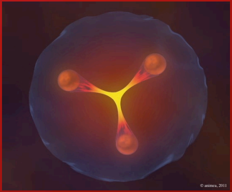
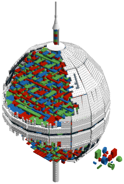
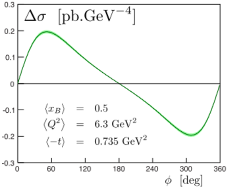
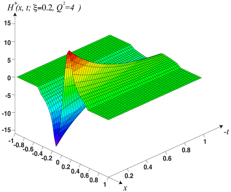
DE LA RECHERCHE À L'INDUSTRIE

Precision 3D hadron structure in the EIC perspective









EICPL | Hervé MOUTARDE

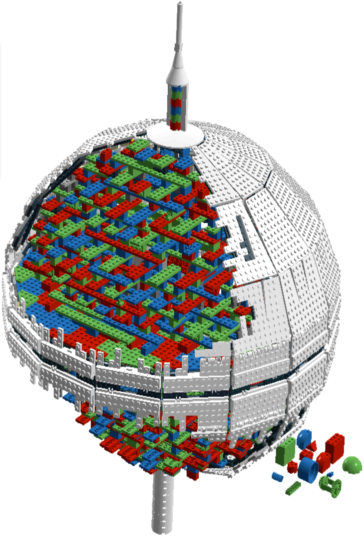
Nov. 15, 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093.

Anatomy of a hadron.

Probing nonperturbative QCD with parton distributions.

How can we recover the well-known characteristics of a hadron from the properties of its **colored building blocks**?



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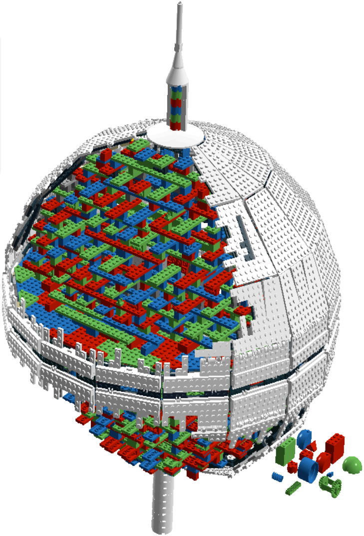
Mass?



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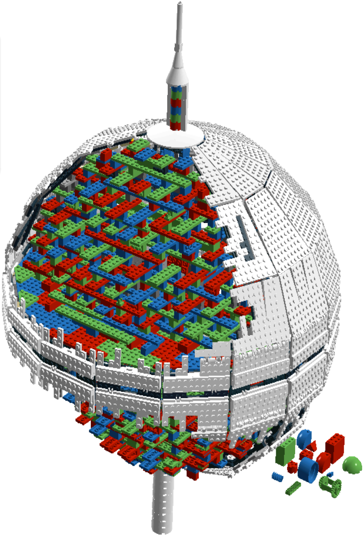
Mass?

Spin?



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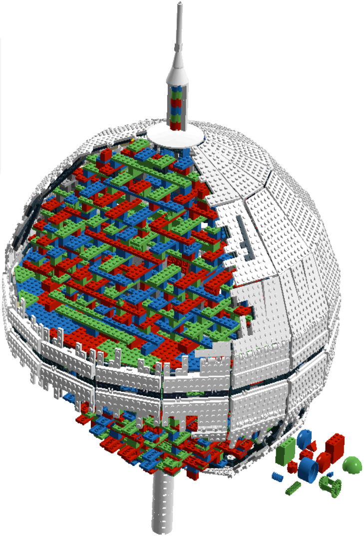
Charge?



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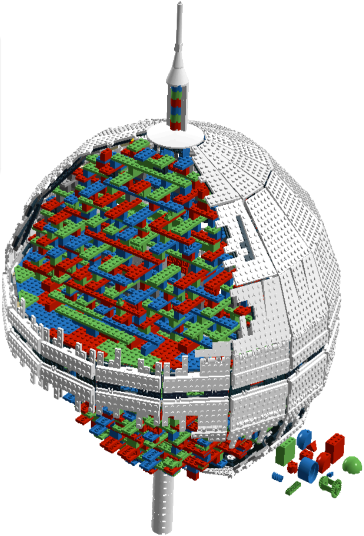

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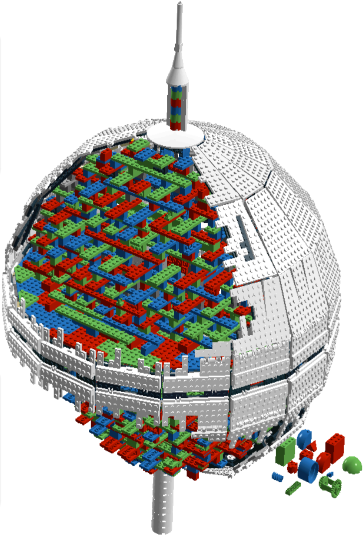
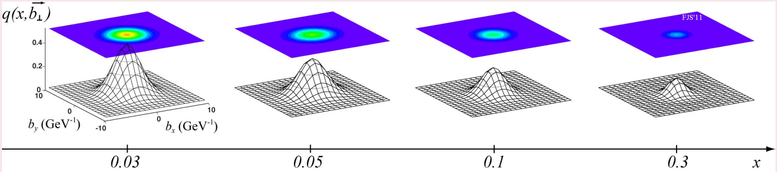


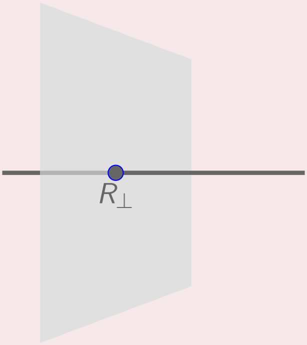
What are the relevant **effective degrees of freedom** and **effective interaction** at large distance?

Structuring questions for the hadron physics community

- QCD **mechanisms** behind the origin of **mass** in the **visible universe**?
- **Cartography** of interactions giving their properties to hadrons?
- **Pressure** and **density** profiles of a hadron as a continuous medium?
- **Localization** of quarks and gluons inside a hadron?
- Possible impact on **initial state** of pp or pA scattering?



Manifestation of 3D hadron structure in scattering processes?



- Transverse center of momentum $R_\perp = \sum_i x_i r_{\perp i}$,

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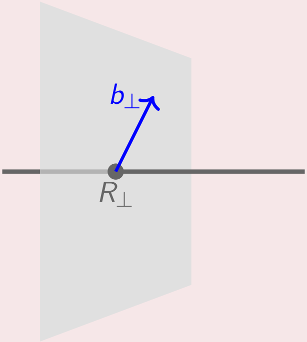

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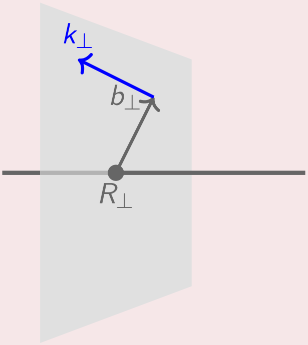

- Transverse center of momentum $R_\perp = \sum_i x_i r_{\perp i}$,
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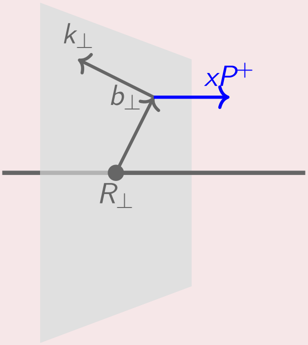

- Transverse center of momentum $R_\perp = \sum_i x_i r_{\perp i}$,
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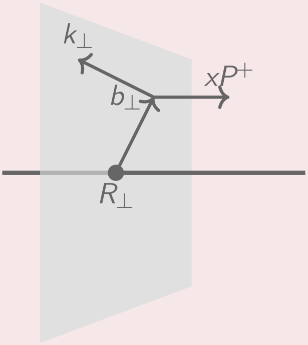

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- What is the distortion brought by spin?

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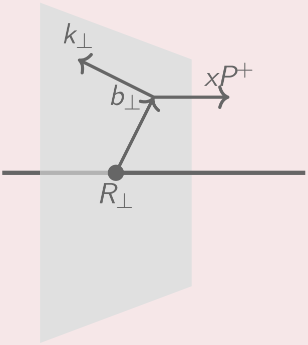

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- Longitudinal momentum .
- What is the distortion brought by spin?

A scattering process will typically involve a bilocal parton matrix operator sandwiched between hadron states with momenta $P \pm \Delta/2$.

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$ $\boxed{\text{GTMD}}$

$$\xi = -\frac{\Delta^+}{2P^+}$$

$$\Delta^2 = -\frac{4\xi^2 M^2 + \vec{\Delta}_\perp^2}{1-\xi^2}$$

The family of 1-quark distributions.

GPDs and TMDs provide complementary 3D information.

$$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp \quad \boxed{\text{GTMD}} \xrightarrow[\xi = 0]{\text{Fourier } \Delta_\perp \leftrightarrow b_\perp} \boxed{\text{Wigner}} \quad x, \vec{k}_\perp, \vec{b}_\perp$$

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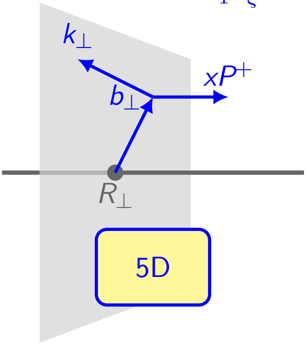

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$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$ GTMD $\xrightarrow[\xi = 0]{\text{Fourier } \Delta_\perp \leftrightarrow b_\perp}$ Wigner $x, \vec{k}_\perp, \vec{b}_\perp$

GTMD $\xrightarrow{\vec{\Delta} = \vec{0}}$ TMD $x, \vec{k}_\perp$

$$\xi = -\frac{\Delta^+}{2P^+}$$

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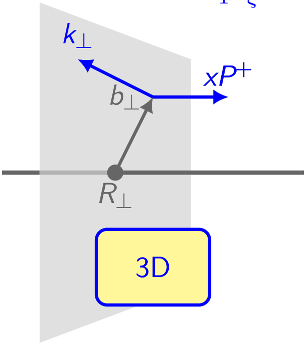

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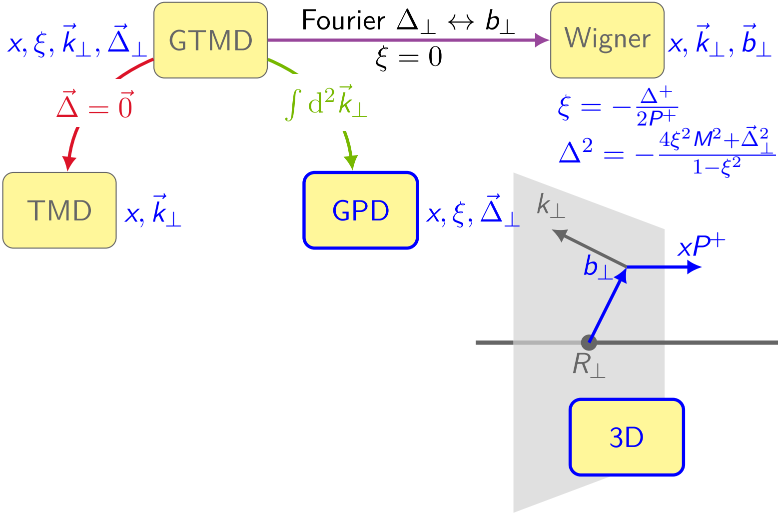

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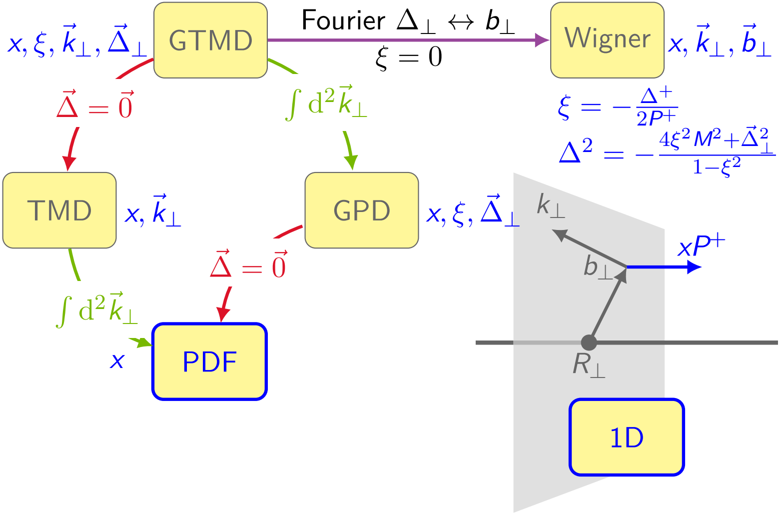

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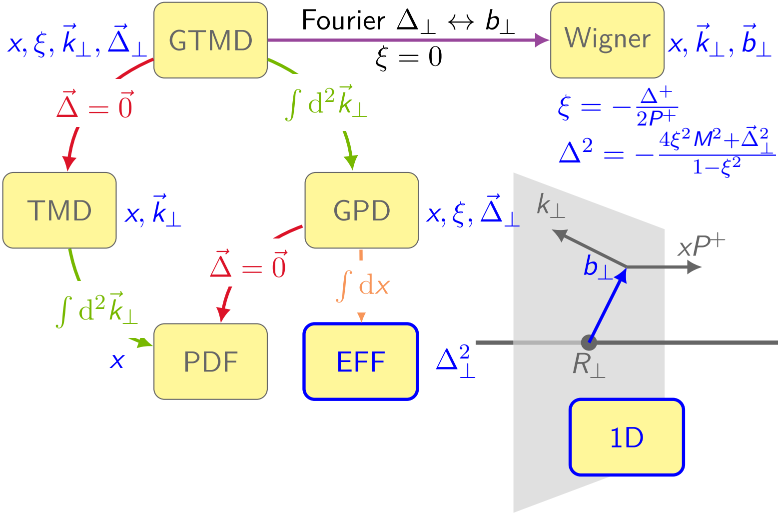

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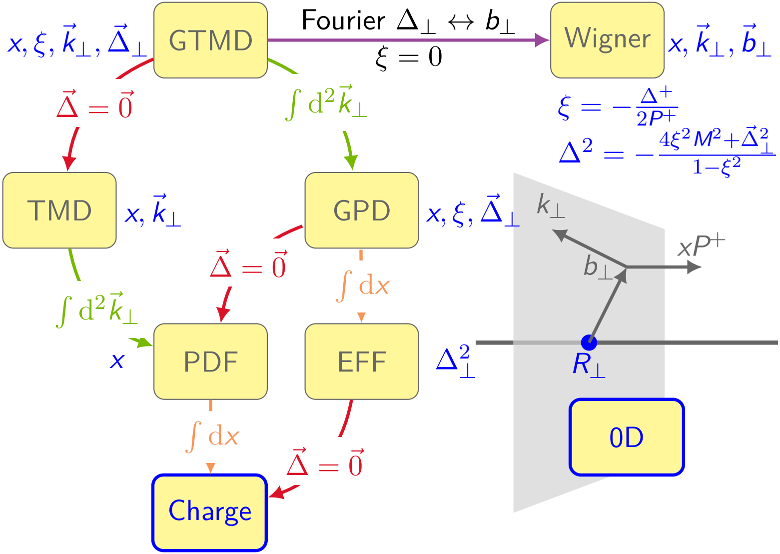

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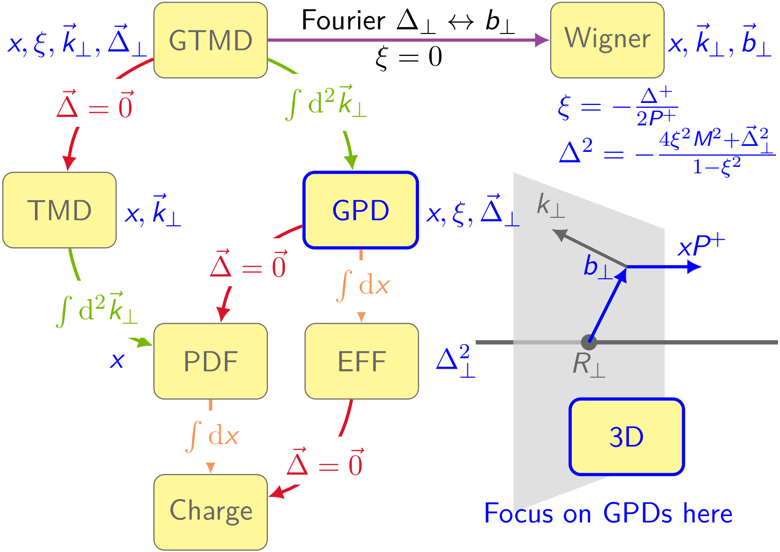
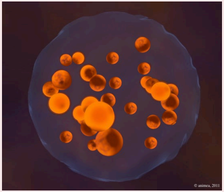


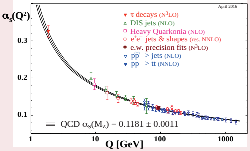
Perturbative and nonperturbative QCD.

Study hadron structure to shed new light on nonperturbative QCD.

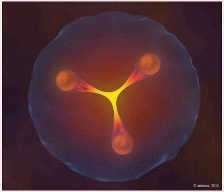
Perturbative QCD



Asymptotic freedom



Nonperturbative QCD



Interface between perturbative and nonperturbative regimes

- Define **universal** objects describing 3D hadron structure: **Generalized Parton Distributions** (GPD).
- Relate GPDs to measurements using **factorization**: **Virtual Compton Scattering** (DVCS, TCS), **Deeply Virtual Meson production** (DVMP).
- Get **experimental knowledge** of hadron structure.

3D hadron imaging.

First results from global fits to world data.

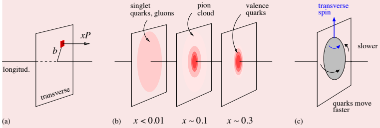
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\left[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda\lambda_N \tilde{H}(x, 0, b_\perp^2) \right]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

✍ Burkardt (2000)

Can we obtain this picture from exclusive measurements?



✍ Weiss (2009)

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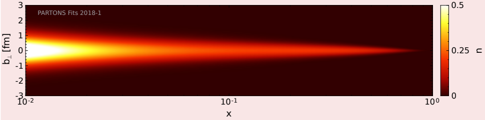
Nucleon tomography in the quark sector



✍ Moutarde *et al.* (2018)

Gravitational form factors.

Definition of pressure.

- **Matrix element** in the Breit frame ($a = q, g$):

$$\left\langle \frac{\Delta}{2} \left| T_a^{\mu\nu}(0) \right| -\frac{\Delta}{2} \right\rangle = M \left\{ \eta^{\mu 0}\eta^{\nu 0} \left[A_a(t) + \frac{t}{4M^2} B_a(t) \right] + \eta^{\mu\nu} \left[\bar{C}_a(t) - \frac{t}{M^2} C_a(t) \right] + \frac{\Delta^\mu \Delta^\nu}{M^2} C_a(t) \right\}$$

- Anisotropic fluid in **relativistic hydrodynamics**:

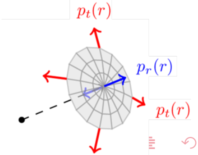
$$\Theta^{\mu\nu}(\vec{r}) = [\varepsilon(r) + p_t(r)]\, u^\mu u^\nu - p_t(r)\eta^{\mu\nu} + [p_r(r) - p_t(r)]\, \chi^\mu \chi^\nu$$

where u^μ and $\chi^\mu = x^\mu / r$.

- Define **isotropic pressure** and **pressure anisotropy**:

$$p(r) = \frac{p_r(r) + 2\, p_t(r)}{3}$$

$$s(r) = p_r(r) - p_t(r)$$



✍ Lorcé *et al.* (2019)

- Write dictionary between quantum and fluid pictures:

$$\frac{\varepsilon_a(r)}{M} = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right]\right\}$$

$$\frac{p_{r,a}(r)}{M} = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{d}{dt}\left(t^{3/2}\, C_a(t)\right)\right\}$$

$$\frac{p_{t,a}(r)}{M} = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{d}{dt}\left[t\frac{d}{dt}\left(t^{3/2}\, C_a(t)\right)\right]\right.$$

$$\frac{p_a(r)}{M} = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t)\right\}$$

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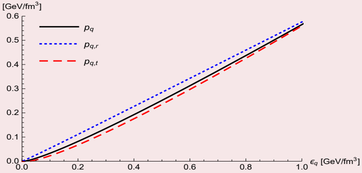
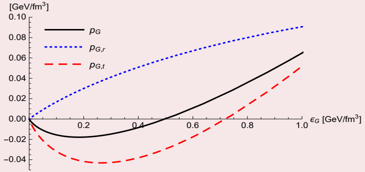
✍ Lorcé *et al.* (2019)

- Evaluate orders of magnitude with naive multiple model:

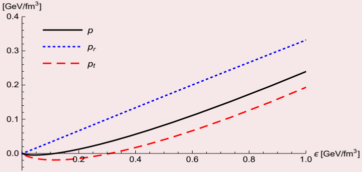
quark



glue

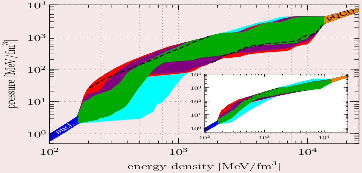


quark + glue



✍ Lorcé *et al.* (2019)

Neutron stars



✍ Annala *et al.* (2018)

Connection to experimental data.

Gravitational form factors from generalized parton distributions.

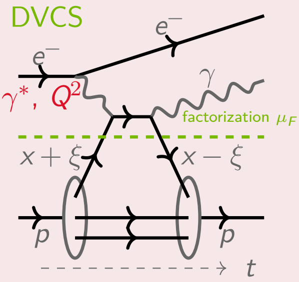
Link between GPDs and GFFs

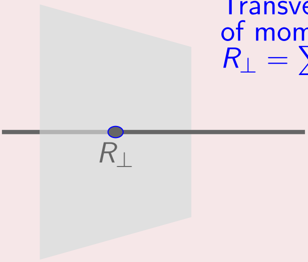
$$\int \mathrm{d}x\, x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x, \xi, t) = B^q(t) - 4\xi^2 C^q(t)$$

✍ Ji (1997), ✍ Goeke (2001)

Deeply Virtual Compton Scattering (DVCS)





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$R_\perp = \sum_i x_i r_{\perp i}$

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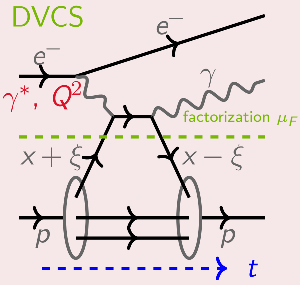

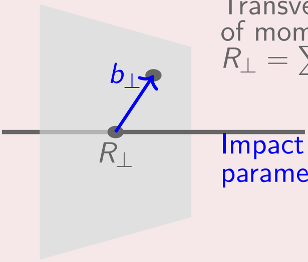

Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Impact parameter $b_\perp$

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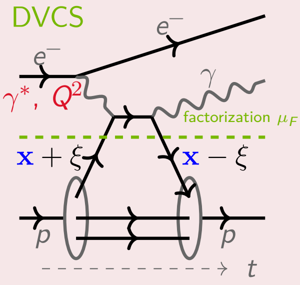

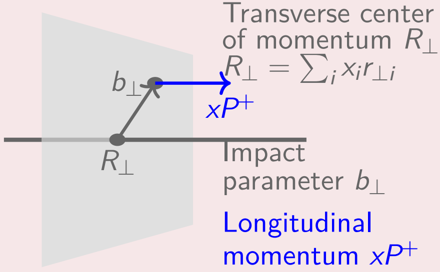

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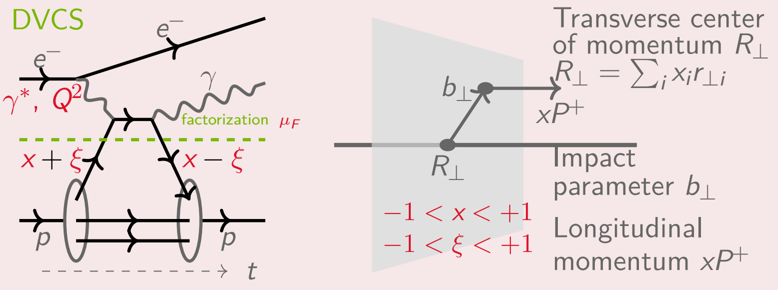
$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

✍ Ji (1997), ✍ Goeke (2001)

Deeply Virtual Compton Scattering (DVCS)



Phenomenology

Compton Form Factors.

DVCS amplitude in the Bjorken regime.

Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, T\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

Almost all existing DVCS data sets.

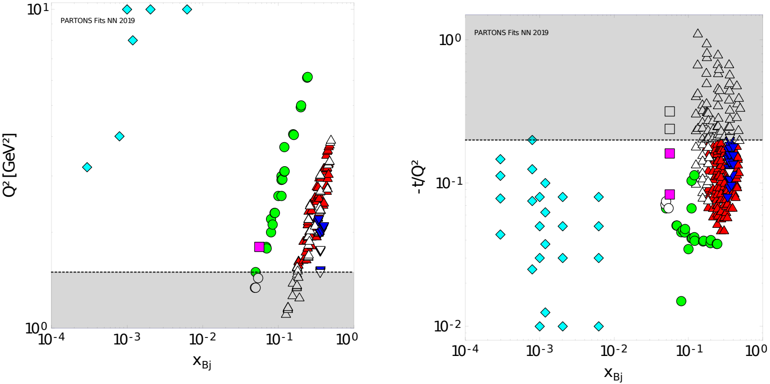
2600+ measurements of 30 observables published during 2001-17.

No.	Collab.	Year	Ref.	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	[40]	A_{LU}^{+}		ϕ	10 / 10
2		2006	[41]	$A_{C}^{\cos i\phi}$	$i=1$	t	4 / 4
3		2008	[42]	$A_{C}^{\cos i\phi}$	$i=0,1$	x_{Bj}	18 / 24
				$A_{UT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\cos i\phi}$	$i=0$		
				$A_{UT,\mathrm{I}}^{\sin(\phi-\phi_S)\cos i\phi}$	$i=0,1$		
				$A_{UT,\mathrm{I}}^{\cos(\phi-\phi_S)\sin i\phi}$	$i=1$		
4		2009	[43]	$A_{LU,\mathrm{I}}^{\sin i\phi}$	$i=1,2$	x_{Bj}	35 / 42
				$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	$i=1$		
				$A_{C}^{\cos i\phi}$	$i=0,1,2,3$		
5		2010	[44]	$A_{UL}^{+,\sin i\phi}$	$i=1,2,3$	x_{Bj}	18 / 24
				$A_{LL}^{+,\cos i\phi}$	$i=0,1,2$		
6		2011	[45]	$A_{LT,\mathrm{DVCS}}^{\cos(\phi-\phi_S)\cos i\phi}$	$i=0,1$	x_{Bj}	24 / 32
				$A_{LT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\sin i\phi}$	$i=1$		
				$A_{LT,\mathrm{I}}^{\cos(\phi-\phi_S)\cos i\phi}$	$i=0,1,2$		
				$A_{LT,\mathrm{I}}^{\sin(\phi-\phi_S)\sin i\phi}$	$i=1,2$		
7		2012	[46]	$A_{LU,\mathrm{I}}^{\sin i\phi}$	$i=1,2$	x_{Bj}	35 / 42
				$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	$i=1$		
				$A_{C}^{\cos i\phi}$	$i=0,1,2,3$		
8	CLAS	2001	[47]	$A_{LU}^{-,\sin i\phi}$	$i=1,2$	—	0 / 2
9		2006	[48]	$A_{UL}^{-,\sin i\phi}$	$i=1,2$	—	2 / 2
10		2008	[49]	A_{LU}^{-}		ϕ	283 / 737
11		2009	[50]	A_{LU}^{-}		ϕ	22 / 33
12		2015	[51]	A_{LU}^{-}, A_{UL}^{-}, A_{LL}^{-}		ϕ	311 / 497
13		2015	[52]	$d^4\sigma_{UU}^{-}$		ϕ	1333 / 1933
14	Hall A	2015	[34]	$\Delta d^4\sigma_{LU}^{-}$		ϕ	228 / 228
15		2017	[35]	$\Delta d^4\sigma_{LU}^{-}$		ϕ	276 / 358
16	COMPASS	2018	[36]	$d^3\sigma_{UU}^{\pm}$		t	2 / 4
17	ZEUS	2009	[37]	$d^3\sigma_{UU}^{+}$		t	4 / 4
18	H1	2005	[38]	$d^3\sigma_{UU}^{+}$		t	7 / 8
19		2009	[39]	$d^3\sigma_{UU}^{\pm}$		t	12 / 12
						SUM:	2624 / 3996

Moutarde *et al.* (2019)

Almost all existing DVCS data sets.

2600+ measurements of 30 observables published during 2001-17.



▼ Hall A ● HERMES ■ COMPASS

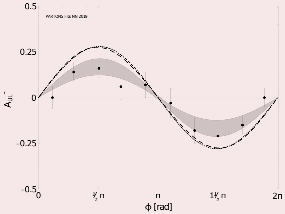
▲ CLAS ♦ H1 and ZEUS

✍ Moutarde *et al.* (2019)

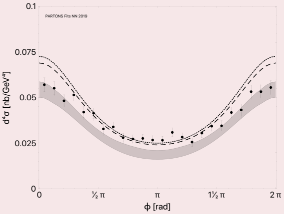
Almost all existing DVCS data sets.

2600+ measurements of 30 observables published during 2001-17.

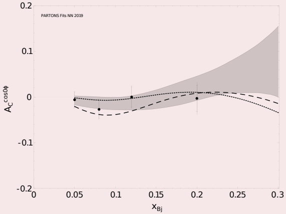
CLAS



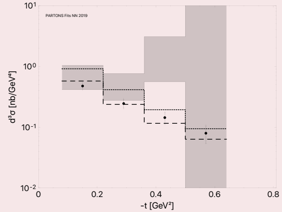
Hall A



HERMES

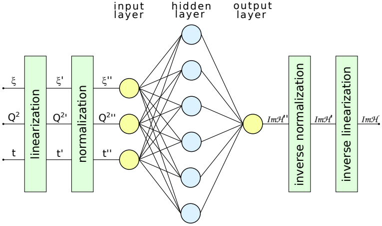


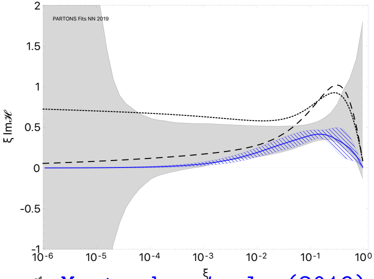
COMPASS



Moutarde *et al.* (2019)

- Real and imaginary parts of CFFs parameterized by **neural networks**.
- Propagation of uncertainties through **replica method** and evaluation of 68 % **confidence levels**.





Moutarde *et al.* (2019)

1. Expand D-term on Gegenbauer polynomials

$$D^q_{\mathrm{term}}(z, t, \mu_F^2) = (1 - z^2) \sum_{\text{odd } n} d^q_n(t, \mu_F^2)\, C_n^{3/2}(z)$$

2. Write dispersion relation for CFF (true at all pQCD orders)

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \, \mathrm{Im}\mathcal{H}(\xi') \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right)$$

3. Compute subtraction constant

$$\mathcal{C}_H^{q,g}(t, Q^2) = \frac{2}{\pi}\int_1^{+\infty} \mathrm{d}\omega \, \mathrm{Im}\, T^{q,g}(\omega) \int_{-1}^1 \mathrm{d}z \, \frac{D^{q,g}(z)}{\omega - z}$$

✍ Diehl & Ivanov (2007)

4. Retrieve GFF

$$d^q_1(t, \mu_F^2) = 5\, C_q(t, \mu_F^2)$$

1. Expand D-term on Gegenbauer polynomials

$$D^q_{\mathrm{term}}(z, t, \mu_F^2) = (1 - z^2) \sum_{\text{odd } n} d_n^q(t, \mu_F^2)\, C_n^{3/2}(z)$$

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3. Compute subtraction constant at LO

$$\mathcal{C}_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

✍ Diehl & Ivanov (2007)

4. Retrieve GFF

$$d_1^q(t, \mu_F^2) = 5\, C_q(t, \mu_F^2)$$

Experimental data

GFF C
Internal pressure

Extracting chain.

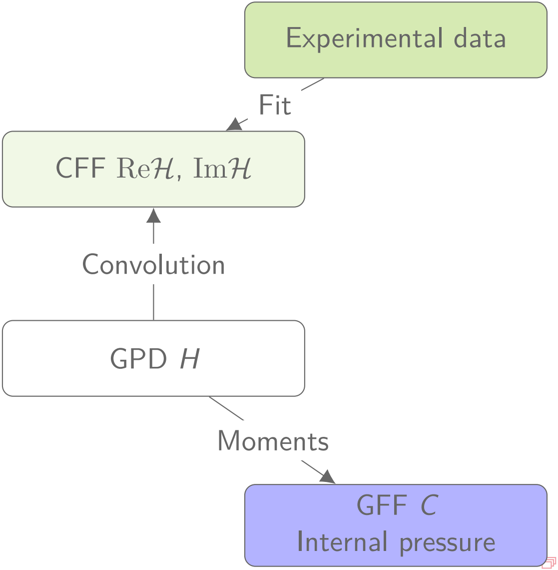
Implementing this first-principle connection.

Experimental data

GPD H

Moments

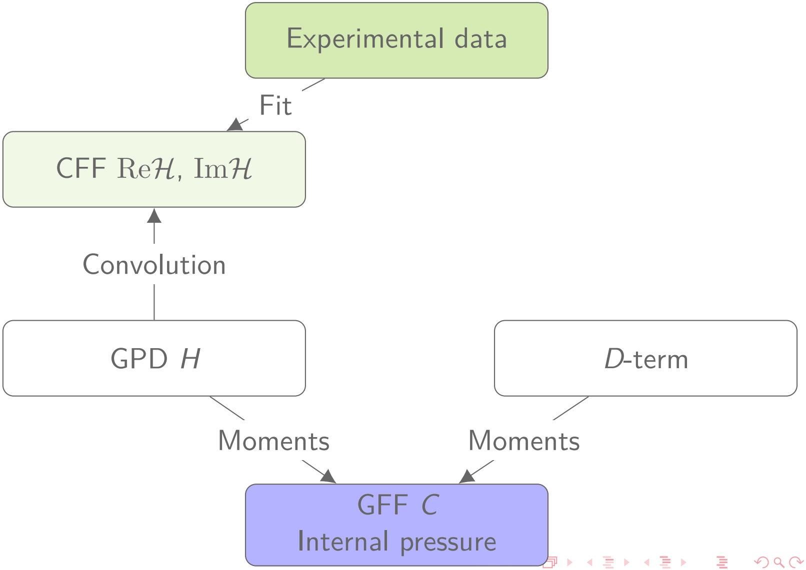
GFF C
Internal pressure

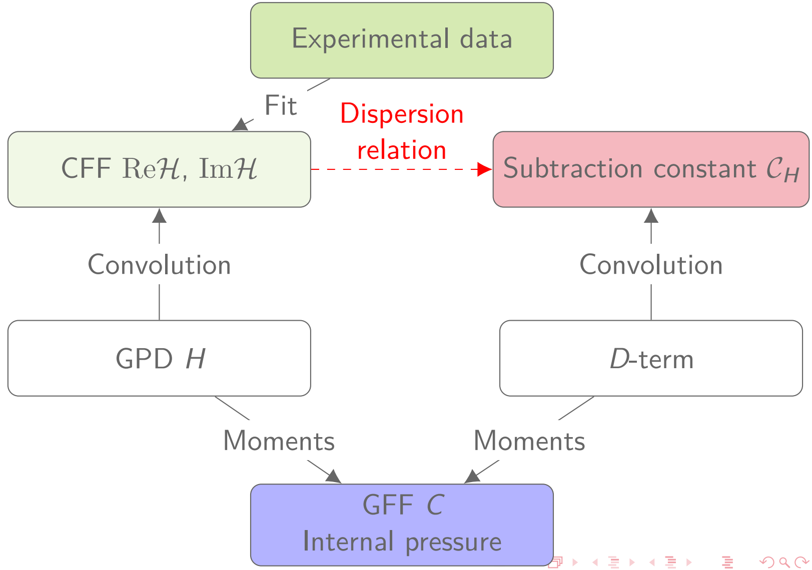


Experimental data
Fit
CFF Re$\mathcal{H}$, Im$\mathcal{H}$
Convolution
GPD H
Moments
GFF C
Internal pressure

Extracting chain.

Implementing this first-principle connection.

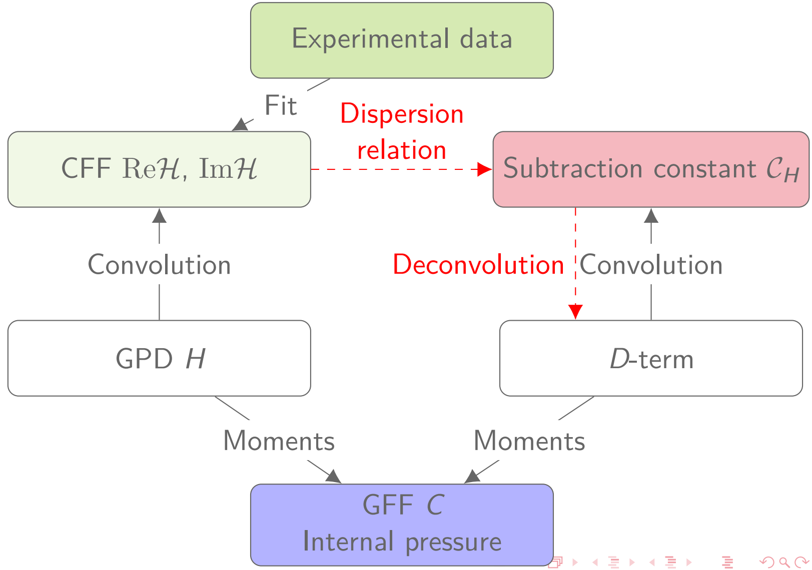




Experimental data
Fit
Dispersion relation
CFF $\mathrm{Re}\mathcal{H}$, $\mathrm{Im}\mathcal{H}$
Subtraction constant $\mathcal{C}_H$
Convolution
Convolution
GPD H
D-term
Moments
Moments
GFF C
Internal pressure

Extracting chain.

Implementing this first-principle connection.



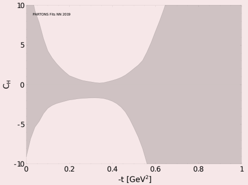
- Range of kinematic variables in neural networks

$$\begin{array}{lclcl} 10^{-6} & < & \xi & < & 1 \\ 0 & < & -t & < & 1 \text{ GeV}^2 \\ 1 & < & Q^2 & < & 100 \text{ GeV}^2 \end{array}$$

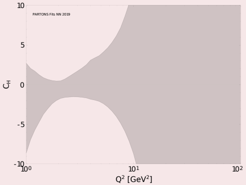
- Implement DVCS dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi}\int_{10^{-6}}^{1} \mathrm{d}\xi' \, \mathrm{Im}\mathcal{H}(\xi) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right)$$

$\xi = 0.2$
$Q^2 = 2 \text{ GeV}^2$



$\xi = 0.2$
$t = -0.3 \text{ GeV}^2$



$t = -0.3 \text{ GeV}^2$
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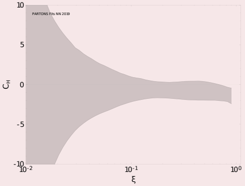


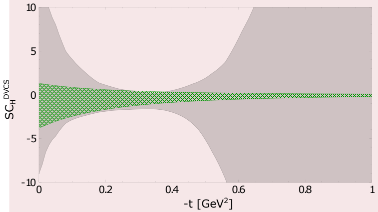
✍ Moutarde *et al.* (2019)

Pressure forces from DVCS measurements.

Working assumptions.

1. Subtraction constant assumed equal to d_1.
2. Equal values for light quark contributions d_1^{uds}.
3. Radiative generation of gluon d_1^g and charm d_1^c contributions.
4. Tripole Ansatz for the t-dependence of d_1.

Tripole Ansatz



Parameter	Value
$d_1^{uds}(\mu_F^2)$	-0.45 ± 0.92
$d_1^c(\mu_F^2)$	-0.0020 ± 0.0041
$d_1^g(\mu_F^2)$	-0.6 ± 1.3

Dutrieux *et al.* (2021)

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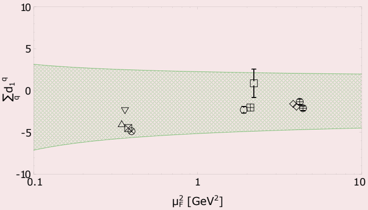
d_1 from DVCS data



Parameter	Value
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Dutrieux *et al.* (2021)

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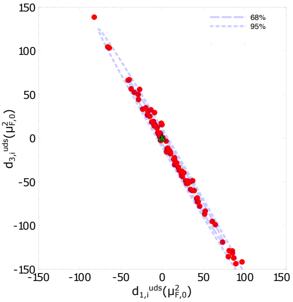
4. Tripole Ansatz for the t-dependence of d_1.

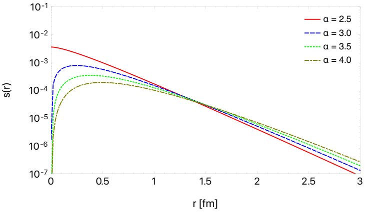
Summary of existing determinations

No.	Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV²	# of flavours	Type	Ref.
1	○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data	[13]
2	□	0.88 ± 1.69	2.2	2	from experimental data	[14]
3	◇	-1.59	4	2	t-channel saturated model	[55]
		-1.92	4	2	t-channel saturated model	[55]
4	△	-4	0.36	3	χQSM	[30]
5	▽	-2.35	0.36	2	χQSM	[10]
6	⊠	-4.48	0.36	2	Skyrme model	[56]
7	⊞	-2.02	2	3	LFWF model	[57]
8	⊗	-4.85	0.36	2	χQSM	[58]
9	⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]
		-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]

✍ Dutrieux *et al.* (2021)

- **No justification to truncate the subtraction constant expansion to its first term** and assume that it is the d_1 coefficient related to the energy-momentum tensor.
- Shape of pressure profile is **fixed** by multipole Ansatz. Actual value is **extremely sensitive** to its parameters.





Dutrieux *et al.* (2021)

- Multipole Ansatz

$$d_n(t, \mu_F^2) = d_n(\mu_F^2) \left(1 - \frac{t}{M^2}\right)^{-\alpha}$$

- Multipole Ansatz

$$d_n(t, \mu_F^2) = d_n(\mu_F^2)\left(1 - \frac{t}{M^2}\right)^{-\alpha}$$

- Leading contributions of d_1 and d_3, higher order terms neglected.

- Multipole Ansatz

$$d_n(t, \mu_F^2) = d_n(\mu_F^2) \left(1 - \frac{t}{M^2}\right)^{-\alpha}$$

- Leading contributions of d_1 and d_3, higher order terms neglected.
- 3 active quark flavors (uds), and radiative generation of c contribution.

- Multipole Ansatz

$$d_n(t, \mu_F^2) = d_n(\mu_F^2) \left(1 - \frac{t}{M^2}\right)^{-\alpha}$$

- Leading contributions of d_1 and d_3, higher order terms neglected.
- 3 active quark flavors (uds), and radiative generation of c contribution.
- 6 parameters

$$d_1^{uds}(\mu_0) \qquad d_3^{uds}(\mu_0) \qquad d_1^{g}(\mu_0) \qquad d_3^{g}(\mu_0) \qquad M \qquad \alpha$$

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\text{GeV}^2)$

$d_1^{g}(0.1\text{GeV}^2)$

$d_3^{uds}(0.1\text{GeV}^2)$

$d_3^{g}(0.1\text{GeV}^2)$

α

M

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

α

M

Free

Fixed

1. Nominal fit:
   - 1 free parameter for light quarks.
   - Gluon and charm radiatively generated.

	LO	NLO
$d_1^{uds}(0.1\mathrm{GeV}^2)$	-0.7 ± 1.7	-0.8 ± 2.0
$d_1^{uds}(2\mathrm{GeV})$	-0.5 ± 1.2	-0.5 ± 1.4
$d_1^{g}(2\mathrm{GeV})$	-0.6 ± 1.6	-0.7 ± 1.9
$d_1^{c}(2\mathrm{GeV})$	-0.002 ± 0.0005	-0.002 ± 0.006

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

α

M

Free

Fixed

2. Alternative fit:
   - 2 free parameters: light quarks and gluons.
   - Charm radiatively generated.

	LO	NLO
$d_1^{uds}(0.1\mathrm{GeV}^2)$	-6.2 ± 14	-0.4 ± 2.3
$d_1^{g}(0.1\mathrm{GeV}^2)$	68 ± 152	6.3 ± 22
$d_1^{uds}(2\mathrm{GeV})$	-0.7 ± 1.2	0.4 ± 2.8
$d_1^{g}(2\mathrm{GeV})$	51 ± 111	5.3 ± 19
$d_1^{c}(2\mathrm{GeV})$	0.2 ± 0.4	0.02 ± 0.06

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

α

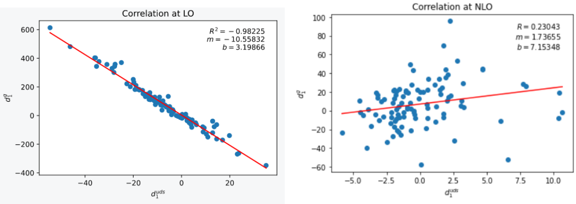
M

Free

Fixed

Investigate 4 fitting scenarios

2. Alternative fit:
   - 2 free parameters: light quarks and gluons.
   - Charm radiatively generated.



Decorrelation of d_1^g and d_1^{uds} at NLO.

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

α

M

Free

Fixed

3. Alternative fit:
   - 2 free parameters: d_1 and d_3 for light quarks.
   - Gluon and charm radiatively generated.

	LO	NLO
$d_1^{uds}(0.1\mathrm{GeV}^2)$	16 ± 37	15 ± 34
$d_3^{uds}(0.1\mathrm{GeV}^2)$	-26 ± 59	-18 ± 39
$d_1^{uds}(2\mathrm{GeV})$	11 ± 25	11 ± 23
$d_1^{g}(2\mathrm{GeV})$	15 ± 34	15 ± 32
$d_1^{c}(2\mathrm{GeV})$	-0.05 ± 0.1	-0.05 ± 0.1
$d_3^{uds}(2\mathrm{GeV})$	-11 ± 26	-7.7 ± 17
$d_3^{g}(2\mathrm{GeV})$	-1.8 ± 3.9	-1.2 ± 2.6
$d_3^{c}(2\mathrm{GeV})$	-0.04 ± 0.01	-0.003 ± 0.007

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

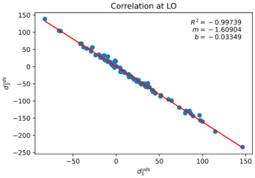
α

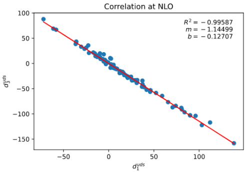
M

Free

Fixed

3. Alternative fit:
   - 2 free parameters: d_1 and d_3 for light quarks.
   - Gluon and charm radiatively generated.





Strong correlation of d_1^{uds} and d_3^{uds} both at LO and NLO.

Precision 3D hadron structure

Investigate 4 fitting scenarios

6 parameters

$d_1^{uds}(0.1\mathrm{GeV}^2)$

$d_1^{g}(0.1\mathrm{GeV}^2)$

$d_3^{uds}(0.1\mathrm{GeV}^2)$

$d_3^{g}(0.1\mathrm{GeV}^2)$

α

M

Free

Fixed

4. Alternative fit:
   - 3 free parameters: complete multipole.
   - Gluon and charm radiatively generated.

	LO	NLO
$d_1^{uds}(0.1\mathrm{GeV}^2)$	-0.6 ± 2.1	-0.7 ± 1.4
α	4.3 ± 4.7	4.0 ± 4.4
M	0.9 ± 0.6	$0.8 \pm 0.6\mathrm{GeV}$
$d_1^{uds}(2\mathrm{GeV})$	-0.4 ± 1.4	-0.5 ± 0.9

Many theoretical constraints on GPDs.

Constraints difficult to implement at the CFF level.

- Reduction to PDFs or elastic form factors.
- Implement *a priori* **positivity** and **polynomiality**. Still uncommon in many models or parameterizations used for phenomenology.

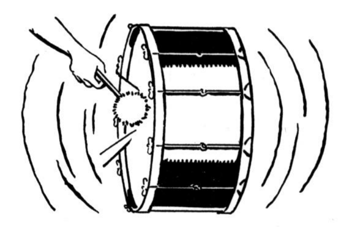
  Sznajder *et al.*, *in progress*
- **General solution** starting from overlap of (potentially effective) light front wave functions.

  ✍ Chouika *et al.* (2017)
- Use of **evolution equations** to implement further constraints on the GPD functional form.
- Work **beyond leading-order** and depart from the parton model...
- Systematic impact study or use of **kinematic corrections** still missing.

"Can one hear the shape of a drum?"

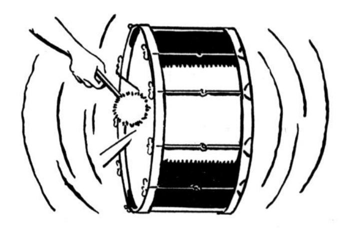
✍ Kac (1966)

"Can one hear the shape of a drum?"



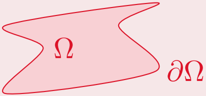
✍ Kac (1966)

"Can one hear the shape of a drum?"



✍ Kac (1966)

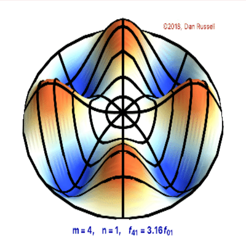
In quantitative terms



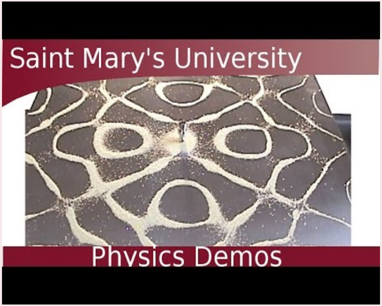
- Dirichlet problem for the Laplacian:

$$\Delta u + \lambda u = 0 \quad \text{and} \quad u_{|\partial\Omega} = 0$$

Vibration patterns



Vibration patterns



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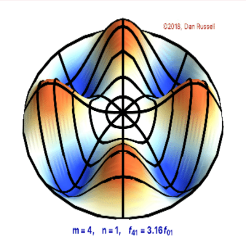

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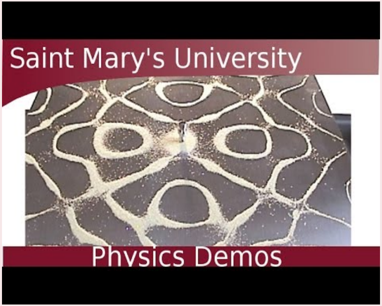
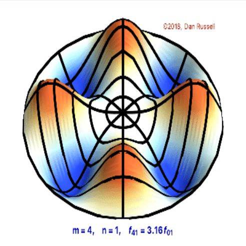


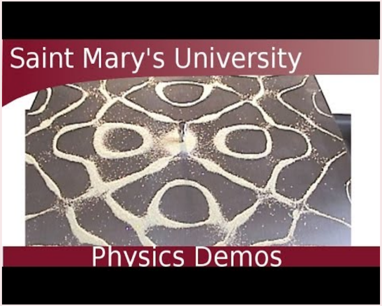
What about the proton?

- "Hit" the proton, *e.g.* with a virtual photon:
- "Listen" to the distribution of produced particles:
- "Measure" harmonics:

Vibration patterns

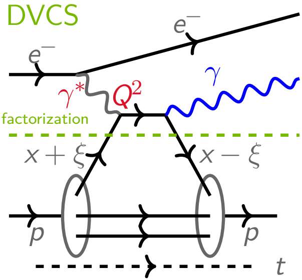


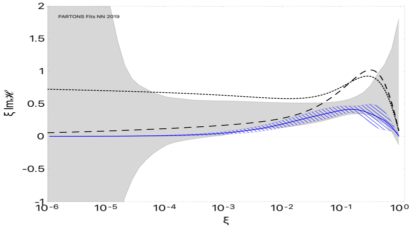
Vibration patterns



What about the proton?

- "Hit" the proton, *e.g.* with a virtual photon: hard
- "Listen" to the distribution of produced particles: exclusive
- "Measure" harmonics: amplitudes (Compton form factors)

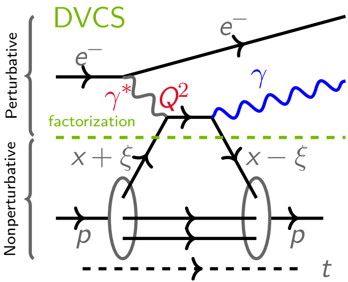


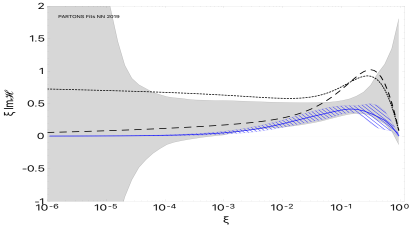


✍ Moutarde *et al.* (2019)

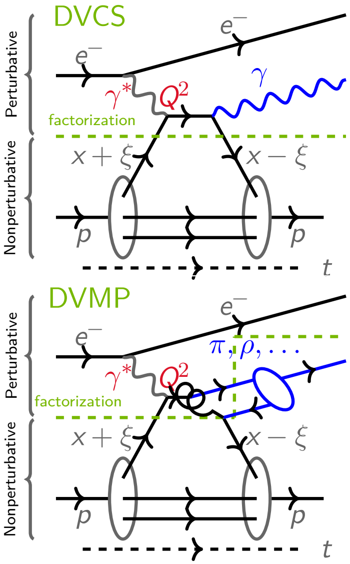
Exclusive processes of current interest.

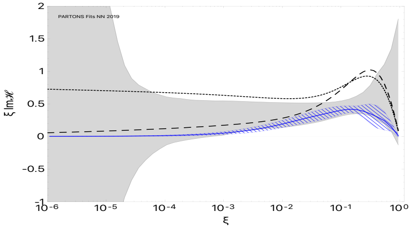
Factorization, universality and event distributions.





✍ Moutarde *et al.* (2019)





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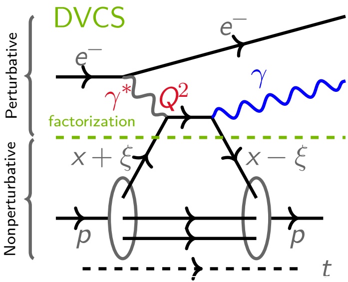

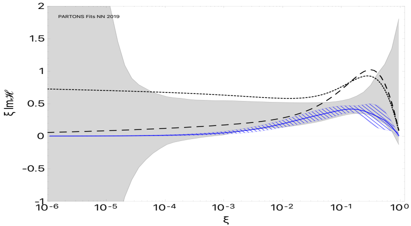

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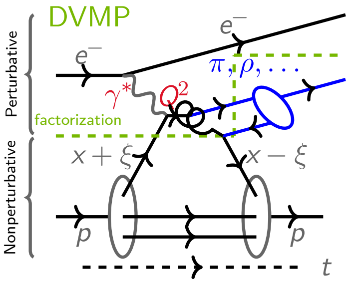
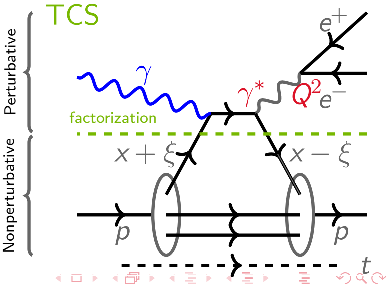
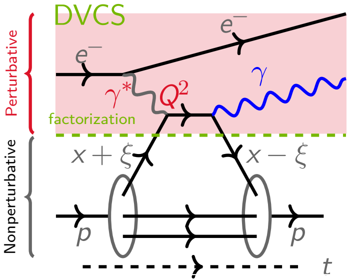
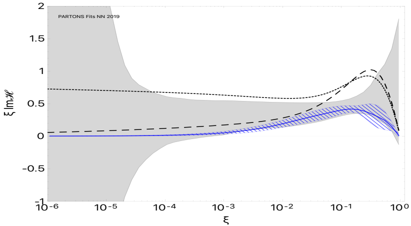
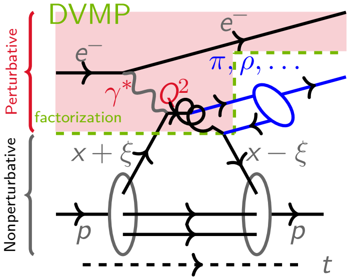


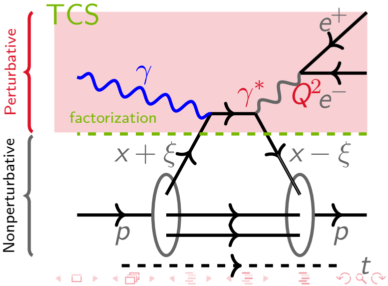






✍ Moutarde *et al.* (2019)





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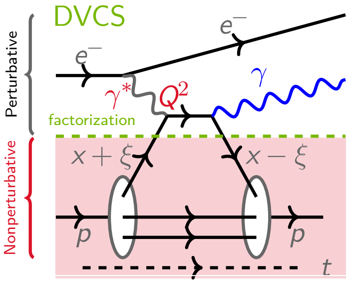
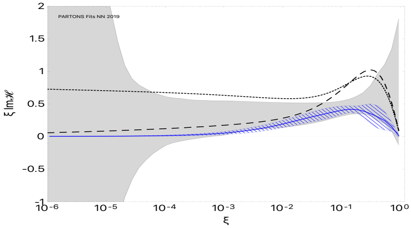
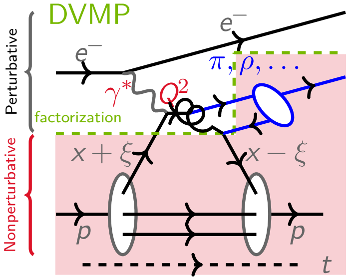
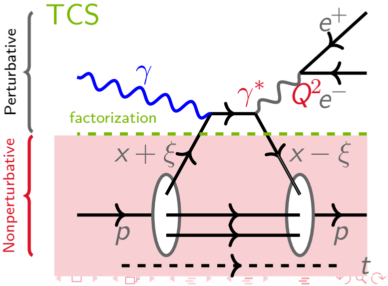




✍ Moutarde *et al.* (2019)





From CFFs to GPDs.

Can we actually recover a GPD from the knowledge of a CFF?!

- Assume CFF $\mathcal{H}$ is perfectly known. Solve inverse problem?

$$\mathcal{H}^q(\xi, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) H^q(x, \xi, \mu^2)$$

- Question raised about 20 years ago and has remained essentially open. Evolution proposed as a crucial element.

✍ Freund (2000)

- There exist **non-zero** GPDs with **vanishing forward limit** and **vanishing CFF** up to order α_s^2.
- The DVCS deconvolution problem is **ill-posed**.

✍ Bertone *et al.* (2021)

- **Same conclusion holds** for several other hard exclusive processes.
- **Define** and **implement** further criterions in fitting strategies to select one solution among infinitely many.

The PARTONS ecosystem

PARtonic
Tomography
Of
Nucleon
Software

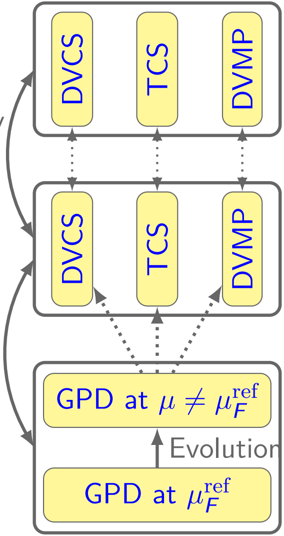
Computing chain design.

Differential studies: physical models and numerical methods.

Full processes
Experimental data and phenomenology

Small distance
Computation of amplitudes

Large distance
First principles and fundamental parameters



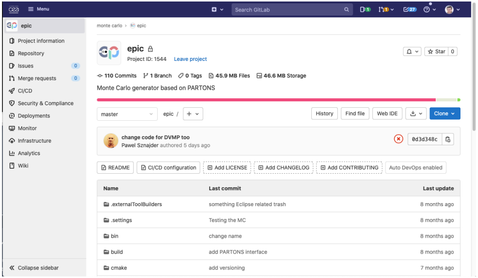
PARtonic **T**omography **O**f **N**ucleon **S**oftware

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Generic exclusive event generators EpIC.

Modular structure compatible with the architecture of PARTONS.

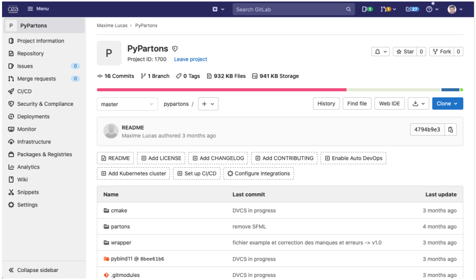
- Includes treatment of radiative corrections.
- **Already used** in the EIC community and run at BNL.
- *Public release soon.*



Python wrapper for PARTONS PyPartons.

Extending the PARTONS ecosystem.

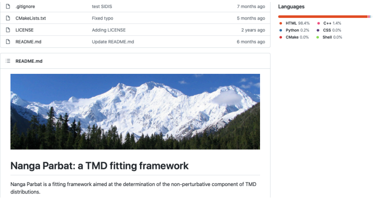
- Simplify connection to **popular libraries** on *e.g.*
  - machine learning,
  - plotting,
  - statistical data analysis.
- Convenient for a wide community of new users.

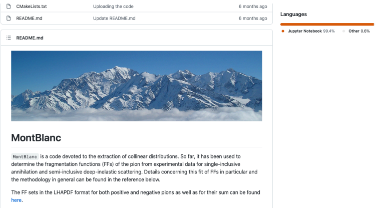


- Facilitates **interactive use**.
- Facilitates **dissemination of research** through *e.g.* Jupyter notebook.
- *Remaining work on documentation before release.*

NangaParbat and MontBlanc.

Tools for TMDs and FFs.



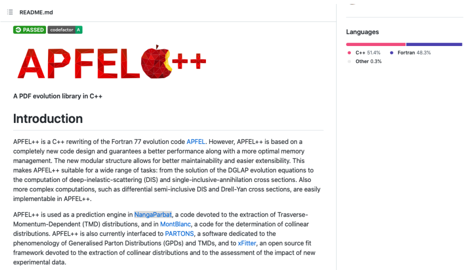


- `NangaParbat` and `MontBlanc` will soon be interoperable with `PARTONS`.
- **Single open source framework** for most of 1-parton distributions.

GPD evolution with APFEL++.

Connecting different computing codes for hadron structure.

- Evolution code for PDFs, GPDs and TMDs.
- APFEL++ numerically solves evolution equations in x-space.
- Fully modular.



README.md

PASSED codefactor A

APFEL++

A PDF evolution library in C++

Introduction

APFEL++ is a C++ rewriting of the Fortran 77 evolution code APFEL. However, APFEL++ is based on a completely new code design and guarantees a better performance along with a more optimal memory management. The new modular structure allows for better maintainability and easier extensibility. This makes APFEL++ suitable for a wide range of tasks: from the solution of the DGLAP evolution equations to the computation of deep-inelastic-scattering (DIS) and single-inclusive-annihilation cross sections. Also more complex computations, such as differential semi-inclusive DIS and Drell-Yan cross sections, are easily implementable in APFEL++.

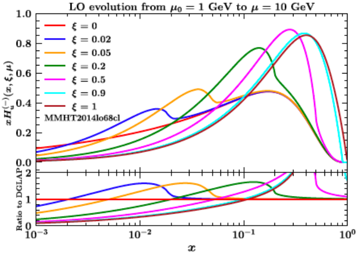
APFEL++ is used as a prediction engine in NangaParbat, a code devoted to the extraction of Trasverse-Momentum-Dependent (TMD) distributions, and in MontBlanc, a code for the determination of collinear distributions. APFEL++ is also currently interfaced to PARTONS, a software dedicated to the phenomenology of Generalised Parton Distributions (GPDs) and TMDs, and to xFitter, an open source fit framework devoted to the extraction of collinear distributions and to the assessment of the impact of new experimental data.

Languages

C++ 51.4% Fortran 48.3% Other 0.3%

Forward limit and DGLAP equations

- When $\kappa \to 0$, one recovers the forward DGLAP equations
  - Only the $\Theta(1-y)\mathcal{P}_1^{-,(0)}(y;\kappa)$ term contributes to the evolution
  - No spurious singularity as $x < y < 1 < 1/\kappa$



- no ξ dependence in input
- Excellent agreement with native Apfel++ DGLAP evolution (red curve)
- Strong ξ dependence generated
- Continuity guaranteed at $x = \xi$ (cusp ?)

First validations

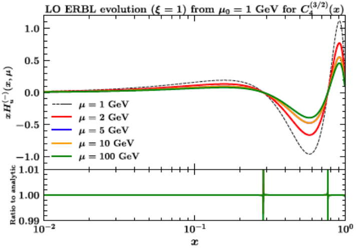
$x = \xi$ continuity, DLGAP limit and spurious divergences handling

GPD evolution with APFEL++.

Connecting different computing codes for hadron structure.

ERBL Limit and Gegenbauer polynomials

- When $\kappa \to 1/x$ (*i.e.* $\xi \to 1$) one recovers the ERBL kernel
  - Eigen basis known $\to$ 3/2-Gegenbauer Polynomials
  - Direct (albeit restricted to $\xi = 1$) comparison between x-space and conformal space evolution



- $H(x, 1) \propto (1 - x^2) C_4^{(3/2)}(x)$
- $\frac{H(x,1,\mu)}{H(x,1,\mu_0)} = \left(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\right)^{\gamma_4/\beta_0}$
- Ratio is independent of x
- Excellent agreement between Apfel++ and conformal evolution in the ERBL limit

Additional validations

conformal evolution when $\xi \to 1$ guaranteeing the ERBL limit

Conclusion

Conclusion and prospects.

The quest towards 3D proton structure.

- Strong **first-principle connection** between concepts and experimental data.
- Need for **multi-channel** analysis **beyond LO** on a **wide kinematic coverage**. EIC much needed!
- The GPD deconvolution problem is **ill-posed**. **Huge sensitivity** to numerical noise or experimental uncertainties.
- Benefiting from new inputs or constraints from **nonperturbative QCD** is highly desirable!
- Development of a **software ecosystem** for 3D hadron structure studies.
- Need for **coordinated effort** involving fits, computing chains *e.g.* PARTONS and continuum or lattice QCD to make the best from experimental data.

Abbreviations

Abbreviations used in this presentation.

ANN	artificial neural network
CFF	Compton form factor
DGLAP	Dokshitzer-Gribov-Lipatov-Altarelli-Parisi
DVCS	deeply virtual Compton scattering
DVMP	deeply virtual meson production
EFF	elastic form factor
ERBL	Efremov-Radyushkin-Brodsky-Lepage
FF	fragmentation function
GFF	gravitational form factor
GPD	generalized parton distribution
LO	leading order
NLO	next-to-leading order
PDF	parton distribution function
TCS	timelike Compton scattering
TMD	transverse momentum dependent PDF

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