High-PT Observables at the EIC with QED and QCD Factorization

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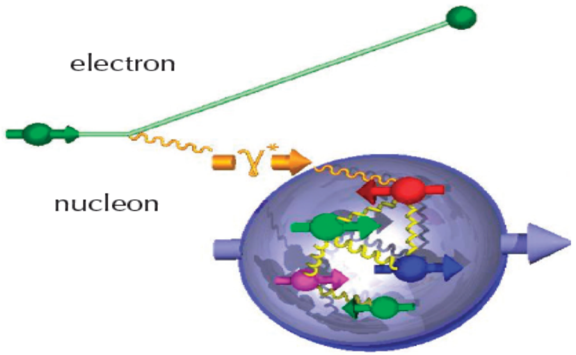
In collaboration with: Tianbo Liu, Wally Melnitchouk, Nobuo Sato, Kazuhro Watanabe, Zhite Yu, …

High energy lepton-hadron scattering

- The new generation of "Rutherford" experiments for probing hadron structure:



✧ A controlled clean "probe" – the virtual photon (x_B, Q^2)

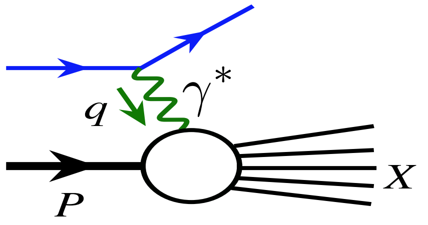
✧ Can either break *or not break* the hadron

Many high-energy lepton-hadron facilities have been built, or to be built, at SLAC, CERN, FNAL, DESY, JLab, BNL, …

✧ <u>Inclusive events</u>: e+p/A → e'+X

Detect only the scattered lepton in the detector

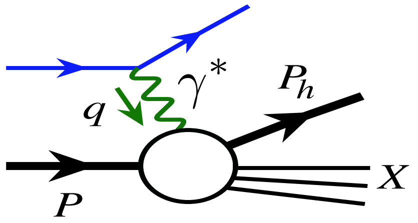
(Modern Rutherford experiment!)



✧ <u>Semi-Inclusive events</u>: e+p/A → e'+h(p,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets

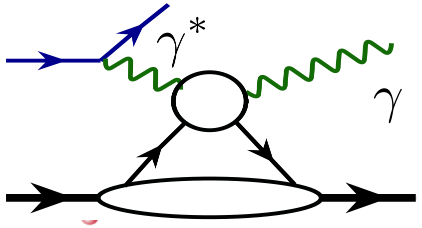
(Initial hadron is broken – confined motion! – cleaner than h-h collisions)



✧ <u>Exclusive events:</u> e+p/A → e'+ p'/A'+ h(p,K,p,jet)

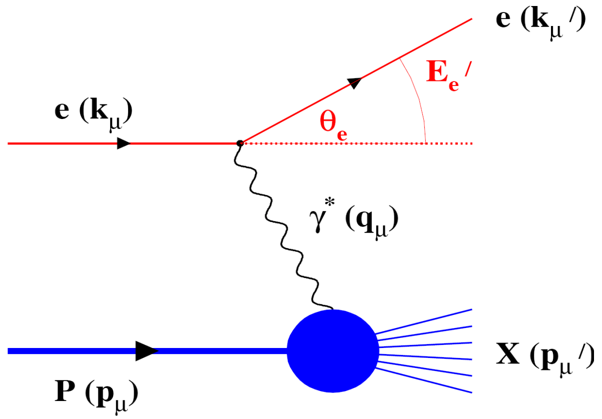
Detect every things including scattered proton/nucleus (or its fragments)

(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)



Lepton-hadron inclusive deep inelastic scattering (DIS)

Approximation of one-photon exchange:



Q^2 = - (k-k')^2 → Measure of the resolution

y = P.(k-k')/P.k → Measure of inelasticity

x_B = Q^2/2P.(k-k') → Measure of momentum fraction of the struck quark in a proton

$Q^2 = S\, x_B\, y$

$$E'\frac{d\sigma}{d^3k'} = \frac{2\alpha_{\rm EM}^2}{s}\frac{1}{Q^4}L^{\mu\nu}(k,k;q)W_{\mu\nu}(q,P)$$

$$L^{\mu\nu}(k,k;q) = 2(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}) + \text{spin...}$$

Deep inelastic scattering (DIS) structure functions:

$$W_{\mu\nu}(q,P) = \frac{1}{4\pi}\sum_X (2\pi)^4\delta^4(P+q-X)\langle P|J_\mu(0)|X\rangle\langle X|J_\nu(0)|P\rangle + \text{spin...}$$

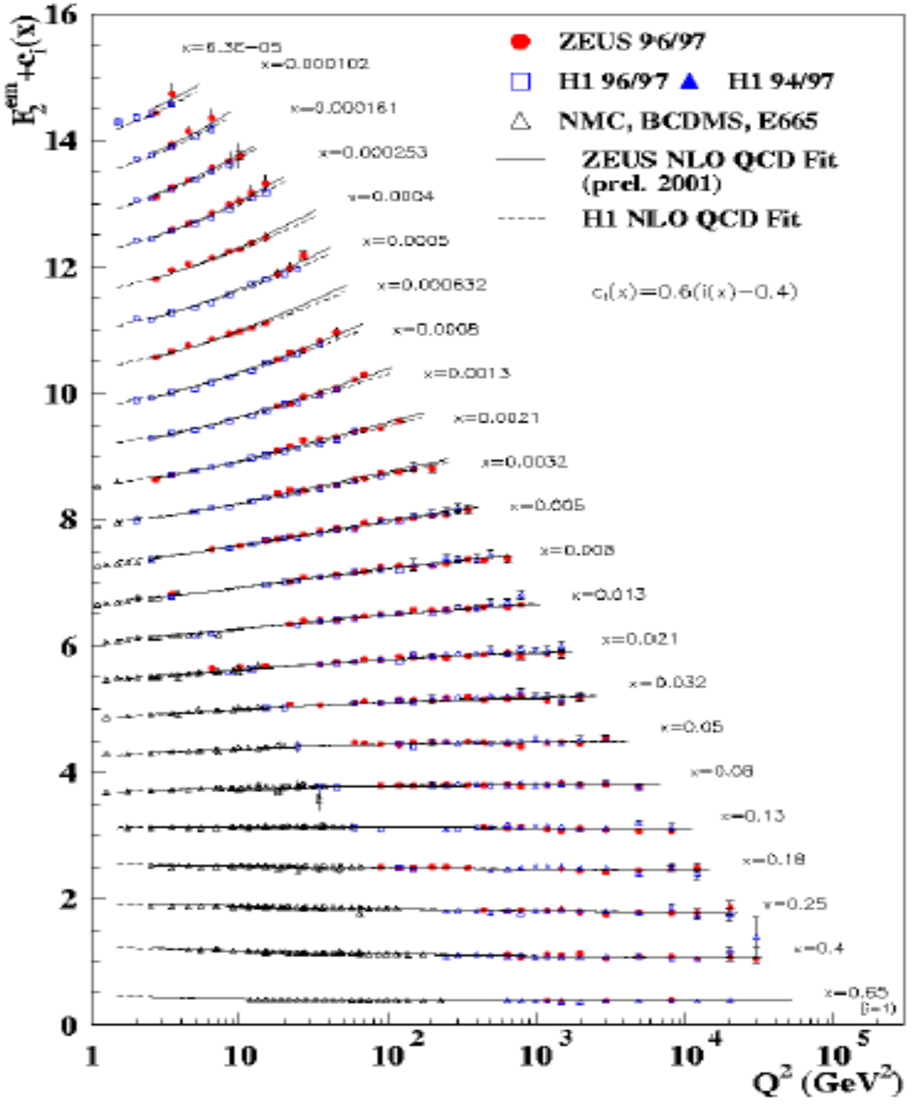
$$= -\widetilde{g}_{\mu\nu}F_1(x_B,Q^2) + \frac{\widetilde{P}_\mu\widetilde{P}_\nu}{P\cdot q}F_2(x_B,Q^2) + \text{spin...}$$

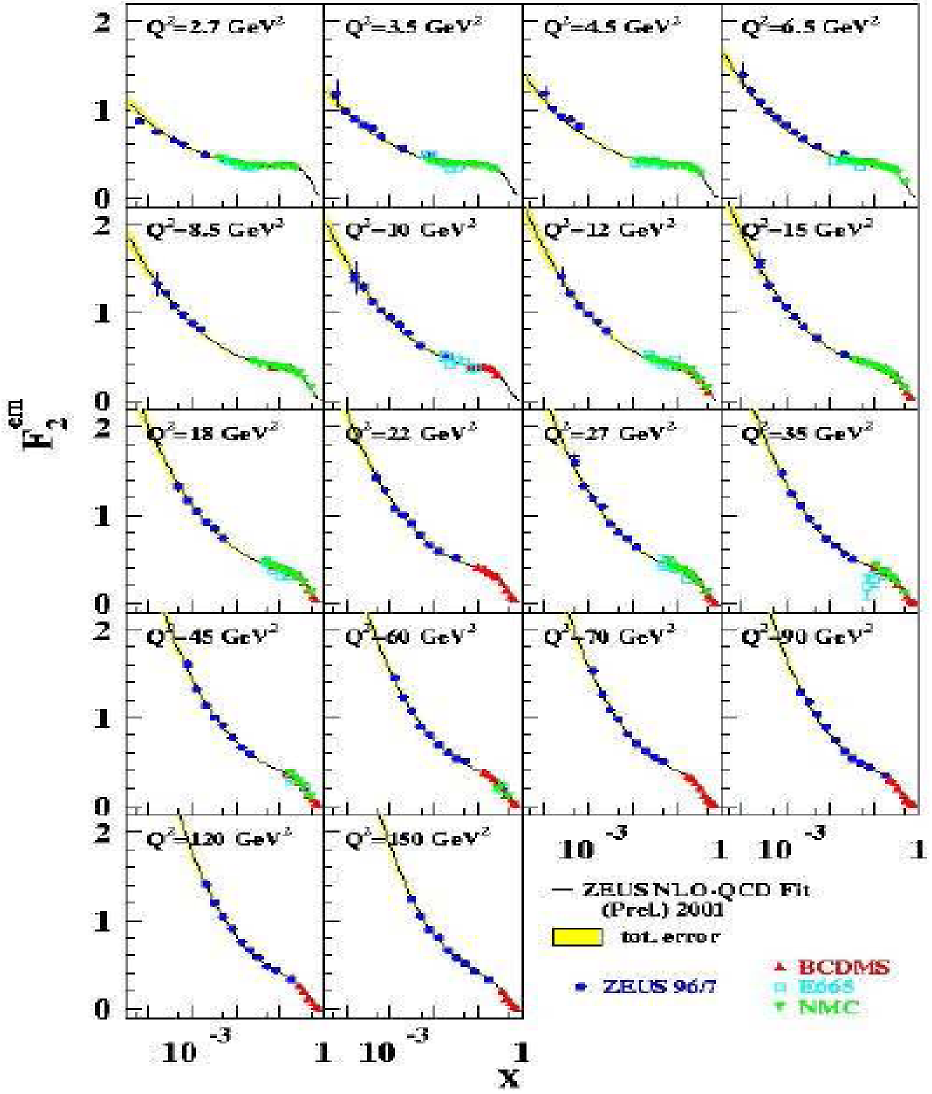
$$\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2 \qquad \widetilde{P}_\mu = \widetilde{g}_{\mu\nu}P^\nu$$

QCD Factorization - Approximation

$$F_i(x_B,Q^2) \approx \sum_f C_{if}(x_B,Q^2;x,\mu^2)\otimes f(x,\mu^2) + \mathcal{O}(1/Q^2)$$

Lepton-hadron inclusive deep inelastic scattering (DIS)





A very successful story of QCD, QCD Factorization, and QCD evolution!

But, this success depends on how precise we were able to treat collision-induced QED radiation!

Outline of the rest of my talk

- **Collision induced QED radiation is an important part of ep Physics at the EIC**
- **Inclusive ep deep inelastic scattering (DIS)**

  => Inclusive production of single high-PT electron in ep collision

  Collinear QED and QCD factorization

- **Single hadron (or jet) photoproduction in ep collision**

  => Inclusive production of single high-PT hadron (or jet) in ep collision

  Collinear QED and QCD factorization

- **Lepton-hadron (ep) Semi-inclusive DIS (SIDIS)**

  => Inclusive production of a pair of high-PT lepton and hadron in ep-collision

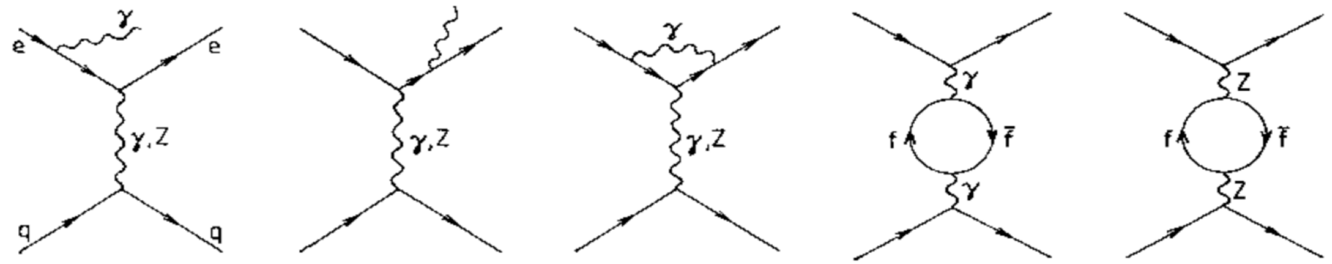
  Hybrid (collinear QED) and (TMD QCD) factorization

- **Summary and outlook**

Collision with a large momentum transfer induces strong QED radiation

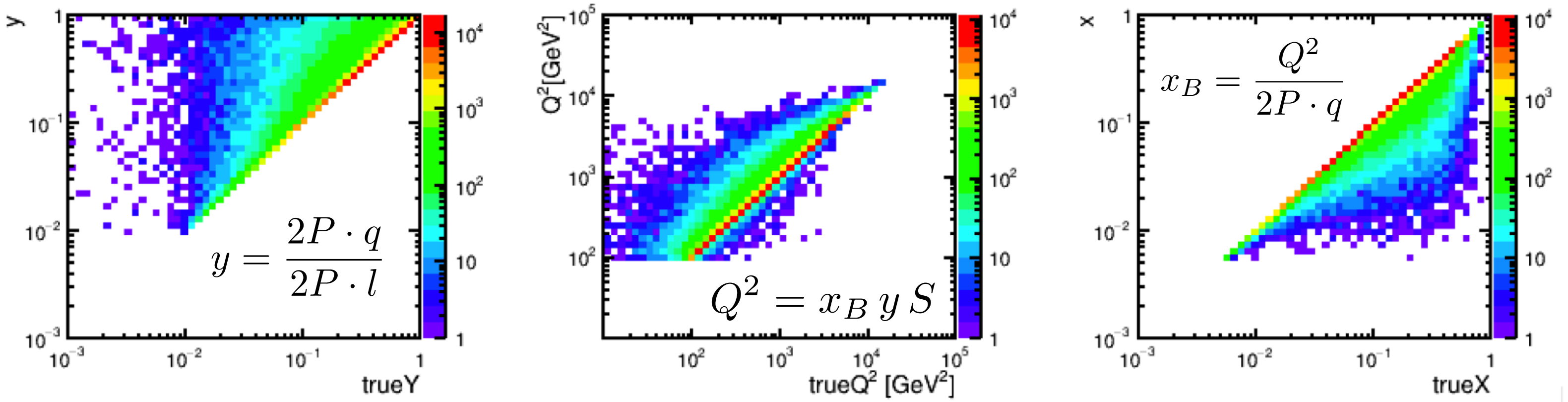
- "Probe" for the hadron is smeared by the induced QED radiation:

Data sample : Int L = 10 fb^{-1}, Kinematics settings: 0.01<y<0.95, 10^2 GeV^2 <Q^2<10^5 GeV^2



initial **final** **vacuum** **loops**

See Xiaoxuan Chu @2nd EIC YR workshop



Instead of a straight line – linear correlation,

the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much

to make them different from what the scattered "quark" experienced!

Ill-defined "photon-hadron" frame?!

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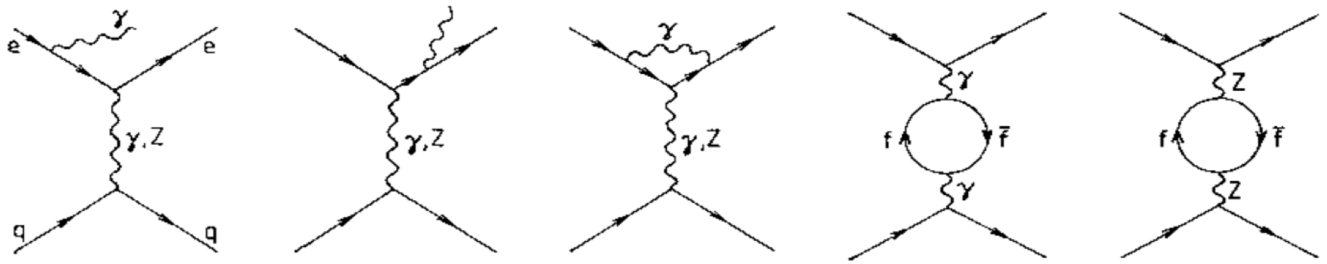

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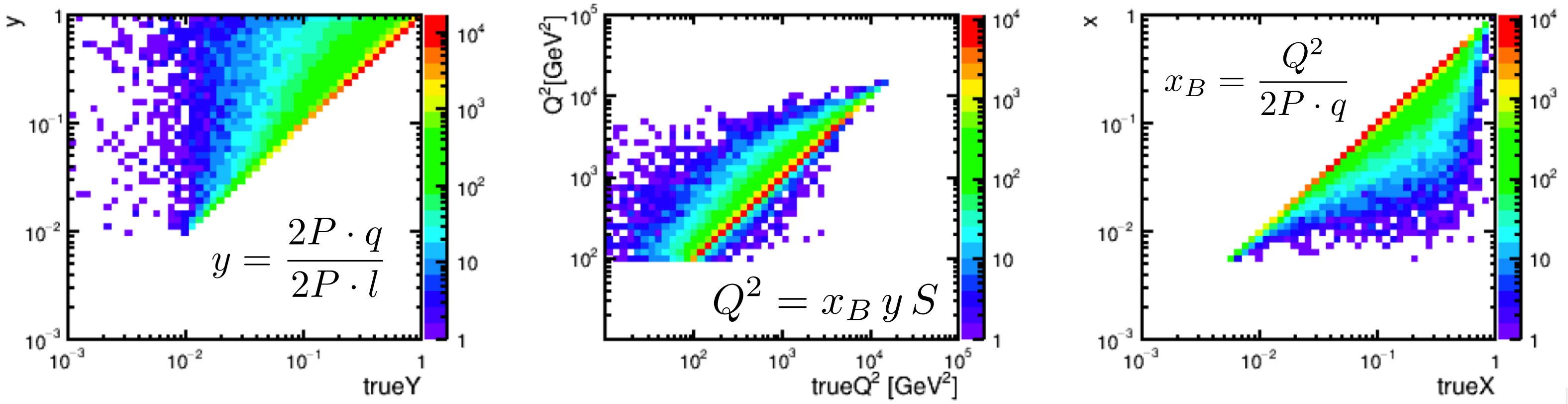
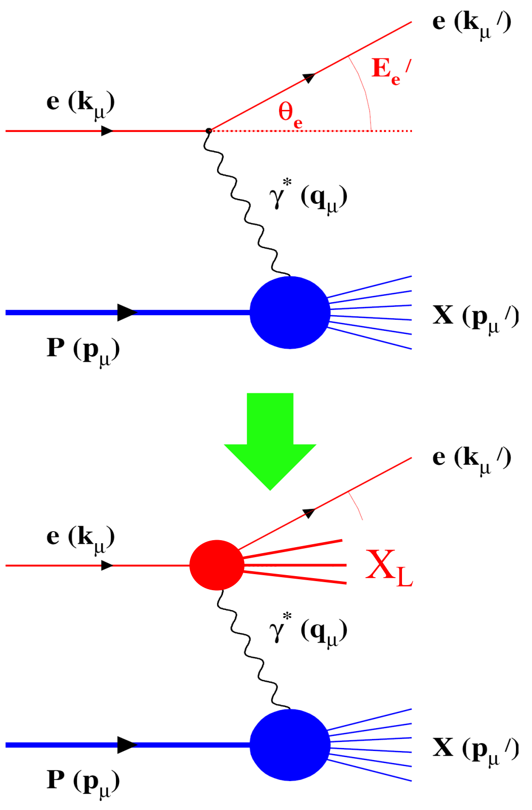




$$q_\mu \to \hat{q}_\mu$$

$$Q^2 = -q^2 \to \widehat{Q}^2 = -\hat{q}^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \to \hat{x}_B = \frac{\widehat{Q}^2}{2P\cdot \hat{q}}$$

Instead of a straight line – linear correlation,

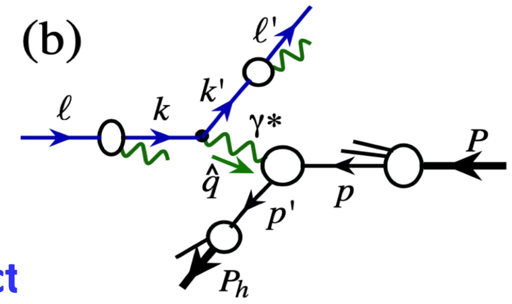
the kinematic variables, y, Q^2 , x_B, from the leptons are smeared so much to make them different from what the scattered "quark" experienced!

Ill-defined "photon-hadron" frame?!

No simple radiative correction for SIDIS

❑ Radiative correction – Born kinematics:

$$\sigma_{\text{Measured}} \equiv \text{RC} \otimes \sigma_{\text{No QED Radiation}}$$



Necessary requirement: RC – Radiative correction factor
does not depend on the hadronic physics that we want to extract

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No simple radiative correction for SIDIS

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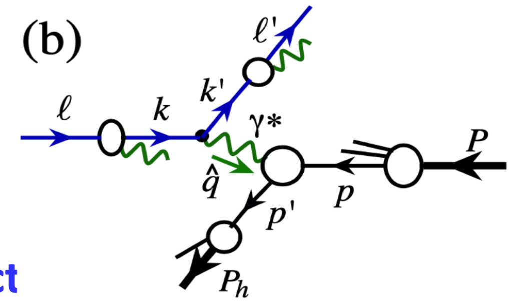
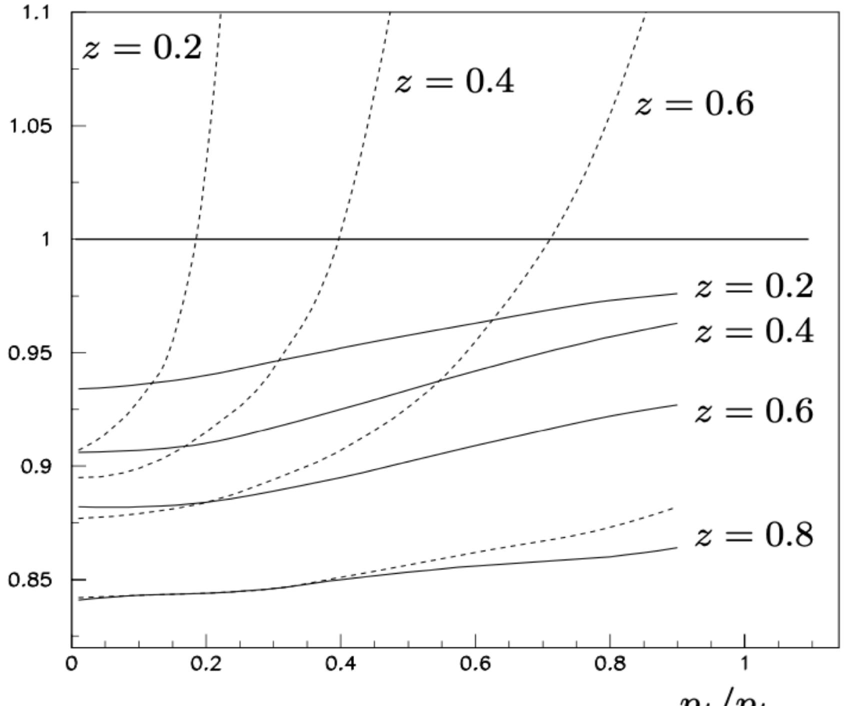


Necessary requirement: **RC – Radiative correction factor does not depend on the hadronic physics that we want to extract**

❑ Impact of QED radiation to SIDIS – order of α_{EM}:

$$\bar{\delta} = \sigma_{\text{Measured}} / \sigma_{\text{No QED Radiation}}$$



$\sqrt{S} = 7.19$ GeV, $x_B = 0.15$, $Q^2 = 4$ GeV^2

$$e(l) + N(P) \rightarrow e'(l') + \gamma(k) + h(P_h) + X$$

I. Akushevich et al. EPJ C10 (1999) 681

Dashed line:

Gaussian pT-dependence

$$b\,\exp(-b\,p_t^2) \qquad \text{where} \quad b = R^2/z^2$$

Solid line:

Power pT-dependence

$$\left[\frac{1}{a + b\,z + p_t^2}\right]^{c+d\,z} \qquad \text{parameters:} \quad R, a, b, c, d$$

$\bar{\delta}$ **depends on physics we want to extract!**

NO simple RC for SIDIS!

QED radiative corrections vs. QED radiative contributions

❑ QED radiative corrections:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\sigma_{\rm obs}(x_B, Q^2) \stackrel{?}{=} R_{\rm QED}(x_B, Q^2; x_{B,{\rm true}}, Q^2_{\rm true}) \times \sigma_{\rm Born}(x_{B,{\rm true}}, Q^2_{\rm true}) + \sigma_X(x_B, Q^2)$$

- The correction factors R_{QED} and σ_x should not depend on the hadron structure that we wish to extract, and they can be systematically calculated in QED to high precision (not satisfied);
- The effective scale Q^2_{true} for the Born cross section σ_{Born} should be large enough to keep the "true" scattering within the DIS regime (questionable);
- Extraction of $\sigma_{\rm Born}$ is an inverse problem

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❑ QED radiative contributions:

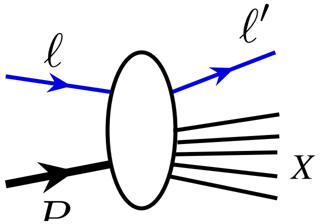
$$\sigma_{\rm obs}(x_B, Q^2) = \sigma^{\rm univ}_{\rm lep}(\mu^2; m_e^2) \otimes \sigma^{\rm univ}_{\rm had}(\mu^2; \Lambda^2_{\rm QCD}) \otimes \widehat{\sigma}_{\text{IR-safe}}(\hat{x}_B, \widehat{Q}^2, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}, \frac{m_e^2}{Q^2}\right)$$

- Infrared sensitive QED contributions – divergent as $m_e/Q \to 0$, are absorbed to universal LDFs and LFFs
- Infrared safe QED contributions – finite as $m_e/Q \to 0$, are calculated order-by-order in power of α
- Power suppressed contributions as $m_e/Q \to 0$, are neglected

Predictive power: Universality of LDFs and LFFs, their evolution, calculable hard parts
Neglect power corrections

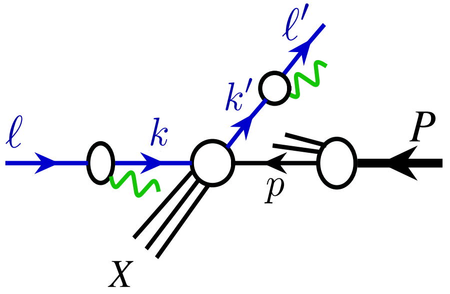
Inclusive lepton-hadron deep inelastic scattering (DIS)

❑ Inclusive production of single high p_T lepton in lepton-hadron collision:



$$e(\ell,\lambda_\ell) + N(P,S) \to e(\ell') + X$$

$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X} = \frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$



$$E'\frac{d\sigma_{\ell P\to\ell' X}}{d^3\ell'} \approx \frac{1}{2s}\sum_{ija}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi} D_{e/j}(\zeta,\mu^2)\, f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x} f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell/\zeta,\mu^2) + \cdots$$

Collinear QED & QCD factorization

Lepton distribution functions (LDFs): $f_{i/e}(\xi,\mu^2)$

Lepton fragmentation functions (LFFs): $D_{e/j}(\zeta,\mu^2)$ $\quad i.j = e,\gamma,\bar{e},\ldots,q,g,\ldots$

Parton distribution functions (PDFs): $f_{a/N}(x,\mu^2)$ $\quad a = q,g,\bar{q},e,\gamma,\bar{e},\ldots$

Short-distance hard coefficients: $\widehat{H}_{ia\to jX}(\xi\ell, xP, \ell/\zeta,\mu^2)$

Photon is charge neutral
QED factorization works

$$\approx \widehat{H}^{(m,n)}_{ia\to jX}(\xi\ell, xP, \ell/\zeta,\mu^2) \approx \mathcal{O}(\alpha^m\alpha_s^n)$$

Nayak, Qiu, Sterman, Phys.Rev.D 72 (2005) 114012

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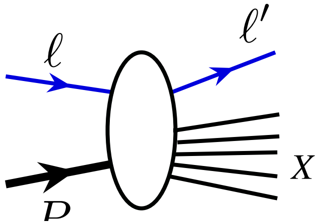

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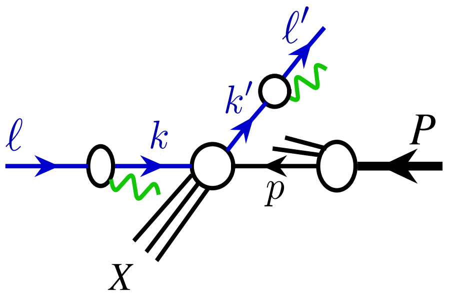

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Photon is charge neutral
QED factorization works

- **No DIS "Structure Functions"!**

  Concept of one-photon exchange

- **QED & QCD contribution are factorized at the same scale:** μ

  $$(x_B, Q^2) \;\to\; (y, \ell'_T)$$

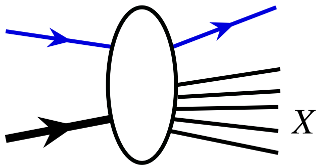
- **Corrections suppressed by power**

  $$(1/\ell'_T)^\alpha$$

Nayak, Qiu, Sterman, Phys.Rev.D 72 (2005) 114012

Inclusive lepton-hadron deep inelastic scattering (DIS)

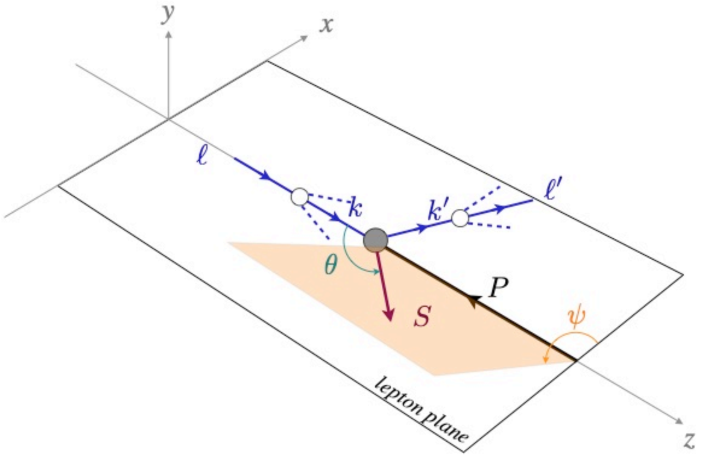
- **Inclusive production of single high p_T lepton in lepton-hadron collision:**



$$e(\ell,\lambda_\ell) + N(P,S) \to e(\ell') + X$$

$$d\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X} = \frac{1}{2s}\left|M_{\ell(\lambda_\ell)P(S)\to\ell' X}\right|^2 d\mathrm{PS}$$

- **Recover the concept of structure functions?**



$$E_{\ell'}\frac{d^3\sigma_{\ell(\lambda_\ell)P(S)\to\ell' X}}{d^3\ell'} \approx \sum_{\lambda_k}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}D_{e/e}(\zeta,\mu^2)\int_{\xi_{\min}}^{1}d\xi\, f_{e(\lambda_k)/e(\lambda_\ell)}(\xi,\mu^2) \times\left[E_{k'}\frac{d^3\hat{\sigma}_{k(\lambda_k)P(S)\to k' X}}{d^3k'}\right]_{k=\xi\ell,\, k'=\ell'/\zeta},$$

$$E_{k'}\frac{d^3\hat{\sigma}_{k(\lambda_k)P(S)\to k' X}}{d^3k'} \approx \frac{2\alpha^2}{\hat{s}\,\widehat{Q}^4}L^{(0)}_{\mu\nu}(k,k',\lambda_k)\,W^{\mu\nu}(\hat{q},P,S)$$

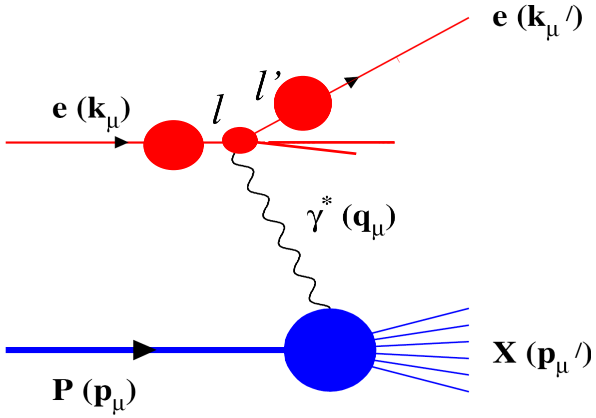
$$W^{\mu\nu}(\hat{q},P,S) = -\widetilde{g}^{\mu\nu}(\hat{q})\,F_1(\hat{x}_B,\widehat{Q}^2) + \frac{1}{P\cdot\hat{q}}\widetilde{P}^{\mu}(\hat{q})\widetilde{P}^{\nu}(\hat{q})F_2(\hat{x}_B,\widehat{Q}^2) + \ldots$$

Structure functions are evaluated at $(\hat{x}_B,\widehat{Q}^2)$ **instead of** (x_B,Q^2)**!**

Collinear factorization for QED radiative contribution

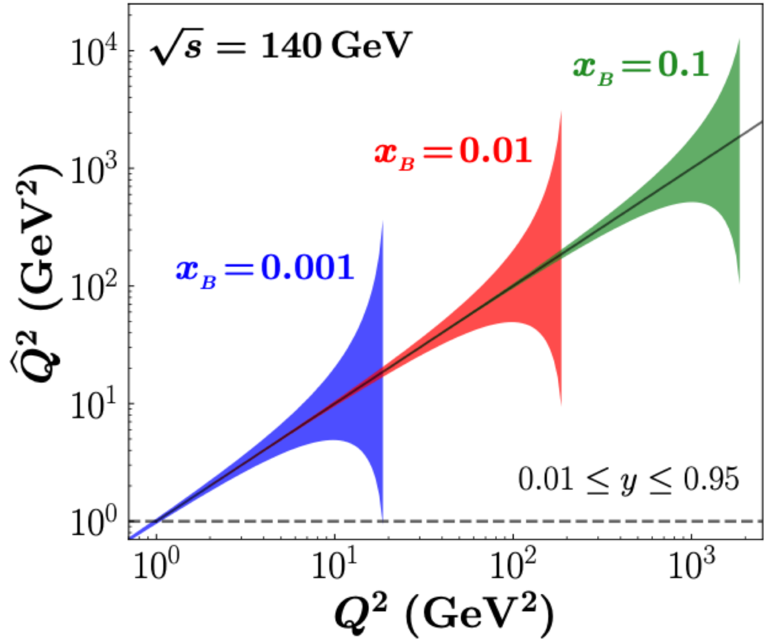
❑ Collinear factorization with the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right] \times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

- QED radiation prevents a well-defined "photon-hadron" frame
- Radiation is CO sensitive as $m_e/Q \to 0$, factorized into LDFs & LFFs
- Hadron is probed by $(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$



$x_B \to \hat{x}_B \in [x_B, 1]$ $\quad$ $\widehat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)}$ $\quad$ $\widehat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$

A simple RC factor at x_B is necessarily sensitive to hadronic information from [x_B, 1] !

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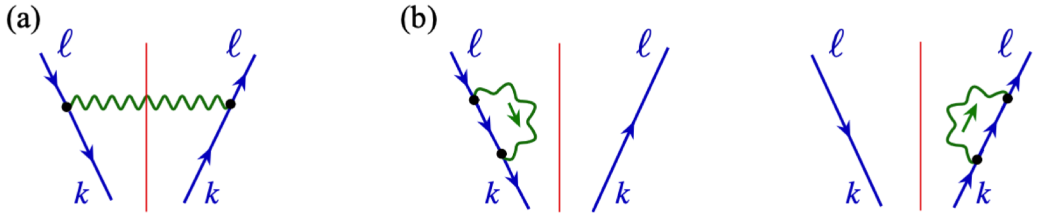
QED Radiative Corrections vs Radiative Contributions

❑ Lepton distribution function:



$$f_{e/e}^{(1)}(\xi,\mu^2) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …

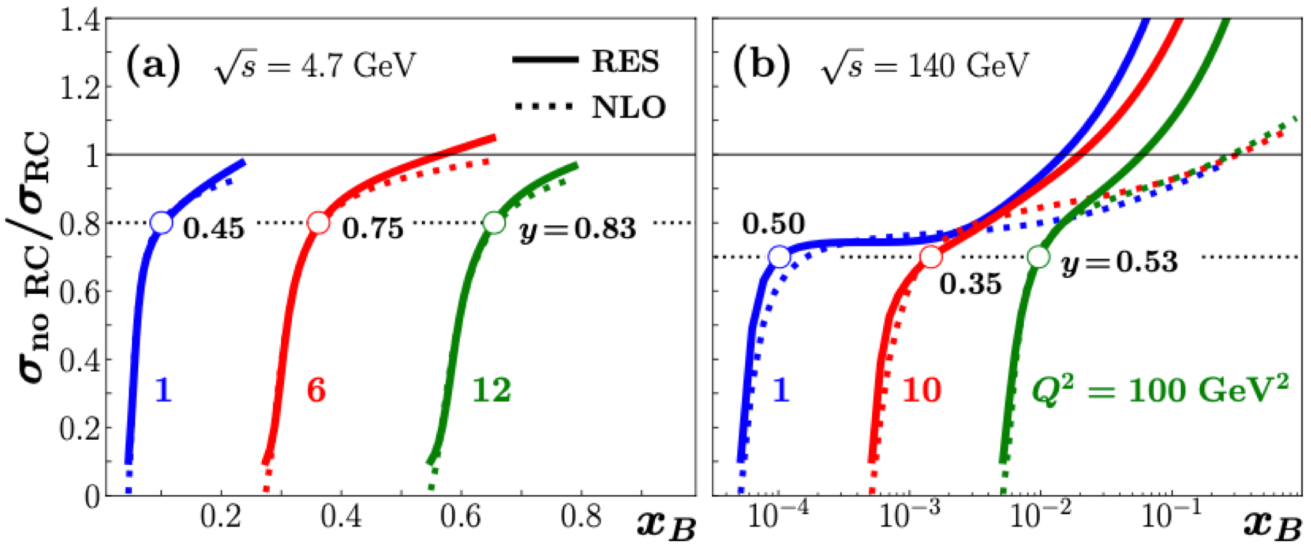
❑ Lepton evolution – e.g., valence:

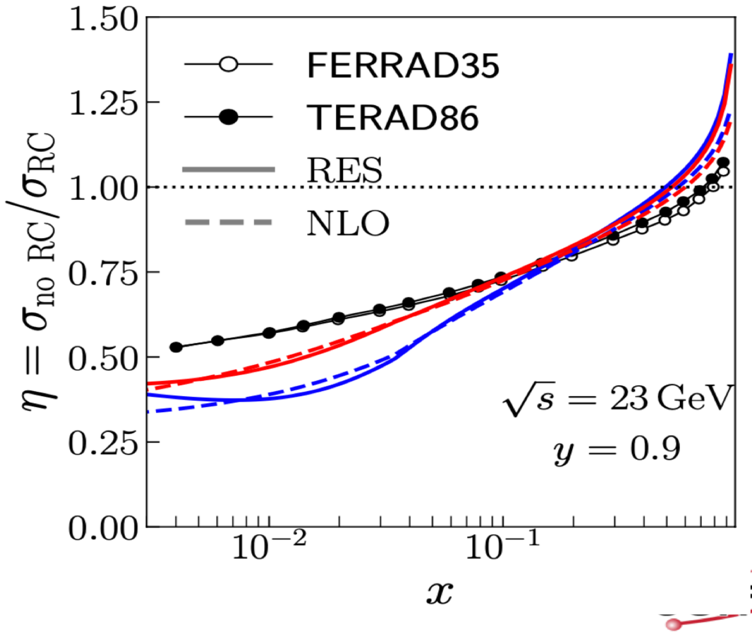
$$\mu^2\frac{\mathrm{d}}{\mathrm{d}\mu^2} f_{e/e}(\xi,\mu^2) = \int_\xi^1 \frac{\mathrm{d}\xi'}{\xi'} P_{ee}\left(\frac{\xi}{\xi'},\alpha\right) f_{e/e}(\xi',\mu^2)$$

❑ Lepton fragmentation function:

$$D_{e/e}^{(1)}(\zeta,\mu) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

+ nonperturbative contributions …





Collinear factorization for QED radiative contribution

Liu, Melnitchouk, Qiu, Sato
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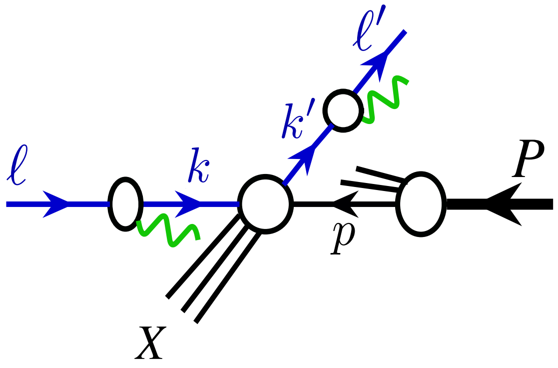
❑ Without the "one-photon" approximation:

~ Inclusive single lepton production at high transverse momentum

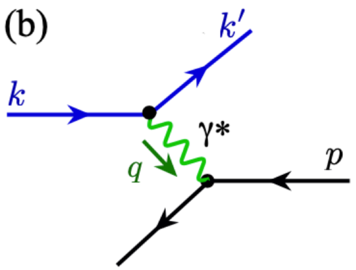


$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2)\;+\;\cdots$$

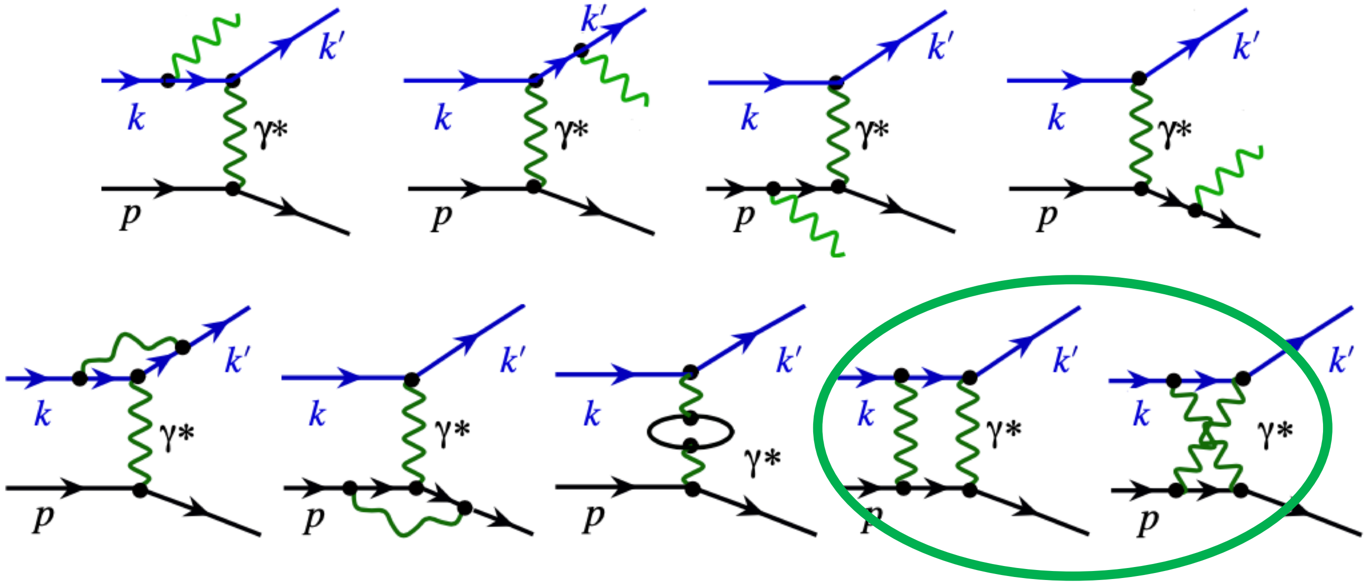
No structure functions, but have PDFs, LDFs, LFFs, …

❑ Calculated hard parts in power of $\alpha^m\alpha_s^n$:

LO



NLO:

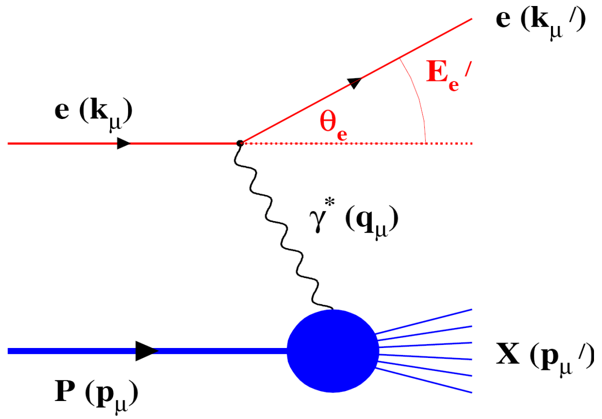


More systematic for PVDIS!

Beyond one-photon exchange

Single hadron (or jet) photoproduction in ep collision

❑ Photoproduction in ep collision is sensitive to how the "photon" is defined:



- Real or quasi-photon is defined by

$$k'_T \leq k_{T_{\rm cut}} \qquad \text{or} \qquad \theta_e \leq \theta_{\rm cut}$$

- Photon flux is derived by

  Evaluating the photon shower with above "cut"
  Weizsaecker-Williiams photon distribution, …

Jefferson Lab

Single hadron (or jet) photoproduction in ep collision

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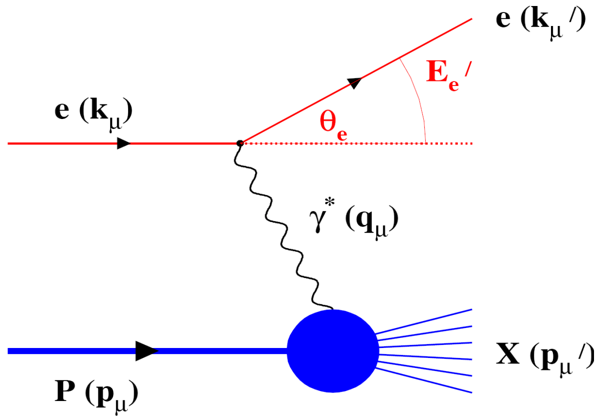


- Real or quasi-photon is defined by

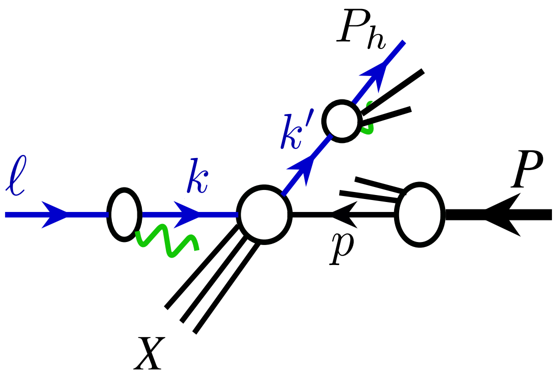
  $k_T' \leq k_{T_{\rm cut}}$ or $\theta_e \leq \theta_{\rm cut}$

- Photon flux is derived by

  Evaluating the photon shower with above "cut"
  Weizsaecker-Williiams photon distribution, …

❑ Inclusive single hadron (jet) production in ep collision:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



With measuring the scattered electron!

Single hard scale, collinear factorization

$$E_h \frac{d\sigma_{\ell P\to P_h x}}{d^3 P_h} = \frac{1}{2s}\sum_{i,a,b}\int_{z_{\rm min}}^1 \frac{dz}{z^2}\int_{\xi_{\rm min}}^1 \frac{d\xi}{\xi} D_{h/b}(z,\mu^2) f_{i/e}(\xi,\mu^2)$$
$$\times \int_{x_{\rm min}}^1 \frac{dx}{x} f_{a/N}(x,\mu^2)\, \widehat{H}_{ia\to bX}(\xi\ell, xP, P_h/z, \mu^2) + \ldots$$

- Universal lepton distribution functions (LDFs)
- No artificial cut to define the "photon"
- Single factorization scale: μ

Single hadron (or jet) photoproduction in ep collision

❑ Evolution of lepton distribution functions (LDFs):

Qiu, Watanabe
In preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $P_{ij}^{(0,0)} = 0\,,\quad N_F,\quad N_l$

- **Factorization scale:**

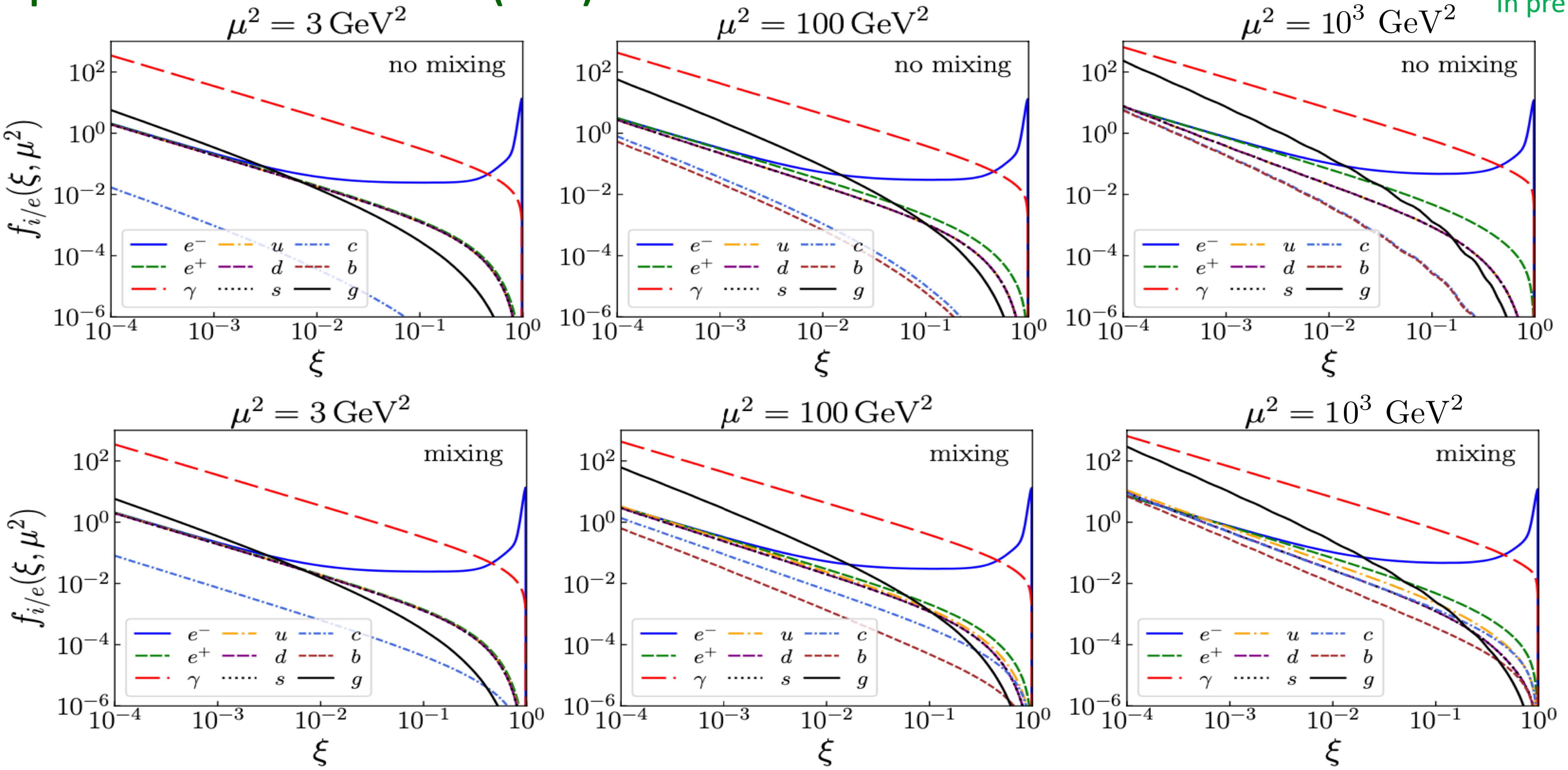
  $\mu^2 \sim m_c^2$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - **…**

Single hadron (or jet) photoproduction in ep collision

- **Lepton distribution functions (LDFs):**

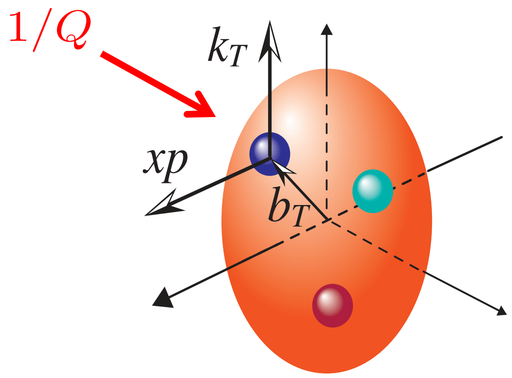
Qiu, Watanabe
In preparation



With LDFs, we calculated single hadron production, including J/ψ production at the EIC

3D-hadron structure – need probes with two scales

- Single-scale hard probe is too "localized":



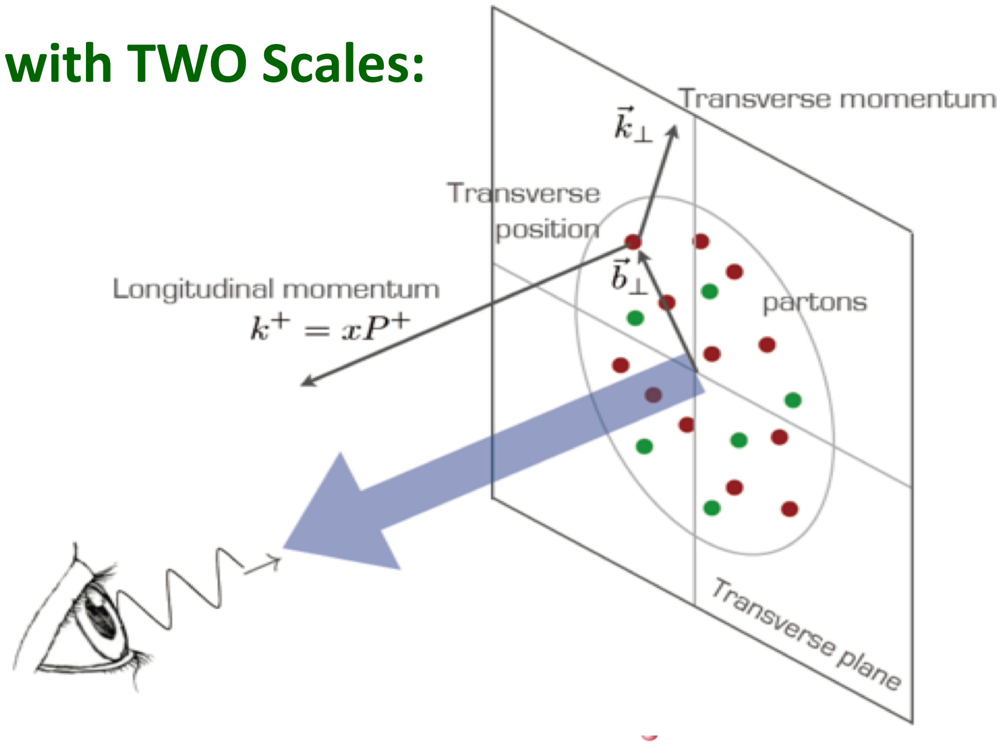
- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Confined transverse motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> $1/Q$

- Need new type of "Hard Probes" – Physical observables with TWO Scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\text{QCD}}$$

Hard scale: Q_1 To localize the probe – factorization
particle nature of quarks/gluons

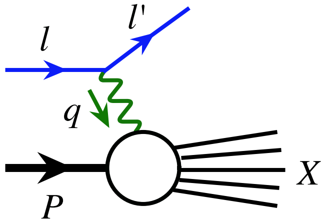
"Soft" scale: Q_2 could be more sensitive to the
hadron structure ~ 1/fm



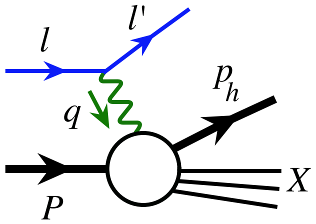
- New challenge:

QCD Factorization for observables with two scales!

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

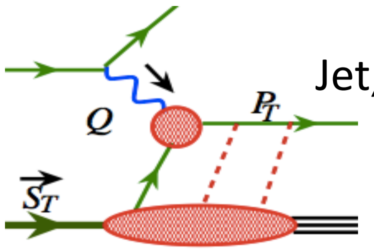


Scale: Q^2 **- PDFs**



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

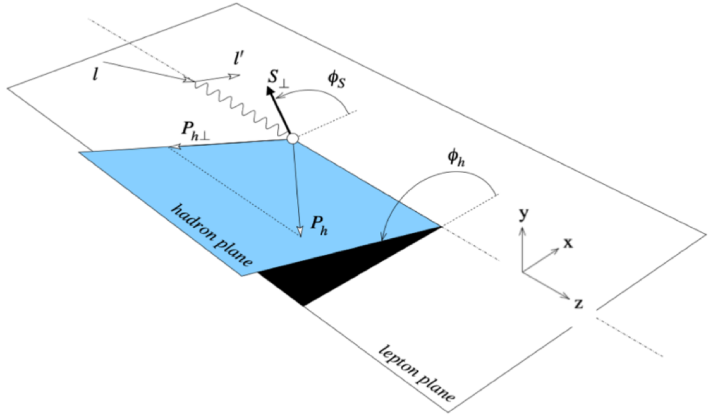


Jet, π, J/ψ, …

$f(x, k_T, Q)$ **- TMDs**

Parton's confined motion, …

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ & F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& +S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
& +S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
& +|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad +\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad +\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& +|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad +\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

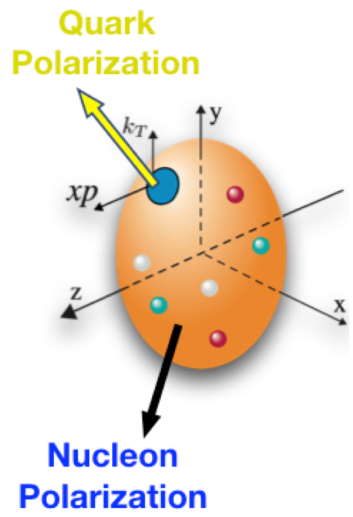


18 SIDIS
Structure Functions

Transverse momentum dependent PDFs (TMDs)

Quark TMDs with polarization:

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^{\perp}(x,k_T^2)$ - Long-Transversity
	T	$f_1^{\perp}(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Trans-Helicity	$h_1(x,k_T^2)$ - Transversity $h_{1T}^{\perp}(x,k_T^2)$ - Pretzelosity



Analogous tables for:

- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

Transverse momentum dependent PDFs (TMDs)

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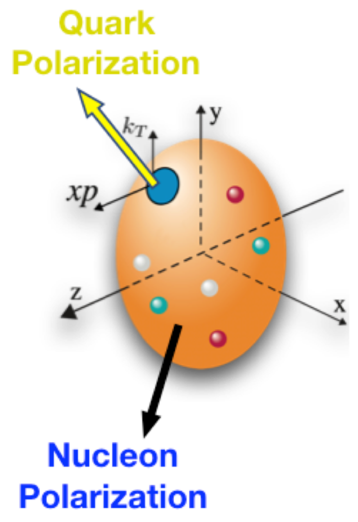
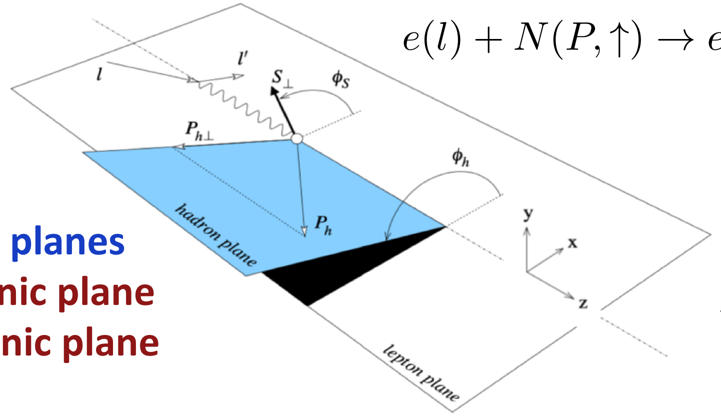
		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ – *Boer-Mulders*
	L		$g_1(x,k_T^2)$ – *Helicity*	$h_{1L}^{\perp}(x,k_T^2)$ – *Long-Transversity*
	T	$f_1^{\perp}(x,k_T^2)$ – *Sivers*	$g_{1T}(x,k_T^2)$ – *Trans-Helicity*	$h_1(x,k_T^2)$ – *Transversity* $h_{1T}^{\perp}(x,k_T^2)$ – *Pretzelosity*



Analogous tables for:

- Gluons $f_1 \rightarrow f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

❑ Polarized SIDIS:



Two planes
Leptonic plane
Hadronic plane

$$e(l) + N(P,\uparrow) \rightarrow e(l') + h(P_h) + X$$

Single Transverse-Spin Asymmetry

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

In photon-hadron frame:

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

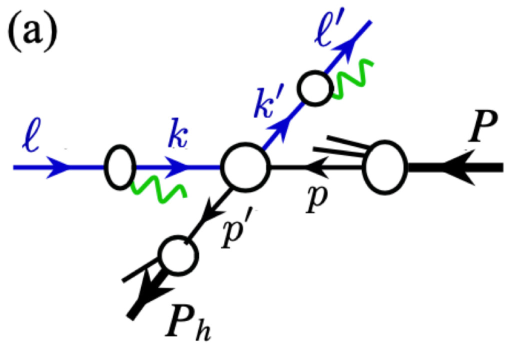
$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

Angular modulation provides the best way to separate TMDs

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Inclusive production of a lepton and a hadron:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371



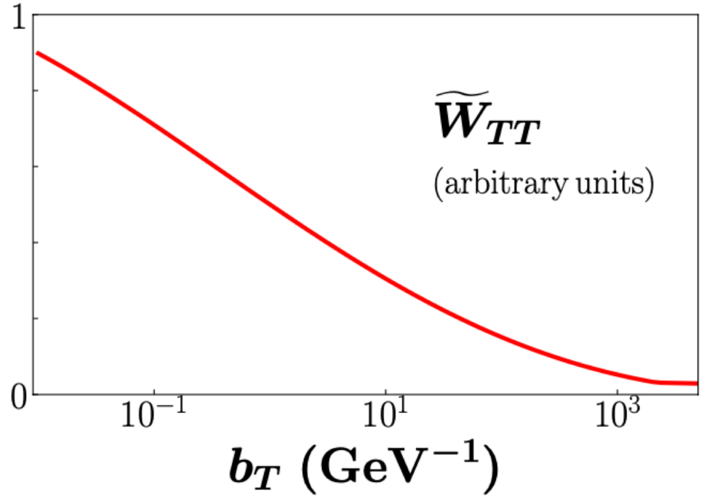
$$e(\ell) + N(P) \to e(\ell') + h(P_h) + X$$

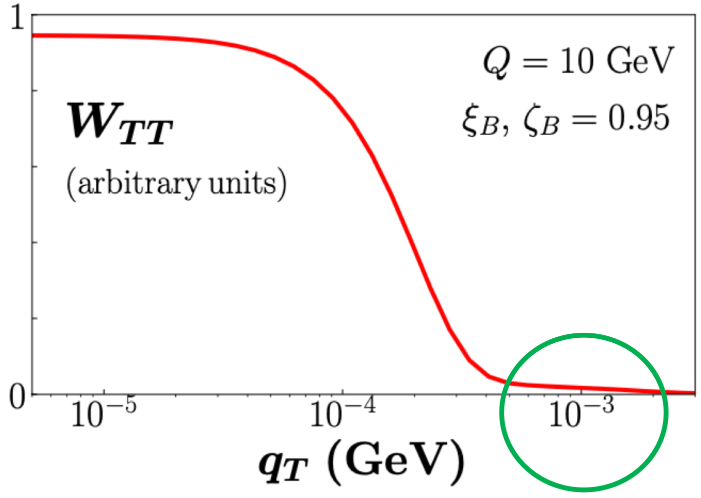
Momentum imbalance between the lepton and the hadron could be sensitive to both parton TMDs and lepton TMDs

Typical parton transverse momentum: $k_T^2 \sim \Lambda_{\rm QCD}^2 + \langle k_T^2 \rangle_{\rm generated\ by\ QCD\ shower}$

- **Estimate of lepton transverse momentum generated by QED shower:**

Resummation to lepton TMD





QED broadening for lepton is so much smaller than typical parton kT!

→ Collinear factorization for high order QED contributions

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

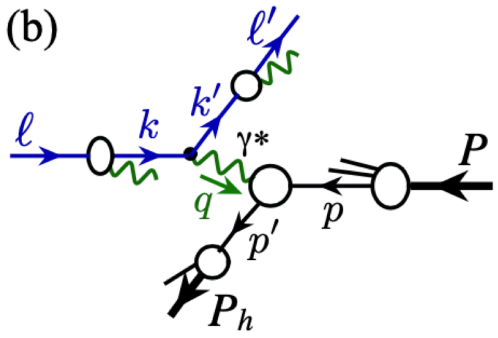
- **QED factorization of collision-induced radiation – collinear:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$E_{\ell'} E_{P_h} \frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to \ell' P_h X}}{d^3\ell'\, d^3P_h} \approx \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} D_{e/j}(\zeta) \int_{\xi_{\min}}^1 d\xi\, f_{i(\lambda_k)/e(\lambda_\ell)}(\xi) \left[E_{k'} E_{P_h} \frac{d^6\hat{\sigma}_{k(\lambda_k)P(S)\to k' P_h X}}{d^3k'\, d^3P_h} \right]_{k=\xi\ell, k'=\ell'/\zeta} + \mathcal{O}\left(\frac{m_e^n}{Q^n}\right)$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \to 0$, factorized into LDFs and LFFs
- IR safe contributions are calculated order-by-order in powers of α
- Neglect m_e/Q power suppressed contributions
- Collinear QED factorization for both inclusive DIS and SIDIS, or e^+e^-, … [global fits of LDFs, LFFs]

- **"One photon"-approximation:**



$$\frac{d^6\sigma_{\ell(\lambda_\ell)P(S)\to \ell' P_h X}}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, dP_{hT}^2} = \sum_{ij\lambda_k} \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^1 \frac{d\xi}{\xi} f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\, D_{e/j}(\zeta)$$
$$\times \frac{\hat{x}_B}{x_B \xi\zeta} \left[\frac{\alpha^2}{\hat{x}_B\, \hat{y}\, \widehat{Q}^2} \frac{\hat{y}^2}{2(1-\hat{\varepsilon})} \left(1 + \frac{\hat{\gamma}^2}{2\hat{x}_B}\right) \sum_n \hat{w}_n F_n^h(\hat{x}_B, \widehat{Q}^2, \hat{z}_h, \widehat{P}_{hT}^2) \right]$$

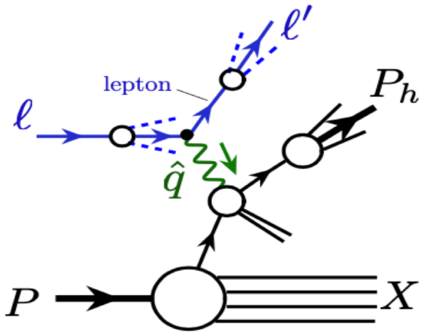
Evaluated in a "virtual photon-hadron" frame

Apply a (ξ, ζ)-dependent Lorentz transformation:

$\{\hat{q}, P, \widehat{P}_h\}$ → (ξ, ζ) → $\{q, P, P_h\}$ In a frame to compare with exp. measurements

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

❑ Two-step approach to SIDIS:



One-photon approximation

1) In "virtual-photon" frame, defined by $\hat{q}(\xi,\zeta) - p$

- TMD factorization when $\widehat{P}_T^2 \ll \widehat{Q}^2$
- CO factorization when $\widehat{P}_T^2 \sim \widehat{Q}^2$
- Matching to get the $\widehat{P}_T$-distribution

2) Lorentz transformation from the "virtual-photon" frame to any experimentally defined frame – lepton-hadron Lab frame, Breit frame (x_B,Q^2), …

QED contribution (not correction) can be systematically improved order-by-order in power α!

❑ Case study F_{UU}:

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = {}& \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ \boxed{F_{UU,T}} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} \\
& + S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h) F_{UL}^{\sin 2\phi_h}\right] \\
& + S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& + |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S) F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S) F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study F_{UU}:**

$$\frac{d\sigma^h_{\rm SIDIS}}{dx_B dy\, dz\, dP^2_{hT}} = \int_{\zeta_{\rm min}}^1 d\zeta \int_{\xi_{\rm min}(\zeta)}^1 d\xi\, D_{e/e}(\zeta)\, f_{e/e}(\xi) \times \left[\frac{\hat{x}_B}{x_B\,\xi\zeta}\right] \left[\frac{(2\pi)^2\,\alpha}{\hat{x}_B \hat{y}\,\widehat{Q}^2}\frac{\hat{y}^2}{2(1-\hat{\varepsilon})} F^h_{UU}(\hat{x}_B, \widehat{Q}^2, \hat{z}, \widehat{P}_{hT})\right]$$

Evaluated in a "virtual photon-hadron" frame

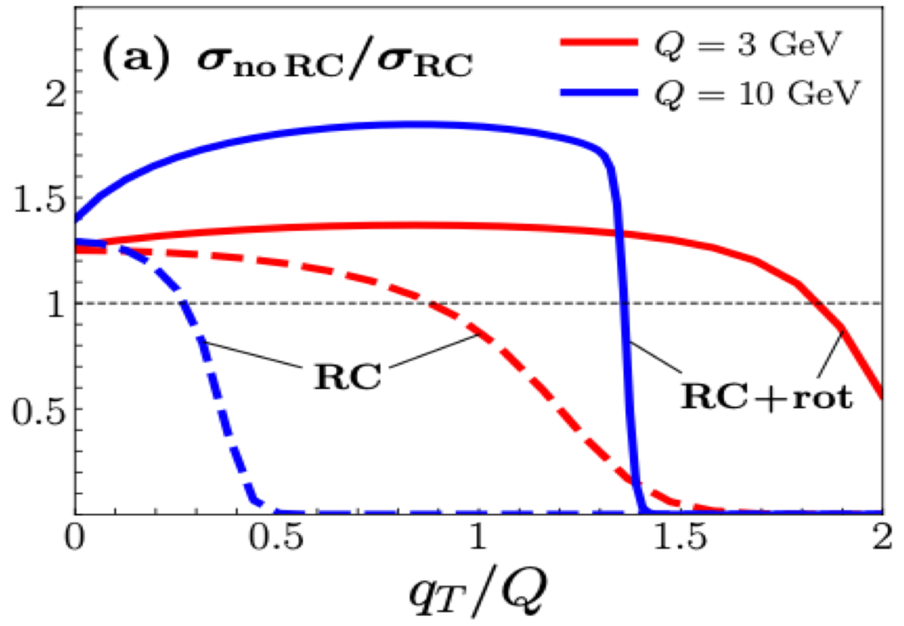
Unpolarized structure function:

$$F^h_{UU} = x_B \sum_q e_q^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{q}_T) \times f_{q/N}(x_B, \boldsymbol{p}_T^2)\, D_{h/q}(z, \boldsymbol{k}_T^2) \qquad \boldsymbol{q}_T = \boldsymbol{P}_{hT}/z$$

(ξ, ζ) - Dependent Lorentz transformation

Effectively, a rotation in hadron-rest frame

Solid – with Lorentz transformation
Dashed – without Lorentz transformation



Lepton-hadron semi-inclusive deep inelastic scattering (SIDIS)

- **Case study – single transverse spin asymmetry:**

Liu, Melnitchouk, Qiu, Sato
2008.02895, 2108.13371

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

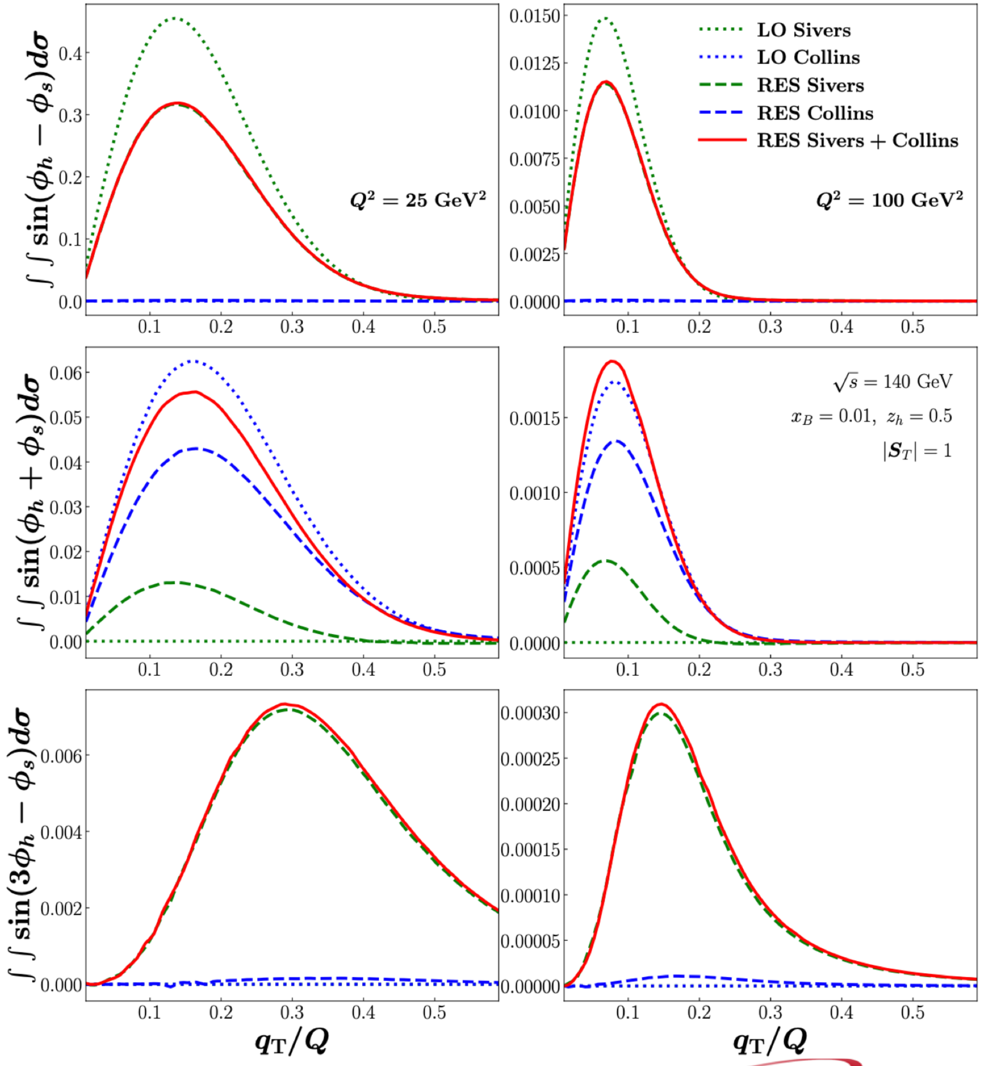
$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$



Summary and Outlook

- ❑ **Collision induced QED radiation is an integrated part of the lepton-hadron collision**
  - ○ Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - ○ No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - ○ Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC
- ❑ **Factorization approach to include both QCD and QED radiative contributions (and shower of weak particles at the LHC energies) provides a consistent and controllable approximation**
  - ○ QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons
  - ○ No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - ○ All perturbatively calculable hard parts are IR safe for both QCD and QED
  - ○ All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Thank you!

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