampling I: one cell, simple box

Sampling is already non-trivial, several works devoted to this case

Werner:1995mx, Becattini:2004rq, Begun:2005qd

Massless particles: analytically known



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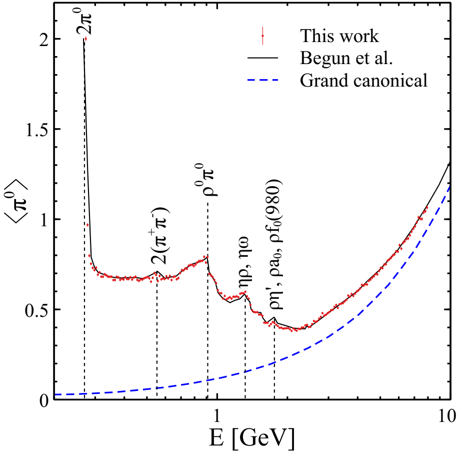
Full hadron gas



Fast open-source microcanonical sampler!

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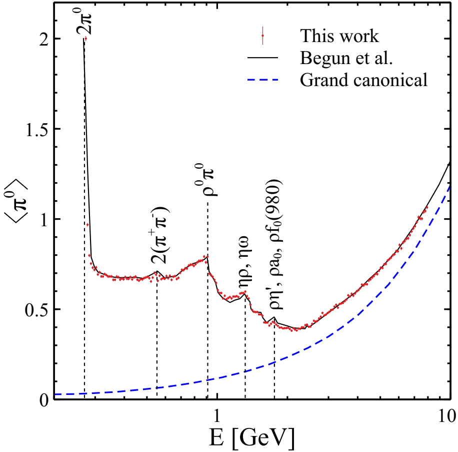
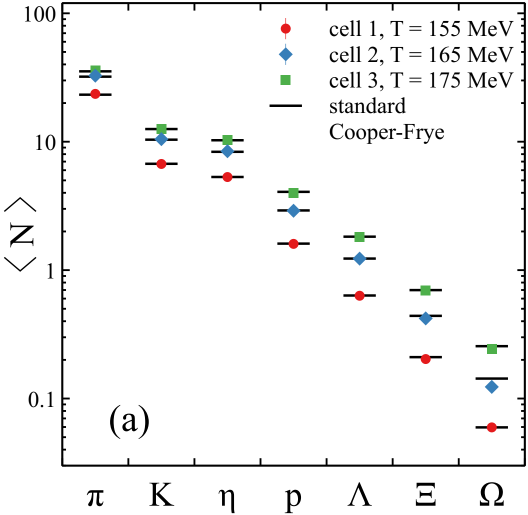


github.com/doliinychenko/microcanonical_cooper_frye

Testing the sampling II

- Patch consisting of 3 cells:
  - $d\sigma_1^\mu = (500.0, 50.0, 20.0, 30.0)\,\mathrm{fm}^3$,
    $d\sigma_2^\mu = (500.0, 40.0, 80.0, 30.0)\,\mathrm{fm}^3$,
    $d\sigma_3^\mu = (500.0, 20.0, 20.0, 20.0)\,\mathrm{fm}^3$
  - $\vec{v}_1 = (0.2, 0.3, 0.4)$, $\vec{v}_2 = (0.1, 0.5, 0.5)$, $\vec{v}_3 = (0.3, 0.4, 0.2)$
  - $T_1 = 0.155$ GeV, $T_2 = 0.165$ GeV, $T_3 = 0.175$ GeV
- Total energy of the patch 1268.2 GeV
- 416 different hadronic species generated ($m < 2.5$ GeV)
- Total energy, momentum, B, S, Q conserved
- Preserving local variations of T, μ, u
- Check local means, scaled variance $\omega \equiv \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ of total multiplicities

Testing the sampling: several cells per patch



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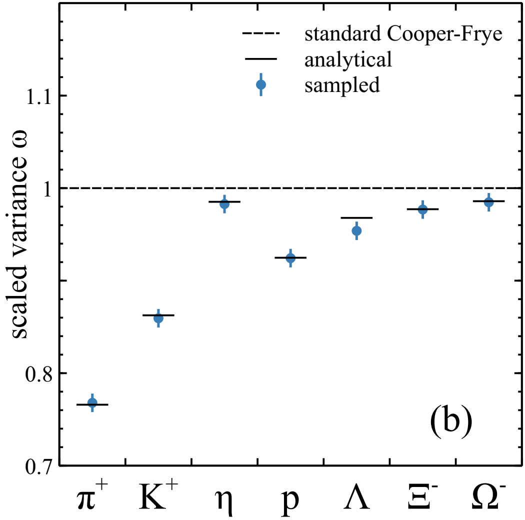

Analytical: M. Hauer, V. V. Begun and M. I. Gorenstein, Eur. Phys. J. C **58**, 83 (2008)

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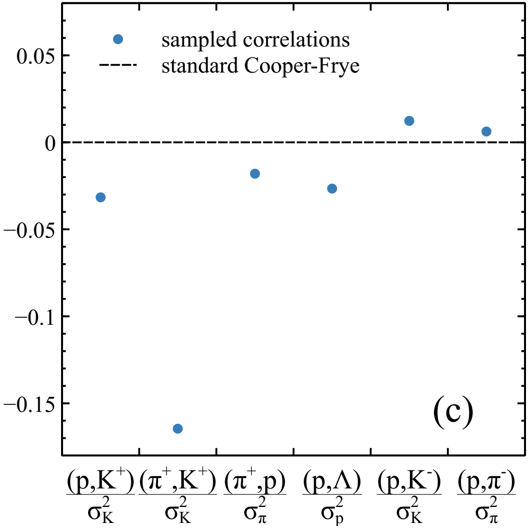
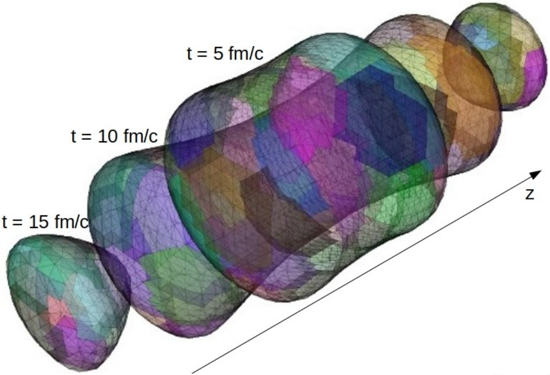


Conclusion so far: sampling works as intended

Partitioning hypersurface into patches

- How big should the patch be?
  - Not too small
  - Contain > 1 particle $\implies$ $> 100 - 1000$ cells per patch
- How to split hypersurface into patches?
- What physics remains after splitting into patches defined by an ad hoc algorithm?



Patch splitting

- Start with particular non-clustered cell, e.g. with smallest τ or η
- Define distance, add closest cells until total rest frame energy E_{patch} reached
- Start new patch

Different algorithms:

- (a) starting with t_{min}, distance $\Delta t^2 + \Delta r^2$
- (b) starting with η_{max}, distance $\Delta t^2 + \Delta r^2$
- (c) starting with η_{max}, distance $\Delta \eta$
- (d) starting with E_{max}, distance $\Delta r^2/d_0^2 + (\Delta T/\sigma_T)^2 + (\Delta \mu_B/\sigma_{\mu_B})^2$
- Additional adjustments to keep patch charges integer

How much do these ad hoc details influence results?

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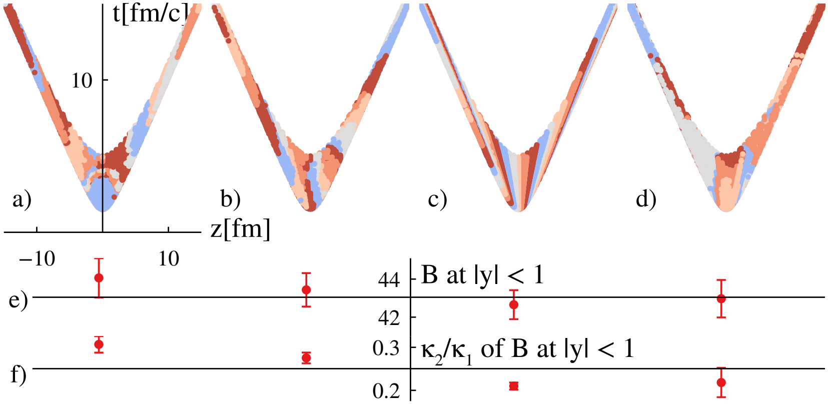
Patch energy E_{patch} – physical parameter, algorithm – systematic error

Effects from splitting algorithm



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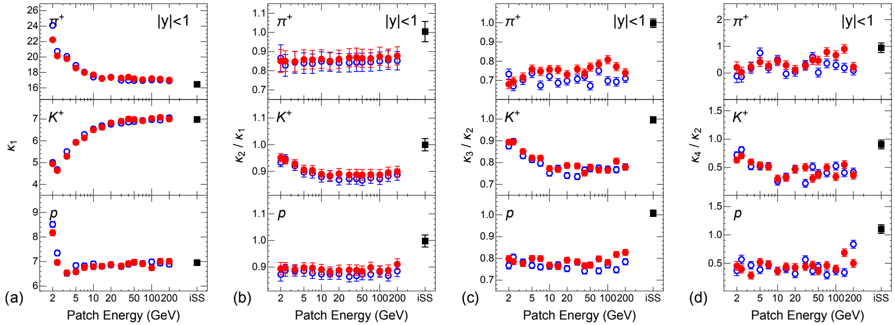
Particle cumulants



Red points – algorithm (d)

Blue points – algorithm (a)

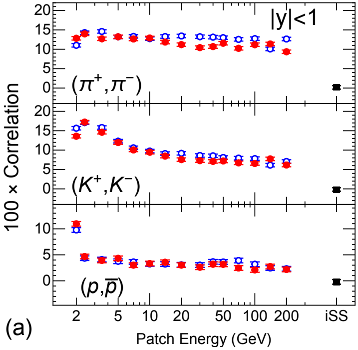
Black points – grand-canonical sampler

Systematic error due to algorithm is tolerable

(Micro-)canonical effects clearly seen even after rapidity cut

Microcanonical sampling: effects and applications

Example: AuAu at 19.6 GeV, 30-40% centrality



- Correlations and fluctuations
- Chiral effects
- Small systems
- Quark-Gluon Plasma in droplets

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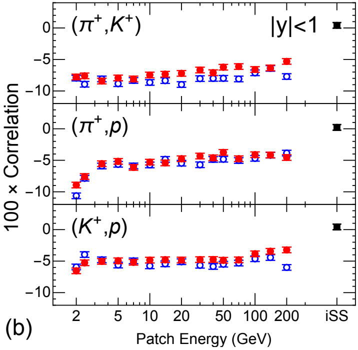

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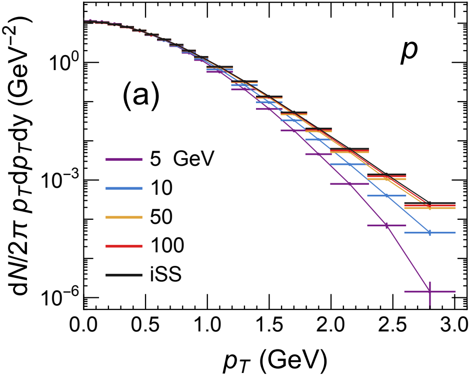

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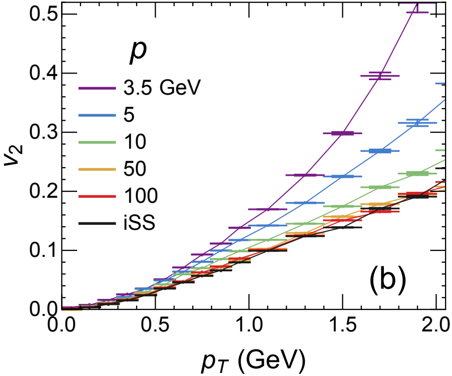

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Summary

- Standard sampling neglects event-by-event conservation laws
- This obfuscates fluctuations
- Reasonably fast method exists to include conservation laws
  - Split hypersurface into patches and conserve on every patch
  - Using Markov chain reminiscent of $2 \leftrightarrow 3$ stochastic collisions to thermalize
  - Passes non-trivial test cases
  - Code publically available github.com/doliinychenko/microcanonical_cooper_frye
- Patch splitting
  - Contains certain degree of arbitrariness
  - To which physics is not too sensitive
  - Patch size is a physical parameter, and it matters
- Microcanonical effects:
  - high-p_T suppression
  - v_2 enhancement
  - Non-trivial correlations
  - Suppression of fluctuations compared to grand-canonical