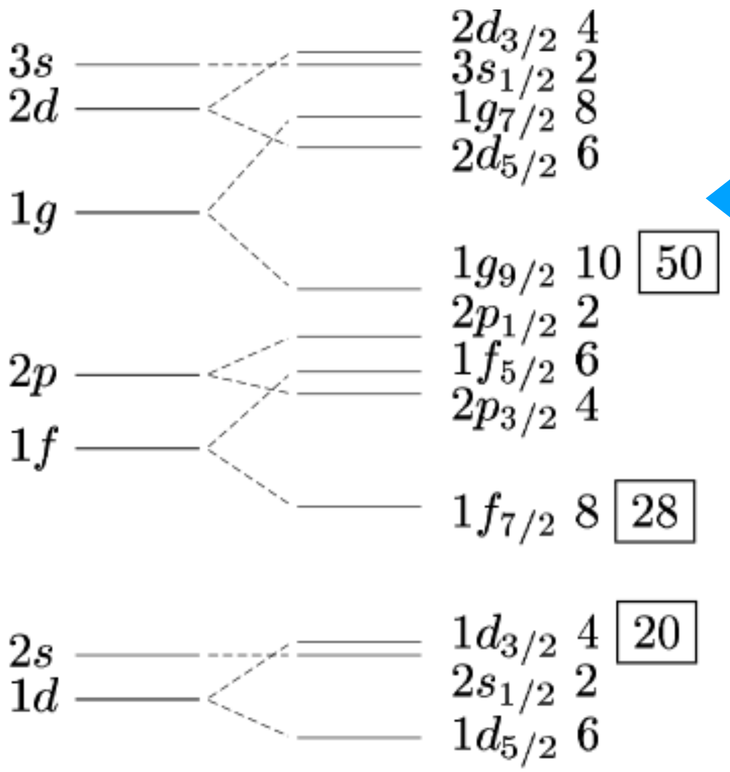
The heavy ions are “preheated” prior to high energy heavy ion collisions

Edward Shuryak
Center for Nuclear Theory, Stony Brook

• e-Print: 2201.11064 [nucl-th]



Why are ^{96}Zr and ^{96}Ru so different? already because 1g_9/2 and 2d_5/2 wave functions are rather different!

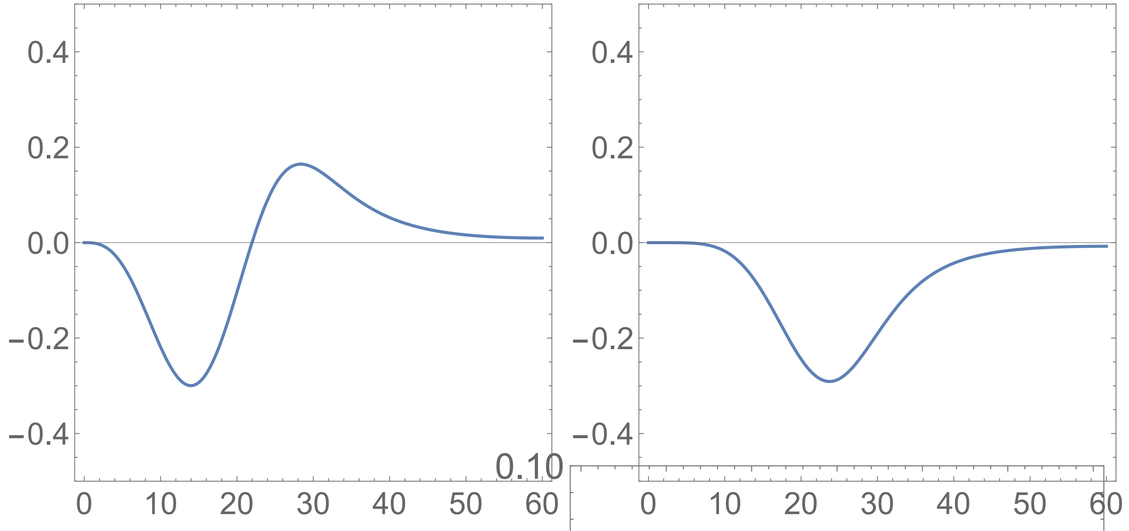
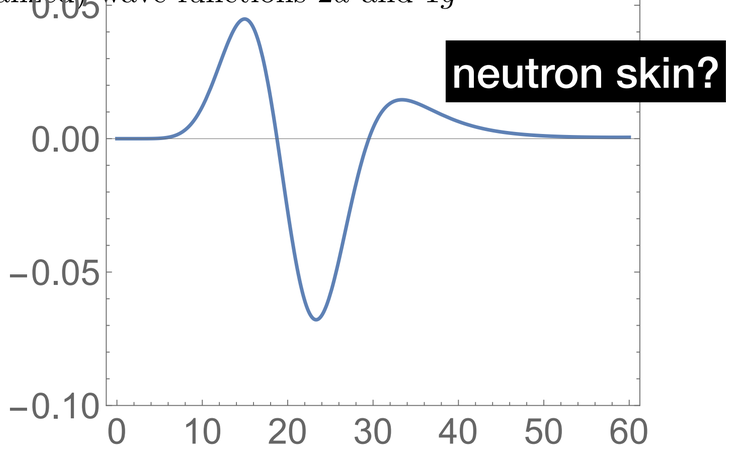


FIG. 9. (Unnormalized) wave functions $2d$ and $1g$

$$Ru = d_{5050}(r) - 6\psi_p^2(r) + 2\psi_n^2(r)$$
$$Zr = d_{5050(r)} - 10\psi_p^2(r) + 6\psi_n^2(r);$$

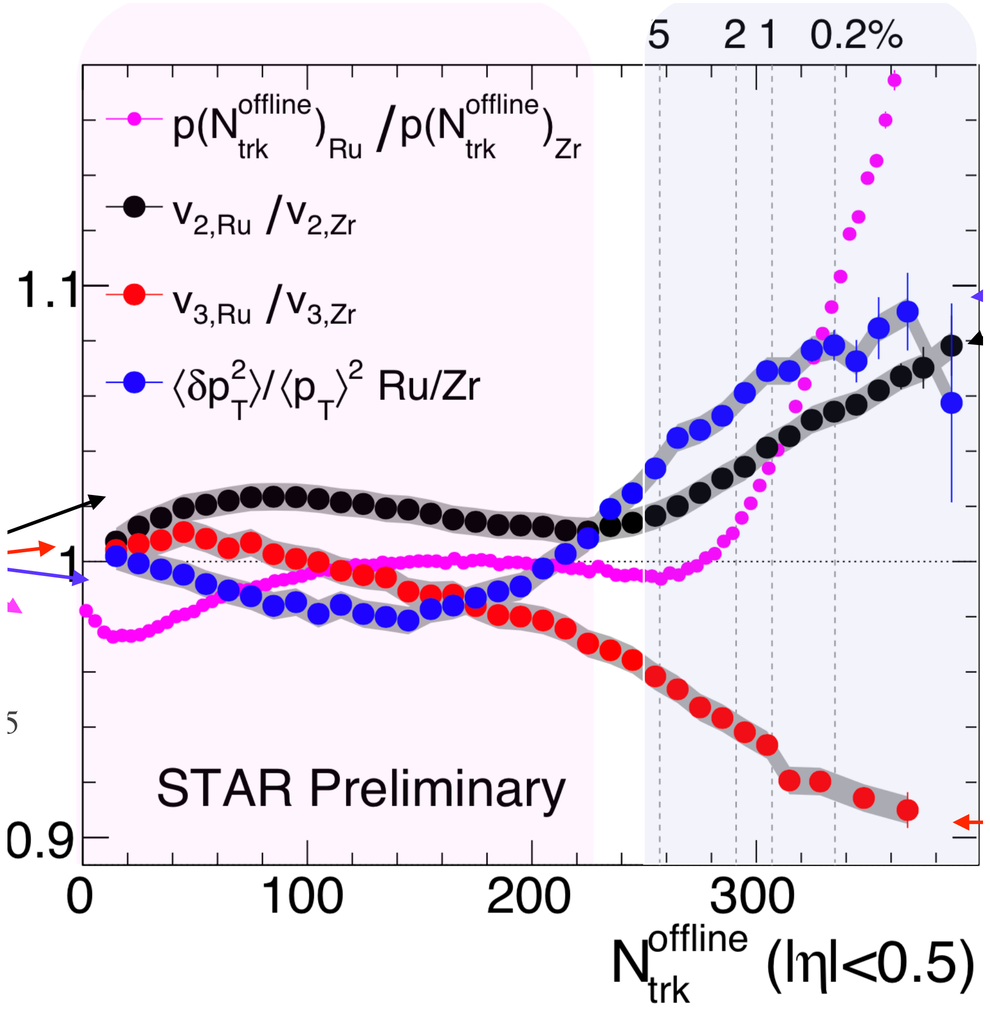


neutron skin?

The now famous STAR experiment has shown that
^{96}Zr and ^{96}Ru turned out to be rather different, more than by 4/96

because, peole concluded,'they have different nuclear shapes

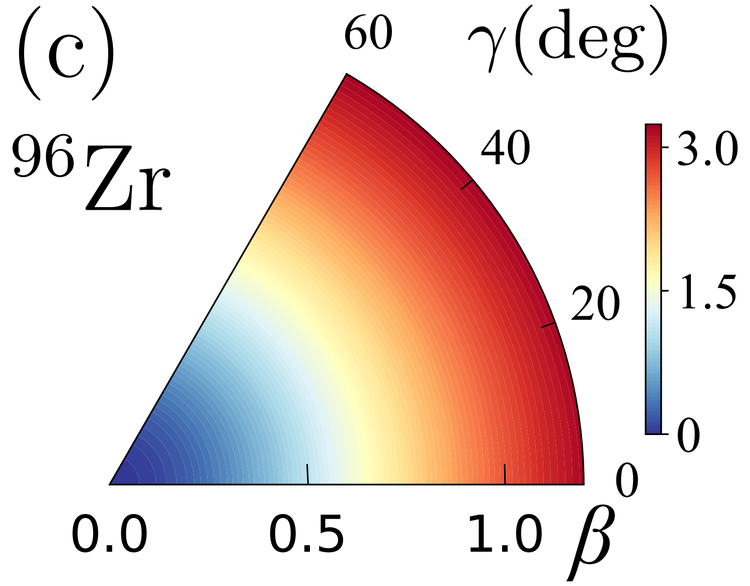
But what exactly are "nuclear shapes"?



Chunjian Zhang and Jiangyong Jia, "Evidence of quadrupole and octupole deformations in ^{96}Zr+^{96}Zr and ^{96}Ru+^{96}Ru collisions at ultra-relativistic energies," (2021), arXiv:2109.01631 [nucl-th].

Shape variables were introduced in nuclear structure calculations,

with the idea that out of states with their fixed values one can construct ground and excited states with needed symmetris



$$E(\beta,\gamma) = \langle \beta,\gamma|\hat{H}|\beta,\gamma\rangle$$

N. Gavrielov, A. Leviatan, and F. Iachello, "The Zr Isotopes as a region of intertwined quantum phase transitions," (2021), arXiv:2112.09454 [nucl-th].

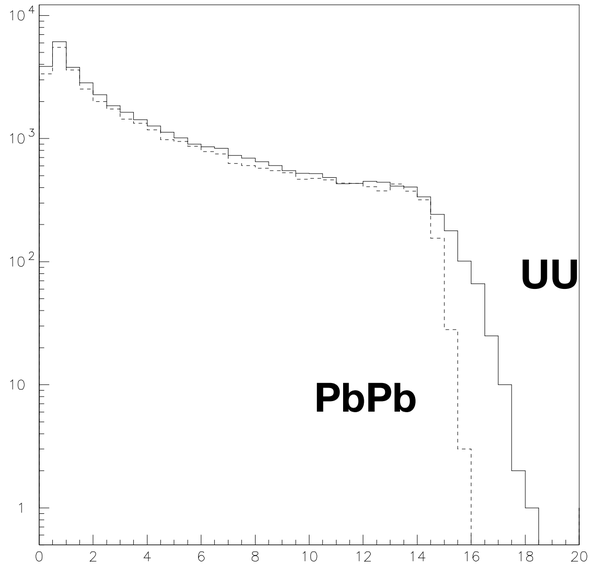
$E_{\langle\beta,\gamma\rangle} = \langle\beta,\gamma|\hat{H}|\beta,\gamma\rangle$

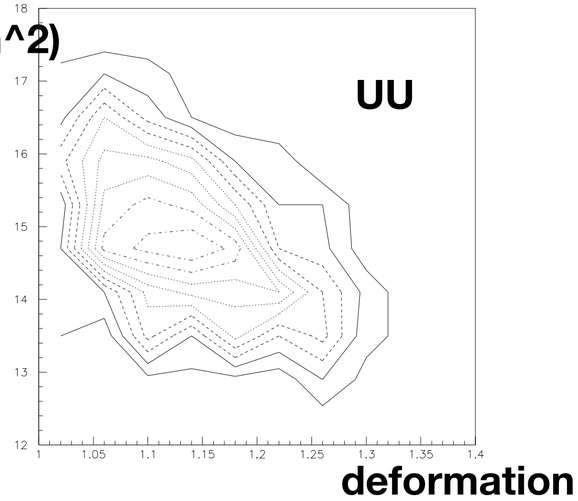
The main question is,
what it has to do with
the initial state
in heavy ion collisions?
Does it help us to find the probability to see given values of collective coordinates?

Edward V. Shuryak, "High-energy collisions of strongly deformed nuclei: An Old idea with a new twist," Phys. Rev. C **61**, 034905 (2000), arXiv:nucl-th/9906062.

$$\frac{R_l}{R_s} = (\frac{1+4\delta/3}{1-2\delta/3})^{1/2}$$

For $\delta_U \approx .27$ this ratio is 1.29,





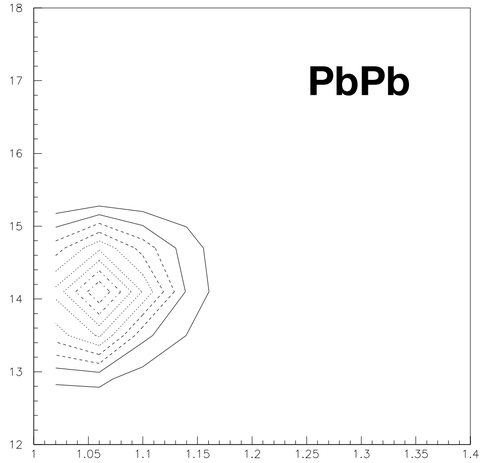


FIG. 3. Distribution over participant density n_p $[fm^{-2}]$ vs deformation R_+/R_-, for (a) UU ($N_{part} > 428$) and (b)) PbPb ($N_{part} > 374$) collisions, respectively.

Proposal of event selection based on magnitude of elliptic flow

Remember, that was before RHIC experiments

Referee: intrinsic deformation is just a technical term used to calculate wave functions U ground state is JP=0+ so U is of spherical shape. Period.

arguments took a year shifted the paper to next millenium!

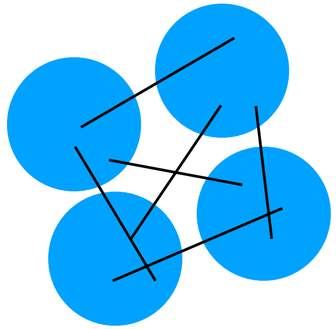
At the collision moment we do not see the ground state, but a wave package made of many excited states

The value of collective variables (such as ellipsoid orientation and deformation) are fixed in each collision

Recently studied problem,
in which there is a virtual state at some moment
which then decays into Hamiltonian bound states

hyperdistance = collective variable

$$\rho^2 \sim \sum_{i>j} R_{ij}^2.$$



12-3=9 variables
3 Jacobi vectors

ppnn system
at freezeout

“precluster”
=>
He4, or He3+n….

$P(\rho)$ **is the density matrix** $\sum_n Tr_x |\psi_n(X,x)|^2 exp(-E_n/T)$

at freezeout the system is well equilibrated, so it was obvious that the density matrix is of the thermal state

The density matrix
traced over all variables except collective ones
(deformation, orientation…)

Hamiltonian states

$$P(X) = \sum_{n=0}^{\infty} \int_{x_i} |\psi_n(X, x_i)|^2 P_n$$

States should be from a ``band" (excitation tree) for which particular collective H applies, see below

What are the weights?

My suggestion:
also thermal
with some effective
temperature T

"PREHEATING" OF NUCLEI BEFORE THE COLLISION MOMENT

Generically, an argument goes like this. Let an act of measurement fixes each coordinate within certain uncertainty $\Delta\vec{x}_i$. The corresponding momenta gets also uncertain, and there should be an uncertainty in the total energy ΔE. All excited states with $E_n < \Delta E$ have as good a reason to contribute to the density matrix (2), as the ground state.

Now, what the probabilities P_n in that expression should be? Here we would like to envoke standard statistical argument. If ΔE is large enough to encompass a large number of state is, then we know that the most important factor in the sum over states would be the density of states itself, or its entropy

$$N(E) \sim exp[S(E)] \tag{3}$$

Standard expansion of it, with $\Delta S/\Delta E = 1/T$, generates Boltzmann weight $exp(-E/T)$. All it

What is the temperature?

The accuracy of localization in the *transverse plane* $\vec{x}_\perp$ for each nucleon is given by a typical impact parameter in NN respective collisions. An estimate of it is

$$\Delta x_\perp \sim \sqrt{\frac{\sigma_{NN}}{\pi}} \sim 1\, fm \tag{4}$$

The uncertainty relation then tells us that each nucleon gets a kick of magnitude $\Delta p_\perp \sim \hbar/\Delta x_\perp \sim 0.2\, GeV$. This corresponds to the nucleon kinetic energy

$$\Delta E_\perp \sim \frac{\Delta p_\perp^2}{2M_N} \sim 20\, MeV \tag{5}$$

Of course, such picture of "preheating" of nuclei does not in general imply that the resulting state is that of thermal equilibrium. It is one with the largest entropy out of pre-selected states in a particular "band" or "excitation tree". Also we know that due to "quantum chaos" phenomenon are "random" in the sense that some single-body distributions are close to thermal density matrices.

(The exchanges of *longitudinal* momenta in NN collisions are much much larger, but they are not relevant for the distribution in the transverse plane we discuss. Both small $T_\perp$ and huge T_{long} will eventually equilibrate into common T_0, but we do know that did not happen at the collision moment. If they would, the state at the collision moment would have very high T and would need a description in terms of quarks and gluons, a la homogeneous CGC gluon state without nucleon correlations. We do know it is *not* so, or else fluctuations of higher angular harmonics would be much much smaller than what it is actuallv observed.)

Proposal at classical level

Let us start from the simplest proposal we have: to use the "deformation potentials" $E(\beta_2, \gamma...)$ calculated by nuclear structure specialists in *classical* Boltzmann distribution

$$P(\beta_2, \gamma..) \sim exp[-\frac{E(\beta_2, \gamma...)}{T_\perp}] \tag{7}$$

in defining the nuclear shape distribution. (Rather than picking up the value of shape coordinates at the potential minimum). Presence of two or more minima are not in this case a problem , nor is it existence of extended flat regions with about the same energy.

Thermal density matrix and path integrals

$$P(x_0;\beta) = \mathcal{N}\int_{x(0)=x_0}^{x(\beta)=x_0} Dx(\tau)\; e^{-S_E[x(\tau)]/\hbar}$$

Feynman, 1950's Matsubara Euclidean time period

$$\beta = \frac{\hbar}{T}\ ,$$

$$P(x_0;\beta) \sim \exp\left[-\frac{m\omega x_0^2}{\coth(\frac{\beta\omega}{2})}\right]\ . \tag{11}$$

with the exponent corresponding to classical "flucton" path

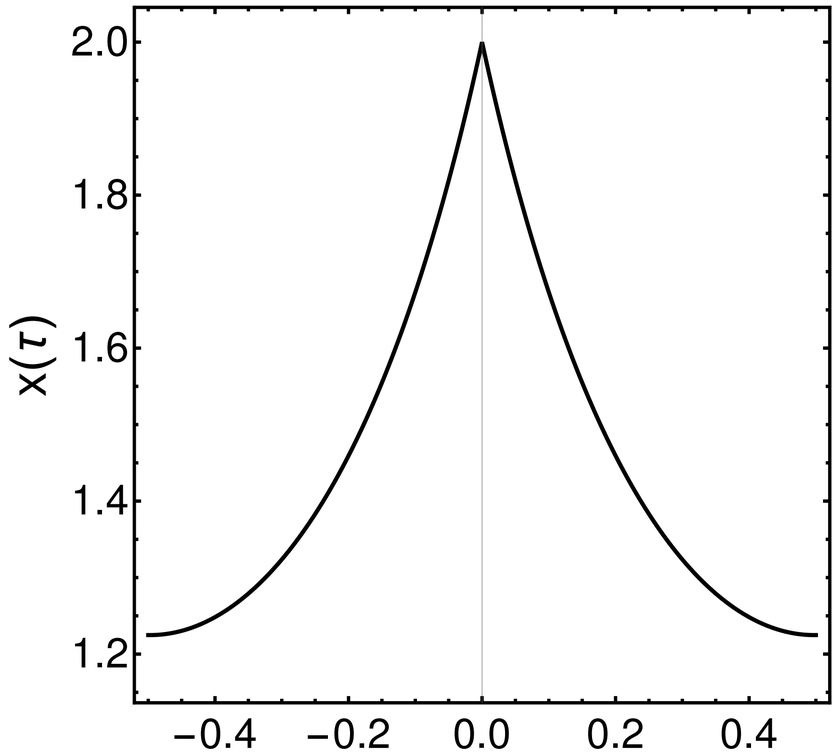
$$x_{\rm fl} = x_0\frac{e^{(\beta-|\tau|)\omega}+e^{|\tau|\omega}}{e^{\beta\omega}+1} \quad , \quad \tau\in[-\beta/2,\beta/2]\ . \tag{12}$$

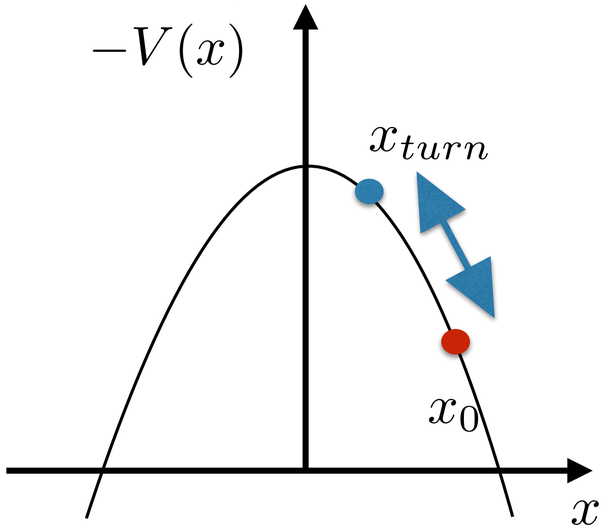
Note that at high $T \gg \omega$ the exponent becomes $m\omega^2 x_0^2/2T = V(x_0)/T$ corresponding to classical Boltzmann factor. In terms of flucton path this limit correspond to the case when particle does not move at all

Proposal at semiclassical level: fluctons

$$S_E[x(\tau)] = \oint d\tau \left(\frac{\dot{x}^2}{2} + \frac{x^2}{2} + \frac{g}{2} x^4 \right) \ .$$

the anharmonic oscillator,





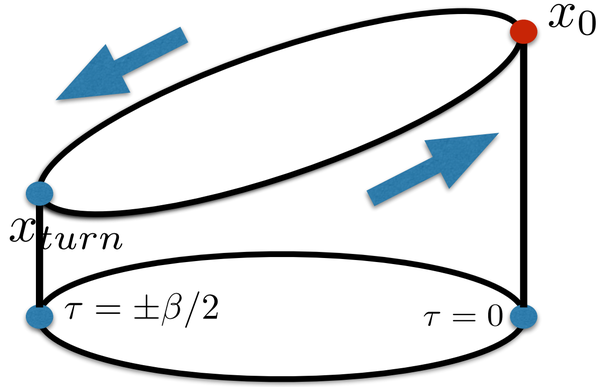


FIG. 2. Two sketches explaining properties of the flucton classical paths. The upper one shows the (flipped) potential $-V(x)$ versus its coordinate. The needed path starts from arbitrary observation point x_0 (red dot), goes uphill, turns back at the turning point x_{turn} (blue dot), and returns to x_0 during the required period $\beta = \hbar/T$ in imaginary time. The lower plot illustrates the same path as a function of Euclidean time τ defined on a "Matsubara circle" with circumference β.

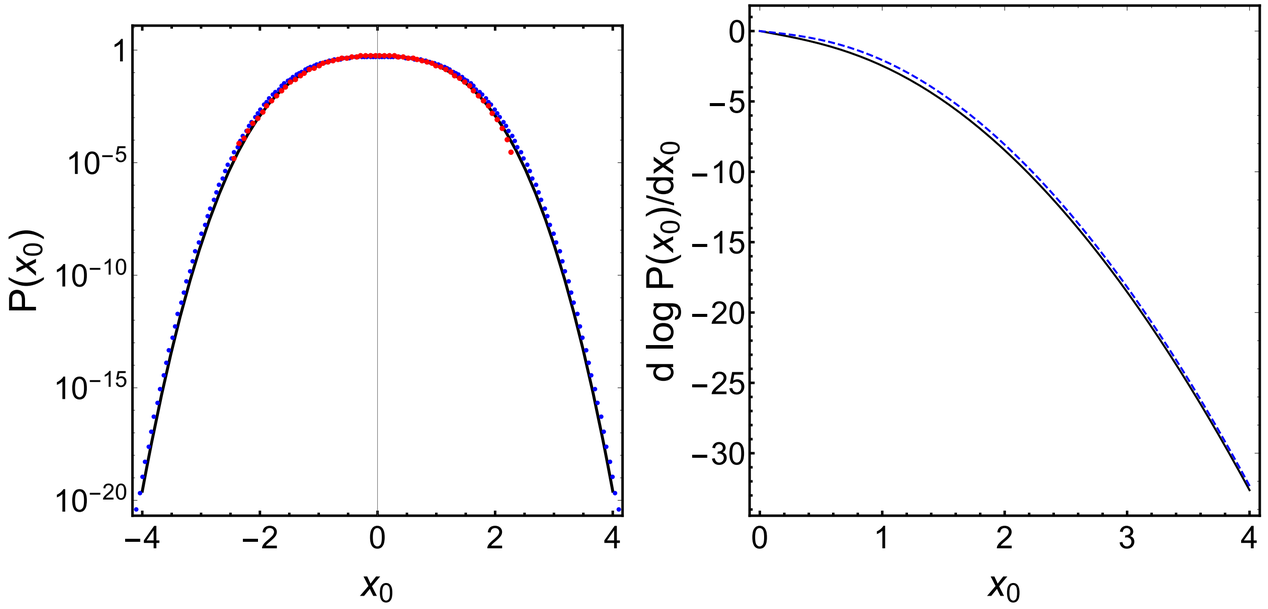


FIG. 4. Left panel: Density matrix $P(x_0)$ vs x_0 for anharmonic oscillator with the coupling $g = 1$, at temperature $T = 1$, calculated via the definition of summing Boltzmann-weighted states (line) and the flucton method (points). The line is based on 60 lowest state wave functions found numerically. Right panel: Comparison of the logarithmic derivative of the density matrix of the upper panel.

Here we present the upper panel of Fig. 4 comparing the summation over 60 squared wav functions, and Boltzmann weighted (solid line), with the result of the flucton method (points) a $T = 1$ (in units of the mass). The coupling is set to $g = 1$. For additional comparison we als got numerical results of a path integral Monte Carlo calculation with the same parameters (nc shown).

Which states to take? “Configurations” and their excitation trees

so we know
not only potential energy
but oscillation frequencies
as well !

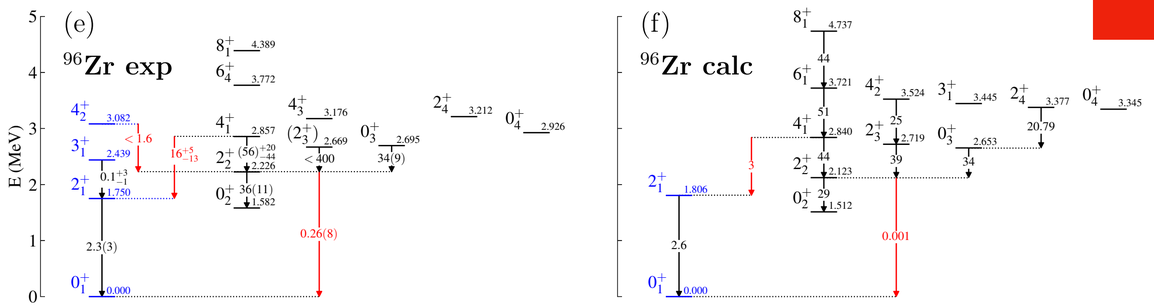


FIG. 5. Blue (left) and black (right) are states corresponding to “excitation trees” growing from configurations A and B, respectively.

“vibrators”
vs the “rotators”

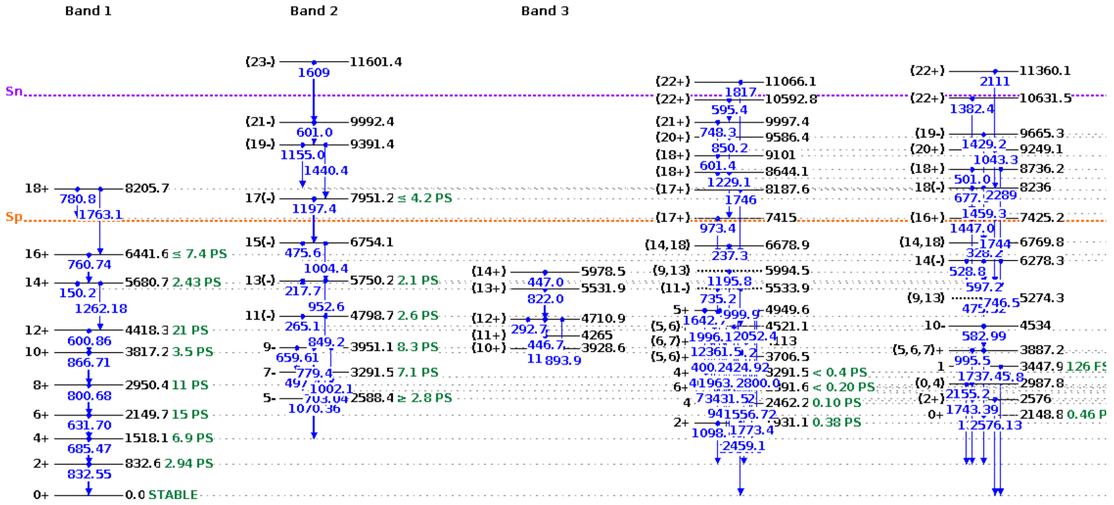


FIG. 7. Excitation levels of ^{96}Ru (replotted from the BNL webpage of nuclear excitations)

Summary

- The main suggestion is to consider a state of the nuclei at time zero
- to be “preheated”

- and therefore calculate density matrices in collective variables
- using thermal methods

- the simplest (or high-T limit) is just Boltzmann exp[- E(beta,gamma…)/T]

- the next is semiclassical “flucton” methods, which are quite accurate