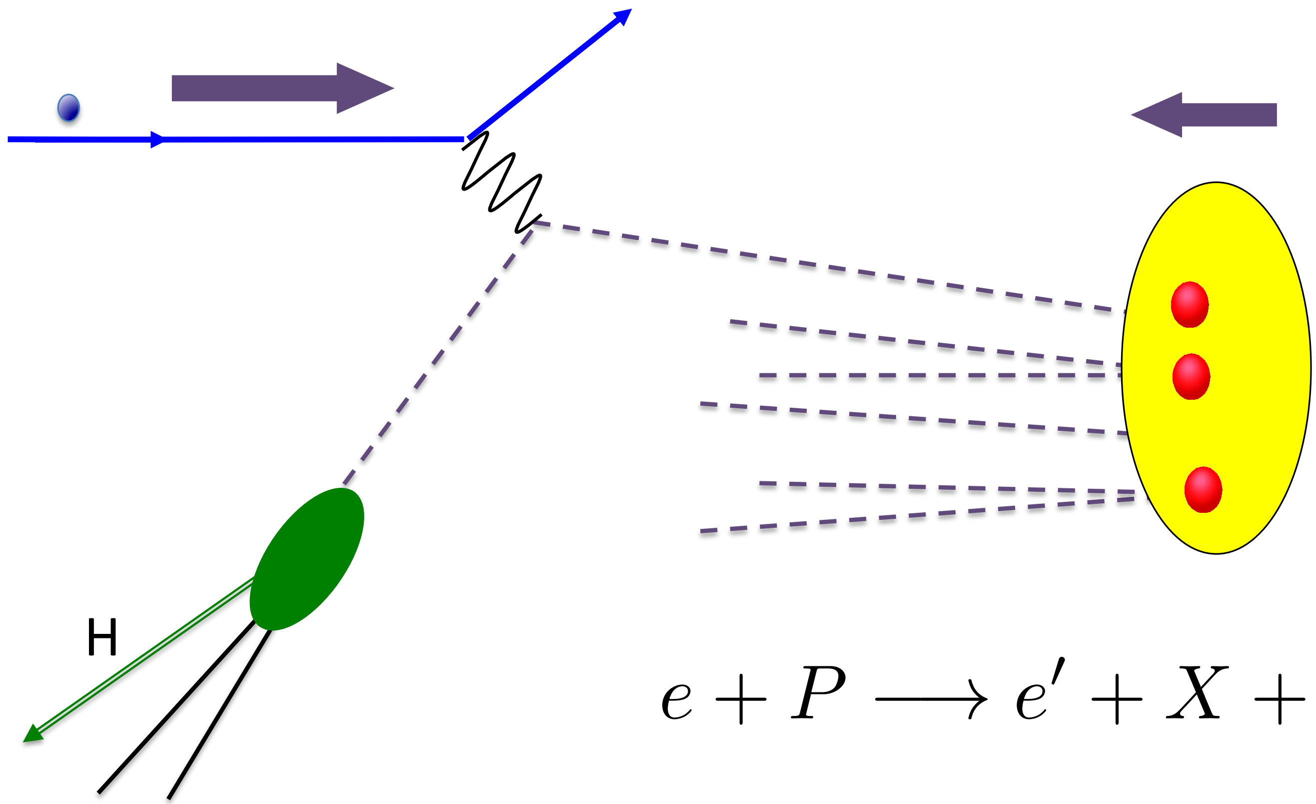
Factorization and Target Fragmentation in SIDIS

Jefferson Lab/Old Dominion University

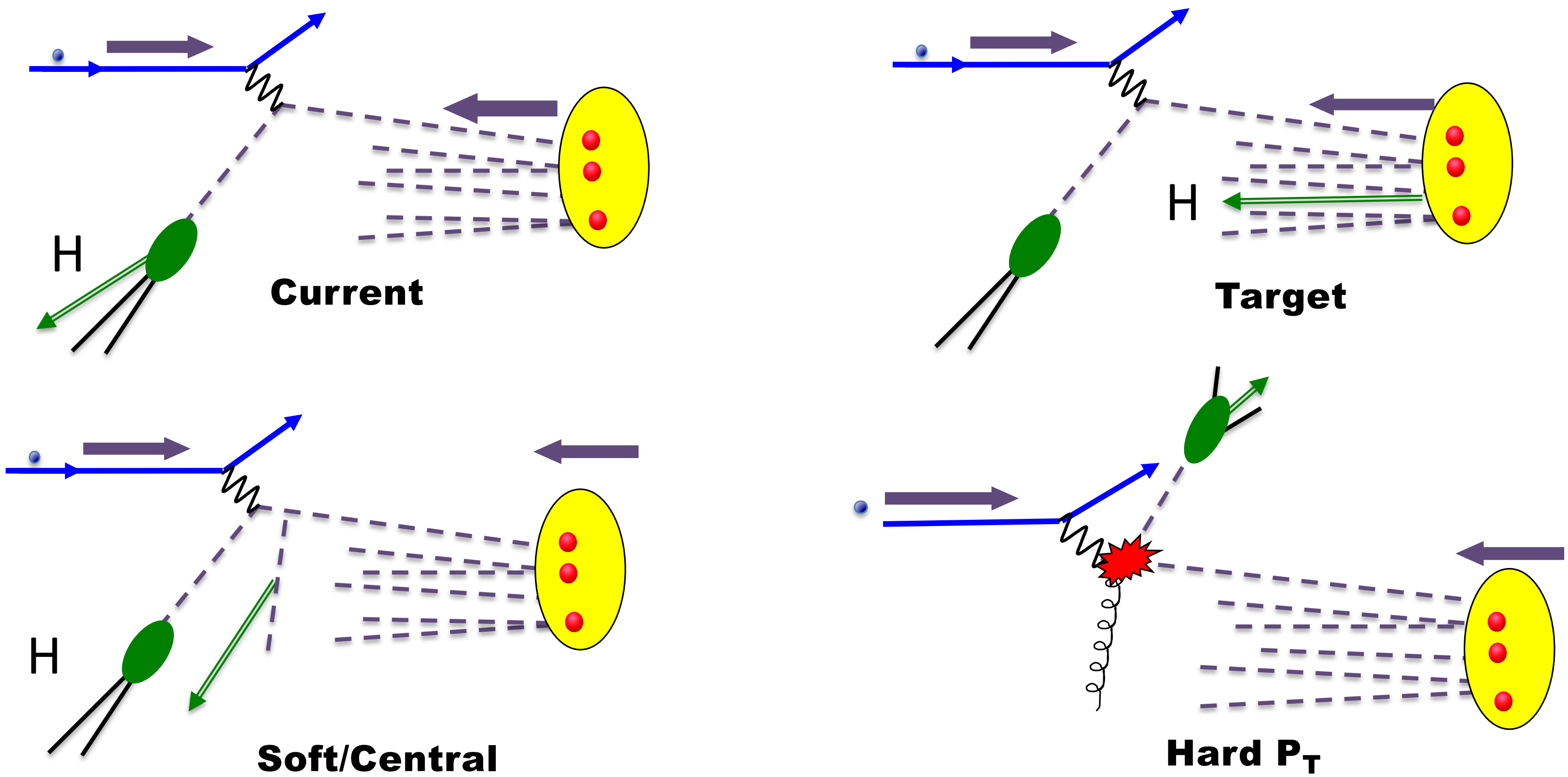
Ted Rogers, Target Fragmentation
Workshop, Spring 2022

SIDIS

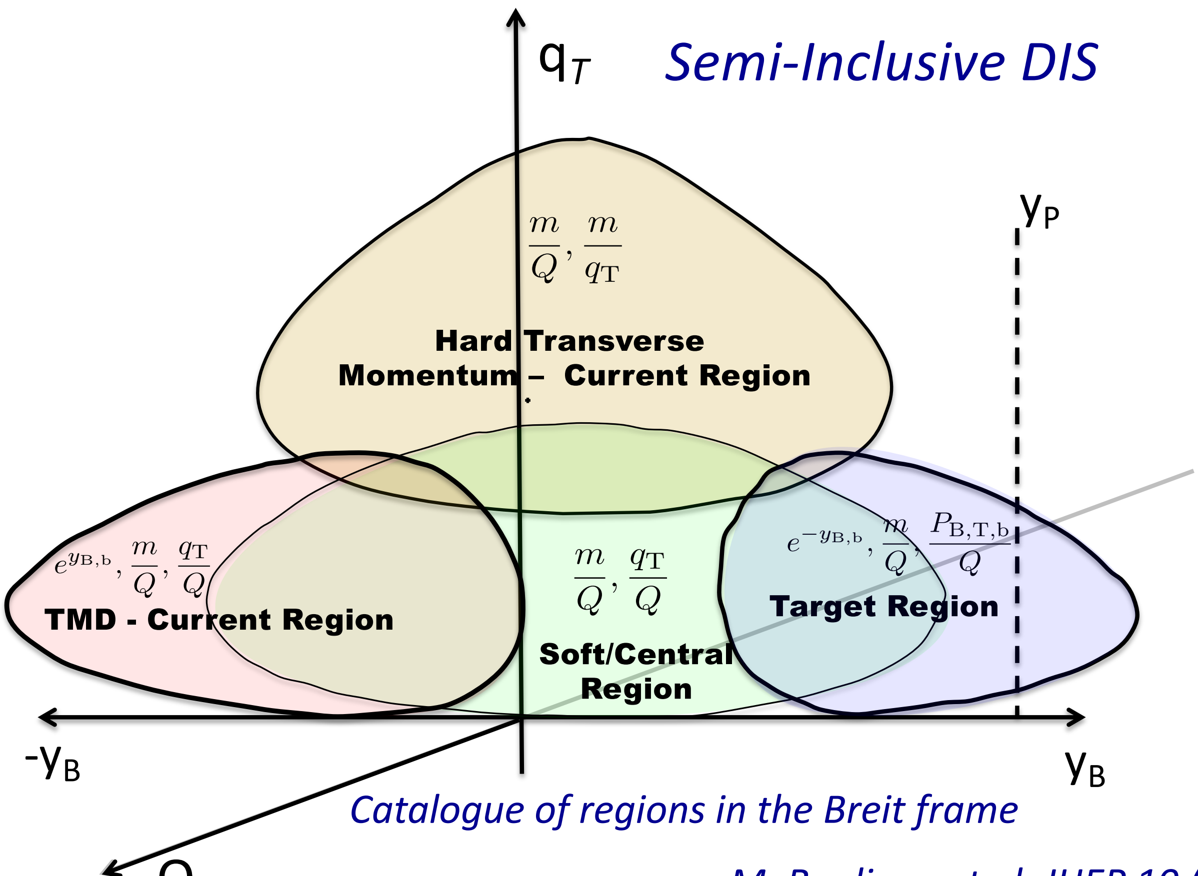


$$e + P \longrightarrow e' + X + P_h$$

Subregions



What is the relevant description?

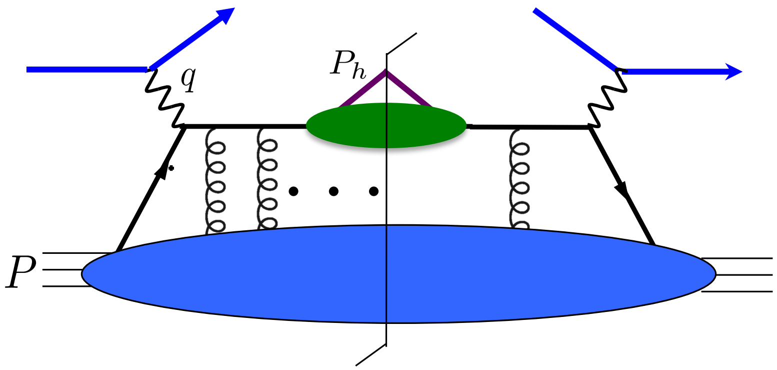


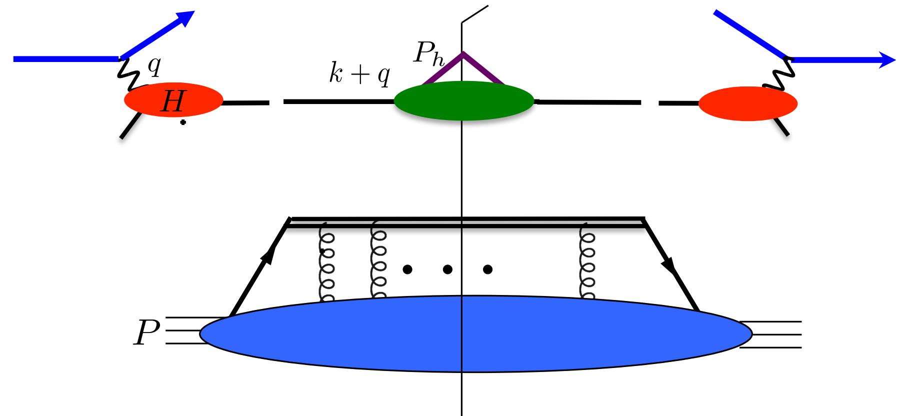
Catalogue of regions in the Breit frame

- Factorization: a power (m/Q) expansion.
- Different factorization theorems apply to different regions of x,z,Q,P_T

M. Boglione et al, JHEP 10 (2019) 122

Current fragmentation factorization





$$\frac{d\sigma}{dx\,dz\,d^2\mathbf{P}_{hT}\,dQ^2} = \hat{\sigma}\, f(x, k_T) D(z, P_{hT})$$

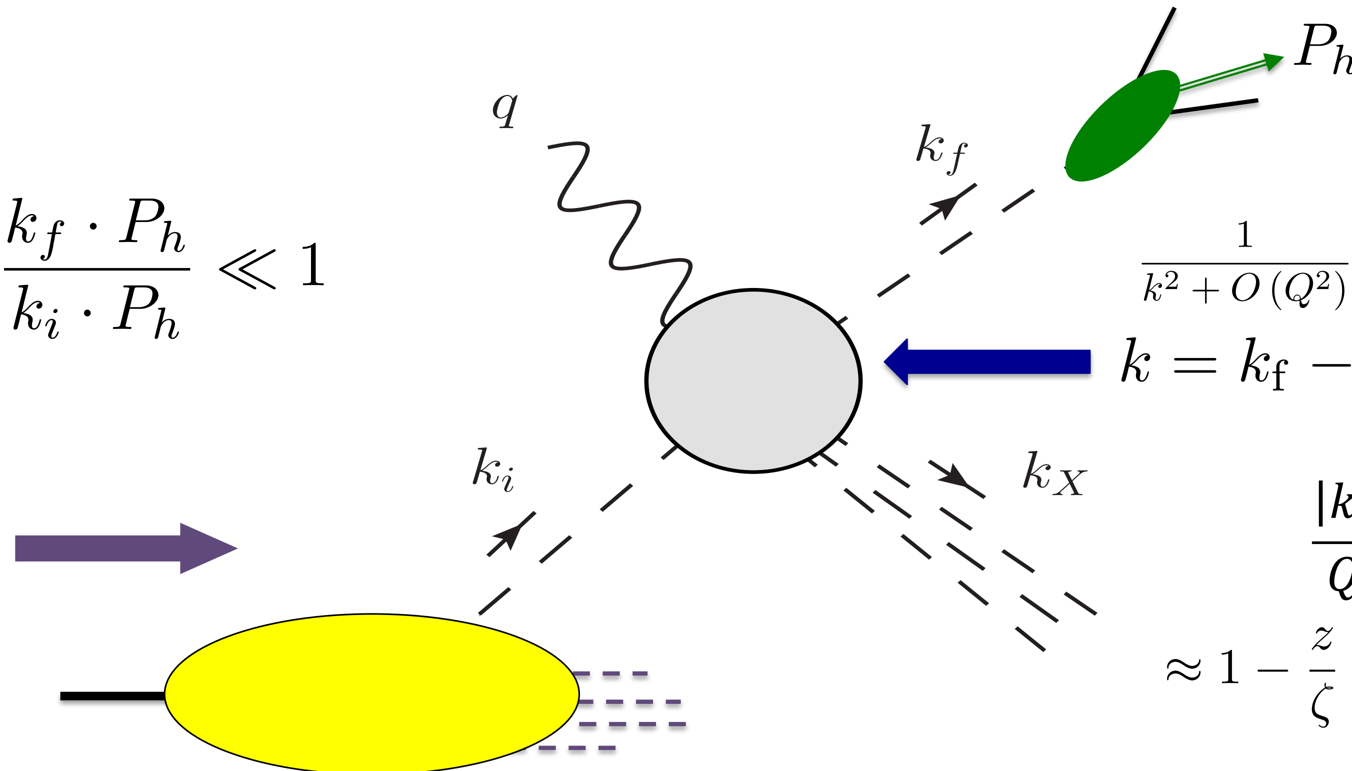
$$f(x, k_T) =$$
$$\frac{1}{2}\int \frac{dw^- d^2\mathbf{w}_T}{(2\pi)^3} e^{-ixP^+w^- + i\mathbf{k}_T\cdot\mathbf{w}_T} \langle P|\overline{\psi}(0, w^-, \mathbf{w}_T)\gamma^+\psi(0)|P\rangle$$

- *Transverse Momentum Dependent (TMD) Factorization at small transverse momentum*

- *Collinear factorization at large transverse momentum*

Current regions

$$\frac{k_f \cdot P_h}{k_i \cdot P_h} \ll 1$$



$$\frac{1}{k^2 + O\left(Q^2\right)}$$

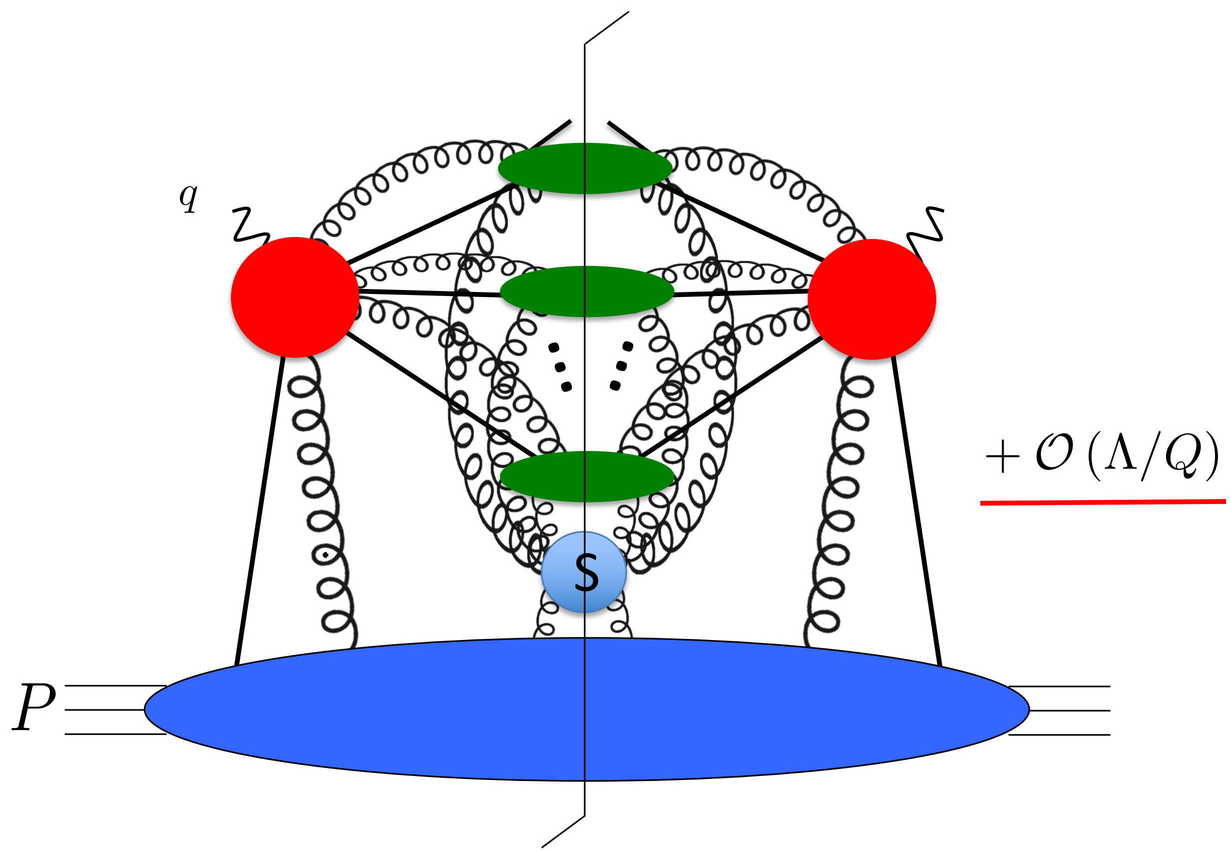
$$k = k_{\rm f} - q$$

$$\frac{|k^2|}{Q^2}$$

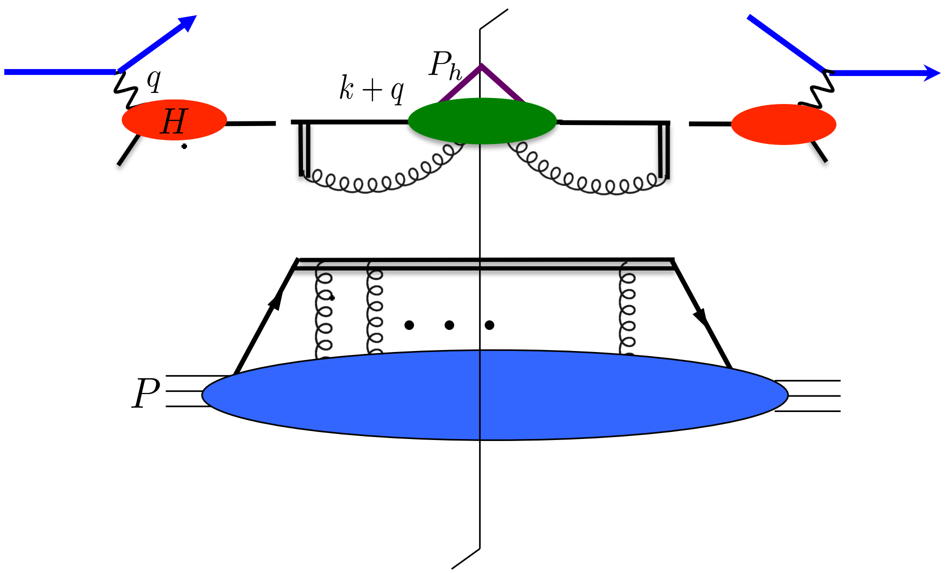
$$\approx 1 - \frac{z}{\zeta} + \frac{z}{\zeta}\frac{q_{\rm T}^2}{Q^2}$$

Semi-Inclusive DIS

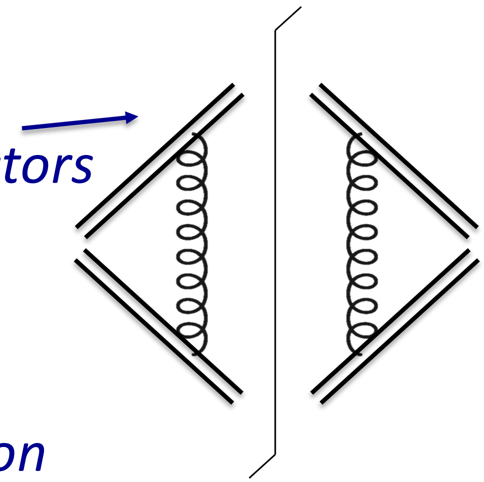
Leading Regions



Current fragmentation factorization



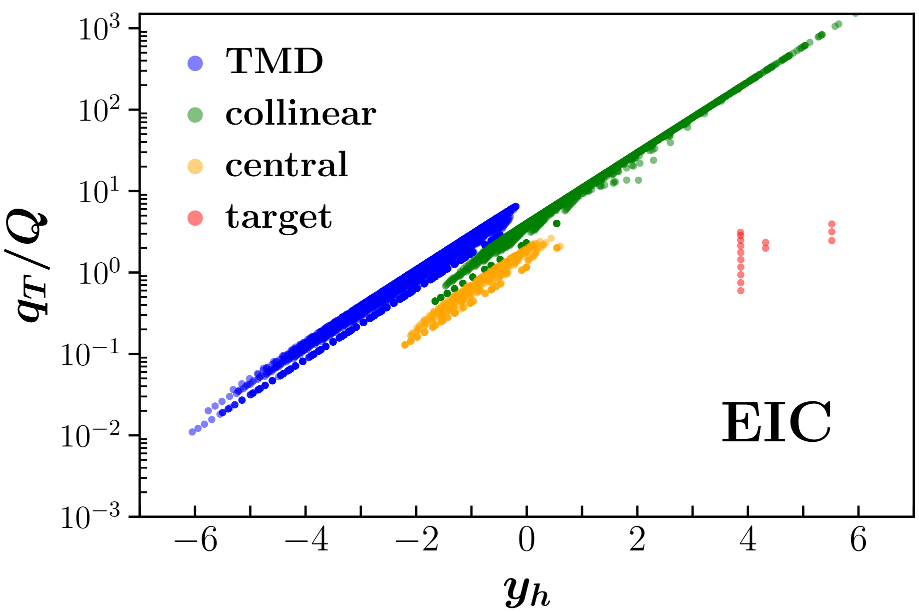
- *Also, soft factors*
- *Wilson lines*
- *TMD evolution*

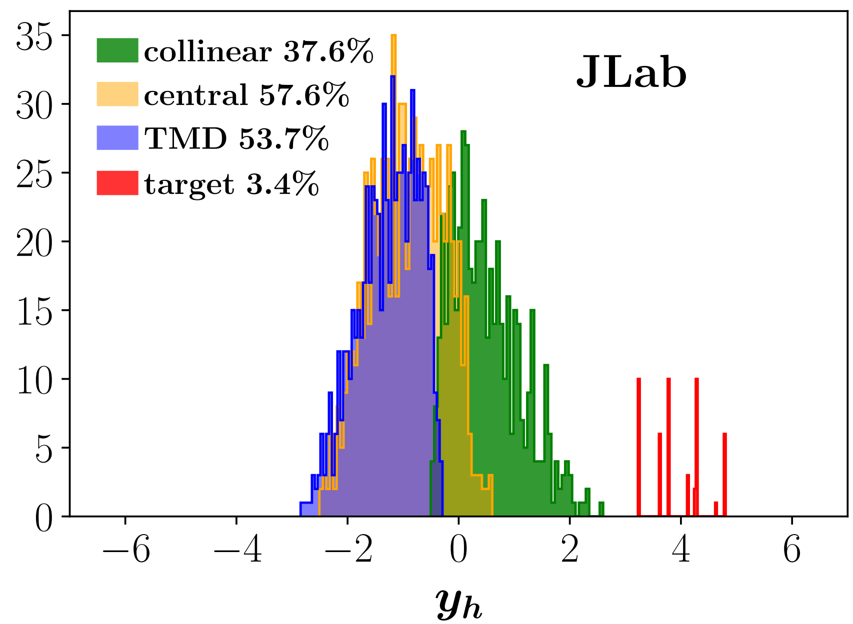


$$D(z, zP_{hT}) =$$

$$\frac{1}{4}\mathrm{Tr}\sum_X \frac{1}{z}\int \frac{dw^- d^2\mathbf{w}_T}{(2\pi)^3} e^{ik^+w^- - i\mathbf{k}_T\cdot\mathbf{w}_T} \langle 0|\gamma^+\psi(w/2)|P_h, X\rangle\langle P_h, X|\overline{\psi}(-w/2)|0\rangle$$

What is the relevant description?



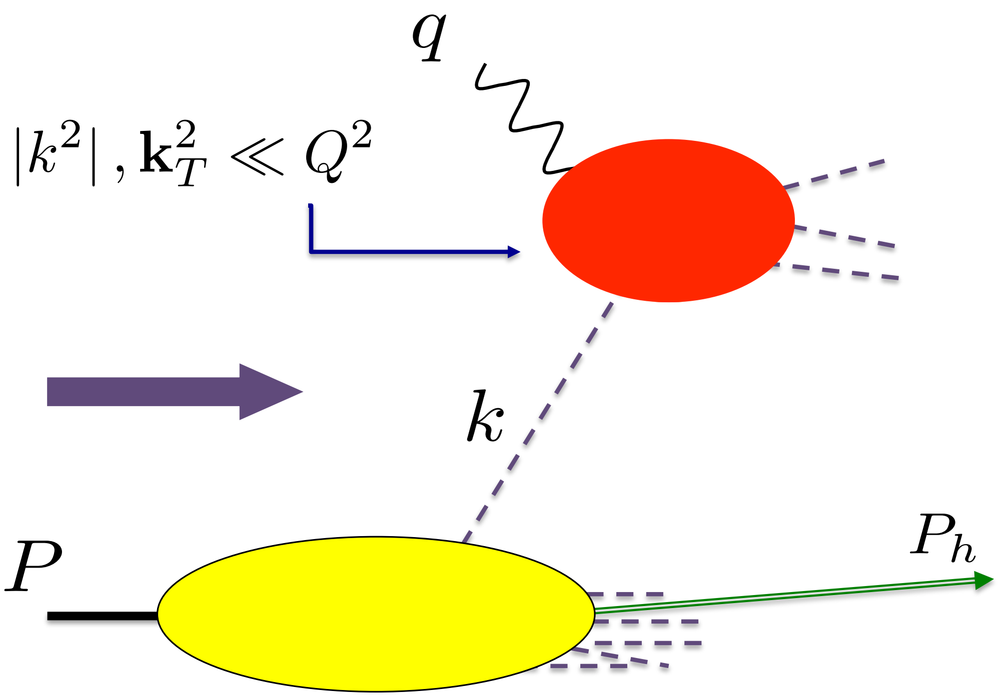


Kinematical bins

From M. Boglione et al, 2201.12197 (2022)

Target fragmentation

$$|k^2|\,,\mathbf{k}_T^2 \ll Q^2$$



$$x_h \equiv \frac{P_h \cdot q}{P \cdot q} \approx \frac{P_h^+}{P^+}$$

$$\frac{P_h \cdot P}{Q^2} \ll 1$$

(Is this really the best criterion?)
$1/x$ *?*

- *<u>Collinear</u> factorization*

Trentadue, Veneziano, Phys. Lett. B323 (1994) 201
Graudenz, Nucl. Phys. B432 (1994) 351
Berera, Soper, Phys.Rev.D 53 (1996) 6162-6179
Grazzini, Trentadue, Veneziano, Nucl. Phys. B519 (1998) 394

Target fragmentation

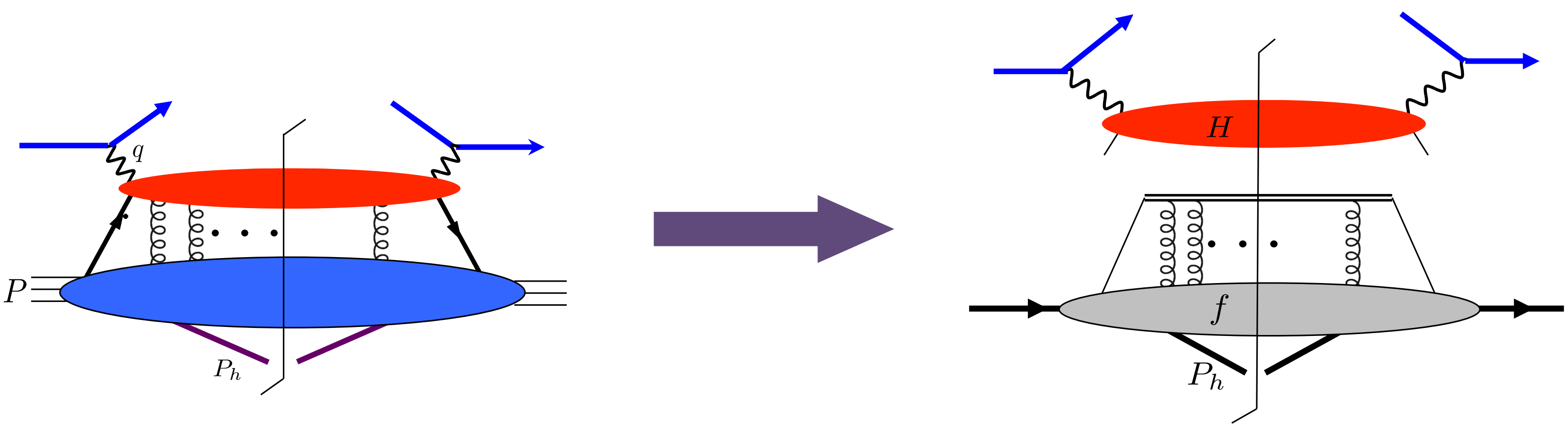
$$f(x) = \frac{1}{2}\sum_X \int \frac{dw^-}{2\pi} e^{-ixP^+w^-} \langle P|\overline{\psi}(0,w^-,\mathbf{0})\gamma^+|X\rangle\langle X|\psi(0)|P\rangle$$

- *Collinear pdf*

Final state hadron

$$M(x,x_h,\mathbf{P}_{hT}) = \frac{1}{2}\sum_X \int \frac{dw^-}{2\pi} e^{-ixP^+w^-} \langle P|\overline{\psi}(0,w^-,\mathbf{0})\gamma^+|X,P_h\rangle\langle X,P_h|\psi(0)|P\rangle$$

- *(Extended) fracture function*



Target fragmentation factorization and applications

- "Fracture functions" integrated over transverse momentum

- "Extended" fracture functions

  Grazzini, Trentadue, Veneziano, Nucl. Phys. B519 (1998) 394

- "TMD fracture functions"
  - Polarization dependent

- Same evolution, same hard factors as ordinary collinear factorization

  Collins, Phys. Rev. D57 (1998) 3051

- SIDIS structure functions

$$\frac{dF_1(x,Q^2,x_h,\mathbf{P}_T)}{dx_h\,d^2\mathbf{P}_T} = \sum_j \int_x^1 \frac{d\xi}{\xi}\hat{F}_1(Q/\mu,x/\xi)\,\frac{M(x,x_h,\mathbf{P}_T)}{x_h}$$

Same fracture function

- Universality:

$$\frac{dF_1(x,Q^2,x_h,\mathbf{P}_T)}{dx_h\,d^2\mathbf{P}_T} = \sum_j \int_x^1 \frac{d\xi}{\xi}\hat{F}_1(Q/\mu,x/\xi)_{\mathrm{dijets}}\,\frac{M(x,x_h,\mathbf{P}_T)}{x_h}$$

Relating fracture functions to PDFs

- Integral relations: $M(x, x_h, \mathbf{P}_{hT}) = \frac{1}{2}\sum_X \int \frac{dw^-}{2\pi} e^{-ixP^+w^-} \langle P|\overline{\psi}(0, w^-, \mathbf{0})\gamma^+|X, P_h\rangle\langle X, P_h|\psi(0)|P\rangle$
  - Differential expression $M(x, x_h, \mathbf{P}_{hT}) = (2\pi)^3 2E_{P_h} \frac{d}{d^3\mathbf{P}_h} f_{P_h}(x)$
  - Integral/sum over $\boldsymbol{P}_{hT}$:

$$\sum_h \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X} |P_h, X\rangle\langle P_h, X| = a_h^\dagger a_h \quad \Longrightarrow \quad \sum_h \int \frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h} M(x, x_h, \mathbf{P}_{hT}) \propto f(x)$$

- Large transverse momentum: $\Lambda_{\mathrm{QCD}} \ll \mathbf{P}_{hT} \ll Q$
  - "refactorization" $M(x, x_h, \mathbf{P}_{hT}) = \int C_F \otimes f(x) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{P_{hT}}\right)$

Chen, Ma, Tong, JHEP 11 (2021) 038

Spin dependent and TMD fracture functions

- TMD fracture function
- 16 fracture functions at leading power

$$
\begin{aligned}
&\mathcal{M}^{[\Gamma]}(x_B, \boldsymbol{k}_\perp, \zeta, \boldsymbol{P}_{h\perp}) \\
&\quad \equiv \frac{1}{4\zeta} \int \frac{\mathrm{d}k^+ \, \mathrm{d}k^-}{(2\pi)^3} \, \delta(k^- - x_B P^-) \, \mathrm{Tr}\,(\mathcal{M}\,\Gamma) \\
&\quad = \frac{1}{4\zeta} \int \frac{\mathrm{d}\xi^+ \, \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^6} \, \mathrm{e}^{\mathrm{i}(x_B P^- \xi^+ - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)} \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3 \, 2E_X} \times \\
&\qquad \langle P, S | \bar{\psi}(0) \Gamma | P_h, S_h; X \rangle \langle P_h, S_h; X | \psi(\xi^+, 0, \boldsymbol{\xi}_\perp) | P, S \rangle
\end{aligned}
$$

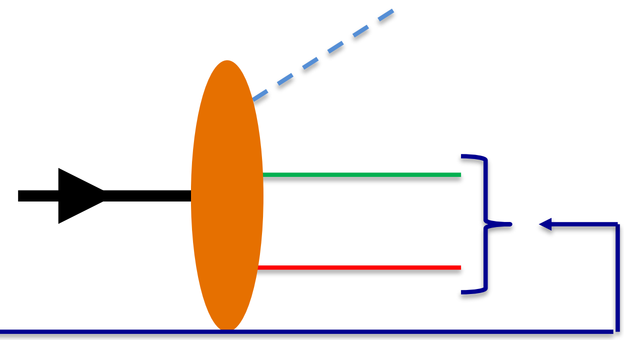
$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B \, \mathrm{d}y \, \mathrm{d}\zeta \, \mathrm{d}^2\boldsymbol{P}_{h\perp} \, \mathrm{d}\phi_S} &= \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y} \left\{ \left(1 - y + \frac{y^2}{2}\right) \right. \\
&\times \sum_a e_a^2 \left[M(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp| \, \frac{|\boldsymbol{P}_{h\perp}|}{m_h} \, M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \, \sin(\phi_h - \phi_S) \right] \\
&+ \lambda_l \, y \left(1 - \frac{y}{2}\right) \sum_a e_a^2 \left[S_\parallel \, \Delta M_L(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \right. \\
&\left. \left. + \; |\boldsymbol{S}_\perp| \, \frac{|\boldsymbol{P}_{h\perp}|}{m_h} \, \Delta M_T^h(x_B, \zeta, \boldsymbol{P}_{h\perp}^2) \, \cos(\phi_h - \phi_S) \right] \right\} .
\end{aligned}
$$

From: Anselmino, Barone, Kotzinian
Phys.Lett.B 699 (2011) 108-118

Interpretations

- Fluctuations of the hadron wavefunction

- How hard is P_{hT}? Correlations between partons (analog of SRCs)?



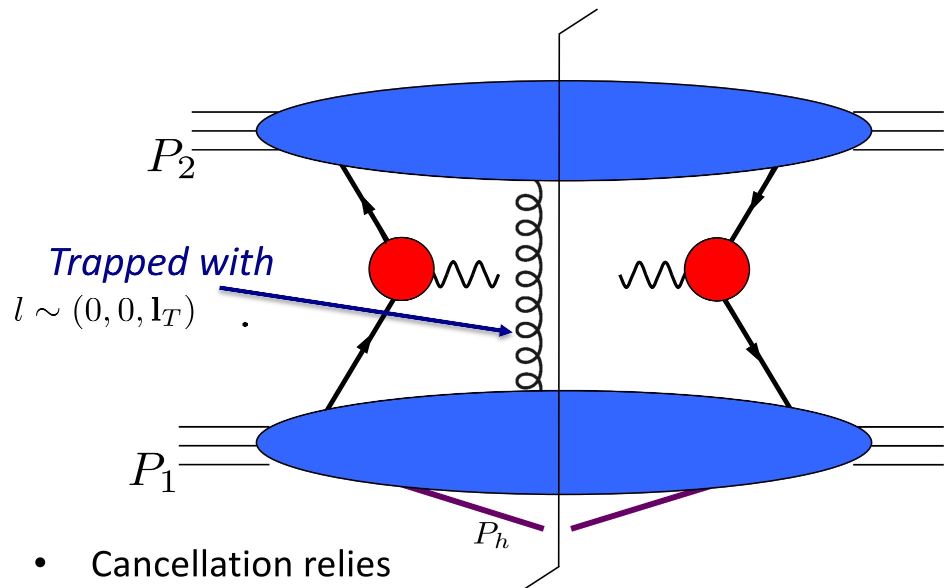
- Is transverse momentum mainly "intrinsic" (e.g., Anselmino, Barone, Kotzinian)

  or radiative

  (e.g., Ceccopieri, Trentadue, Phys. Lett. B636 (2006) 310 & Phys. Lett. B660 (2008) 43)

Limits of target region factorization

- Glauber gluons in hadron-hadron collisions:



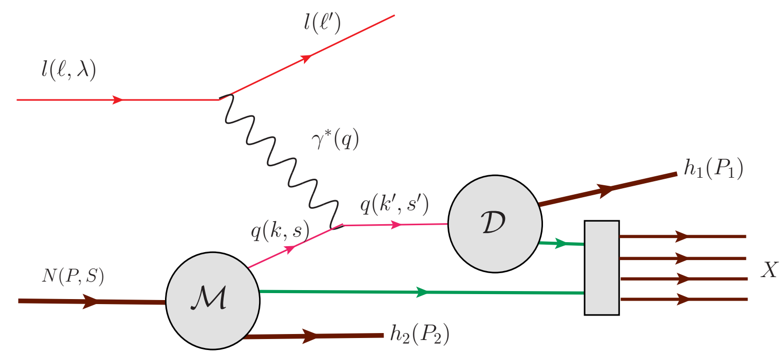
- Cancellation relies on inclusive sum

Photon rather than hadron?

Chai, Chen, Ma, Tong, JHEP 10 (2019) 285

- TMD-sensitive processes:

  - Double inclusive processes



Anselmino, Barone, Kotzinian
Phys.Lett.B 699 (2011)

Do standard TMD factorization derivations carry over to this situation?

Phenomenological analysis of fracture functions

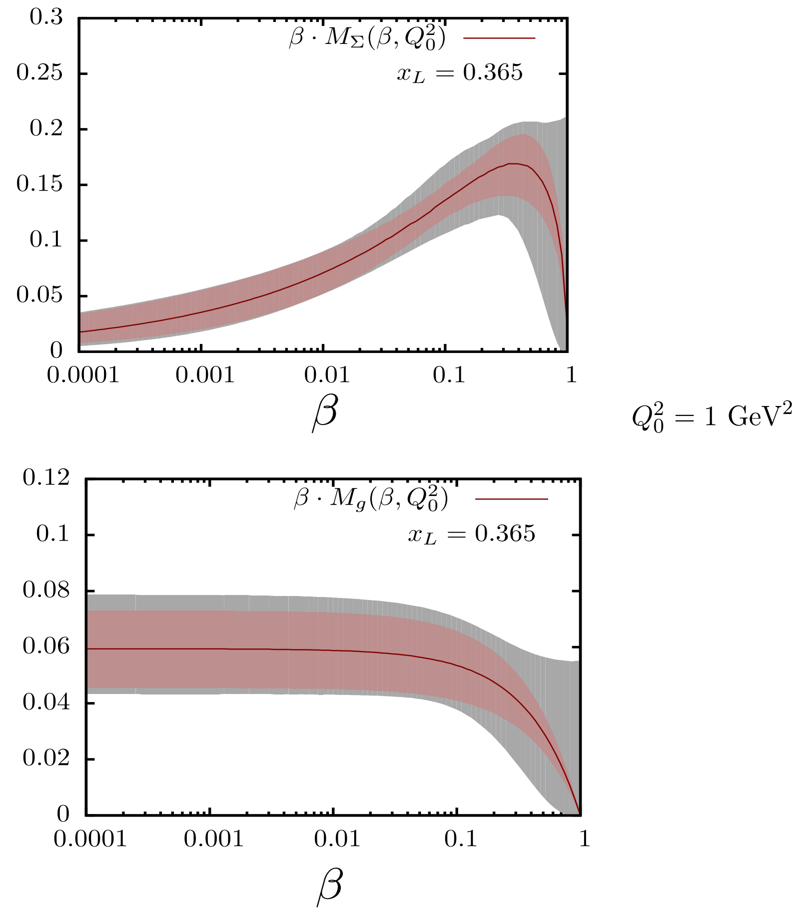
- Forward Λ production

  Ceccopieri, Mancusi, Eur.Phys.J.C 73 (2013) 2435

- Forward neutrons at HERA

  Ceccopieri, Eur.Phys.J.C 74 (2014) 8, 3029

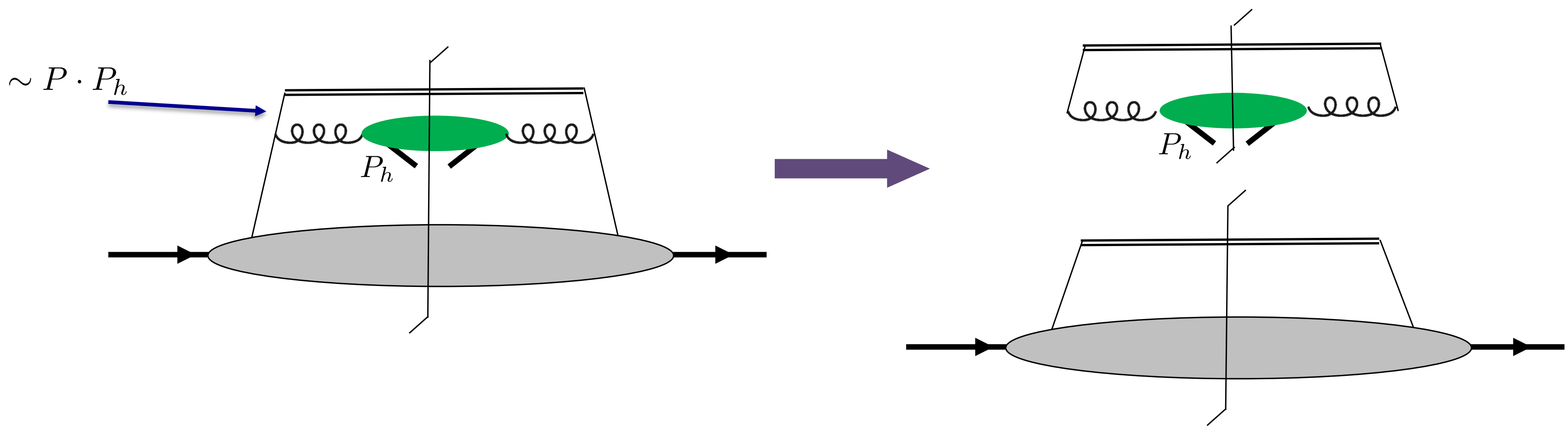
$$\beta = \frac{x}{1 - x_h}$$



TMD Refactorization?

- $\Lambda^2_{\rm QCD} \ll P\cdot P_h \ll Q^2\,, P^2_{hT} \ll P\cdot P_h$

$$\sum_h \int \frac{d^3\mathbf{P}_h}{(2\pi)^3 2E_h} M(x, x_h, \mathbf{P}_{hT}) \propto f(x)$$



Conjecture:

$$M(x, x_h, \mathbf{P}_{hT}) = H(x_h, x) D(z, \mathbf{k}_{2T}) f(x', \mathbf{k}_{1T})$$

$$\int d^2\mathbf{k}_T f(x, k_T) \sim f(x)$$

Conclusion

- Questions for target fragmentation and fracture functions
  - Exact kinematical boundary between target and other regions?
  - Interpretation? (e.g., short range correlations)
  - Check factorization with double hadron processes
  - "Intrinsic" transverse momentum or radiation?
  - Relationship to other functions?
    - Integral relations
    - collinear refactorization
    - TMD refactorization