Few-Nucleon Systems in Lattice QCD

Michael Wagman

Brookhaven National Laboratory

High Energy / Nuclear Theory / RIKEN seminar

February 18, 2022
Quarks, gluons, and nuclei

At low energies, nuclei “look like” collections of protons and neutrons

— Low-energy EFTs with nucleon degrees of freedom make accurate predictions

At high energies, nuclei “look like” collections of quarks and gluons

— Perturbative QCD makes accurate predictions for high-energy physics

Medium-energy interactions, as well as EFT input parameters and quark and gluon structure functions, contain rich nonperturbative physics

— Lattice QCD can provide accurate nonperturbative predictions

New physics and nuclei

Nuclei are useful experimental targets, cross-sections often grow with baryon number.

Converting between nuclear- and nucleon-level cross-sections requires:

- Nuclear models (uncertainty quantification?)
- Direct LQCD calculations (impractical)
- LQCD informed EFT + modeling

Many low-energy searches for fundamental symmetry violation require nuclear matrix elements to relate experimental observables to new physics theory parameters.

Standard Model predictions with controlled uncertainties essential for next-generation accelerator neutrino experiments aiming for few-percent systematic uncertainties.
**Neutrino masses**

Experimentally observed neutrino oscillations require neutrinos to be massive.

Solar, atmospheric, reactor, and accelerator neutrino experiments have constrained the 3 mixing angles in the PMNS matrix with few-percent precision.

\[ \nu_e \quad \nu_\mu \quad \nu_\tau \]

\[ U_{PMNS} \]

\[ (\nu_1) \quad (\nu_2) \quad (\nu_3) \]

\( CP \)-violating phase is more poorly constrained.

Neutrino masses require physics beyond the Standard Model: right-handed sterile neutrinos and/or lepton-number-violating Majorana neutrino masses.

\[ L_5 \supset - \frac{1}{\Lambda} \varepsilon_{ab}(\bar{\ell}_{La}H_b)\varepsilon_{cd}(H^T\bar{C\ell}^{T}_{Ld}) + \text{h.c.} = - \frac{\nu^2}{\Lambda} (\bar{\nu}C\bar{\nu}^T + \nu^T C\nu) + \ldots \]

Weinberg, PRL 43 (1979)
Lepton number violation

If lepton number is not conserved, then matter-antimatter asymmetry could be generated through leptogenesis


Low-energy signature of lepton-number violation: $0\nu\beta\beta$

Experiments directly measure half-lives $T^{0\nu}_{1/2}$ of specific nuclei ($A \geq 48$)

Using these results to constrain neutrino mass and mixing parameters requires nuclear matrix elements

$$\left(\frac{1}{T^{0\nu}_{1/2}}\right)^{-1} = G^{0\nu} |\mathcal{M}^{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$

Challenging to calculate from first principles, but significant recent theory progress
Neutrino $CP$ violation

Next-generation neutrino experiments DUNE and Hyper-Kamiokande aim to precisely measure neutrino $CP$ violation, mass hierarchy, ... 

Relating measured final-state event rates to incoming neutrino energy distribution requires theory input on $\nu A$ cross-section

$$\frac{N_{\text{near}}(E_\nu)}{N_{\text{far}}(E_\nu)} = \frac{\int dE'_\nu \Phi_{\text{near}}(E'_\nu) \sigma(E'_\nu)}{\int dE'_\nu \Phi_{\text{far}}(E'_\nu) \sigma(E'_\nu)}$$

DUNE aims to have few-percent theory uncertainty

— estimates show going from 2% to 3% theory uncertainty means 50% longer runtime to measure $CP$ violation at a given precision

Abi et al (DUNE), arXiv 1807.10334
Neutrino-nucleus scattering

Accelerator neutrino flux covers a wide range of energies with different dominant physics processes:

- Quasi-elastic
- Resonance production
- Transition region
- Deep inelastic scattering

Nuclear models exist that can describe most of these regions, but precision x-sec predictions require precise knowledge of few-nucleon input parameters:

- Nucleon form factors
- Transition form factors
- Two-body currents
Lattice QCD, EFT, and $\nu A$

LQCD can provide results for few-nucleon quantities that inform nuclear effective theories and are complementary to experiment.

Easy for LQCD:
- Axial vs vector currents
- Isovector vs isoscalar
- Pions

Hard for LQCD:
- Large baryon number
- Real-time dynamics
- Multi-hadron states
- (Light quark masses)

Lattice QCD and $\nu A$

$\nu A$ scattering amplitudes factorize into leptonic and hadronic parts

$$\mathcal{M}_{\nu A \to \ell f} \propto (\overline{u}_\ell \gamma_\mu \gamma_5 u_\nu) \langle f | \overline{q} \gamma_\mu \gamma_5 q | A \rangle + \ldots$$

Generic Euclidean hadronic matrix elements calculable (in principle) using lattice QCD

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}q \mathcal{D}\bar{q} e^{-S_{QCD}(U,q,\bar{q})} \mathcal{O}(U,q,\bar{q}) \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}(U_i)$$

Quark fields integrated analytically, “just” need to solve for propagators

Monte Carlo sample gluon fields with probability $\propto e^{-S_{\text{eff}}}$
LQCD and nuclear matrix elements

LQCD spectrum determined from 2-point correlation functions

\[ C_A(t) = \langle A(t) A^\dagger(0) \rangle = \sum_n \langle 0 | A(0) e^{-Ht} | n \rangle \langle n | A^\dagger(0) | 0 \rangle + \ldots \]

\[ = \sum_n |Z_n|^2 e^{-E_n t} \]

Nuclear matrix elements determined from 3-point correlation functions including a local operator insertion
Nucleon form factors

Vector and axial form factors recently calculated using nearly physical quark masses:

Recent vector / axial form factor studies show the importance of excited-state effects in nucleon form factor calculations with light quark masses.

Remaining LQCD systematic uncertainties arise from lattice spacing effects, finite-volume effects, excited-state effects.

Park et al [NME], arXiv:2103.05599
$N\pi$ scattering

Recent calculations in $I = 3/2$ channel including $\Delta$ and $N_\pi$ interpolating operators use variational methods to extract ground- and excited-states in many angular momentum channels (cubic irreps) 

Silvi et al, PRD 23 (2021)

Finite-volume quantization conditions relate energy levels to p-wave $N_\pi$ scattering phase shifts
Axial currents in nuclei

Axial-current responses of nucleons are modified in nuclei

Modern calculations including multi-nucleon correlations and currents with e.g. chiral EFT can reproduce experiment without “quenching” $g_A$

Few-nucleon LQCD results can constrain two-body currents in nuclear models and EFTs


Pastore et al, PRC 97 (2018)
Proton-proton fusion

Axial current transition matrix element between spin-singlet and spin-triplet $np$ systems computed using fixed-order background fields

Savage, MW et al [NPLQCD], PRL 119 (2017)

LQCD results matched to pionless EFT by computing same background-field correlation function

Results used to constrain LEC for two-body axial current operator in pionless EFT

Same operator relevant for proton-proton fusion and other reactions, future LQCD calculations could improve phenomenological predictions
Axial matrix elements

Flavor decomposition of axial matrix elements of up to three nucleon systems computed with $m_\pi = 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)

Fractional differences from naive shell model expectations show that multi-nucleon correlations lead to percent-level effects on axial charges of light nuclei for these quark masses

$N_f = 3, \ m_\pi = 806(9)$ MeV, $a = 0.145(2)$ fm
**Triton $\beta$ decay**

Triton $\beta$ - decay rate governed by Gamow-Teller matrix element

\[
g_A(3H) = |\langle 3\text{He}|A_3^+|3\text{H}\rangle| = |\langle 3\text{H}|A_3^+|3\text{H}\rangle|
\]

Computed in ChEFT \hspace{2cm} Baroni et al, PRC 98 (2018)

After fitting LECs to experimental triton $\beta$-decay rate predicts

\[
\frac{|\langle 3\text{He}|A_3^+|3\text{H}\rangle|}{g_A} = 0.951(13)
\]

Deviations from 1 arise from two-body currents and multi-nucleon interactions

NLO calculations in pionless EFT relate nuclear effects to the two-body axial current coupling $L_{1A}$ appearing in proton-proton fusion

De-Leon, Platter, Gazit (2016)
Triton $\beta$ decay from LQCD

LQCD calculations of triton recently performed using $m_\pi = 450$ MeV

Parreño, MW et al [NPLQCD] PRD 103 (2021)

Signal-to-noise problem makes calculations exponentially noisier at lighter quark masses

Results consistent with bound triton obtained on 3 volumes

Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation to physical point

Several systematic uncertainties remain, but encouraging agreement with experiment seen

Matching to finite-volume pionless EFT used to constrain $L_{1A}$

Parreño, MW et al [NPLQCD] PRD 103 (2021)

Detmold and Shanahan, PRD 103 (2021)
PDFs and LQCD

LQCD can also compute quantities relevant to high-energy scattering

Large momentum effective theory connects Euclidean matrix elements to light-cone PDFs

Current LQCD results can improve global analyses of isovector polarized PDFs

\langle x \rangle_{\text{proton}}^q \text{ and } \langle x \rangle_{\text{proton}}^g \text{ calculated by several groups}


Bringewatt et al [JAM], arXiv:2010.00548
Nuclear momentum fractions

First calculations of gluon and isovector quark momentum fractions of light nuclei

Winter, MW et al [NPLQCD], PRD 96 (2017)  
Detmold, MW et al [NPLQCD] PRL 126 (2021)

Results matched to poinless EFT to determine two-body current operator that governs isovector EMC effects relevant to neutrino DIS

Cloët, Bentz, Thomas, PRL 102 (2009)

Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, …) demonstrates potential for LQCD to usefully constrain nuclear PDFs

![Graphs and plots](image)
Beyond-Standard-Model Interactions

Nuclear matrix elements for $2\nu\beta\beta$ calculated similarly with background fields

$$\langle pp|A_3^+ A_3^+ |nn\rangle$$

Shanahan, MW et al [NPLQCD], PRL 119 (2017)

Pion analogs of $0\nu\beta\beta$ such as $\pi^- \rightarrow \pi^+ e^- e^-$ computed

**Short distance** Nicholson et al, PRL 121 (2018)


Detmold and Murphy, arXiv:2004.07404

Other nuclear matrix elements of BSM currents computed with $m_\pi \approx 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)

Scalar current nuclear matrix elements relevant to dark matter direct detection

Tensor current nuclear matrix elements relevant to nuclear electric dipole moments

<table>
<thead>
<tr>
<th>$\Delta R_S^{(u-d)}$</th>
<th>$\Delta R_S^{(u+d+s)}$</th>
<th>$\Delta R_S^{(u+d-2s)}$</th>
<th>$\Delta R_S^{(s)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2T_3$</td>
<td>$B$</td>
<td>$B$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scalar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuteron</td>
</tr>
</tbody>
</table>

$g_S$
Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects
Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- **Excited-state effects**

Gap between ground and two-nucleon finite-volume “scattering” states becomes small for large volumes, ground-state dominance relies on overlap factors

\[
Z_0 e^{-E_0 t} \left( 1 + \frac{Z_1}{Z_0} e^{-\delta t} + \ldots \right) \quad \delta \sim \frac{4\pi^2}{M L^2}
\]

For non-positive-definite correlation functions, cancellations between the ground and excited-state could in principle conspire to form a “false plateau”

See e.g. Iritani et al, JHEP 10 (2016)

First studies using positive-definite correlation functions (enabled by distillation / stochastic LapH) give results that suggest tensions with previous studies

The variational method

Correlation-function matrices for an interpolator set including both local “hexaquark” and bilocal “dibaryon” operators can generalize calculations performed to date.

Variational bounds on energy spectrum obtained by diagonalizing these matrices.

All LQCD nuclear matrix element calculations.

Although application of variational methods to multi-nucleon systems has long been advocated, it has only recently become computationally feasible through methods such as distillation and propagator sparsening.

Peardon et al PRD 80 (2009)  
Morningstar et al PRD 83 (2011)  
Detmold, MW, PRD 104 (2021)  
Li et al, PRD 103 (2021)
Hexaquark operators

Known from $\pi\pi$ scattering studies near the $\rho$ resonance that local and nonlocal operators can be nearly orthogonal

Dudek et al, PRD 87 (2013)

Calculations with $t \sim$ few fm neglecting one type of operator show plateau-like behavior but energy spectra with “missing levels” (compared to more complete calculations)

Analog of $\bar{q}(x) \Gamma q(x)$ operators - local (up to Gaussian smearing) “hexaquark”

$$H_{0ns}(t) = \sum_{\vec{x} \in \Lambda_s} \psi_n^{[H]}(\vec{x}) \varepsilon^{abcdef} \frac{1}{2} \left[ p_{1s}^{abc}(\vec{x}, t) n_{2s}^{def}(\vec{x}, t) - p_{2s}^{abc}(\vec{x}, t) n_{1s}^{def}(\vec{x}, t) + n_{1s}^{abc}(\vec{x}, t) p_{2s}^{def}(\vec{x}, t) - n_{2s}^{abc}(\vec{x}, t) p_{1s}^{def}(\vec{x}, t) \right]$$

Quark exchange symmetries very useful for reducing the number of weights

Detmold and Orginos, PRD 87 (2013)  2880 $\rightarrow$ 21
Dibaryon operators

Non-interacting two-baryon FV energy eigenstates involve color singlet baryons

\[ D_{\rho ms}(t) = \sum m \psi_m^{[D]}(\vec{x}_1, \vec{x}_2) \sum \nu^\rho_{\sigma \sigma'} \frac{1}{\sqrt{2}} \left[ p_{\sigma s}(\vec{x}_1, t) n_{\sigma' s}(\vec{x}_2, t) + (-1)^{1-\delta_{\rho 0}} n_{\sigma s}(\vec{x}_1, t) p_{\sigma' s}(\vec{x}_2, t) \right] \]

With plane-wave product wave functions

\[ \psi_m^{[D]}(\vec{x}_1, \vec{x}_2) = e^{i \vec{k}_m \cdot (\vec{x}_1 - \vec{x}_2)} \quad \vec{k}_m = \frac{2 \pi \vec{n}_m}{L} \]

Quark propagator sparsening leads to incomplete Fourier projection and mixing with higher modes, but these are negligible compared to other excited states

Amarasinghe, MW et al, arXiv:2108.10835
Quasi-local operators

What about loosely bound systems like the deuteron?

Finite-volume EFT wavefunction:

\[
\sum_{\vec{n} \in \mathbb{Z}_3} e^{-\kappa |\vec{x}_1 - \vec{x}_2 + n \vec{L}|} \left( \frac{\mathcal{A}}{|\vec{x}_1 - \vec{x}_2 + \vec{n}\vec{L}|} + \cdots \right)
\]

See e.g. Koning, Lee, and Hammer, Annals Phys. 327, 1450 (2012)

Briceño, Davoudi, Lee and Savage, PRD 88 (2013)

 Doesn’t factorize into product of single-baryon wavefunctions, no baryon blocks…

Factorizable approximation:

\[
\psi_m^{[D]}(\vec{x}_1, \vec{x}_2) = \sum_{\tau \in \mathbb{T}_S} e^{-\kappa_m |\tau(\vec{x}_1) - \vec{R}|} e^{-\kappa_m |\tau(\vec{x}_2) - \vec{R}|}
\]

\[
x_1 - x_2 \in \Lambda_S/\Lambda
\]

\[
\sum_{\vec{n} \in \mathbb{Z}^3} e^{-\kappa_m |\vec{x}_1 - \vec{x}_2 + \vec{n}\vec{L}|}
\]

\[
\psi_m^{[Q]}(\vec{x}_1, \vec{x}_2)
\]

Amarasinghe, MW et al, arXiv:2108.10835
Two nucleons in a box

Diagonalization of correlation-function matrices can be used to remove excited-state contamination from states strongly overlapping with other operators.

Each energy level dominantly overlaps with one operator structure, sub-dominant operators collectively 30%.
Building a deuteron

Spin-orbit coupling leads to the appearance of many different “orbital angular momentum” wavefunctions transforming in the same cubic group irrep

Different dibaryon and hexaquark interpolating operator structures are again approximately orthogonal

Interpolating operator sets including quasi-local alongside dibaryon operators are degenerate in both channels (at current statistical precision)

Amarasinghe, MW et al, arXiv:2108.10835
Deuteron variational energy levels

Mostly hexaquark

Cubic analog of $S$-wave

Cubic analog of $D$-wave, $G$-wave, ...

Amarasinghe, MW et al, arXiv:2108.10835
Interpolating-operator dependence

Removing the operator structure with maximum overlap on to a given energy level leads to “missing energy levels”

Even with 10s of interpolating operators, possible to “miss” ground-state — valid lower bound on ground-state energy, but best-fit results can differ by 5+ $\sigma$

Consistent with various dibaryon and hexaquark operators being approximately orthogonal

Much larger ($t \gtrsim 1/\delta \sim 5$ fm) source/sink separations would be needed to resolve spectrum using interpolating-operator set missing dominant operators

Amarasinghe, MW et al, arXiv:2108.10835
Missing state toy model

Toy model: 3 state system, 2 interpolating operators

\[ Z_n^{(A)} = (\epsilon, \sqrt{1 - \epsilon^2}, 0) \quad Z_n^{(B)} = (\epsilon, 0, \sqrt{1 - \epsilon^2}) \]

- Both operators have small overlap \( \epsilon \) with ground state
- Both have \( \approx 0 \) overlap with the state that the other has maximum overlap with

Spectrum

\[ E_0^{(AB)} = \eta - \Delta \quad E_1^{(AB)} = \eta \quad E_2^{(AB)} = \eta + \delta \]

Correlation-function matrix

\[ C^{(AB)}(t) = \sum_n \begin{pmatrix} \left( Z_n^{(A)} \right)^2 & Z_n^{(A)} Z_n^{(B)} \\ Z_n^{(A)} Z_n^{(B)} & \left( Z_n^{(B)} \right)^2 \end{pmatrix} e^{-E_n^{(AB)} t} \]

Eigenvalues:

\[ \lambda_0^{(AB)} = e^{-(t-t_0)\eta} \left[ 1 + \epsilon^2 (e^{t\Delta} - e^{t_0\Delta}) + \mathcal{O}(\epsilon^4) \right] \]

\[ \lambda_1^{(AB)} = e^{-(t-t_0)(\eta+\delta)} \left[ 1 + \epsilon^2 (e^{t(\Delta+\delta)} - e^{t_0(\Delta+\delta)}) + \mathcal{O}(\epsilon^4) \right] \]
Variational predictions

Dibaryon-dibaryon, \([D, D]\), correlation functions \(~ 95\%\) ground-state contribution in reconstruction using same operator set

Reconstructions of \([D, H]\) correlation functions using spectrum from variational methods can reproduce LQCD results

Variational method results provide model of spectrum in which \([D, H]\) correlation functions approach ground-state from below

Amarasinghe, MW et al, arXiv:2108.10835
For a given interpolating-operator set, two-nucleon finite-volume energy spectrum can be extracted in various cubic irreps associated with S-wave, D-wave, and higher-partial-wave interactions.
Variational phase shift results

Finite-volume spectrum can be mapped to S-wave, P-wave and higher-partial-wave scattering phase shifts using generalizations of Lüscher’s quantization condition.
S-wave phase shift results using variational methods and symmetric dibaryon correlation functions consistent among several groups

Discrepancies with previous results using dibaryon-hexaquark correlation functions on the same gauge-field ensemble from multiple groups

Further variational studies are needed to conclusively determine whether two-nucleon systems bind with heavier-than-physical quark masses
LQCD calculations of nuclear matrix elements can constrain EFTs and nuclear models relevant for neutrino-nucleus scattering, double-beta decay, and other new physics searches.

Low-energy excited states lead to significant and hard-to-quantify systematic uncertainties in multi-nucleon correlation functions.

Future variational studies exploring the Hilbert space possibly associated with a bound state are required to conclusively determine with two-nucleon systems bind with heavier than physical quark masses.