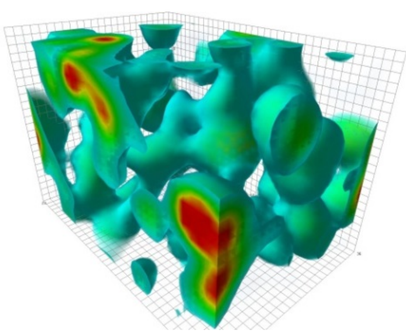
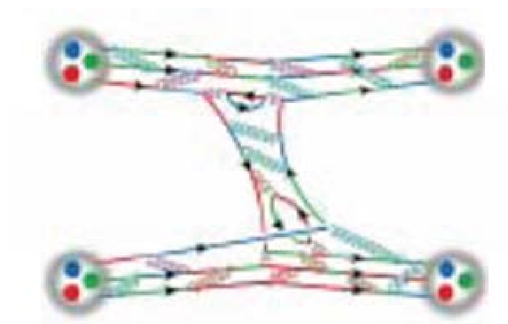
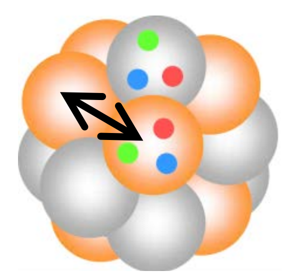
# Hadron Interactions from Lattice QCD

**Takumi Doi**

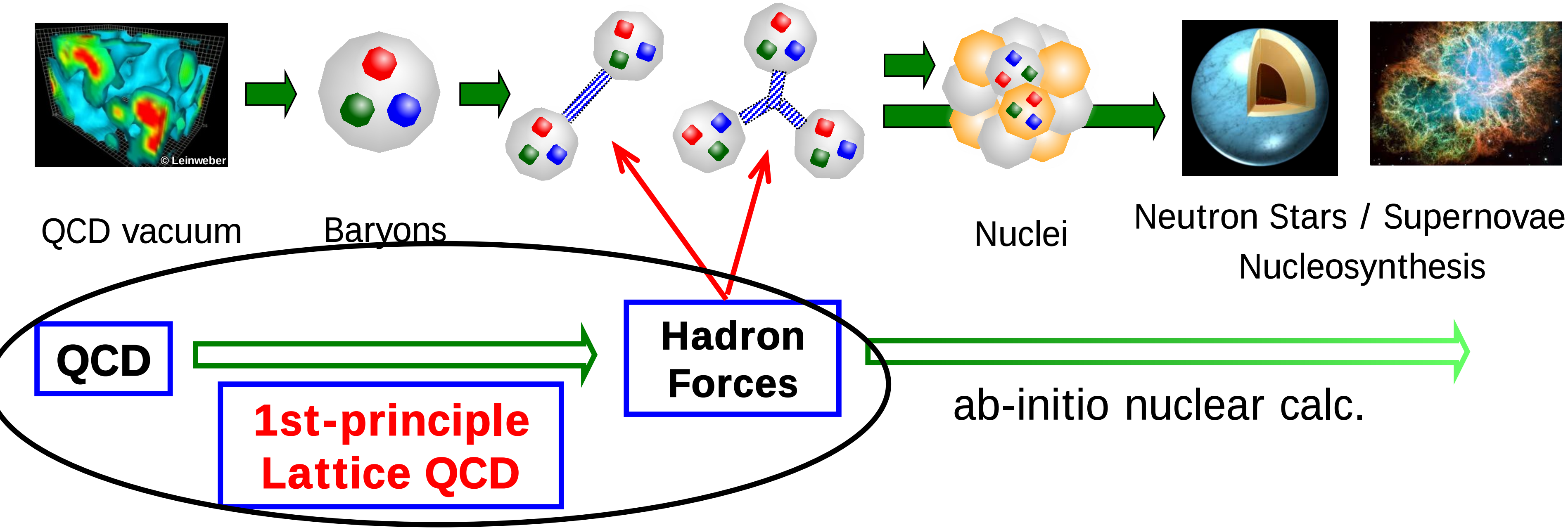
(RIKEN Nishina Center / iTHEMS)

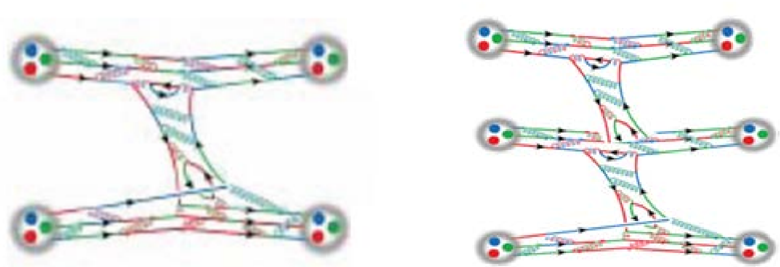






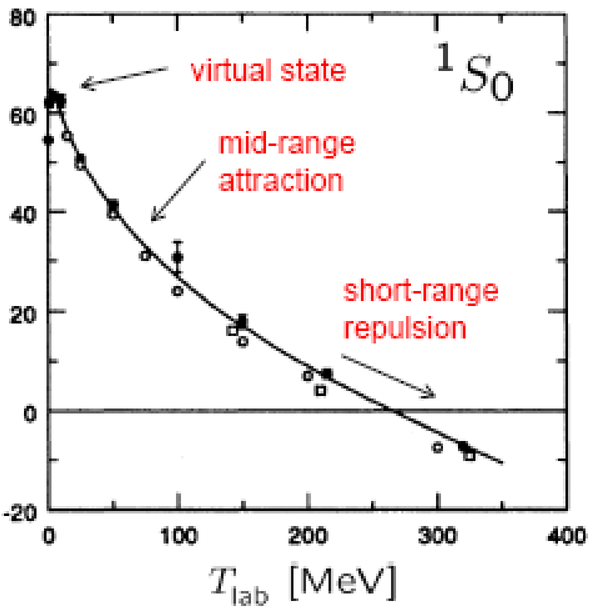
# *The Odyssey from Quarks to Universe*



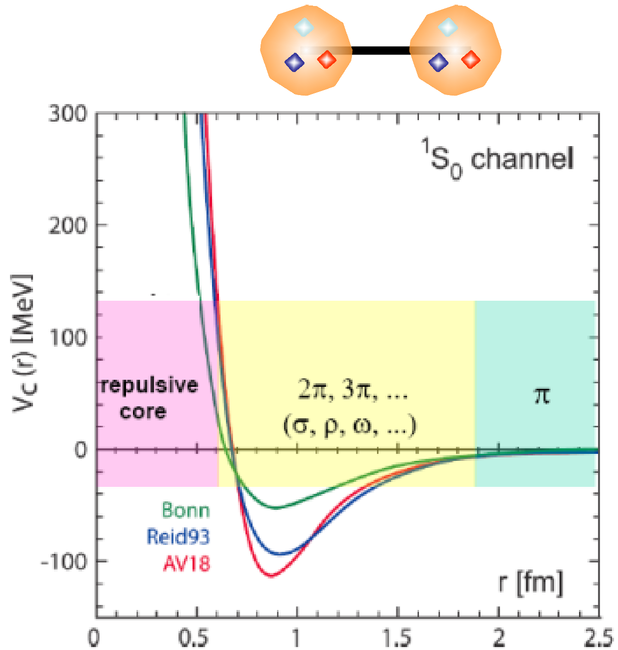



**Nuclear Physics directly based on QCD**

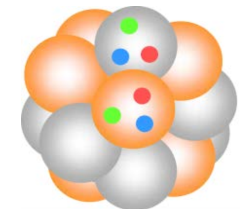
# Nuclear Forces: Foundation of nuclear physics



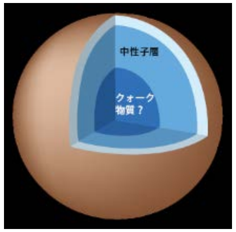

**NN phase shifts**
**from experiments**



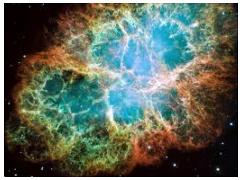

**Phenomenological**
**Nuclear Forces**



*Nuclei*



*Neutron Stars*



*Super Novae*

**Various applications**

- ***Nuclear Forces* play *crucial roles***
  - *Yet, no clear connection to QCD so far*

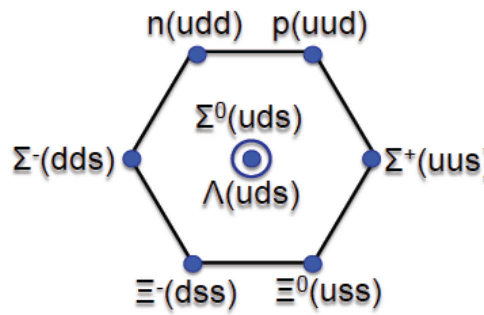
**Phen. NN potentials: #params = 30~40**

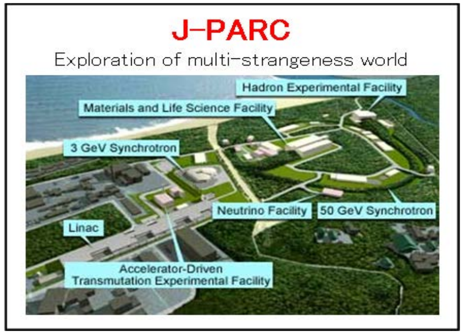
**←→ QCD: #inputs = 6 : quark masses ($m_u$, $m_d$, $m_s$, $m_c$, $m_b$) & coupling $\alpha_s$**

# Nuclear Forces → Baryon Forces (incl. Hyperons)

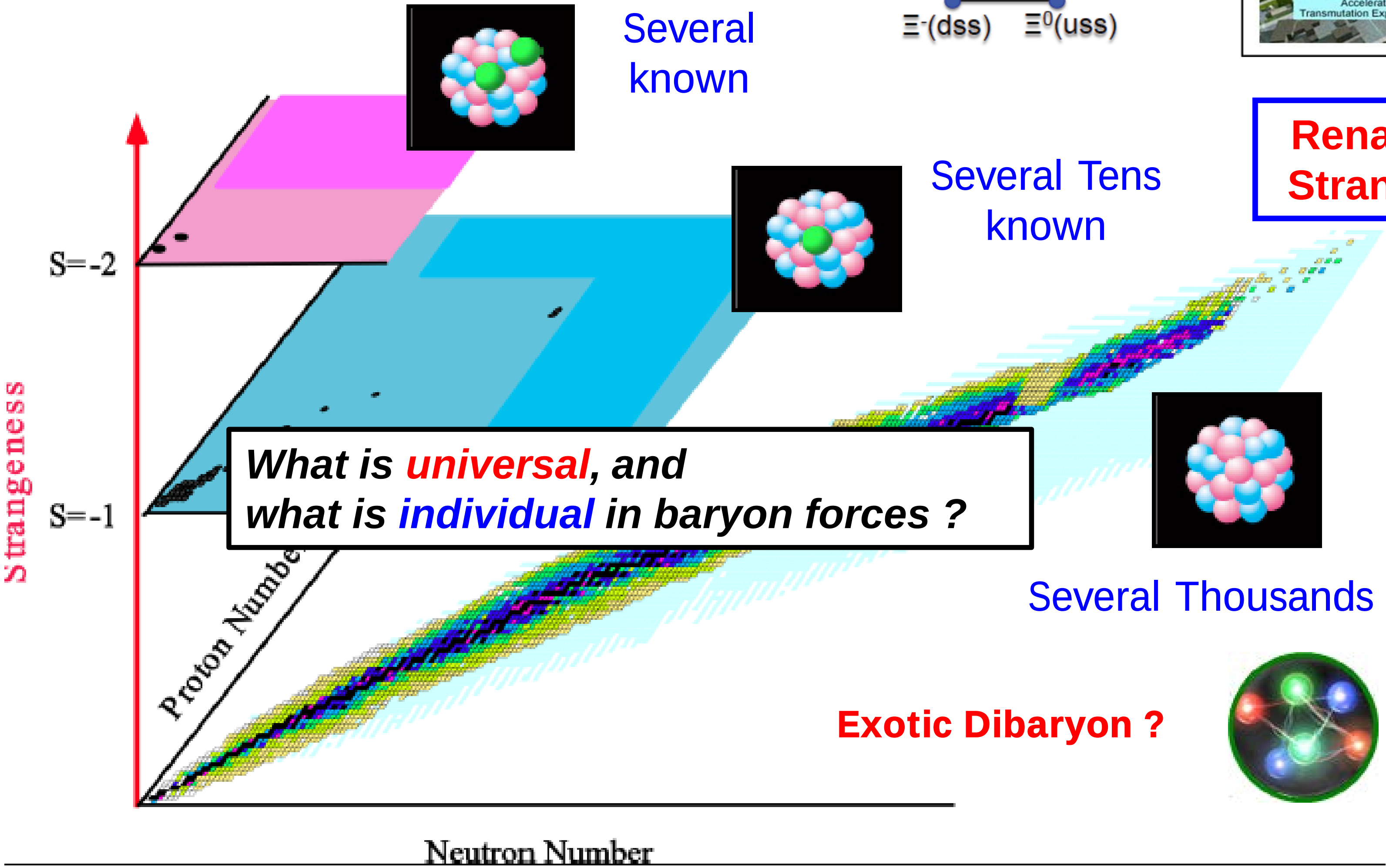
**3D Nuclear Chart**

Nucleons : u, d quarks
Hyperons : u, d, s quarks




**Renaissance in Strange World !**




Several known

Several Tens known

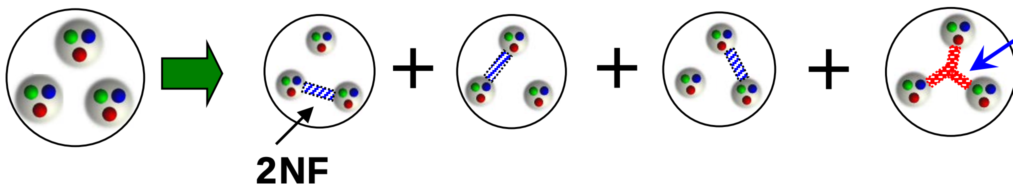
Several Thousands known

***What is universal, and***
***what is individual in baryon forces ?***

**Exotic Dibaryon ?**

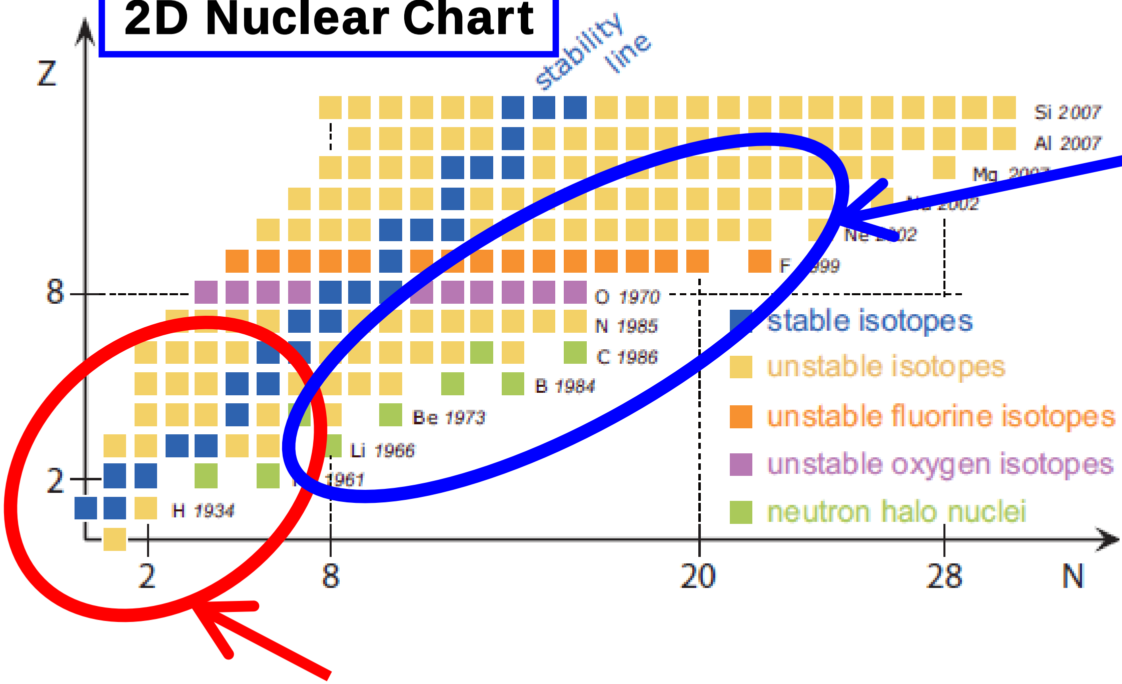
# Nuclear Forces ➔ Thee-Nucleon Forces (3NF)

## What is 3NF ?




**3NF: Forces which cannot be explained by pair-wise 2NF**

## 2D Nuclear Chart




Precise ab initio calculations show 3NF is indispensable

**Paradigm Shift** in **Unstable Nuclei** (New Magic Numbers !)

🡐 **Important role of 3NF**

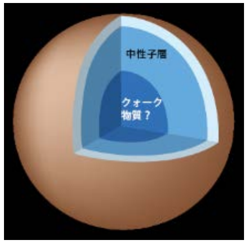
T.Otsuka et al., PRL105(2010)032501

➔ **r-process Nucleosynthesis**

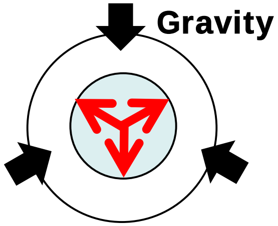
RIBF/FRIB

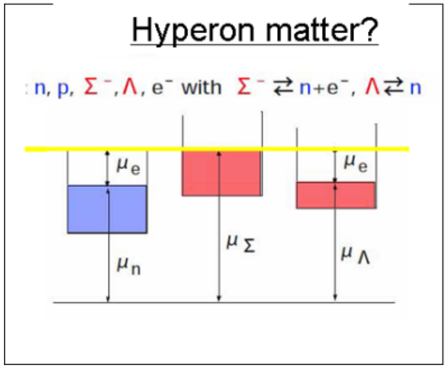
# Dense Matter ← Interactions of YN, YY, + NNN, YNN,… are crucial

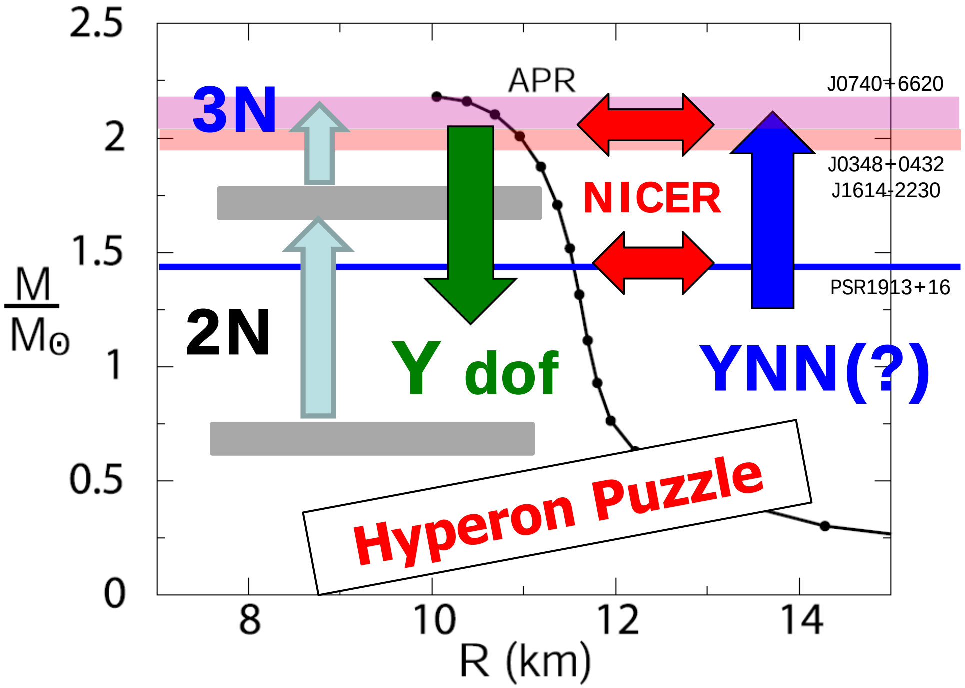
- Neutron Stars, Supernovae
  ↔ EoS of dense matter




~10km
1-2 Msolar




*How to sustain a neutron star against gravitational collapse ?*

***Quark matter ?***

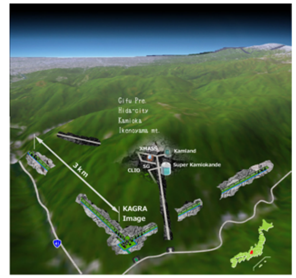
G. Baym et al., Rep.Prog.Phys 81(2018)056902

Akmal et al.('98), Nishizaki et al.('02), Takatsuka et al.('08)

# Observation of NS-NS merger (GW170817)

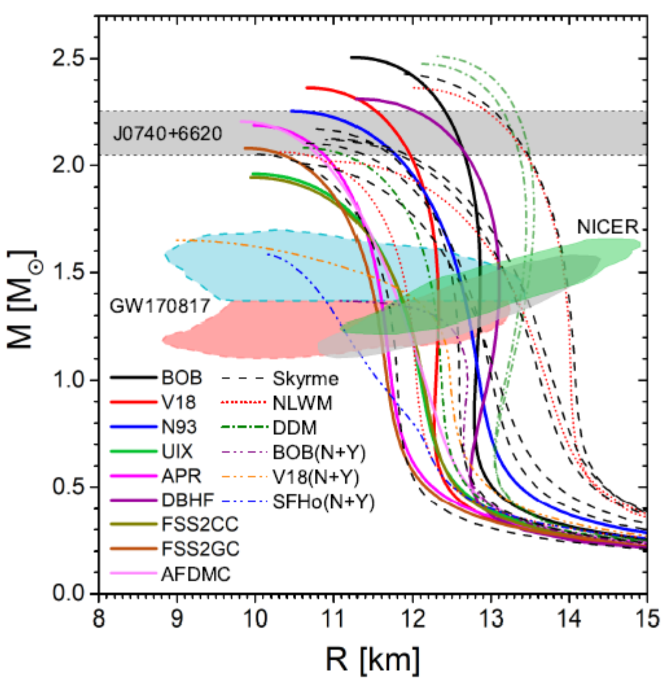
**GW**

**NS-NS merger**

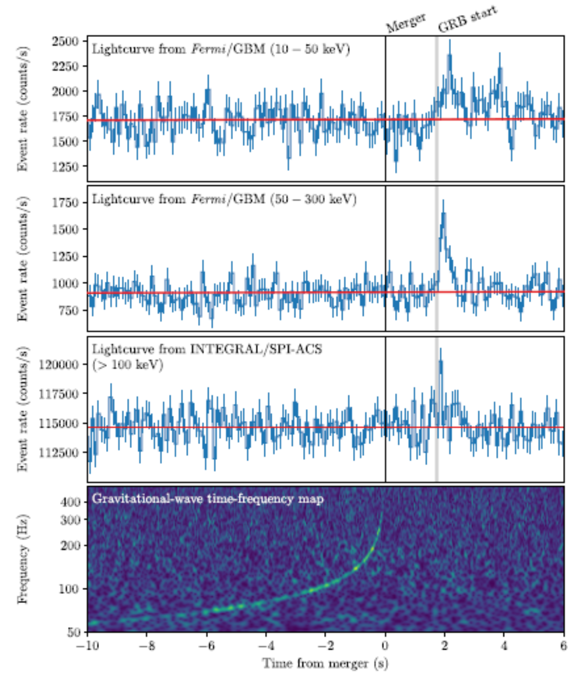


**LIGO/Virgo**
(KAGRA)

**Impact on EoS**




G. F. Burgio+, PPNP120(2021)103879




Gravitational Wave
& EM Wave observed

Nucleosynthesis of heavy
elements observed

*Much more will come!*

# Nuclear/Hyperon Forces → Charmed Forces

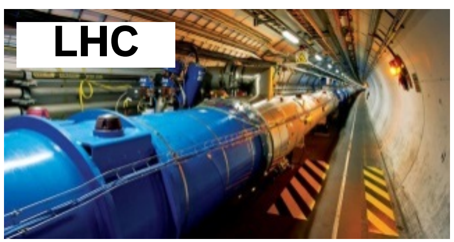
| light | ~ a few MeV | ~100 MeV | ~300 MeV | ~1.3 GeV | ~4.2 GeV | heavy |
|---|---|---|---|---|---|---|
| | up,down quarks | strange quark | $\Lambda_{QCD}$ | charm quark | bottom quark | top |

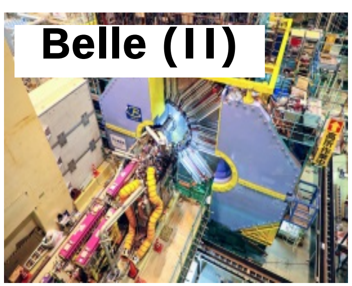
<u>chiral symmetry</u>

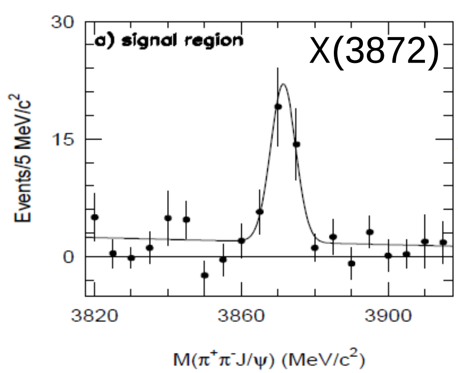
<u>heavy quark symmetry</u>

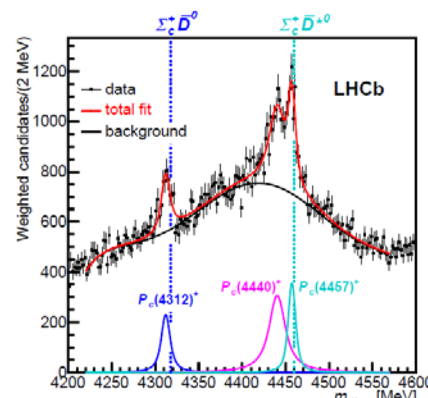
Heavy quarks: New doorway to the mysteries of QCD

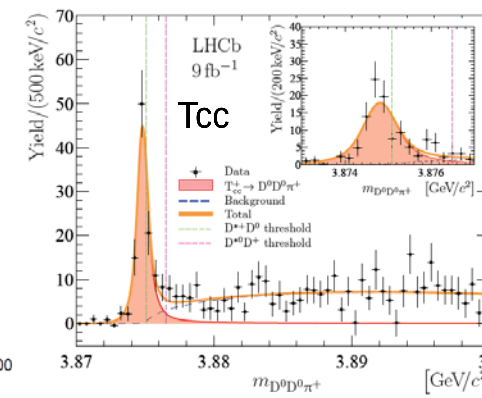
## <u>Many new exotic particles being reported!</u>



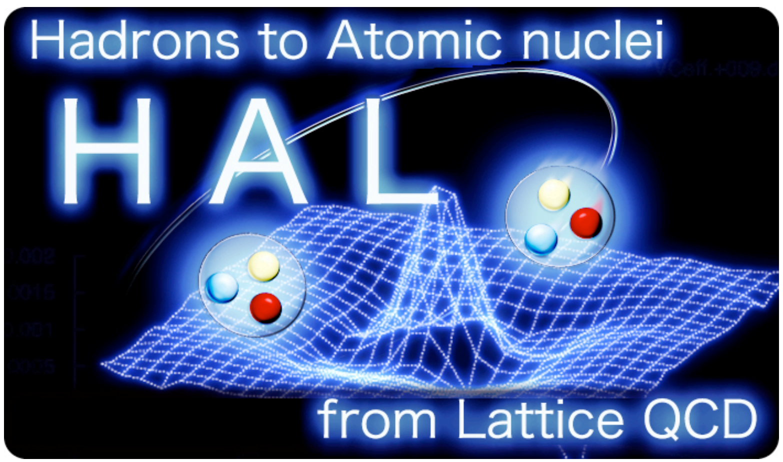




Hadron interactions crucial to understand these “signals” !

e.g., Zc(3900) from HAL LQCD → threshold cusp

Y. Ikeda et al. (HAL), PRL117(2016)242001

# Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)

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**T. Inoue** (Nihon Univ.)
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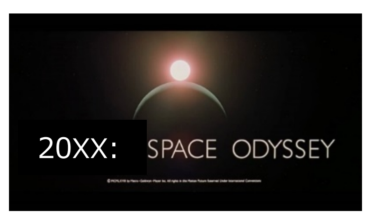
**H. Tong** (Tianjin Normal Univ.)

「20XX年宇宙の旅」

***from Quarks to Universe***




**+**
**E. Itou** (RIKEN)
**I. Kanamori** (RIKEN)
**K.-I. Ishikawa** (Hiroshima Univ.)

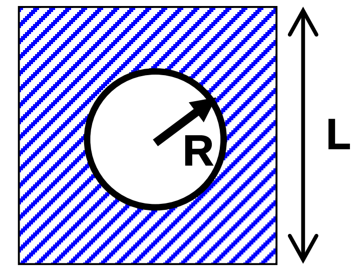
- **<u>Outline</u>**
  - ~~Introduction~~
  - Theoretical framework
    - Luscher's formula
    - HAL QCD method
    - Reliability test of LQCD methods
  - (Results at heavy quark masses)
  - Results near physical quark masses
  - Summary / Prospects

# Luscher's formula: Scatterings on the lattice

- Consider Schrodinger eq at asymptotic region

$$(\nabla^2 + k^2)\psi_k(\boldsymbol{r}) = mV_k(\boldsymbol{r})\psi_k(\boldsymbol{r})$$

$$V_k(\boldsymbol{r}) = 0 \text{ for } r > R$$

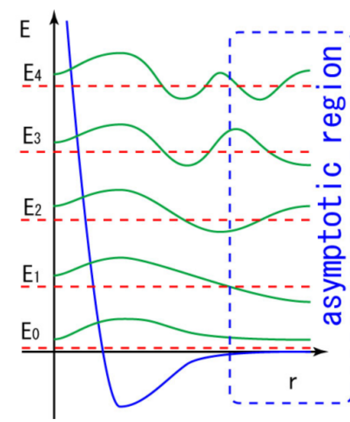



  - (periodic) Boundary Condition in finite V
    ➔ constraint on energies of the system
  - Energy E and phase shift (at E) are related

$$E = 2\sqrt{m^2 + k^2} \qquad \text{(QFT: } \psi_k(r) \to \text{NBS w.f.)}$$

$$k \cot \delta_{\mathbf{E}} = \frac{2}{\sqrt{\pi} L} Z_{00}(1; q^2), \qquad q = \frac{kL}{2\pi}$$

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\boldsymbol{n} \in \boldsymbol{Z}^3} \frac{1}{(\boldsymbol{n}^2 - q^2)^s}$$




Large V expansion

$$\Delta E = E - 2m = -\frac{4\pi \mathbf{a}}{mL^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \mathcal{O}\left(\frac{1}{L^3}\right)\right]$$

$c_1, c_2$: geometric constants

**a**: scattering length

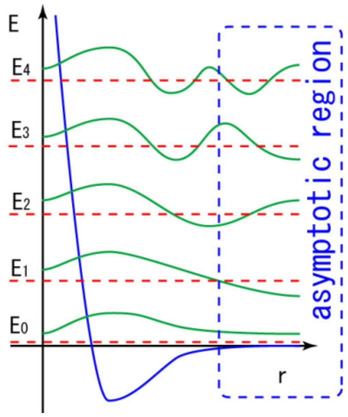
# [HAL QCD method]

- "Potential" defined through phase shifts (S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r})N(\vec{x})|N(k)N(-k);W\rangle$$

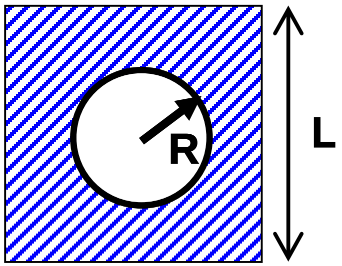
$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R \qquad W = 2\sqrt{m^2+k^2}$$




– Wave function ←→ phase shifts

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

(below inelastic threshold)



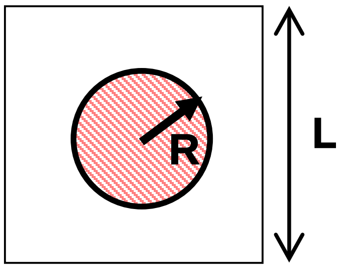

<u>**Extended to multi-particle systems**</u>


M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504
Ishizuka, Pos LAT2009 (2009) 119
Aoki-Hatsuda-Ishii PTP123(2010)89
S.Aoki et al., PRD88(2013)014036

# "Potential" as a representation of S-matrix

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$




  – U(r,r'): faithful to the phase shift by construction
    - U(r,r'): NOT an observable, but well defined
    - U(r,r'): E-independent, while non-local in general

      – "Proof of Existence": Explicit form can be given as

$$U(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{m} \sum_{n,n'}^{n_{\rm th}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r}) \mathcal{N}_{nn'}^{-1} \psi_{n'}^*(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r} \psi_n^*(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

      – Non-locality ➔ derivative expansion

Okubo-Marshak(1958)

$$U(\vec{r}, \vec{r'}) = \left[V_c(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}V_{LS}(r) + \mathcal{O}(\nabla^2)\right]\delta(\vec{r} - \vec{r'})$$

LO LO NLO NNLO

Aoki-Hatsuda-Ishii PTP123(2010)89



Check on convergence: K.Murano et al., PTP125(2011)1225

# Three-Body Forces

- Unitarity of S-matrix

S.Aoki et al. (HAL Coll.), PRD88(2013)014036
Gongyo-Aoki PTEP2018(2018)093B03

$$T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} \boxed{e^{i\delta_{[L]}(Q)} \ \sin\delta_{[L]}(Q)}$$

c.f. R.B. Newton (1974) for n = 3

**Similar formula to 2-body system**
(w/ diagonalization matrix U which includes dynamics)

- NBS wave function

$$\psi_\alpha([x]) =_{\rm in} \langle 0|\phi([x])|\alpha\rangle_{\rm in} =_{\rm in} \langle 0|N(\vec{x}_1)N(\vec{x}_2)\cdots N(\vec{x}_n)|\alpha\rangle_{\rm in}$$

$$\psi_{[L],[K]}(R,Q_A) \propto \sum_{[N]} U_{[L][N]}(Q_A) \boxed{e^{i\delta_{[N]}(Q_A)} \frac{\sin(Q_A R - \Delta_L + \delta_{[N]}(Q_A))}{(Q_A R)^{(D-1)/2}}} U^\dagger_{[N][K]}(Q_A)$$

**Similar asymptotic behavior to 2-body system**

arbitrary n-body
(non-rela approx.)

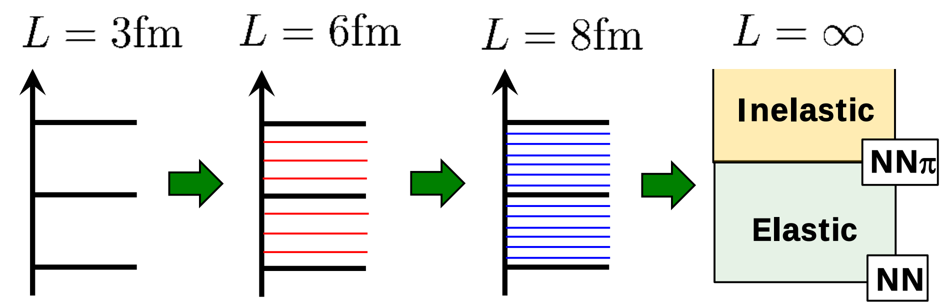
c.f. Finite V spectrum, n=3 only, relativistic: Hansen, Sharpe, Briceno, ...

# *The Challenge in multi-baryons on the lattice*

Existence of elastic scatt. states

→ (almost) No Excitation Energy

→ LQCD method based on G.S. saturation impossible




Signal/Noise issue

"Sign Problem"

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi('84), Lepage('89)

L=8fm @ physical point $(E_1 - E_0) \simeq 25\mathrm{MeV} \Longrightarrow t > 10\mathrm{fm}$

$$S/N \sim 10^{-32}$$

Direct method (naïve plateau fitting at t ~ 1fm)

→ Does it really reliable?

# Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

***E-indep of potential U(r,r')*** ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

## Original (t-indep) HAL method

$$G_{NN}(\vec{r},t) = \langle 0|N(\vec{r},t)N(\vec{0},t)\overline{\mathcal{J}_{\rm src}(t_0)}|0\rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum A_{W_i}\psi_{W_i}(\boldsymbol{r})e^{-(W_i-2m)t}$$

➔ Many states contribute

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_0}(\boldsymbol{r}')} = (\underline{E_{W_0}} - H_0)\underline{\psi_{W_0}(\boldsymbol{r})}$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{\psi_{W_1}(\boldsymbol{r}')} = (\underline{E_{W_1}} - H_0)\underline{\psi_{W_1}(\boldsymbol{r})}$$

. . .

Partial solution of Sign Problem

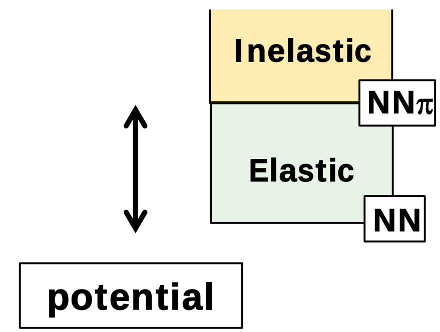
## New t-dep HAL method

All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\underline{R(\boldsymbol{r}',t)} = \underline{(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2}} - H_0)\underline{R(\boldsymbol{r},t)}$$

**~~G.S. saturation~~ ➔ "Elastic state" saturation**

**[Exponential Improvement]**



# Reliability test of LQCD methods

## NN @ heavy quark masses

HAL method (HAL) : unbound

Direct method (NPL/CalLat/PACS-CS(Yamazaki et al.)): bound

***Inconsistent!***

## What is the most plausible systematics ?

### HAL QCD method

Energy-indep potential: “signal” from all elastic states

Non-locality of potential: derivative expansion could introduce systematics

### Direct method (= plateau fitting + Luscher’s formula)

Plateau fitting at t ~ 1fm (much less than $1/(E_1-E_0)$)

➔ Excited states give “noises”

# Examine the reliability of the HAL QCD method

Convergence of the derivative expansion of potential

LQCD data: $\Xi\Xi(^1S_0)$ @ $m\pi$=0.51GeV

wall source & smeared source

Same confs in Yamazaki et al.('12)

T. Iritani et al. (HAL) PRD99(2019)014514

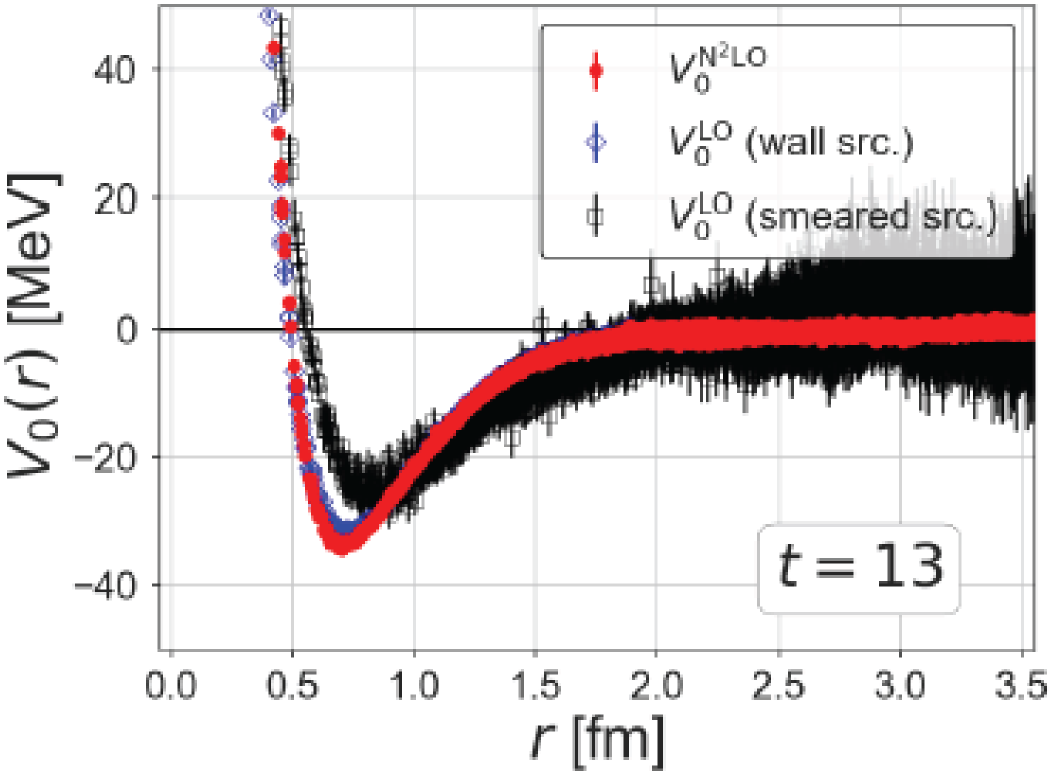
# Higher Order Approximation (N²LO) (2)

$$U(r,r') \simeq \left[V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\nabla^2\right]\delta(r-r')$$

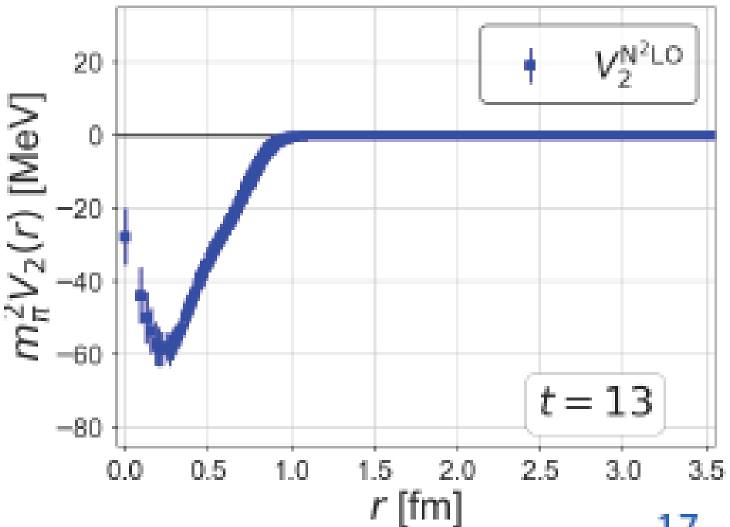
**wall src.** –> small $\mathbf{V_2\nabla^2}$ **correction**
**smeared src.** –> large $\mathbf{V_2\nabla^2}$ **correction**

→ $V_2(r)\nabla^2 R^{\mathrm{wall/smear}}(r)$

dep. on shape of **R**

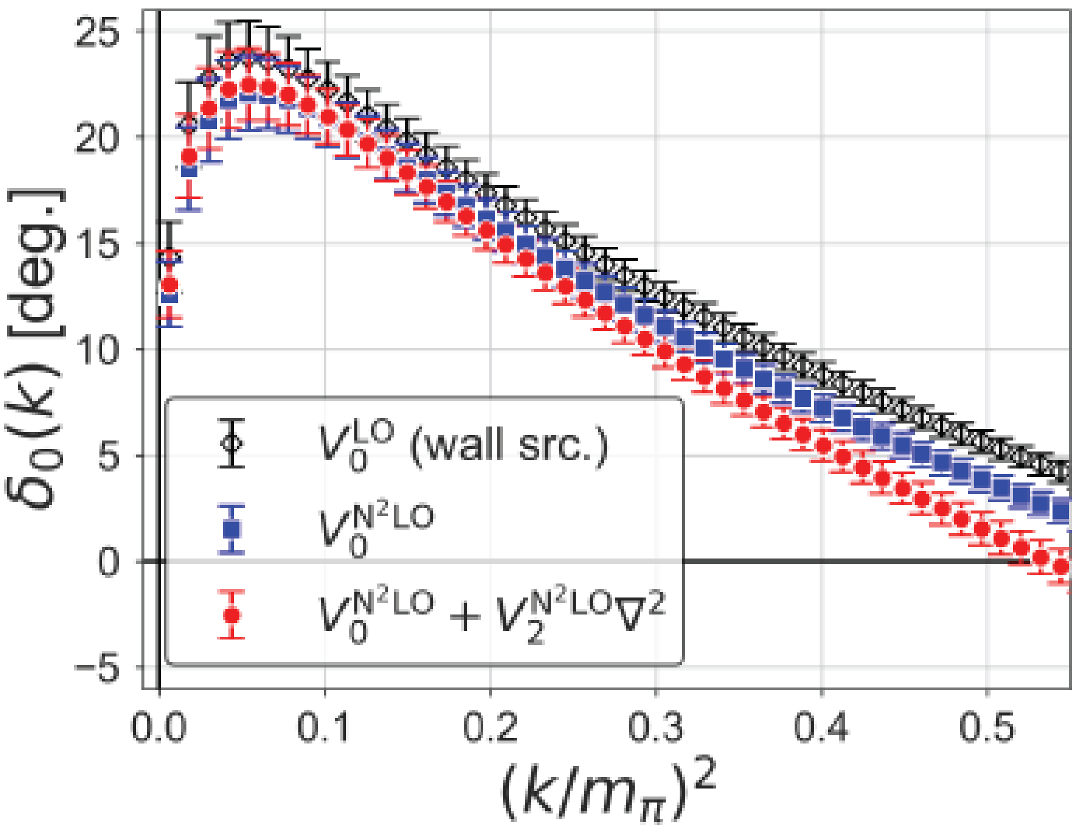


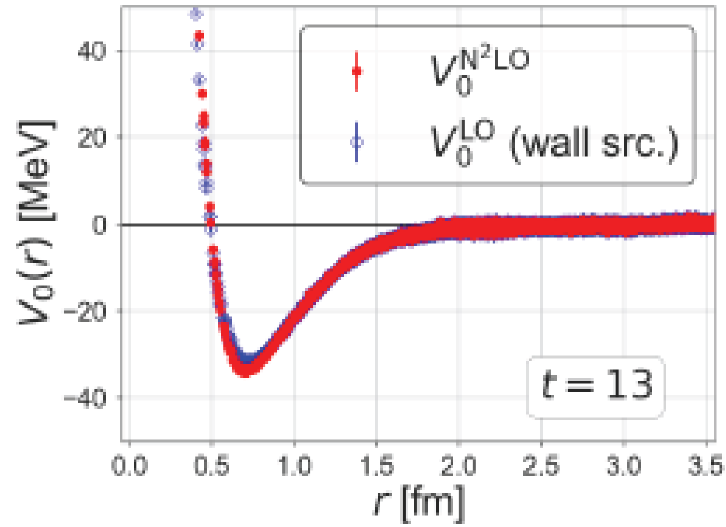

**V₂-pot.**

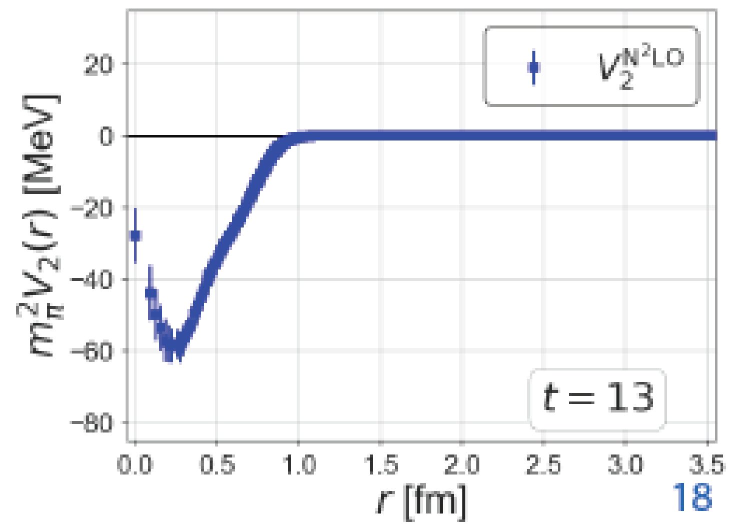


# Phase Shift and Uncertainties in Velocity Expansion

- **Wall src. LO approx.** (standard of HAL QCD studies) **works well at low energy.**
- **$V_2$ correction** at high energy



# Examine the reliability of the Direct method

LQCD data: $\Xi\Xi(^1S_0)$ @ $m\pi$=0.51GeV

wall source & smeared source

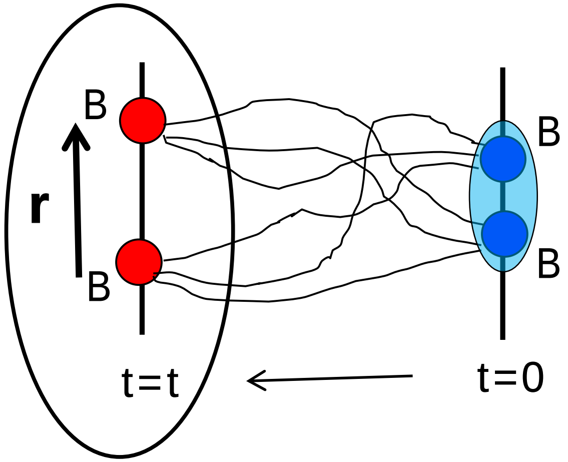
Same confs in Yamazaki et al.('12)

T. Iritani et al. (HAL Coll.) JHEP1610(2016)101

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

# Operator dependence in the direct method

$$C_{2B}(t) = \langle 0|\mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t)\overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)]|0\rangle$$

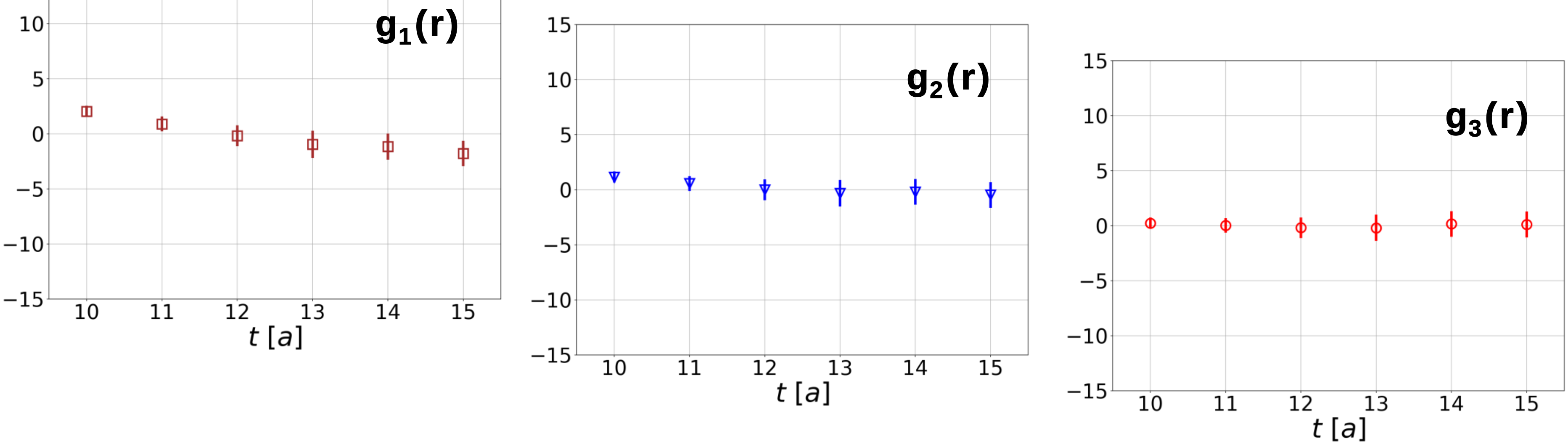



Study sink op dep
w/ smeared src tuned in single-baryon

$$\mathcal{J}^{2B}_{\mathrm{sink}} = \sum_{\vec{r}} g(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$

Usual direct method: g(r)=1 only

**Effective Energy shift ΔE**

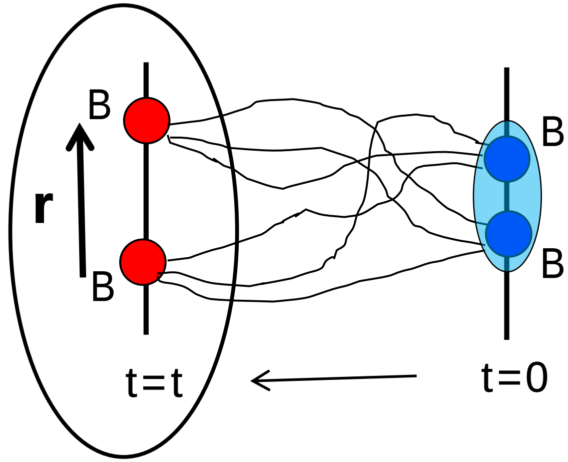



**All plateaux "look" reliable**

**In reality, I shift data vertically "by hand"**

# Operator dependence in the direct method

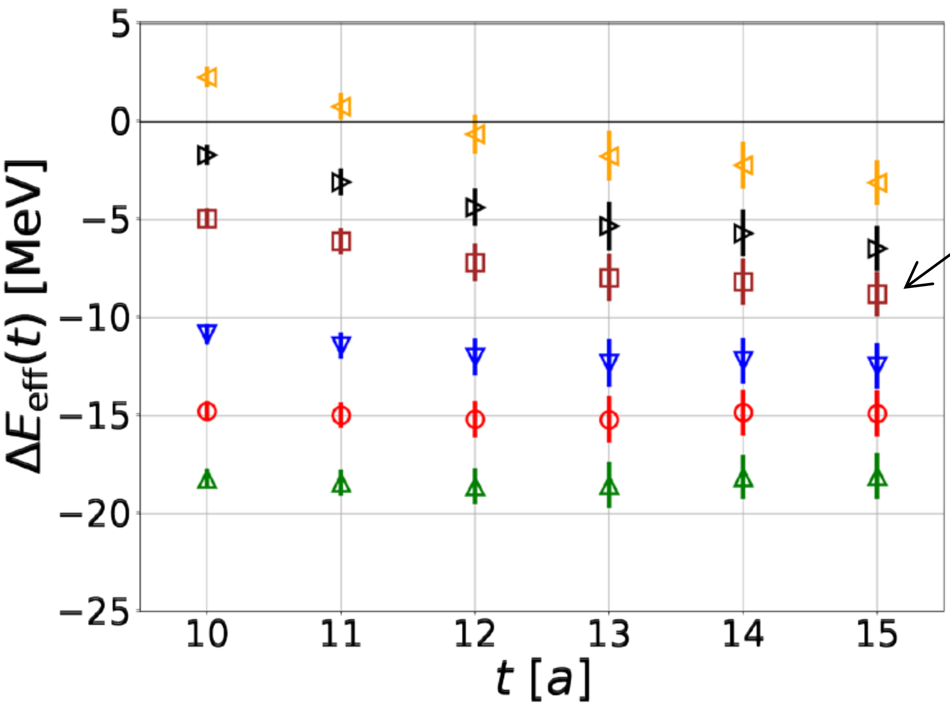
$$C_{2B}(t) = \langle 0|\mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t)\overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)]|0\rangle$$




Study sink op dep
w/ smeared src tuned in single-baryon

$$\mathcal{J}^{2B}_{\mathrm{sink}} = \sum_{\vec{r}} g(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$

Usual direct method: g(r)=1 only




The results depend on
the choice of sink op
→ predictive power is lost!

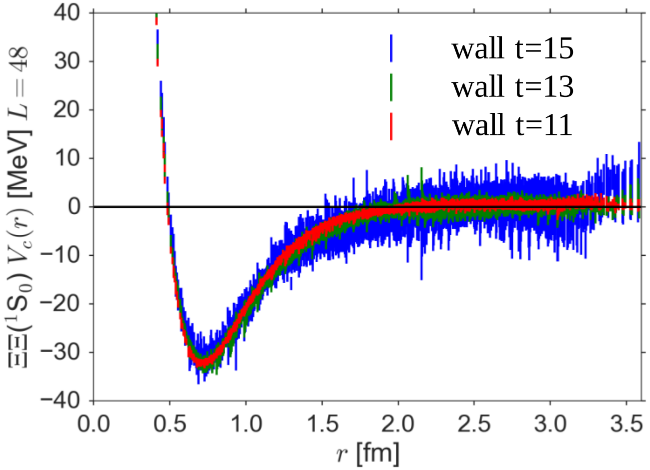
T. Iritani et al. JHEP10(2016)101 + update

# Anatomy of the Direct method
# and
# the consistency between
# Luscher's formula and HAL method

T. Iritani et al. (HAL) JHEP03(2019) 007

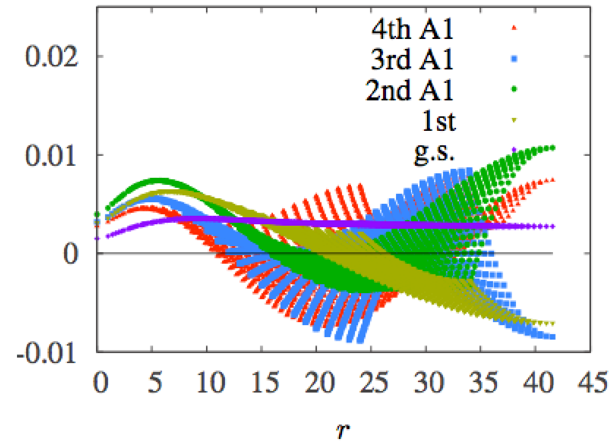
# Understand the origin of “pseudo-plateaux”

## Potential




**Solve Schrodinger eq. in Finite V**

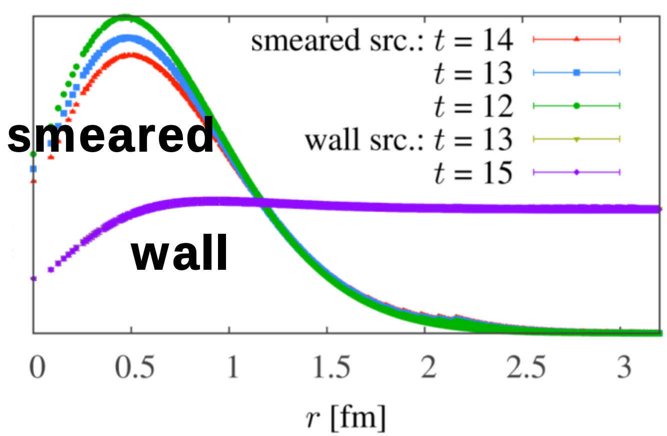
## Eigen-wave functions




## Eigen-energies

| $n$-th A1 | $\Delta E_n$ [MeV] |
|---|---|
| 0 | -2.58(1) |
| 1 | 52.49(2) |
| 2 | 112.08(2) |
| 3 | 169.78(2) |
| 4 | 224.73(1) |

## NBS correlator R(r,t)



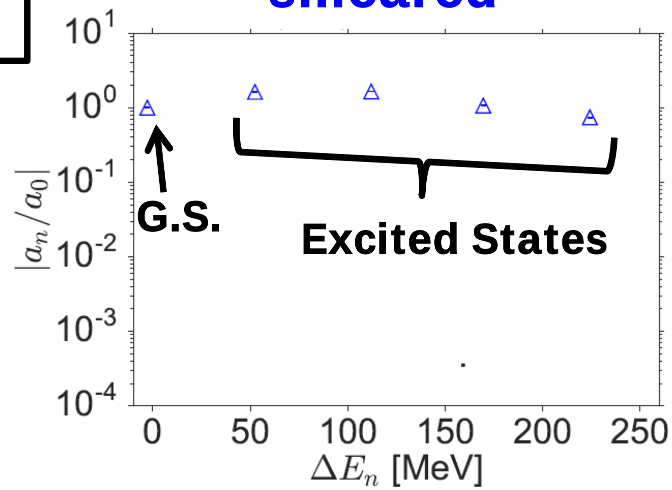

**Decompose NBS correlator to each eigenstates**

Decompose NBS correlator
to each eigenstates

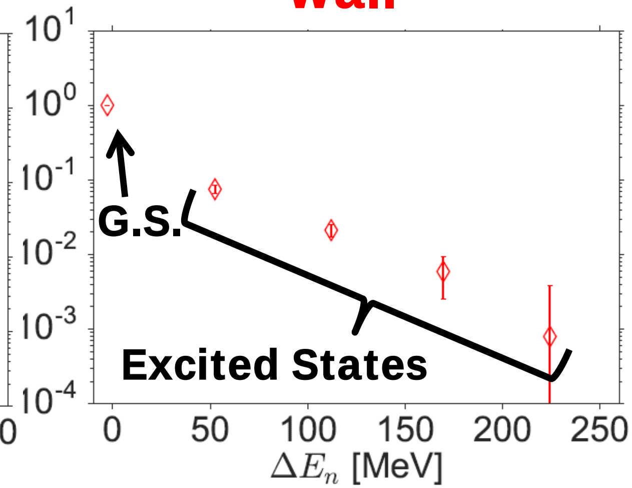
**NBS correlator R(r,t)**

**Contribution from each (excited) states (@ t=0)**

**smeared**




**wall**



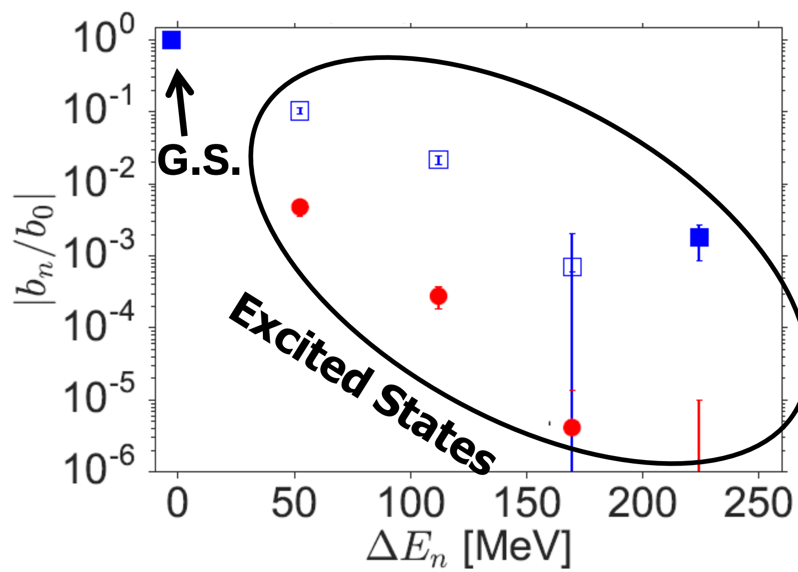

excited states NOT suppressed

excited states suppressed

**Temporal-correlator**
**R(t) = $\Sigma_r$ R(r,t)**

(R(t) w/ smeared has been used in Direct method)

**Contribution from each (excited) states (@ t=0)**

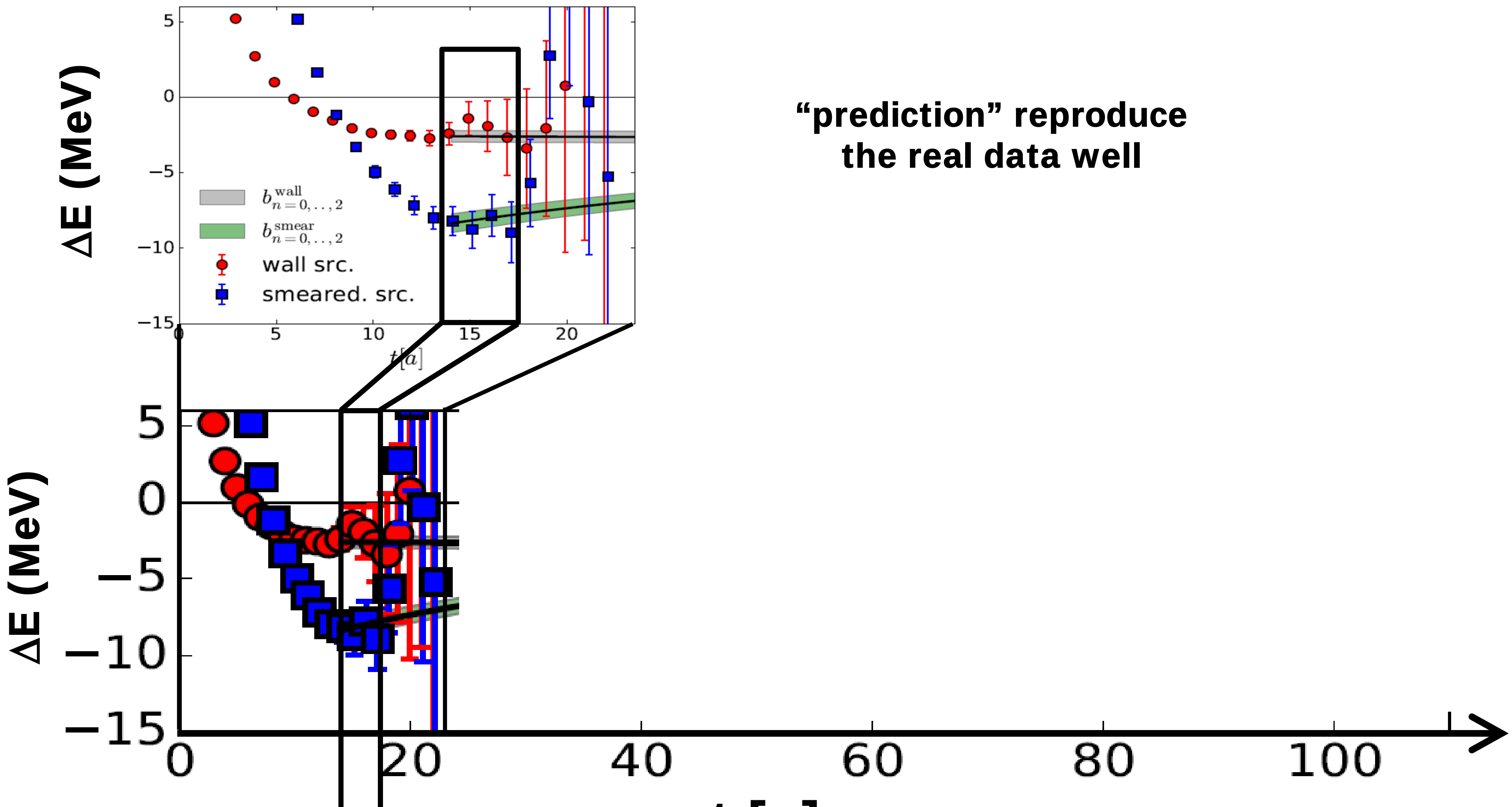



Blue: smeared

Red: wall

# Understand the origin of "pseudo-plateaux"

**We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"**




**"prediction" reproduce the real data well**

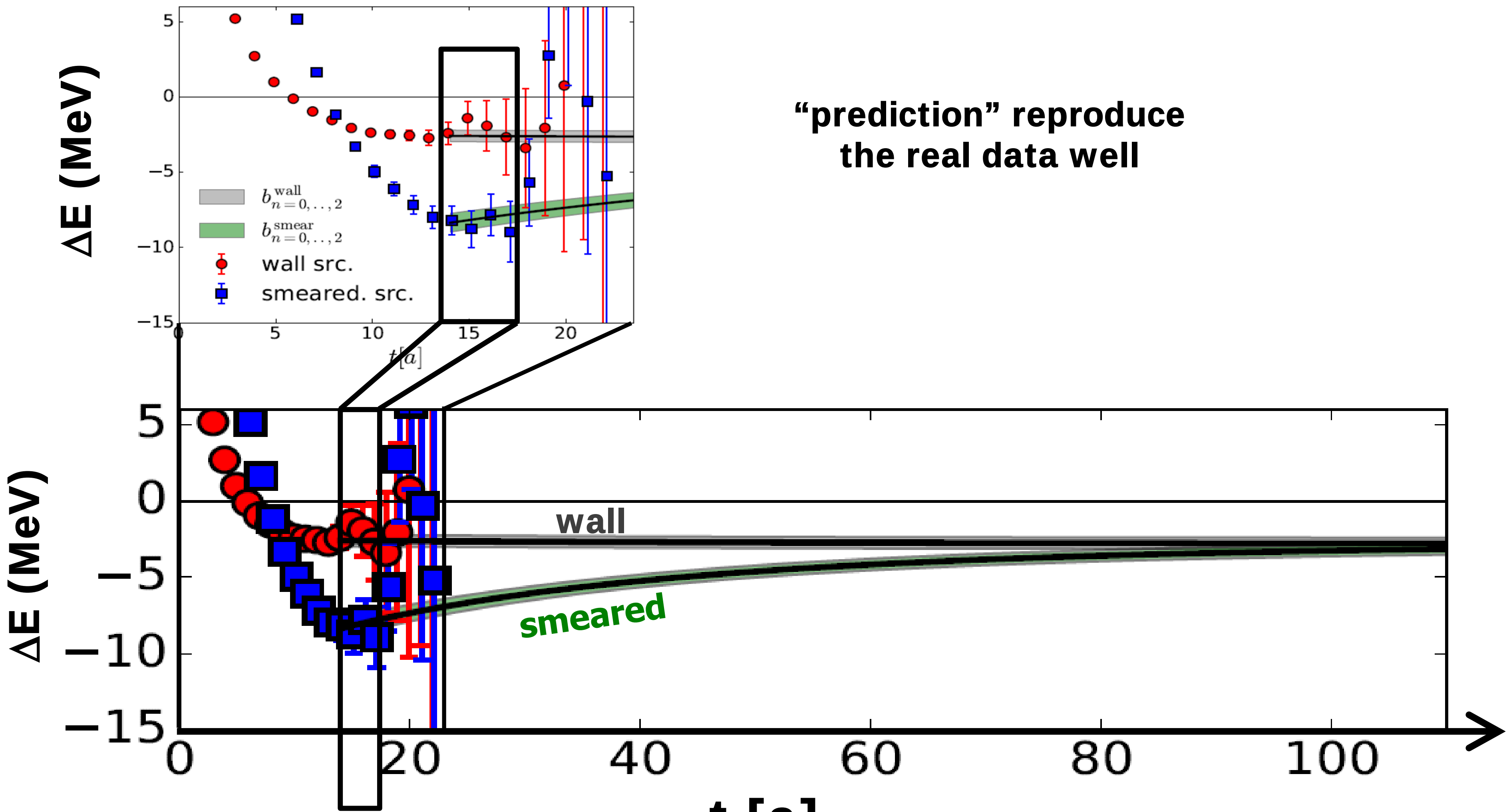
**"pseudo-plateaux" at t ~ 1fm**

**"real plateau" at t ~ 10fm**

($E_1$-$E_0$=50MeV)

# Understand the origin of "pseudo-plateaux"

**We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"**




**"prediction" reproduce the real data well**

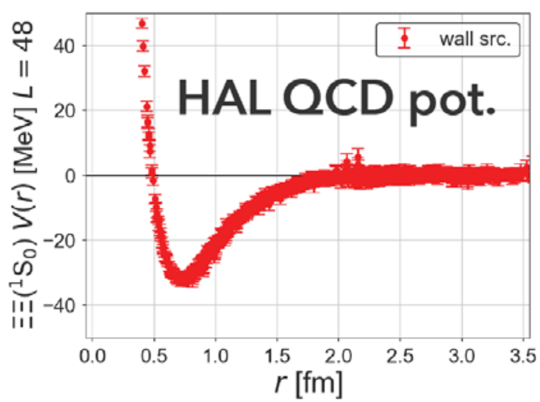
**"pseudo-plateaux" at t ~ 1fm**

**"real plateau" at t ~ 10fm**

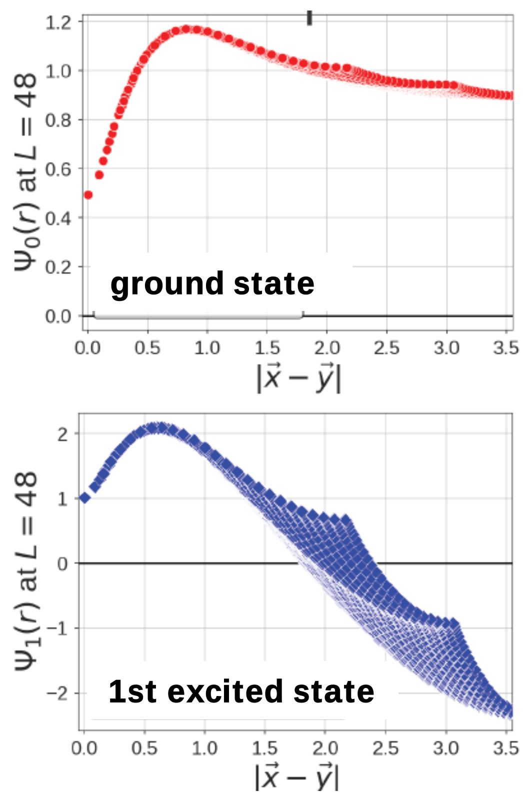
($E_1$-$E_0$=50MeV)

# Operator optimized for 2-body system by HAL

- HAL method ➔ HAL pot ➔ 2-body wave func. @ finite V
- 2-body wave func. ➔ optimized operator
  - Applicable for sink and/or src op : Here we apply for sink op
- While utilizing info by HAL, formulation is Luscher's method

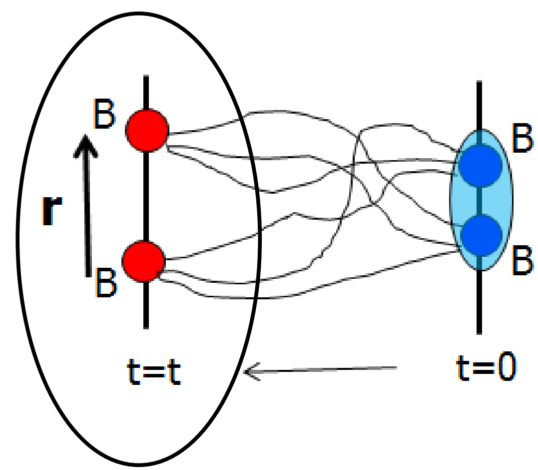



**wave func. ψ(r)**




**HAL-optimized sink op**

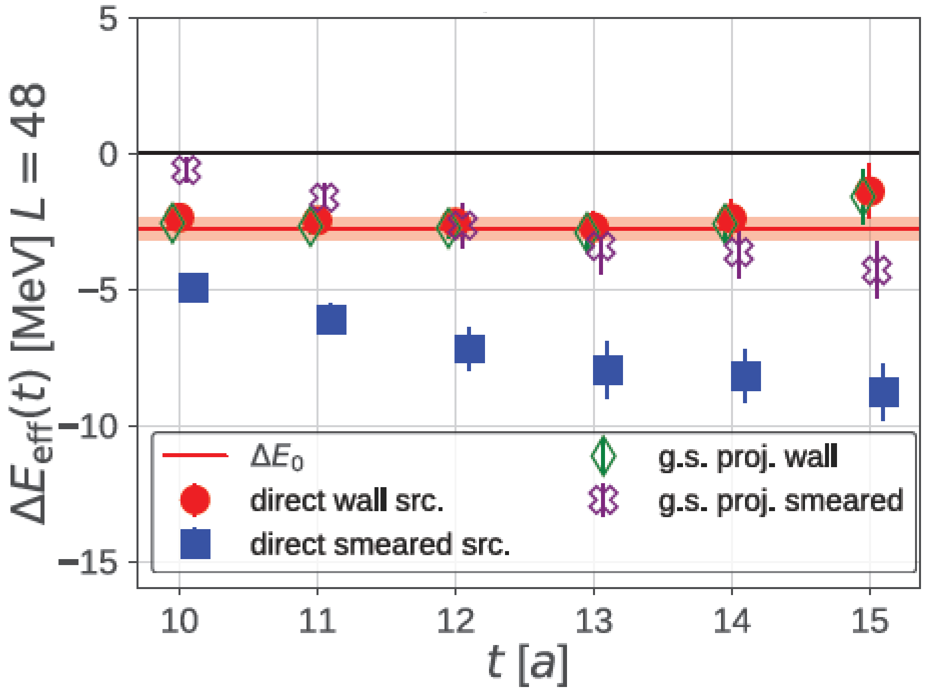
$$\mathcal{J}^{2B}_{\rm sink} = \sum_{\vec{r}} \psi^\dagger(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$



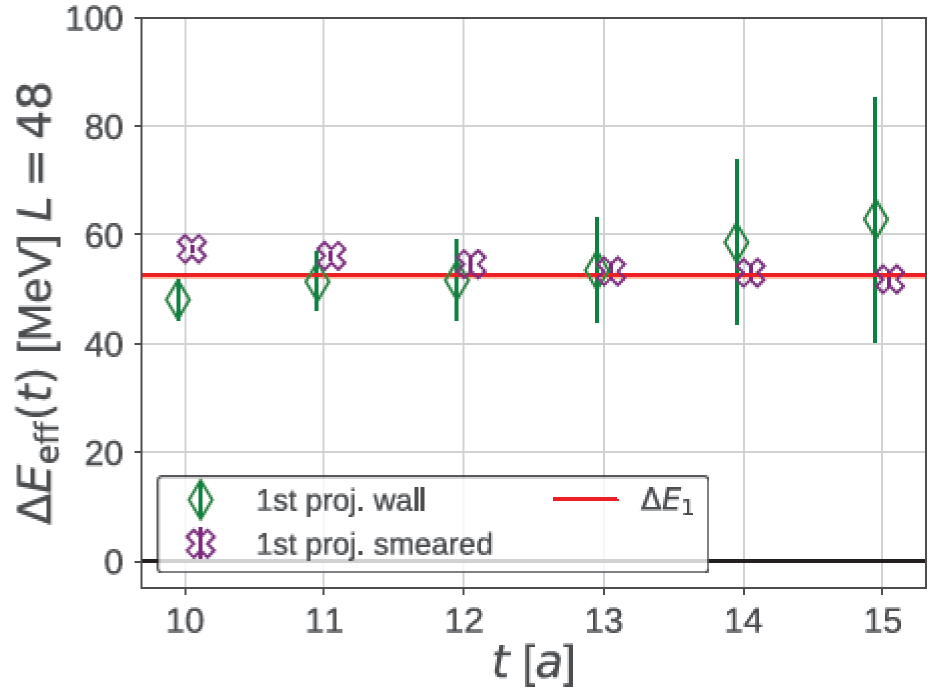
# Effective energy shift ΔE from “HAL-optimized op”

HAL-optimized sink op ➔ projected to each state ➔ “True” plateaux

**Ground State**




**1st excited state**




**HAL QCD pot = Luscher’s formula w / proper projection**

**≠ Direct method w/ naïve plateau fitting**

**Luscher’s formula requires state-projection (a la HAL) or variational calc**

# Reliability test of LQCD methods

## NN @ heavy quark masses

HAL method (HAL) : unbound

Direct method (NPL/CalLat/PACS-CS(Yamazaki et al.)): bound

***Inconsistent!***

T. Iritani et al. (HAL QCD Coll.) JHEP10(2016)101, PRD96(2017)034521, PRD99(2019)014514, JHE03(2019)007

Semi-improved calc w/ Luscher's formula (Mainz2019) : unbound

Variational calc w/ Luscher's formula (CalLat2020) : unbound

Variational calc w/ Luscher's formula (NPL2021) : (unbound)

***Issue was essentially settled***

- **Outline**
  - Introduction
  - Theoretical framework
    - Luscher's formula
    - HAL QCD method
    - Reliability test of LQCD methods
  - (Results at heavy quark masses)
  - Results near physical quark masses
  - Summary / Prospects

- Baryon Forces from LQCD — Ishii-Aoki-Hatsuda (2007)
- Exponentially better S/N — Ishii et al. (2012)
- Coupled channel systems — Aoki et al. (2011,13)

**[Theory] = HAL QCD method**

# Baryon Interactions near the Physical Point

## [Hardware]

**= K-computer [10PFlops]**

**+ FX100 [1PFlops] @ RIKEN**
**+ HA-PACS [1PFlops] @ Tsukuba**

- HPCI Field 5 / Post K Priority Issue 9

## [Software]

**= Unified Contraction Algorithm**

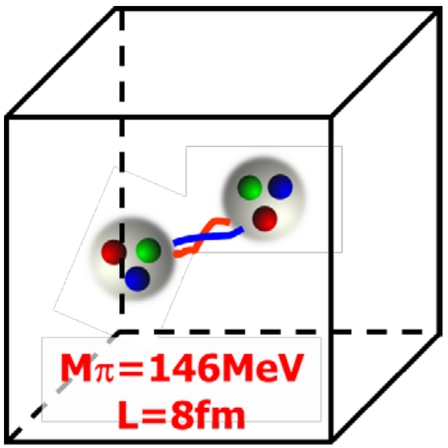
- Exponential speedup — Doi-Endres (2013)

$$^3\mathrm{H}/^3\mathrm{He} \quad : \quad \times 192$$
$$^4\mathrm{He} \quad : \quad \times 20736$$
$$^8\mathrm{Be} \quad : \quad \times 10^{11}$$

# Lattice QCD Setup

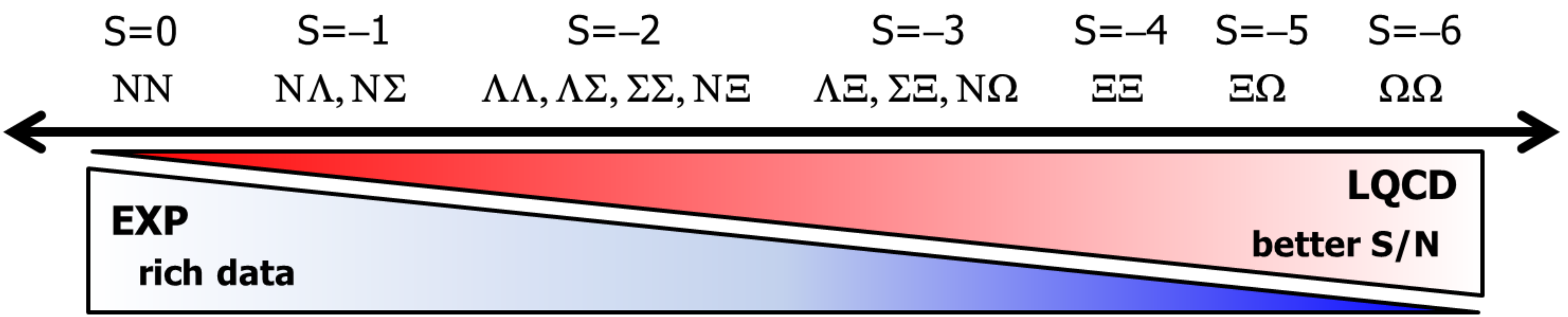
- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - $V=(8.1\text{fm})^4$, a=0.085fm (1/a = 2.3 GeV)
  - **m(pi) ~= 146 MeV, m(K) ~= 525 MeV**
  - #traj ~= 2000 generated

PACS Coll., PoS LAT2015, 075

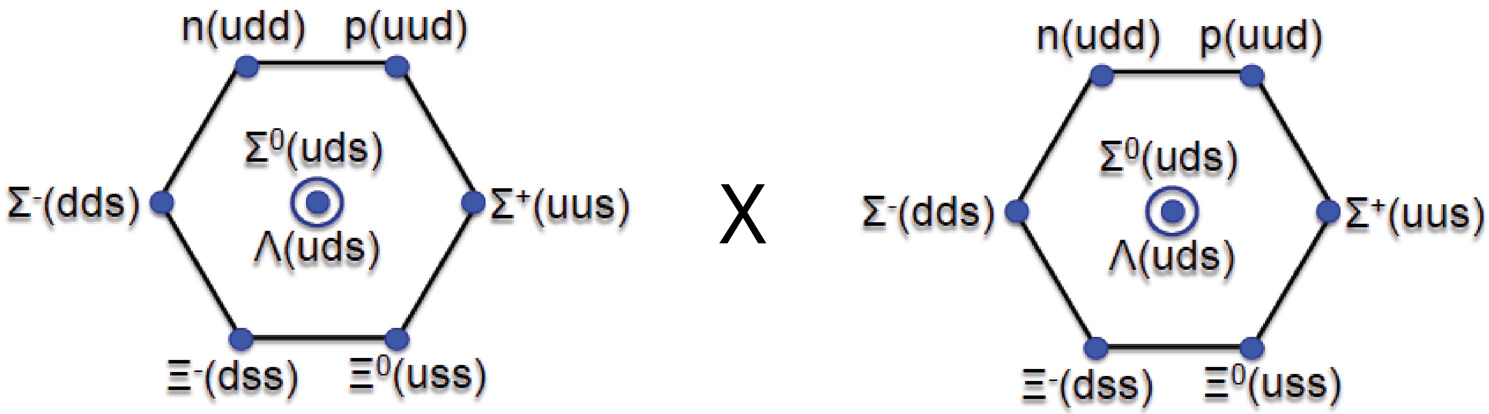



- **Measurement**
  - All of NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)

**Predictions for Hyperon forces**



# Birds-eye View
# classification w/ flavor SU(3)-irrep base




$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{anti-symmetric}}$$

**NN channel** (→ 27, 10*)

**c.f. Exact SU(3) limit LQCD calc @ heavy masses**

T.Inoue et al. (HAL.), PTP124(2010)591

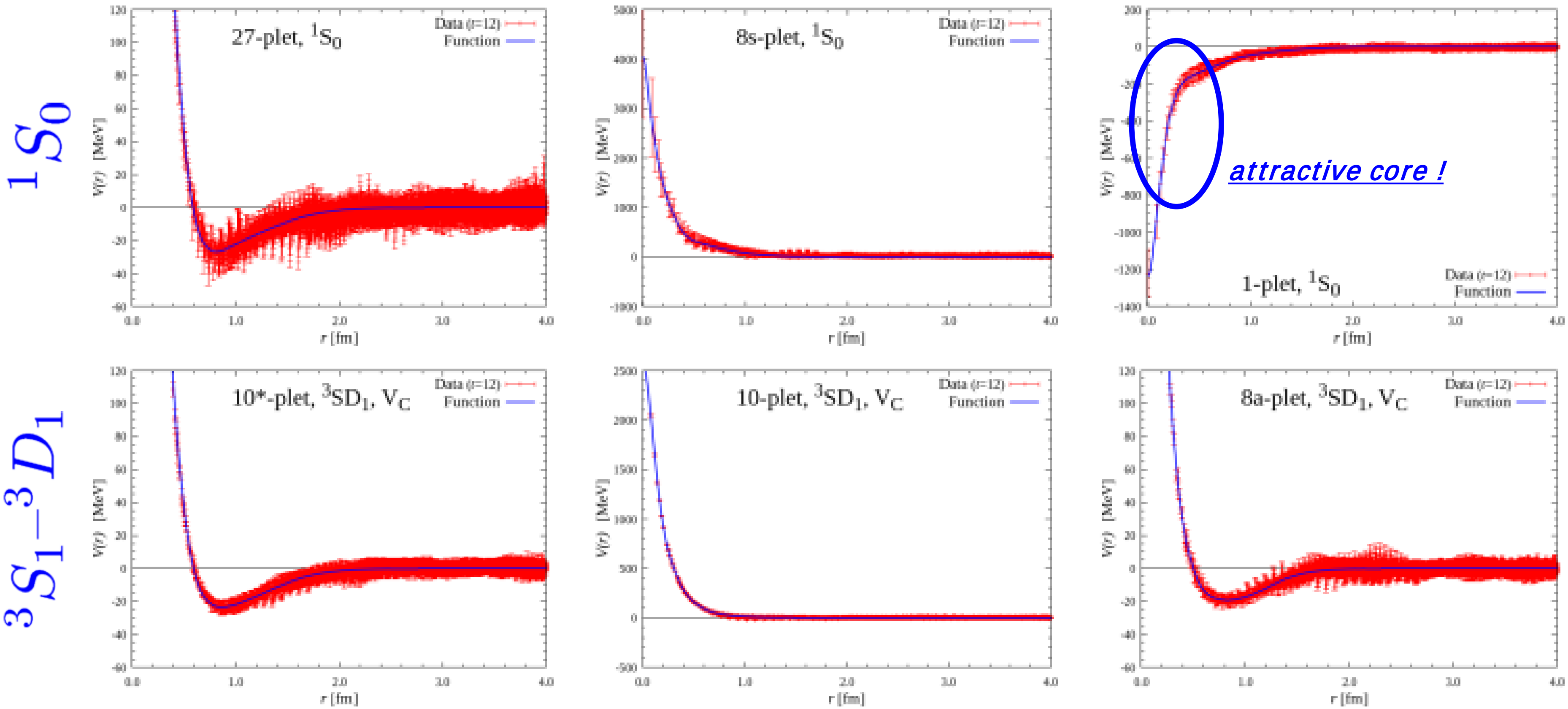
T.Inoue et al. (HAL.), NPA881(2012)28

# Diagonal Potentials in SU(3)f-irrep base in S=-2

(only) S= -2 can access all irreps

off-diag pot relatively small

T.Inoue (HAL), AIP Conf. Proc. 2130 (2019) 020002




**27,10*:**
**NN-type**

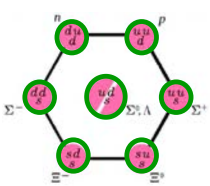
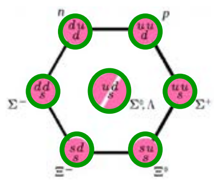
**8s,10:**
**strong repulsive core**

**1s: deep attractive pocket**
**8a: weak repulsive core**

**Quark Pauli repulsion + OGE for short range**

M.Oka et al., NPA464(1987)700

# Candidates of di-baryons

 X 

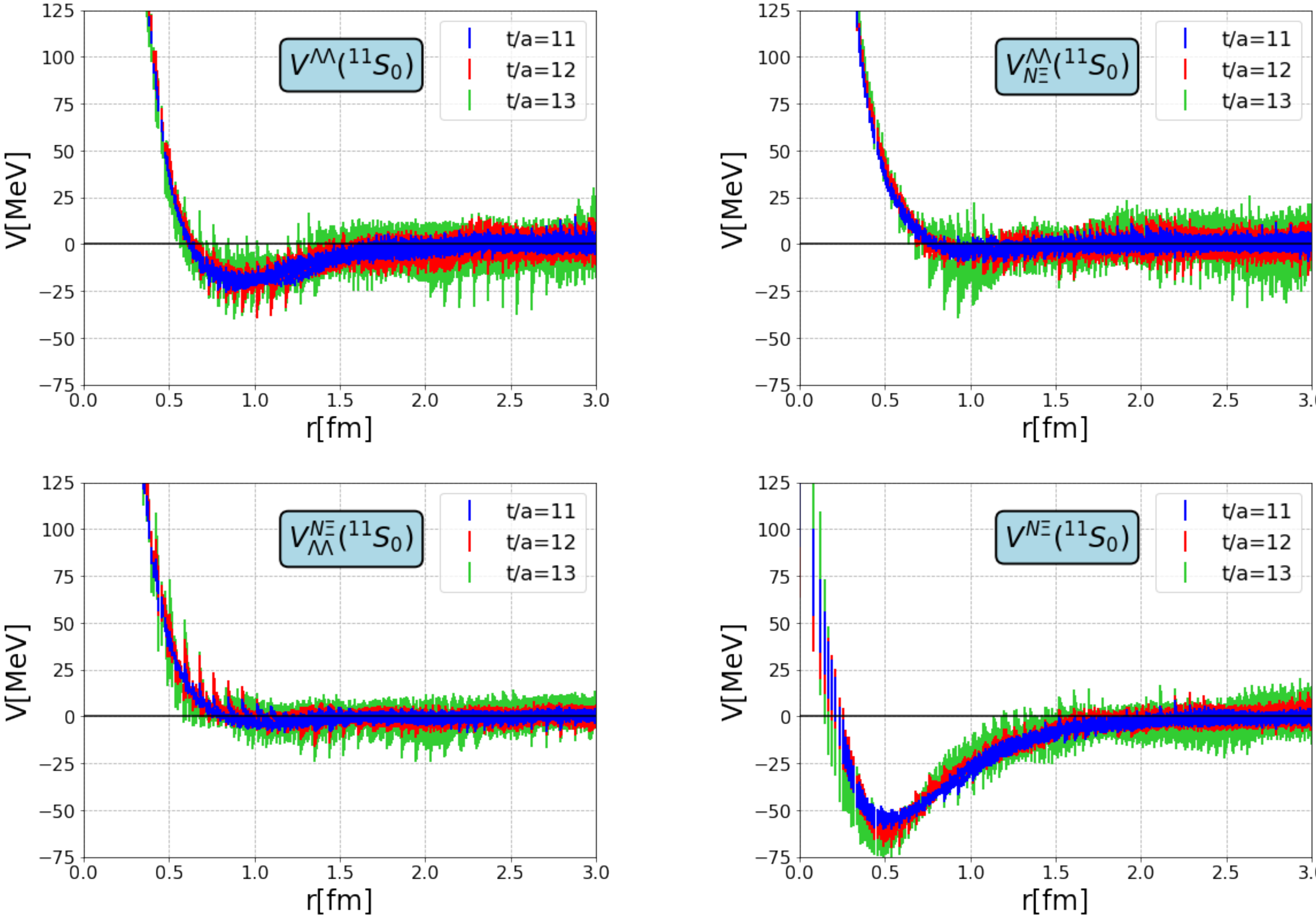
$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

- 27: dineutron, ΞΞ etc. (J=0)
- 1: H-dibaryon (J=0)
- 10*: Deuteron (J=1)

There may also exist S= -2 hypernuclei relevant to these strong attractions

➔Detailed study w/ SU(3) breaking effects (particle-base)

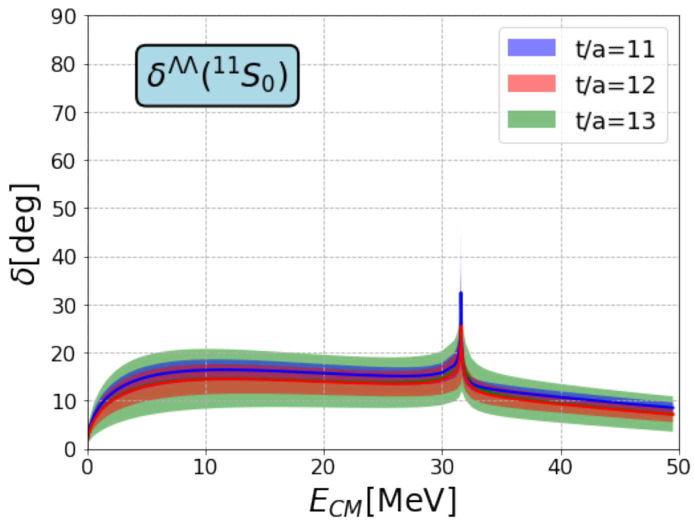
# ΛΛ, NΞ (effective) 2x2 coupled channel analysis

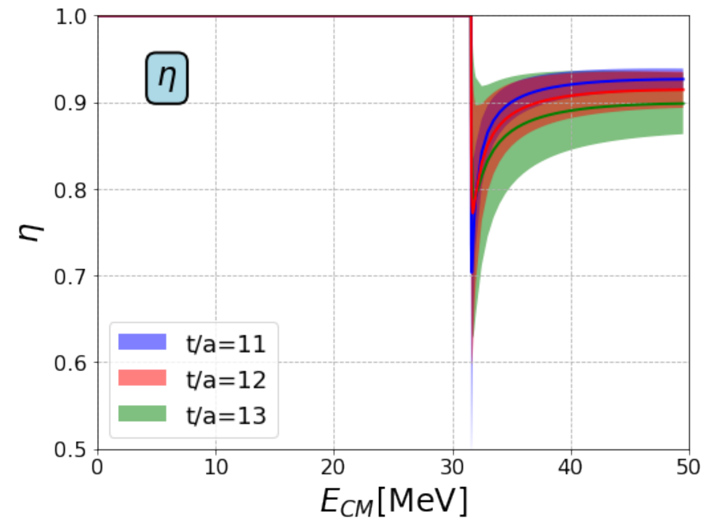



NΞ ($^{11}S_0$) channel is attractive
NΞ−ΛΛ coupling is small

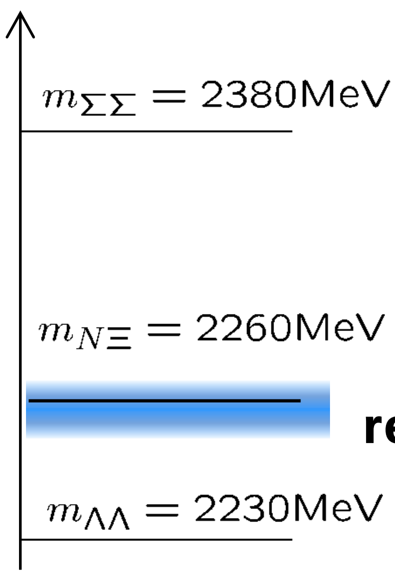
K. Sasaki et al., (HAL Coll.), NPA998(2020)12137

# ΛΛ, NΞ 2x2 coupled channel analysis



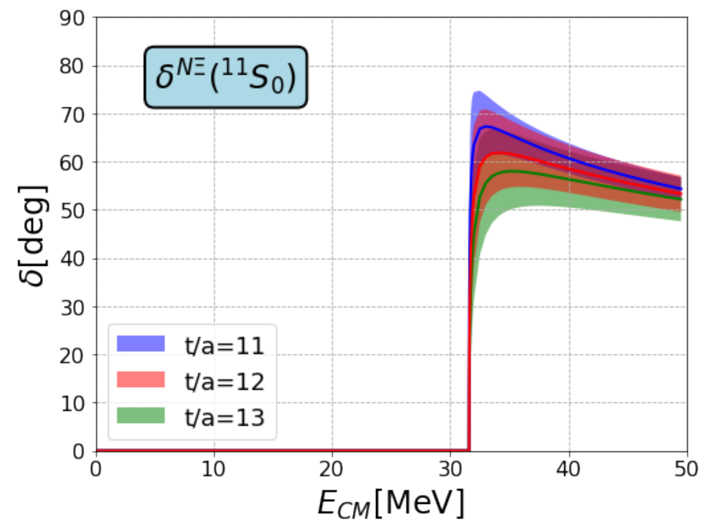




$a_0$ = -0.81(23)(+0.00/-0.13) [fm]
$r_{eff}$ = 5.47(78)(+0.09/-0.55) [fm]




**J-PARC E42 (Ahn)**

**NΞ ~ unitary limit**
**remnant of "H-dibaryon"**

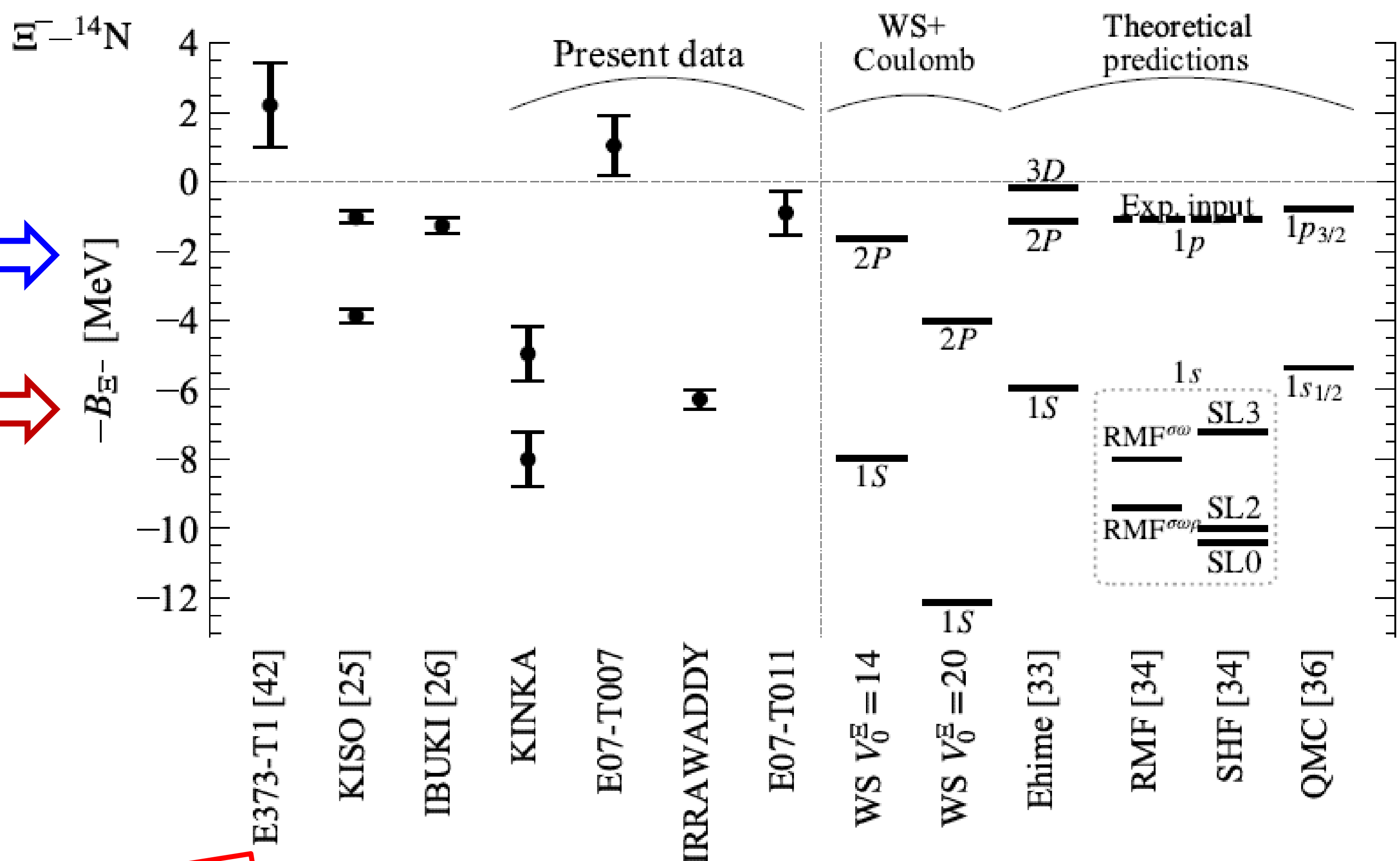



(N.B. NΞ = 1rep 50%, 27rep 30% in SU(3))

# Recent experimental progress on Ξ–Hypernuclei

**Excited state?**

**Ground state?**




**J-PARC E07, E70, more in HEF-EX**

M. Yoshimoto et al., PTEP2021, 073D02

Attractive NΞ–int well established
Small NΞ–ΛΛ coupling indicated

# Baryon-Baryon correlation in HIC

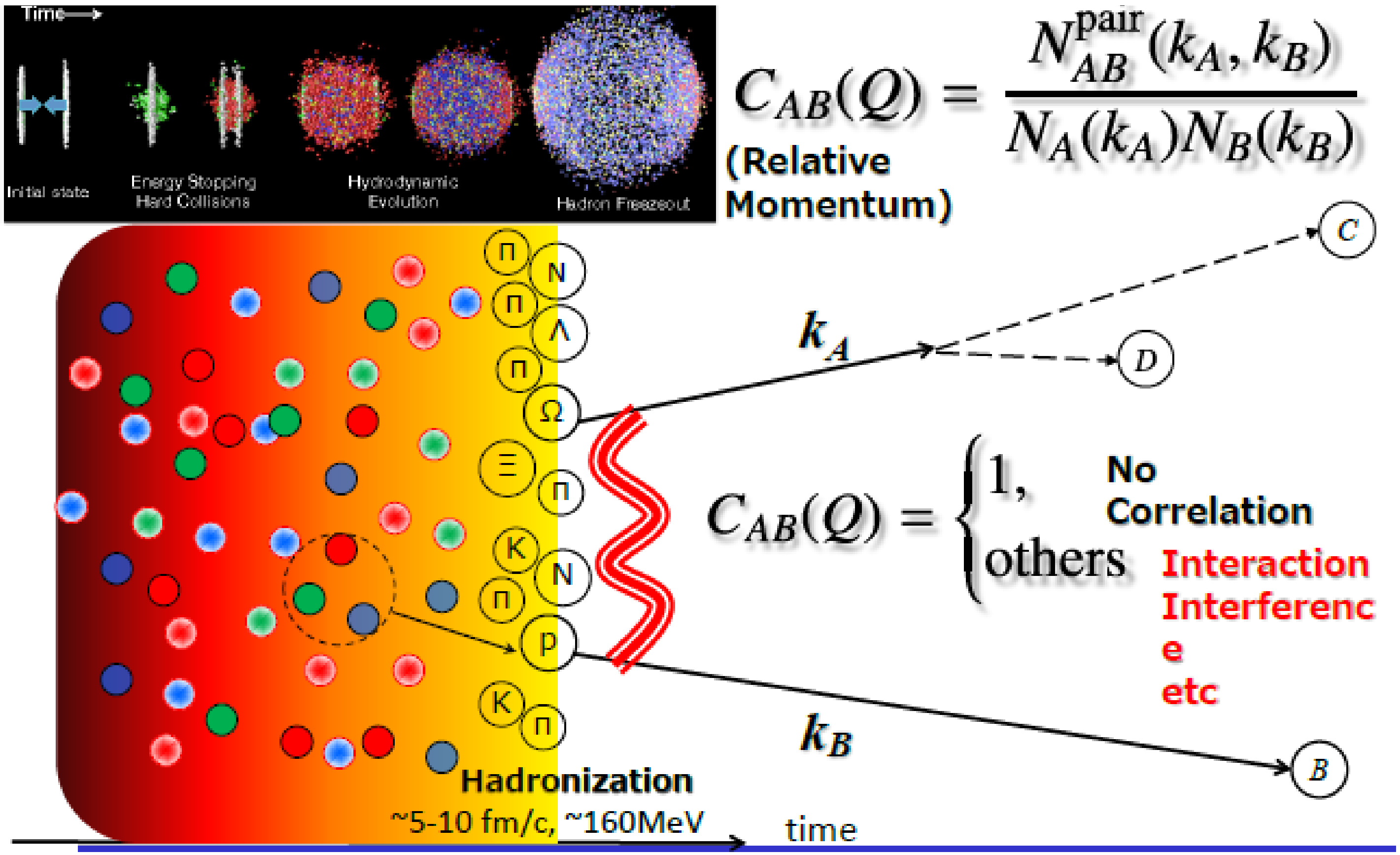


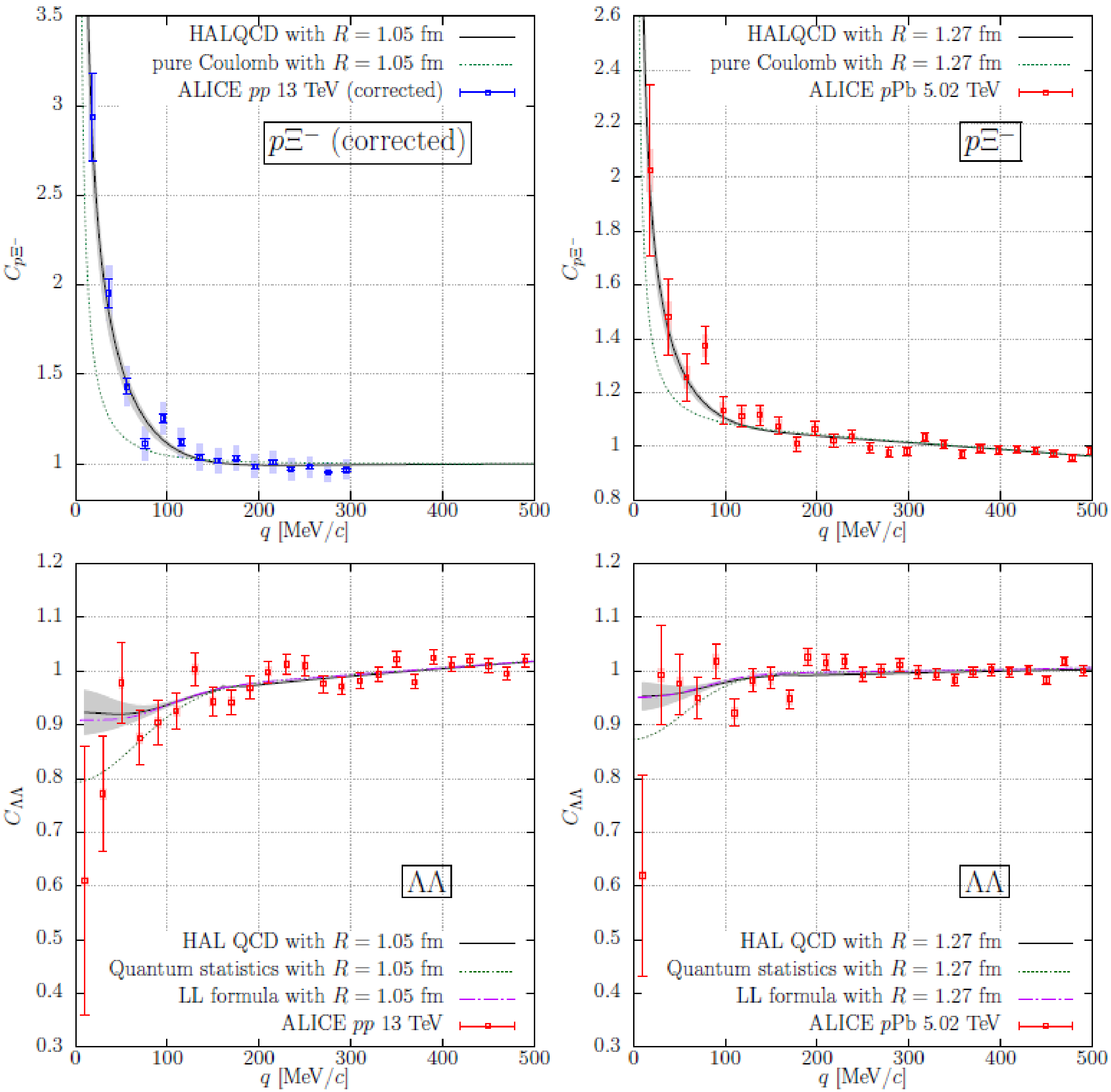

Fig. from K. Morita

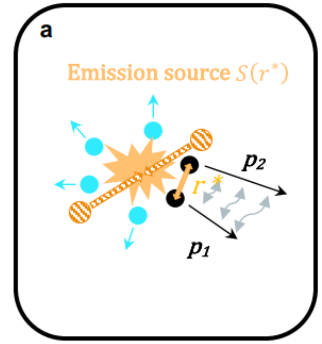
**BB-correlation**
**←→ BB- (final state) interaction**

Ratio of correlation between small/large source size is useful to mask Coulomb effect

K. Morita et al., PRC94(2016)031901
K. Morita et al., PRC102(2020)015201

# Femtoscopy from nucleus collisions




(Fig from ALICE)

⇦ $\mathbf{p\Xi^-}$

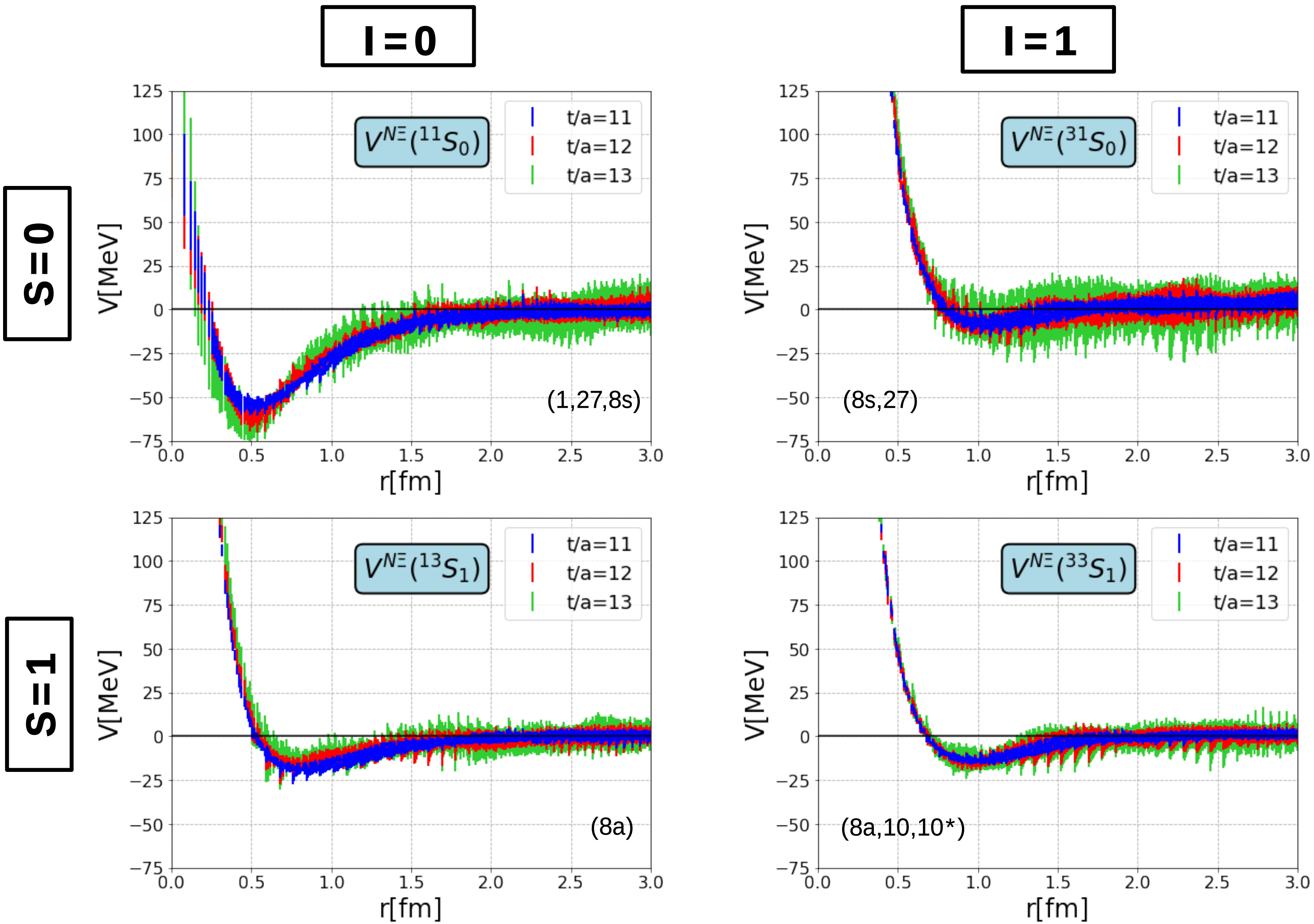
LQCD prediction confirmed by experiment!

⇦ $\Lambda\Lambda$

Y. Kamiya et al., PRC105(2022)014915

See also ALICE Coll., PLB797(2019)134822, PRL123(2019)112002, Nature 588(2020)232

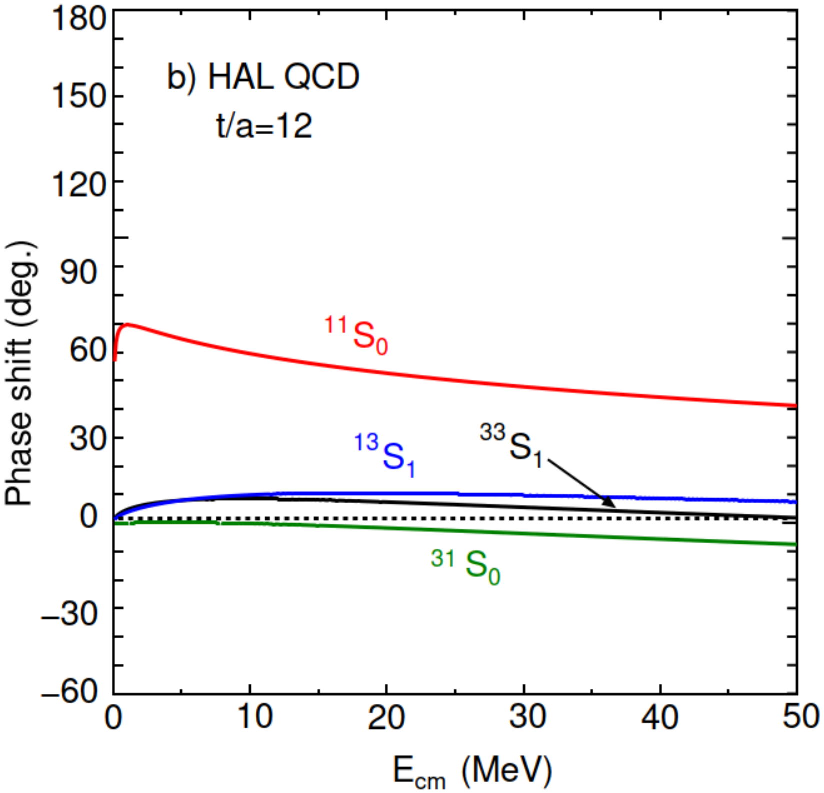
# Spin-Isospin dependence of NΞ potentials

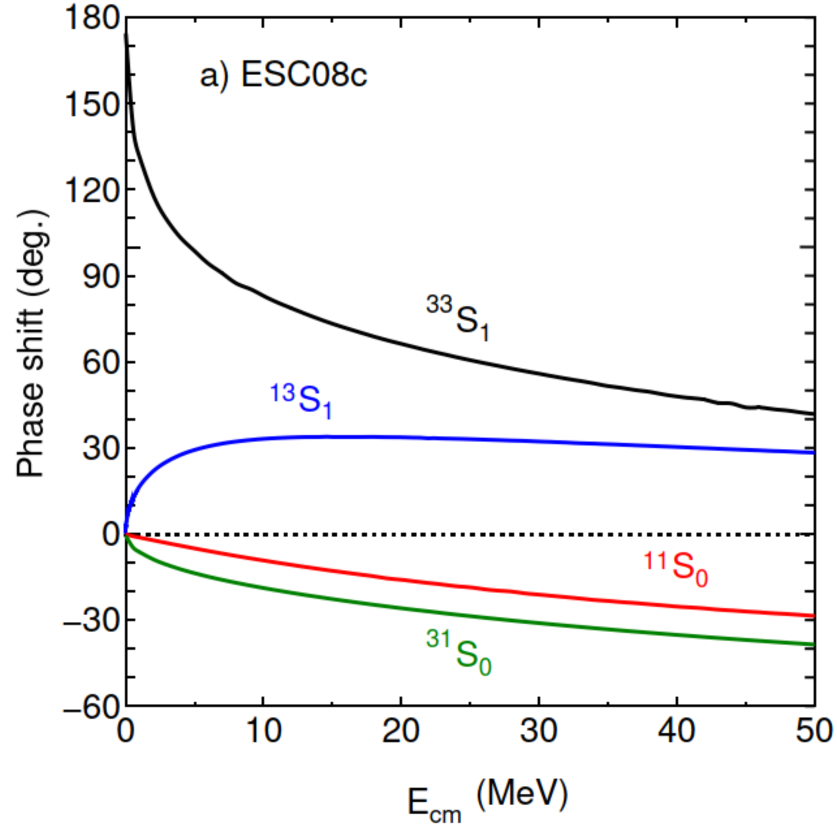



K. Sasaki et al., (HAL Coll.), NPA998(2020)12137

# Prediction of NΞ scattering phase shifts

E. Hiyama et al., PRL124(2020)092501



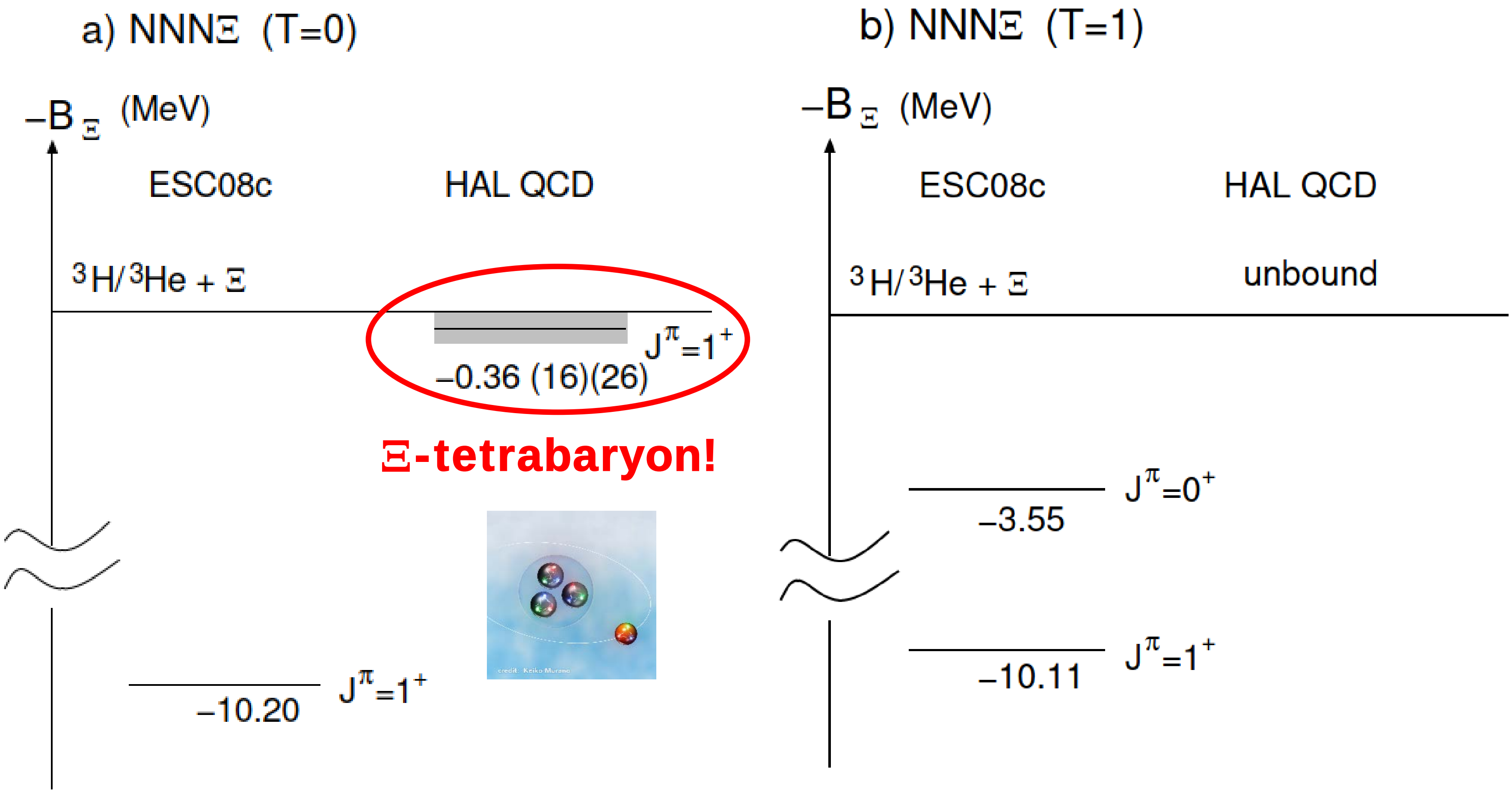

**[HAL]**

I=0, S=0: **attractive**
I=0, S=1: weakly attractive
I=1, S=0: weakly repulsive
I=1, S=1: **weakly attractive**

**[Nijmegen (ESC08c)]**

I=0, S=0: **repulsive**
I=0, S=1: attractive
I=1, S=0: repulsive
I=1, S=1: **strongly attractive**

# S= -2 Light Hypernuclei from LQCD

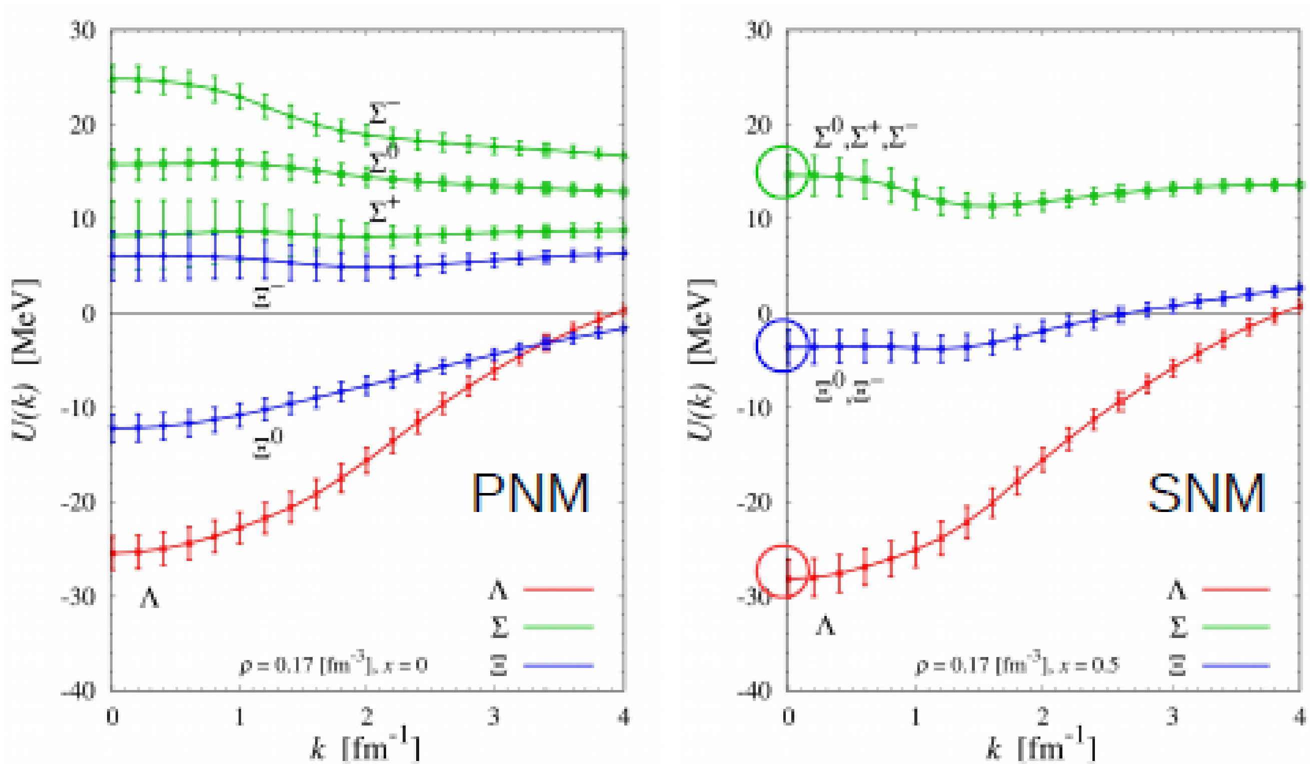



Guiding experiments

HIC@LHC, J-PARC, …

E. Hiyama et al., PRL124(2020)092501

# "Super-super heavy nuclei": Dense matter from LQCD
# Hyperon single-particle potential




$@\rho = 0.17\,[\mathrm{fm}^{-3}]$

Preliminary

T. Inoue (HAL Coll.)
PoS INPC2016, 277

- obtained by using $YN, YY$ S-wave forces form QCD.
- Results are compatible with experimental suggestion.

$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10?, \quad U_\Sigma^{\mathrm{Exp}}(0) \geq +20? \quad [\mathrm{MeV}]$$

attraction　　attraction small　　repulsion

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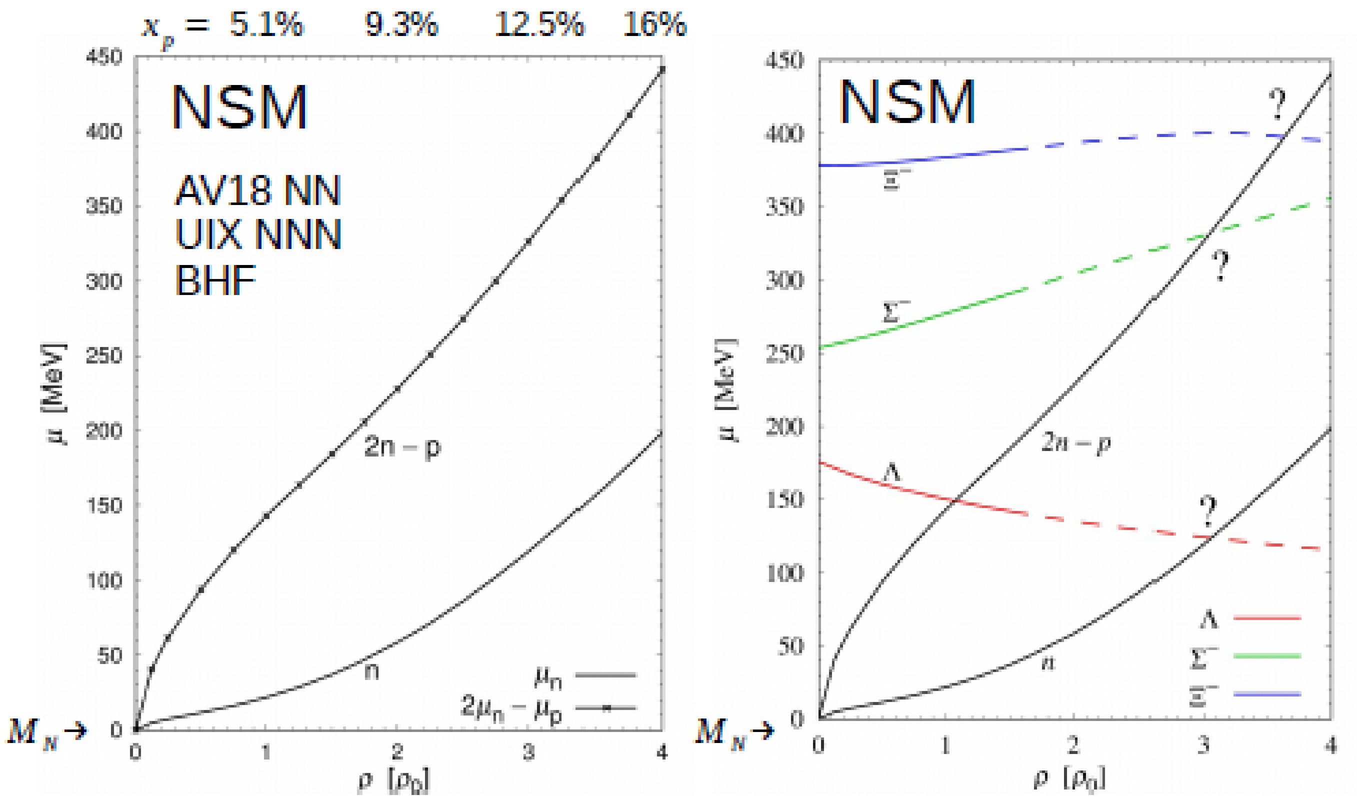
(YN/YY pot from SU(3)f-irrep diag used)

**[ T. Inoue ]**

# Hyperon onset in NSM (just for fun)




S-wave *YN* only

Preliminary

- Result indicate $\Lambda$, $\Sigma^-$, $\Xi^-$ appear around $\rho$ = 3.0 - 4.0 $\rho_0$
- However,
  - $YN^{L=1,2...}$ and $YNN$ force could be imp
  - We may need to compare with more s BHF.

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**[Challenges]**
**Precision for |S|<=1**
**3-baryon forces**
**P-wave/LS forces**

**[ T. Inoue ]**

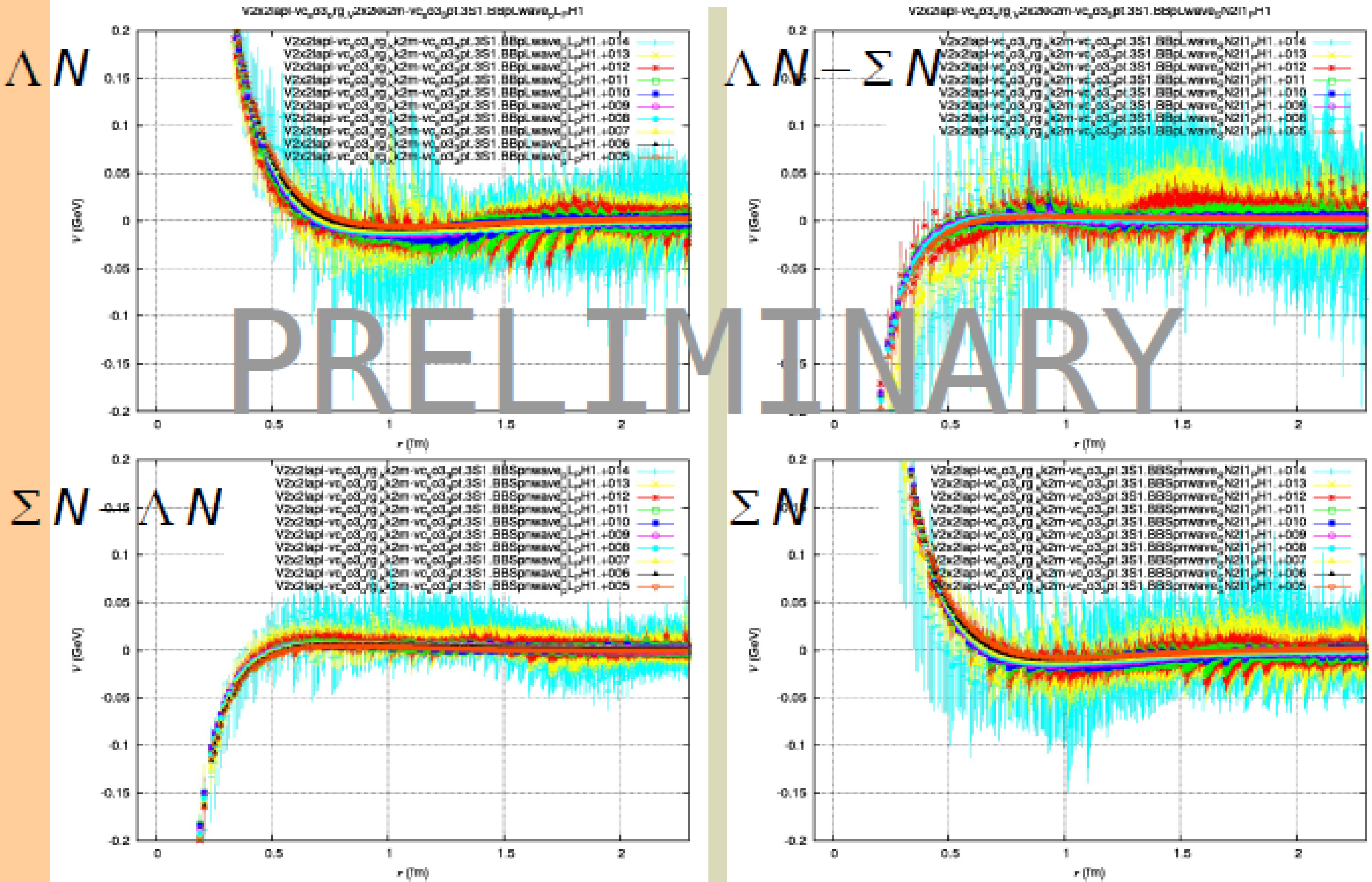
# ΛN–ΣN Vc potential in $^3S_1$–$^3D_1$

H. Nemura
(Lat2021)

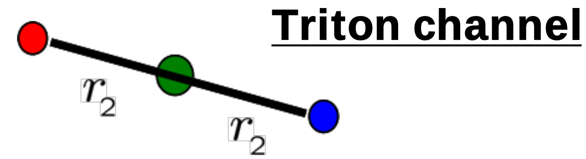
Very preliminary result of LN potential at the physical point

$V_C(\,^3S_1 - \,^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$

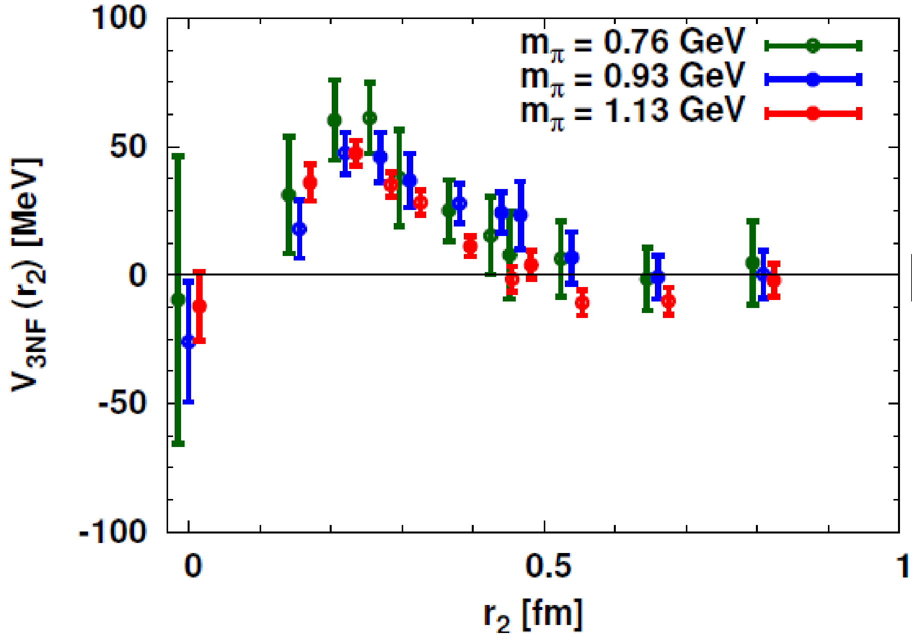


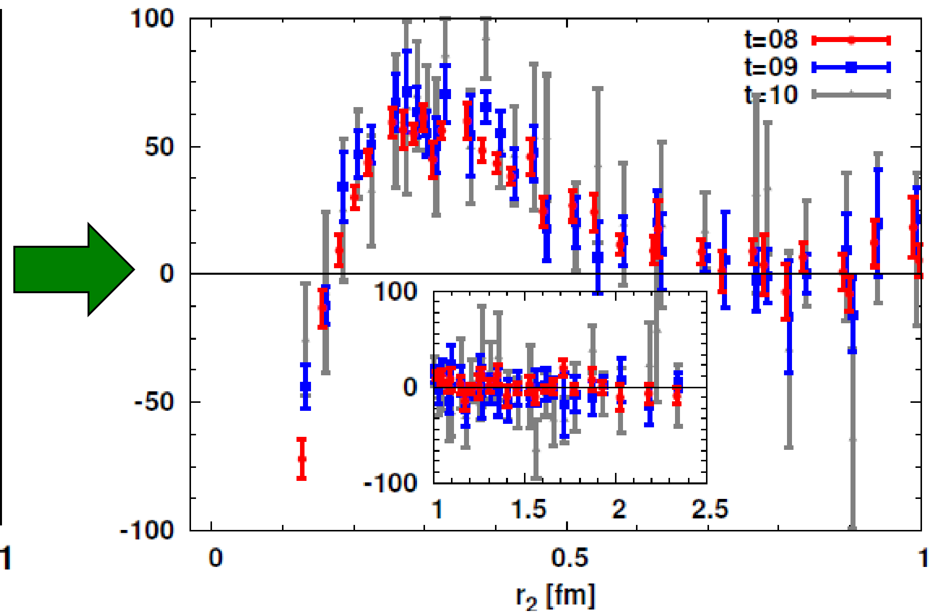
# 3N-forces (3NF)




## Nf=2, mπ=0.76-1.1 GeV

## Nf=2+1, mπ=0.51 GeV



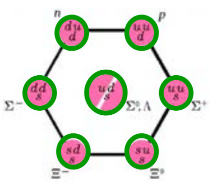
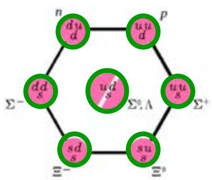




Magnitude of 3NF is similar for all masses
Range of 3NF tend to be enlarged for m(pi)=0.5GeV

Next challenge: **Calc of P-wave 2BF** : better subtraction of 2BF in 3-body systems
YNN (w/o or w/ P-wave 2BF) : gauge conf generation on Fugaku

T.D. et al. (HAL Coll.) PTP127(2012)723
Aoki-Doi Front.Phys.8 (2020) 307

# Candidates of di-baryons

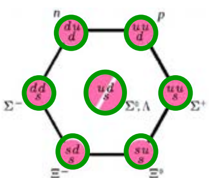
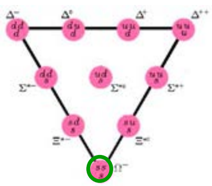
 X 

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8s$$

dineutron, ΞΞ etc. (J=0)

H-dibaryon (J=0)

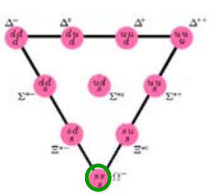
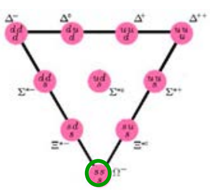
Deuteron (J=1)

 X 

$$8 \times 10 = 35 + 8 + 10 + 27$$

NΩ (J=2)

Goldman et al. ('87)
Oka ('88)

 X 

$$10 \times 10 = 28 + 27 + 10^* + 35$$

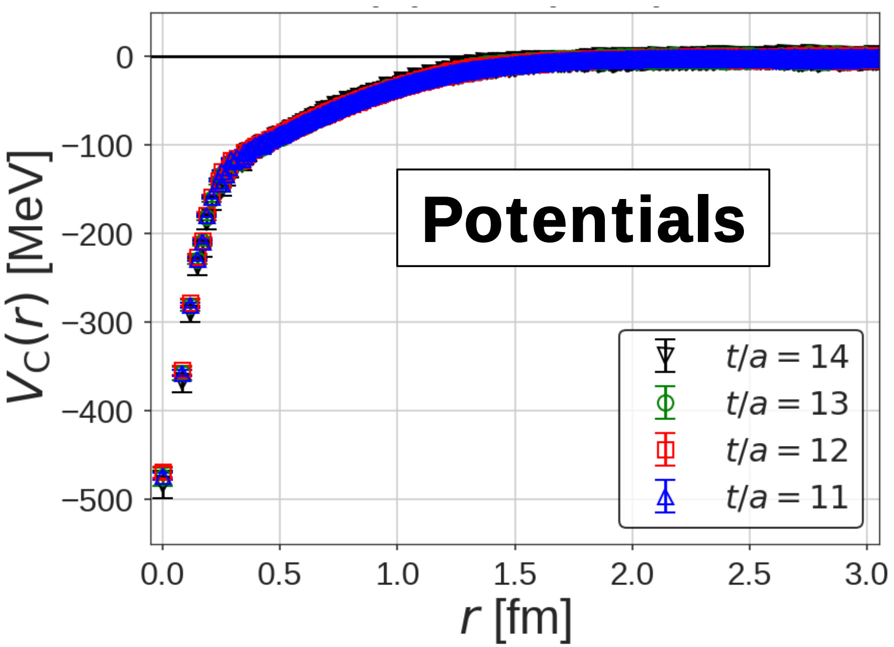
ΩΩ (J=0)

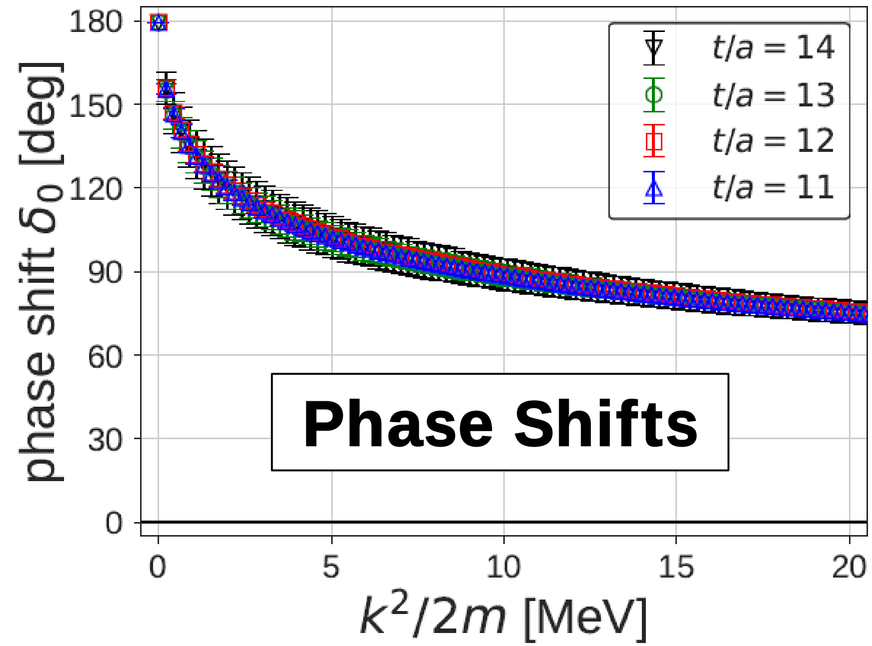
ΔΔ (J=3)

Zhang et al. ('97)

Dyson-Xuong ('64)
Kamae-Fujita ('77)
Oka-Yazaki ('80)

# NΩ system ($^5S_2$)



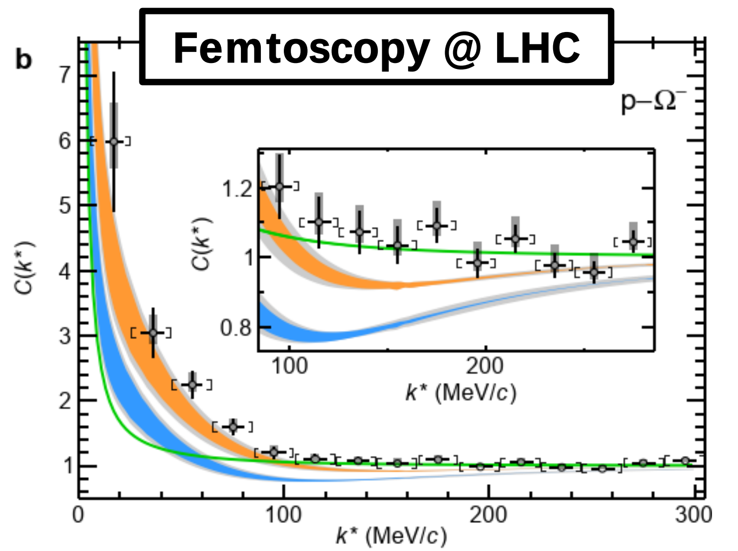




**(Quasi) Bound state**

**[~ Unitary limit ]**

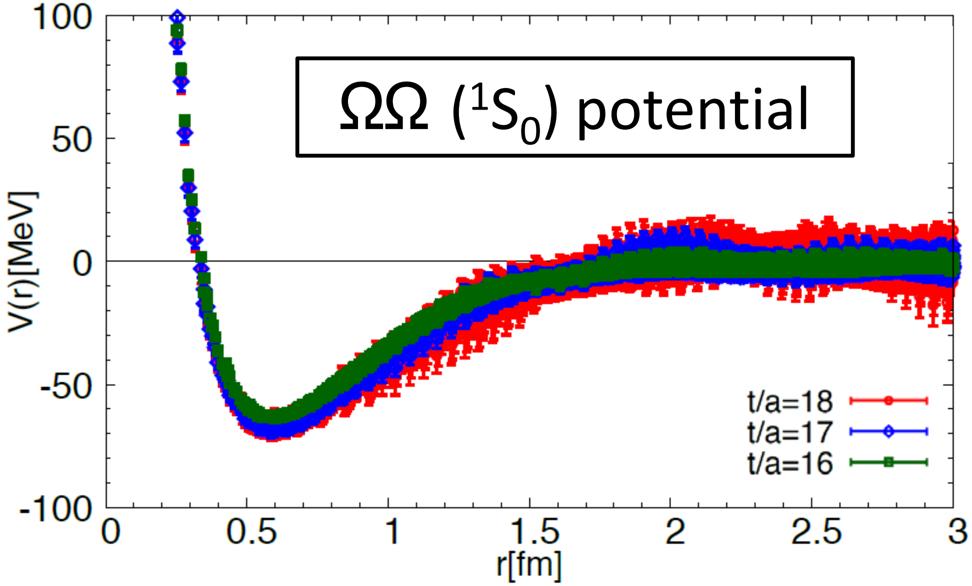
$$B_{N\Omega} = 1.54(0.30)(^{+0.04}_{-0.10}) \text{ MeV}$$
$$B_{p\Omega-} = 2.46(0.34)(^{+0.04}_{-0.11}) \text{ MeV}$$

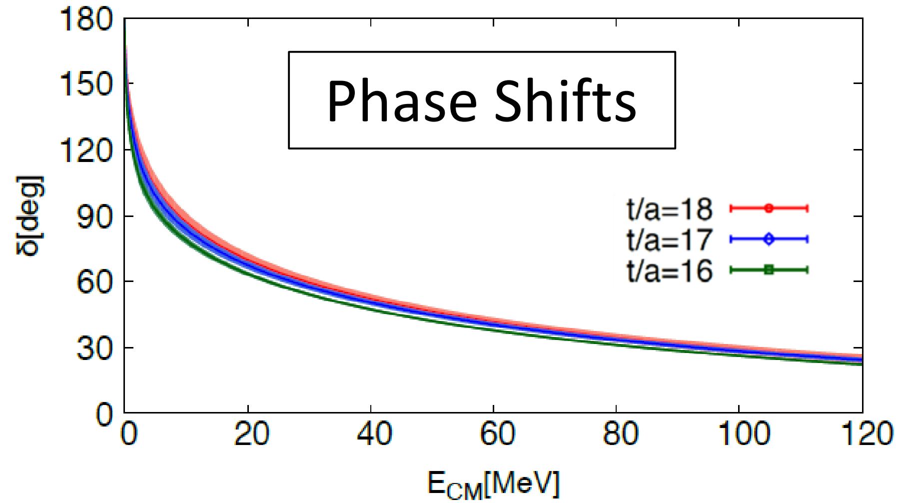
T. Iritani et al. (HAL Coll.), PLB792(2019)284




ALICE Coll., Nature 588 (2020) 232

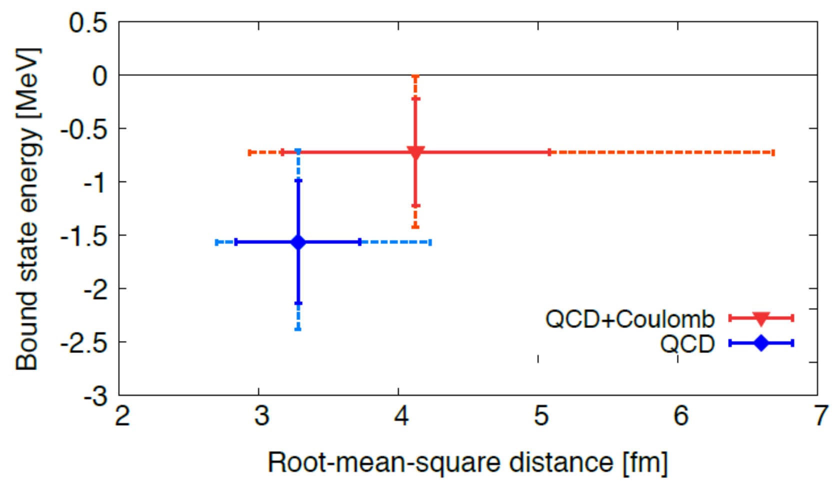
# “Most Strange” Dibaryon : ΩΩ




**“di-Omega”**

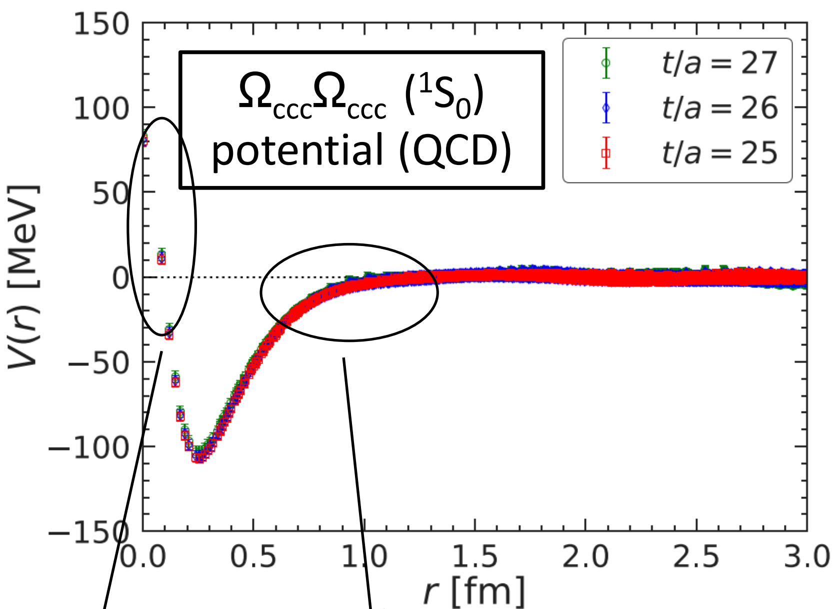
**[~ Unitary limit]**

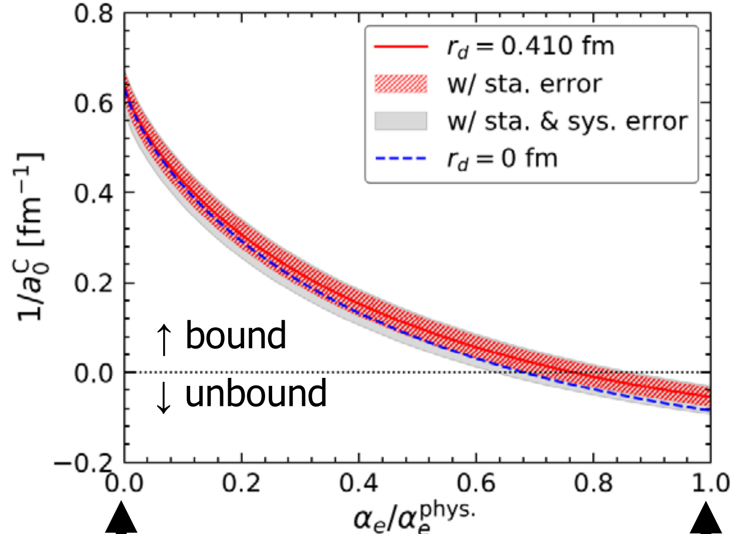



could be searched in LHC RUN3

S. Gongyo et al. (HAL Coll.), PRL120(2018)212001

# “Most Charming” Dibaryon : $\Omega_{ccc}\Omega_{ccc}$



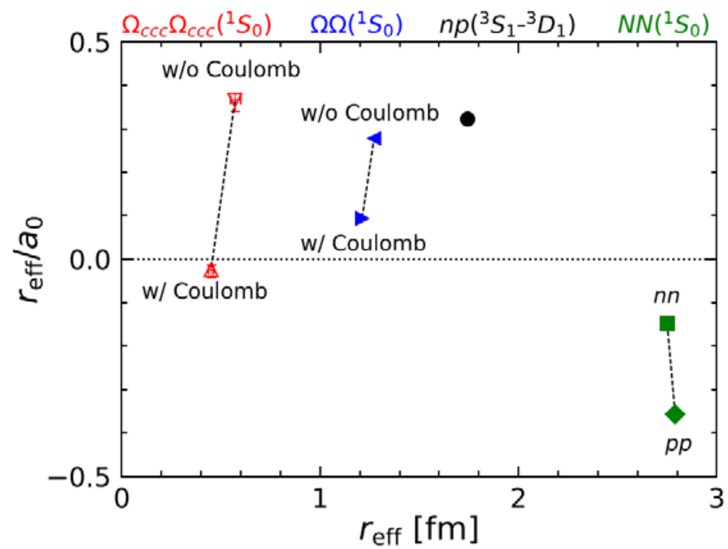




QCD only

QCD+Coulomb

Attraction short-ranged

Color-magnetic int suppressed
by $(m_s/m_c)^2 \sim 0.1$




**Dibaryon closest to the unitarity**

Y. Lyu, H. Tong et al., PRL127(2021)072003

# *Hadron Interactions from Lattice QCD*

## *Where are we going?*

***Challenges, new development and future prospects***
***w/ new supercomputer "Fugaku"***

# New calc on Fugaku: physical point simulation!

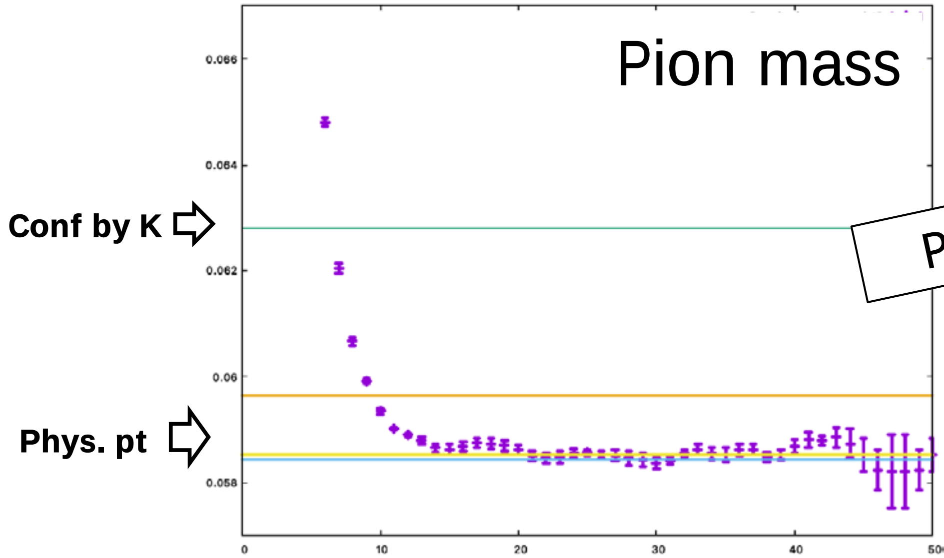
**Fugaku (富岳) : 440 PFlops**

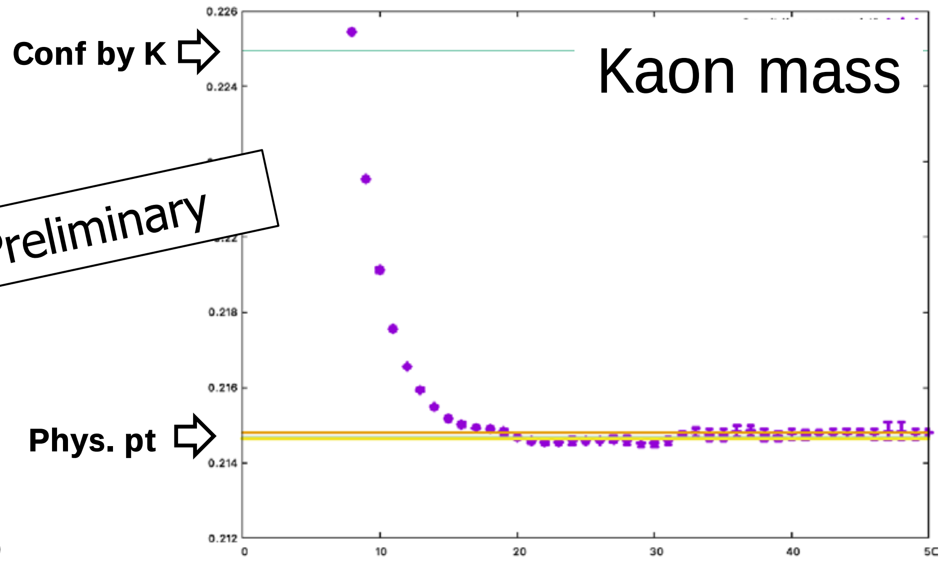
Successor of K-computer
Developed since 2014-,
Full operation since 2021-

Codesign of hardware/software
(LQCD was one of 9 targets)


**Fastest in the world! (2020-)**

Our Efficiency = ~17% (w/ naïve double prec count)




**[ E. Itou+ ]**

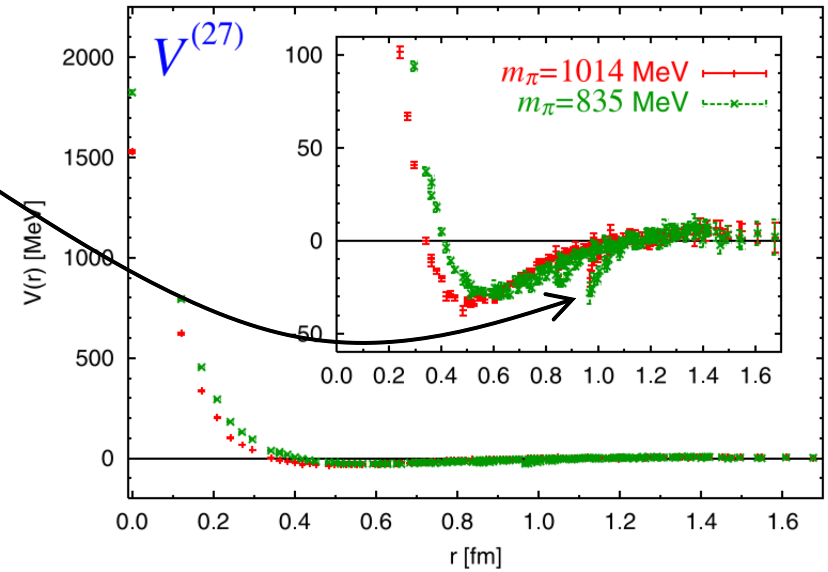
**We are on the physical point!**

# S/N improvement by partial wave decomposition

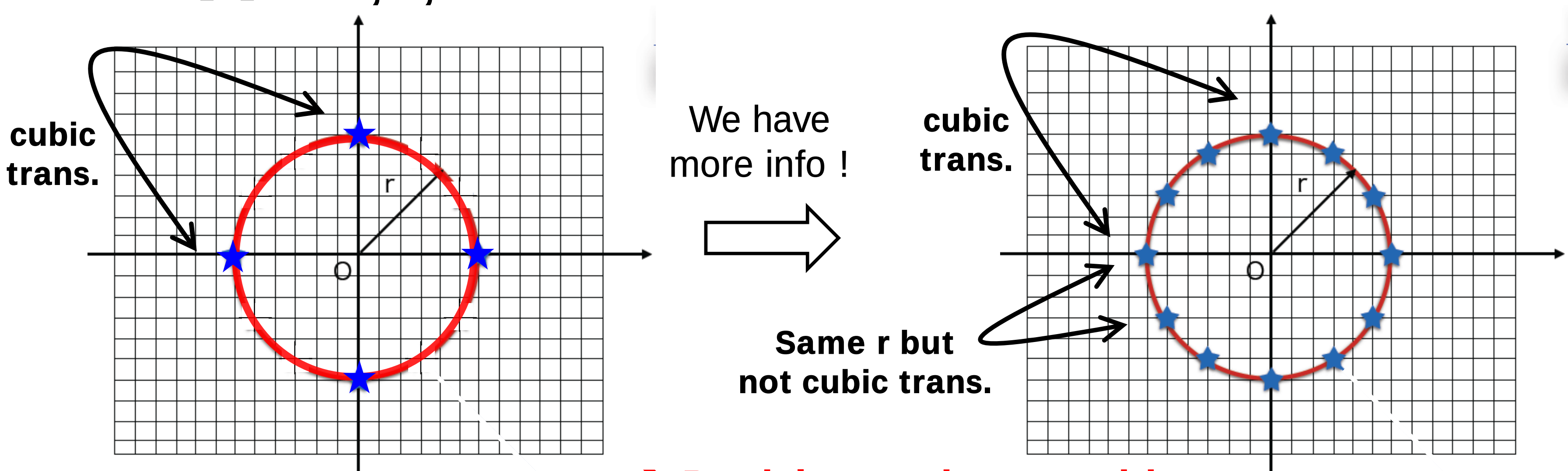
T. Miyamoto et al. (HAL Coll.), PRD101(2020)074514

- Cubic group irrep on lattice
  → sys err by mixing of partial waves
- Partial wave decomposition
  can cure the problem

C. W. Misner, Class. Quantum Grav. 21 (2004) S243-S247



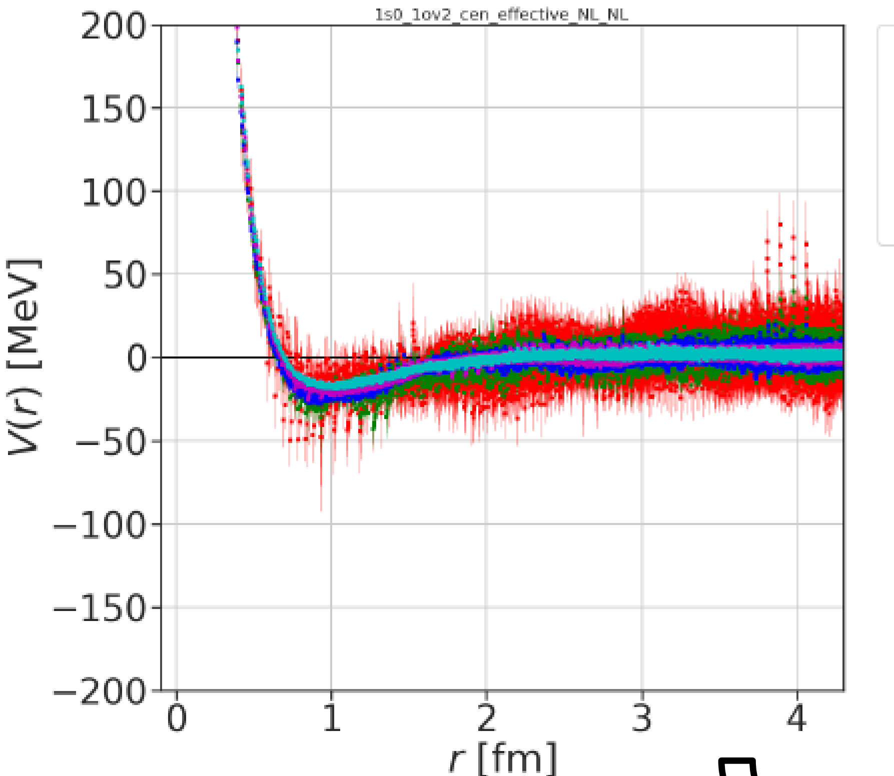

**$A_1^+$ -irrep in cubic group**
**↔ L=0, 4, ...**



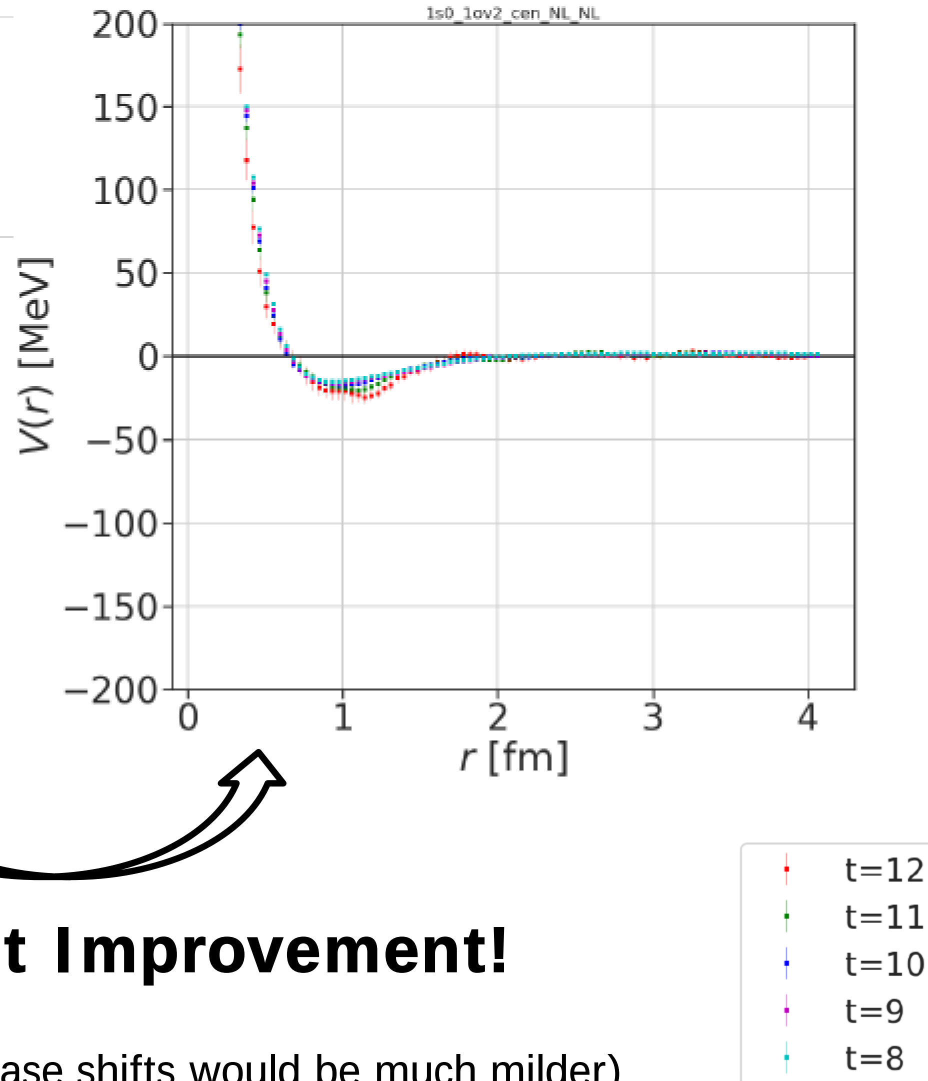

**→ Partial wave decomposition**

# Effective $N\Lambda$ central potential in I=1/2, $^1S_0$ channel

**w/o partial wave decomp.**




**w/ partial wave decomp.**



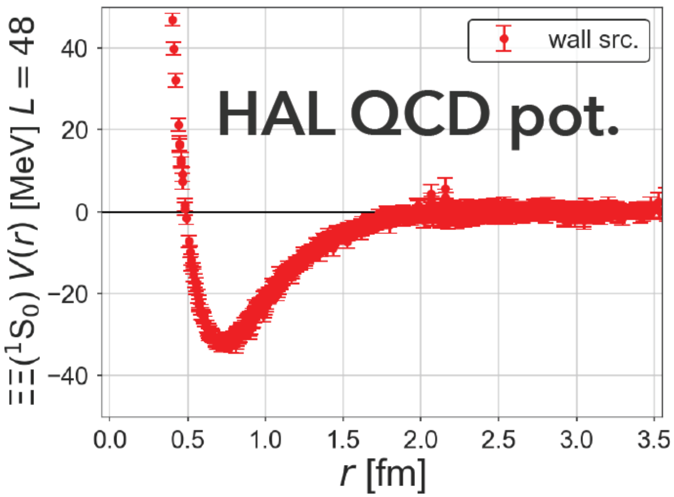

$N\Lambda$ scatt exp @ J-PARC

**Significant Improvement!**

**[ T. M. Doi ]**

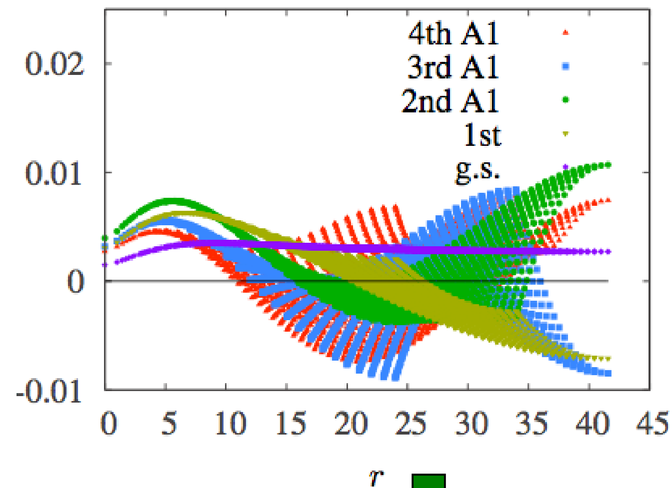
(N.B. improvement in phase shifts would be much milder)

# New method to examine sys err in HAL potential




**Solve Schrodinger eq. in Finite V**

## Eigen wave functions



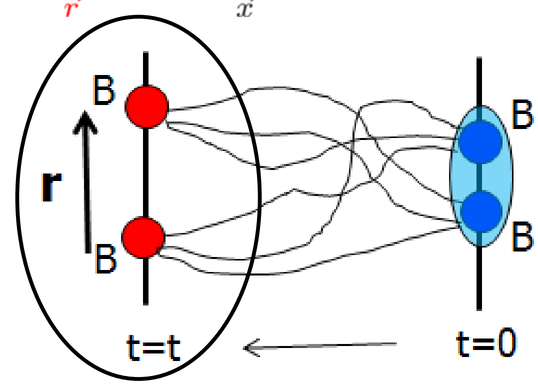

**Solve in FV**

## Eigen energies = FV spectrum

| $n$-th A1 | $\Delta E_n$ [MeV] |
|---|---|
| 0 | -2.58(1) |
| 1 | 52.49(2) |
| 2 | 112.08(2) |
| 3 | 169.78(2) |
| 4 | 224.73(1) |

## Optimized (sink) op

$$\mathcal{J}^{2B}_{\text{sink}} = \sum_{\vec{r}} \psi^{\dagger}(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$

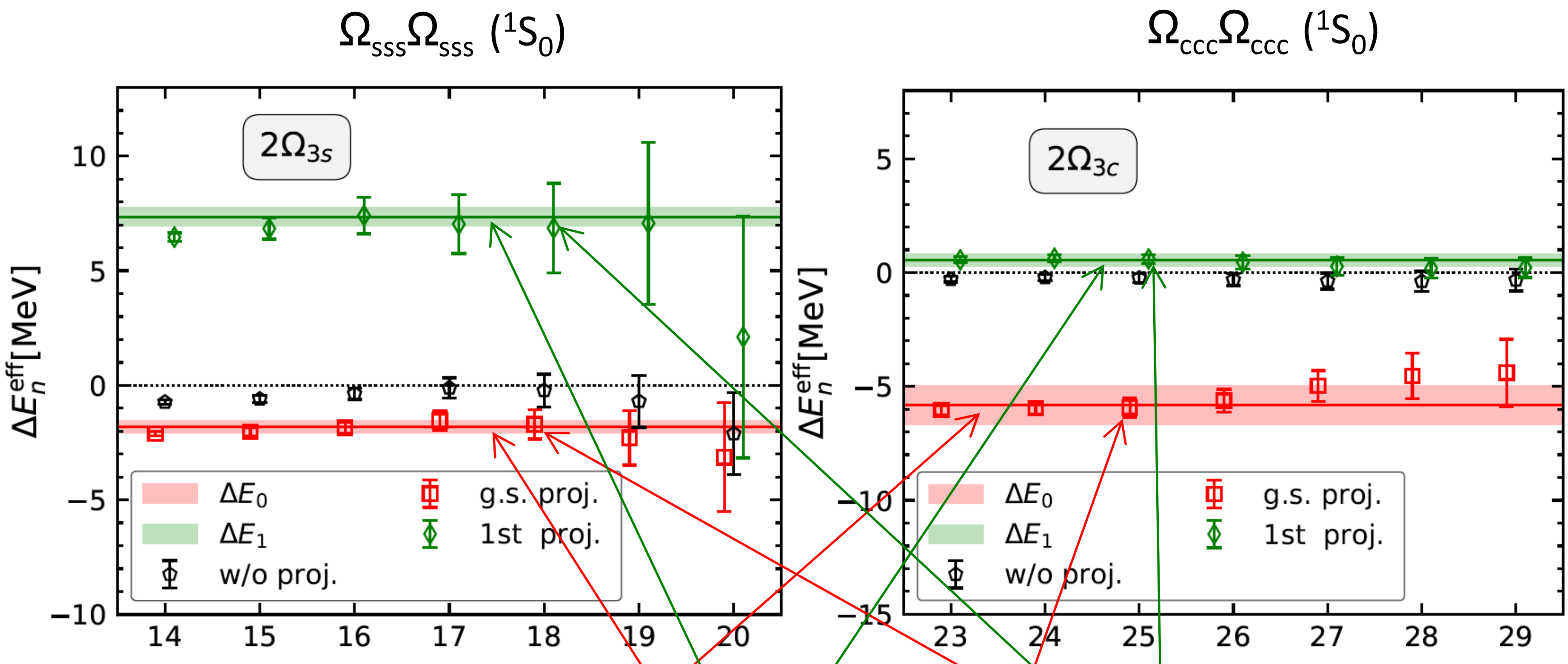



**FV spectrum from Optimized temporal corr**

**Consistency Check!**

T. Iritani et al. (HAL), JHEP03(2019)007

# Example: Comparison of FV spectrum for di-Omegas




**ΔE of g.s. from HAL pot** ⟺ **ΔE from opt. t-corr for g.s.**

**ΔE of 1st from HAL pot** ⟺ **ΔE from opt. t-corr for 1st.**

**Good agreement!**

**Sys err in HAL pot under control**

$\Omega_{sss}\Omega_{sss}$ : g.s. dominant
$\Omega_{ccc}\Omega_{ccc}$ : 1st. dominant

Y. Lyu et al., arXiv:2201.02782

# Toward P-wave int by all-to-all method

## LapH method

M. Peardon et al., PRD80(2009)054506

$$\mathcal{S}^{ab}(x,y) = \sum_{l=0}^{N_l} \omega_l \, v_l^a(x) v_l^{*b}(y)$$

$$\Delta^{ab}(x,y;U) = \sum_{k=1}^{3} \left\{ U_k^{ab}(x)\delta(y,x+\hat{k}) + U_k^{ba}(y)^*\delta(y,x-\hat{k}) - 2\delta(x,y)\delta^{ab} \right\}$$

$$= \begin{pmatrix} v_1 & v_2 & \cdots & v_N \end{pmatrix} \begin{pmatrix} \lambda_1 & & & \mathbf{0} \\ & \lambda_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \lambda_N \end{pmatrix} \begin{pmatrix} v_1^\dagger \\ v_2^\dagger \\ \vdots \\ v_N^\dagger \end{pmatrix}$$

Approximate all-to-all prop by $N_l$ low-modes in gauge covariant Laplacian

## New improvement: Free LapH method

T. Sugiura et al., PoS LAT2021,565

gauge cov. Laplacian → free Laplacian

$$\Delta(x,y) = \sum_{k=1}^{3} \left\{ \delta(y,x+\hat{k}) + \delta(y,x-\hat{k}) - 2\delta(x,y) \right\}$$

Comput. Cost can be reduced from O(Nc^4 x $N_l$^4) → Nc! Nc O($N_l$^3)

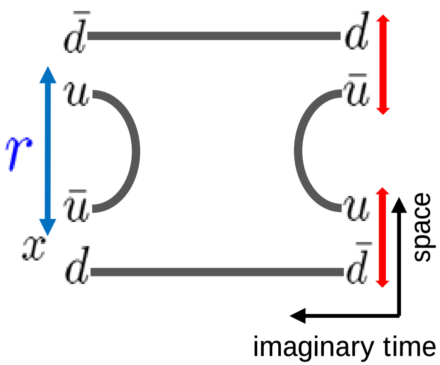
→ Typically O(100) speedup!

Explicit calc for NN in progress

[T. Sugiura]

➔ P-wave int, LS-forces, better systematics

# Resonances, Exotics




e.g., I=1 pipi system

Challenge: calc of quark-annihilation diagram

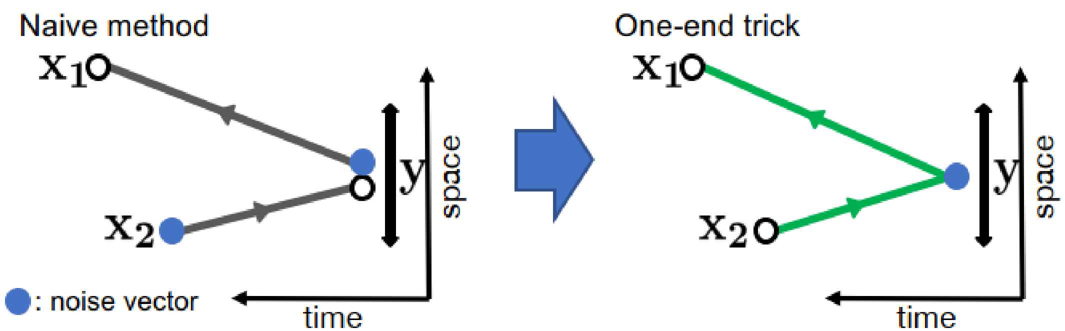
←→ all-to-all propagator required

x O(L^4) cost

Hybrid method (low-modes + stochastic estimate) is noisy

Y. Akahoshi et al., (HAL Coll.), ('19, '20)

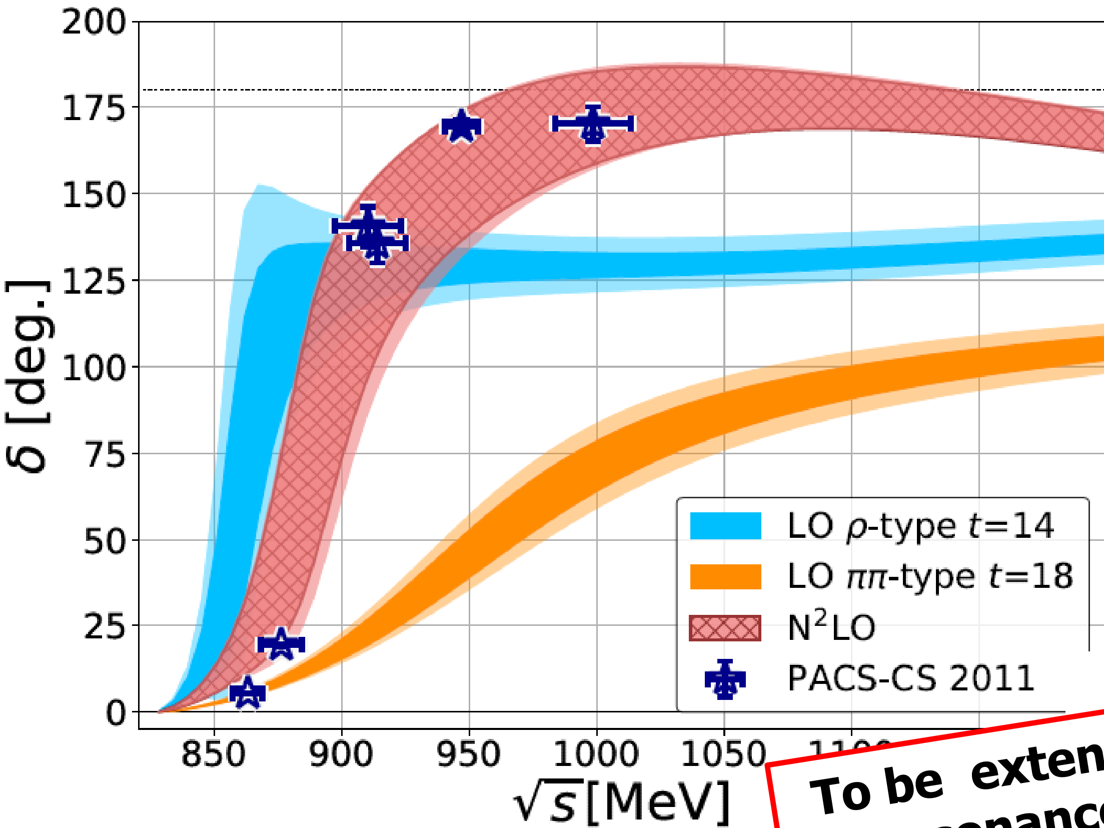
## New method w/ one-end trick




McNeile-Michael (2006)

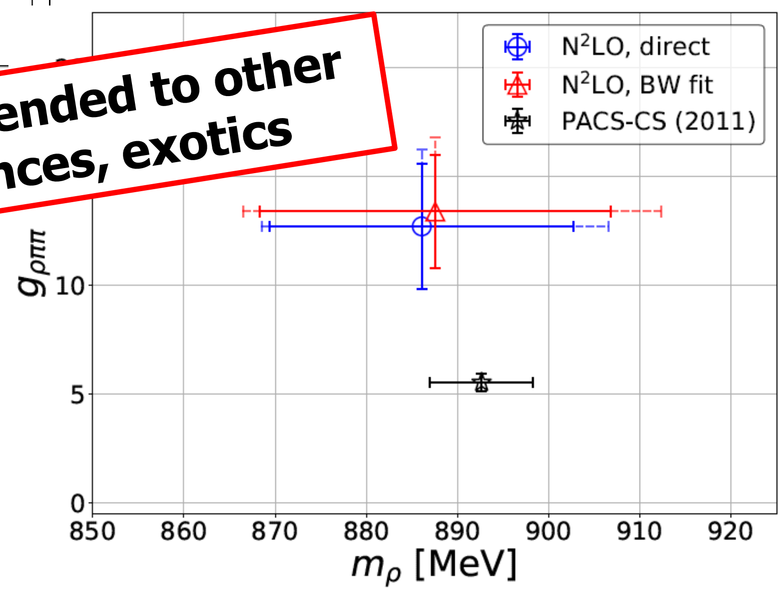
**# of noise vectors: 2 (naive method) -> 1 (one-end trick)** **→ x1/10 stat error!**

# Study of I=1 pipi system: phase shifts




m(pi) ~= 0.40GeV
m(rho) ~= 0.89GeV

**mass & g(ρππ)**




**To be extended to other resonances, exotics**

Rho-resonance from N2LO analysis
mass: consistent w/ FV method
width: improvement of N2LO necessary

Y. Akahoshi et al., PRD104(2021)054510

2019 2020 2021 2022 2030s

**From "near the physical point"**
**To "on the physical point"**

(uncertainty from quark mass dep
~ B.E. of dibaryon, hypernuclei)

**Config generation**

**Dibaryons**

**Hyperon forces**

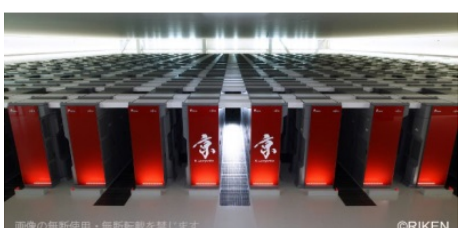
**Charmed forces**

**Exotics, Resonances**

**P-wave**
**LS-forces**
**3-body forces**

K comp ←→ ~ 8yr
(K: 5.5yr + HOKUSAI: 2.5yr)

Fugaku ←→ ~ 2yr
(S/D-waves)




**11 PFlops**

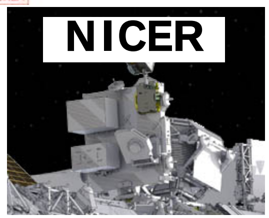
**440 PFlops**

2019 — 2020 — 2021 — 2022 — 2030s


YN, YY, (YNN)
Exotic hyper-nuclei

**J-PARC ExHEF** (2028(?)-)

**HIAF** (2024-)


GW in NS merger
NS radius

➔ EoS


3NF (I=1/2, 3/2)
r-process in NS merger

**FRIB** (2022-)

**FAIR** (2025(?)-)


Baryon correlation

Exotic hadrons

**LHC RUN3** (2022-24)


Exotic hadrons


→


→

# Summary

- Renaissance in particle/nuclear/astro-physics
  - **Observations** of neutron stars (LIGO-Virgo-KAGRA, NICER, …)
  - **Experiments** of hadrons/nuclei → J-PARC, LHC, Belle II, …
  - **Theory** by **LQCD calc of hadron interactions**
- The 1st LQCD for Baryon Interactions near the phys. point
  - Central/Tensor forces for NN/YN/YY in P=(+) channel
  - Dibaryons, Applications to Hypernuclei, EoS
- Prospects
  - **Baryon forces on the physical point by Fugaku supercomputer**
  - Dibaryons, Hypernuclei, charmed systems
  - Partial wave decomposition, all-to-all methods
  - Future: P=(-) channel, LS-forces, 3-baryon forces, etc., & EoS
  - Resonances & Exotics