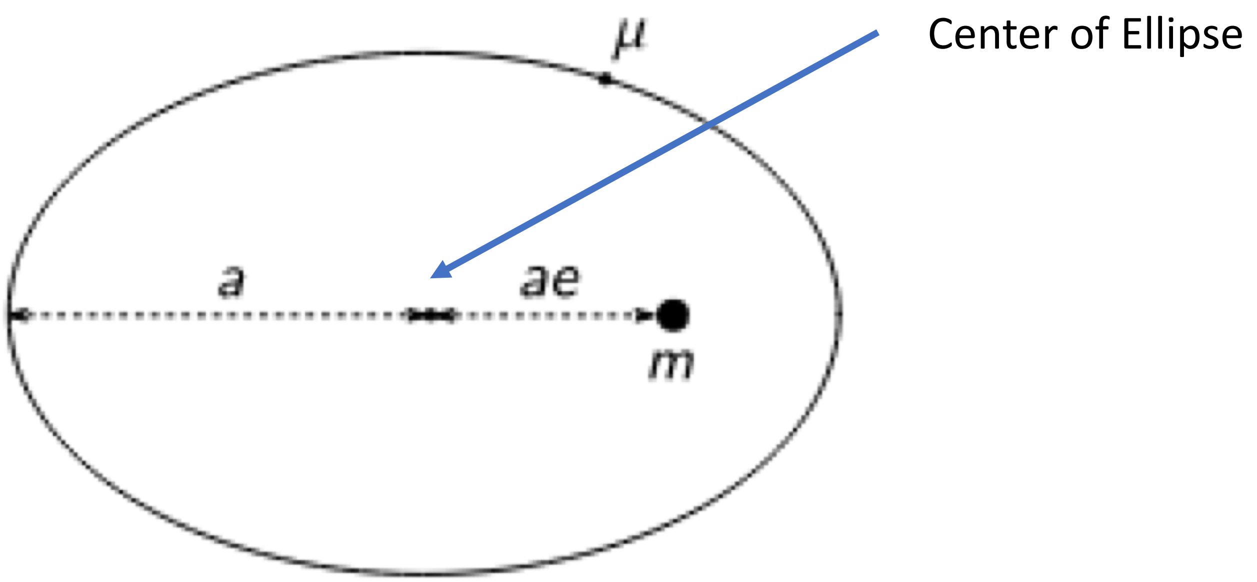
Keplerian solution for bound orbits

Keplerian elements defining the relative orbit (two point masses or spherically symmetric mass distributions), no external perturbations, Newtonian gravity

- eccentricity e
- semi-major axis a

In this image, one of the masses is m and is at one focus of the ellipse
The diagram could be draw with the other object at the focus.



Presentation describing Post Keplerian Orbital Elements
http://ipta.phys.wvu.edu/files/student-week-2017/letiec.pdf

Traditional Orbital Elements

a semimajor axis of relative orbit

e eccentricity of relative orbit

i inclination of orbital plane to line of sight

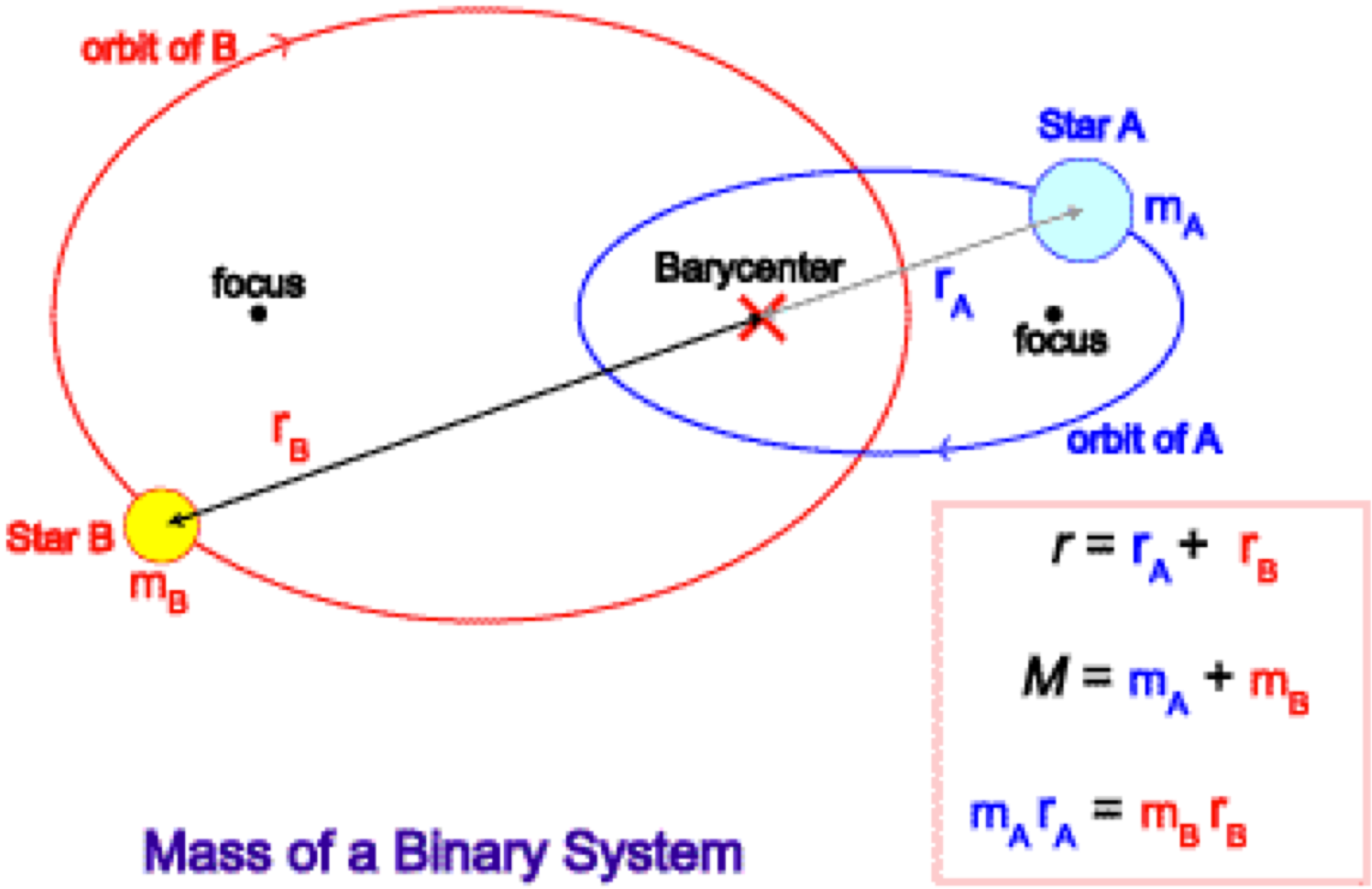
ω longitude of peri apsis from line of nodes

Ω angle between line of sight and line of nodes

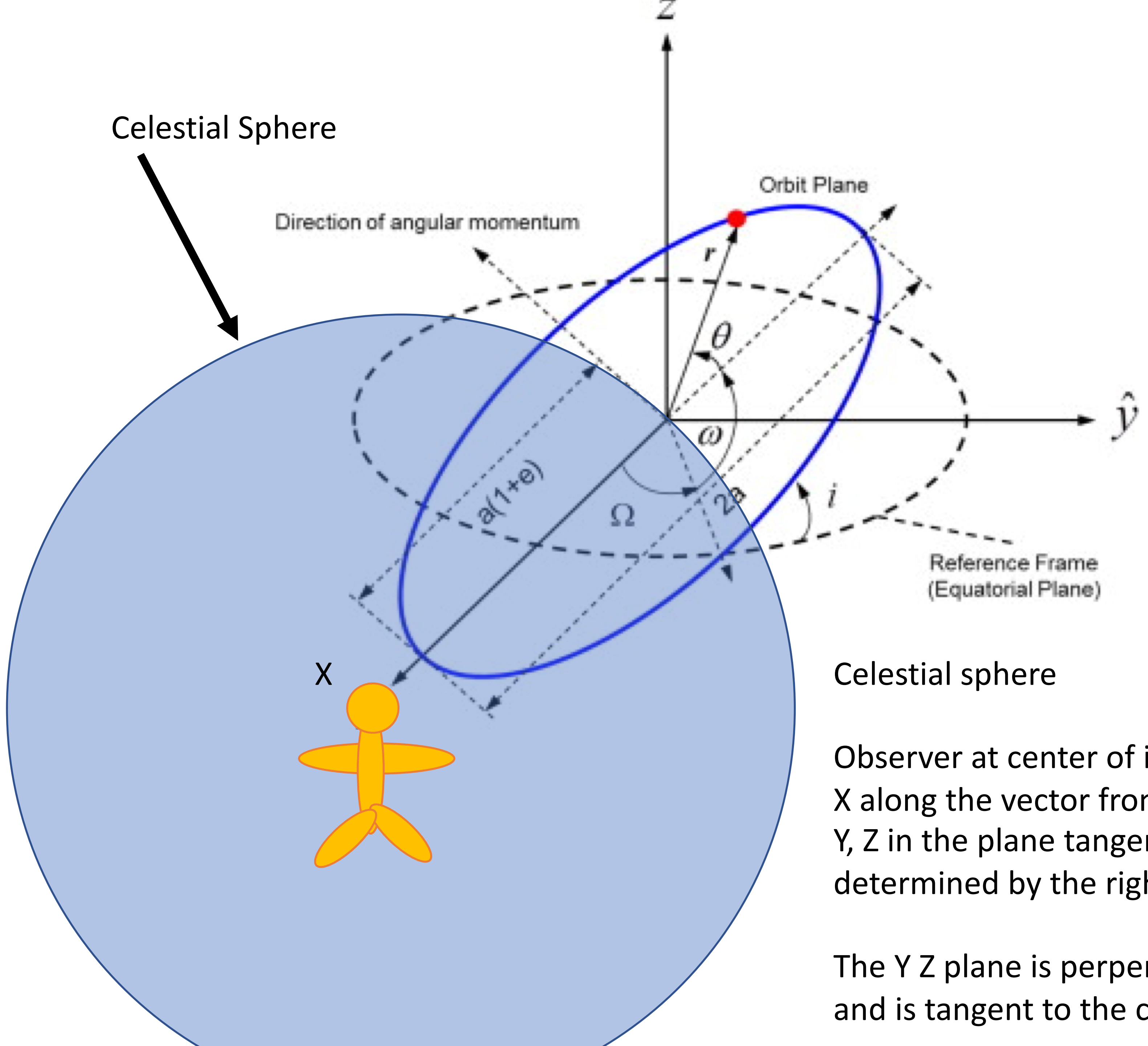
T time of peri apsis passages

The “extra” two are masses of the objects in this paper

The same thing results in visualized in the center of mass coordinate system.
The two objects move with the same period, with amplitudes of motion in inverse
Ratio to the masses.



Determination of mass of a binary system. In this system $m_A > m_B$.



Celestial sphere

Observer at center of infinite Celestial Sphere
X along the vector from Observer (center of Celestial Sphere
Y, Z in the plane tangent to the Celestial sphere, perpendicular to X, determined by the right hand rule with Z to north

The Y Z plane is perpendicular to the line of sight from the observer and is tangent to the celestial sphere

Keplerian solution for bound orbits

Keplerian elements

- eccentricity e
- semi-major axis a

Constants of motion

- energy $E = -Gm\mu/(2a)$
- ang. mom. $L = \mu\sqrt{Gma(1-e^2)}$

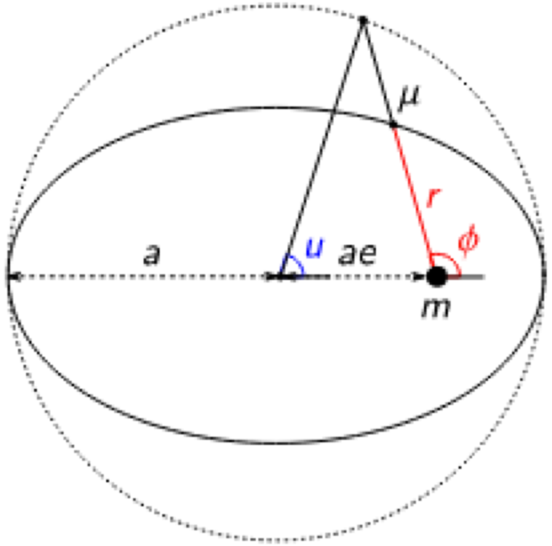
Parametric solution

$$r(u) = a\,(1 - e\cos u)$$

$$\phi(u) = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{u}{2}\right)$$

$$\nu(u) = u - e\sin u = n\,(t - t_0)$$

Kepler's third law: $n^2 a^3 = Gm$



u eccentric anomaly

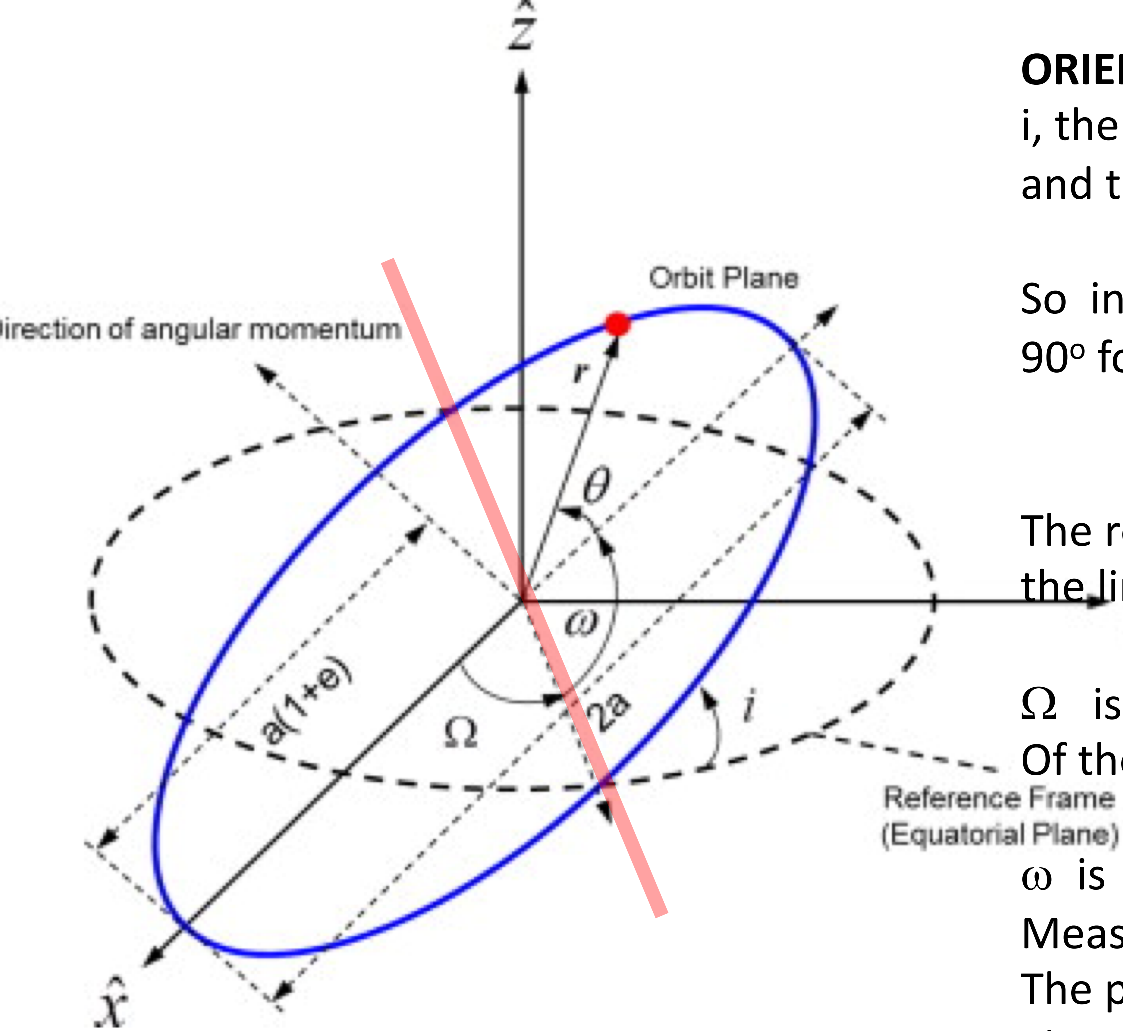
$\mu = \frac{m_1 m_2}{(m_1+m_2)}$

ϕ Is the true anomaly
(called θ elsewhere in this presentation)

ν Is the mean anomaly

Vis Viva Equation, Relative orbit

$$V_2^2 = Gm_1\sqrt{\left(\frac{2}{r} + \frac{1}{a}\right)}$$



ORIENTATION OF THE ORBIT

i, the inclination is the angle between the plane of the orbit and the tangent to the celestial sphere

So inclination is zero for a face on orbit as seen by the observer, 90° for an edge-on orbit

$$i = \arcsin(L * x)$$

The reference for the inclination is the line of nodes, the line of intersection between the orbital plane and the XY plane

Ω is the angle from from the X axis counterclockwise toward Y Of the line of nodes, measured I the XY plane

ω is the angle from the line of nodes to the major axis at periastron, Measured in the plane of the orbit (I have seen the Ω +ω given as The parameter that defines direction of periapsis, when done that way, The angles are not in the same plane, but are added)

POSITION OF OBJECTS

θ the true anomaly in this picture i.e. angle past the peri apsis (ϕ elsewhere)

T, time of periastron passage

Other Descriptive Parameters   Describing the position of the system, vs physics

TABLE III. VLBA astrometric results. Listed are right ascension (R.A.), declination (Dec.), proper motion in both coordinates, parallax and position epoch.

R.A., α (J2000)	$07^{h}37^{m}51\overset{s}{.}247(1)$
Dec, δ (J2000)	$-30°39'40\overset{''}{.}68(1)$
Parallax, π_v (mas)	$1.30^{+0.13}_{-0.11}$
Proper motion in R.A., μ_α (mas yr^{-1})[a]	-2.567 ± 0.030
Proper motion in Decl., μ_δ (mas yr^{-1})[a]	2.082 ± 0.038
Position epoch (MJD)	58000

[a]A prior based on the timing proper motion was applied as described in Appendix A.

The stationary situation is affected by tidal forces, relativistic effects etc.

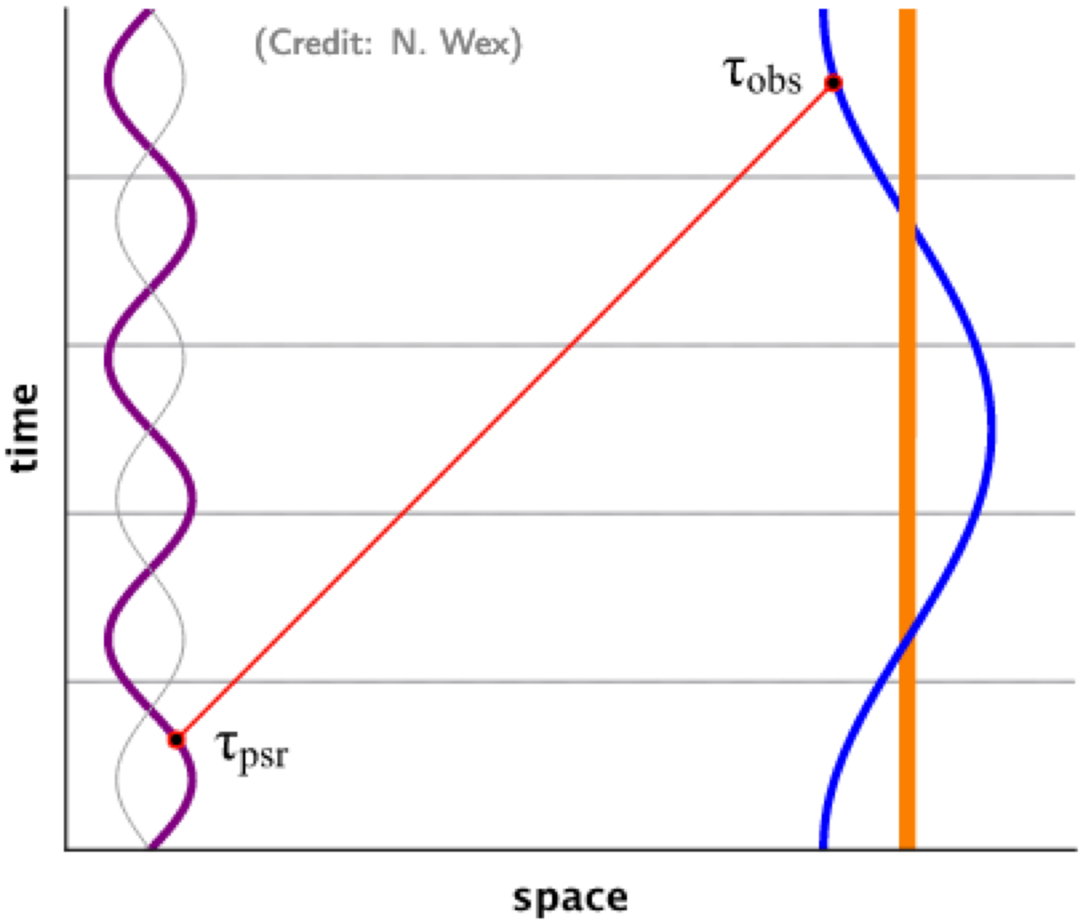
External force, general relativity result in changes to the orbital elements, and there are forms for the rate of change of each element due to force in a particular direction.

The elements remain defined even as they are changing. Sometimes the instantaneous situation is defined as Osculating elements.

What is observed for a pulsar?

Time of arrival of pulses.

The problem of binary pulsar timing



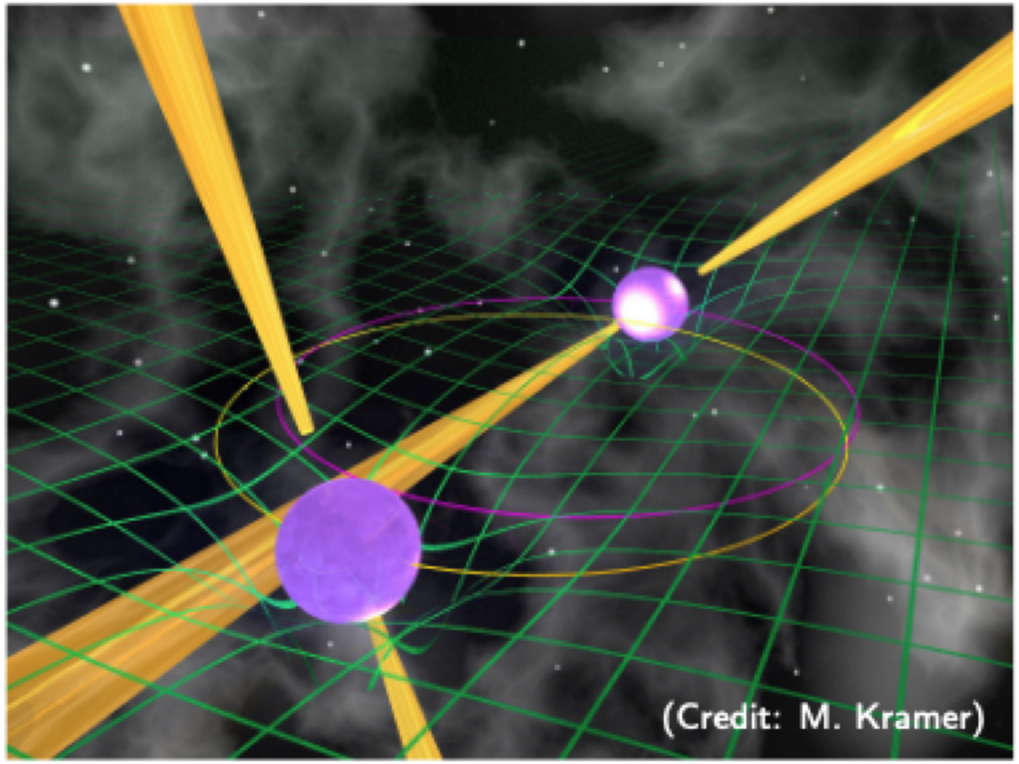
Component of pulsars' orbital motion in the direction to the observer causes ongoing dominant variation of time of arrival (TOA).

Observer's position has several periodic (and not) changes in distance to the center of mass of a binary pulsar.

Orbit around the Sun has yearly variation, Earth's rotation has at least sidereal rotation period variation, plus a variety of lower amplitude, more complex variation.

The double pulsar PSR J0737-3039

[Burgay *et al.*, Nature 2003]



23-ms pulsar ("A") and the second-born 2.8-s pulsar ("B")
Relative orbit 2.45 hr.