RBRC Workshop:
Physics Opportunities from the RHIC Isobar Run

Fluid dynamics of multiple conserved charges

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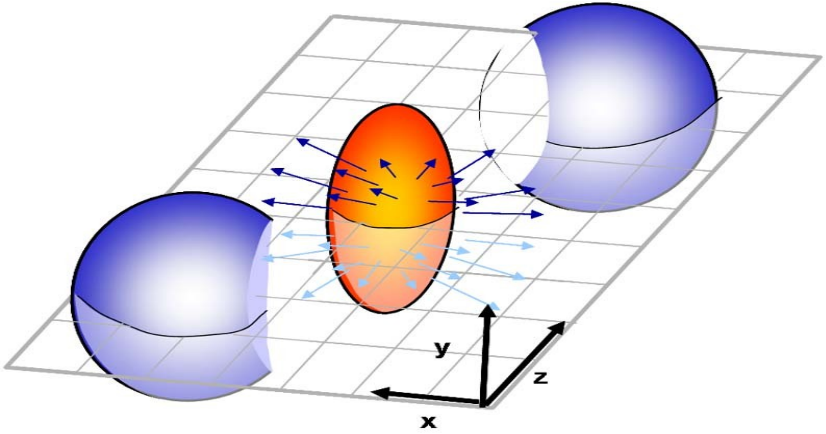
Fluid dynamics of multi-component systems

Traditionally:

Viewed as ‘blob’ of one type of matter (single component) with one velocity field

- usually ‘blob’ of energy
  with conserved particle number



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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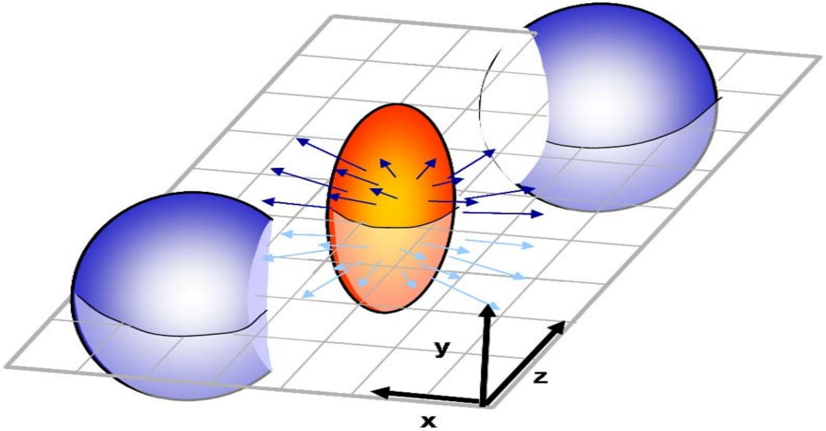

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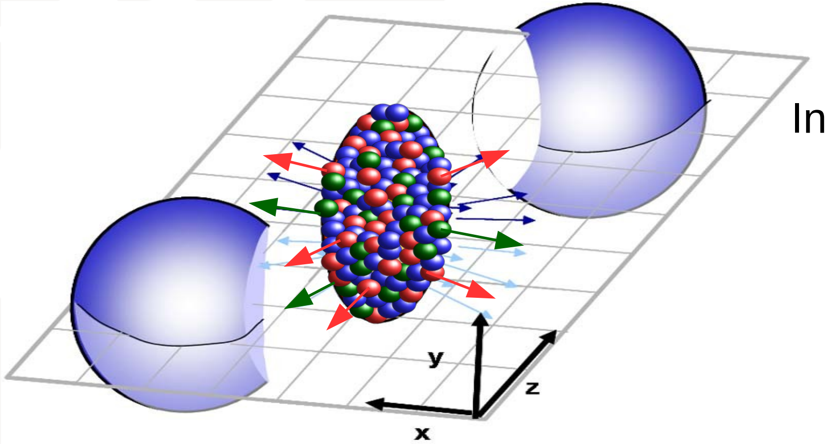

In general:

Consists of <u>multiple components</u> with <u>various properties</u> with <u>multiple velocity fields</u>

- with **multiple conserved quantities**
  (e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry ⟶ **<u>coupled charge currents!</u>**

Fluid dynamics of multi-component systems

<u>Hydrodynamics:</u> macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

$$T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_q^\mu = \sum_i q_i N^\mu = n_q u^\mu + V_q^\mu$$

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q-th conserved charge (eg. B,Q,S)

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$$10 + 4N_{\text{ch}} \text{ degrees of freedom}, \quad 4 + N_{\text{ch}} \text{ equations} \quad \rightarrow \quad 6 + 3N_{\text{ch}} \text{ unknowns}$$

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What needs to be known:

- Equation of state $P_0 = P_0(\epsilon, n_q), \quad T = T(\epsilon, n_q), \quad \alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q)$
- Equations of motion for dissipative fields & transport coefficients $\Pi, V_q^\mu, \pi^{\mu\nu}$
- Initial state
- Freeze-out and δf-correction

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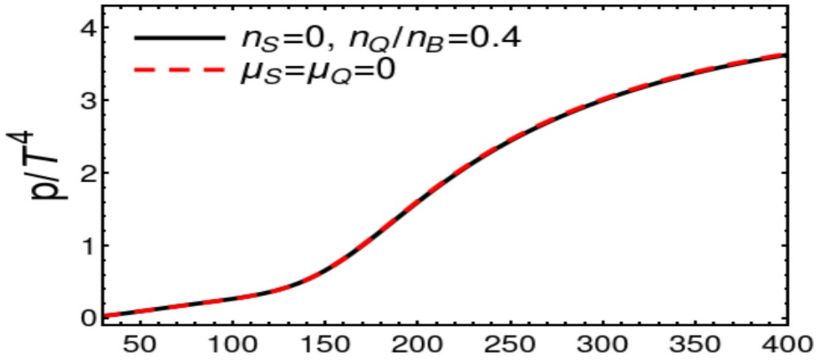
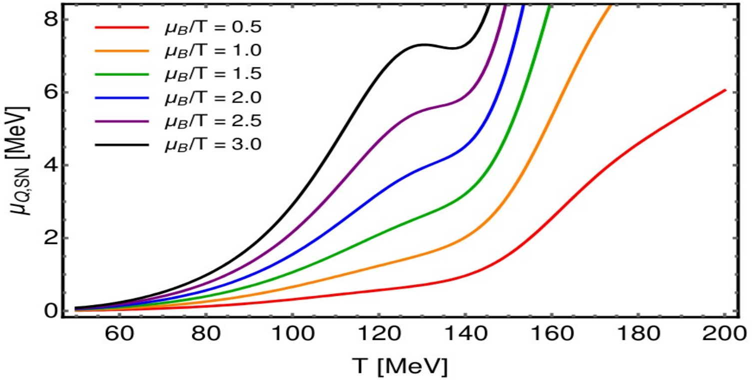
Fluid dynamics with conserved baryon number:
Denicol et al., PRC 98, 034916 (2018)
Li et al., PRC 98, 064908 (2018)
Du et al., Comp. Phys. Comm. 251, 107090 (2020)

Equation of state with multiple conserved charges

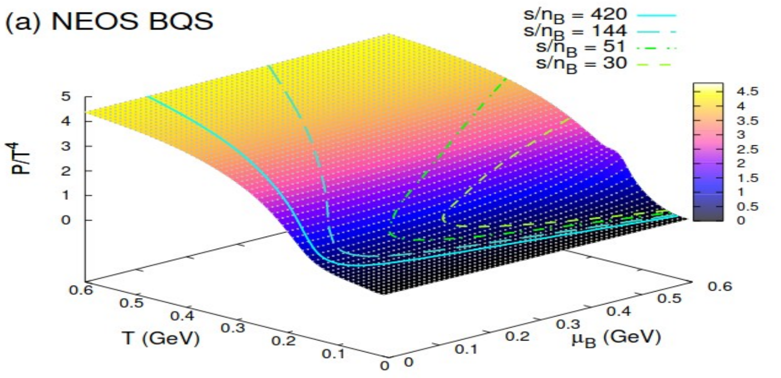
$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$$

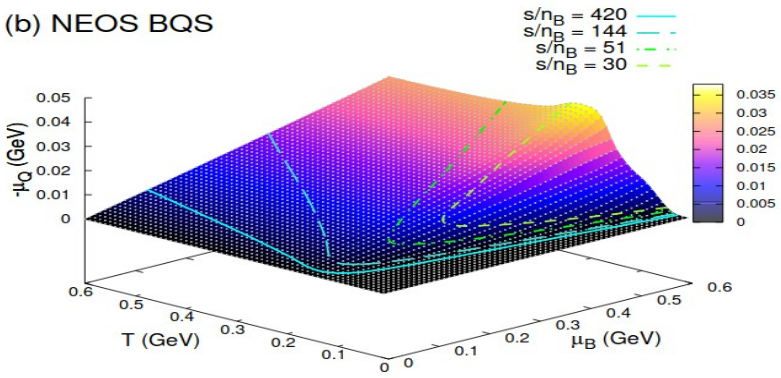
Noronha-Hostler et al., PRC 100, 064910 (2019)



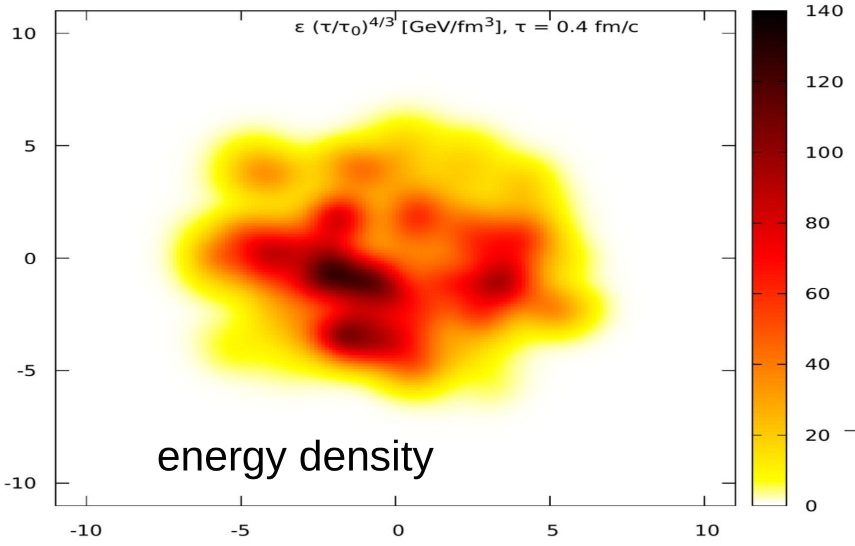


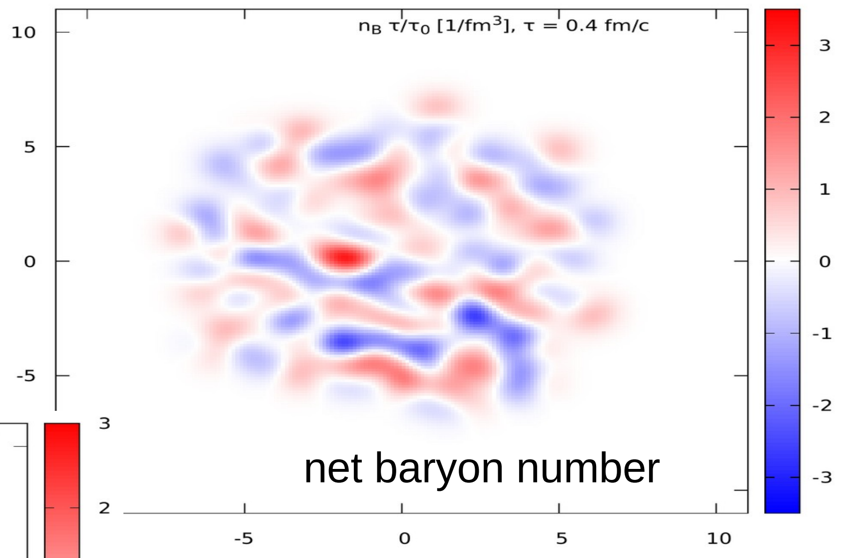
Monnai et al., PRC 100, 024907 (2019)

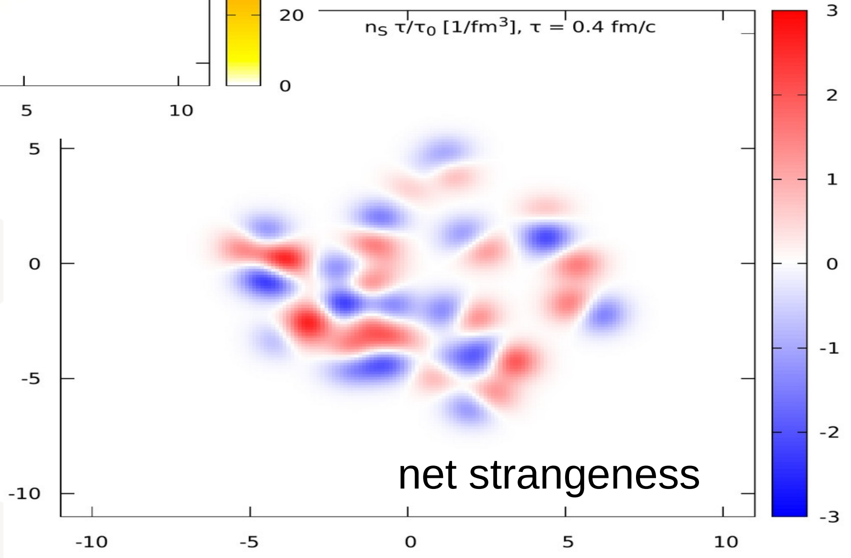




Initial state with multiple conserved charges







Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments
→ **upcoming publication!** (Fotakis, Molnár, Niemi, Rischke, Greiner)

Also refer to: Monnai, Hirano, Nucl. Phys. A847:283-314 (2010) or Kikuchi et al. PRC 92, 064909 (2015)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

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equilibrium off-equilibrium

$$f_{i,\mathbf{k}} = \boxed{f^{(0)}_{i,\mathbf{k}}} + \boxed{\delta f_{i,\mathbf{k}}}$$

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Irreducible off-equilibrium moments obey Boltzmann eq.:

$$f_{i,\mathbf{k}} = \underbrace{\boxed{f_{i,\mathbf{k}}^{(0)}}}_{\text{equilibrium}} + \underbrace{\boxed{\delta f_{i,\mathbf{k}}}}_{\text{off-equilibrium}}$$

Problem: infinitely many coupled PDEs.

$$\rho_{i,n}^{\mu\nu} = \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^n k_i^{\langle\mu} k_i^{\nu\rangle} \boxed{\delta f_{i,\mathbf{k}}}$$

Aim: Truncate in a well-defined manner (perturbation theory)

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Aim: Truncate in a well-defined manner (perturbation theory)

"Order-of-magnitude approximation":
relate them to the dissipative fields with constituent's transport coefficients

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta}\pi^{\mu\nu} + \mathcal{O}(2)$$

Counting scheme:

Gradients in velocity, temperature etc. $\sigma^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Kn})$
Dissipative fields $\pi^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Rn}^{-1})$

Fluid dynamics of multicomponent systems

2^{nd}-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

upcoming publication!

$$\tau_\Pi \dot{\Pi} + \Pi = S_\Pi$$
$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = S_q^\mu$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = S_\pi^{\mu\nu}$$

Relaxation equations (Israel-Stewart-type causal theory)

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Relaxation equations (Israel-Stewart-type causal theory)

$$\begin{aligned} S_q^\mu = &\sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu}\omega^{\nu\mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'}^\mu \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu}\sigma^{\mu\nu} \\ &- \ell_{V\Pi}^{(q)} \nabla^\mu \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q)} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q)} \pi^{\mu\nu} \dot{u}_\nu \\ &+ \sum_{q'} \lambda_{V\Pi}^{(q,q')} \Pi \nabla^\mu \alpha_{q'} - \sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu\nu} \nabla_\nu \alpha_{q'} \end{aligned}$$

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$$- \ell_{V\Pi}^{(q)} \nabla^\mu \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q)} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q)} \pi^{\mu\nu}\dot{u}_\nu$$
$$+ \boxed{\sum_{q'} \lambda_{V\Pi}^{(q,q')} \Pi \nabla^\mu \alpha_{q'}} - \boxed{\sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu\nu}\nabla_\nu \alpha_{q'}}$$

Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term

2nd order terms: couples all currents to each other; depend on all gradients!

Coupled charge-transport

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \tau^{(1)}_{ij,0n} q_i \left(q'_j J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)$$

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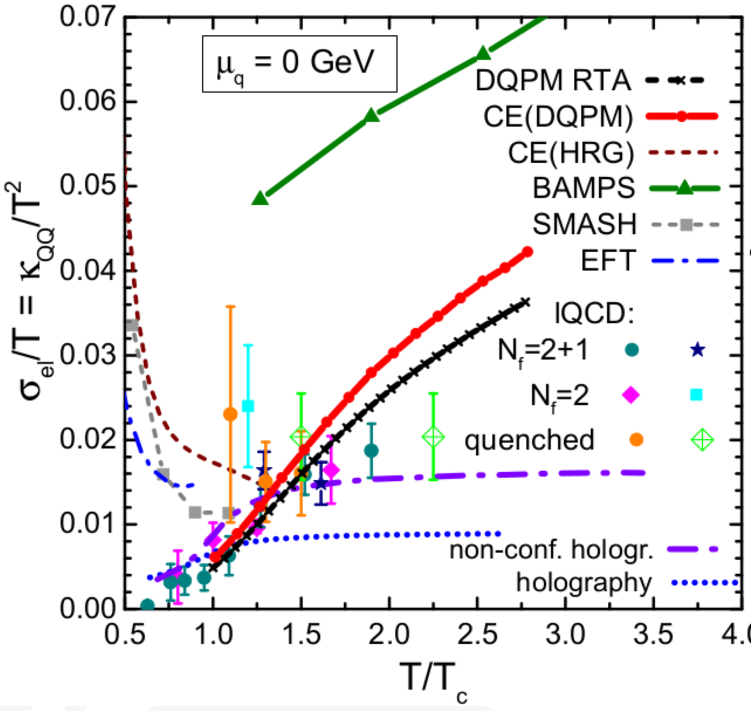
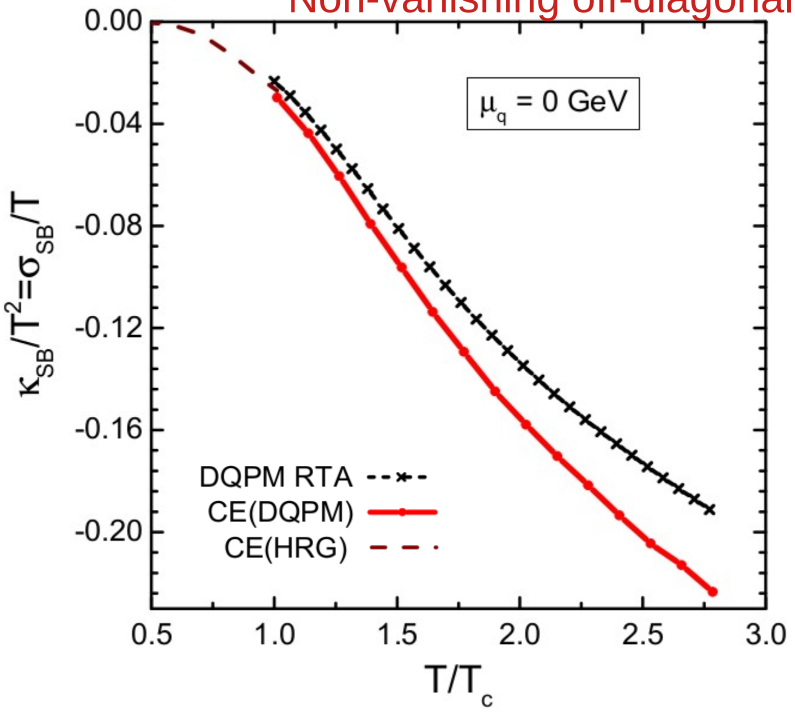
Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Example: introduction of features from LQCD via the usage of DQPM



Non-vanishing off-diagonal entries!



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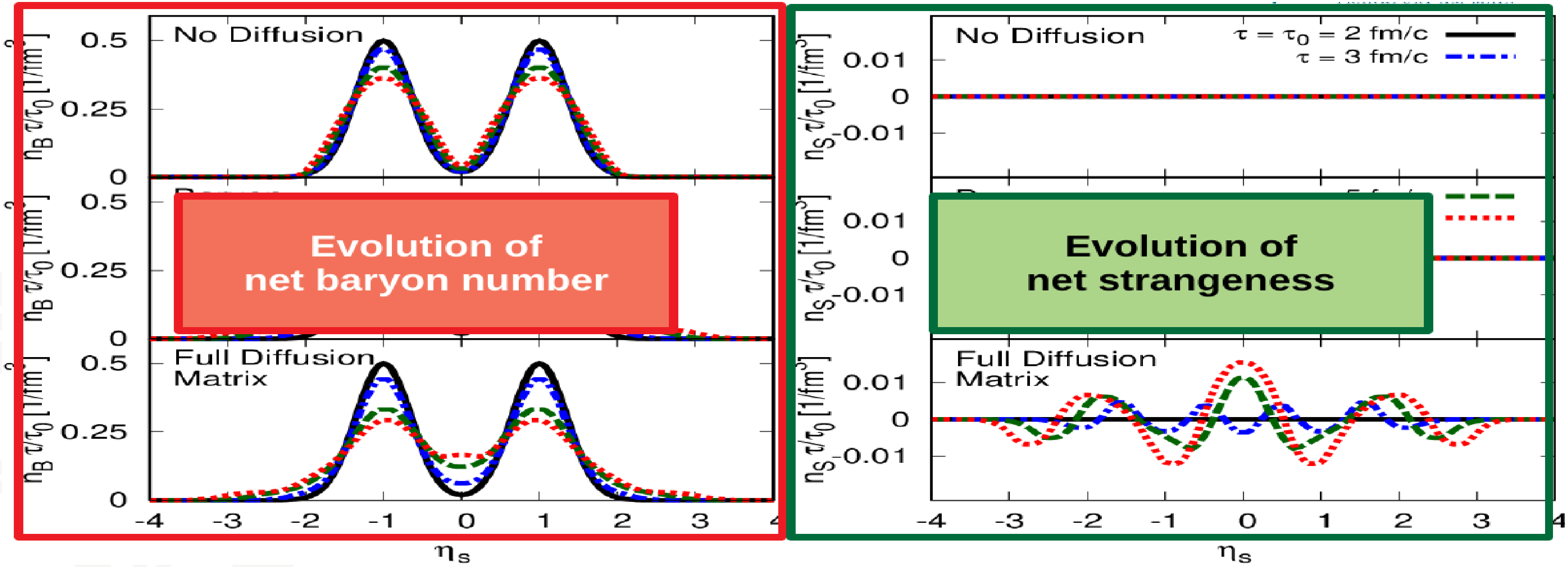

Simplistic case study: no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation with SHASTA

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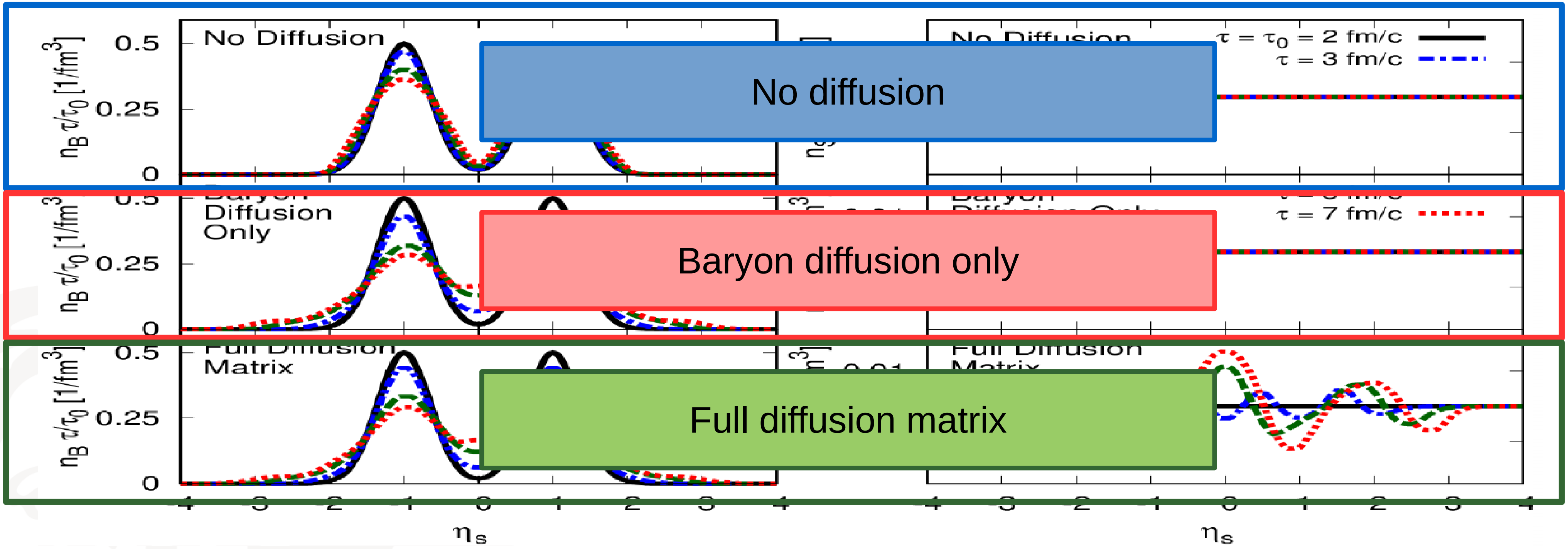

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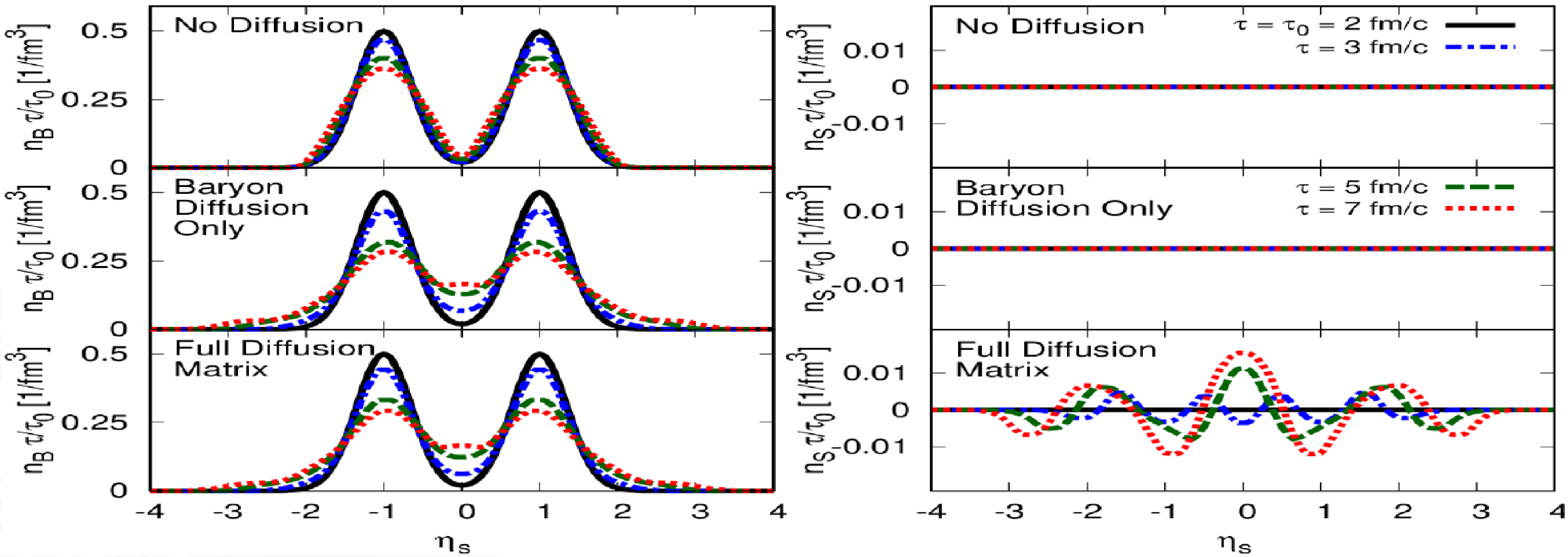

Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

→ Generation of domains of non-vanishing local net charge (here net strangeness)!

e.g.:

$$\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$$

$$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$$

Single-component vs. Multicomponent system

A potentially problematic term in single-component systems $$\mathrm{S}_q^{\mu} = (\ldots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + (\ldots)$$

Ultrarelativistic, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

Used in simulations of heavy-ion collisions!

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\mathrm{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

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Multicomponent system: $$\ell_{V\pi}^{(q)} = \frac{9}{68\sigma P}\left(\frac{1}{4}\left(17 - 9N_{\mathrm{spec}}\right)c_q - \frac{8}{5}\sum_{i=1}^{N_{\mathrm{spec}}} q_i\right) \overset{\text{single}}{\rightarrow} \ell_{n\pi} = \tau_n/(20T)$$

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The problem: system with conserved net-charge and **each** constituent has anti-particle partner at **vanishing** chemical potential:

$$\ell_{V\pi}^{(q)} = 0 \neq \ell_{n\pi} = \tau_n/(20T)$$

Single-component vs. Multicomponent system

Run simulation of system with conserved baryon number and strangeness with shear viscosity and diffusion; account for second-order terms

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Second-order coefficients in ultrarelativistic, single-component limit from Denicol (2012)

$$\tau_n \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q,\nu}\omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu}\sigma^{\mu\nu} + \boxed{\frac{\tau_n}{20T}\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T}\pi^{\mu\nu}\dot{u}_\nu} - \frac{\tau_n}{20T}\pi^{\mu\nu}\nabla_\nu \alpha_q$$

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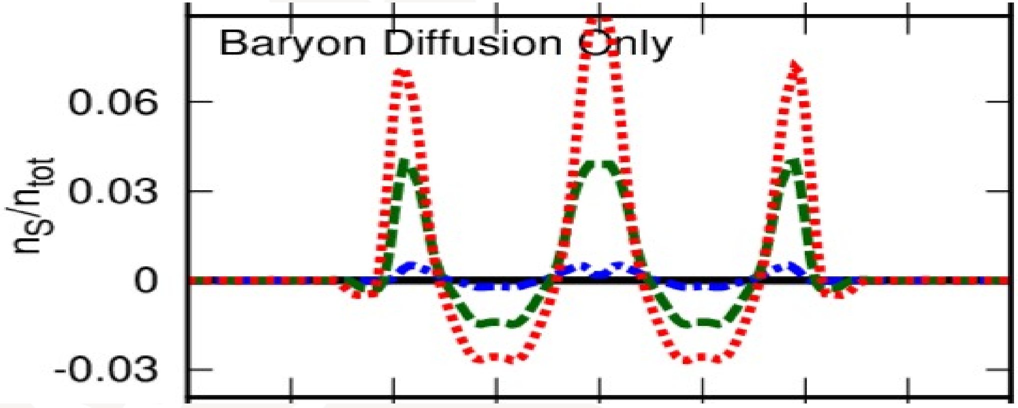
Here (plot): baryon diffusion only: $\kappa_{\mathrm{BB}} \neq 0, \qquad \kappa_{\mathrm{SS}} = 0 = \kappa_{\mathrm{SB}}$



Second-order transport coefficients not consistent with assumed system

$\longrightarrow$ generation of unphysical charge currents

Consistency is important in charge transport!

Conclusion

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- Derived 2nd-order relativistic fluid dynamic theory for multicomponent systems from the Boltzmann equation
- Transport coefficients given explicitly containing all information about particle interactions
- Mixed chemistry couples diffusion currents to each other $\longrightarrow$ coupled charge-transport
- Consistency of EoS, 1st and 2nd transport coefficients is important!
- Thermal features from lQCD can be adapted in transport coefficients
- Implemented derived fluid dynamic theory in (3+1)D-hydro code

Outlook

- Evaluate 2nd order transport coefficients for more realistic systems
- Use more realistic initial state and equation of state (see above)
- Apply freeze-out routines, take δf-correction
- Find observables sensitive to charge-coupling $\longrightarrow$ investigate impact

Backup

Computation of transport coefficients (Example: diffusion coefficients)

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On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

→ **upcoming publication!** (Fotakis, Molnár, Niemi, Rischke, Greiner)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty} \sum_j \boxed{\mathcal{C}_{ij,nm}^{(1)}} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

Computation of transport coefficients (Example: diffusion coefficients)

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Entries of „collision matrix“ (for diffusive moments)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{(1)}\right)^{-1}_{ij,0n} q_i \left(q_j' J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1}\right)$$

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Equation of State - details

- Hadronic system including lightest 19 species

$$\pi^{\pm}, \pi^0, K^{\pm}, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^{\pm}, \bar{\Sigma}^{\pm}$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

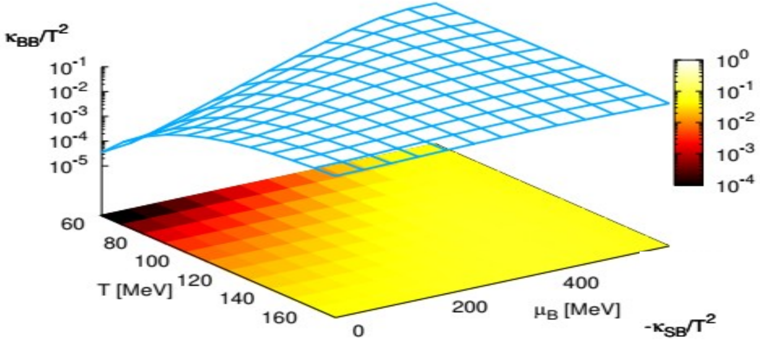
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

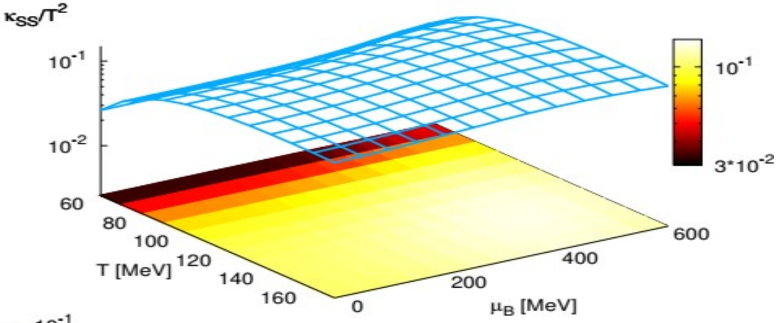
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. ***37****, 405 (1931) & Phys. Rev.* ***38****, 2265 (1931)*

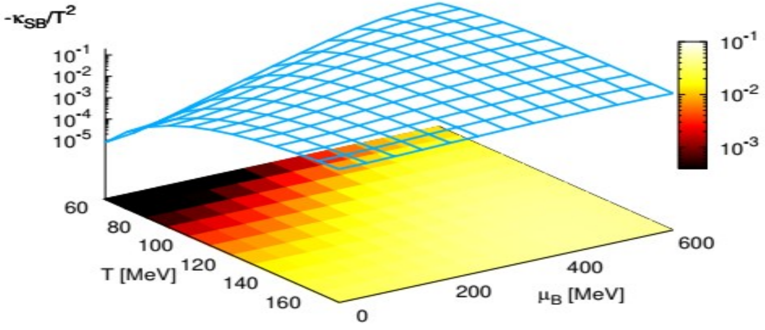


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

$\Longrightarrow$ significant coupling?



- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

Initial conditions - details

- $\tau_0 = 2 \text{ fm/c}$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density