Subeikonal corrections in the Color Glass Condensate

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based on [T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska - arXiv:2012.03886]

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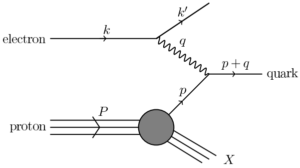
Outline

- Introduction to small x physics and CGC
- Quark propagator at NEik accuracy
- Application to forward qA scattering:
  unpolarized X-section and helicity asymmetry
- Summary and outlook

High energy scattering in QCD

"hard" scattering	"soft" scattering
→ large momentum exchange	→ small momentum exchange
→ weakly coupled	→ strongly coupled
→ perturbative	→ non-perturbative

DIS in QCD :



Three Lorentz invariant quantities :

1. $q^2 = -Q^2 \equiv$ virtuality of the incoming photon
2. $x = \frac{Q^2}{2P \cdot Q} \equiv$ longitudinal momentum fraction carried by the parton
3. $s \simeq 2P \cdot Q \equiv$ energy of the colliding $\gamma - p$ system

increasing the energy $(s = Q^2/x)$ of the system:

Bjorken limit fixed x, $Q^2 \to \infty$

- density of partons decreases.
- system becomes more dilute!
- evolution is given by DGLAP.

Regge-Gribov limit fixed Q^2, $x \to 0$

- density of partons increases.
- system becomes dense!
- causes saturation !

Bjorken limit and DGLAP evolution

In the Bjorken limit: description given by *(QCD-improved) parton model*

proton $\simeq$ set of independent free partons (assumed to be dilute).

γ-p X-section (collinear factorization):

$$\sigma_{T,L}^{\gamma p \to X}(x_{Bj}, Q^2) \propto \sum_{i=q_f,\bar{q}_f,g} \int_{x_{Bj}}^{1} \frac{dz}{z} C_{T,L;i}\left(\frac{x_{Bj}}{z}, \alpha_s(Q^2)\right) f_i\left(z, Q^2\right)$$

$C_{T,L;i}\left(\frac{x_{Bj}}{z}, \alpha_s(Q^2)\right) \equiv$ perturbatively calculable coefficient function

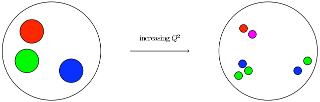
$f_i\left(z, Q^2\right) \equiv$ Parton Distribution Function (PDF): *The number density of partons of type i in the proton seen with transverse resolution $1/Q^2$, carrying a momentum fraction z.*

In the infinite momentum frame:

- transverse size of the photon $\sim 1/Q$ (very small probe).
- can scatter off a quark with the size of $\sim 1/Q$.

$\Rightarrow$ Q is the resolution scale!

$\cdots$ *with increasing Q^2:*



- more substructure resolved by the probe,
- target effectively contains more partons,
- HOWEVER, density of partons decreases!!

DGLAP evolution:

$$\frac{d}{d\log Q^2} f_i(x, Q^2) = \frac{\alpha(Q^2)}{\pi} \int_0^1 \frac{dz}{z} \left\{ P_{q\leftarrow q}(z) f_i\left(\frac{x}{z}, Q\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z}, Q\right) \right\}$$

Regge-Gribov limit: decreasing x at fixed Q^2

[Balitsky, Fadin, Kuraev, Lipatov - 1977, 1978]

First approach: BFKL equation - evolution wrt rapidity $Y = \ln(1/x)$

$$\frac{\partial\varphi(Y,q)}{\partial Y} = \frac{\alpha_s N_c}{\pi^2}\int d^2k\left[\frac{q^2}{k^2(q-k)^2}\varphi(Y,k) - \frac{1}{2}\frac{q^2}{k^2(q-k)^2}\varphi(Y,q)\right]$$

$\varphi(Y,q) \equiv$ unintegrated gluon density $\rightarrow$ $$xf_g(x,Q) = \int_0^{Q^2}\frac{d^2k}{k^2}\varphi(x,k)$$

At very high energies BFKL equation has two major problems:

- Froissart Bound : $\sigma^{total} < \frac{\pi}{m^2}Y^2$
    - X-section calculated by the solution of BFKL equation : $\sigma^{total} \sim e^{cY}$
    - to solve this problem information from the infrared scale of QCD needed.
- violation of unitarity
    - scattering probability grows without a bound, exceeding unity at rapidities of order $Y \simeq \frac{1}{\alpha_s}\ln(1/\alpha_s)$
    - this problem can be addressed by taking into account gluon saturation effects.

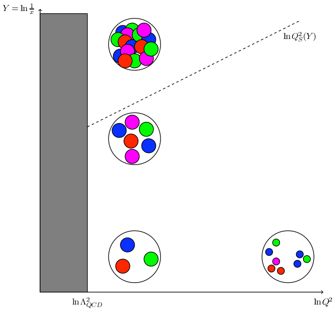
$\cdots$ *decreasing x at fixed Q^2 (rapidity evolution):*

- Nb. of partons increase due to splitting
- Transverse scale doesn't change
- Mother and daughter partons have the same size

$\Rightarrow$ *density of partons increases and causes saturation.*

Color Glass Condensate (CGC) - I

High energy scattering in QCD:



- Regge-Gribov limit : $x \to 0$
- at small $x \to$ **saturation**!
  - $Q_s \equiv$ saturation scale
    $\equiv \alpha_s \times$ (gluon density per unit area)
  - Q_s is a measure of the strength of the gluon interaction processes that may occur when the gluon density becomes large.
  - $Q_s \gg \Lambda_{QCD} \Rightarrow$ weak coupling methods can still be applied !

[McLerran, Venugopalan - hep-ph/9309289 / hep-ph/9311205]

In the saturation regime the prescription of scattering process: Color Glass Condensate (CGC)

CGC description of a process: "effective degrees of freedom" with respect to a cut off Λ^+

- fast partons : $k^+ > \Lambda^+$ $\to$ described by color sources: $J^\mu(x) = \delta^{\mu+}\rho(x^-, x_\perp)$
- slow partons: $k^+ < \Lambda^+$ $\to$ described by color fields $A^\mu(x)$

interaction between fast and slow partons: $\int d^4x J^\mu(x) A_\mu(x)$

Color Glass Condensate (CGC) - II

Within the CGC framework:

expectation value of an observable $\mathcal{O}$ $\Rightarrow$ $\langle\mathcal{O}\rangle \equiv \int [D\rho]\; W[\rho]\; \mathcal{O}[\rho]$

$W[\rho] \equiv$ distribution function for the color sources ρ^a.

[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner, 1997-2002]

Rapidity (Y) evolution of the distribution function is governed by the JIMWLK evolution equation:

$$\partial_Y W_Y[\rho] = -\mathcal{H}_{JIMWLK}\left[\rho, \frac{\delta}{\delta\rho}\right] W_Y[\rho]$$

interaction between the projectile and the target - **Eikonal scattering**:

each parton picks up a Wilson line during the interaction with the target:

$$U_{\mathcal{R}}(\mathbf{x}) = \mathcal{P}_+ \exp\left[ig \int dx^+ T^a_{\mathcal{R}} A^-_a(x^+, \mathbf{x})\right]$$

dipole operator appears in the observable:

$$d_{\mathcal{R}}(\mathbf{x},\mathbf{y}) = \frac{1}{D_{\mathcal{R}}} \mathrm{tr}\left[U_{\mathcal{R}}(\mathbf{x}) U^\dagger_{\mathcal{R}}(\mathbf{y})\right]$$

$D_{\mathcal{R}} \equiv$ the color dimension of the representation.

Nonlinear evolution of the dipole operator

JIMWLK equation for the dipole operator

$$\frac{\partial \langle d(\mathbf{x},\mathbf{y})\rangle}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2}\int d^2\mathbf{z}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}\Big\{\langle d(\mathbf{x},\mathbf{y})\rangle - \langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle\Big\}$$

Dipole scattering probability: $d(\mathbf{x},\mathbf{y}) = 1 - N(\mathbf{x},\mathbf{y})$

$\langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle \equiv$ simultaneous scattering of both dipoles off the target.

[Balitsky - 1996, Kovchegov -1999]

Assumption: *The areas of the target on which the dipoles are uncorrelated*:

$\boxed{\langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle \Rightarrow \langle d(\mathbf{x},\mathbf{z})\rangle\ \langle d(\mathbf{z},\mathbf{y})\rangle} \Rightarrow \boxed{\text{JIMWLK equation} \Rightarrow \text{BK equation}}$

possible applications in the gluon saturation regime:

- *dilute-dilute scattering* : No saturation effects / BFKL formalism
  - can be applied to: $\gamma^* - \gamma^*$, DIS on p, pp at moderate energies
- *dilute-dense scattering* : saturated target / CGC formalism
  - can be applied to: DIS on A , pA collisions, forward particle production in pp.
- *dense-dense scattering:* saturated projectile and target / non-linear dynamics of Yang-Mills fields
  - can be applied to: pp at very high energies, heavy ion collisions.

saturation sensitive observables in pA collisions:

⋆ *forward particle/jet production* ⋆ *two particle correlations*

Dilute-Dense Scattering within CGC

High energy scattering within the CGC :

- Semi-classical approximation :
  - dense target $\equiv$ classical background field $\mathcal{A}^\mu_a(x) = O\left(\frac{1}{g}\right)$ at weak coupling g with finite support
  - dilute projectile $\equiv$ color charge $J^\mu_a(x) = O(g)$
- Eikonal approximation:
  - take the high energy limit $s \to \infty$.
  - drop power-suppressed contributions.

Coupling between the projectile and the target $\to \int d^4x\ J^a_\mu(x)\ \mathcal{A}^\mu_a(x)$

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...

Eikonal Dilute-Dense Scattering

Boosting the target:

$$A^\mu_a(x) \mapsto \begin{cases} \gamma_t\, A^-_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t} A^+_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ A^i_a\left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

- $A^-_a \gg A^i_a \gg A^+_a$ in a generic gauge
- in the light-cone gauge:

$A^\mu_a(x) = \delta^{\mu -}\delta(x^+)A^-_a(\mathbf{x})$

target is localized at $x^+ = 0$ $\rightarrow$ shockwave!!

independent of x^-

Boosting the projectile :

$$J^\mu_a(x) \mapsto \begin{cases} \frac{1}{\gamma_p}\, J^-_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ \gamma_p\, J^+_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ J^i_a\left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \end{cases}$$

- $J^+_a \gg J^i_a \gg J^-_a$
- slow x^+ dependence due to Lorentz time dilation

$J^\mu_a(x) \propto \delta^{\mu +}\delta(x^-)\rho^a(\mathbf{x})$

projectile is localized at $x^- = 0$ $\rightarrow$ shockwave!!

Corrections beyond eikonal accuracy

At the level of the background field, the eikonal approximation amounts to

1. $\mathcal{A}_a^\mu(x) \simeq \delta^{\mu-}\mathcal{A}_a^-(x)$
2. $\mathcal{A}_a^\mu(x) \simeq \mathcal{A}_a^\mu(x^+, \mathbf{x})$
3. $\mathcal{A}_a^\mu(x) \propto \delta(x^+)$

Relaxing any of these approximations will give correction to the strict eikonal limit!
Three sources of corrections to eikonal approximation :

1. other components of the target background field $\mathcal{A}_a^\mu(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}_a^\mu(x)$
3. Finite width L^+ of the target along x^+

Background field in the strict eikonal limit:

$$\boxed{\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \simeq \delta^{\mu-}\, A^-(x^+, \mathbf{x}) \propto \delta(x^+)}$$

Beyond the Eikonal approximation

There are several motivations to go beyond the Eikonal approximation :

- **Theory :** Study power-suppressed corrections to better understand the foundations of the CGC and its domain of validity
- **Pheno :** Not so high energies at EIC and RHIC $\Rightarrow$ Power-suppressed corrections can be sizable
- **Spin :** Eikonal scattering is blind to spin $\Rightarrow$ QCD Spin physics at low x driven by non-eikonal contributions

Beyond eikonal approximation, at Next-to-Eikonal accuracy (NEik):

* Target with finite width $\Rightarrow$ transverse motion of the parton within the medium
* Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$

High-energy expansion with small parameters $\sim \frac{L^+}{k^+} Q^2$ (L^+ : target width, k^+ : parton momentum, Q^2 : any transverse scale in the problem)

DISCLAIMER: In this study, x^- dependence of $\mathcal{A}^\mu(x)$ still neglected. I will discuss about this issue at the end of the talk.

Quark propagator - basics

Full quark Feynman propagator in background field $\mathcal{A}^{\mu}(x)$

$$S_F(x,y)_{\alpha\beta} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}$$

free propagator + corrections due to interactions
with the background field

Free quark Feynman propagator:

$$S_{0,F}(x,y)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{i(\not{k}+m)}{[k^2 - m^2 + i\epsilon]}$$

Corrections:

- **at the eikonal order**

$$\delta S_F\Big|^{\text{Eik}} \equiv \delta S_F\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

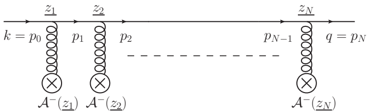
- **at the next-to-eikonal order**

$$\delta S_F\Big|^{\text{NEik}} \equiv \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Quark propagator in the eikonal limit

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure } \mathcal{A}^-}$$

In eikonal limit, the quark already interacts with arbitrarily many $\mathcal{A}^-$ fields



Eikonal interactions with the medium resummed into the Wilson lines:

$$\mathcal{U}_F(x^+,y^+;\mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\,\mathcal{P}_+\left[-ig\int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z^+,\mathbf{z})\right]^N$$

Quark propagator in the eikonal limit

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = S_{0,F}(x,y)_{\alpha\beta} + \delta S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}}_{\text{pure }\mathcal{A}^-}$$

In eikonal limit, the quark already interacts with arbitrarily many $\mathcal{A}^-$ fields
For generic x and y, with notations $\underline{k} \equiv (k^+, \mathbf{k})$, and $\check{k}$ on-shell version of k:

$$S_F(x,y)_{\alpha\beta}\Big|^{\text{Eik}} = \mathbf{1}_{\alpha\beta}\,\delta^{(3)}(\underline{x}-\underline{y})\,\text{sgn}(x^- - y^-)\,\frac{\gamma^+}{4}$$
$$+\int\frac{d^3\underline{q}}{(2\pi)^3}\int\frac{d^3\underline{k}}{(2\pi)^3}\,2\pi\delta(q^+-k^+)e^{-ix\cdot\check{q}+iy\cdot\check{k}}\,\frac{(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)}{(2k^+)^2}$$
$$\times\int d^2\mathbf{z}\,e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\theta(k^+)\theta(x^+-y^+)\,\mathcal{U}_F(x^+,y^+;\mathbf{z})_{\alpha\beta}$$
$$-\theta(-k^+)\theta(y^+-x^+)\,\mathcal{U}_F^\dagger(y^+,x^+;\mathbf{z})_{\alpha\beta}\Big\}$$

Eikonal interactions with the medium resummed into the Wilson lines:

$$\mathcal{U}_F(x^+,y^+;\mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty}\frac{1}{N!}\,\mathcal{P}_+\left[-ig\int_{y^+}^{x^+}dz^+\,t\cdot\mathcal{A}^-(z^+,\mathbf{z})\right]^N$$

- background field $\mathcal{A}_a^-(z^+,\mathbf{z})$ has a finite support $[-L^+/2, L^+/2]$ - this is where the non-trivial medium contributions come from in the interval $[y^+, x^+]$
- if there is no support the propagator reduces to the Feynman propagator in vacuum

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = \underbrace{S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-}}_{S_F\big|_{\text{pure } \mathcal{A}^-}} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

From now on, always $x^+ > L^+/2$ and $y^+ < -L^+/2$: quark propagating through the whole target

Quark propagator in pure $\mathcal{A}^-$ background field up to next-to-eikonal order for positive energy:

$$\begin{aligned}
S_F(x,y)_{\alpha\beta}\Big|_{\text{pure } \mathcal{A}^-} = & \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}+iy\cdot\check{k}}\, (\check{\not{q}} + m)\gamma^+(\check{\not{k}} + m) \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \mathcal{U}_F\Big(\frac{L^+}{2}, -\frac{L^+}{2}; \mathbf{z}\Big) \\
& - \frac{(\mathbf{q}^j + \mathbf{k}^j)}{4k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\Big(\frac{L^+}{2}, z^+; \mathbf{z}\Big) \overleftrightarrow{\partial_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+, -\frac{L^+}{2}; \mathbf{z}\Big) \right] \\
& - \frac{i}{2k^+} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\Big(\frac{L^+}{2}, z^+; \mathbf{z}\Big) \overleftarrow{\partial_{\mathbf{z}^j}}\, \overrightarrow{\partial_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+, -\frac{L^+}{2}; \mathbf{z}\Big) \right] \Bigg\}
\end{aligned}$$

NEik corrections: transverse drift term + transverse Brownian motion term

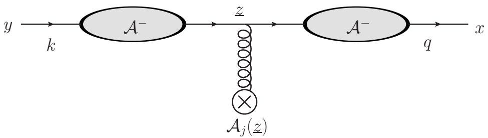
Analog to earlier results on the gluon propagator with subeikonal corrections:
Altinoluk, Armesto, Beuf, Martinez, Salgado, JHEP **1407**, 068 (2014)
Altinoluk, Armesto, Beuf, Moscoso, JHEP **1601**, 114 (2016)

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace one $\gamma^+ \mathcal{A}^-_a$ interaction by $\gamma^j \mathcal{A}^a_j$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int d^4z \; S_F(x,z)\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^a\right] \mathcal{A}^a_j(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$



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Replace one $\gamma^+ \mathcal{A}_a^-$ interaction by $\gamma^j \mathcal{A}_j^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int d^4z \; S_F(x,z)\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^a\right] \mathcal{A}_j^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$

Subeikonal correction due to an interaction with $\mathcal{A}_\perp$:

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3} \int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$
$$\times (\check{\not q} + m)\gamma^j\gamma^+\gamma^i\, (\check{\not k} + m) \int d^3\underline{z} \left[e^{-i\mathbf{z}\cdot\mathbf{q}}\, \mathcal{U}_F\Big(\frac{L^+}{2}, z^+; \mathbf{z}\Big)\right]$$
$$\times \left[\overleftarrow{\partial_{\mathbf{z}^j}} \left[gt\cdot\mathcal{A}_i(\underline{z})\right] - \left[gt\cdot\mathcal{A}_j(\underline{z})\right] \overrightarrow{\partial_{\mathbf{z}^i}} \right] \left[\mathcal{U}_F\Big(z^+, -\frac{L^+}{2}; \mathbf{z}\Big)\, e^{i\mathbf{z}\cdot\mathbf{k}} \right]$$

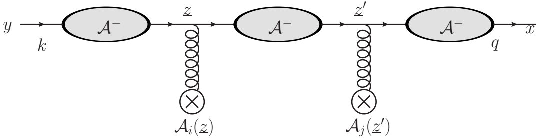
Reminder: $x^+ > L^+/2$ and $y^+ < -L^+/2$:
quark propagating through the whole medium

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace two $\gamma^+ \mathcal{A}_a^-$ interactions by $\gamma^j \mathcal{A}_j^b$ and $\gamma^i \mathcal{A}_i^a$

$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp} = \int d^4z \int d^4z'\; S_F(x,z')\Big|^{\text{Eik}} [-ig\,\gamma^j\, t^b]\; \mathcal{A}_j^b(\underline{z}') \times S_F(z',z)\Big|^{\text{Eik}} [-ig\,\gamma^i\, t^a]\; \mathcal{A}_i^a(\underline{z}) \quad S_F(z,y)\Big|^{\text{Eik}}$$



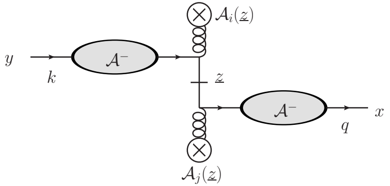
Naively of NNEik order, but instantaneous contribution in the middle Eikonal propagator produces a NEik contribution

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure } \mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single } \mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp}$$

Replace two $\gamma^+ \mathcal{A}^-_a$ interactions by $\gamma^j \mathcal{A}^b_j$ and $\gamma^i \mathcal{A}^a_i$

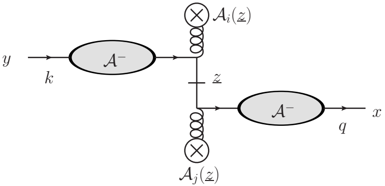
$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double } \mathcal{A}_\perp} = \int d^4z \int d^4z' \; S_F(x,z')\Big|^{\text{Eik}} \left[-ig\,\gamma^j\, t^b\right] \mathcal{A}^b_j(\underline{z}') \times S_F(z',z)\Big|^{\text{Eik}} \left[-ig\,\gamma^i\, t^a\right] \mathcal{A}^a_i(\underline{z}) \;\; S_F(z,y)\Big|^{\text{Eik}}$$



Instantaneous double $\mathcal{A}_\perp$ insertion at NEik

Full next-to-eikonal quark propagator:

$$S_F\Big|^{\text{NEik}} = S_F\Big|^{\text{Eik}} + \delta S_F\Big|^{\text{NEik}}_{\text{pure }\mathcal{A}^-} + \delta S_F\Big|^{\text{NEik}}_{\text{single }\mathcal{A}_\perp} + \delta S_F\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp}$$



$$\delta S_F(x,y)\Big|^{\text{NEik}}_{\text{double }\mathcal{A}_\perp} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, \frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$
$$\times\, (\check{\not{q}} + m)\gamma^j\gamma^+\gamma^i\, (\check{\not{k}} + m) \int d^3\underline{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$
$$\times\, (-i)\;\; \mathcal{U}_F\Big(\frac{L^+}{2}, z^+; \mathbf{z}\Big)\, [gt\cdot\mathcal{A}_j(\underline{z})]\, [gt\cdot\mathcal{A}_i(\underline{z})] \mathcal{U}_F\Big(z^+, -\frac{L^+}{2}; \mathbf{z}\Big)$$

Next-to-eikonal quark propagator - spinor structure

Spinor structure:

- $\mathcal{A}^-$ field associated with $(\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$
- $\mathcal{A}_\perp$ field associated with $(\check{\not{q}}+m)\gamma^j\gamma^+\gamma^i\,(\check{\not{k}}+m)$

  - separate symmetric and anti-symmetric parts:

$$\gamma^j\gamma^+\gamma^i = \delta^{ij}\,\gamma^+ + \gamma^+\frac{[\gamma^i,\gamma^j]}{2}$$

(helicity independent + helicity dependent)

Helicity dependence:

$$[\gamma^i,\gamma^j] = -4i\epsilon^{ij}S^3$$

S^3 - helicity operator, acting on spinors as:

$$S^3 u(\check{k},h) = hu(\check{k},h)$$
$$S^3 v(\check{k},h) = -hv(\check{k},h)$$

Quark propagator

$$S_F(x,y) = S_F(x,y)\Big|_{\text{unpol.}} + S_F(x,y)\Big|_{\text{h. dep.}}$$

Unpolarized part

$$S_F(x,y)\Big|_{\text{unpol.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^2}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, (\check{\not{q}}+m)\gamma^+(\check{\not{k}}+m)$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$
$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$
$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Big\}$$

Helicity-dependent part

$$S_F(x,y)\Big|_{\text{h. dep.}} = \int \frac{d^3\underline{q}}{(2\pi)^3}\int \frac{d^3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+-k^+)\,\frac{\theta(k^+)}{(2k^+)^3}\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}$$
$$\times(\check{\not{q}}+m)\gamma^+\frac{[\gamma^i,\gamma^j]}{4}(\check{\not{k}}+m)\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$
$$\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(\underline{z})\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)$$

The final result fully gauge invariant (due to covariant derivatives):

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt\cdot\mathcal{A}_\mu(\underline{z});\quad \overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}}^\dagger;\quad \overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}}$$

Longitudinal chromo-magnetic field of the target associated with helicity:

$$\mathcal{F}^a_{ij}(\underline{z}) \equiv \partial_{\mathbf{z}^i}\mathcal{A}^a_j(\underline{z}) - \partial_{\mathbf{z}^j}\mathcal{A}^a_i(\underline{z}) - gf^{abc}\,\mathcal{A}^b_i(\underline{z})\,\mathcal{A}^c_j(\underline{z})$$

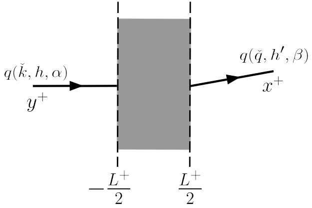
Scattering amplitude from quark propagator

- Simplest observable where NEik correction matter: quark-target cross section
- Need for the relevant scattering amplitude

S-matrix element

Formal definition:
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \langle 0|\hat{b}_{\text{out}}(\check{q},h,\beta)\hat{b}^{\dagger}_{\text{in}}(\check{k},h,\alpha)|0\rangle = (2k^+)2\pi\delta(q^+ - k^+)\, i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})$$

LSZ reduction:
$$S_{q(\check{q},h',\beta)\leftarrow q(\check{k},h,\alpha)} = \lim_{x^+\to\infty}\lim_{y^+\to-\infty}\int d^2\mathbf{x}\int dx^-\int d^2\mathbf{y}\int dy^- \times e^{ix\cdot\check{q}-iy\cdot\check{k}}\,\bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$



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$$\times\, e^{ix\cdot\check{q} - iy\cdot\check{k}}\,\bar{u}(\check{q},h')\gamma^+ S_F(x,y)_{\alpha\beta}\gamma^+ u(\check{k},h)$$

Quark-target scattering amplitude

$$i\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q}) = \delta_{hh'}\int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\Big\{\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z}\Big)$$
$$+\frac{\epsilon^{ij}h}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\Big(-igt\cdot\mathcal{F}_{ij}(\underline{z})\Big)\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$
$$-\frac{i}{2k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\,\overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]$$
$$-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}}\,\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{z}\Big)\Big]\Big\}_{\alpha\beta}$$

Unpolarized cross section

Differential cross section for quark scattering on the target is

$$\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}} = \frac{1}{(2\pi)^2}\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Cross section averaged over the target $(\mathbf{z}-\mathbf{z}')\equiv\mathbf{r}$ and $(\mathbf{z}+\mathbf{z}')\equiv 2\mathbf{b}$

$$\left\langle\frac{d^2\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r})$$
$$+\Big(\bar{O}(\mathbf{r})-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r})-\mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big]-\frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r})-\mathcal{O}^\dagger_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

Dipole operator:

$$d_F(\mathbf{r}) = 1-\bar{P}(\mathbf{r})+\bar{O}(\mathbf{r})$$
$$d_F(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\Big\langle\mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\mathcal{U}_F\Big(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

Decorated dipole operators:

$$\mathcal{O}^j_{(1)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dz^+\Big\langle\mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)$$
$$] \qquad \times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftrightarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A$$

$$\mathcal{O}_{(2)}(\mathbf{r}) = \frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2}dz^+\Big\langle\mathrm{tr}\Big[\mathcal{U}_F^\dagger\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)$$
$$\times\,\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\overleftarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\,\overrightarrow{\mathcal{D}_{\mathbf{b}^j+\frac{\mathbf{r}^j}{2}}}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A.$$

Anti-quark–target differential cross section:

$$\left\langle \frac{d^2\sigma^{\bar{q}A\to\bar{q}+X}}{d^2\mathbf{q}} \right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\; e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\Big\{1-\bar{P}(\mathbf{r})$$

$$-\Big(\bar{O}(\mathbf{r}) - \frac{(\mathbf{q}^j+\mathbf{k}^j)}{4k^+}\Big[\mathcal{O}^j_{(1)}(\mathbf{r}) - \mathcal{O}^{\dagger j}_{(1)}(\mathbf{r})\Big] - \frac{i}{2k^+}\Big[\mathcal{O}_{(2)}(\mathbf{r}) - \mathcal{O}^{\dagger}_{(2)}(\mathbf{r})\Big]\Big)\Big\}$$

- By definition : $d_F(\mathbf{r})^\dagger = d_F(-\mathbf{r})$
  $\Rightarrow$ Decomposition $d_F(\mathbf{r}) = 1 - \bar{P}(\mathbf{r}) + \bar{O}(\mathbf{r})$ with
  - Real part $1 - \bar{P}(\mathbf{r})$ even in $\mathbf{r}$
  - Imaginary term $\bar{O}(\mathbf{r})$ odd in $\mathbf{r}$

- **Signature transformation ($\mathcal{U} \to \mathcal{U}^\dagger$) and charge conjugation ($q \to \bar{q}$):**

  $\bar{P}(\mathbf{r})$: Pomeron is **even** under both transformations
  $\bar{O}(\mathbf{r})$: Odderon is **odd** under both transformations

  * **Eikonal terms contain both Pomeron and Odderon**
  * **Next-to-eikonal corrections are of Odderon-type**

Quark helicity asymmetry

The difference between the cross sections for a quark of positive and negative helicity scattering on the nucleus target is:

$$\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}} \equiv \frac{1}{(2\pi)^2}\frac{1}{2N_c}\sum_{h,h'}\sum_{\alpha,\beta}(2h)\,\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})^\dagger\mathcal{M}^{hh'}_{\alpha\beta}(\underline{k},\mathbf{q})\Big|_{q^+=k^+}$$

Quark helicity asymmetry averaged over the target

$$\left\langle\frac{d^2\Delta\sigma^{qA\to q+X}}{d^2\mathbf{q}}\right\rangle_A = \frac{1}{(2\pi)^2}\int d^2\mathbf{r}\,e^{-i(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}}\frac{(-i)}{4k^+}\Big[O_{(3)}(\mathbf{r})-O^\dagger_{(3)}(\mathbf{r})\Big]$$

New decorated dipole operator:

$$\begin{aligned}O_{(3)}(\mathbf{r}) \;=\; &\frac{1}{N_c}\int d^2\mathbf{b}\int_{-L^+/2}^{L^+/2}dz^+\Big\langle \mathrm{Tr}\Big[\mathcal{U}^\dagger_F\Big(\mathbf{b}-\frac{\mathbf{r}}{2}\Big)\\ &\times\mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big\{\epsilon^{ij}\Big[gt\cdot\mathcal{F}_{ij}\Big(z^+,\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\}\mathcal{U}_F\Big(z^+,-\frac{L^+}{2};\mathbf{b}+\frac{\mathbf{r}}{2}\Big)\Big]\Big\rangle_A\end{aligned}$$

The anti-quark helicity asymmetry is identical!

Remarks:

- $O_{(3)}(\mathbf{r})$ behaves neither like Pomeron nor like Odderon (odd under signature transformation but even under charge conjugation)
- $\mathcal{F}_{ij}$ and $O_{(3)}(\mathbf{r})$ vanish for unpolarized target, but not for longitudinally polarized target $\Rightarrow$ Double longitudinal spin asymmetry A_{LL}

Dynamics of the target: x^- dependence of the field

T. Altinoluk, G. Beuf - arXiv: 2109.01620

x^- dependence is included in both scalar and quark propagators:

$$S_F(x,y) = \int \frac{d^{D-1}\underline{q}}{(2\pi)^{D-1}} \int \frac{d^{D-1}\underline{k}}{(2\pi)^{D-1}} \theta(q^+)\theta(k^+)\, e^{-i\check{q}\cdot x}\, e^{i\check{k}\cdot y} \int dz^-\, e^{iz^-(q^+-k^+)} \int d^{D-2}\mathbf{z}\; e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}$$
$$\times \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \Bigg\{ \mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{z},z^-\right)$$
$$- \frac{(\mathbf{q}^j+\mathbf{k}^j)}{2(q^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z},z^-\right) \overleftrightarrow{D}_{\mathbf{z}^j} \mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z},z^-\right)\right]$$
$$- \frac{i}{(q^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z},z^-\right) \overleftarrow{D}_{\mathbf{z}^j} \overrightarrow{D}_{\mathbf{z}^j} \mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z},z^-\right)\right]$$
$$+ \frac{1}{(q^+ + k^+)} \frac{1}{4} [\gamma^i,\gamma^j] \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2},z^+;\mathbf{z},z^-\right) gF_{ij}(z)\, \mathcal{U}_F\left(z^+,-\frac{L^+}{2};\mathbf{z},z^-\right)\right] \Bigg\} \frac{(\check{\not{k}}+m)}{2k^+}$$

- generalized eikonal expression including the z^- dependence.
- can be expanded to isolate the effects of the dynamics of the target.

Summary and final remarks

- Full next-to-eikonal (NEik) expression for the quark propagator through the background field derived
  - Corrections due to transverse motion (drift and Brownian) of the quark while crossing the Lorentz contracted target
  - Corrections due to interaction with the $\mathcal{A}_\perp$ components of the background field
  - Recently, including the effects of x^- dependence of the $\mathcal{A}_\mu$.
- Gauge covariant expression: covariant derivatives and field strength insertions in Wilson lines
- Eikonal limit blind to spin, whereas one NEik term is proportional to helicity
- Helicity piece consistent with results of Kovchegov *et al.* (2016-2020)
- qA (and $\bar{q}A$) cross section (unpolarized and helicity asymmetry) studied at NEik
- The NEik quark propagator is a building block for scattering processes at NEik (calculation of DIS di-jet production in progress - T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska)