TRENTo initial condition model and the isobar collisions

RBRC virtual Workshop, Physics Opportunities from the RHIC Isobar Run

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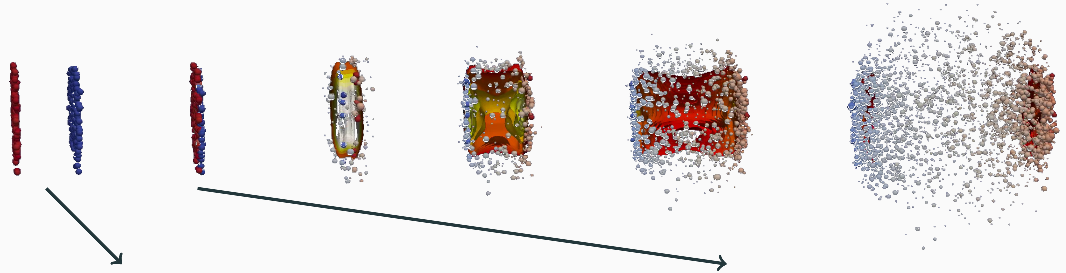
TRENTo initial condition model

Initial condition is still a major uncertainty in heavy-ion collisions



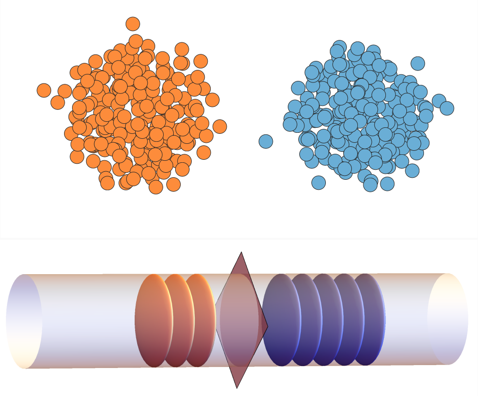
Uncertainty in nuclear structure

- Woods Saxon parametrization, deformation, radial profiles.
- Correlations.
- Isospin.

Uncertainty in energy deposition.

- Transverse ($x_\perp$) structure.
- Longitudinal (η_s) structure.
- Baryon number, initial flow ...

The idea of TRENTo (middle rapidity)



Assumption: $\gamma \to \infty$

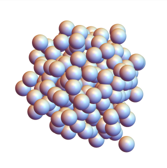
In central region with boost invariance

$$\frac{dE}{dx_\perp^2\, d\eta_s}(\eta_s = 0) = f(T_A(x_\perp), T_B(x_\perp))$$

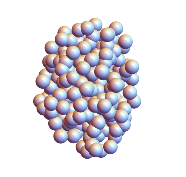
A flexible parametric approach to $f(T_A, T_B)$ [JS Moreland, JE Bernhard, SA Bass, PRC 92, 011901 (2015)].

No dynamics, but useful to quickly estimate the effect of initial state uncertainty.

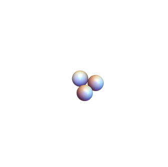
Nuclear configuration: current public TRENTo (2D)[1]



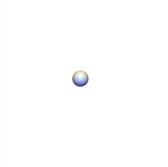
^{208}Pb



^{238}U



^{3}He



p

- No isospin, just nucleons.
- One-nucleon density: Woods-Saxon form $\frac{1}{1+\exp\left(\frac{r-R}{a}\right)}$
  - R: radius, a: diffuseness
  - Deformation: current public version only includes β_2, β_4.

$$R \to R\left[1 + \beta_2 Y_{20}(\theta,\phi) + \beta_4 Y_{40}(\theta,\phi)\right]$$

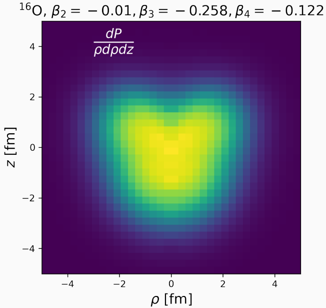
  - Parameters [Atom.Data Nucl.Data Tabl. 109-110 (2016) 1].
- $\min r_{ij} > d_{\min}$ to mimic short-range repulsion.
- Light nuclei: load samples of nuclear configurations $|\Psi|^2(r)$, e.g., ^{3}He [PLB 680, 225–230 (2009)], ^{16}O.

[1] http://qcd.phy.duke.edu/trento/index.html

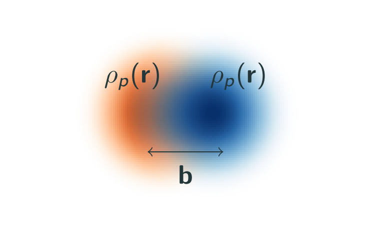
Nuclear configuration: will enable more density profile

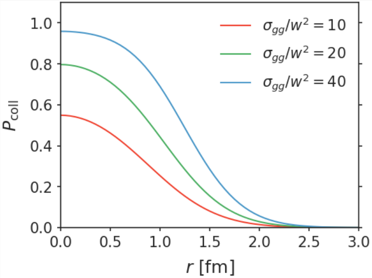
- Allow direct input to Woods-Saxon parameters $R, a, \beta_n, \ldots$.
- Including β_3 deformation.
- $\frac{1}{1+e^{(r-R_{\theta,\phi})/a}} \rightarrow \frac{1+b(r/r_0)^2}{1+e^{(r-R_{\theta,\phi})/a}}$

  Example: Oxygen with a large $|\beta_3|$ and nonzero b and r_0 ▷



Nucleon profile and N-N inelastic cross section





Nucleon model #1: Gaussian proton

$$\rho_p(\mathbf{r}, z) = \frac{e^{-\frac{\mathbf{r}^2+z^2}{2w^2}}}{(2\pi w)^{3/2}} \xrightarrow{\int dz} \rho_p(\mathbf{r}) = \frac{e^{-\frac{r^2}{2w^2}}}{2\pi w^2}$$

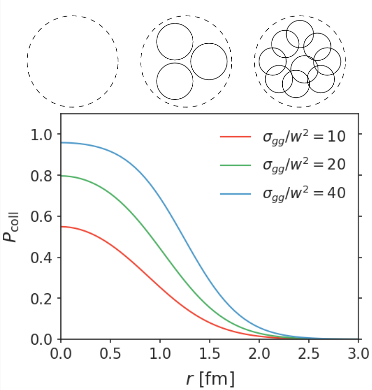
Probability of inelastic collisions at fixed impact parameter.

$$\begin{aligned} T_{pp}(b) &= \int \rho_p(\mathbf{r} - \mathbf{b}/2)\rho_p(\mathbf{r} + \mathbf{b}/2) d\mathbf{r}^2 \\ P_{\text{coll}}(b) &= 1 - \exp\{-\sigma_{gg} T_{pp}(b)\} \end{aligned}$$

σ_{gg}: effective opacity parameter tuned to reproduce $\sigma_{pp}^{\text{inel}}(\sqrt{s})$

$$\sigma_{pp}^{\text{inel}} \sqrt{s} = \int P_{\text{coll}}(\mathbf{b}; \sigma_{gg}(\sqrt{s})) d\mathbf{b}^2$$

Nucleon profile and N-N inelastic cross section



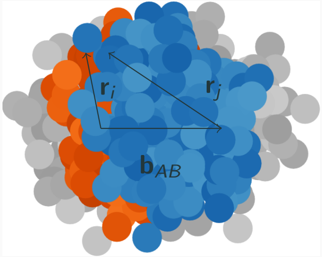
Nucleon model #2: with substructures [JS moreland, JE Bernhard, SA Bass, PRC 101, 024911]

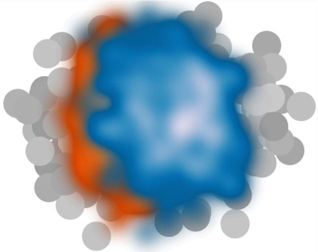
$$\rho_p(r) = \frac{1}{N}\sum_{i=1}^{N} \frac{e^{-\frac{(r-r_i-R_{\mathrm{cm}})^2}{2w_c^2}}}{2\pi w_c^2}, r_i \sim \frac{e^{-\frac{r_i^2}{2w'^2}}}{2\pi w'^2}$$

R_{cm} fix the center of mass.

σ_{gg} solved in a MC way to reproduce $\sigma_{pp}^{\mathrm{inel}}(\sqrt{s})$.

Binary collisions and fluctuating participants density



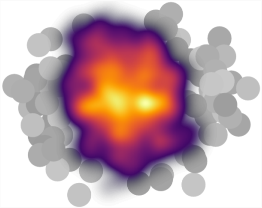


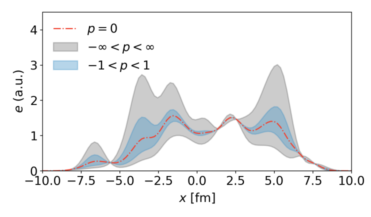
- Participant nucleons determined by sampling binary collision probability $P_{\text{coll}}(b = |\mathbf{r}_j - \mathbf{b}_{AB} - \mathbf{r}_i|)$.
- Fluctuating participant density:

$$T_{A \text{ or } B}(\mathbf{r}) = \sum_{i \in \text{Part. } A \text{ or } B} \gamma_i \rho_p(\mathbf{r} - \mathbf{r}_i)$$

- $P(\gamma_i) \propto \gamma^{k-1} e^{-k\gamma}$. Emulate fluctuation in *pp* measurement, can change with kinematic cuts!

Energy density production at mid-rapidity





$$\frac{dE_T}{dx_\perp^2\, d\eta_s}(x_\perp, \eta_s = 0) = \mathrm{Norm} \times f(T_A(x_\perp), T_B(x_\perp))$$

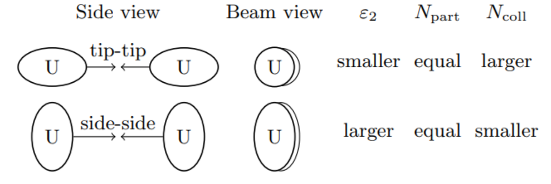
TRENTo assumes

$$f(T_A, T_B) = \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

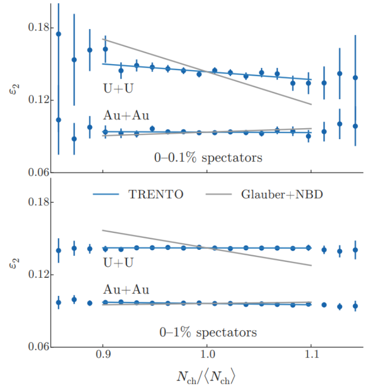
known as “generalized mean” (p-mean) ansatz.

One motivation of using p-mean

p-mean is "homogeneous" $f(kT_A, kT_B) = kf(T_A, T_B)$.
Binary collisions $(T_A T_B)$ is not.



If N_{coll} involved, fine binning of N_{ch} should differentiate ϵ_2 ▷.

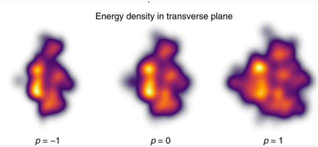


[JS Moreland, JE Bernhard, SA Bass, PRC 92, 011901 (2015)]

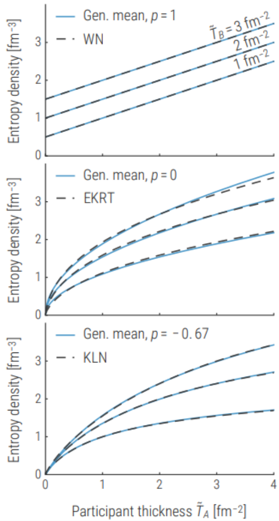
p-mean is a class of energy deposition consistent with this observation.

Two-component Glauber $N_{\text{ch}} \propto (1-x)N_{\text{part}} + xN_{\text{coll}}$ is not consistent.

Connections to scaling of other models



Still, only a subclass of exisiting models.



- Wounded nucleon model

$$\frac{dS}{dy\, d^2r_\perp} \propto \tilde{T}_A + \tilde{T}_B$$

- EKRT model PRC 93, 024907 (2016)

  after brief free streaming phase

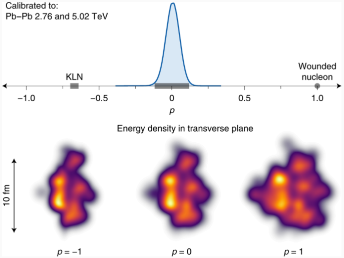
$$\frac{dE_T}{dy\, d^2r_\perp} \sim \frac{K_{\text{sat}}}{\pi} p^3_{\text{sat}}(K_{\text{sat}}, \beta; T_A, T_B)$$

- KLN model PRC 75, 034905 (2007)

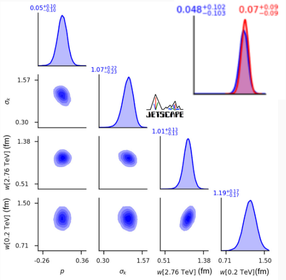
$$\frac{dN_g}{dy\, d^2r_\perp} \sim Q^2_{s,\min}\left[2 + \log\left(\frac{Q^2_{s,\max}}{Q^2_{s,\min}}\right)\right]$$

[JS Moreland]

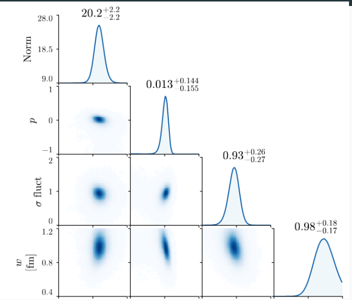
Energy deposition ansatz



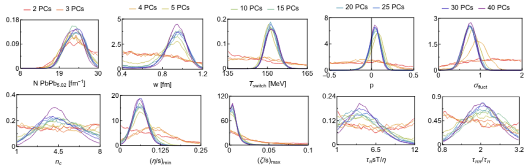
Round proton, AA@LHC
[Duke PRC 94 024907]



Round p, RHIC&LHC, δf uncertainty
[JETSCAPE PRC 103, 054904]



Fluctuating proton AA and pA
[Duke PRC 101, 024911]



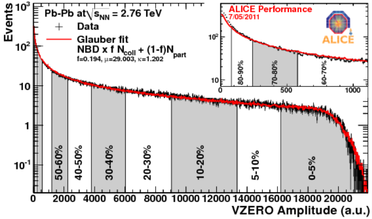
Fluctuating proton AA and pA, p_T-diff obs, refined centrality class [Trajectum PRC 103, 054909,]

The p-parameter is always tightly constrain with high likelihood at $p = 0$.

$p = 0$ implies $e = \sqrt{T_A T_B}$, can be motivated by $E_{\rm cm} = \sqrt{T_A p^+ T_B p^-} = \sqrt{T_A T_B s}$ [C Shen, S Alzhrani PRC 102 014909]

Nuclear/nucleon configurations & total cross-section

Centrality: percentage of minimum-bias hadronic cross section Pb-Pb@2.76 TeV $770 \pm 10(\mathrm{stat.})^{+60}_{-50}(\mathrm{sys.})\mathrm{fm}^2$ **8% level**. [ALICE PRL 109 252302, PRC 88 044909].

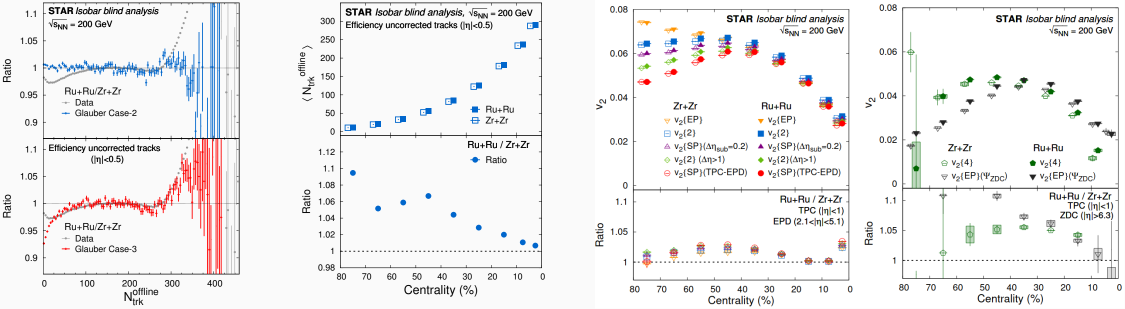


In Glauber-based models, including TRENTo

- Gaussian nucleon w and β can affect the total cross section: $\sigma_{\mathrm{PbPb}}^{\mathrm{TRENTo}}[w = 0.5\ \mathrm{fm}] = \mathbf{782 \pm 4}\ \mathrm{fm}^2$ vs $\sigma_{\mathrm{PbPb}}^{\mathrm{TRENTo}}[w = 0.8\ \mathrm{fm}] = \mathbf{833 \pm 4}\ \mathrm{fm}^2$
- Some reasons that σ_{AA} is not used as a constraint in analysis before:
  - *pp* and nuclear inelastic cross-section have large uncertainty.
  - No exact match of geometry model to the experimental minimum-bias trigger.
  - Different IC models have different minimum-bias criteria ...
- Can we make use of the precision measurement cross sections in isobar collisions?

Isobar collisions

Some isobar results from STAR Collaboration [arXiv:2109.00131].



- Very high precision measurements.
- Can be very challenging for models. Previous Global fits usually agree with multiplicity and flow data within 5-10% uncertainty.

Perturbations in nuclear deformation

Use isobar to maximize the sensitivity to nuclear geometry [J Jia, C-J Zhang, 2111.15559 and J Jia PRC 105 014905].

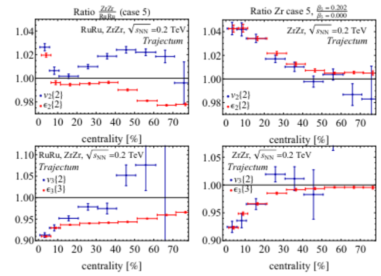


FIG. 7. It is an interesting question if our $v_2\{2\}$ (top) and $v_3\{2\}$ (bottom) could have been anticipated by initial geometric differences of $\epsilon_n\{n\}$, as in [7]. We show such comparisons for ZrZr/RuRu (case 5, left) and for the case of appendix B, where we divide $\beta_3 = 0.202$ with the case $\beta_3 = 0$ (right).

Linearized response of v_n to ϵ_n

$$\begin{aligned} v_2 &\approx k_{22}\epsilon_2 \\ v_3 &\approx k_{23}\epsilon_3 \end{aligned}$$

Best scenario: isobar systems only differ in higher orders in the response coefficients k_{22}, k_{23}.

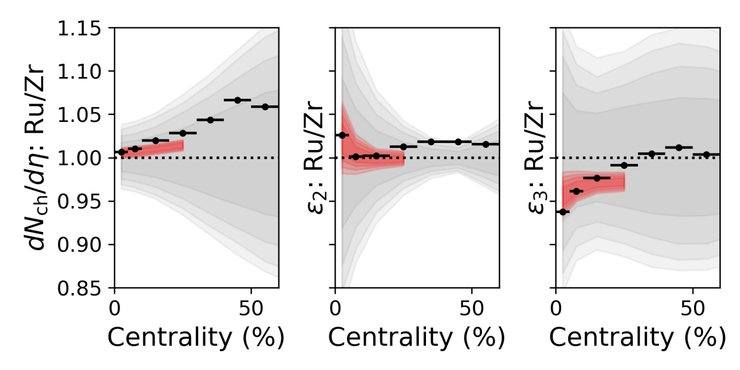
- Unfortunately, hydrodynamical response not entirely canceled when $R_A \neq R_{\bar{A}}$ [◁ G. Nijs, W. van der Schee 2112.13771] except for very central region.

An initial-state study (0-25%)

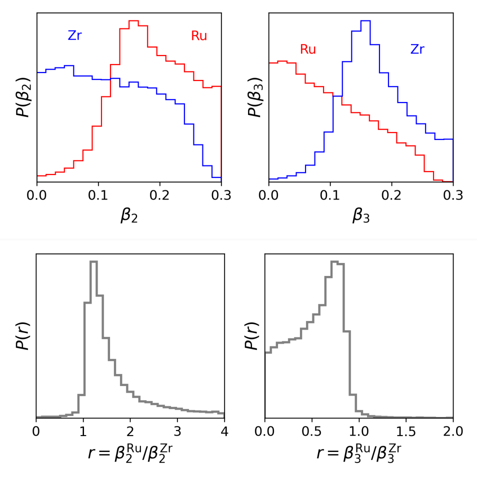
First, fixing the energy deposition parameter $p = 0$, nucleon width $w = 0.6$ fm, fluctuation parameter $k = 1$, and repulsion distance d_{min}. Just vary Woods-Saxon parameters

- $0 < \beta_{2,\mathrm{Ru}}, \beta_{2,\mathrm{Zr}} < 0.3$.
- $0 < \beta_{3,\mathrm{Ru}}, \beta_{3,\mathrm{Zr}} < 0.3$.
- $4.9 < R_{\mathrm{Ru}}, R_{\mathrm{Zr}} < 5.2$ fm.
- $0.4 < a_{\mathrm{Ru}}, a_{\mathrm{Zr}} < 0.6$ fm.

$$\begin{cases} \dfrac{(dN_{\mathrm{ch}}/d\eta)_{\mathrm{Ru\text{-}Ru}}}{(dN_{\mathrm{ch}}/d\eta)_{\mathrm{Zr\text{-}Zr}}} \\ \dfrac{(v_2)_{\mathrm{Ru-Ru}}}{(v_2)_{\mathrm{Zr\text{-}Zr}}} \\ \dfrac{(v_3)_{\mathrm{Ru\text{-}Ru}}}{(v_3)_{\mathrm{Zr\text{-}Zr}}} \end{cases} \quad v.s. \quad \begin{cases} \dfrac{\left(\int e d^2\mathbf{x}\right)_{\mathrm{Ru\text{-}Ru}}}{\left(\int e d^2\mathbf{x}\right)_{\mathrm{Zr\text{-}Zr}}} \\ \dfrac{(\epsilon_2)_{\mathrm{Ru-Ru}}}{(\epsilon_2)_{\mathrm{Zr\text{-}Zr}}} \\ \dfrac{(\epsilon_3)_{\mathrm{Ru\text{-}Ru}}}{(\epsilon_3)_{\mathrm{Zr\text{-}Zr}}} \end{cases}$$



Nuclear deformation with "only" information from HIC

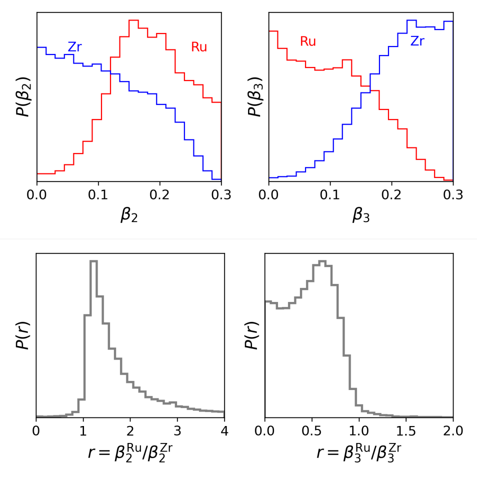


Apart from the sign of β, no prior knowledge from nuclear strucutre used.

◁ Not very sensitive to the absolute value of β without using the magnitude of v_n. High confidence: $\beta_{2,\mathrm{Ru}}/\beta_{2,\mathrm{Zr}} > 1, \beta_{3,\mathrm{Ru}}/\beta_{3,\mathrm{Zr}} < 1$

Is this conclusion robust when other TRENTO parameters vary?

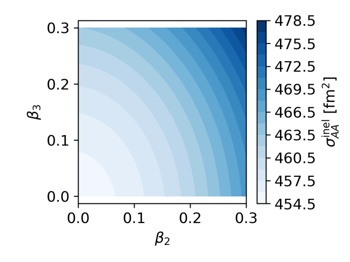
Vary both TRENTo parameters and the nuclear deformation and Woods-Saxon parameter.



- $0 < \beta_{2,\mathrm{Ru}}, \beta_{2,\mathrm{Zr}} < 0.35$.
- $0 < \beta_{3,\mathrm{Ru}}, \beta_{3,\mathrm{Zr}} < 0.35$.
- $4.9 < R_{\mathrm{Ru}}, R_{\mathrm{Zr}} < 5.1$ fm.
- $0.4 < a_{\mathrm{Ru}}, a_{\mathrm{Zr}} < 0.6$ fm.
- $p \sim e^{-\frac{(p-0.05)^2}{2\times 0.06^2}}$ informative prior from previous study.
- $0.4 < w < 1.0$ fm, nucleon width.
- $1/3 < k < 3$, fluctuation.
- $0 < d < 1.5$ fm, nucleon repulsion distance.

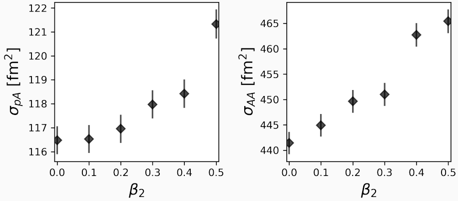
Relatively robust conclusion on β_2 and β_3, considering uncertainties in TRENTo parameters.

Nuclear cross section

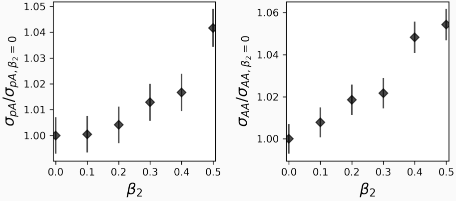


- AA cross section changes significantly with current parametrization of β (and a_0, w).
- Cross sections as an independent constraint.
- Precise values may depend on "minimum bias" definition + other systematic. Do they cancel in isobar ratio?

Total cross-section for AA and pA:



Ratio of two isobars with one has $\beta_2 = 0$.



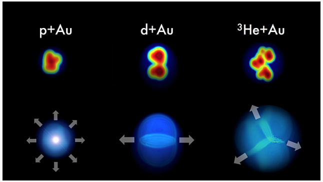
3D developments

How can we use isobars in asymmetric collisions?

- Total cross sections of pA vs $p\bar{A}$.
- Longitudinal decorrelations for rapidity evolution of geometry.
- Collisions of large nuclei and isobar, e.g. Au+Ru vs Au+Zr.

  $R_{\mathrm{Au}} \approx 6.5$ fm. $R_{\mathrm{Ru,Zr}} \approx 5.0$ fm.

  Eliminate one deformed object in ultra-central collisions.

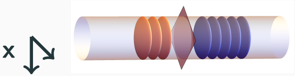


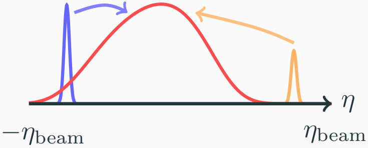
Ru, Zr

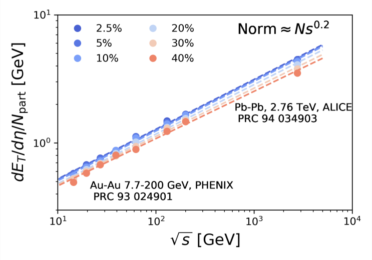
[Fig. Javier Orjuela Koop, University of Colorado, Boulder]

Extra efforts: 3D initial condition + 3+1D simulation (order of magnitude expensive).

TRENTo: from middle to finite rapidity







- New[2] TRENTO 3D parametrization is constructed exclusively for $p=0$. Near middle rapidity

$$e(\mathbf{x}, \eta_s = 0) \propto \left[\frac{T_A(\mathbf{x})^p + T_B(\mathbf{x})^p}{2}\right]^{\frac{1}{p}} \rightarrow N\sqrt{s}^{\alpha}\sqrt{T_A T_B}$$

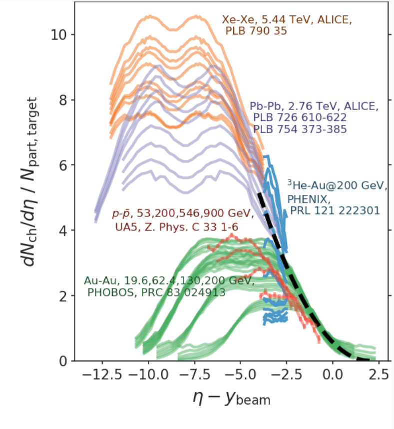
- Extend to finite rapidity, but away from $y_{\rm beam}$

$$\begin{aligned} e(\mathbf{x}, |\eta_s| \ll y_b) &= e(\mathbf{x}, 0) e^{-\frac{(\eta_s - \eta_{c.m.})^2}{2y_b}} \\ \eta_{c.m.}(\mathbf{x}) &= \frac{1}{2}\ln\frac{T_A e^{y_b} + T_B e^{-y_b}}{T_A e^{-y_b} + T_B e^{y_b}} \end{aligned}$$

width$\sim\sqrt{y_b}$ (Landau picture of particle production).

[2] Earlier 3D extension, WK, JS Moreland, JE Bernhard, SA Bass, PRC 96, 044912 (2017).

Scaling of particle production near y_{beam}



Limiting fragmentation assumption[3]:
$dN_{ch}/d\eta/N_{part,target} \approx F(\eta - y_b)$

- Form of $dF(\eta - y_b)$ motivated by parton distribution function of the broken target[4].
- Assume energy deposition $y \approx y_b$ scales as

$$\frac{de_{\mathrm{F/B}}}{d\eta} \sim C_{\mathrm{F/B}}\left[T_A(\mathbf{x})F(y_{\mathrm{b}} - \eta) + T_B(\mathbf{x})F(y_{\mathrm{b}} + \eta)\right]$$

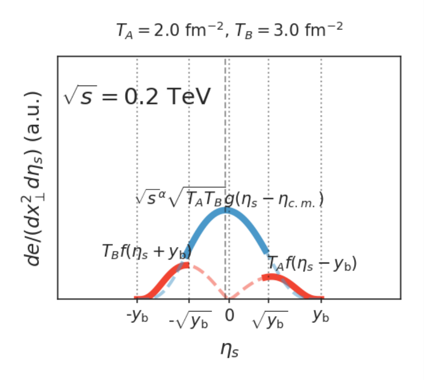
- Interpolate to midrapidty fireball $(N\sqrt{s}^{\alpha}\sqrt{T_A T_B}g(\eta - \eta_{\mathrm{cm}}))$, with longitudinal energy-momentum conservation.

[4] J Benecke, TT Chou, CN. Yang, E Yen Phys. Rev. 188 (1969) 2159. PHOBOS PRL 91 (2003) 052303.
[4] J Jalilian-Marian, PRC 70, 027902; SA Bass, B Müller, DK Srivastava PRL 91 052302

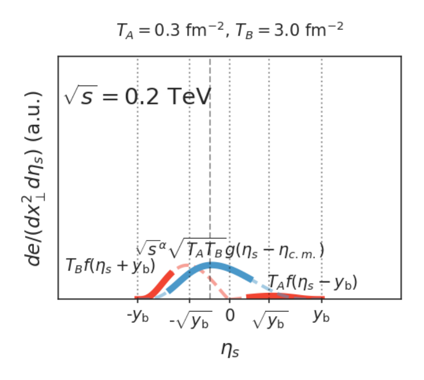
Impact on rapidity-dependent geometric properties

- Geometric properties will evolve from fragmentation region (T_A, T_B) to central region ($\sqrt{T_A T_B}$).
- Central fireball becomes increasingly important at high $\sqrt{s}$.

Typical T_A, T_B for A-A collisions



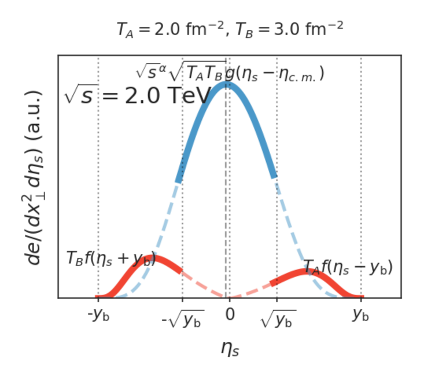
Typical T_A, T_B for p-A collisions



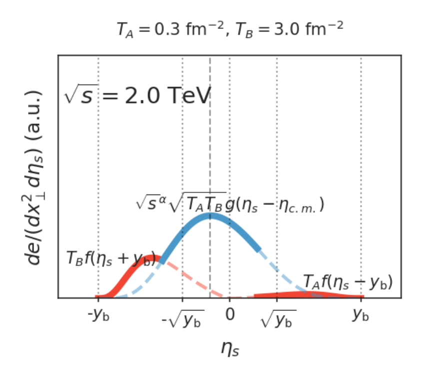
Impact on rapidity-dependent geometric properties

- Geometric properties will evolve from fragmentation region (T_A, T_B) to central region ($\sqrt{T_A T_B}$).
- Central fireball becomes increasingly important at high $\sqrt{s}$.

Typical T_A, T_B for A-A collisions



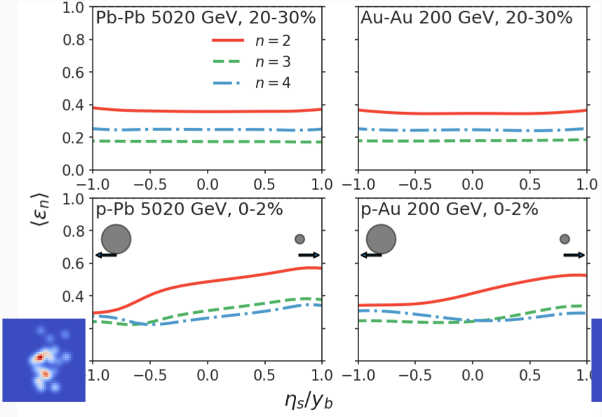
Typical T_A, T_B for p-A collisions

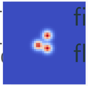


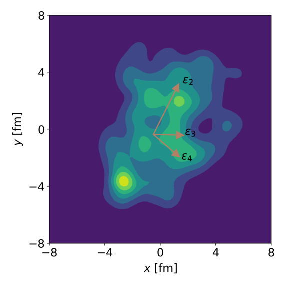
Spacetime-rapidity evolution of the event geometry

Rapidity evolution of the eccentricity:

$$\color{red}{\epsilon_n(\eta_s)}\color{black}{e^{in\Phi_n(\eta_s)}} = \frac{\int dx_\perp^2\, r^n e^{in\phi} e(x_\perp, \eta_s)}{\int dx_\perp^2\, r^n e(x_\perp, \eta_s)}$$

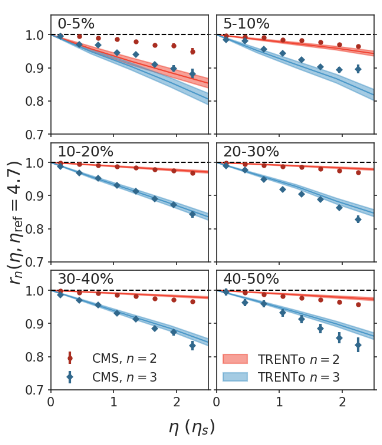






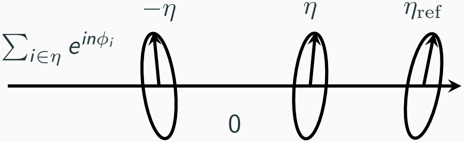
- $\langle \epsilon_n \rangle(\eta_s) \sim \text{const.}$ in AA collisions.
- In p-A collisions, ϵ_n interpolates proton-shape fluctuation, central fireball, and nuclear participant fluctuation.

Longitudinal factorization ratio of participant planes



Pb-Pb 2.76 TeV, CMS, PRC 92 034911

$Q_n(\eta) = \sum_{i \in \eta} e^{in\phi_i}$



$$r_n = \frac{\langle Q_n(-\eta) Q_n^*(\eta_{\rm ref}) \rangle}{\langle Q_n(\eta) Q_n^*(\eta_{\rm ref}) \rangle} \approx \frac{\langle \cos(n[\Psi_n(-\eta) - \Psi_n(\eta_{\rm ref})]) \rangle}{\langle \cos(n[\Psi_n(\eta) - \Psi_n(\eta_{\rm ref})]) \rangle}$$

- Approximate Ψ_n with Φ_n of ϵ_n.
- Agreement for mid-central collisions. TRENTo results in too much decorrelation in 0-5% collisions.

  Other studies: AMPT+hydro, LG Pang et al Eur.Phys.J.A 52 (2016) 97; 3D-Glasma, B Schenke, S Schlichting; Torque Glauber, P Bozek, W Broniowski, PLB 752 (2016) 206-211

[7] Pb-Pb 2.76 TeV, CMS, PRC 92 034911. Pb-Pb 5.02 TeV, ATLAS, EPJC 78 142; Au-Au 200 & 27 GeV, STAR Preliminary QM18 (NPA 982 403-406),QM19(2005.03252)

Ongoing works

Ongoing efforts with **Derek Soeder (Duke)**, Jean Francois Paquet, Steffen Bass.

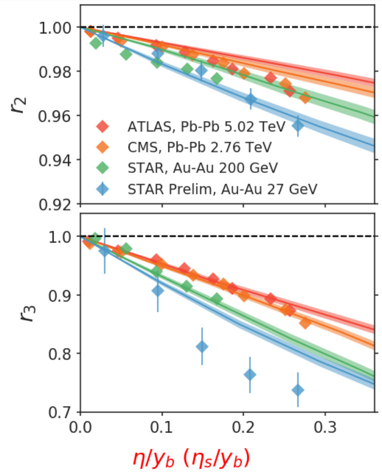
- Calibrate new 3D TRENTo + (1+1D) dynamics to charged particle pseudorapidity density.
- To do: calibrate with JETSCAPE (3+1)D simulation of soft sector [JETSCAPE Phys.Rev.C 103 (2021) 5, 054904, https://jetscape.org/sims/].
    - TRENTo (2d/3d)
    - Pre-equilibrium dynamics (Free streaming).
    - 3+1D viscous hydrodynamics (MUSIC).
    - Particlization (IS3D).
    - Hadronic transport (SMASH).

Summary

- TRENTo: parametric initial condition available in 2D and 3D (developing).
- New 3D model:
    - TRENTo-2D near middle rapidity is interpolated to limiting fragmentation region near beam rapidity.
    - Analysis with dynamical models underway.
- Isobar measurements are sensitive to nuclear geometry.
- A simple Bayes study of Ru/Zr of $N_{\mathrm{ch}}, v_2, v_3$ at the initial condition (IC) level:
    - Results are robust within our current uncertainty in energy deposition model.
    - IC calculation without dynamics may not have the required accuracy.
- Are total cross-section ratios feasible in isobar collisions $AA/\bar{A}\bar{A}$, $pA/p\bar{A}$, $AB/\bar{A}B$ to constrain Glauber-based models?

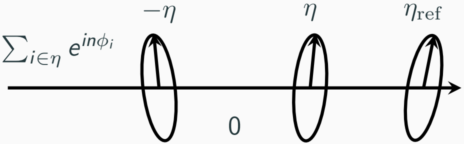
Questions?

Longitudinal factorization ratio of participant planes



Pb-Pb 2.76 TeV, CMS, PRC 92 034911

$$Q_n(\eta) = \sum_{i\in\eta} e^{in\phi_i}$$



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- $\sqrt{s}$-dependent r_n in 10-40%[5], to be improved with dynamical evolution.

Is this conclusion robust when other TRENTO parameters varies?

Vary both TRENTo parameters and the nuclear deformation and Woods-Saxon parameter.

- $0 < \beta_2 < 0.35$.
- $0 < \beta_3 < 0.35$.
- $4.9 < R_{\mathrm{Ru}}, R_{\mathrm{Zr}} < 5.1$ fm.
- $0.4 < a_{\mathrm{Ru}}, a_{\mathrm{Zr}} < 0.6$ fm.
- $p \sim e^{-\frac{(p-0.05)^2}{2\times 0.06^2}}$ informative prior from previous study.
- $0.4 < w < 1.0$ fm, nucleon width.
- $1/3 < k < 3$, fluctuation.
- $0 < d < 1.5$ fm, nucleon repulsion distance.