

**PRECISION QCD PREDICTIONS IN EP
PHYSICS AT EIC WORKSHOP**

STONYBROOK UNIVERSITY

AUGUST 3, 2022



Istituto Nazionale di Fisica Nucleare



**UNIVERSITÀ
DI PAVIA**

S T O N Y B R O O K
STATE UNIVERSITY OF NEW YORK

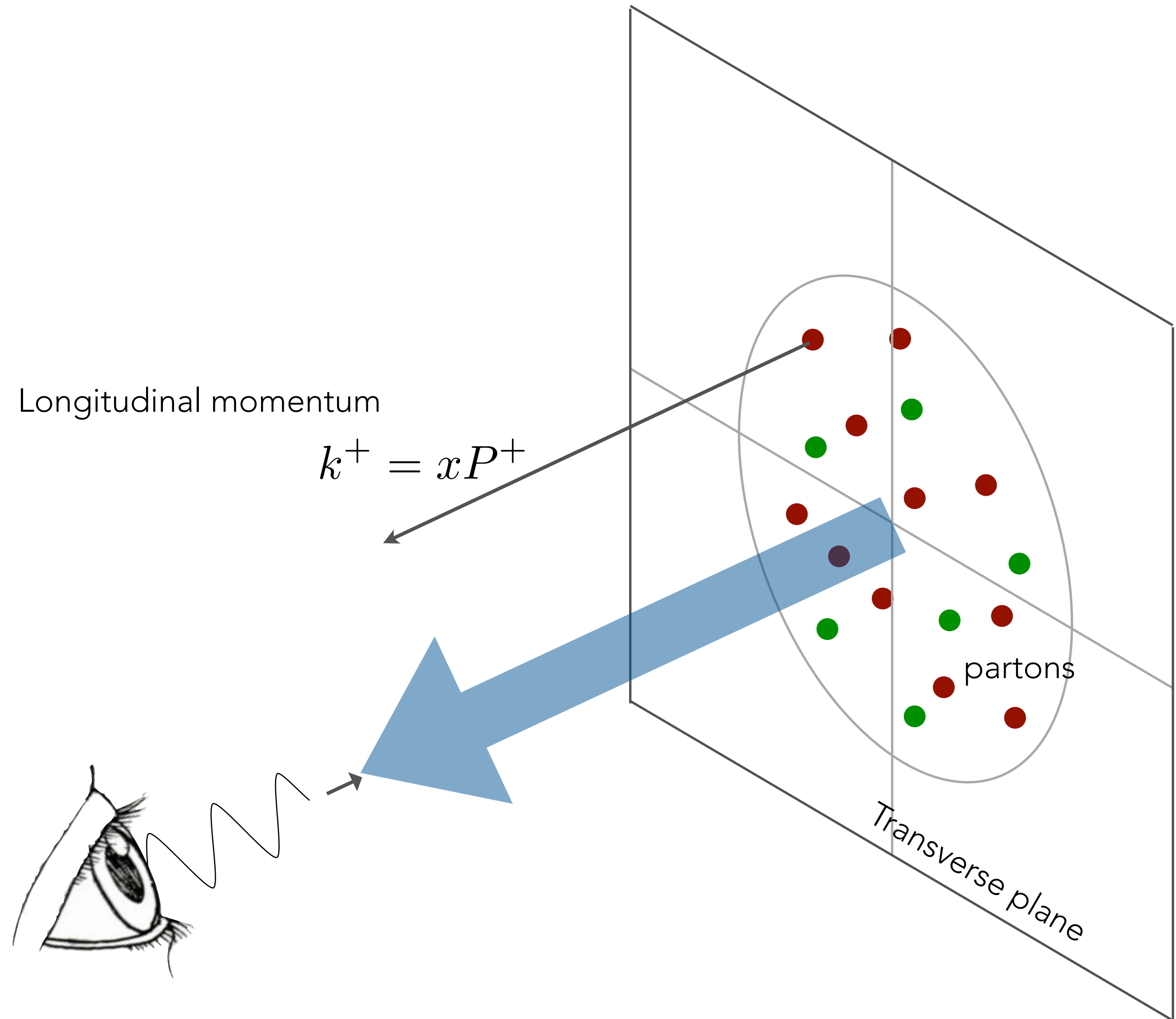
MAPTMD22

A new precise extraction of unpolarized TMDs

MATTEO CERUTTI

MAP COLLABORATION

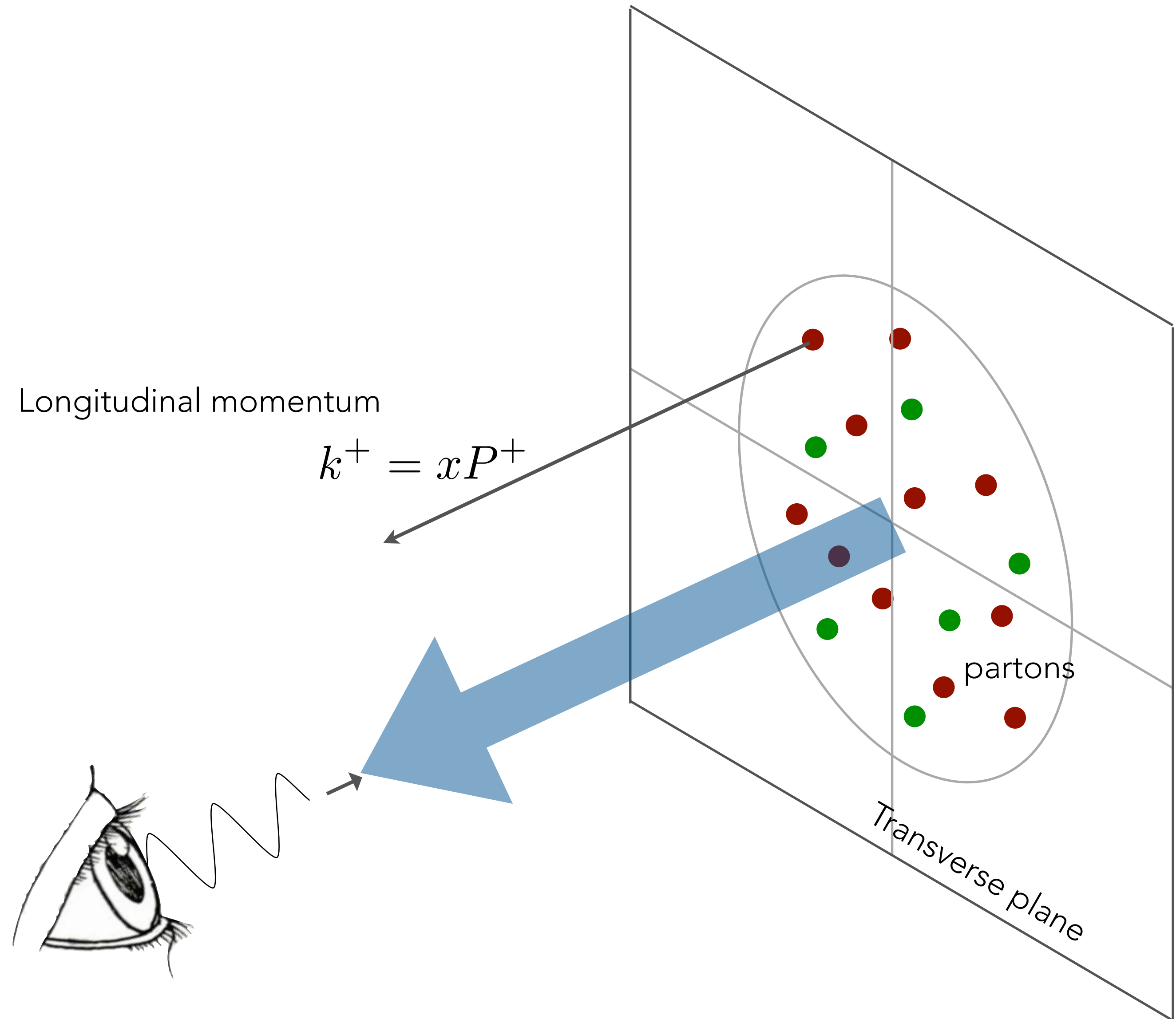
Parton Distribution Functions (PDFs)



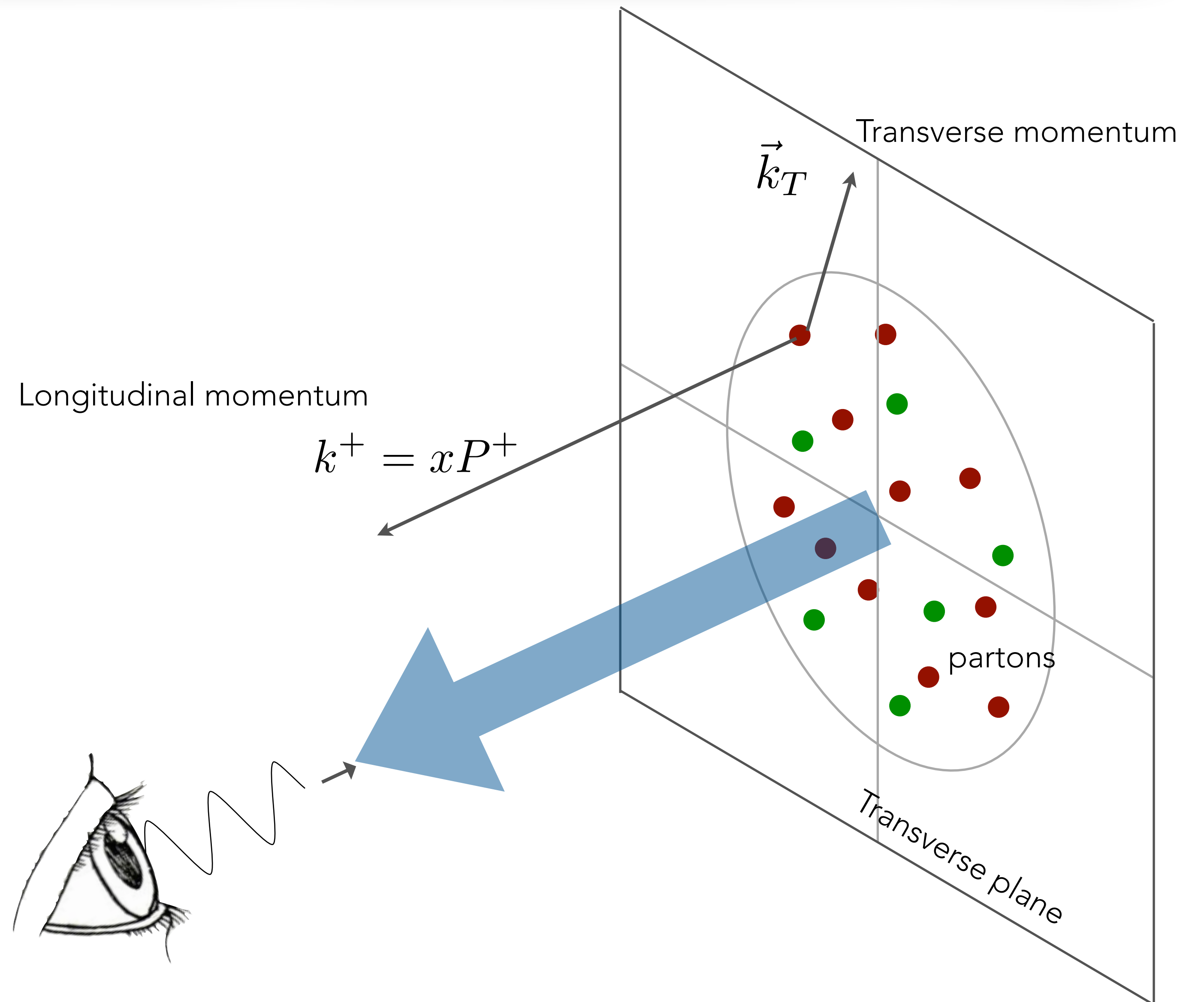
Parton Distribution Functions (PDFs)

1-D maps of the internal structure of the nucleon

$$f(x)$$



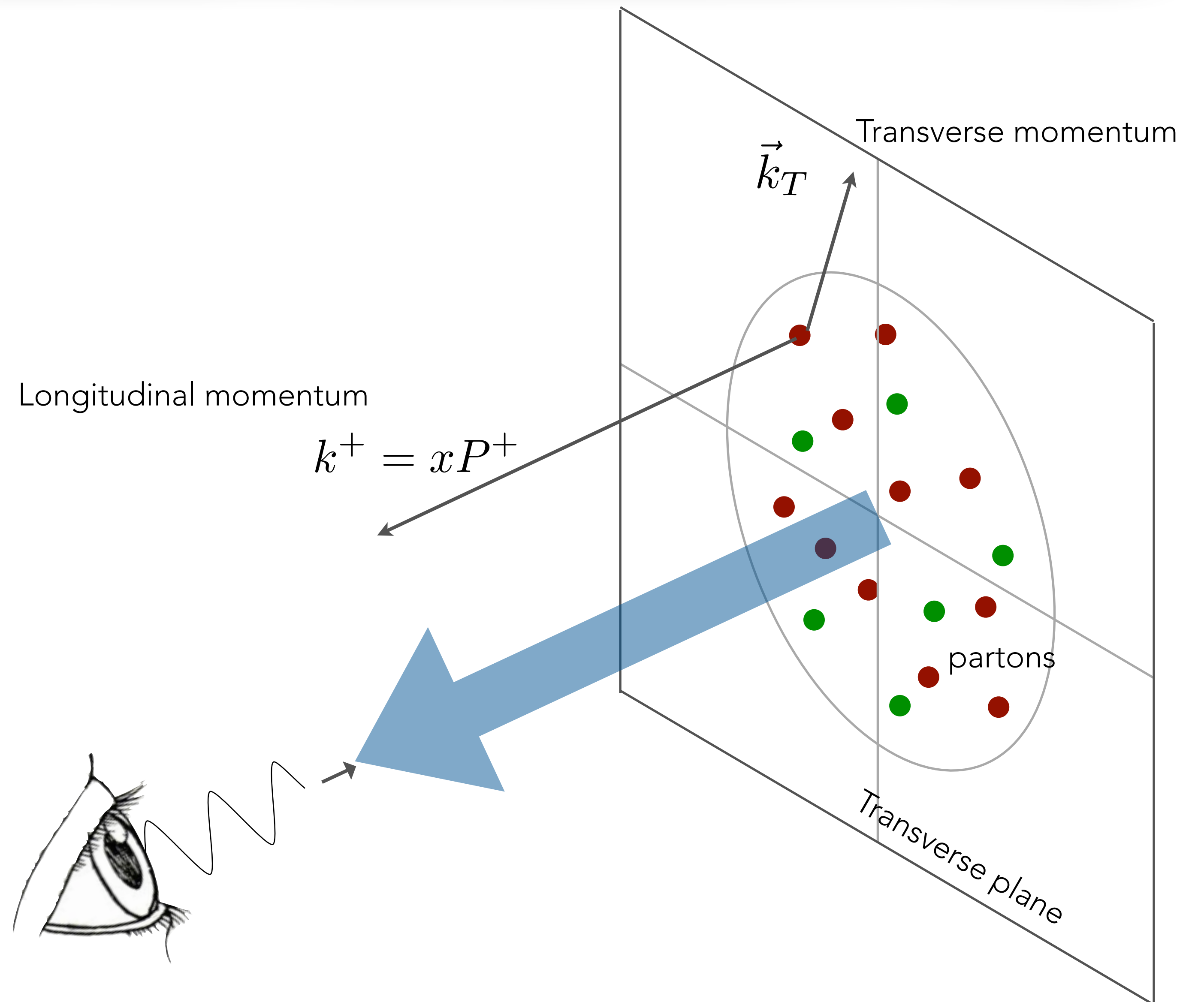
Transverse Momentum Distributions (TMDs)



Transverse Momentum Distributions (TMDs)

3-D maps of the internal structure of the nucleon

$$f(x, \vec{k}_T)$$

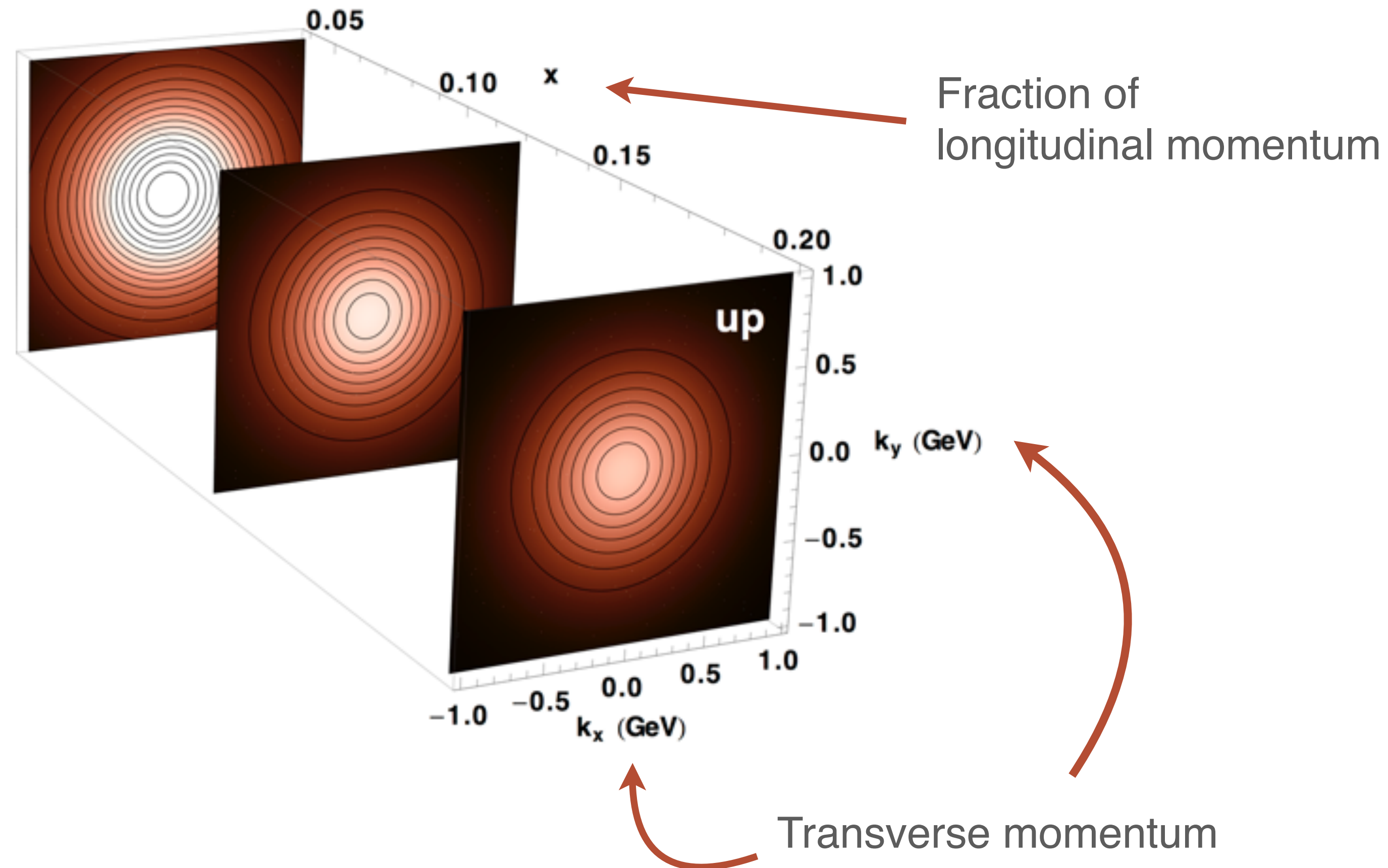


Transverse Momentum Distributions (TMDs)

TMDs describe the distribution of partons in three dimensions in momentum space

They can be extracted through global fits

There are attempts to calculate them in lattice QCD



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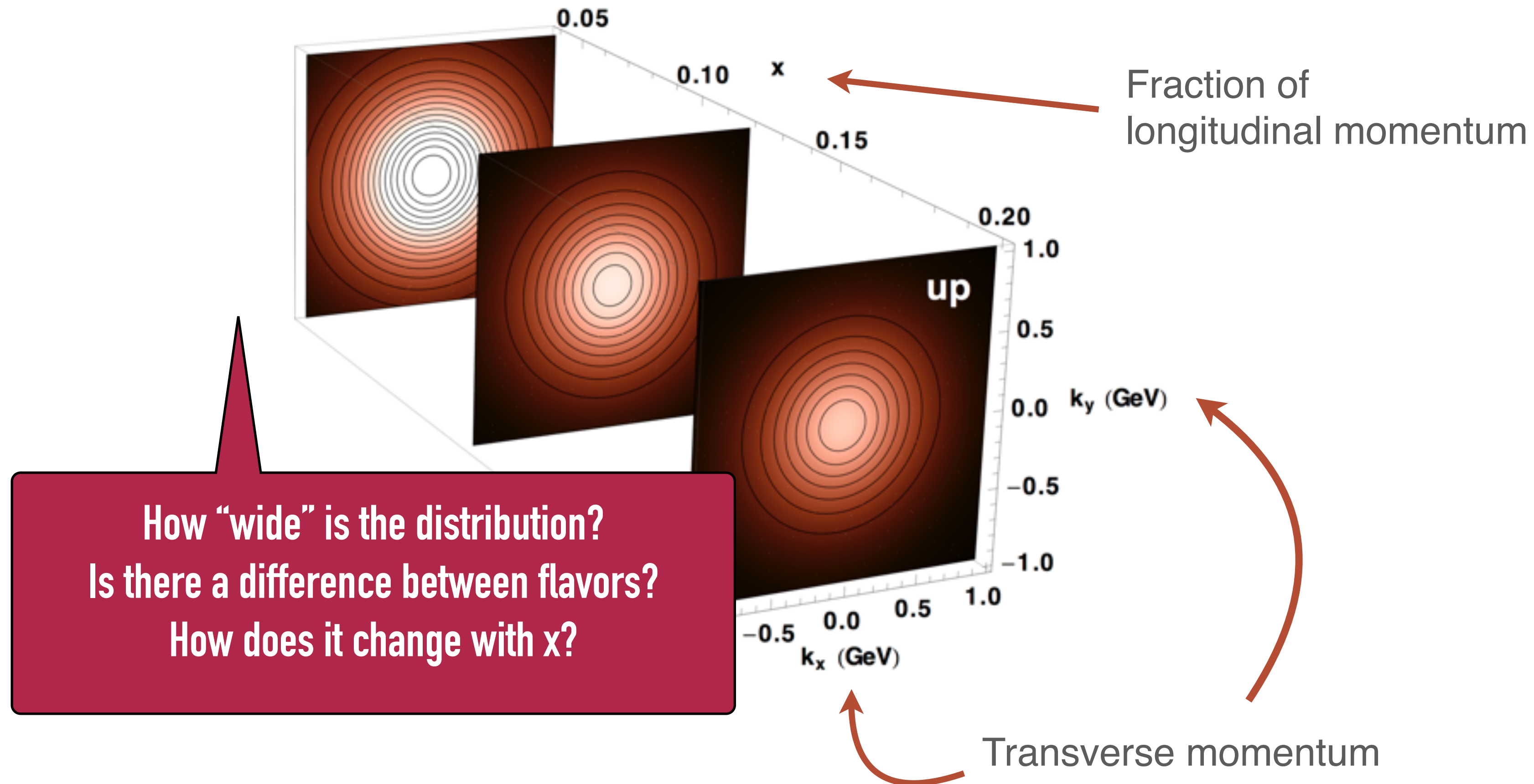


Table of TMD PDFs

Quark Polarization

	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	$h_1 \quad h_{1T}^\perp$

Survive upon integration over transverse momentum

Time-reversal odd

Time-reversal even

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Nucleon Pol.

Survive upon integration over transverse momentum

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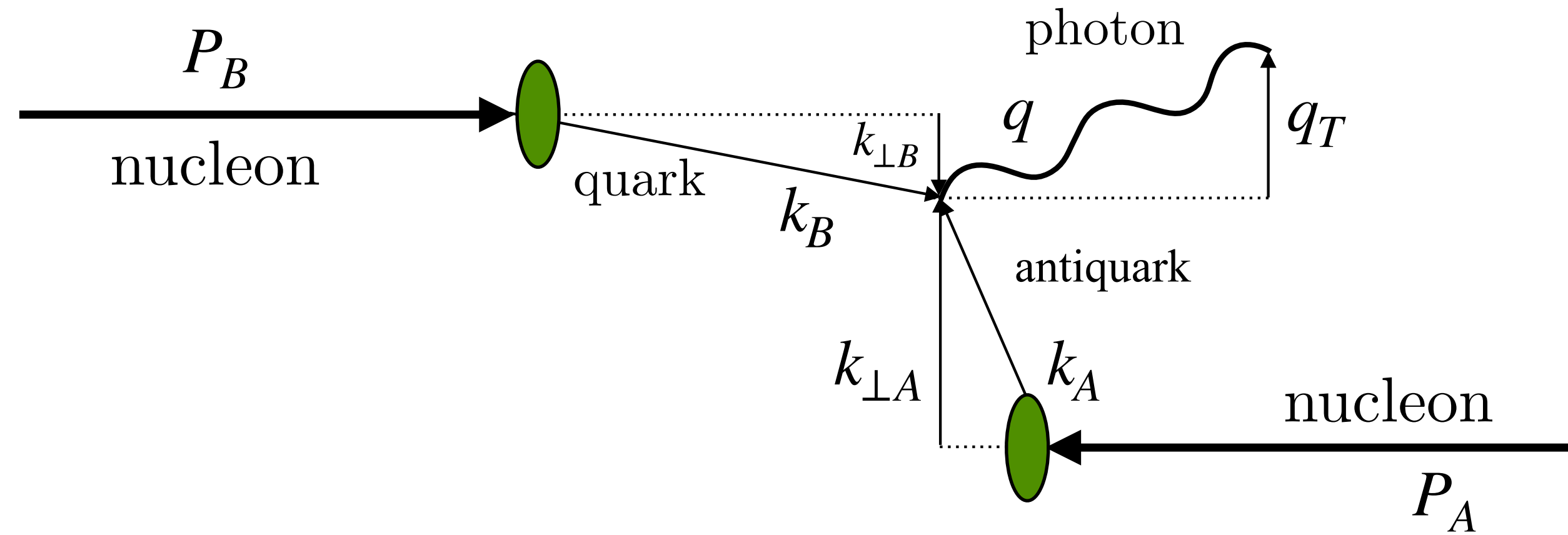
Time-reversal even

Main topic of this talk

On top of them, there are gluon-TMDs and twist-3 functions

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

TMD factorization – Drell-Yan process



W term

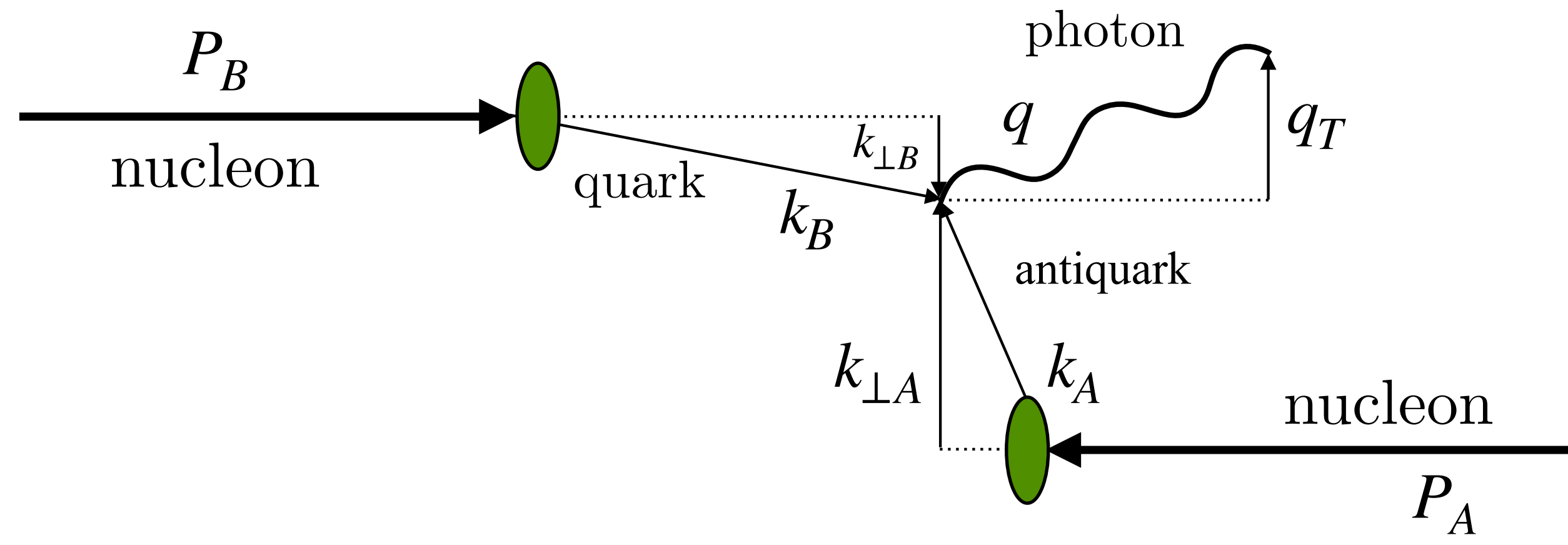
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^a(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{a}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$+ Y_{UU}^1(Q^2, \mathbf{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

TMD factorization — Drell-Yan process



W term

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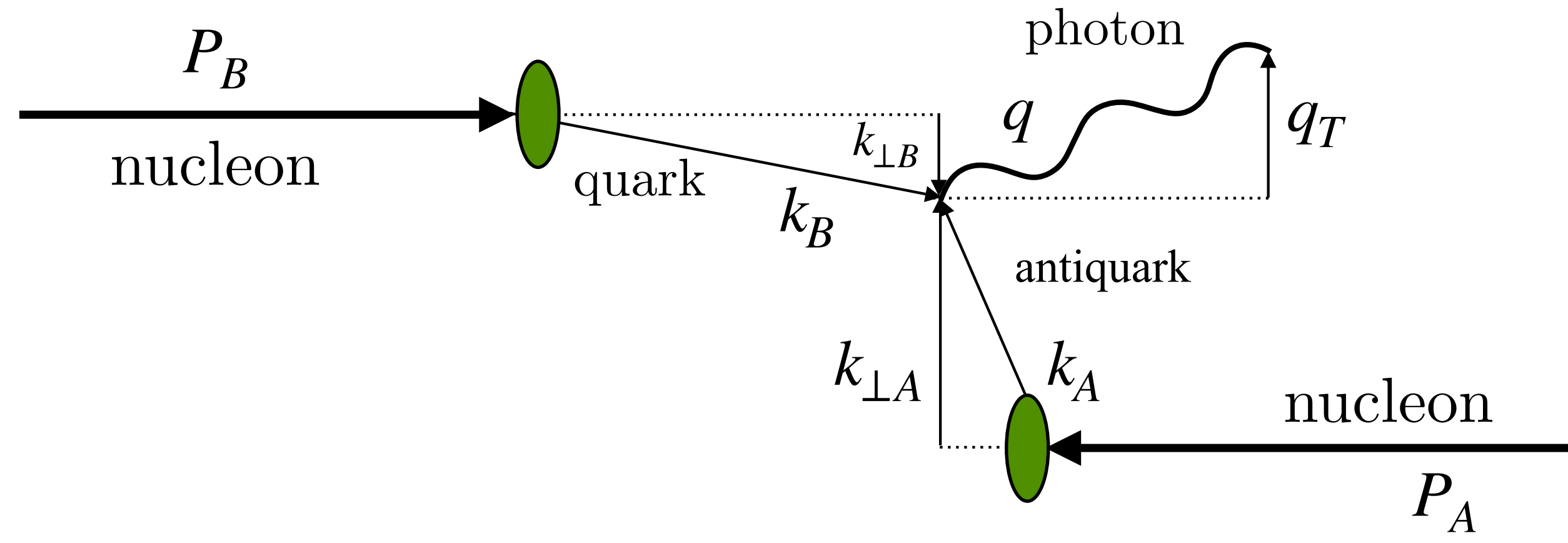
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📍 The **W term** dominates in the region where $\mathbf{q}_T \ll Q$

TMD factorization – Drell-Yan process



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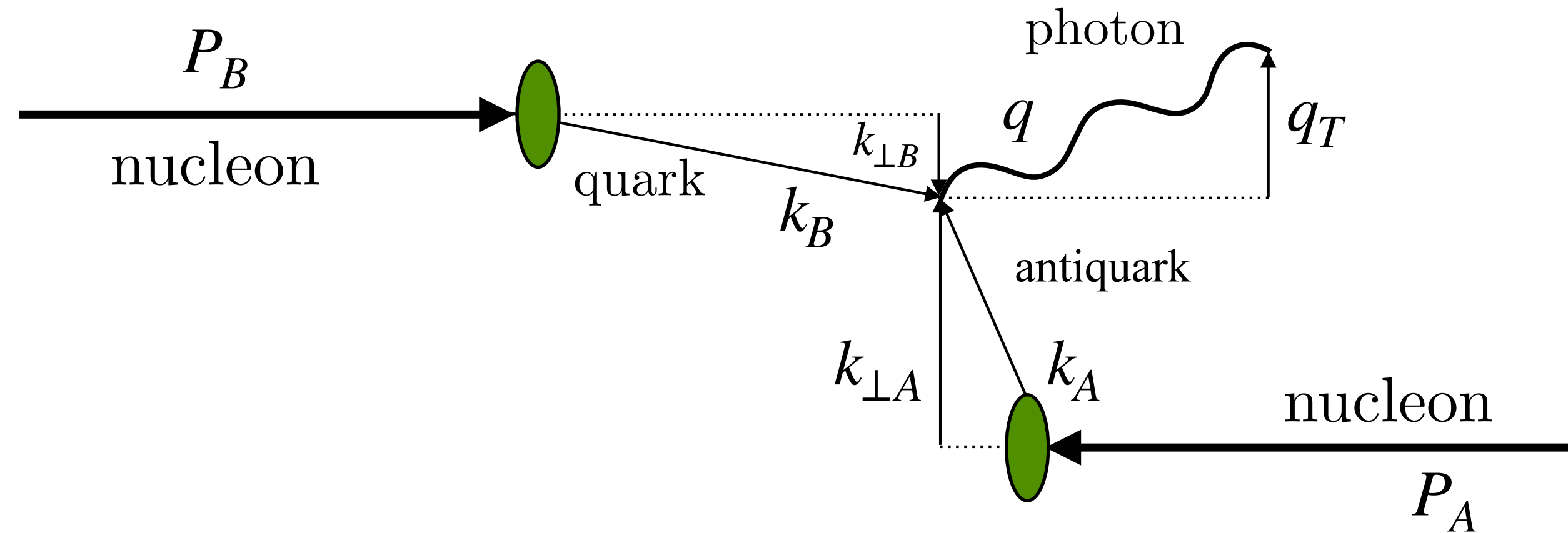
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Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

- The **W term** dominates in the region where $\mathbf{q}_T \ll Q$
- Y term dominates in the complementary region

TMD factorization — Drell-Yan process



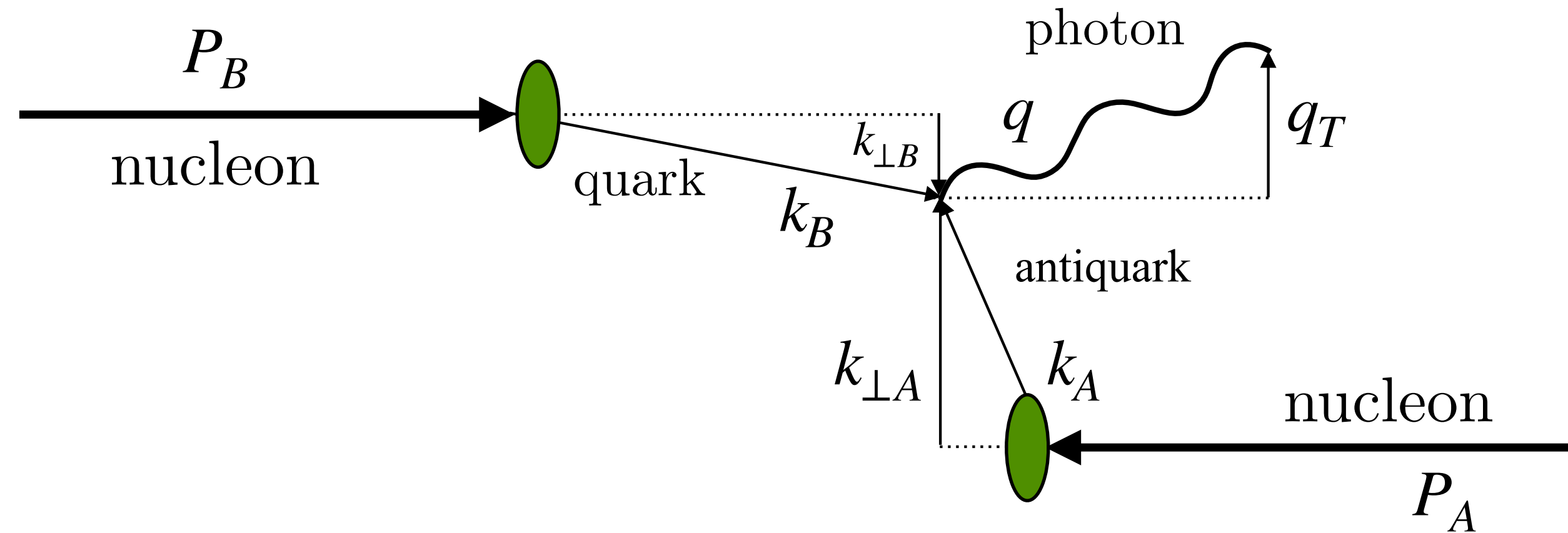
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2)$$

$$\approx \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^q(x_A, \mathbf{k}_{\perp A}^2; \mu^2) f_1^{\bar{q}}(x_B, \mathbf{k}_{\perp B}^2; \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} - \mathbf{q}_T + \mathbf{k}_{\perp B})$$

$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2) \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

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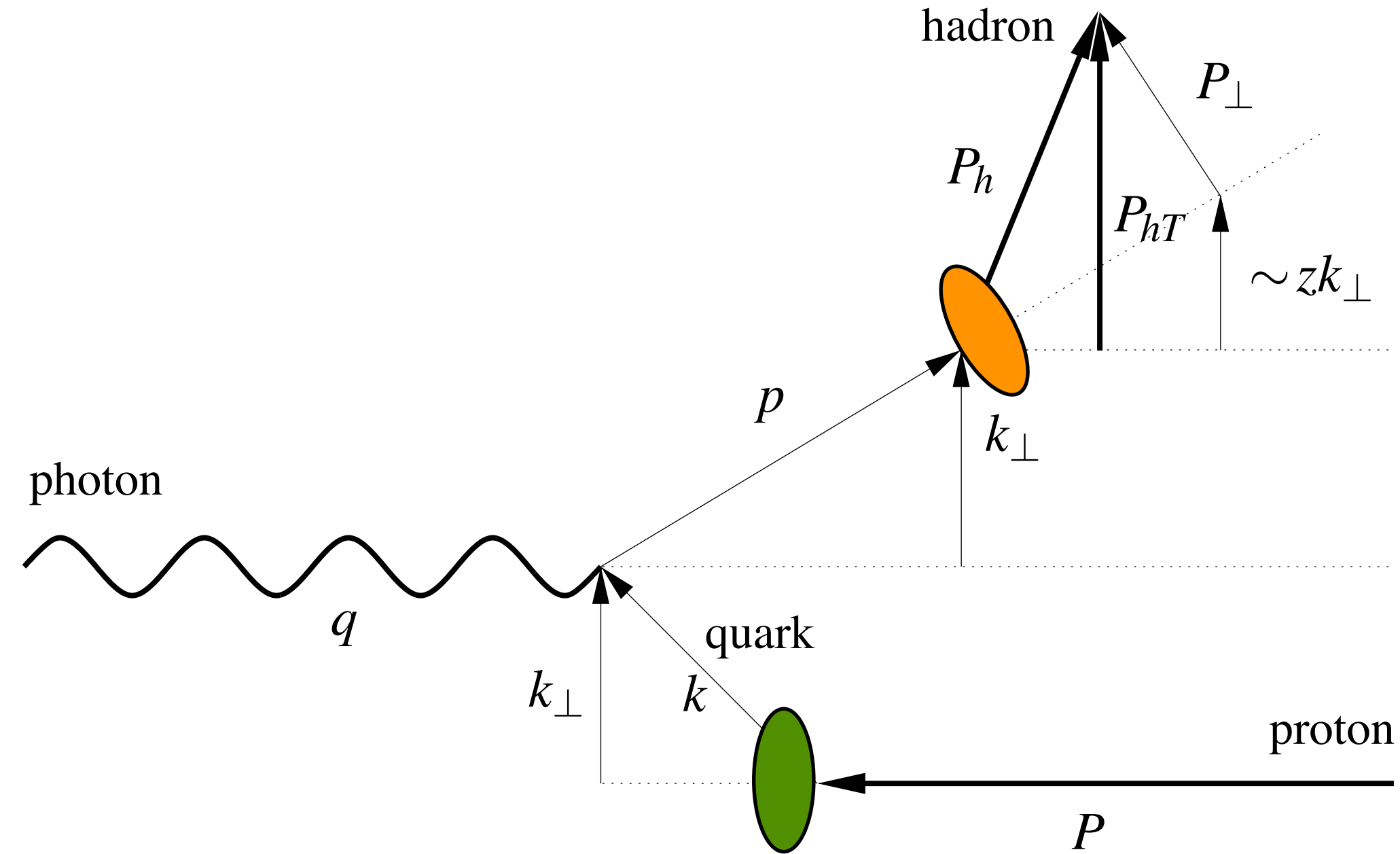
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Arnold, Metz and Schlegel, Phys.Rev.D 79 (2009)

- Fourier-transformed space to avoid convolutions
- TMDs formally depend on two scales, but we set them equal

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta)$$

TMD factorization – SIDIS process



$$F_{UU,T}(x, z, \mathbf{P}_{hT}^2, Q^2)$$

$$= x \sum_q \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp} d^2 \mathbf{P}_{\perp} f_1^a(x, \mathbf{k}_{\perp}^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_{\perp}^2; \mu^2) \delta(z \mathbf{k}_{\perp} - \mathbf{P}_{hT} + \mathbf{P}_{\perp})$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

$$= x \sum_a \mathcal{H}_{UU,T}^q(Q^2, \mu^2) \int db_T b_T J_0(b_T |\mathbf{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_{\perp}^2; \mu^2) \hat{D}_1^{a \rightarrow h}(z, b_{\perp}^2; \mu^2)$$

Available codes – NangaParbat

<https://github.com/MapCollaboration/NangaParbat>



☰ README.md



Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

<https://github.com/MapCollaboration/NangaParbat>

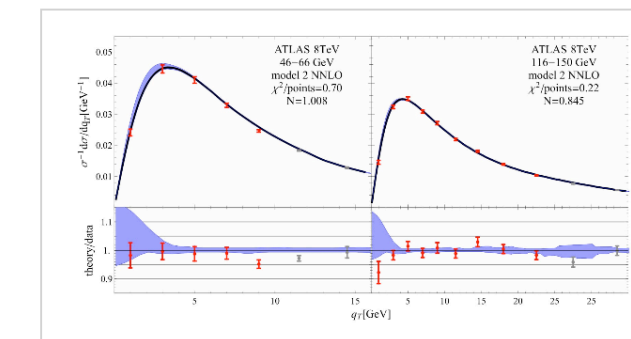
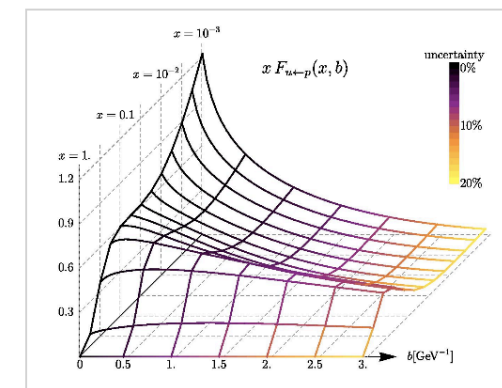
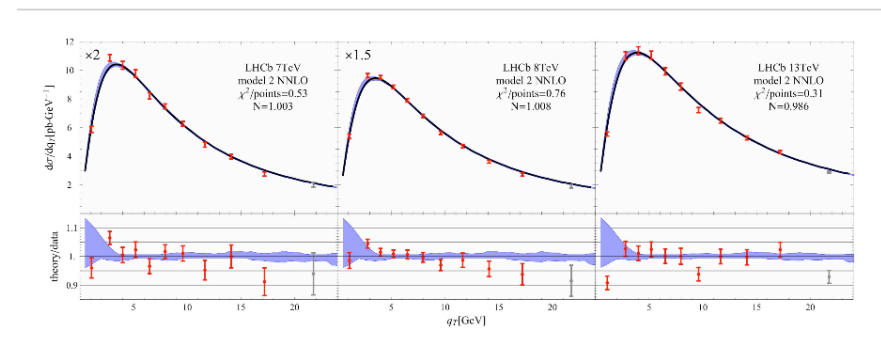
For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

Available codes – NangaParbat

<https://teorica.fis.ucm.es/artemide/>

arTeMiDe



News



12 Dec 2019: Version 2.02 released (+manual update).

23 Feb 2019: Version 1.4 released (+manual update).

21 Jan 2019: Artemide now has a [repository](#).

[Archive of older links/news.](#)

Articles, presentations & supplementary materials



[Extra pictures for the paper arXiv:1902.08474](#)

[Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.](#)

[Link to the text in Inspire.](#)

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About us & Contacts



If you have found mistakes, or have suggestions/questions, please, contact us.

Some extra materials can be found on [Alexey's web-page](#)

Alexey Vladimirov Alexey.Vladimirov@physik.uni-regensburg.de

Ignazio Scimemi ignazios@fis.ucm.es

TMD factorization — expression of a TMD

Structure of a TMD in b_T -space

$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} e^{i \mathbf{b}_T \cdot \mathbf{k}_\perp} f_1^q(x, k_\perp^2; \mu, \zeta)$$


$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} (\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu})} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}} \right)^{K_{\text{resum}} + g_K} f_{1NP}(x, b_T^2; \zeta_f, Q_0)$$

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matching coefficients
(perturbative)

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perturbative Sudakov
form factor

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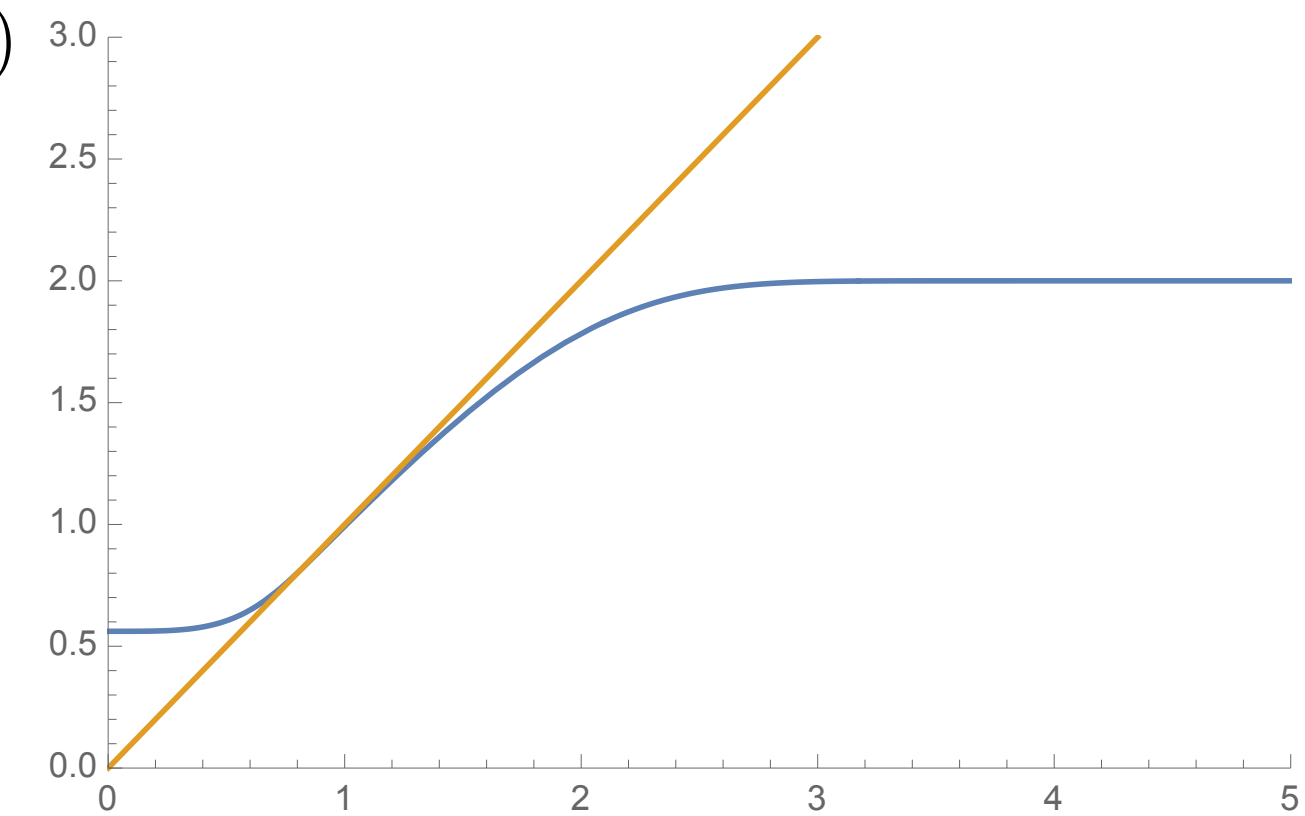
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$$b_*(b_T) = b_{\text{max}} \left(\frac{1 - \exp\left(-\frac{b_T^4}{b_{\text{max}}^4}\right)}{1 - \exp\left(-\frac{b_T^4}{b_{\text{min}}^4}\right)} \right)^{\frac{1}{4}}$$

$$b_{\text{max}} = 2e^{-\gamma_E}$$

$$b_{\text{min}} = \frac{2e^{-\gamma_E}}{Q}$$



Collins, "Foundations of Perturbative QCD"

b_T

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nonperturbative part
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nonperturbative part
of evolution

nonperturbative part
of TMD

Perturbative accuracy

Orders in powers of α_S

-

Perturbative accuracy

Orders in powers of α_S

Accuracy	Hard factor and matching coefficient	Ingredients in perturbative Sudakov form factor		PDFs/FFs and α_S evol.
	H and C	K and γ_F	γ_K	
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N ³ LL ⁻	2	3	4	NLO (FF only)
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Collinear fragmentation functions available beyond NLO only recently

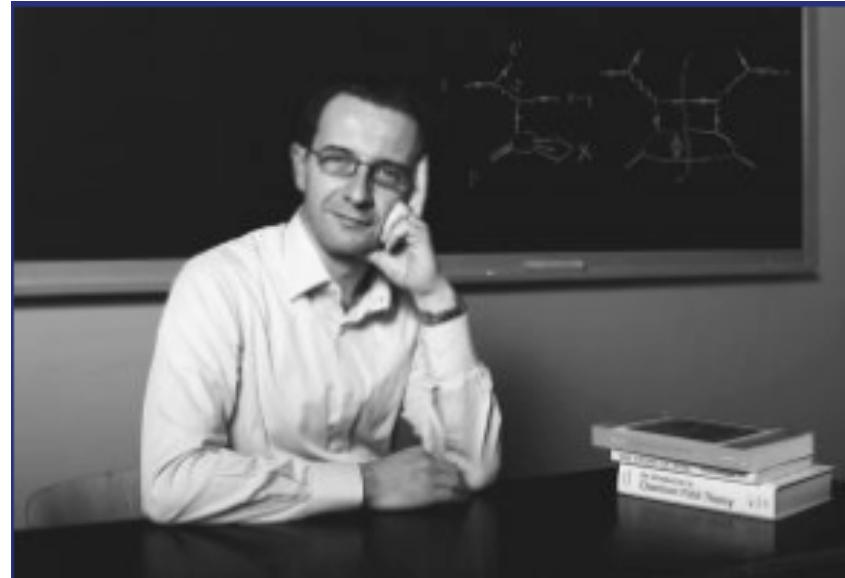
Borsa et al., 2202.05060
Khalek et al., 2204.10331

Available global fits

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	$N^3\text{LL}^-$	✓	✓	✓	1039	1.06
MAPTMD22	$N^3\text{LL}^-$	✓	✓	✓	2031	1.06

A new extraction of unpolarized quark TMDs

Alessandro Bacchetta



Marco Radici



Andrea Signori



Valerio Bertone



Chiara Bissoletti



Giuseppe Bozzi



Fulvio Piacenza



A new extraction of unpolarized quark TMDs

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy: **N^3LL^-**
- ***Normalization*** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description: **$\chi^2 / N_{data} = 1.06$**

Main differences with SV19

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- Different implementation of TMD evolution

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Collins-Soper-Sterman vs ***zeta-prescription***

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- Different criteria of data selection
- Different choice of nonperturbative functional form

MAPTMD22 — Datasets included

Drell-Yan

Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

$9 \lesssim Q \lesssim 11$ GeV excluded (Υ resonance)

$$q_T|_{\max} = 0.2Q$$

484 experimental points

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SIDIS

HERMES data

COMPASS data

$$Q > 1.3 \text{ GeV}$$

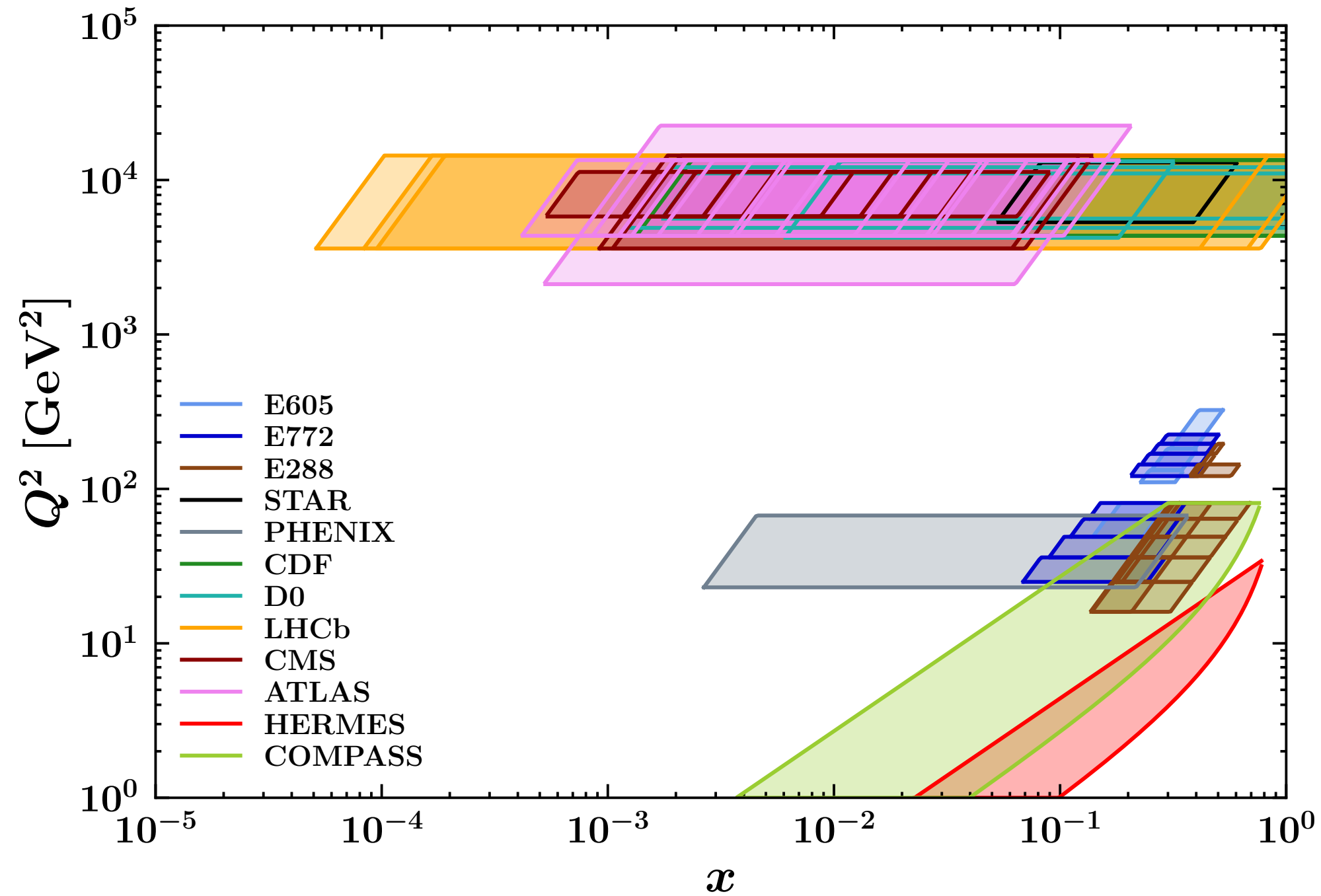
$$0.2 < z < 0.7$$

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

1547 experimental points

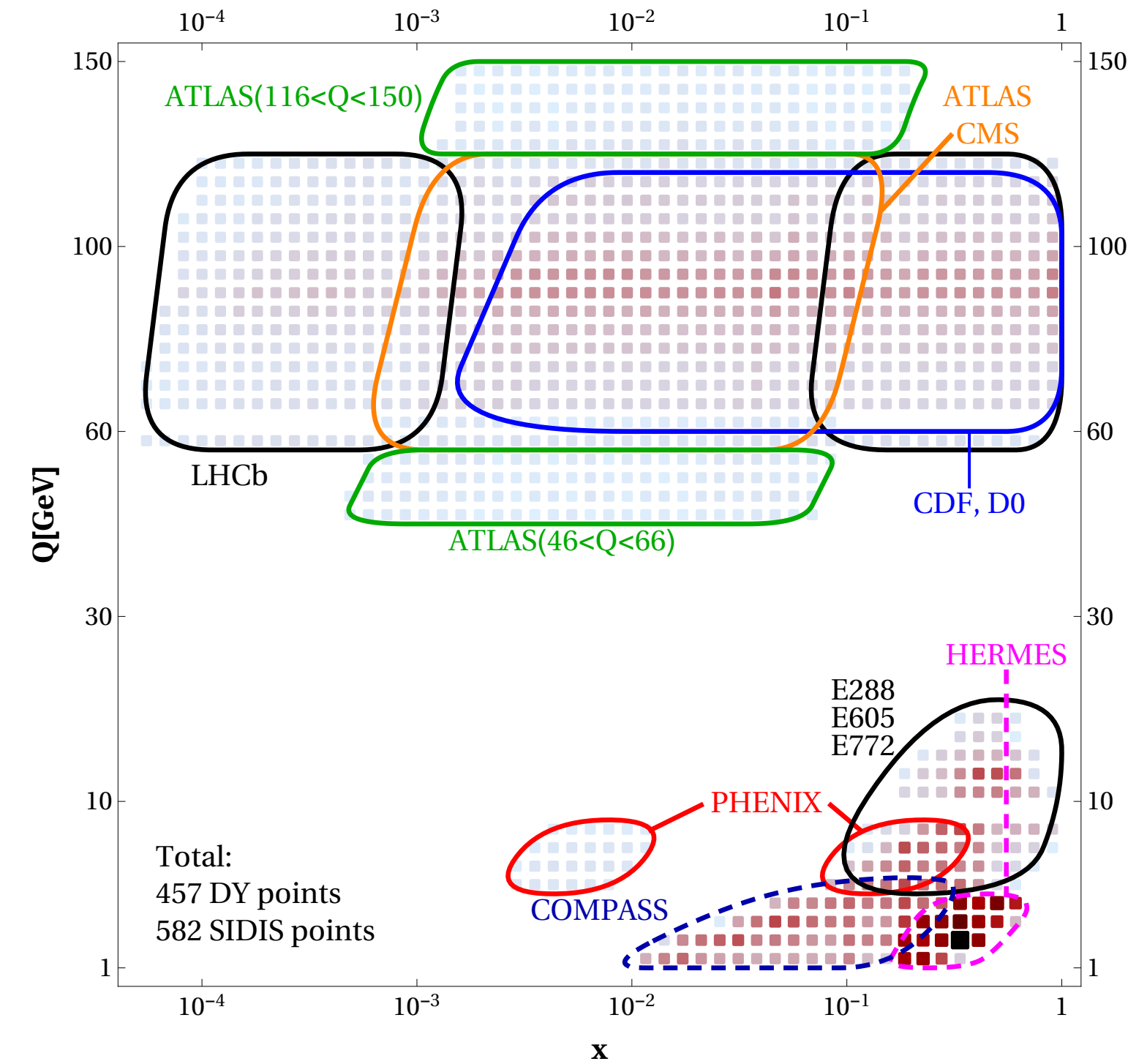
Comparison of datasets included

MAPTMD22



484(DY) + 1547(SIDIS) = 2031 fitted data

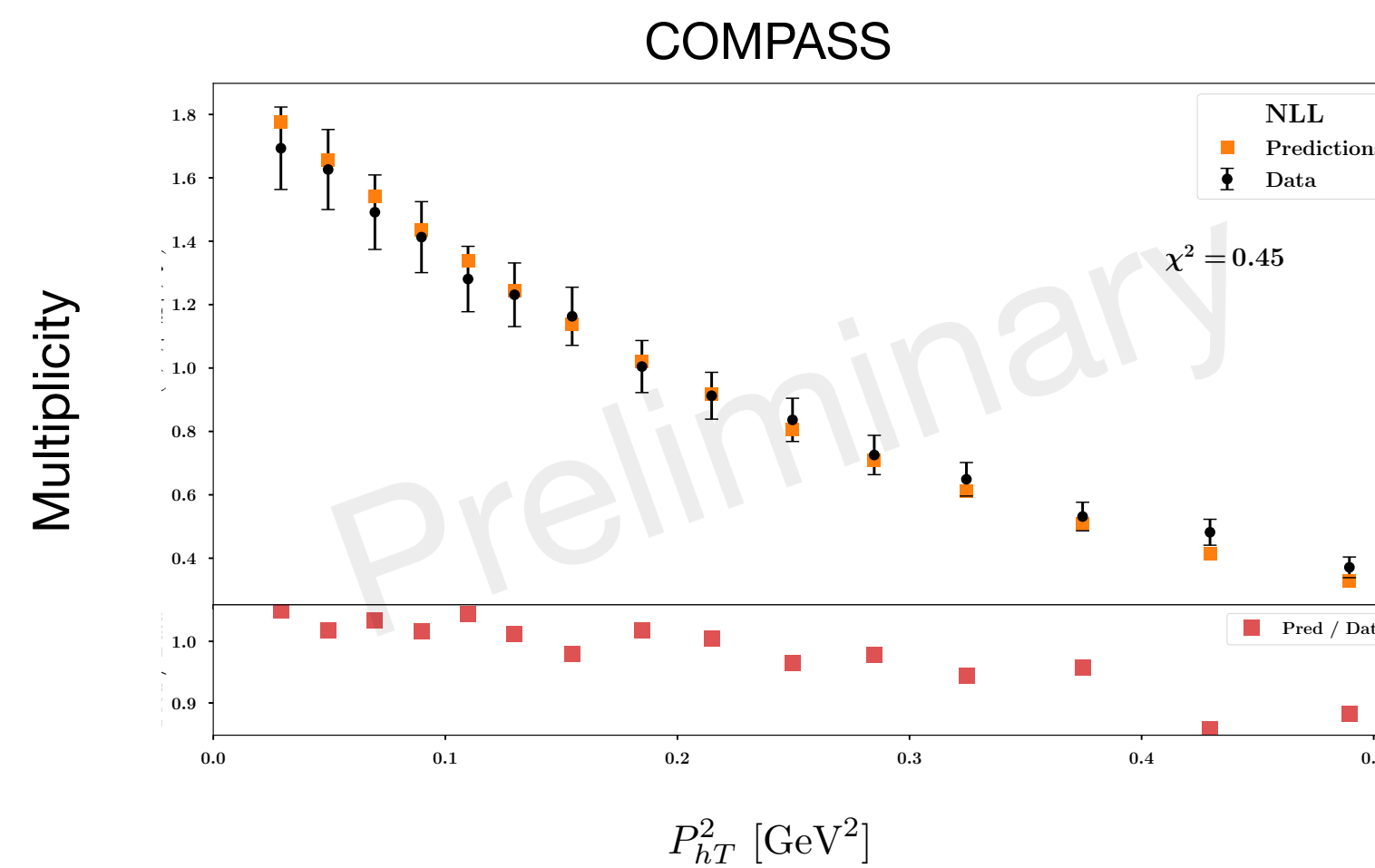
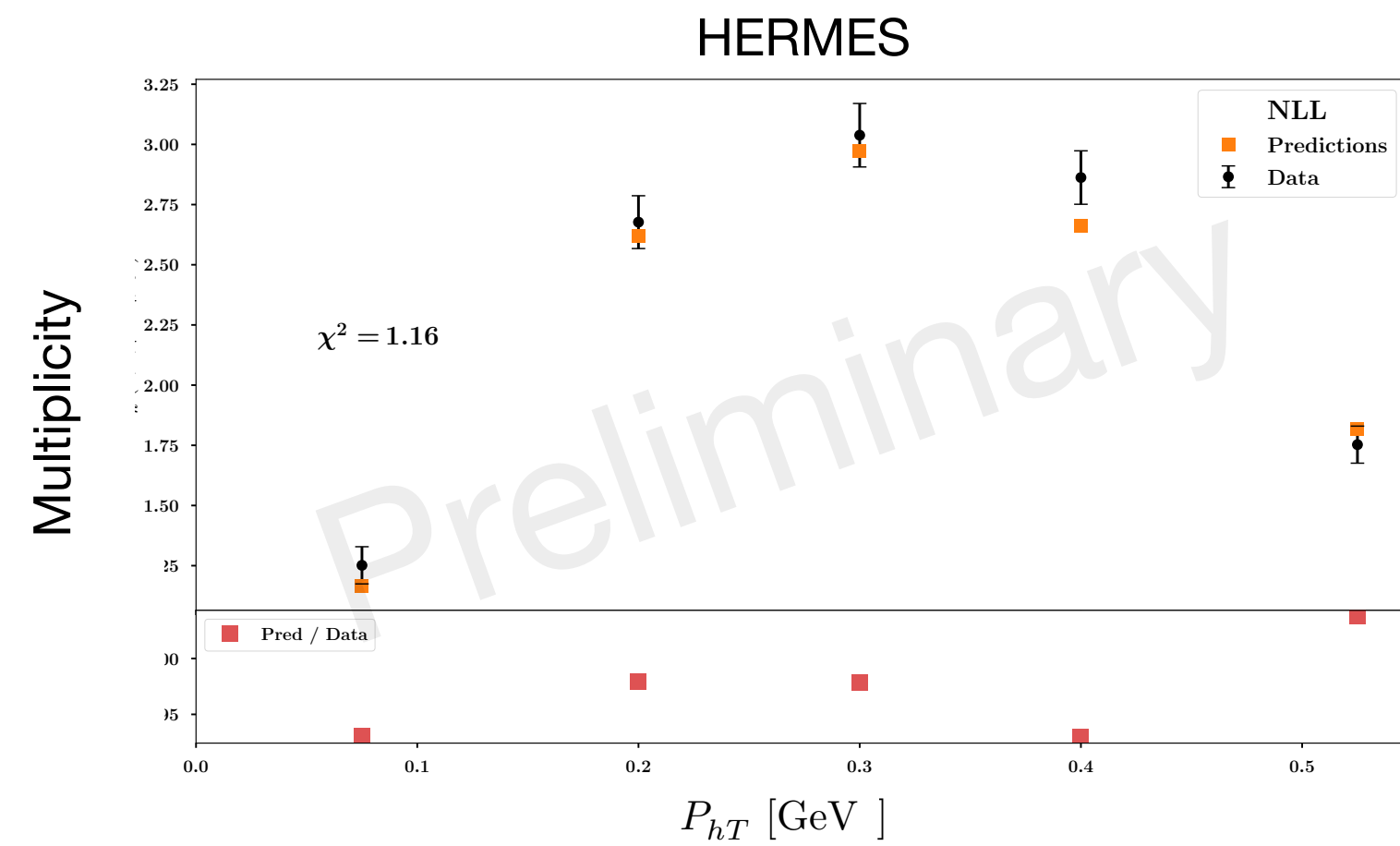
SV19



457(DY) + 582(SIDIS) = 1039 fitted data

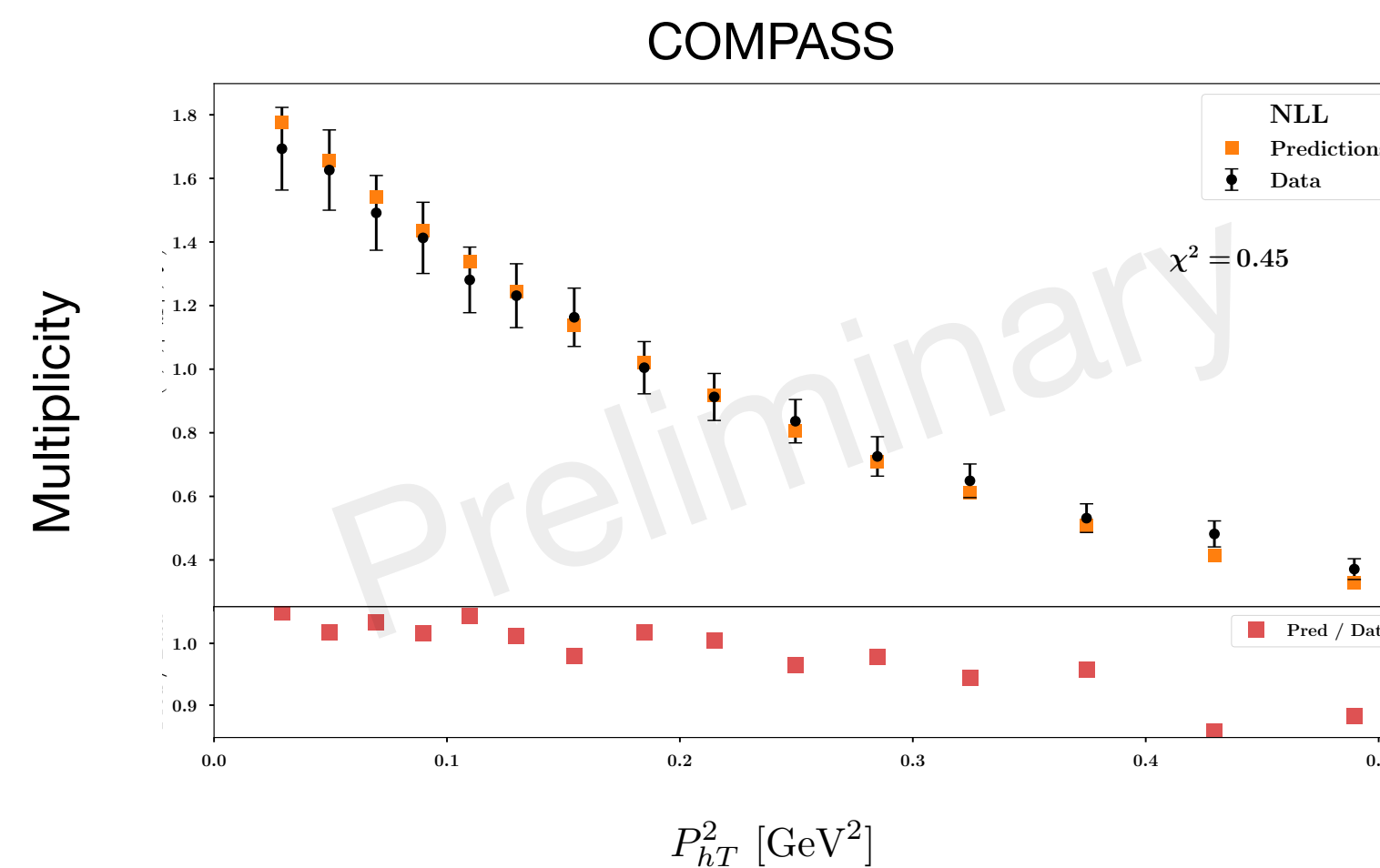
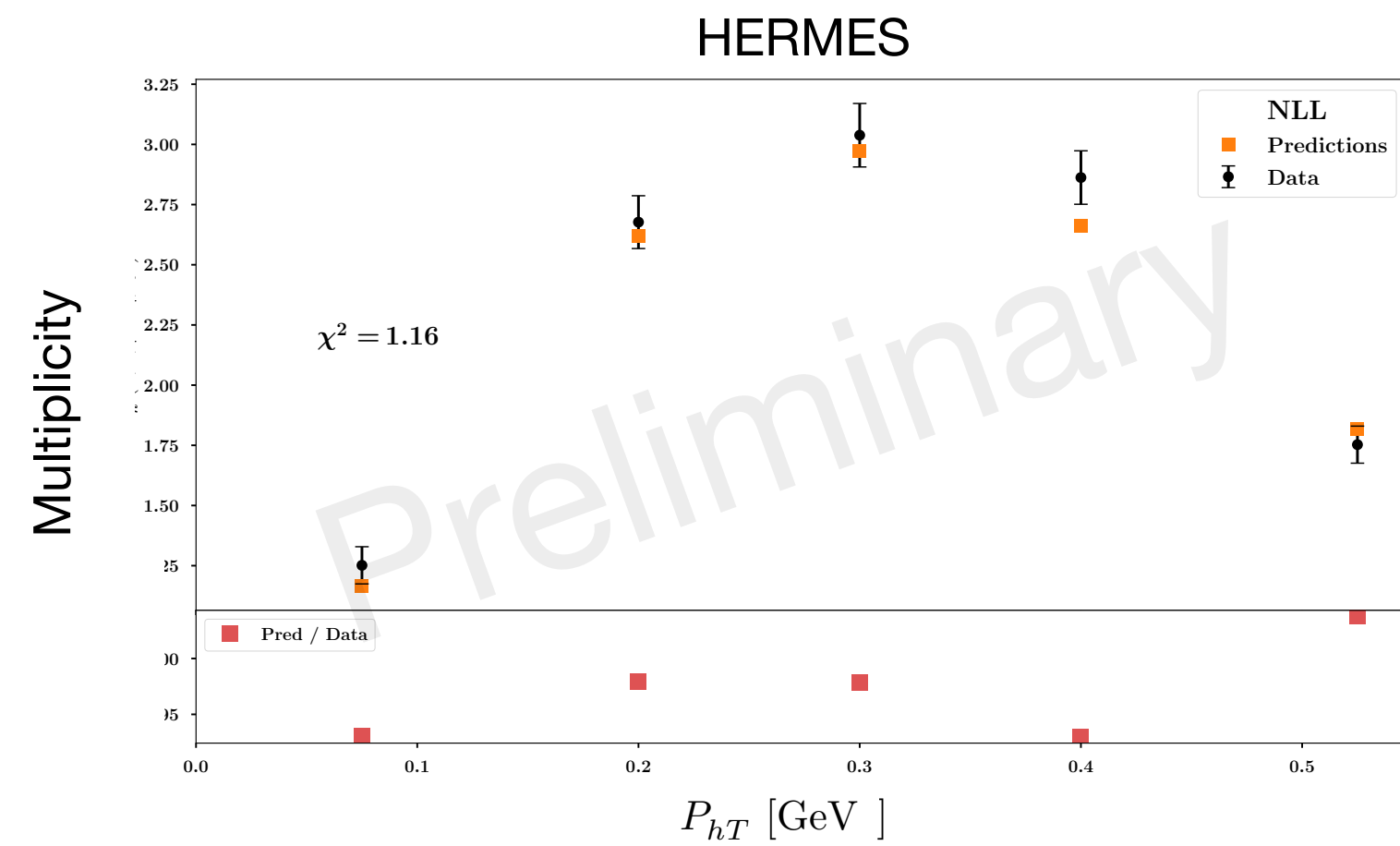
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities at NLL

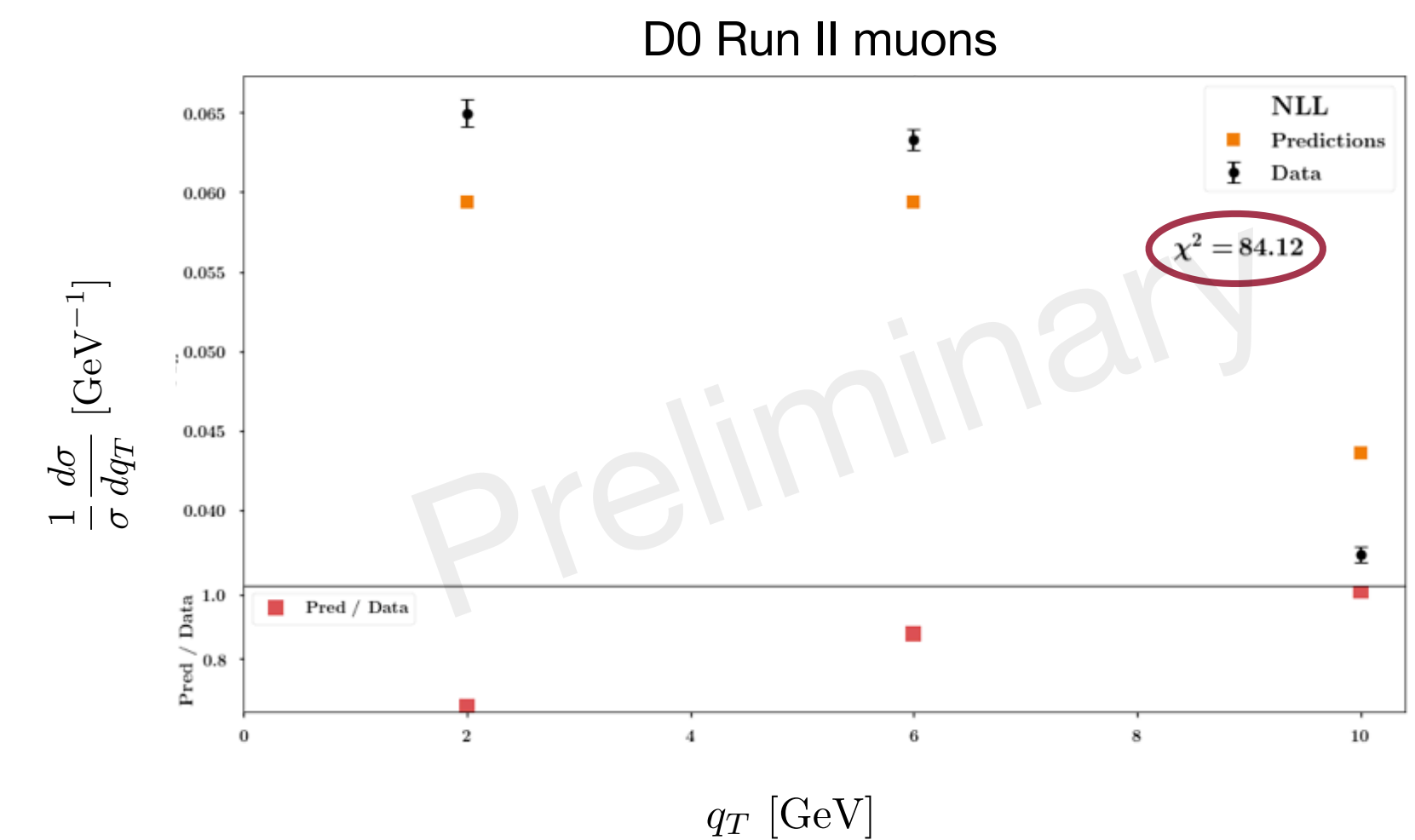
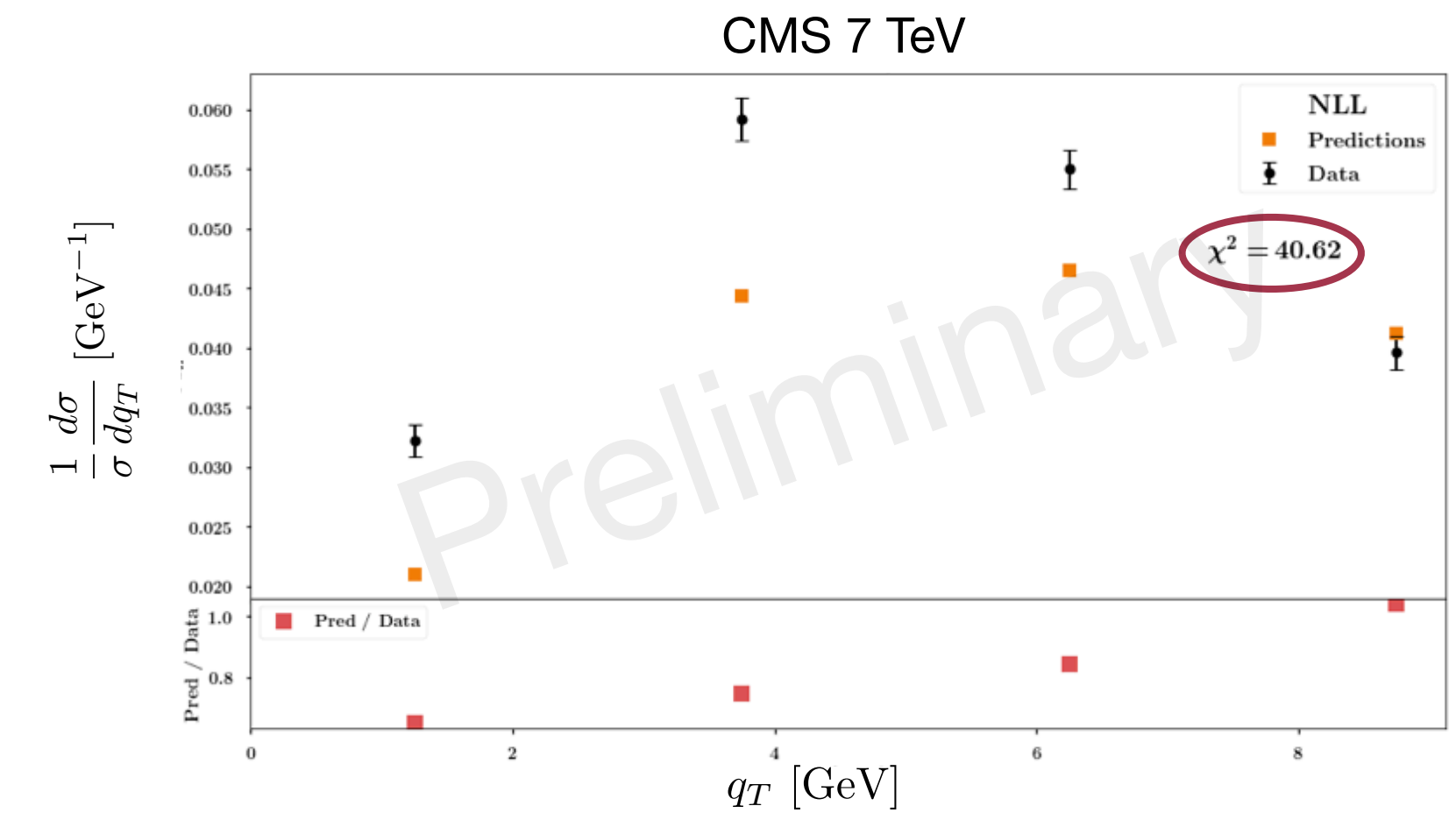


MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities at NLL



High-Energy Drell-Yan at NLL

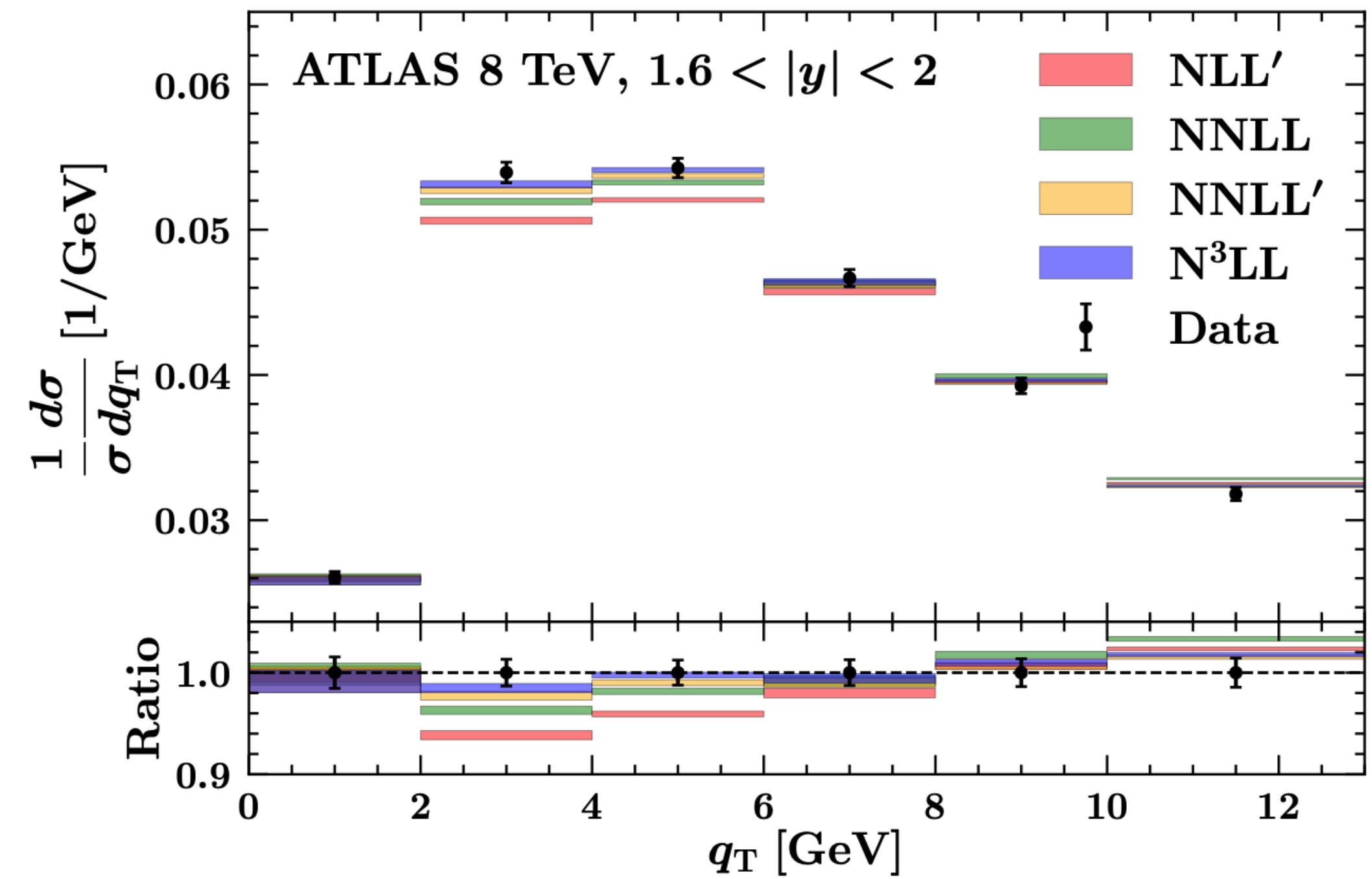


MAPTMD22 – Normalization of SIDIS

MAPTMD22 – Normalization of SIDIS

High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$



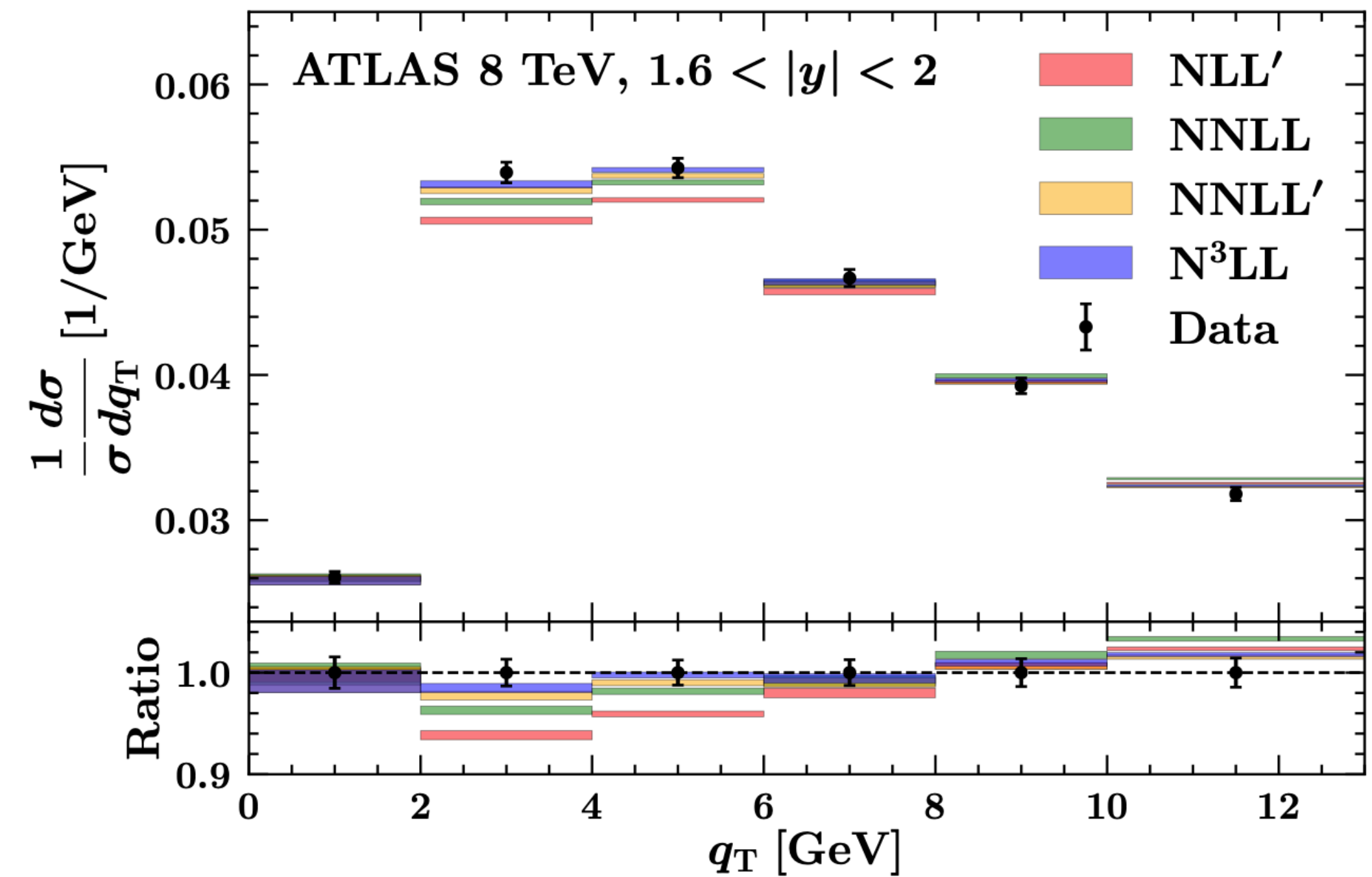
Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$



Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

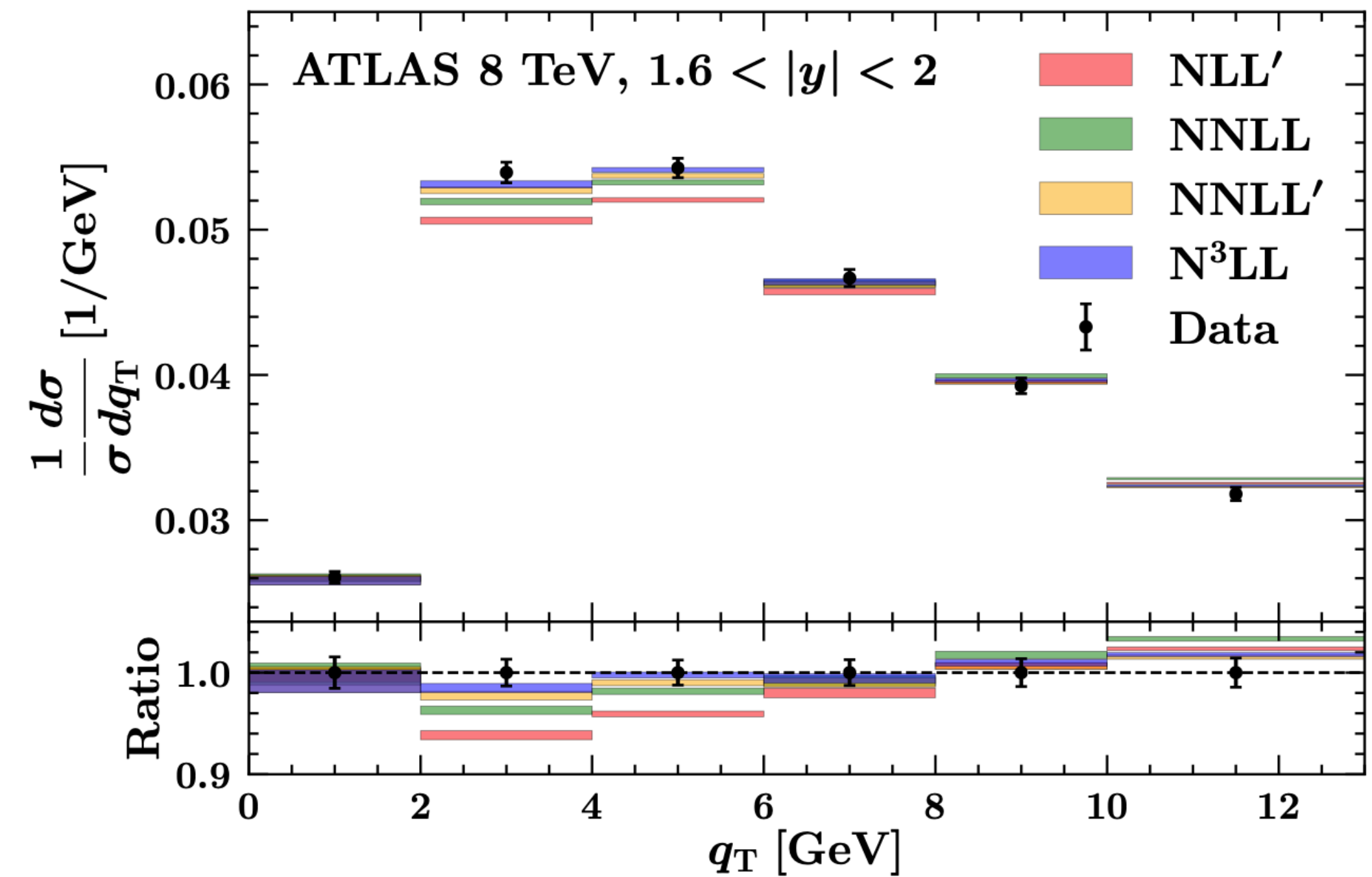
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$

High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$

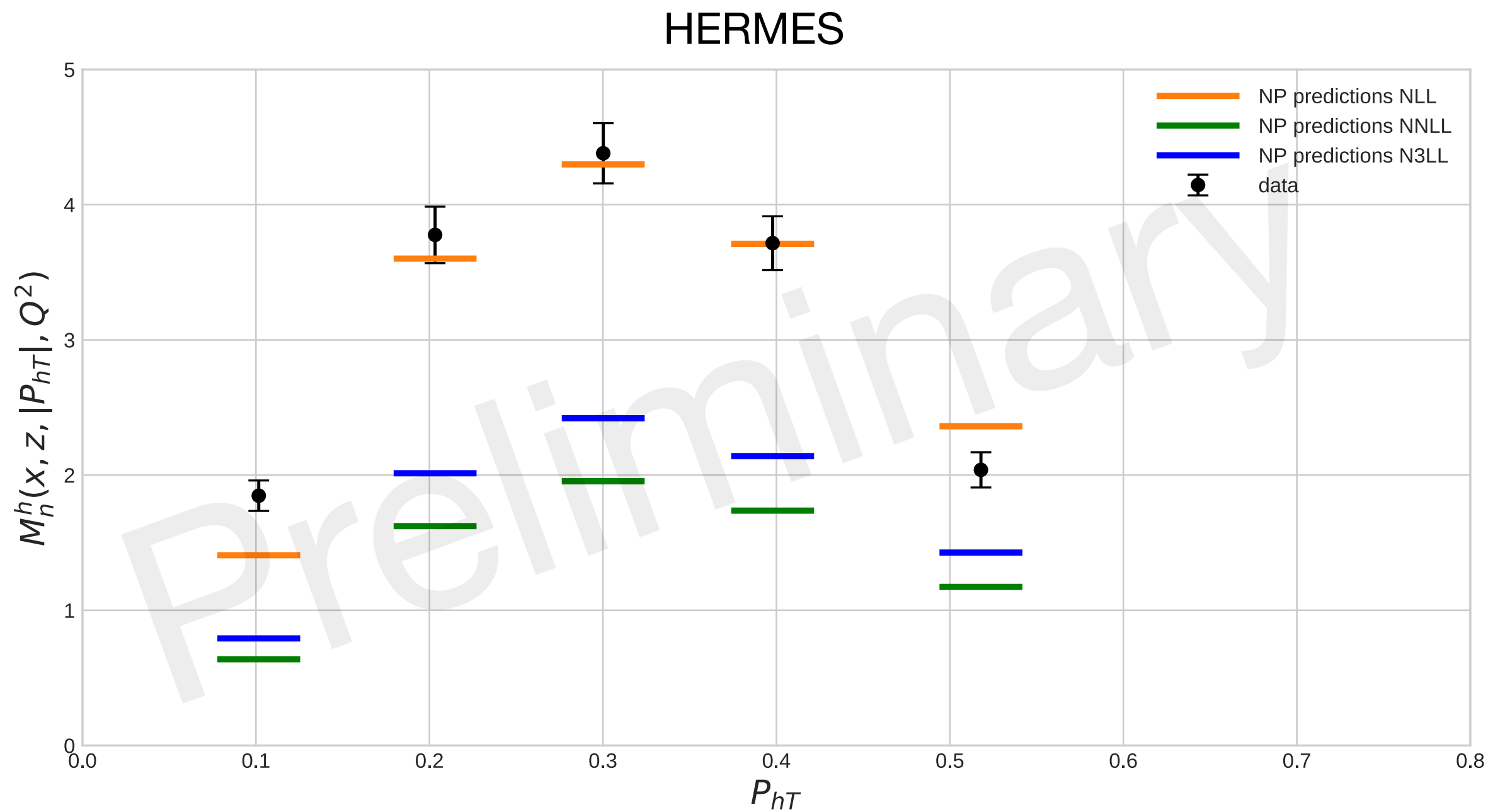


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

MAPTMD22 – Normalization of SIDIS

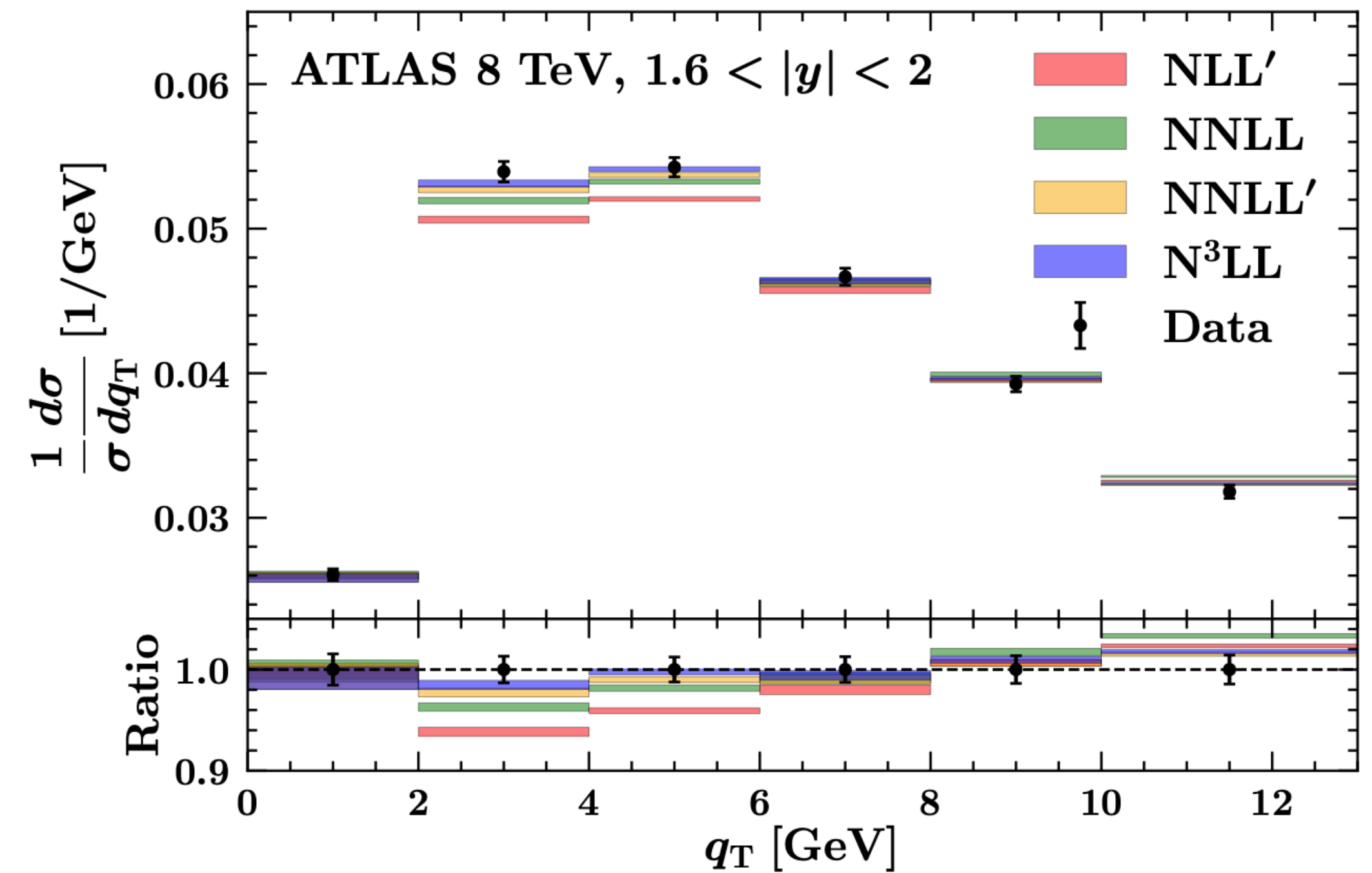
SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$



High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$

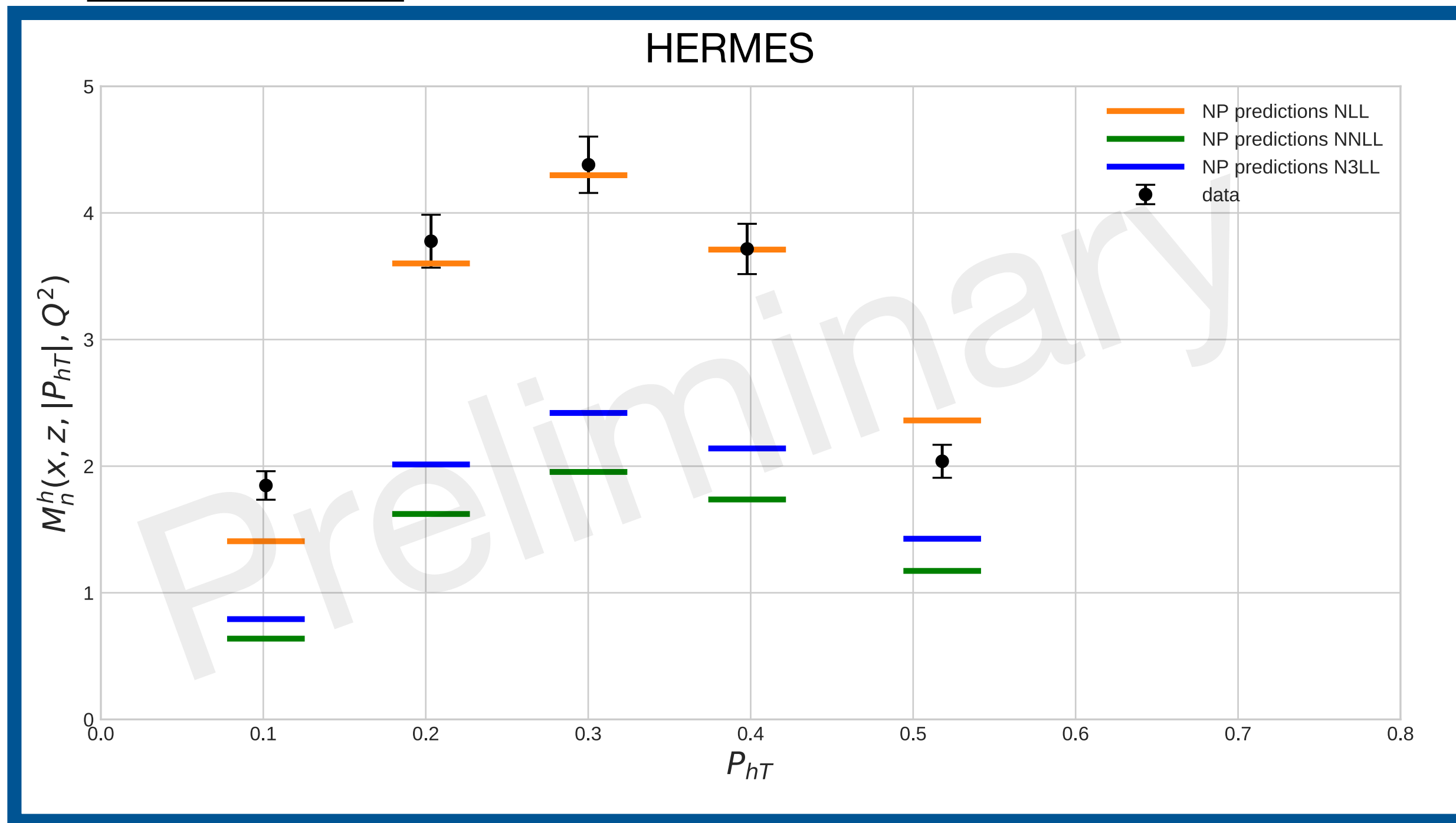


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

MAPTMD22 – Normalization of SIDIS

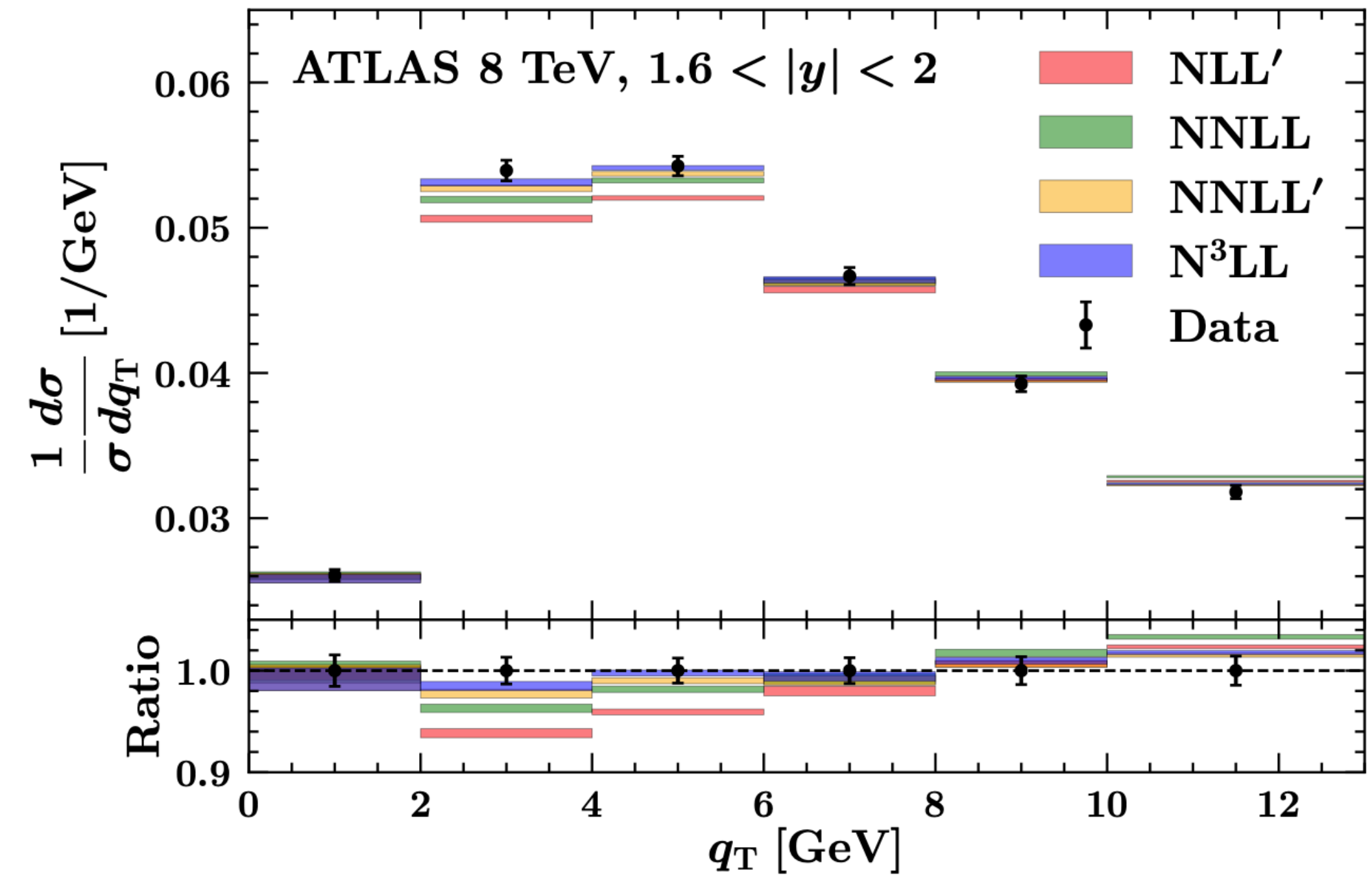
SIDIS multiplicities beyond NLL

$Q \sim 2 \text{ GeV}$



High-Energy Drell-Yan beyond NLL

$Q \sim 100 \text{ GeV}$



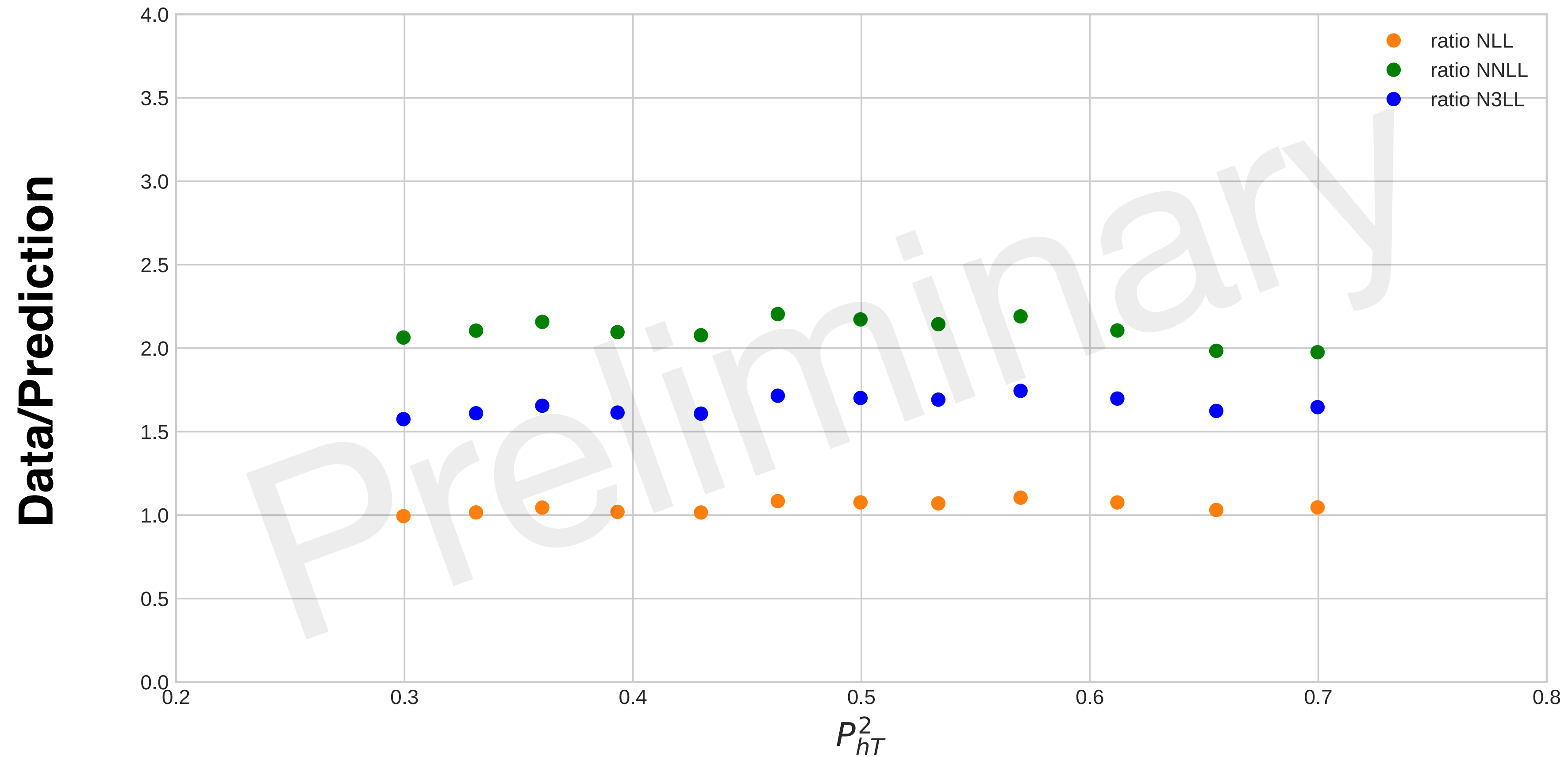
The description considerably worsens at higher orders!!

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

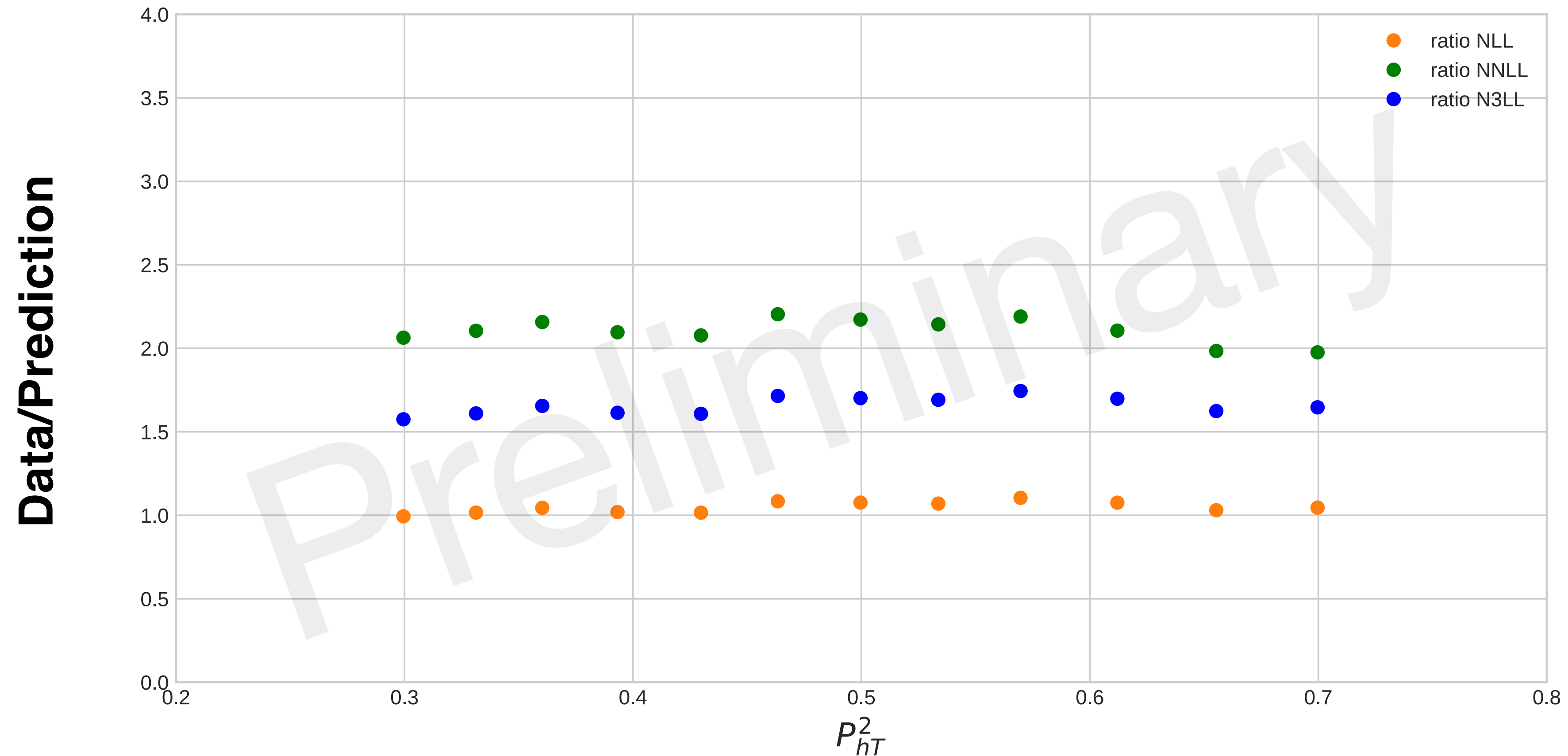
J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176



MAPTMD22 – Normalization of SIDIS

COMPASS multiplicities (one of many bins)

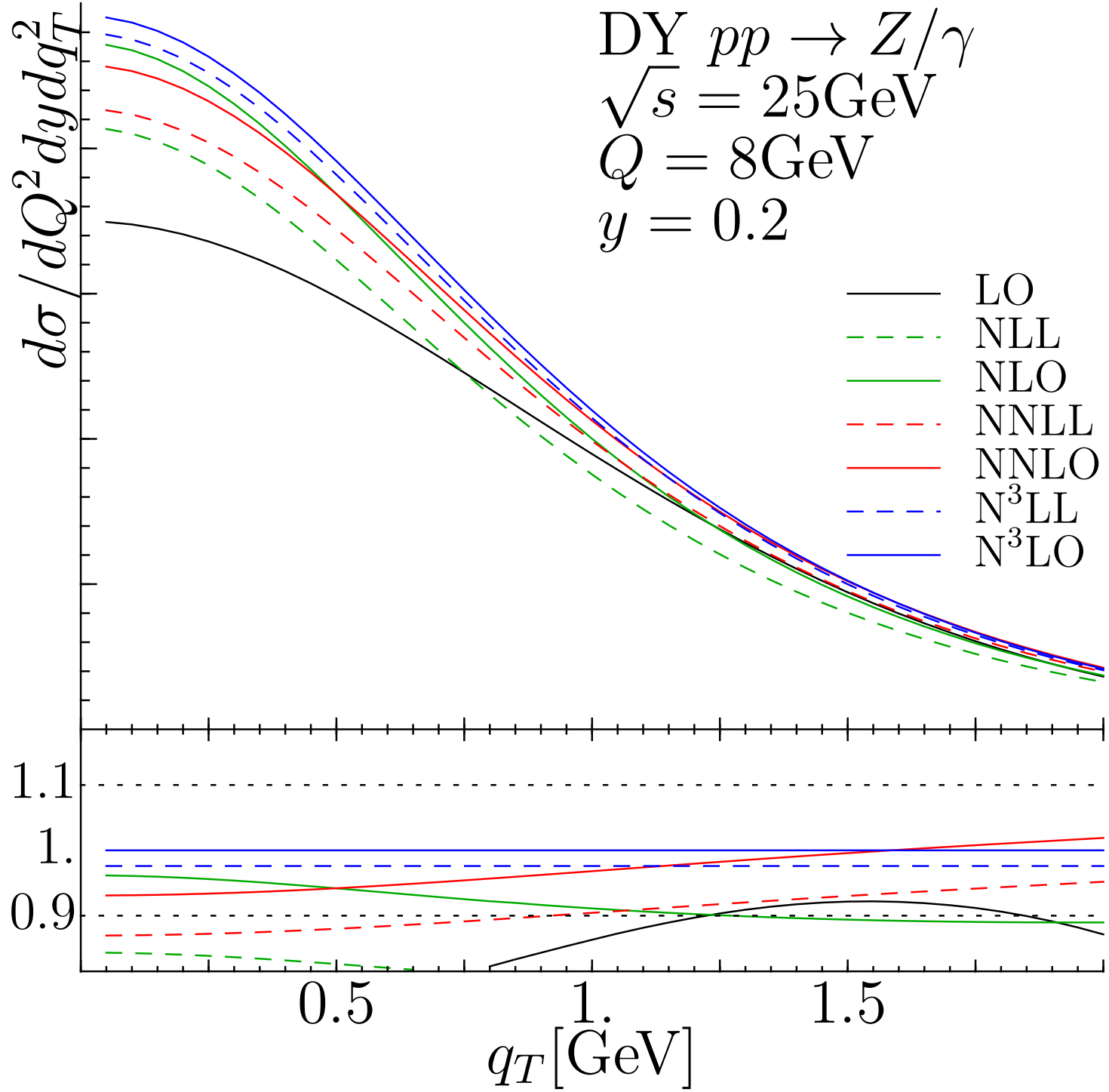
J.O. Gonzalez-Hernandez, PoS DIS2019 (2019) 176



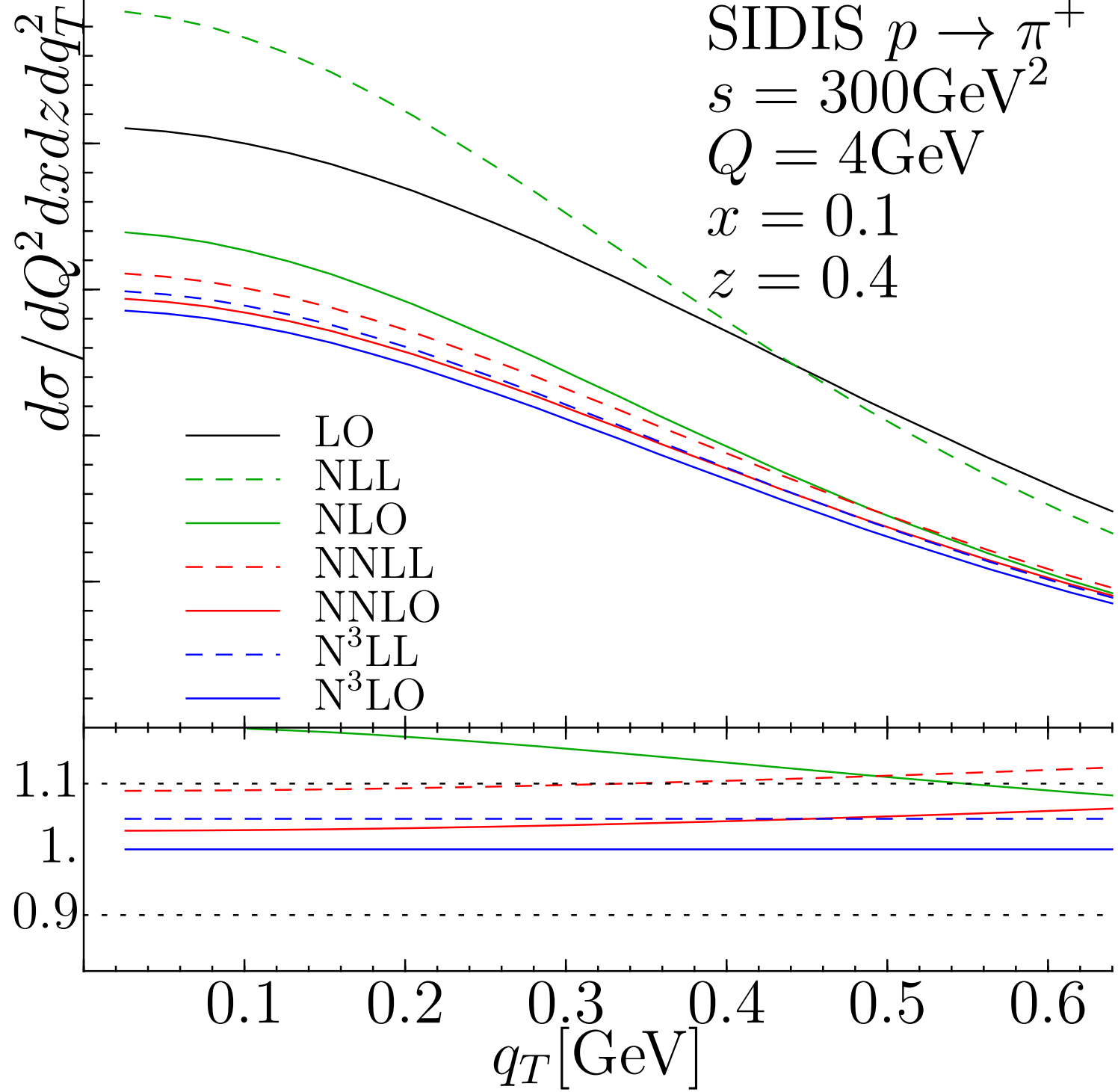
The discrepancy amounts to an almost constant factor!!

Comparison of different orders – SV19

Scimemi, Vladimirov, arXiv:1912.06532



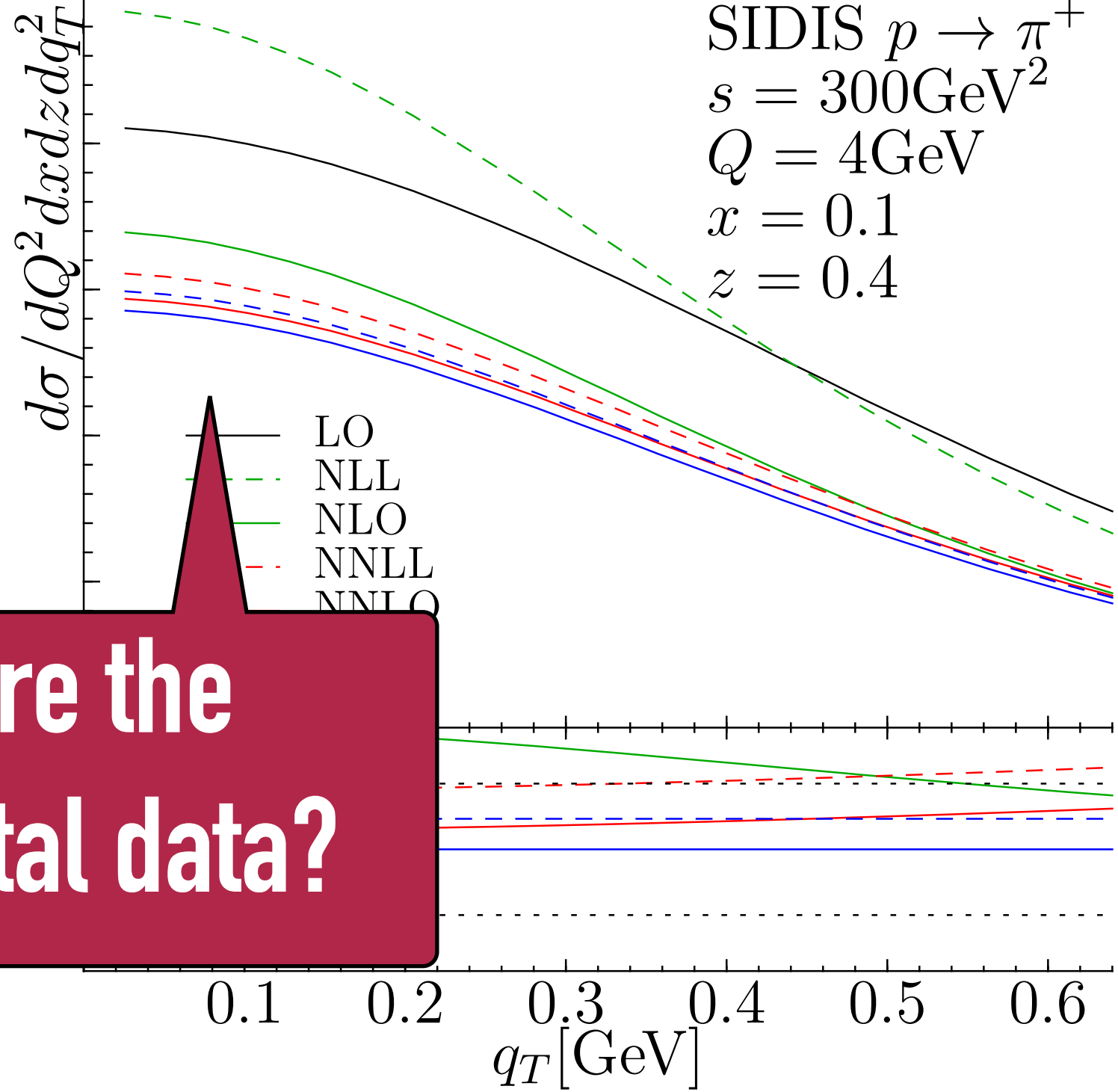
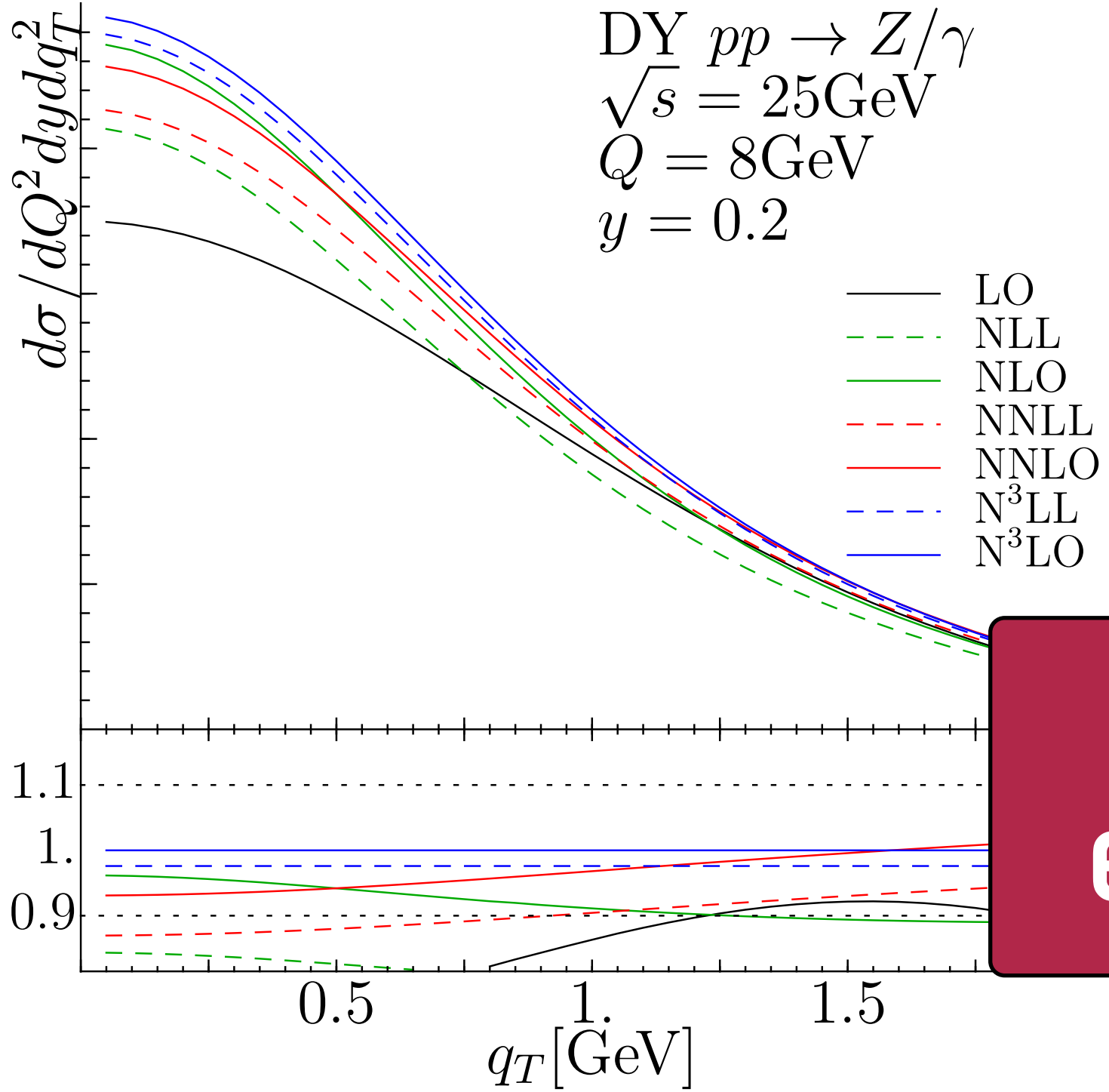
Drell-Yan



SIDIS

Comparison of different orders – SV19

Scimemi, Vladimirov, arXiv:1912.06532



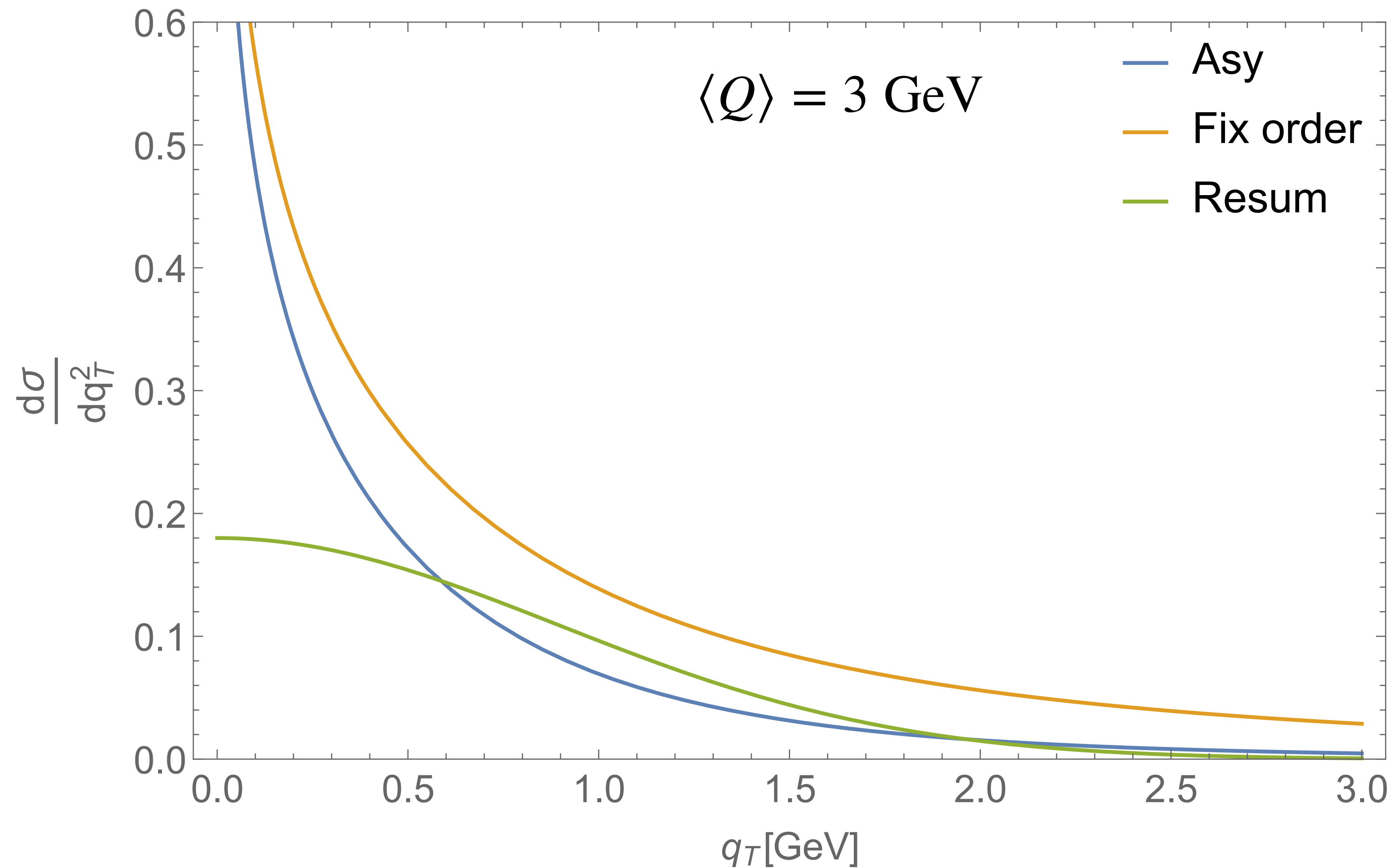
Where are the experimental data?

Drell-Yan

SIDIS

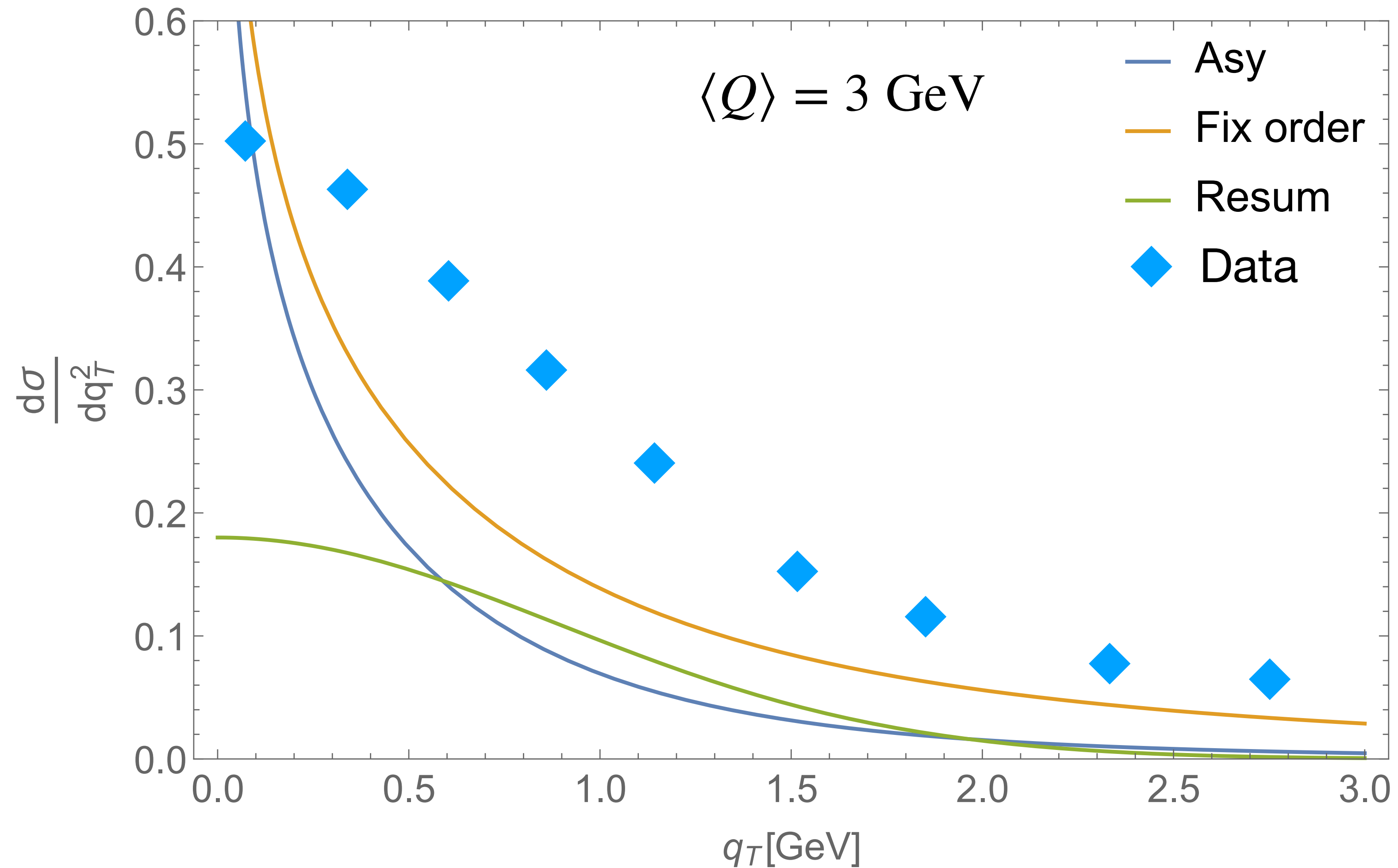
MAPTMD22 – Normalization of SIDIS

According to our formalism



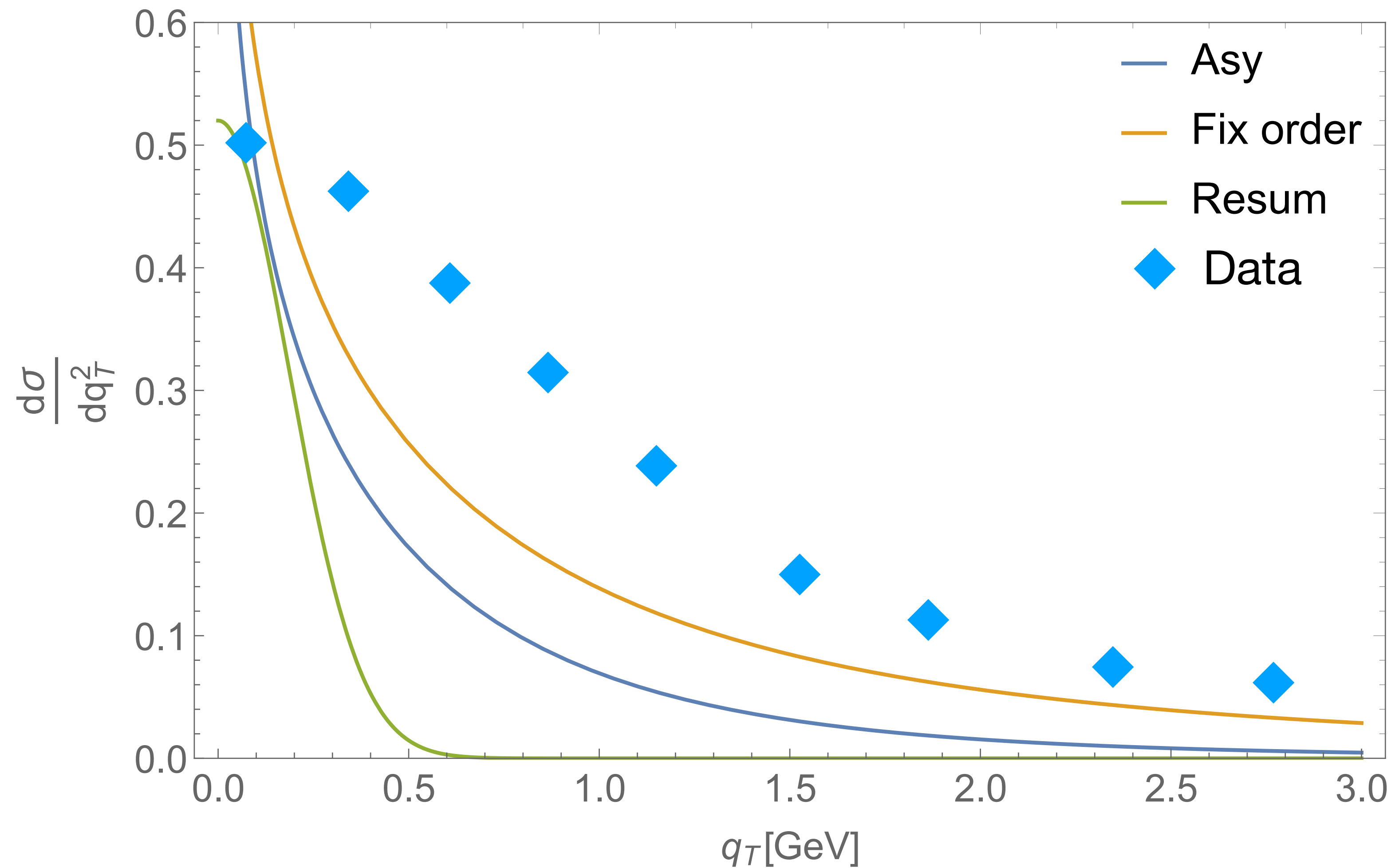
MAPTMD22 – Normalization of SIDIS

According to our formalism



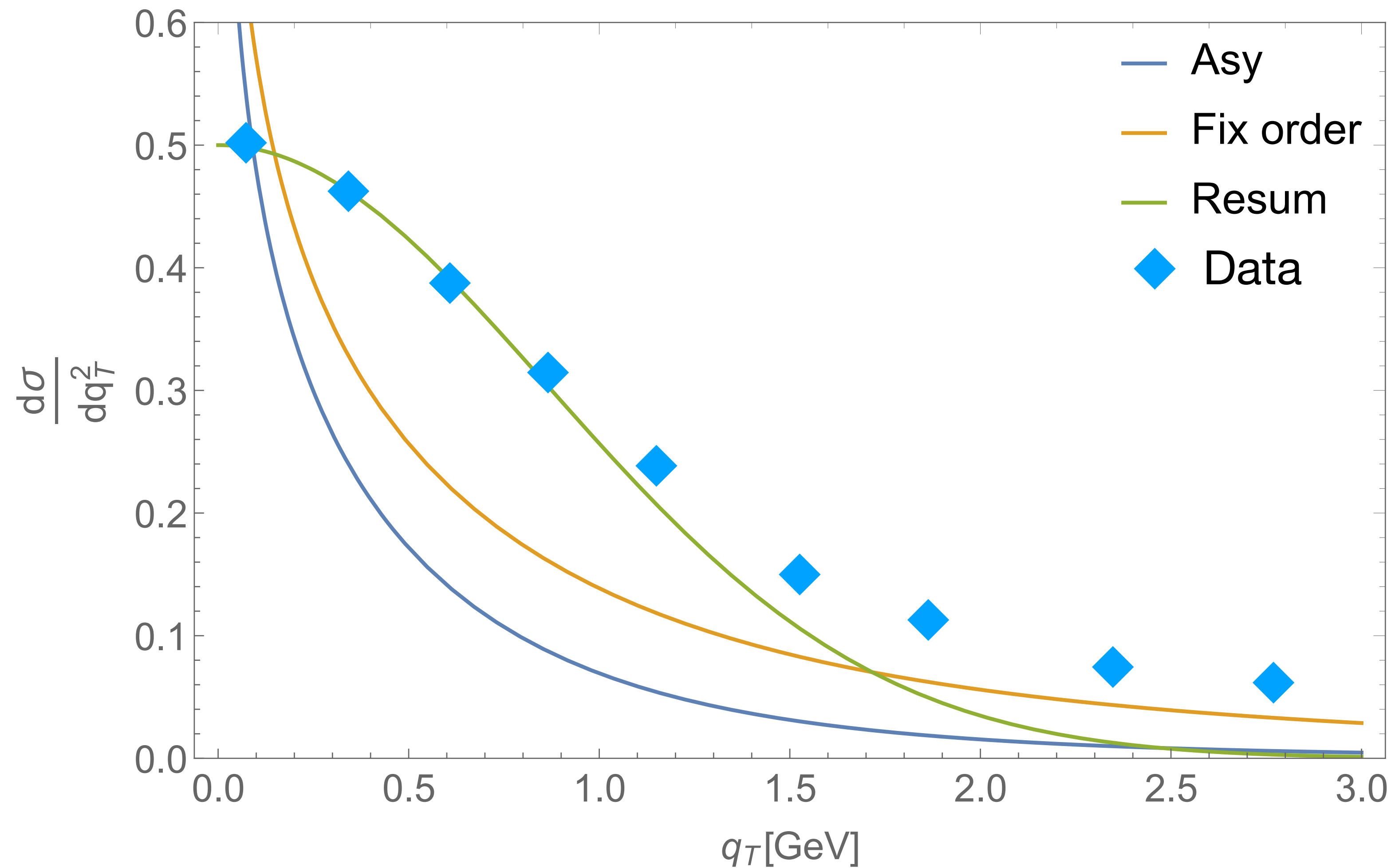
MAPTMD22 – Normalization of SIDIS

Solution1: restrict the TMD region



MAPTMD22 – Normalization of SIDIS

Solution2: enhance TMD contributions



MAPTMD22 – Normalization of SIDIS

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$$

MAPTMD22 – Normalization of SIDIS

SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} / \frac{d\sigma}{dx dQ}$

Collinear SIDIS cross section $\frac{d\sigma}{dx dQ dz}$

MAPTMD22 – Normalization of SIDIS

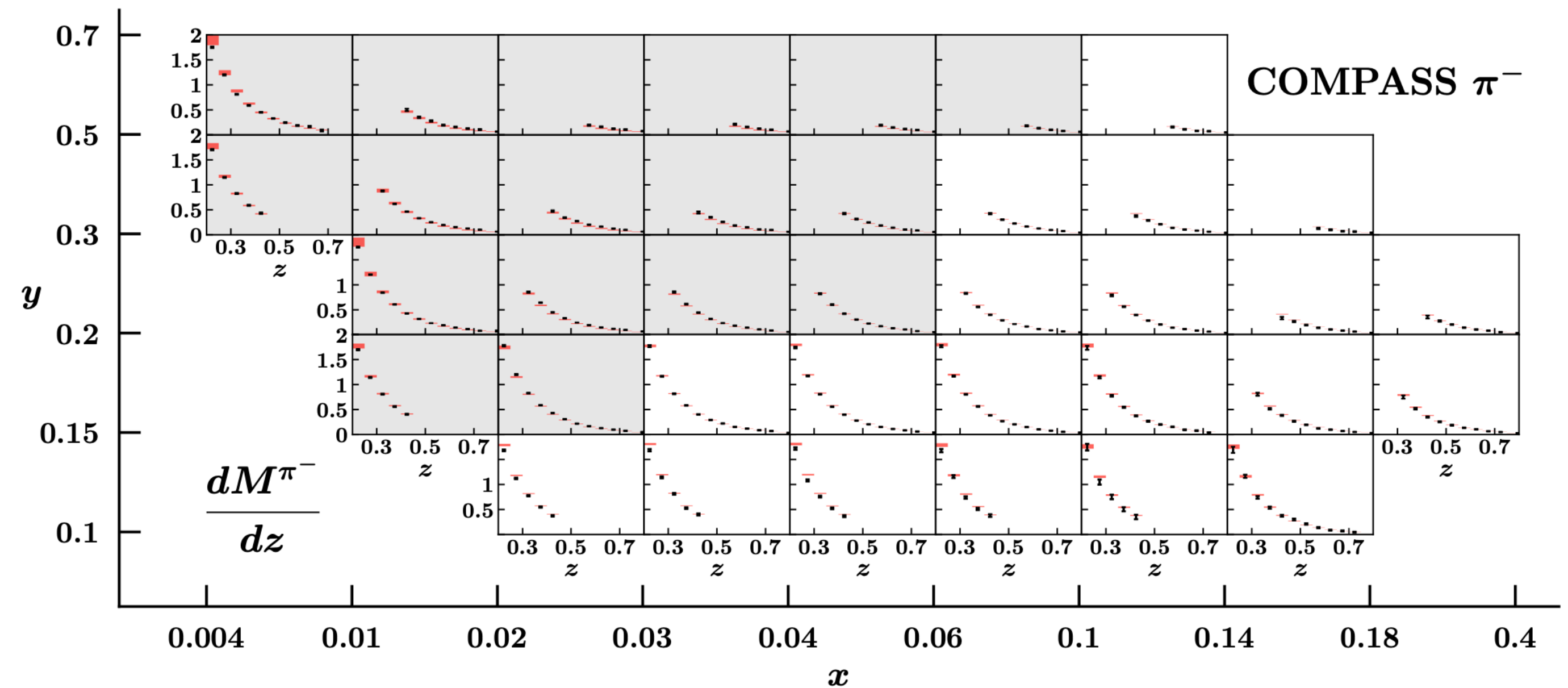
SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ dz}$$

No problems of normalization!!



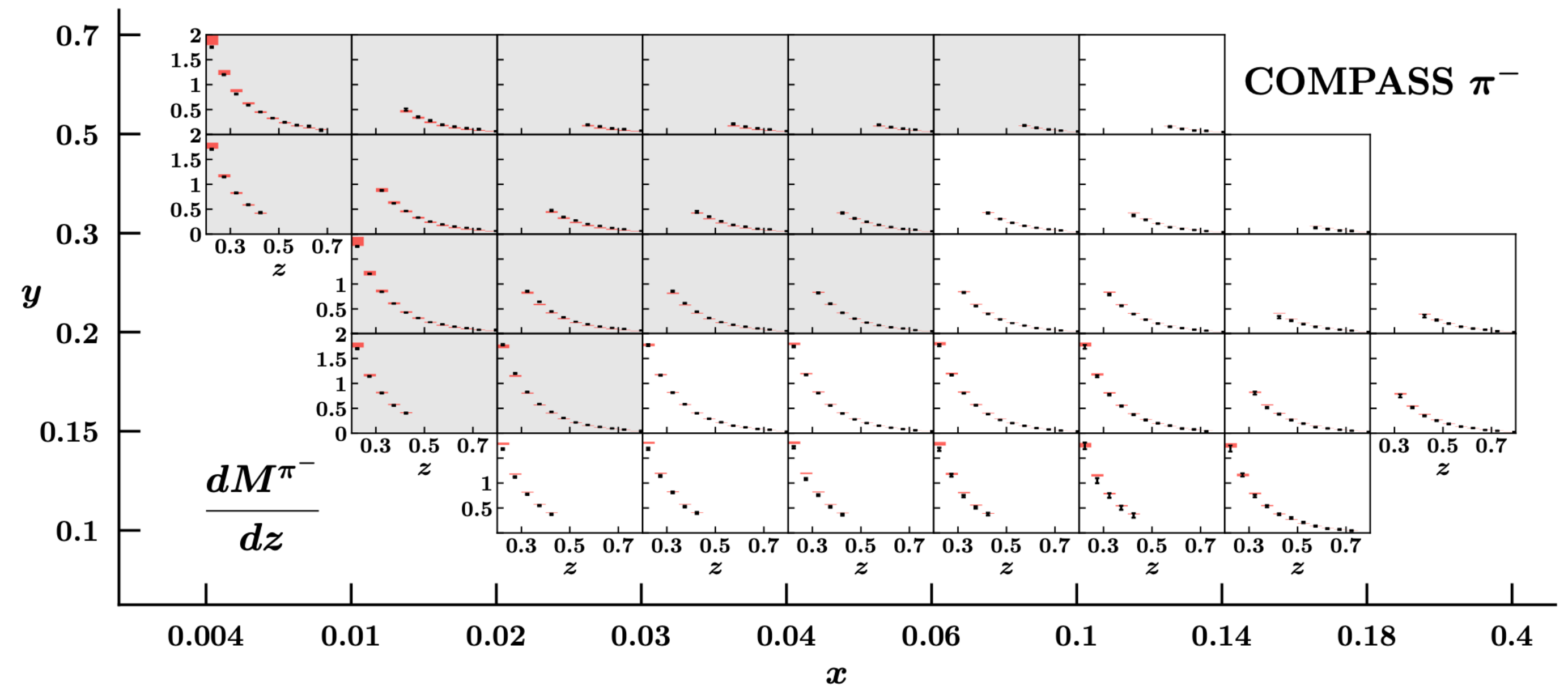
MAPTMD22 – Normalization of SIDIS

SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$

Collinear SIDIS cross section $\frac{d\sigma}{dx dQ dz}$ **No problems of normalization!!**

Normalization of prediction such that

$$\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}} = \frac{d\sigma}{dx dQ dz}$$



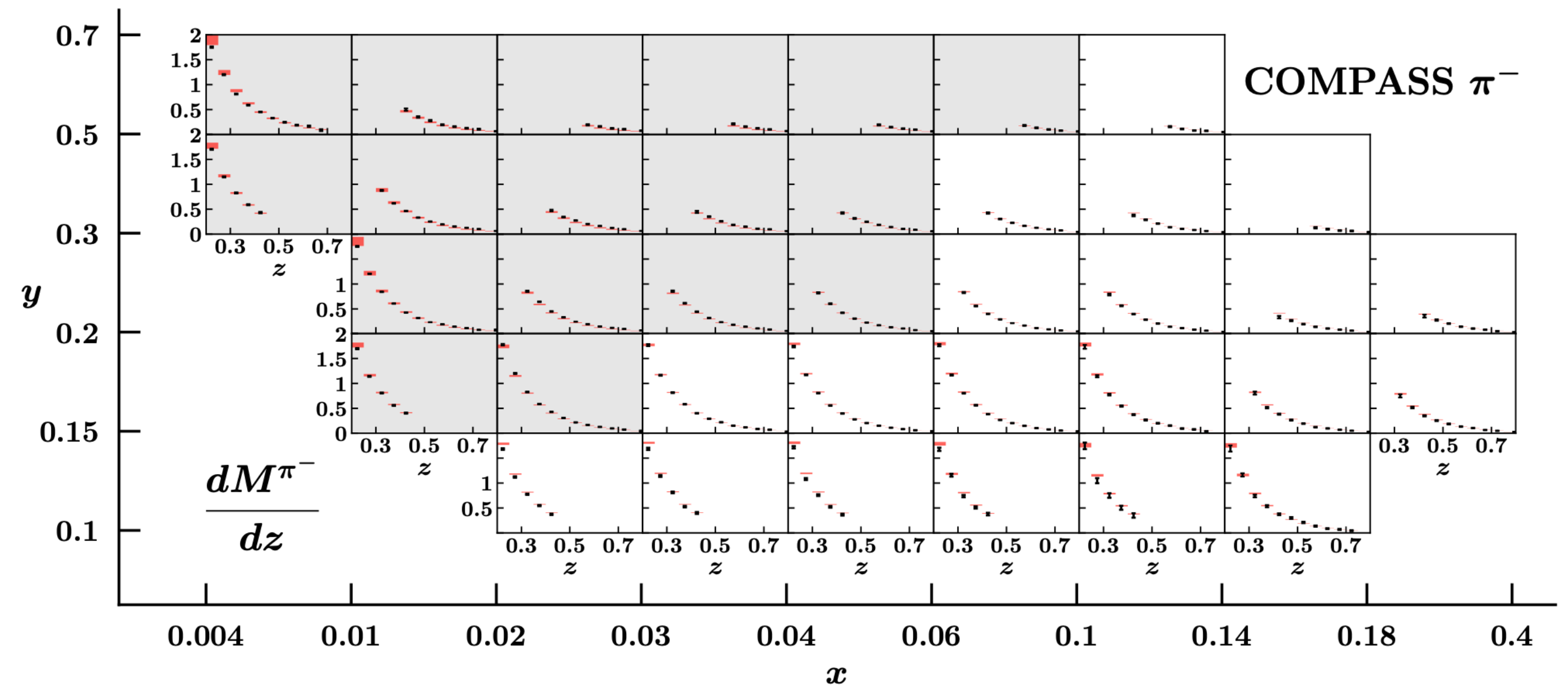
MAPTMD22 – Normalization of SIDIS

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Normalization of prediction such that

$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \bigg/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$



MAPTMD22 – Normalization of SIDIS

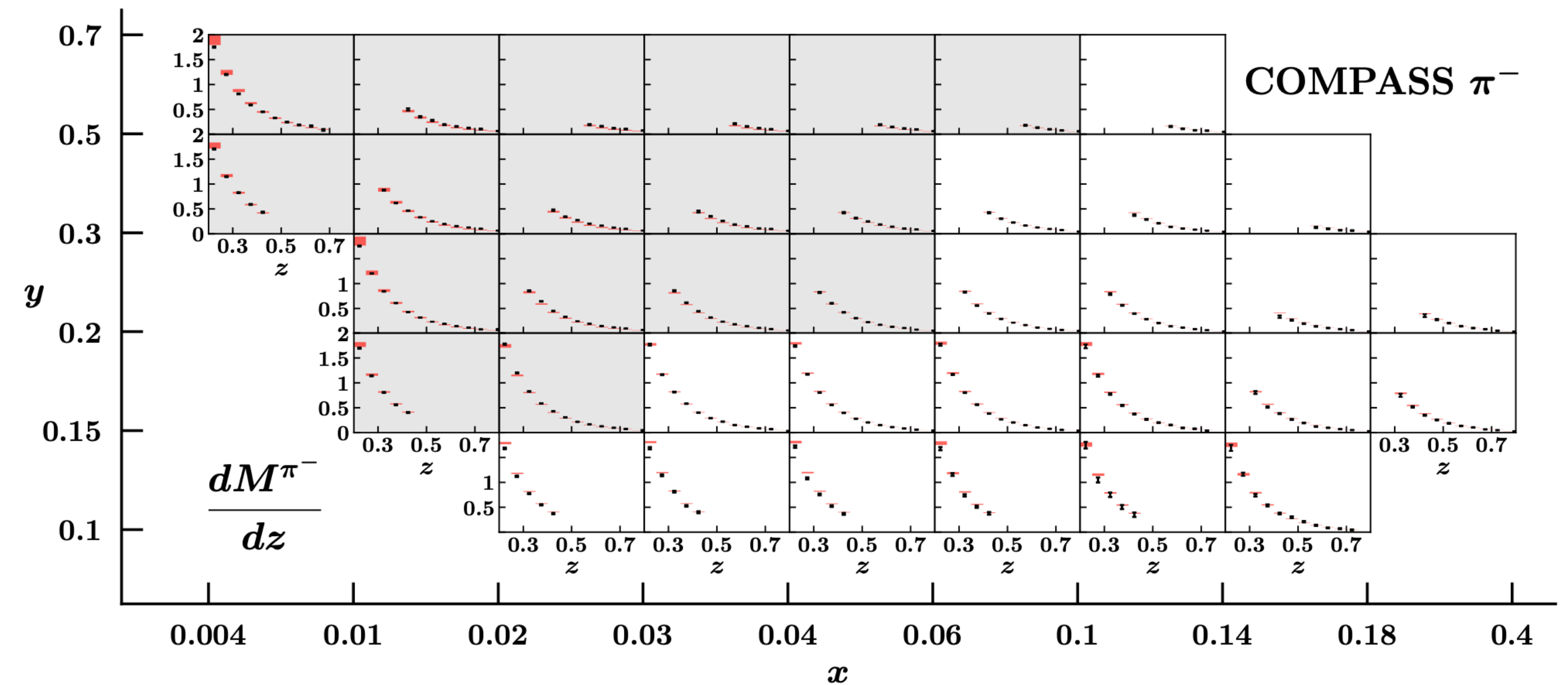
SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$

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Normalization of prediction such that

$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \bigg/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$

$$M(x, z, P_{hT}, Q) = w(x, z, Q) \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$$



MAPTMD22 – Normalization of SIDIS

SIDIS multiplicity $M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \bigg/ \frac{d\sigma}{dx dQ}$

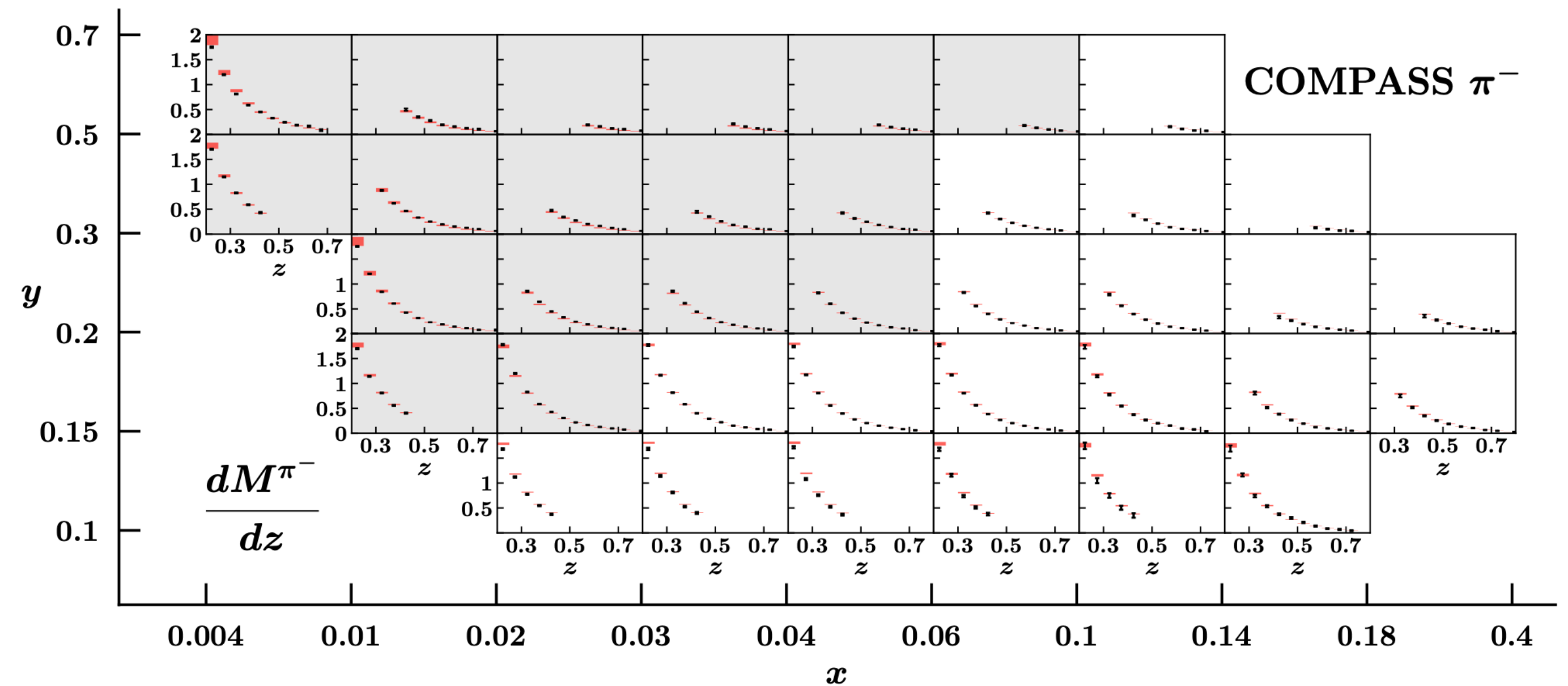
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Independent of the fitting parameters!!

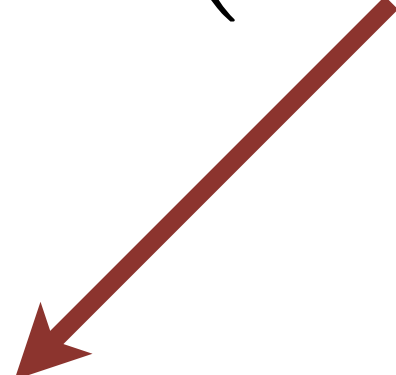


MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_{\perp}^2}{g_{1A}}} + \lambda_B k_{\perp}^2 e^{-\frac{k_{\perp}^2}{g_{1B}}} + \lambda_C e^{-\frac{k_{\perp}^2}{g_{1C}}} \right)$$


$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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MAPTMD22 – Parameterization of TMDs

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MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_{\perp}^2}{g_{1A}}} + \lambda_B k_{\perp}^2 e^{-\frac{k_{\perp}^2}{g_{1B}}} + \lambda_C e^{-\frac{k_{\perp}^2}{g_{1C}}} \right)$$

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$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

MAPTMD22 – Parameterization of TMDs

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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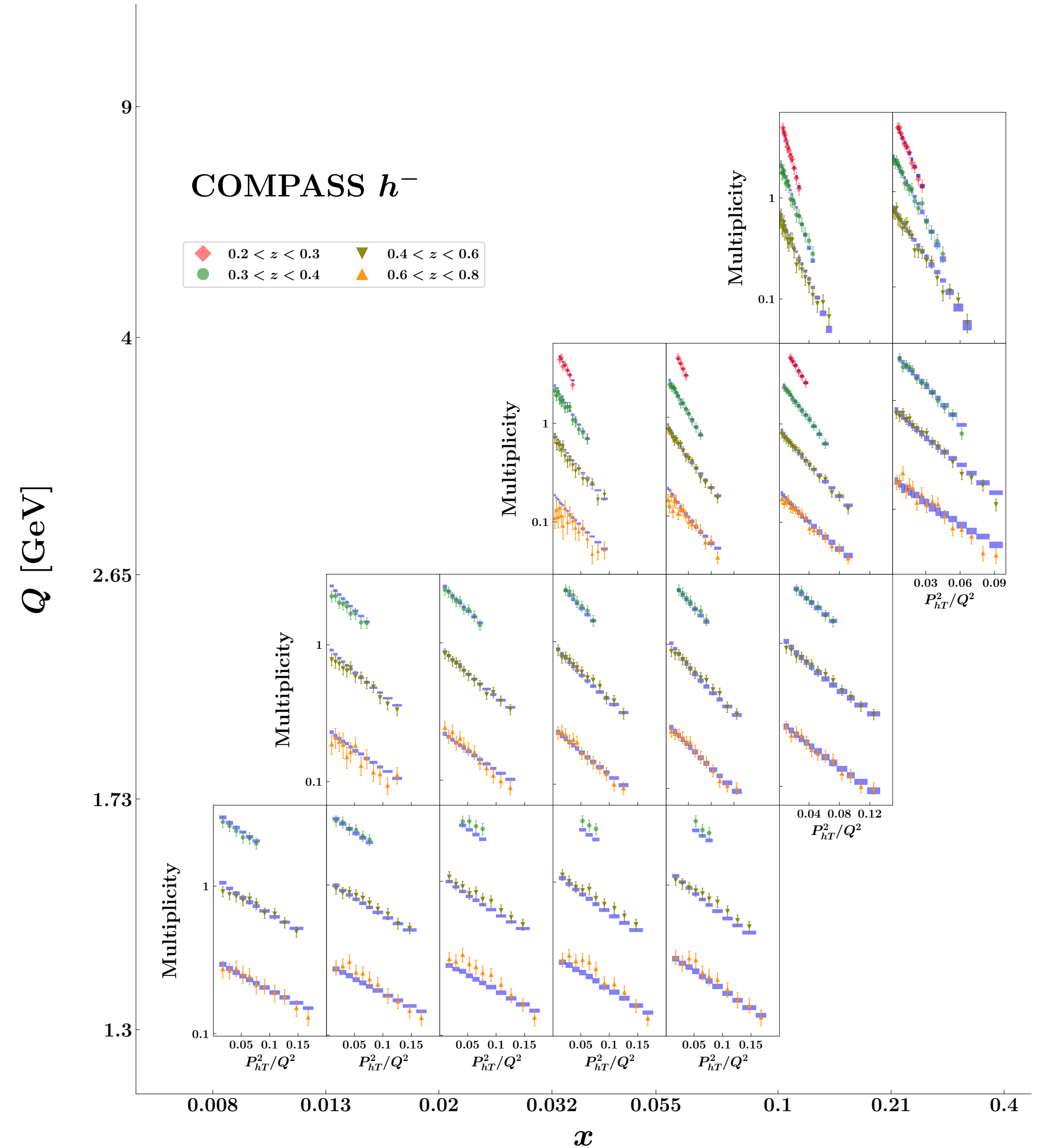
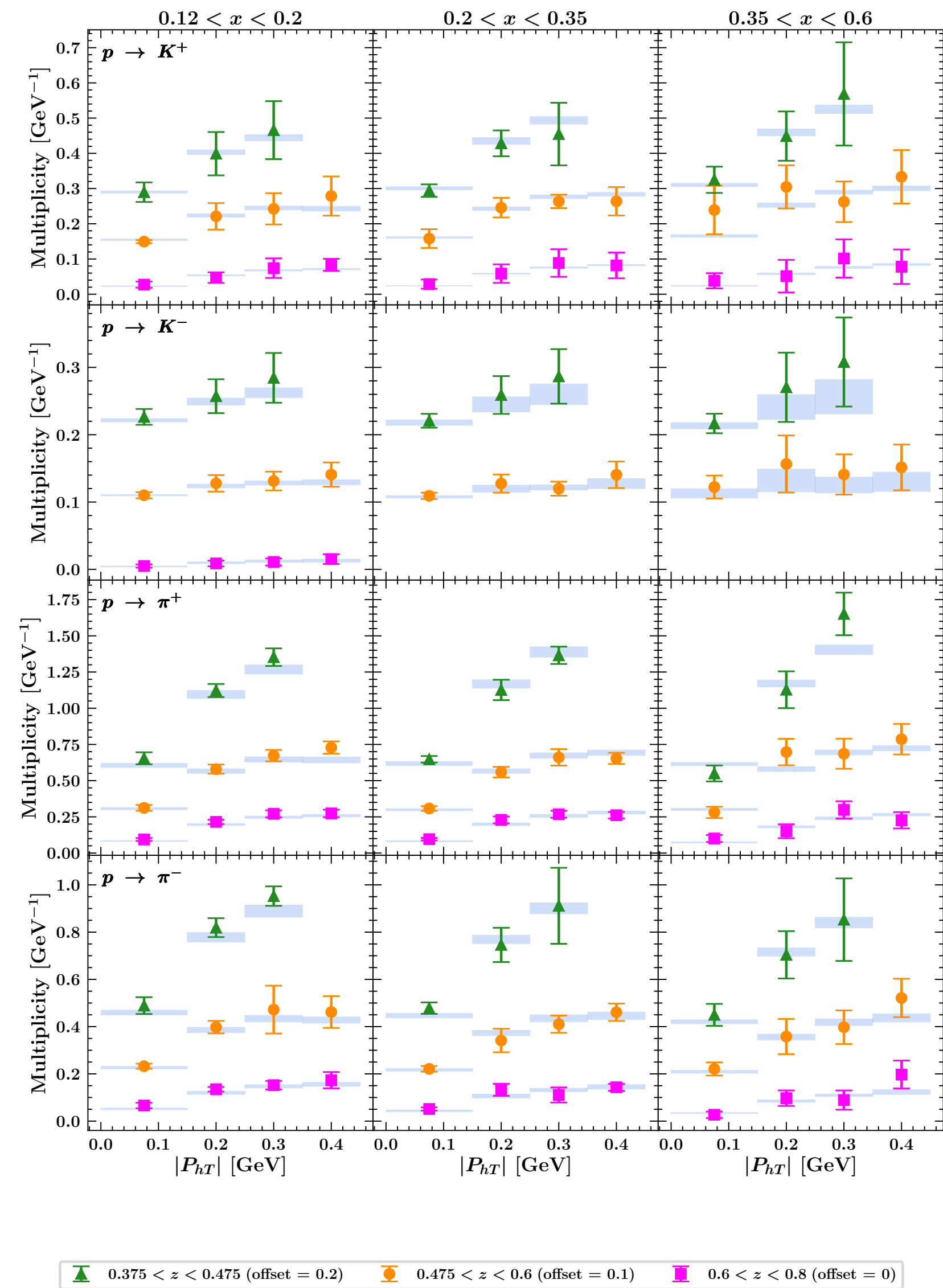
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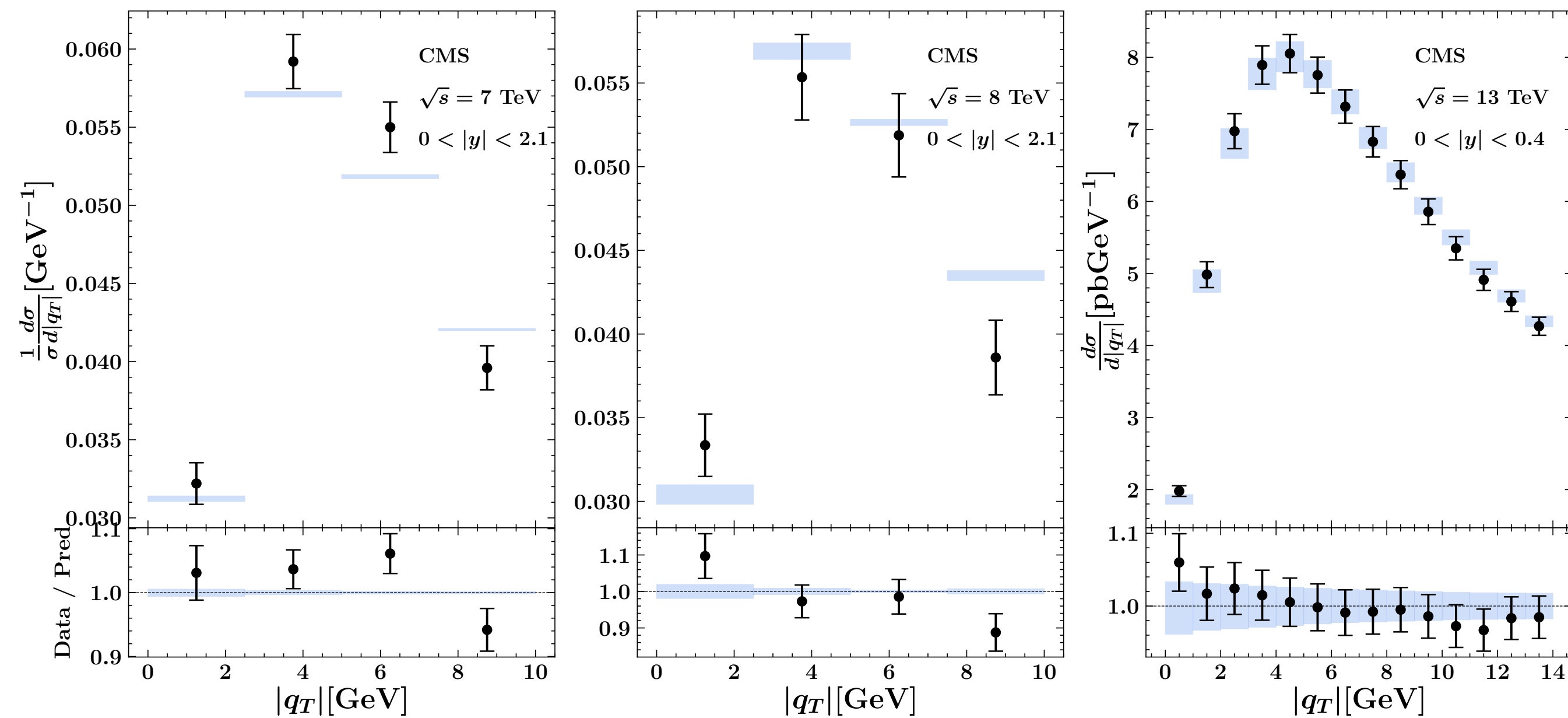
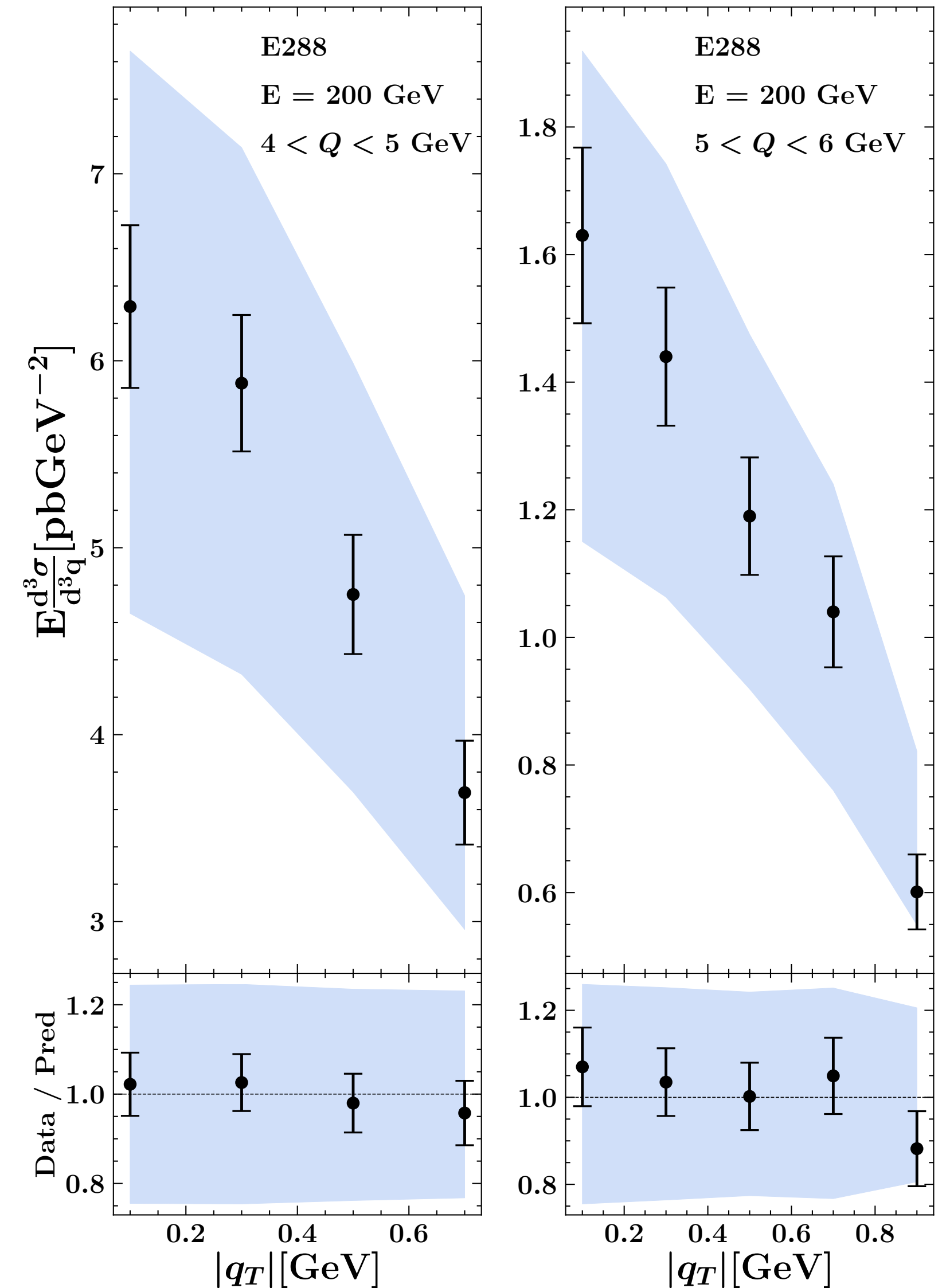
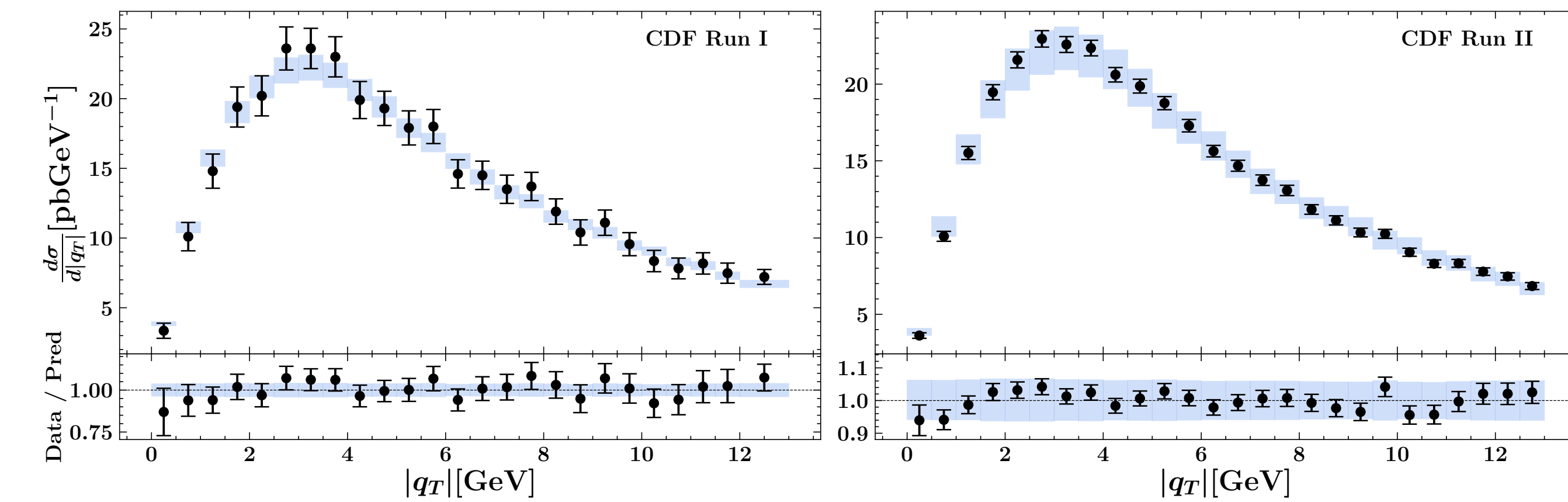
11 parameters for TMD PDF
 + 1 for NP evolution + 9 for TMD FF
 = 21 free parameters

MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$

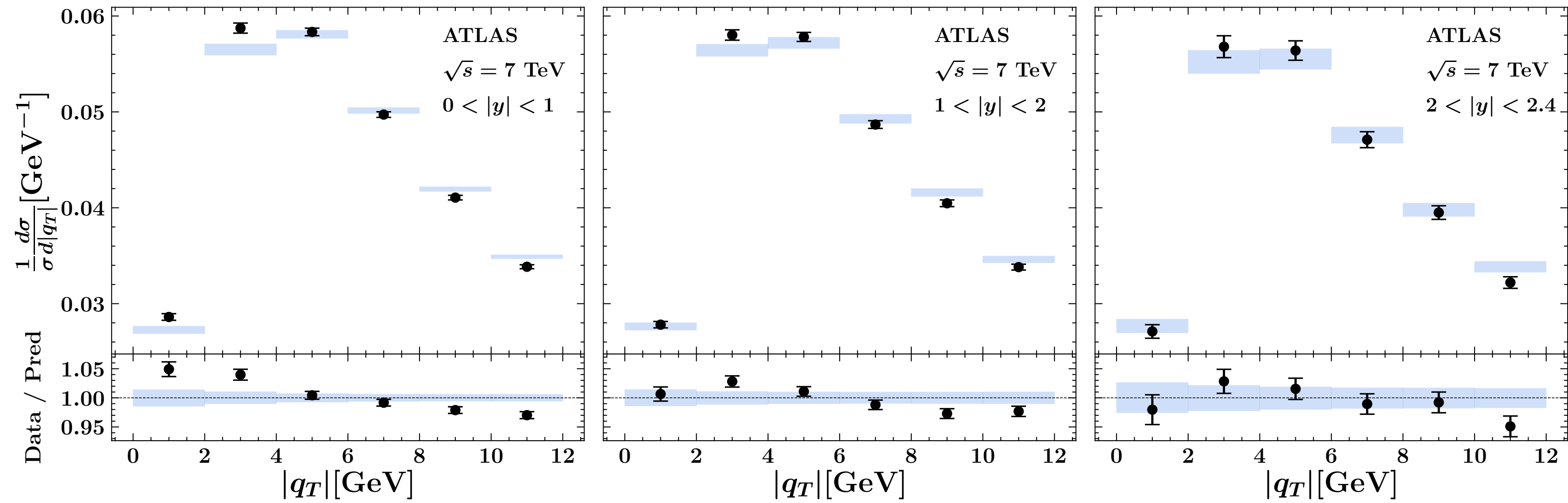
HERMES



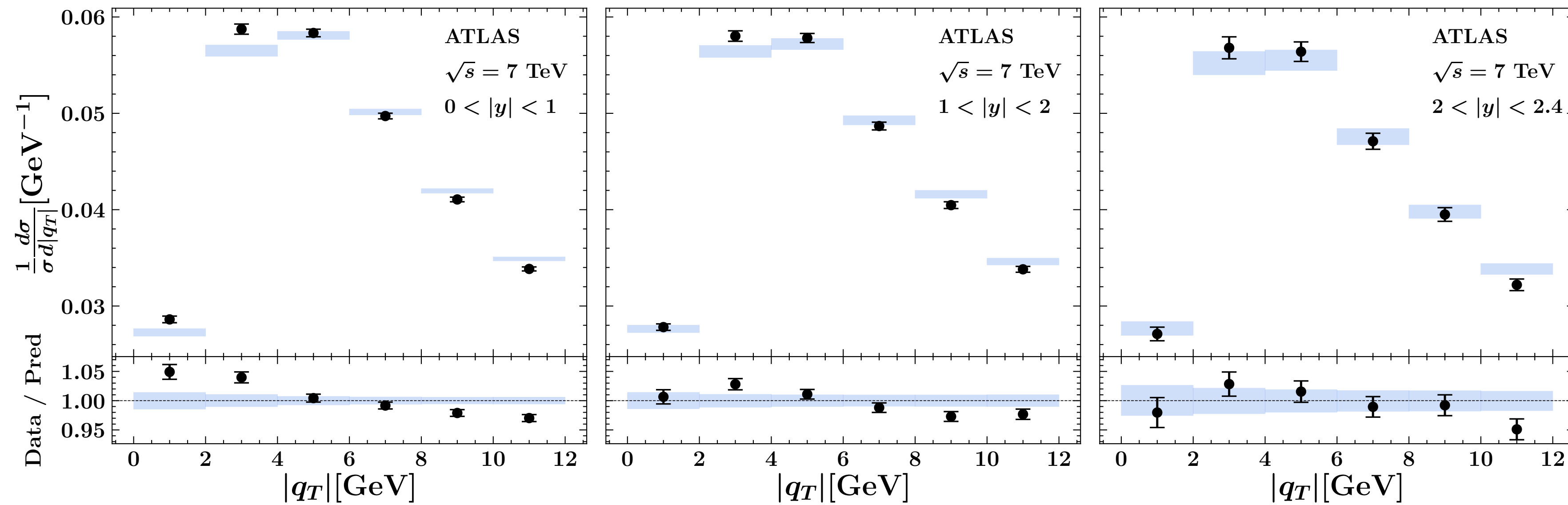
MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$



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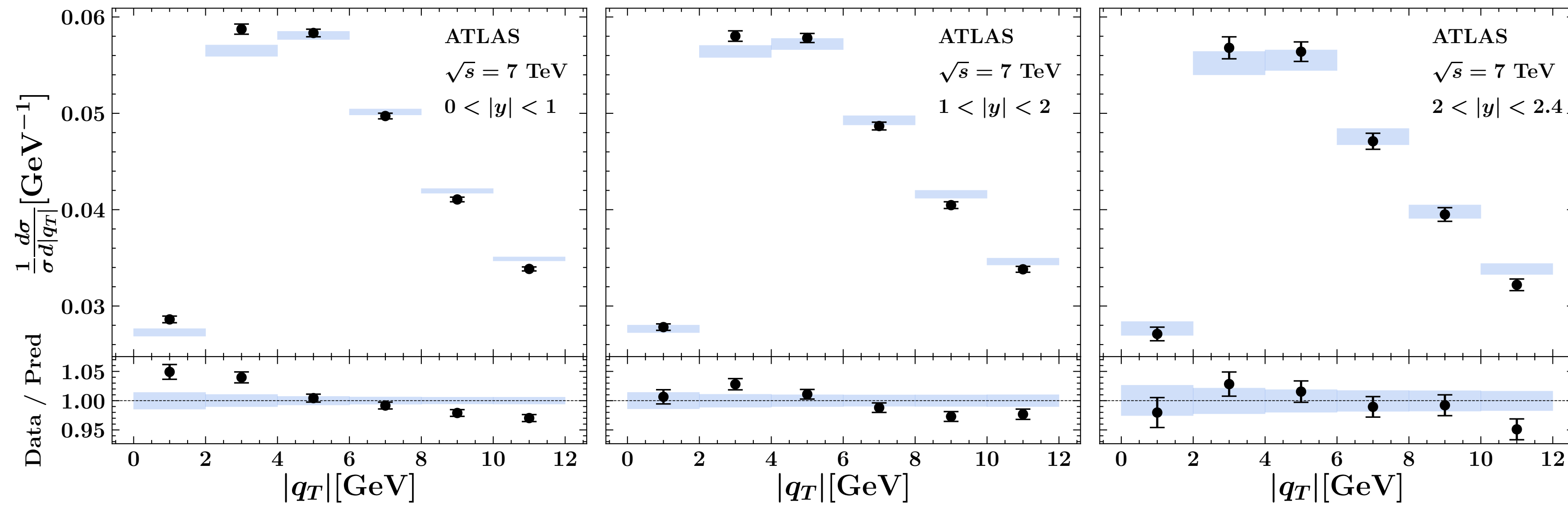


MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$



Possible justifications:

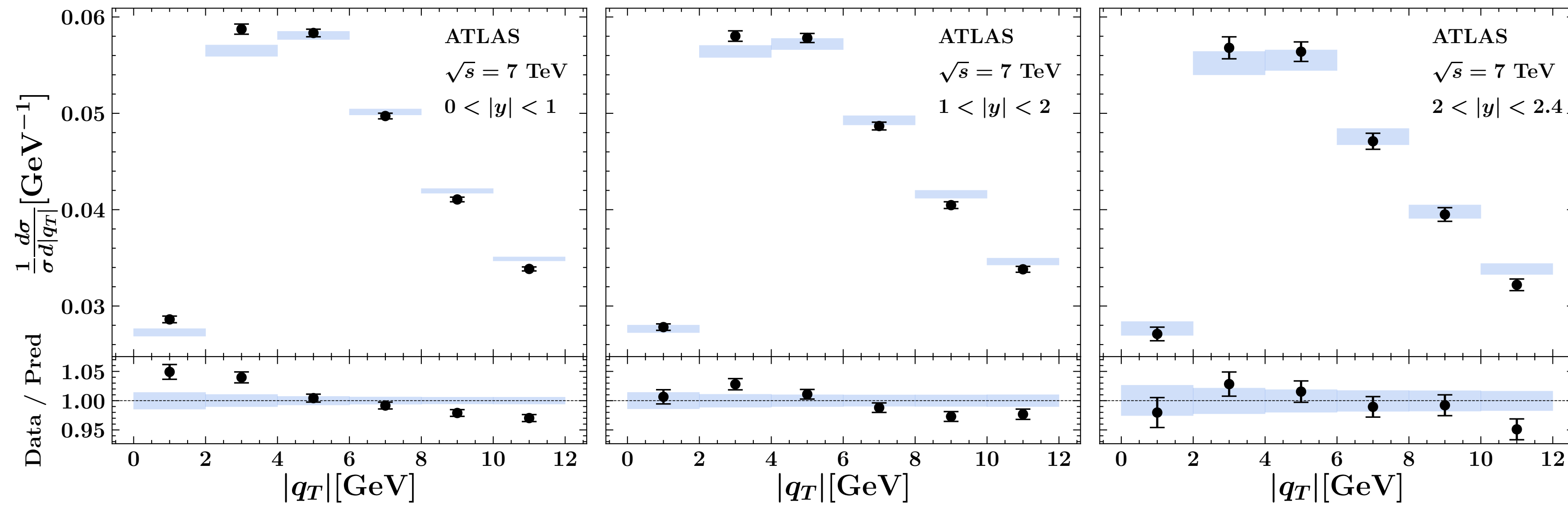
MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$



Possible justifications:

- ⊖ Small experimental uncertainties

MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$

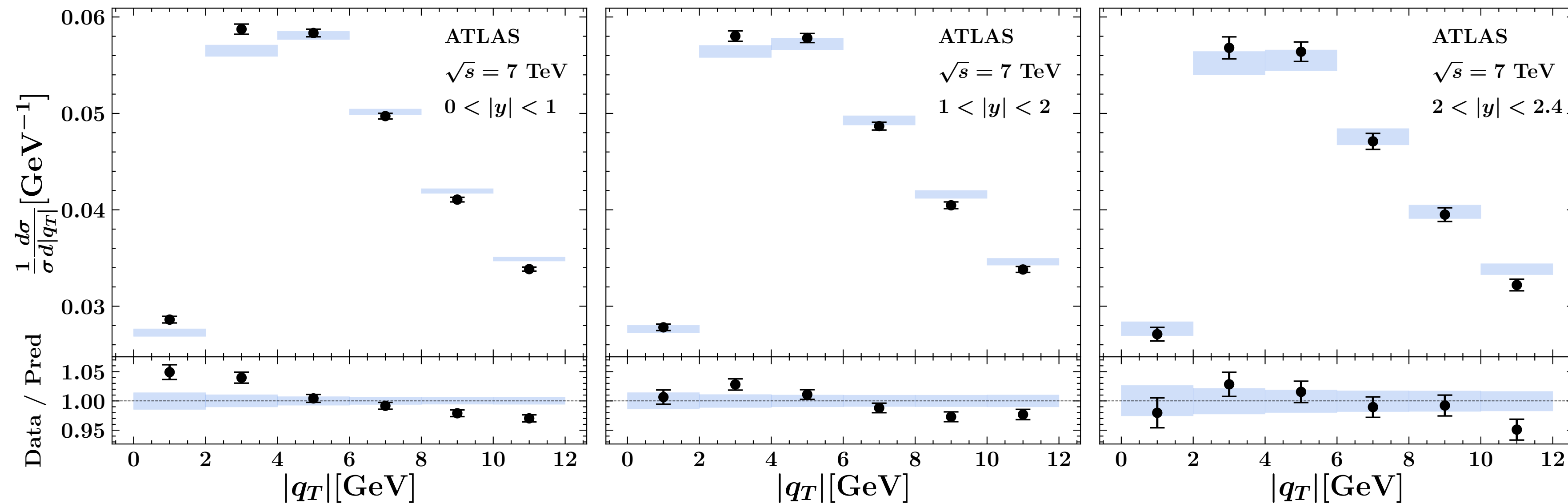


Possible justifications:

⊖ Small experimental uncertainties

⊖ Implementation of lepton cuts

MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$



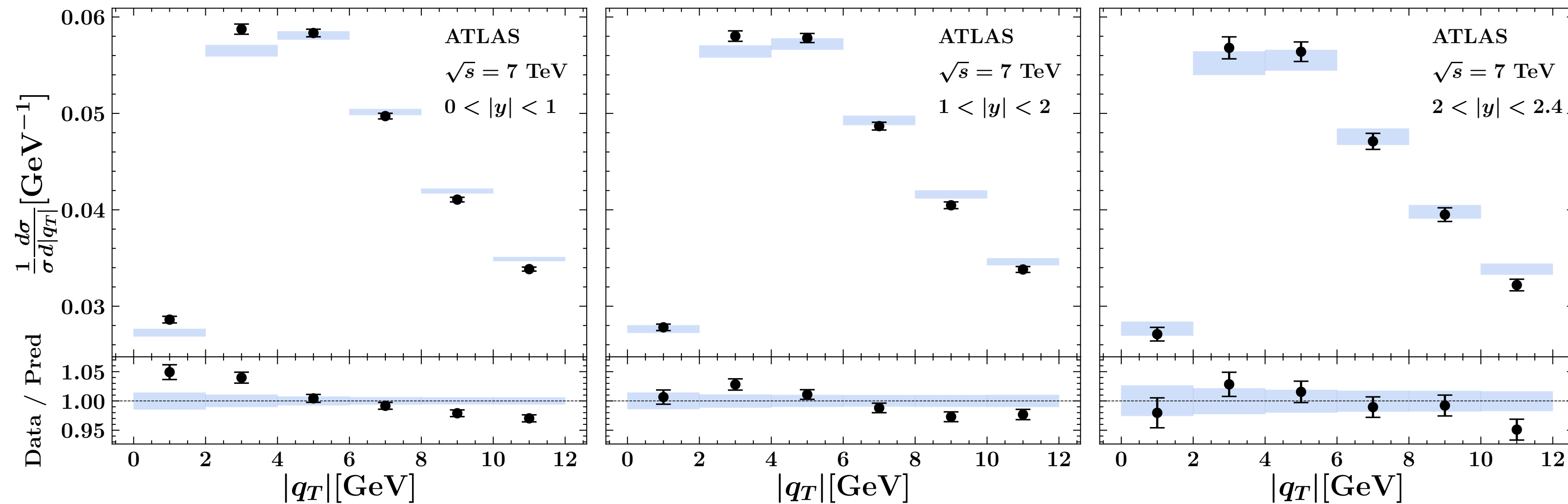
Possible justifications:

⊖ Small experimental uncertainties

⊖ Effects of power corrections

⊖ Implementation of lepton cuts

MAPTMD22 — Results of the fit $\chi^2/N_{data} = 1.06$

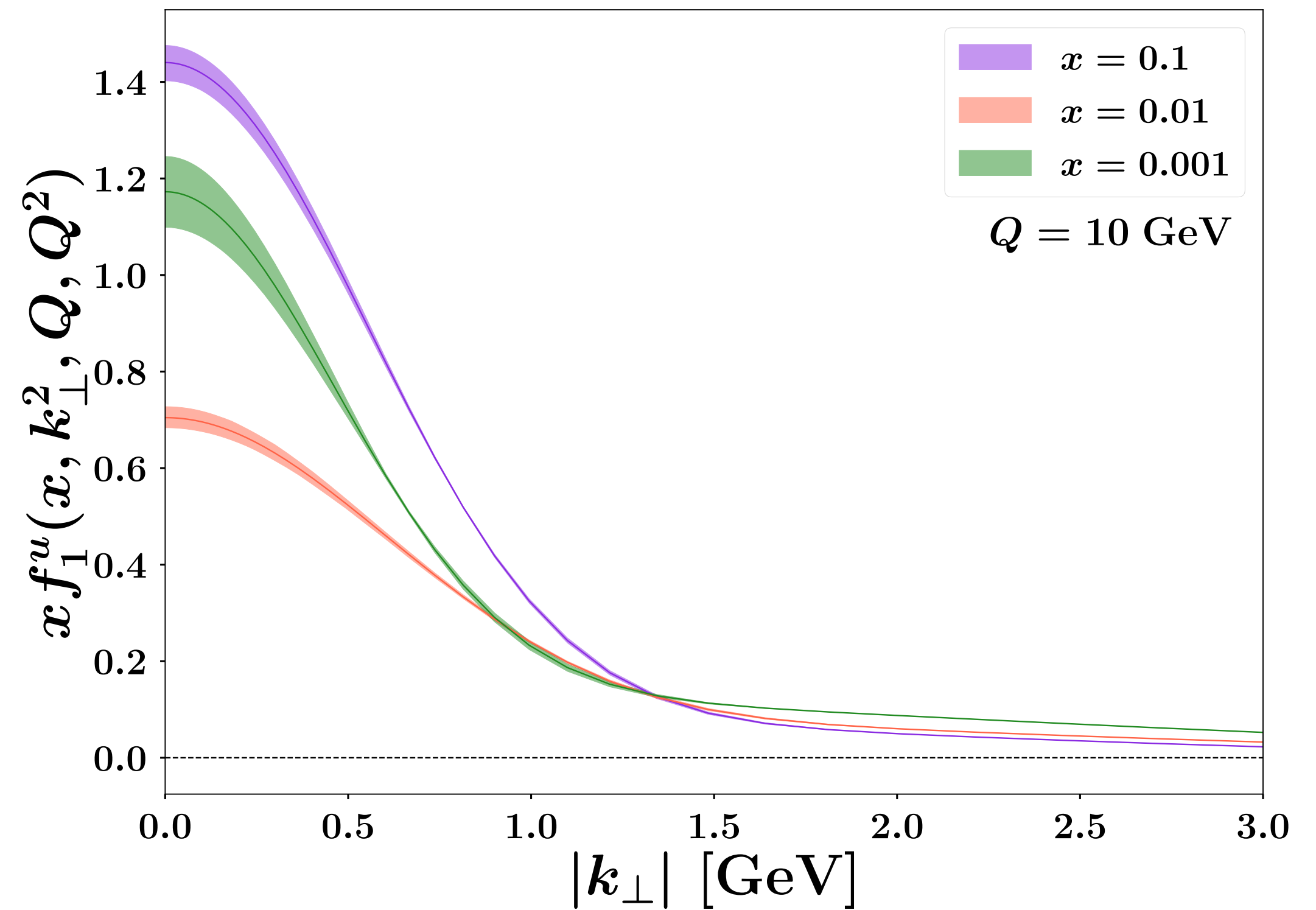
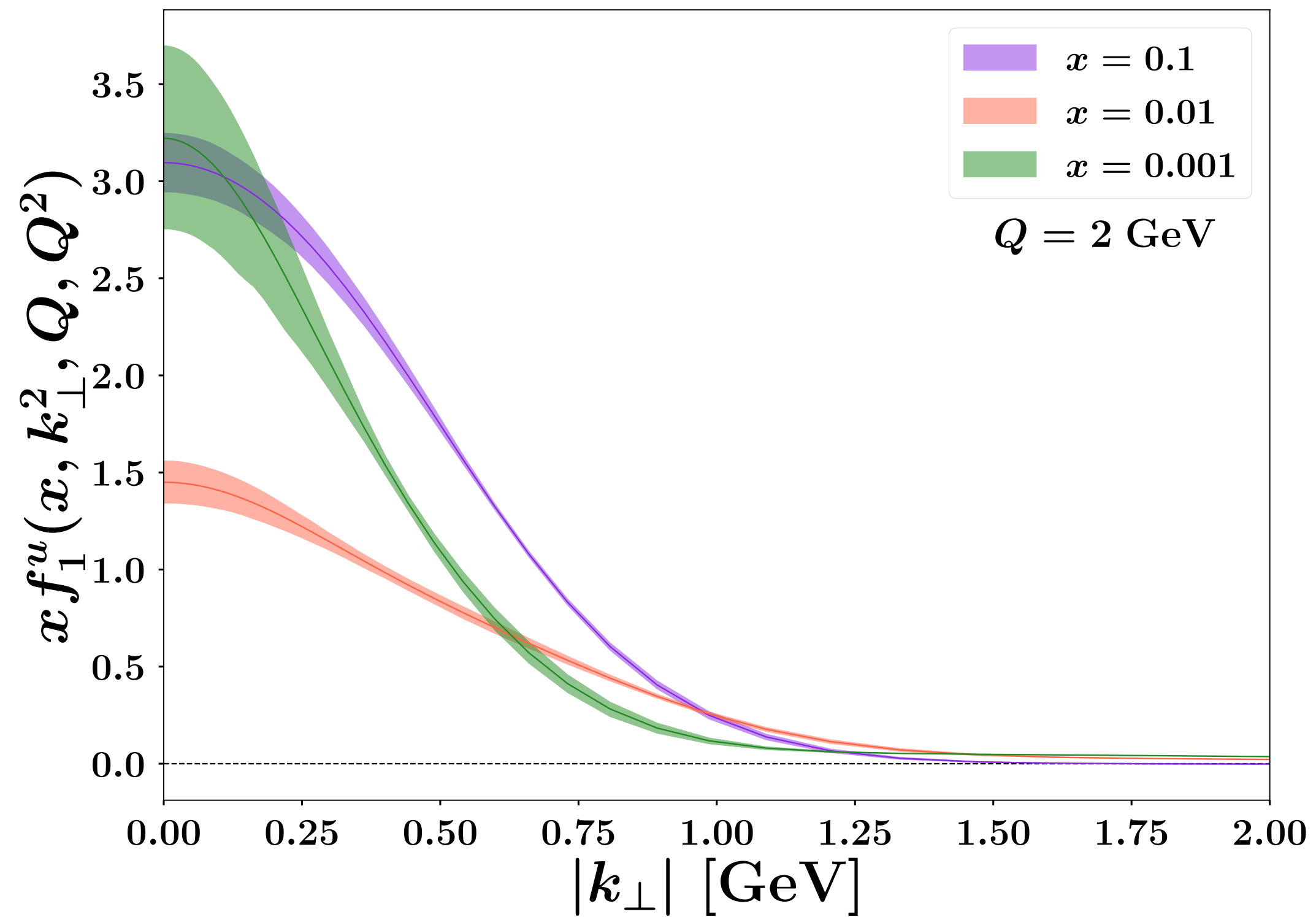


Possible justifications:

- Small experimental uncertainties
- Effects of power corrections
- Implementation of lepton cuts
- Effects of the matching between perturbative and non-perturbative physics

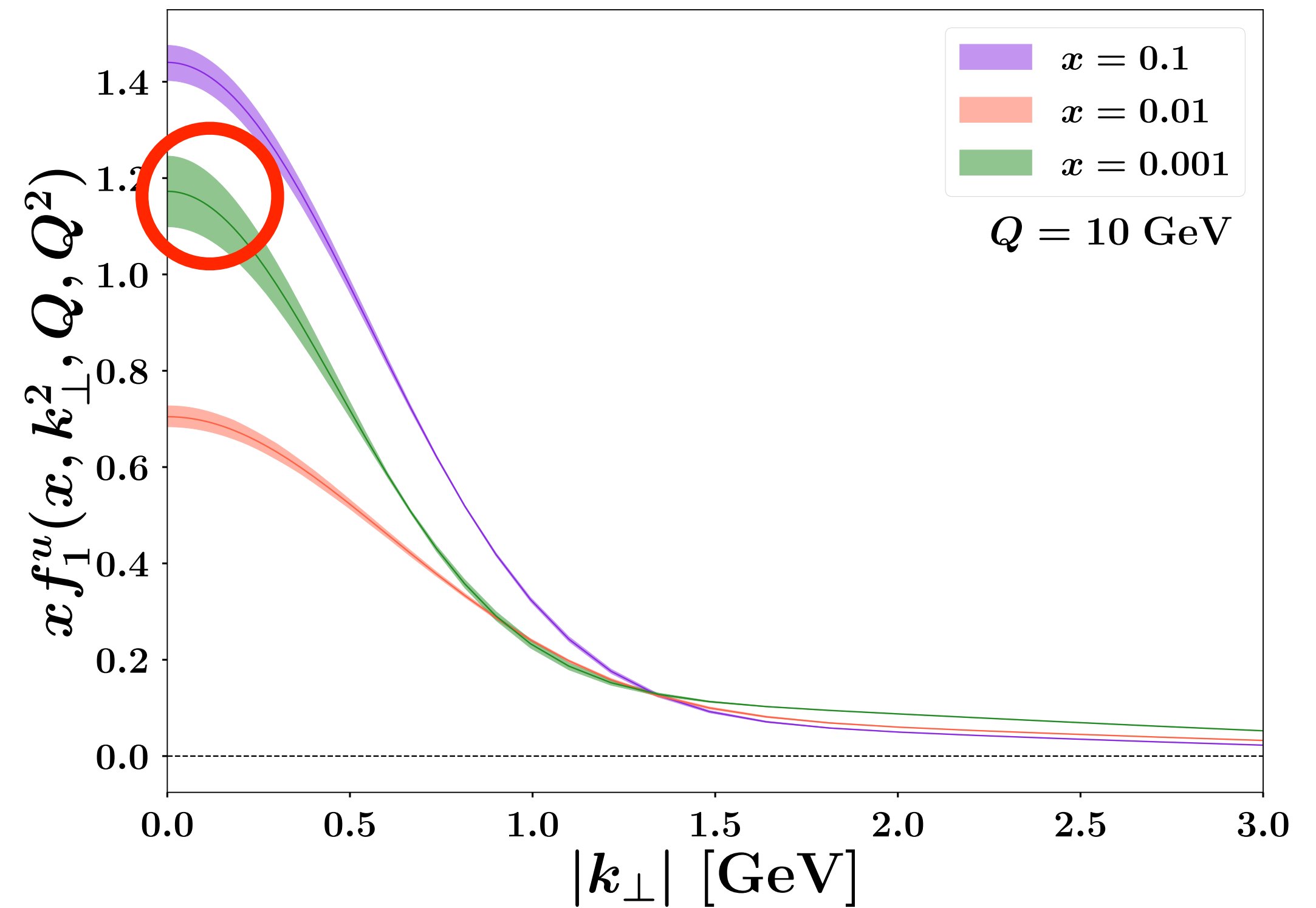
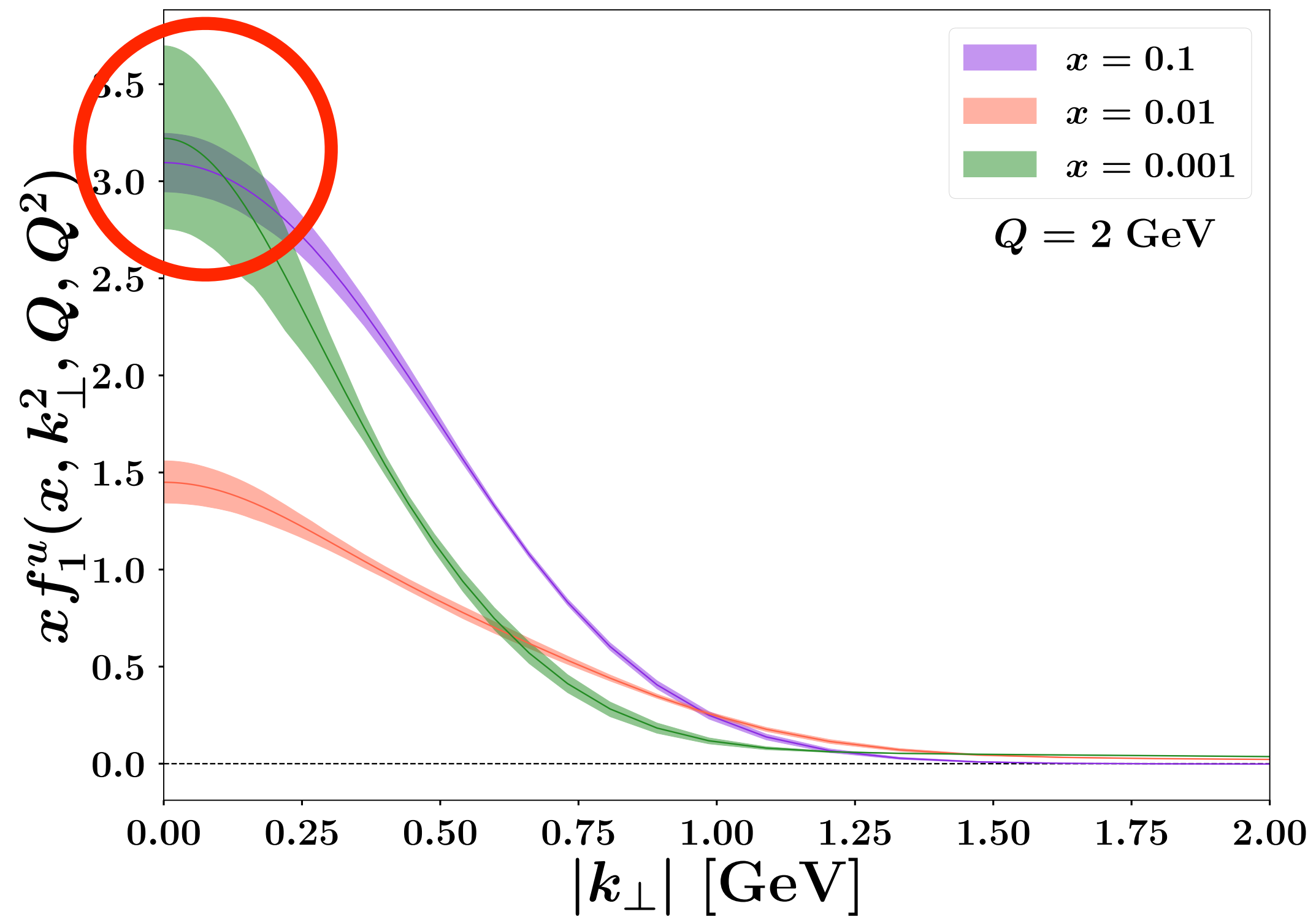
MAPTMD22 — Output of the fit

Visualisation of TMD PDFs



MAPTMD22 — Output of the fit

Visualisation of TMD PDFs



MAPTMD22 – Output of the fit

Collins-Soper kernel

MAPTMD22 – Output of the fit

Collins-Soper kernel

Kernel of the rapidity evolution equation

$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

MAPTMD22 – Output of the fit

Collins-Soper kernel

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perturbatively calculable

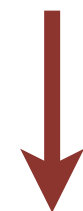
MAPTMD22 – Output of the fit

Collins-Soper kernel

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to be fitted

perturbatively calculable

MAPTMD22 – Output of the fit

Collins-Soper kernel

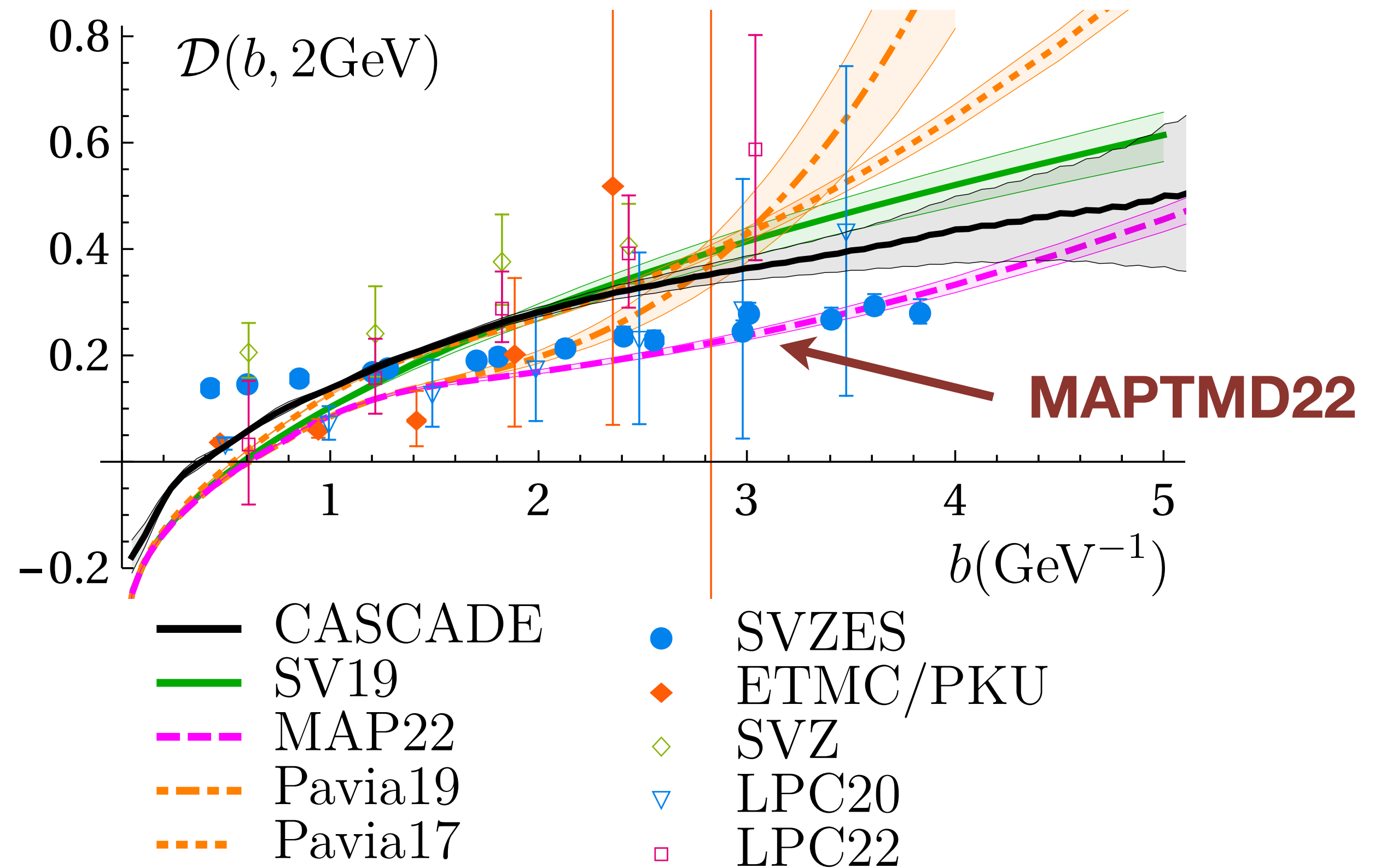
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$$\frac{\partial \ln \hat{f}_1(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = K(b_T, \mu)$$

$$K(b_T, \mu_{b_*}) = K(b_*, \mu_{b_*}) + g_K(b_T)$$

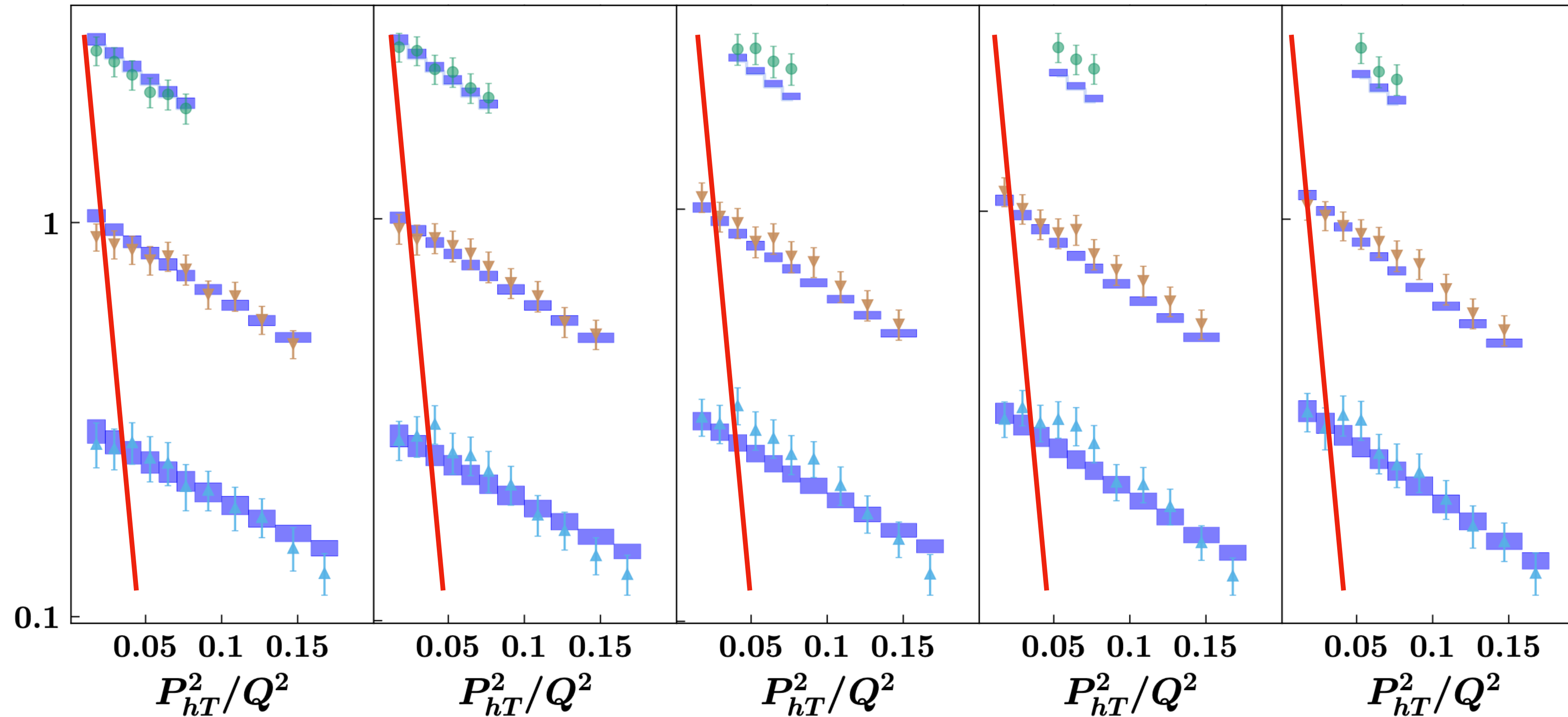
to be fitted

perturbatively calculable



SIDIS data selection

$$\left. \frac{P_{hT}}{zQ} \right|_{\max} = 0.25$$



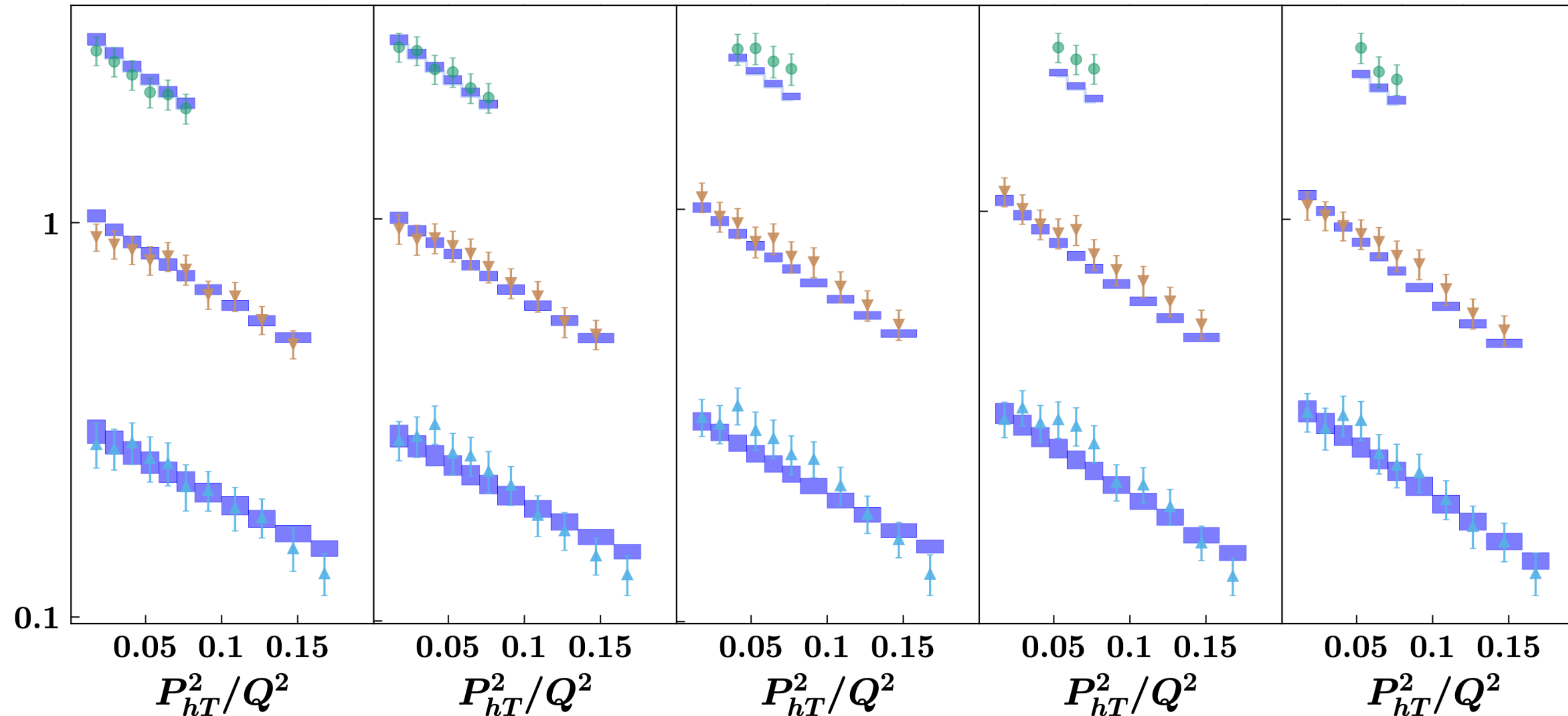
COMPASS multiplicities
(different x-bins)

$1.3 < Q < 1.73$ GeV

- $0.3 < z < 0.4$ (offset = 0)
- ▼ $0.4 < z < 0.6$ (offset = 0)
- ▲ $0.6 < z < 0.8$ (offset = 0)

SIDIS data selection

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$



COMPASS multiplicities
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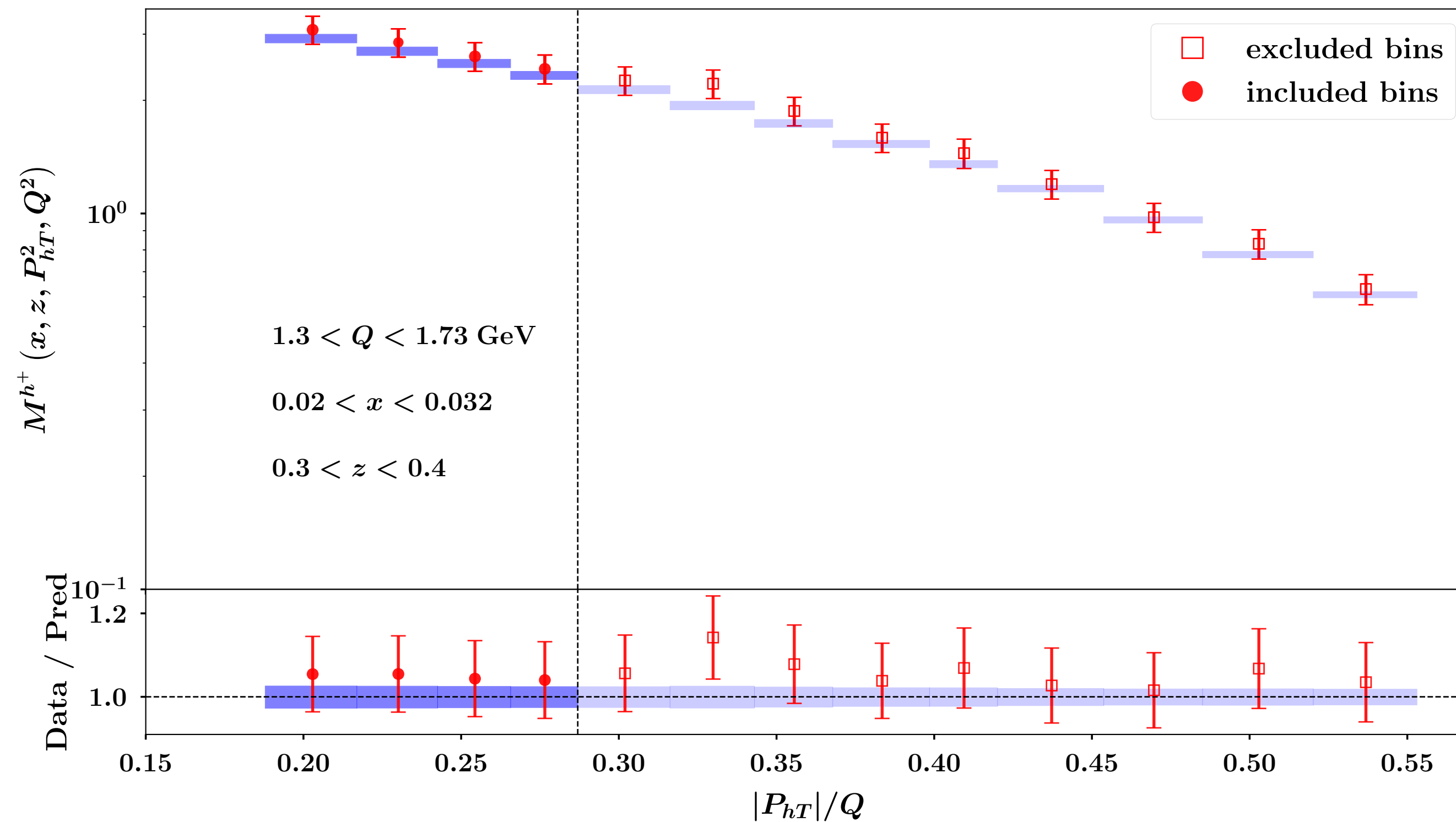
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MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

$$P_{hT}|_{\max} = \min[0.2Q, 0.7zQ] + 0.5 \text{ GeV}$$

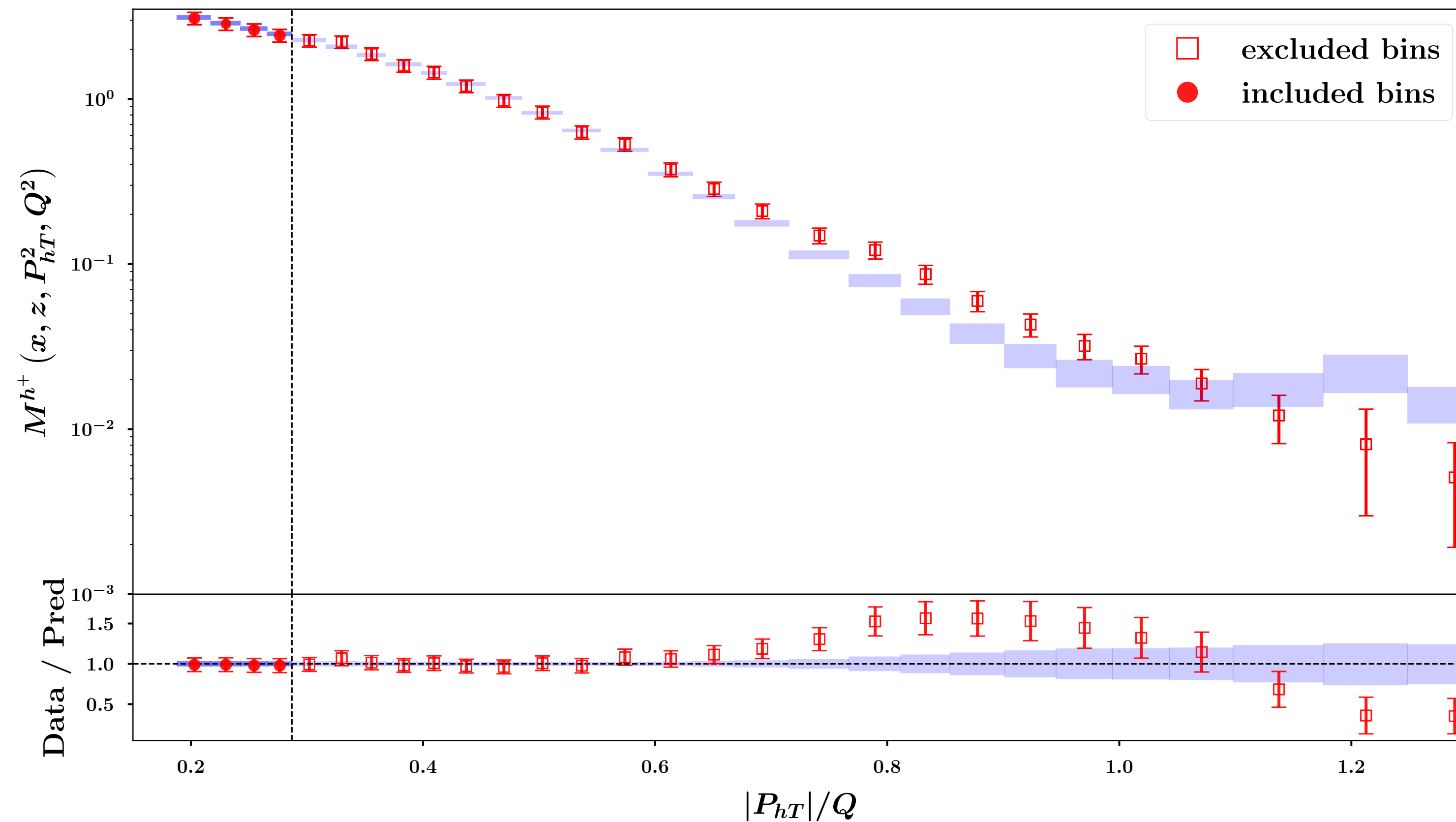


MAPTMD22 – SIDIS data selection

COMPASS multiplicities (one of many bins)

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Total number of points



Impact of power corrections

Results obtained with the arTeMiDe framework

include (m/Q)
include (M/Q)
include (q_T/Q) in kinematics
include (q_T/Q) in x_S, z_S

Impact of power corrections

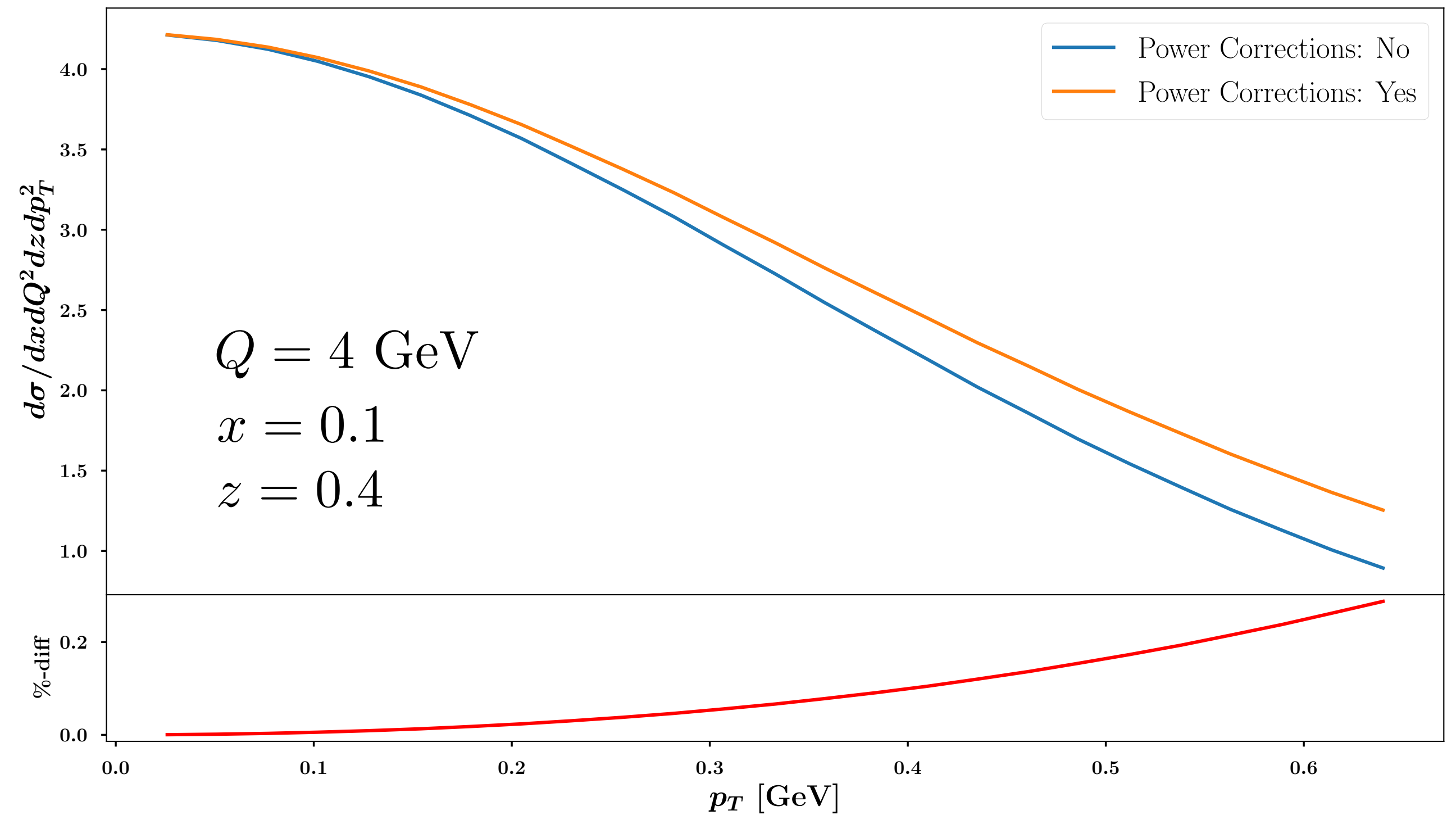
Results obtained with the arTeMiDe framework

```
# -----  
# ---- PARAMETERS OF TMDX-SIDIS ----  
# -----  
*10 :  
*p1 : initialize TMDX-SIDIS module  
T  
*A : ---- Main definitions ----  
*p1 : Order of coefficient function  
T  
*p2 : Use transverse momentum corrections in kinematics  
T → F  
*p3 : Use target mass corrections in kinematics  
T → F  
*p4 : Use product mass corrections in kinematics  
T → F  
*p5 : Use transverse momentum corrections in x1 and z1  
T → F
```

Impact of power corrections

Results obtained with the arTeMiDe framework

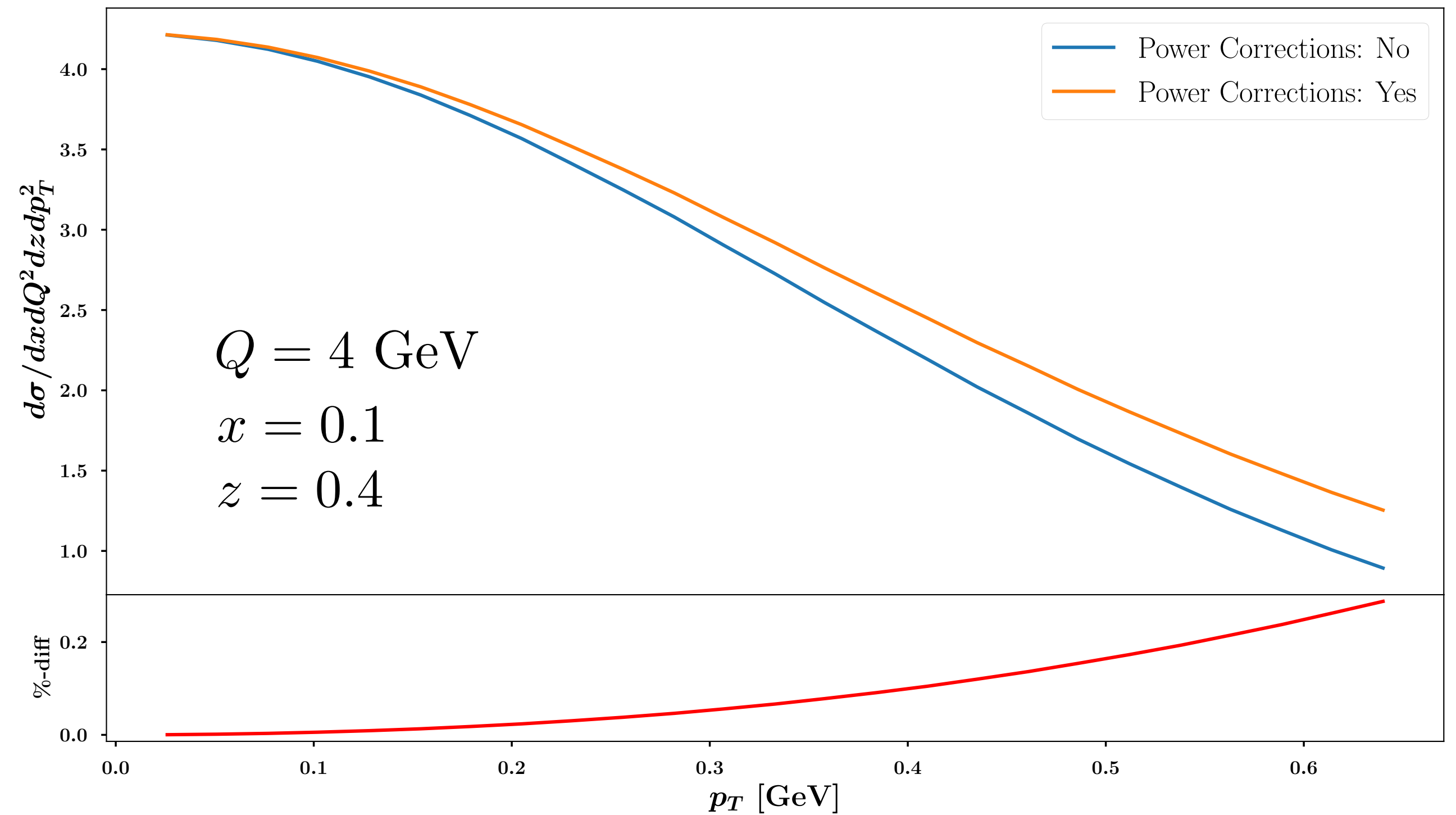
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```



This is NOT a constant factor

Summary

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MAPTMD22 GLOBAL FIT - A new precise extraction of unpol TMDs

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points

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Summary

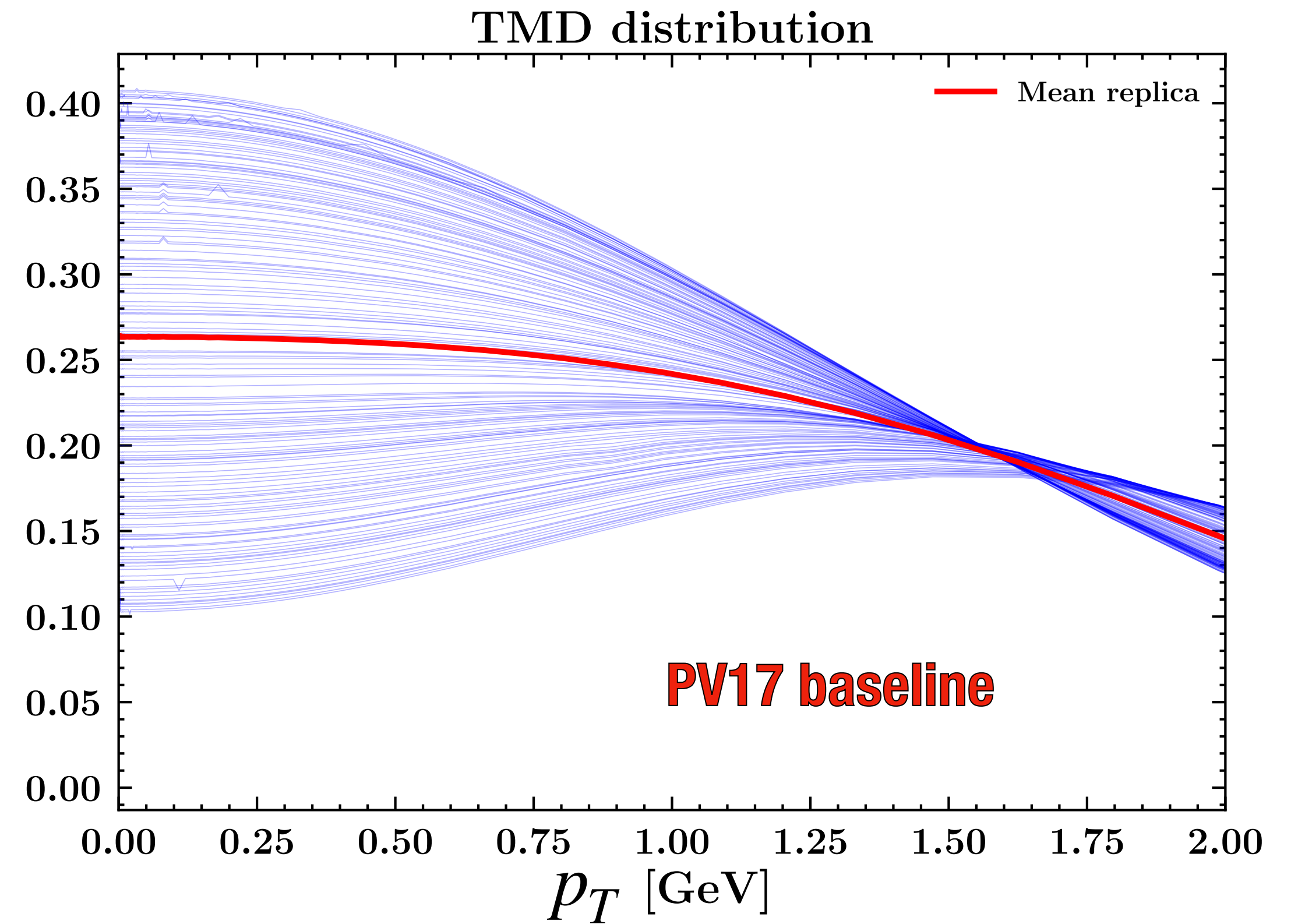
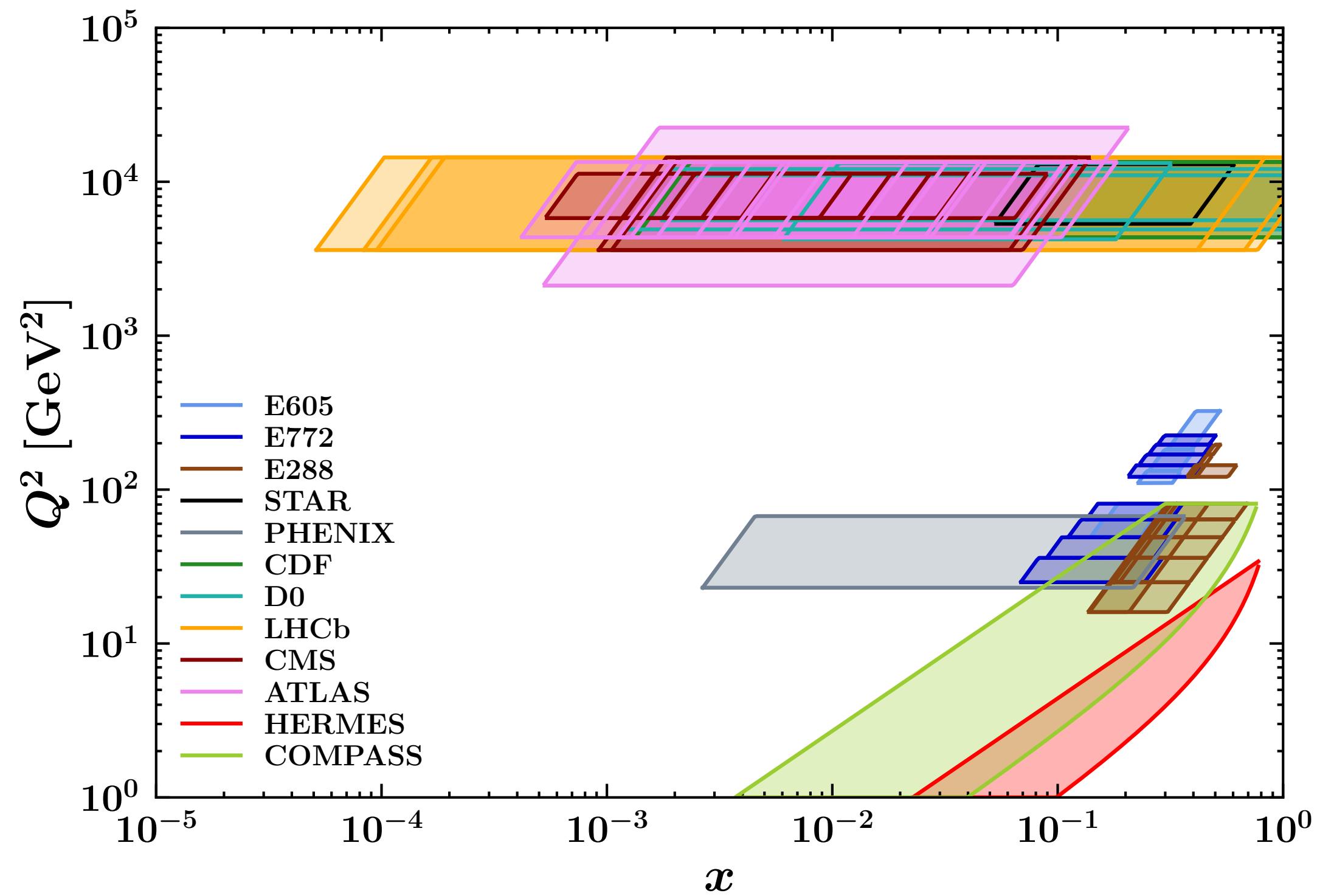
MAPTMD22 GLOBAL FIT - A new precise extraction of unpol TMDs

- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy: **N^3LL^-**
- ***Normalization*** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description: **$\chi^2 / N_{data} = 1.06$**

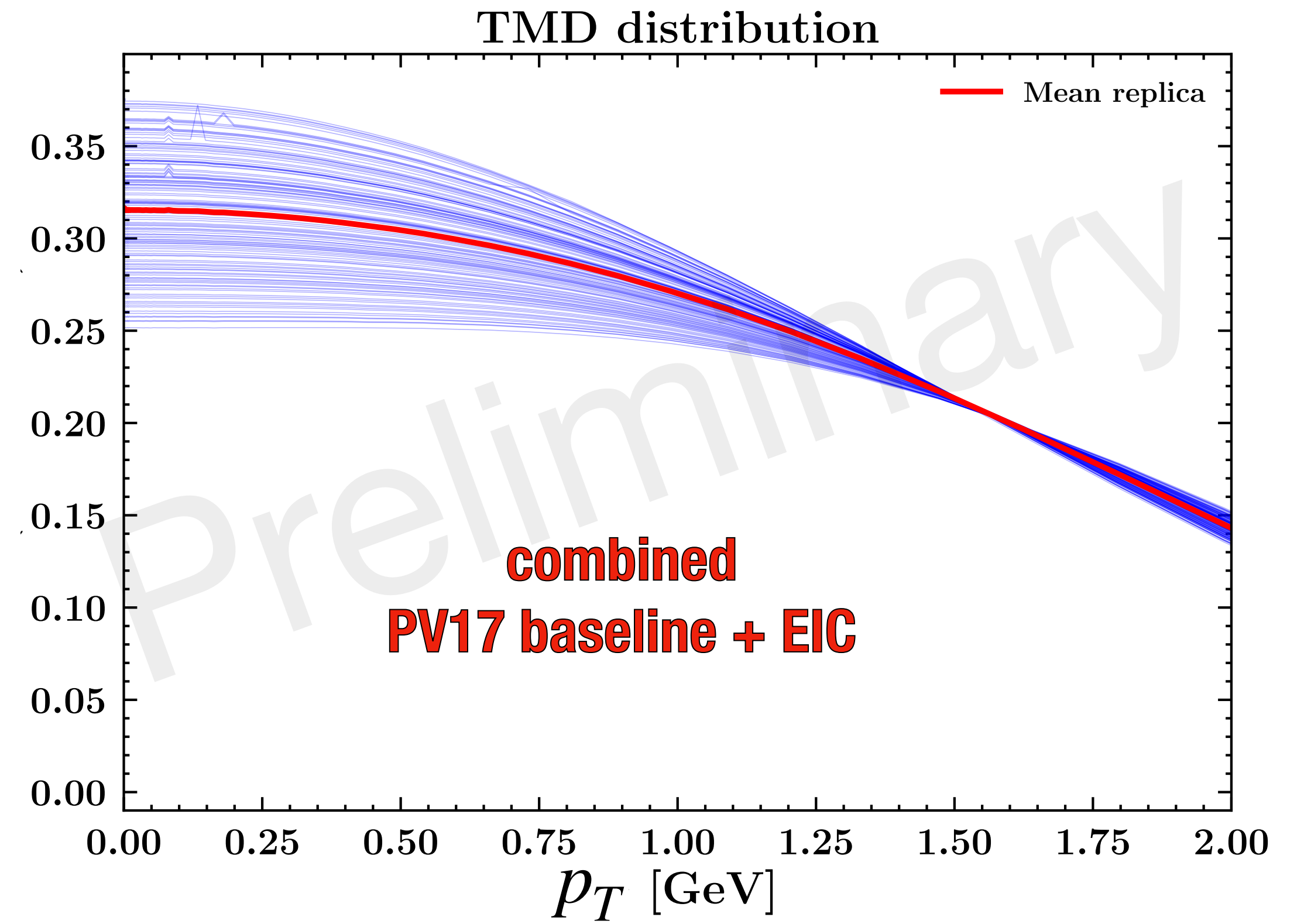
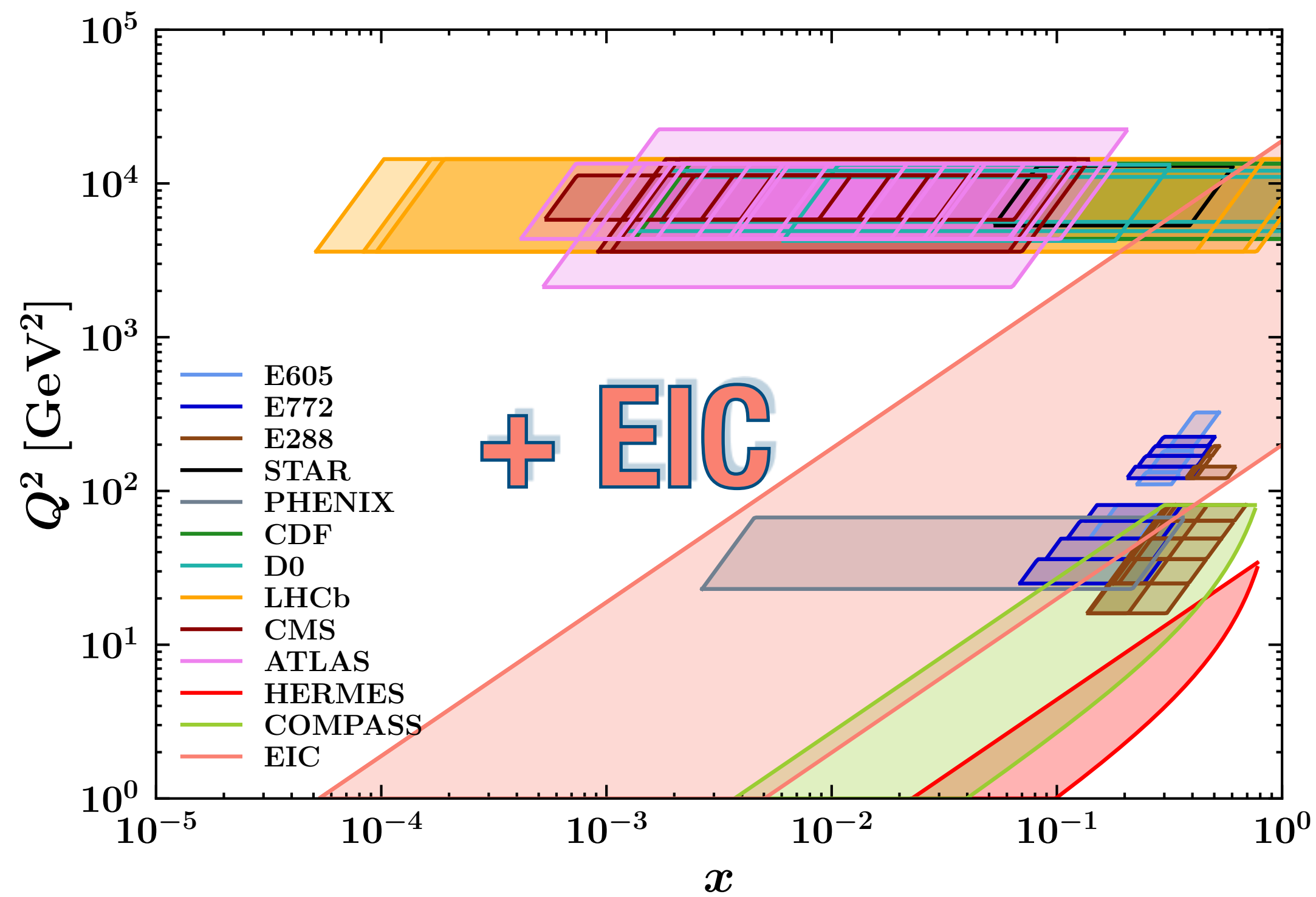


What is the role of the EIC?

Impact of EIC pseudodata on TMDs



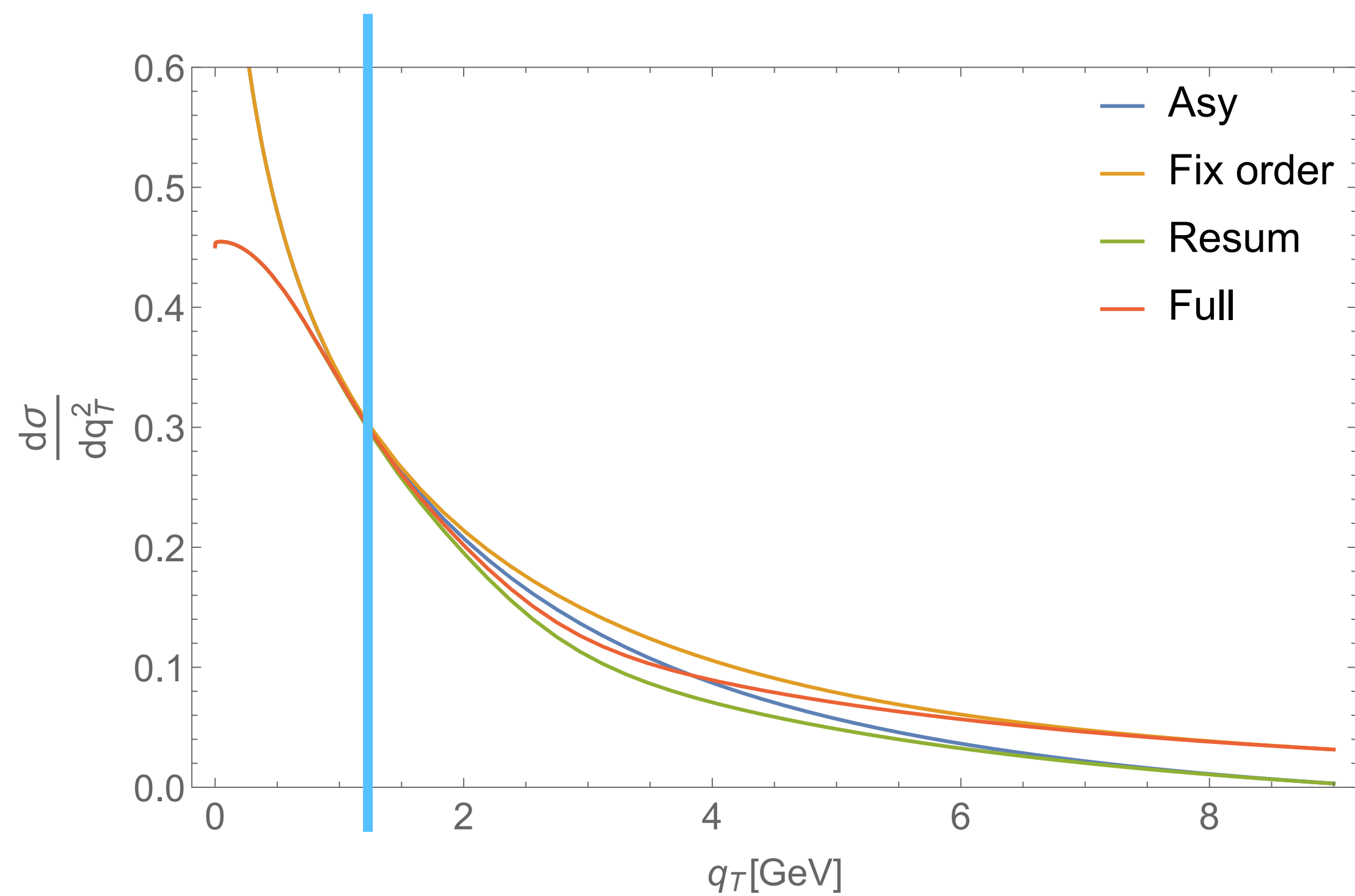
Impact of EIC pseudodata on TMDs



BACKUP SLIDES

Source of W-term suppression

Ideal situation at high Q

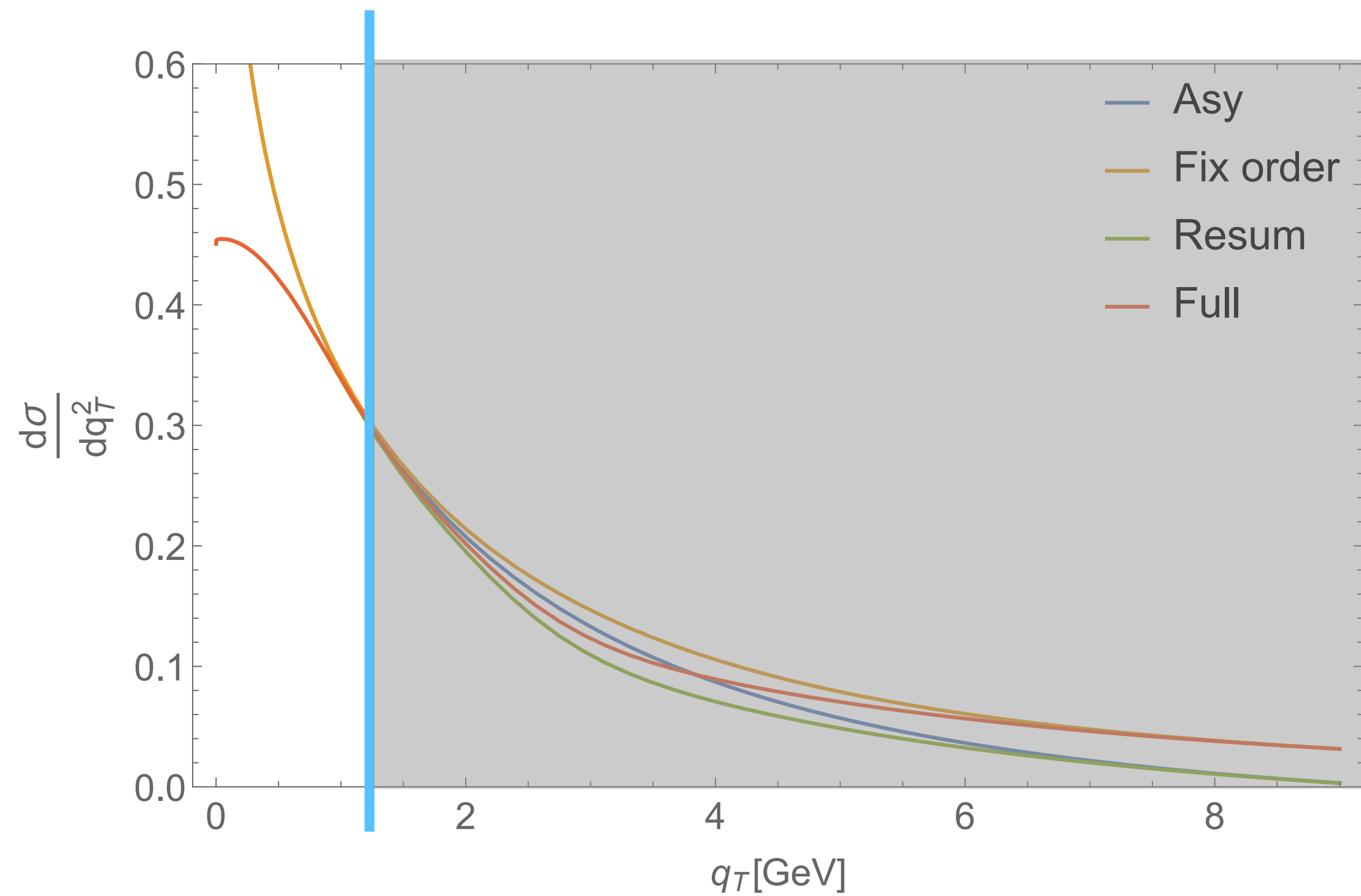


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order

Source of W-term suppression

Ideal situation at high Q

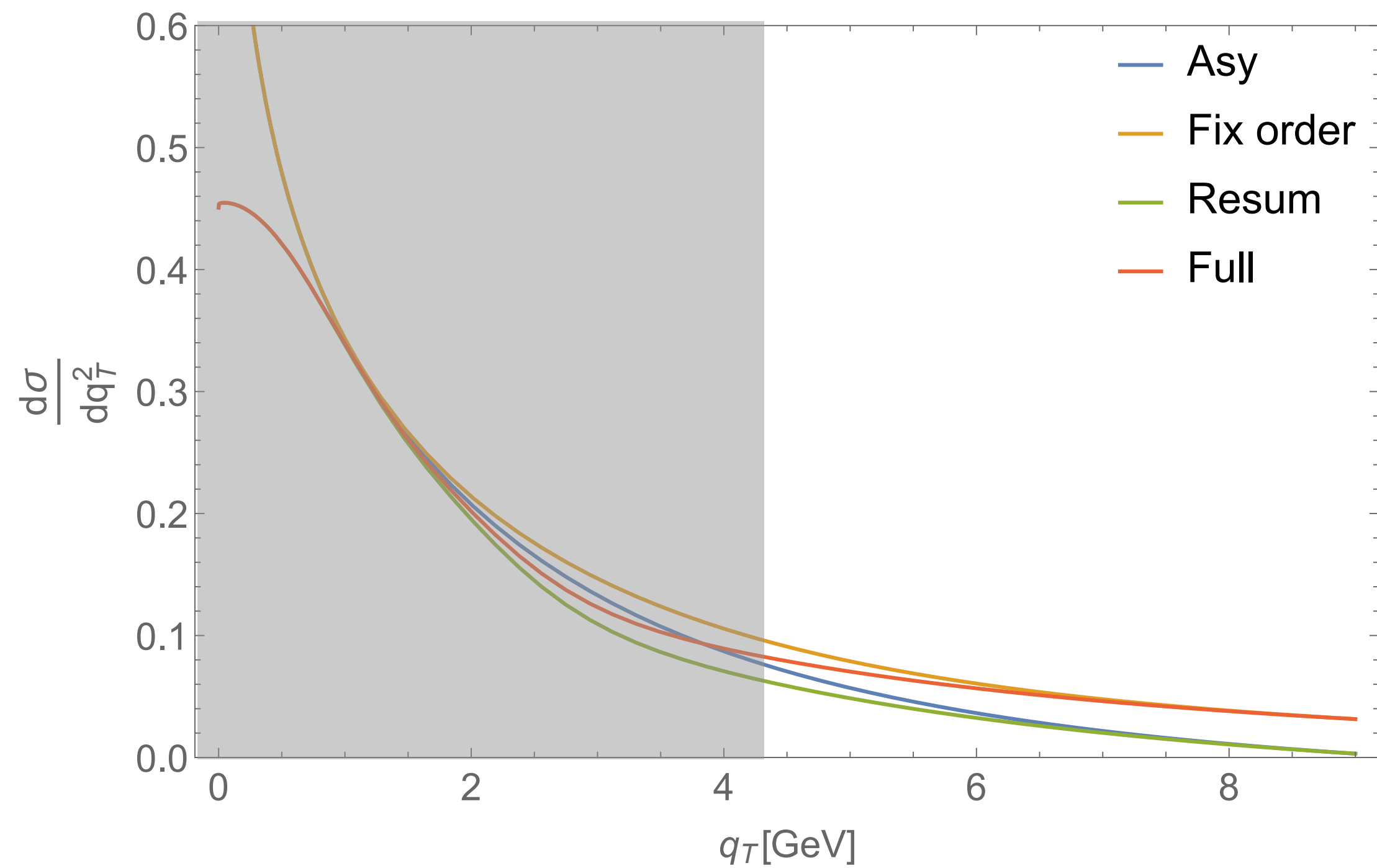


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order → **TMD Region**

Source of W-term suppression

Ideal situation at high Q

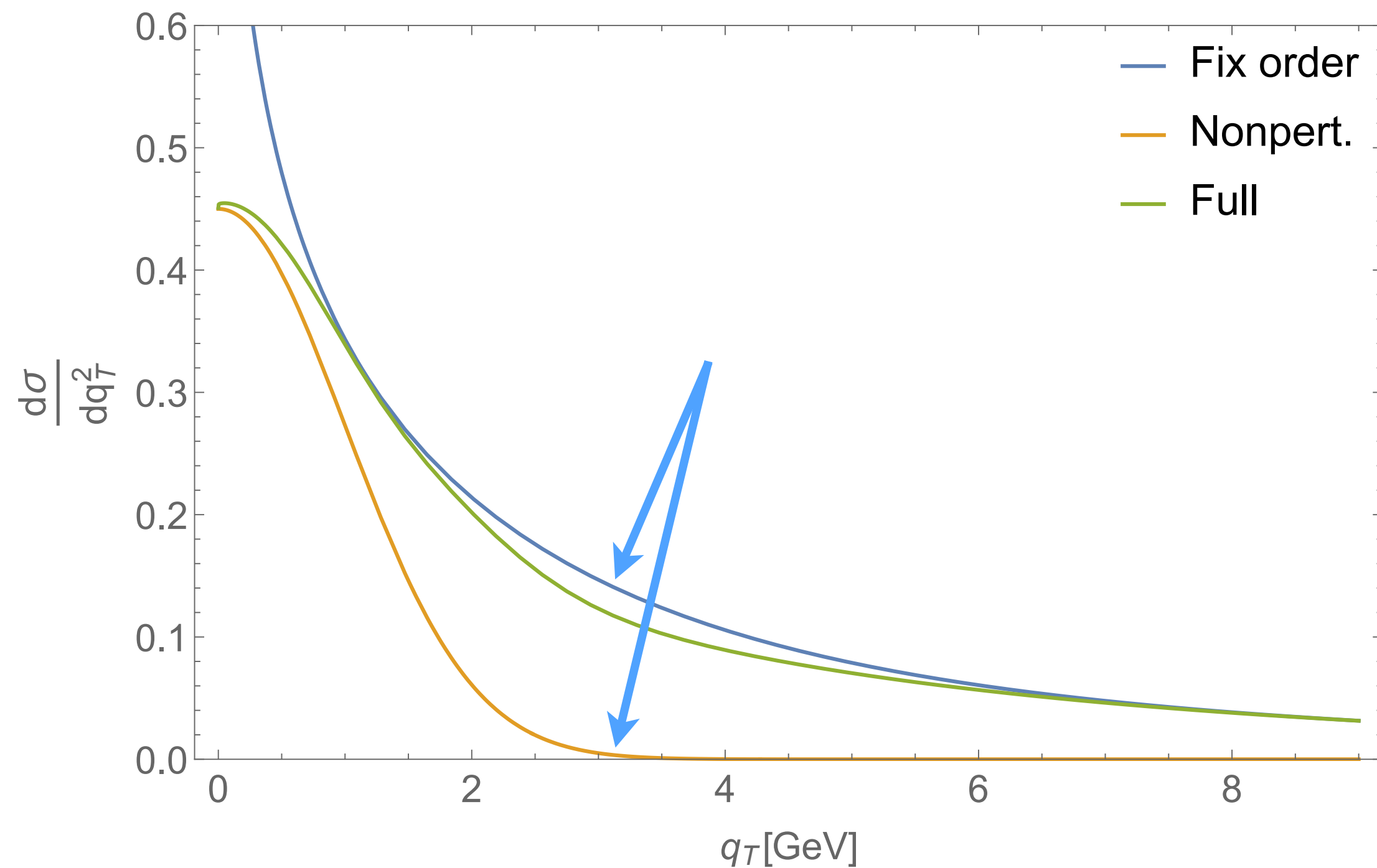


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order
- From a certain value of q_T the total cross section follows the Fixed Order term

Source of W-term suppression

Ideal situation at high Q

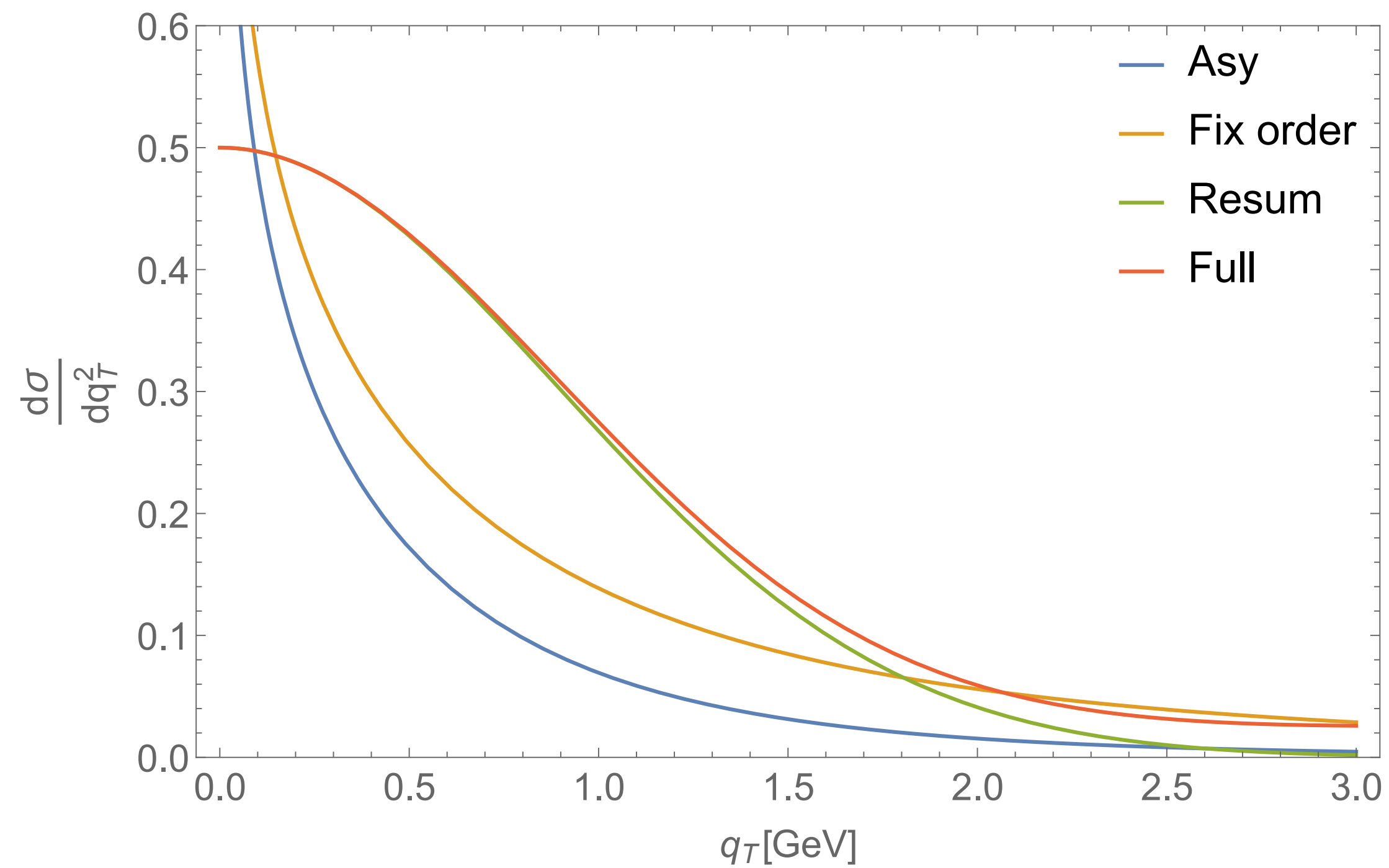


Standard approach

- Collinear result is mostly given by the integral of the Fixed Order
- The Non-Perturbative term is only a small correction

Source of W-term suppression

Ideal situation at low Q

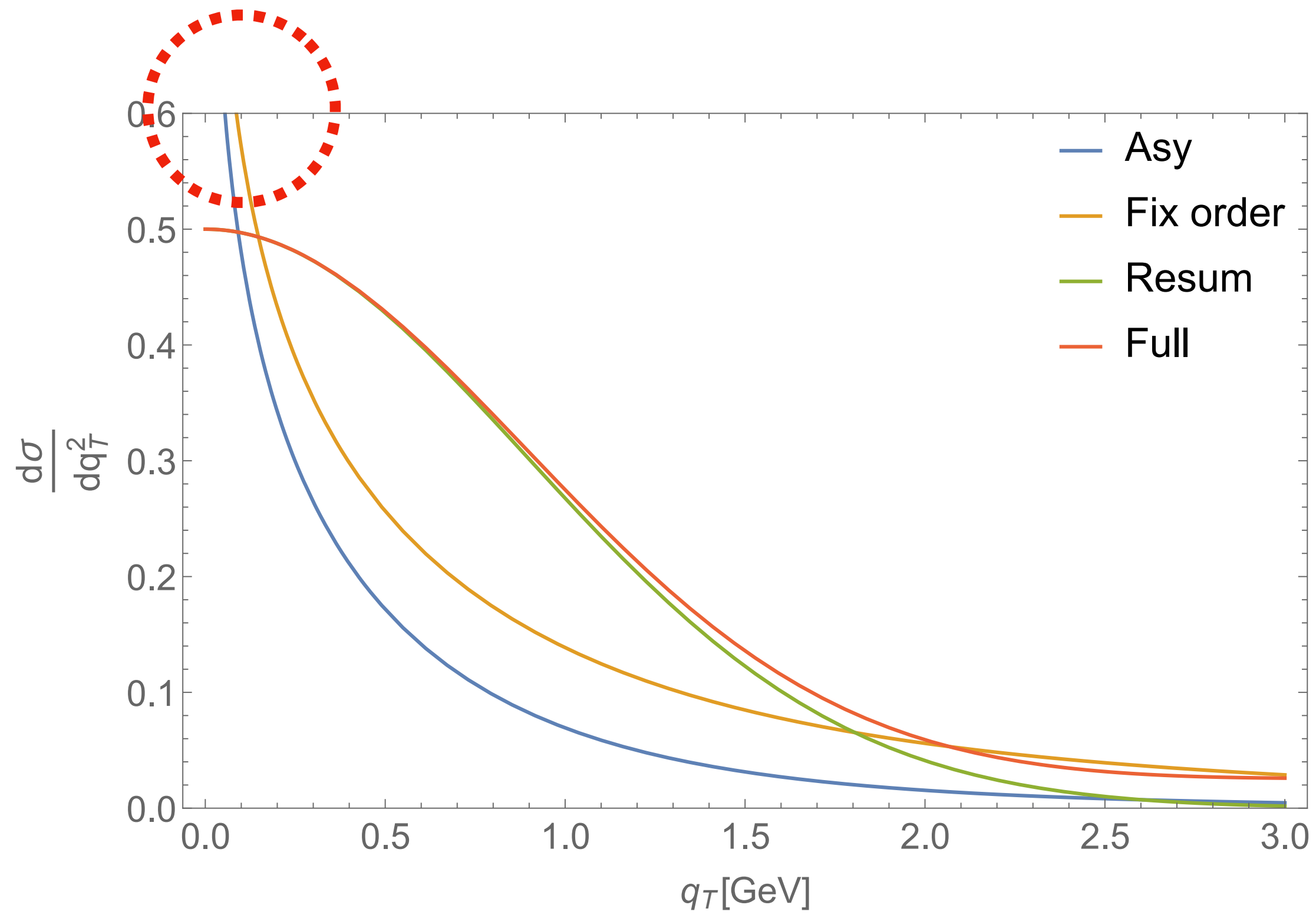


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order

Source of W-term suppression

Ideal situation at low Q

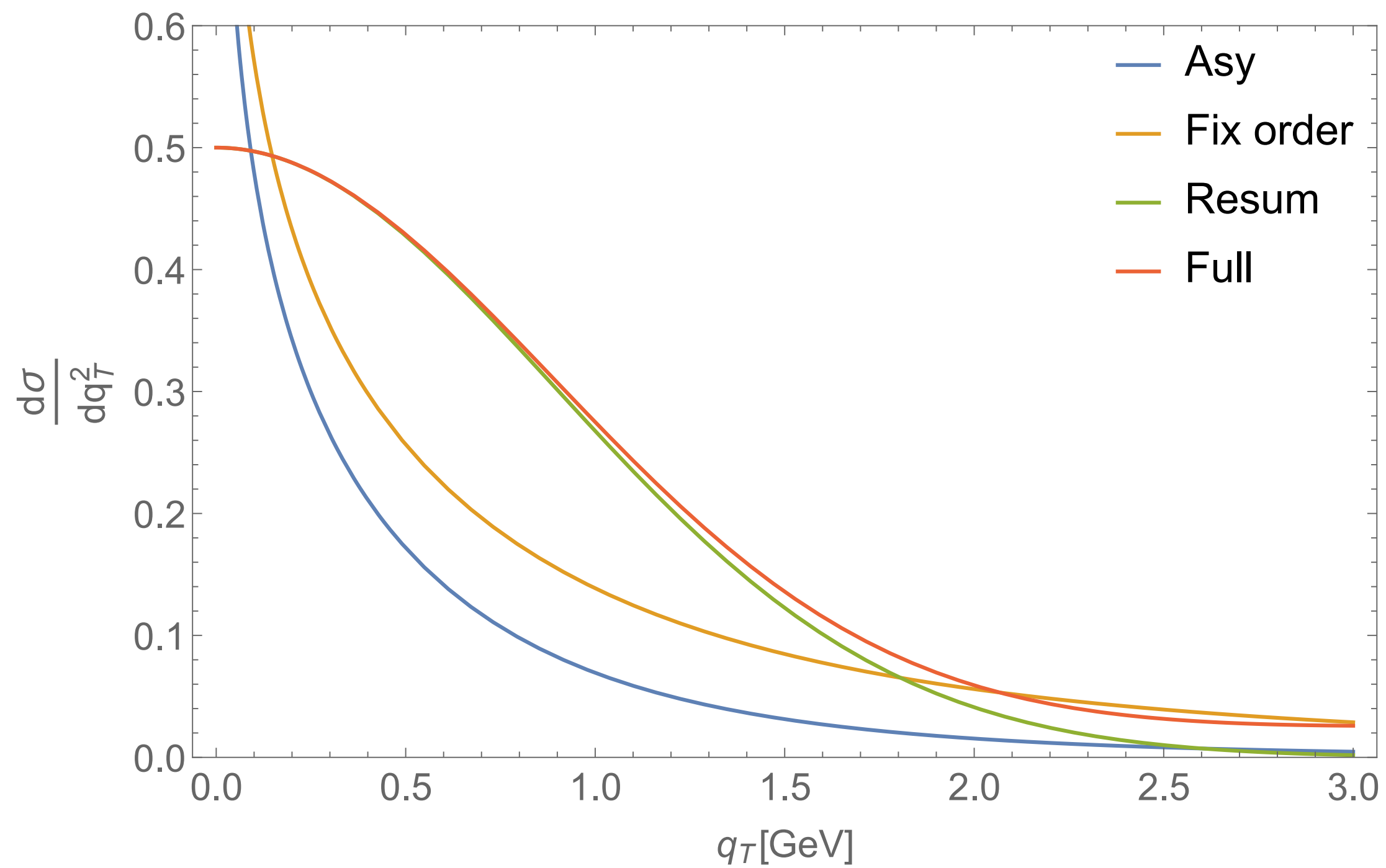


Standard approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order → **TMD Region?**

Source of W-term suppression

Ideal situation at low Q

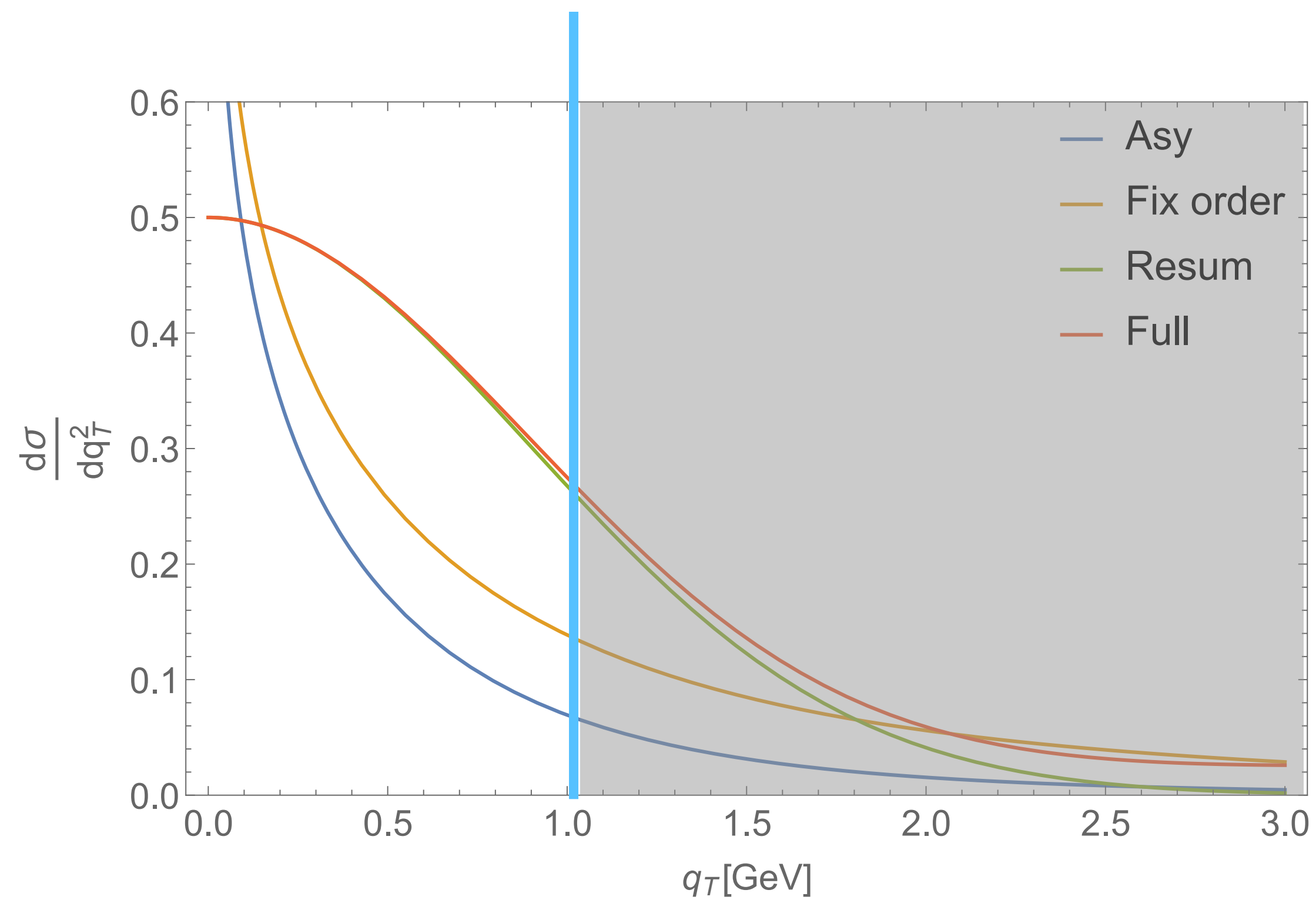


Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates

Source of W-term suppression

Ideal situation at low Q



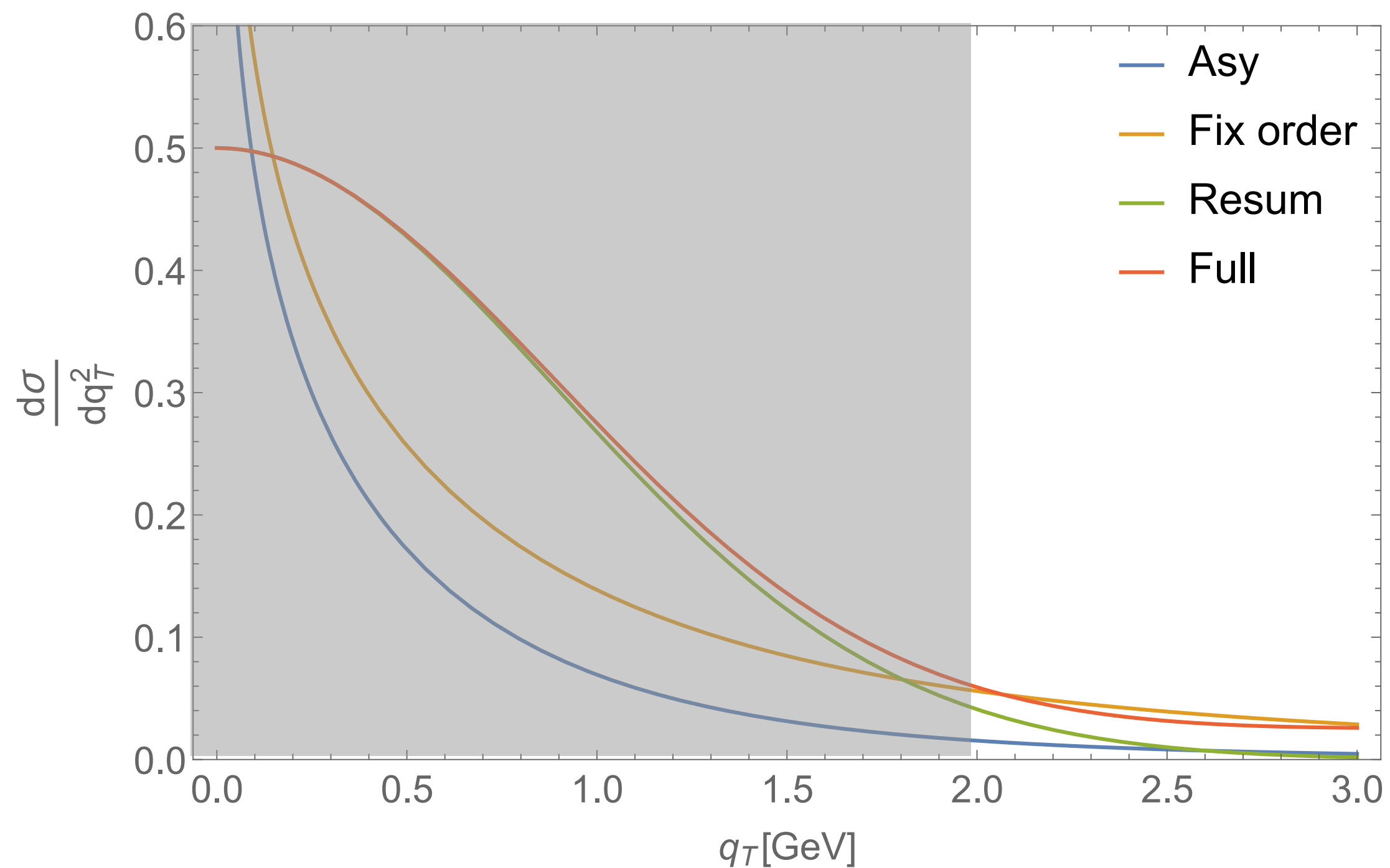
Non-Perturbative approach

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→ TMD Region

Source of W-term suppression

Ideal situation at low Q

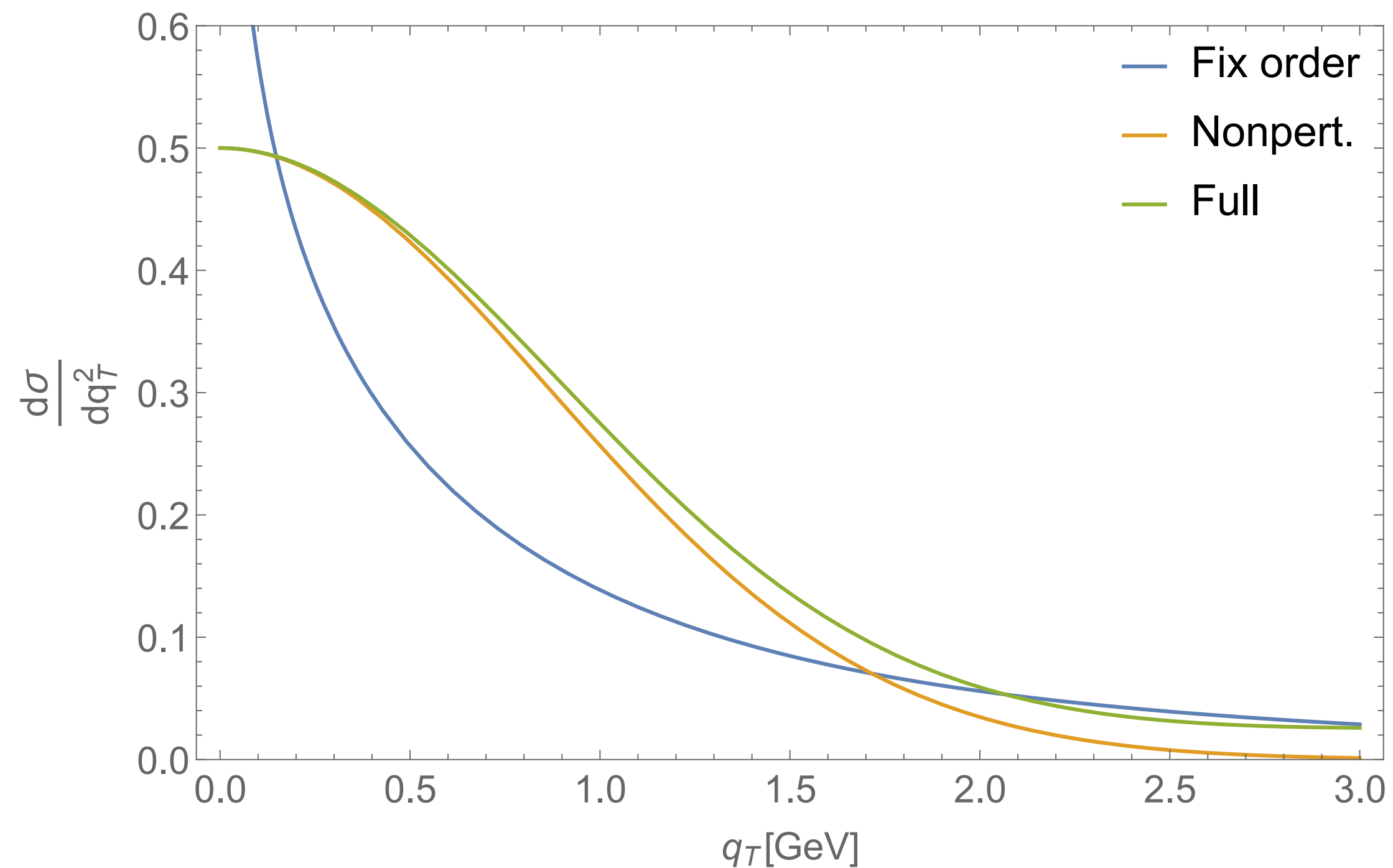


Non-Perturbative approach

- Resummed contribution is dominant where the Asymptotic term is close to the Fixed Order OR the Non-Perturbative contributions dominates
- From a certain value of q_T the cross section follows the Fixed Order term

Source of W-term suppression

Ideal situation at low Q



Non-Perturbative approach

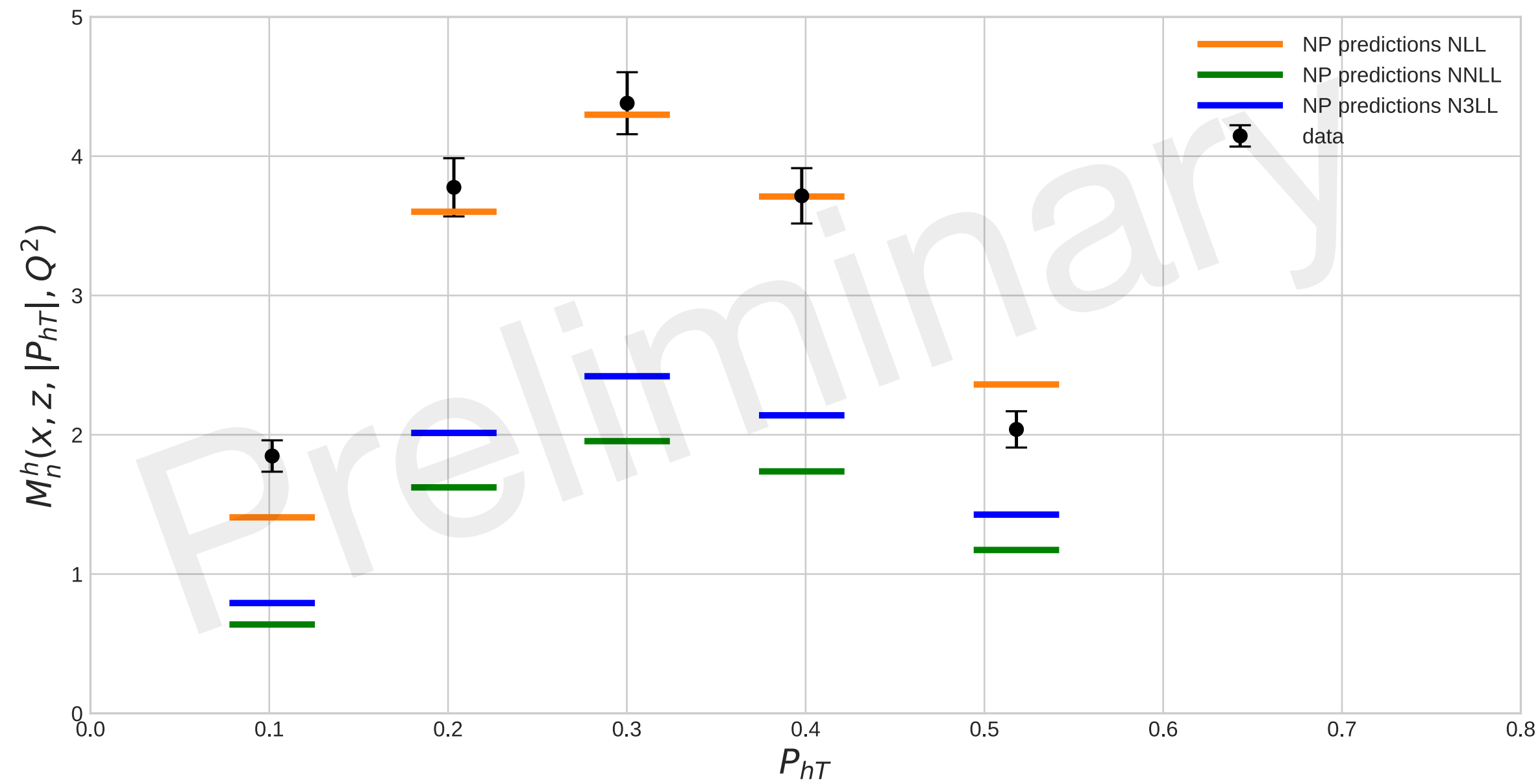
- Collinear result is no more mostly given by the integral of the Fixed Order
- The Non-Perturbative term is not only a small correction, but is even larger than the Fixed Order contribution

Source of W-term suppression

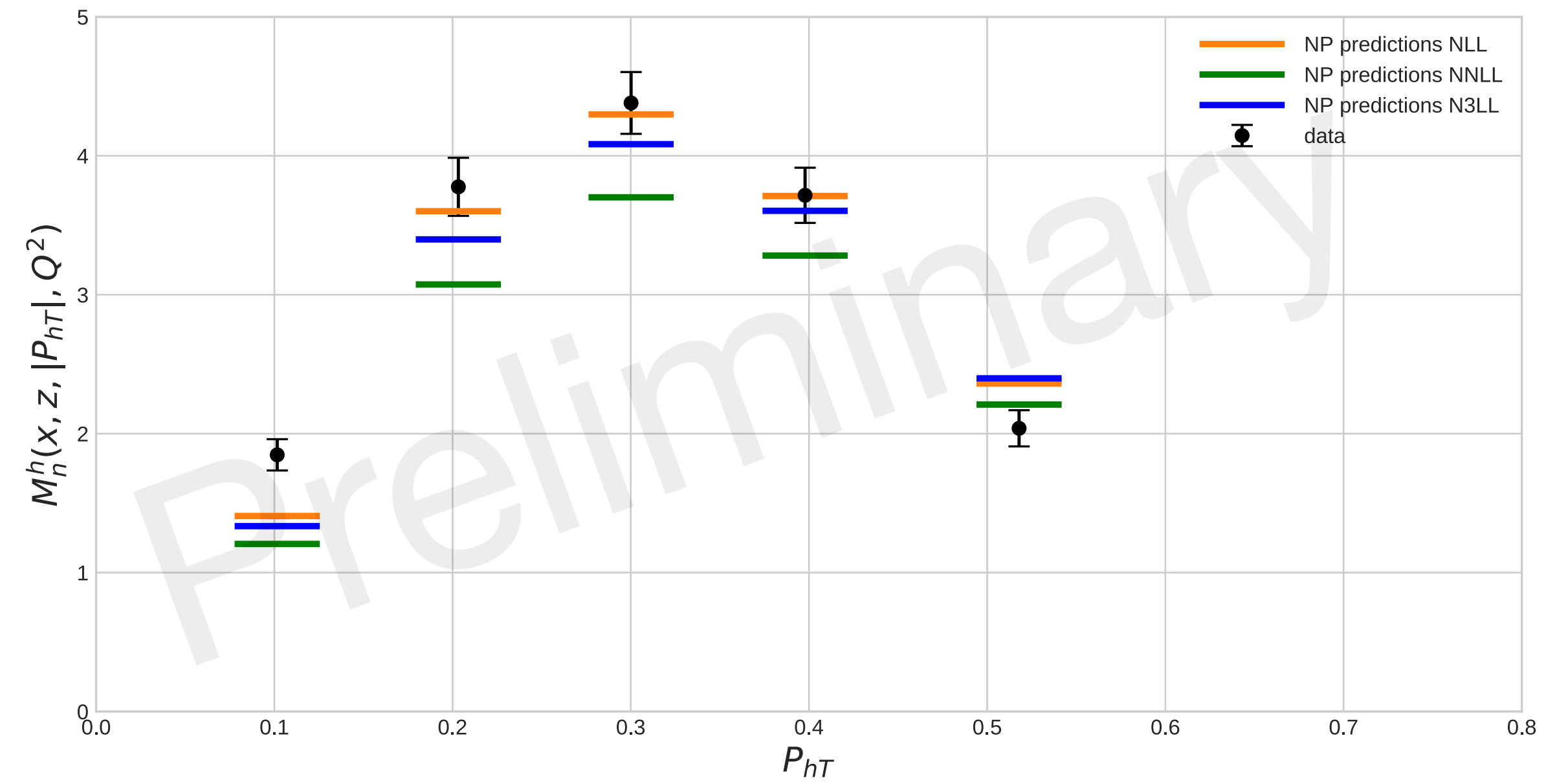
Present situation at low Q

HERMES multiplicity

Full Hard Factor



Hard Factor = 1

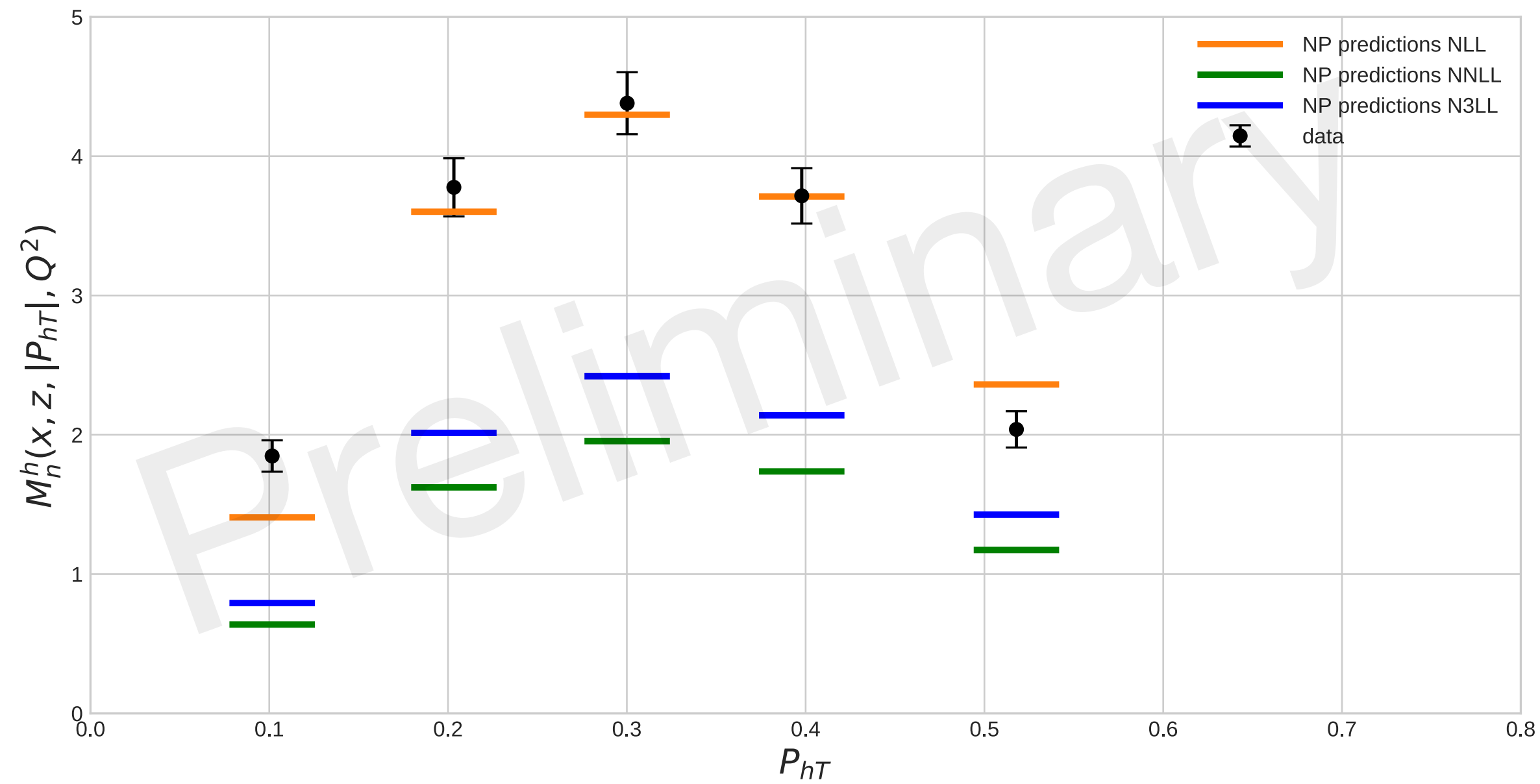


Source of W-term suppression

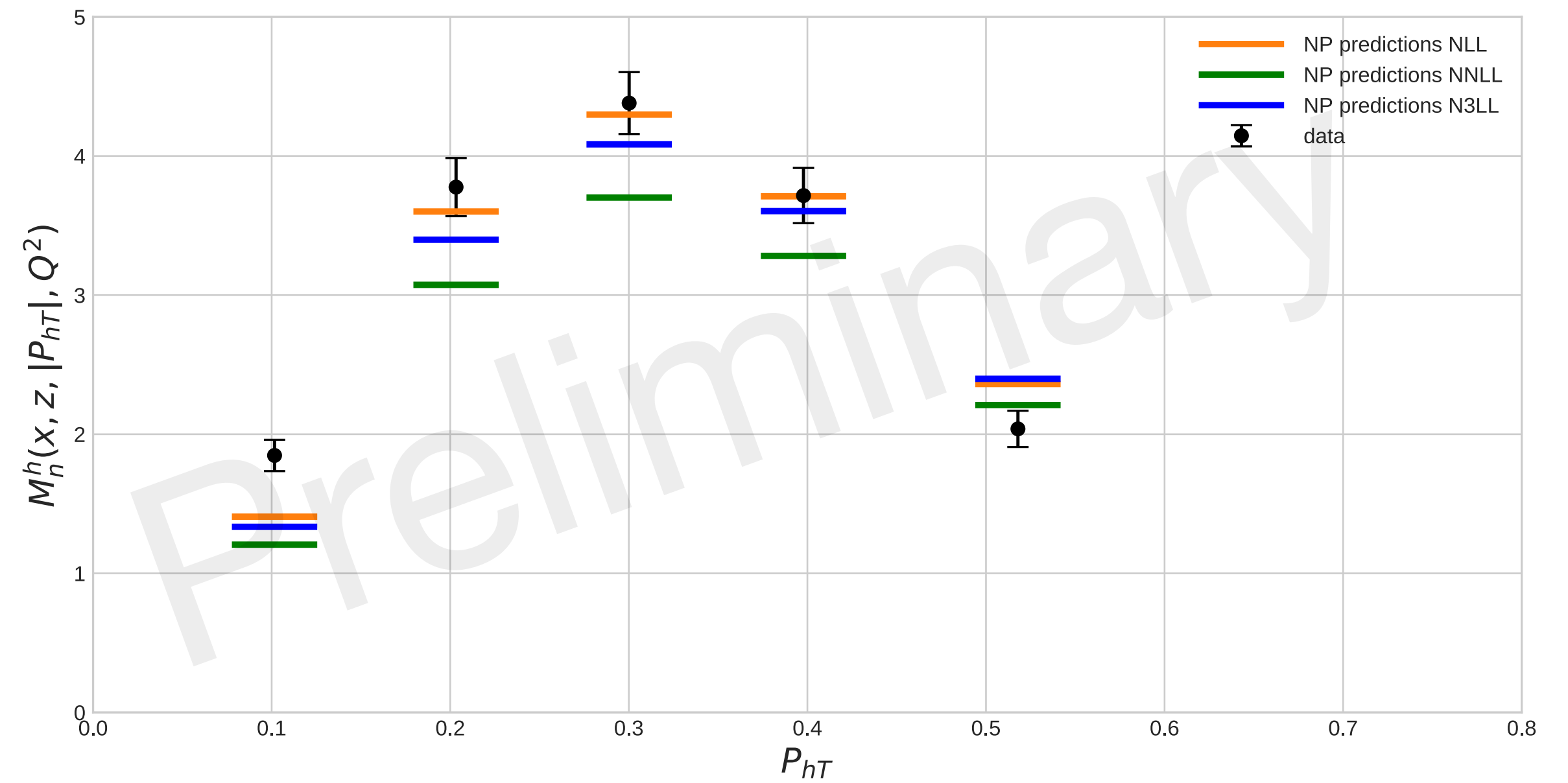
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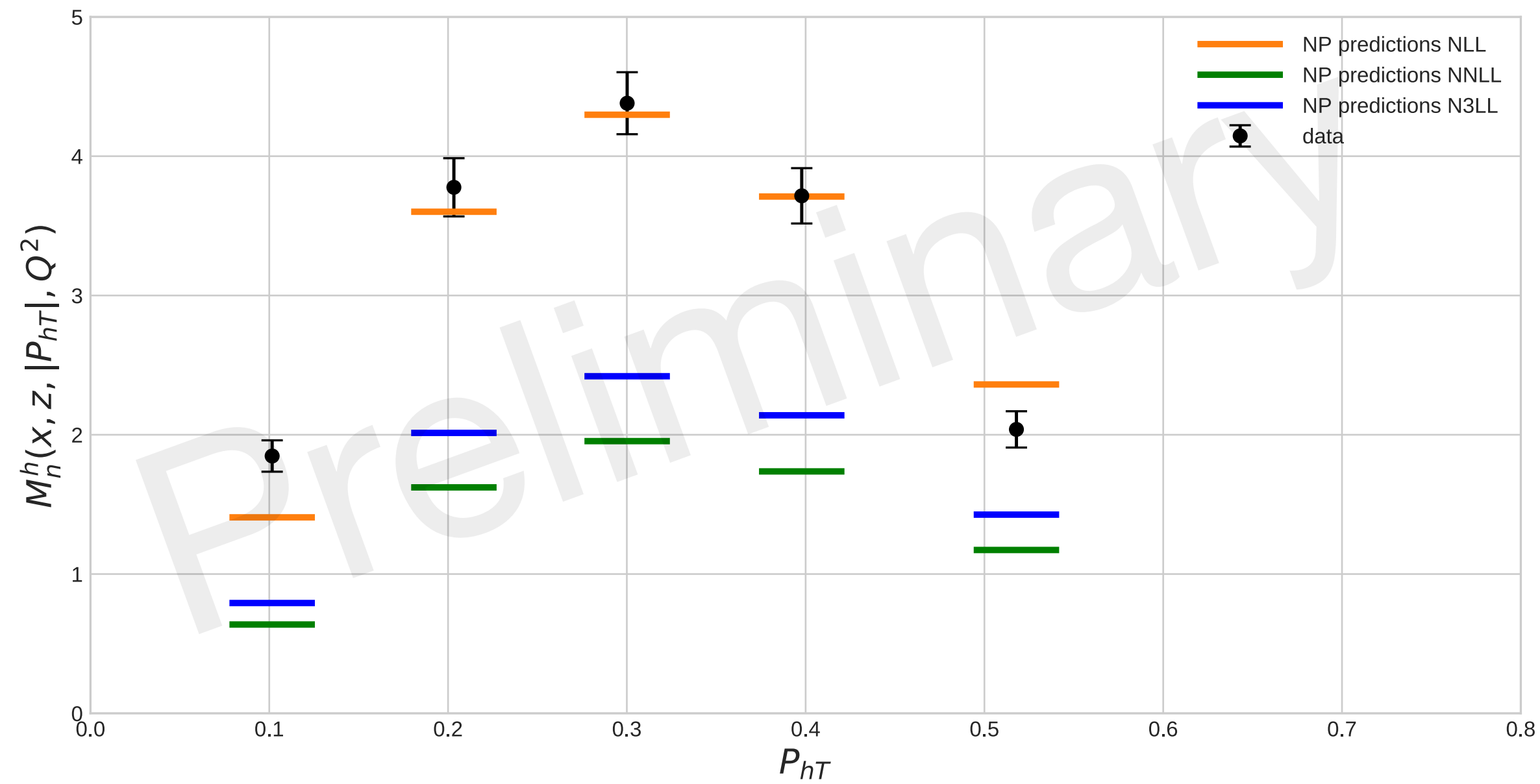


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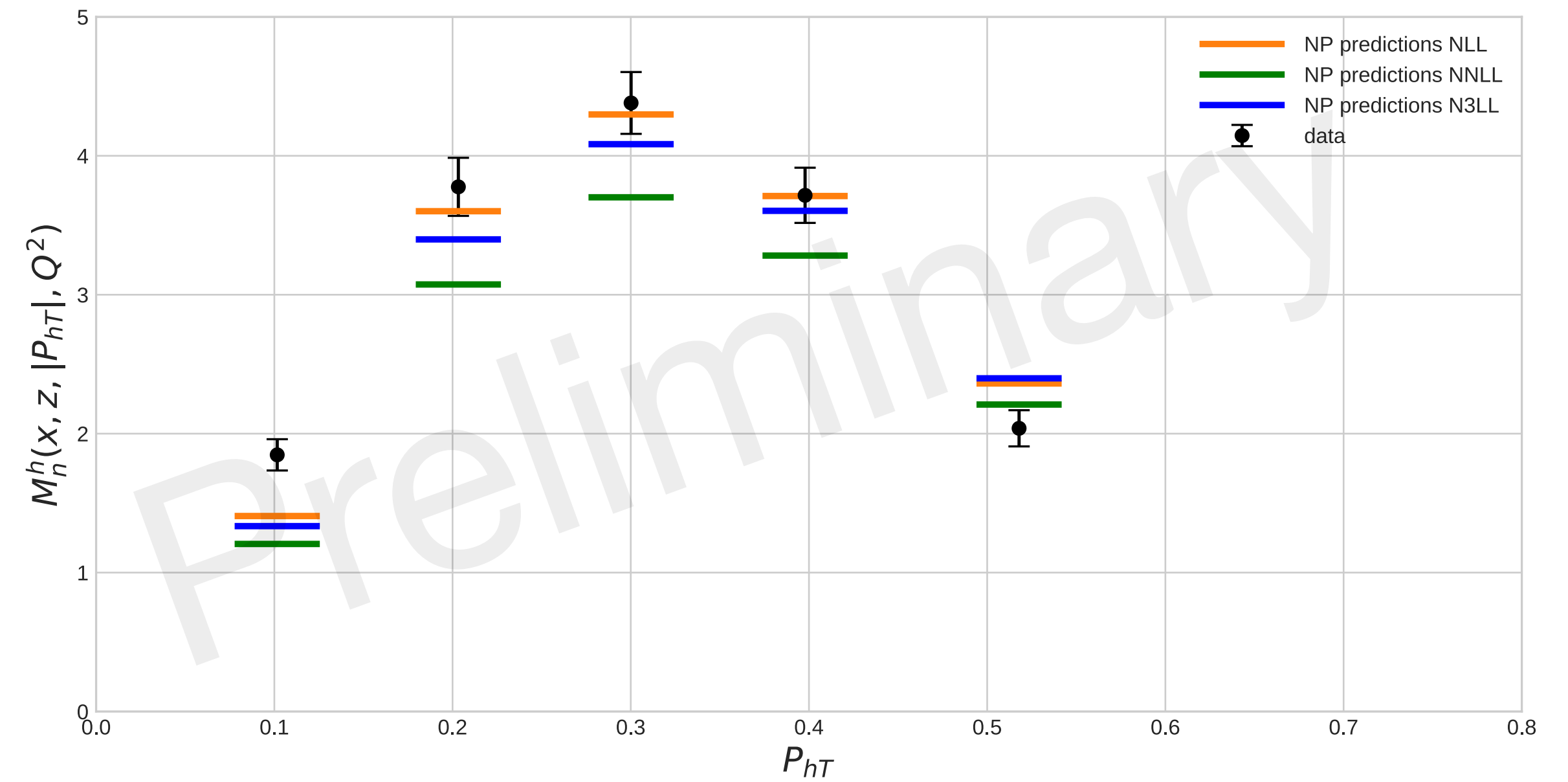
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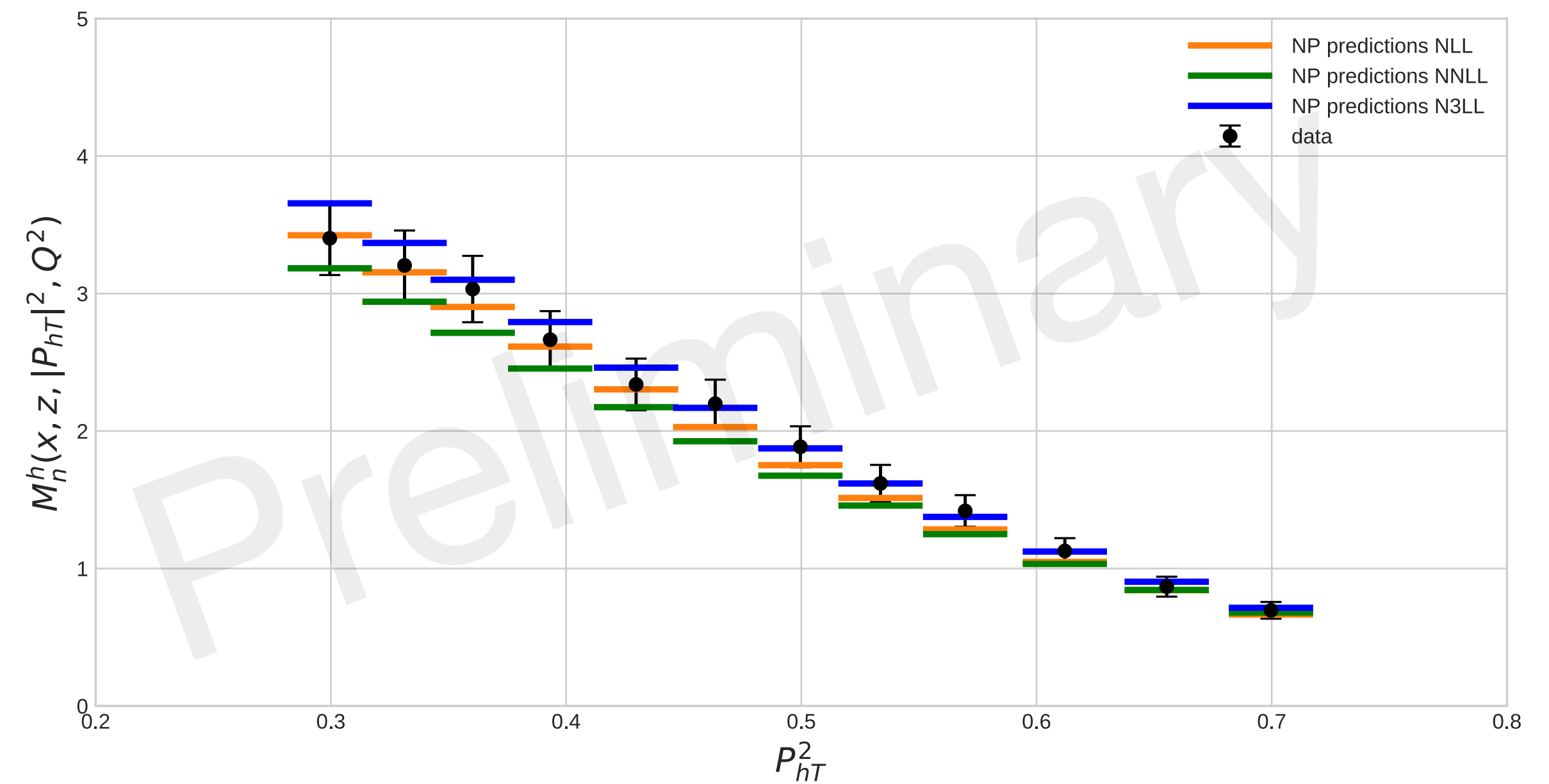
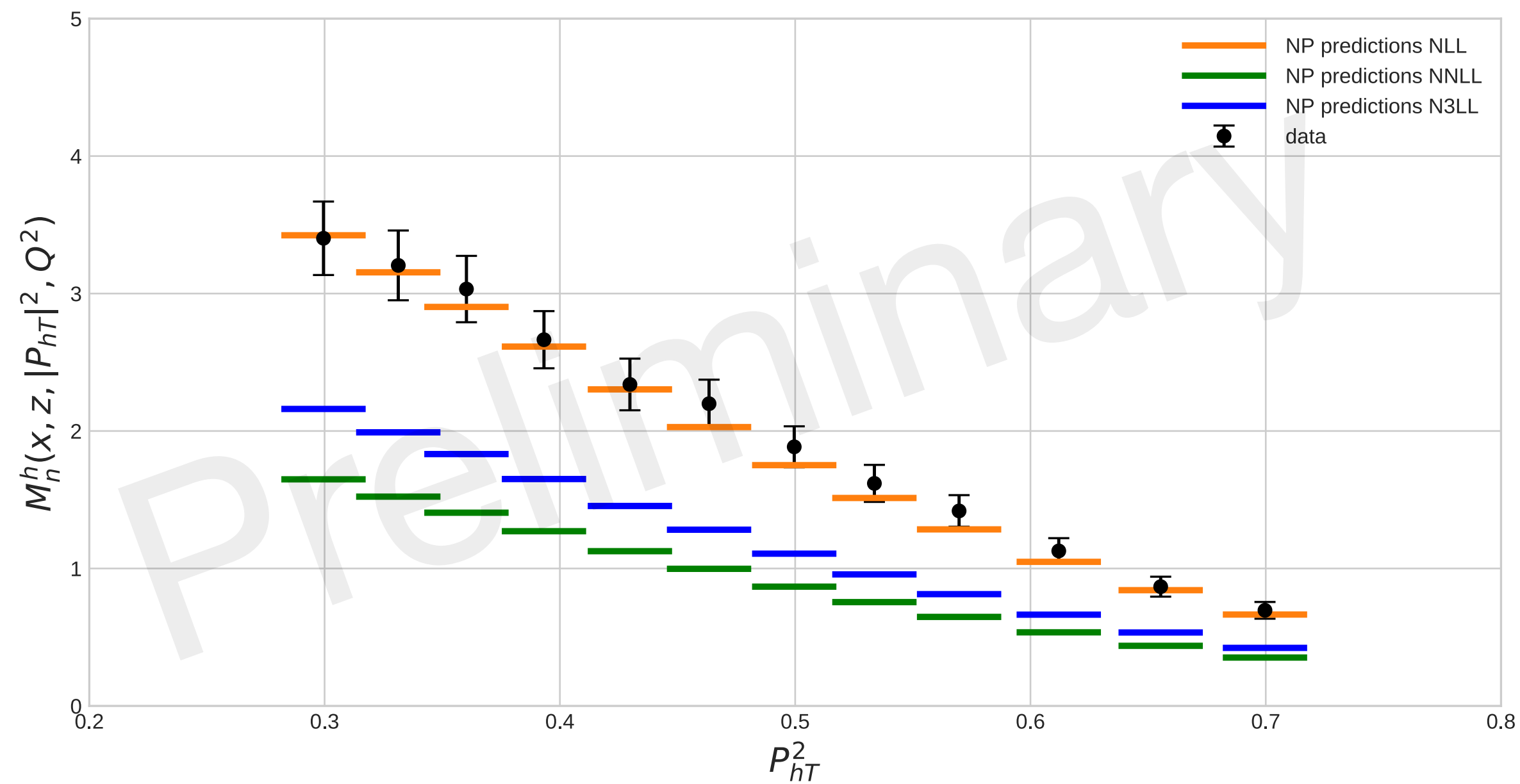
Source of W-term suppression

Present situation at low Q

COMPASS multiplicity

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Hard Factor = 1



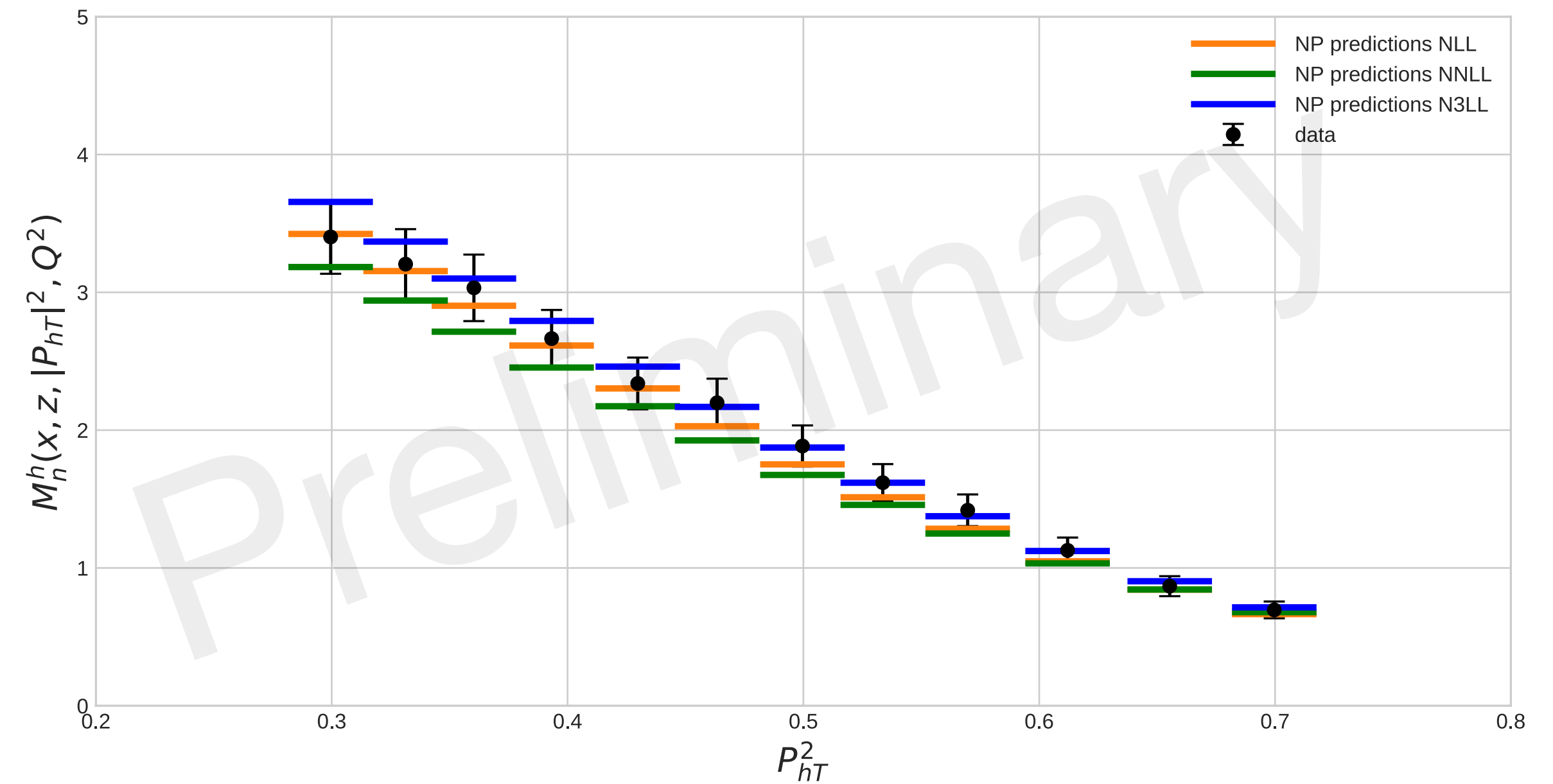
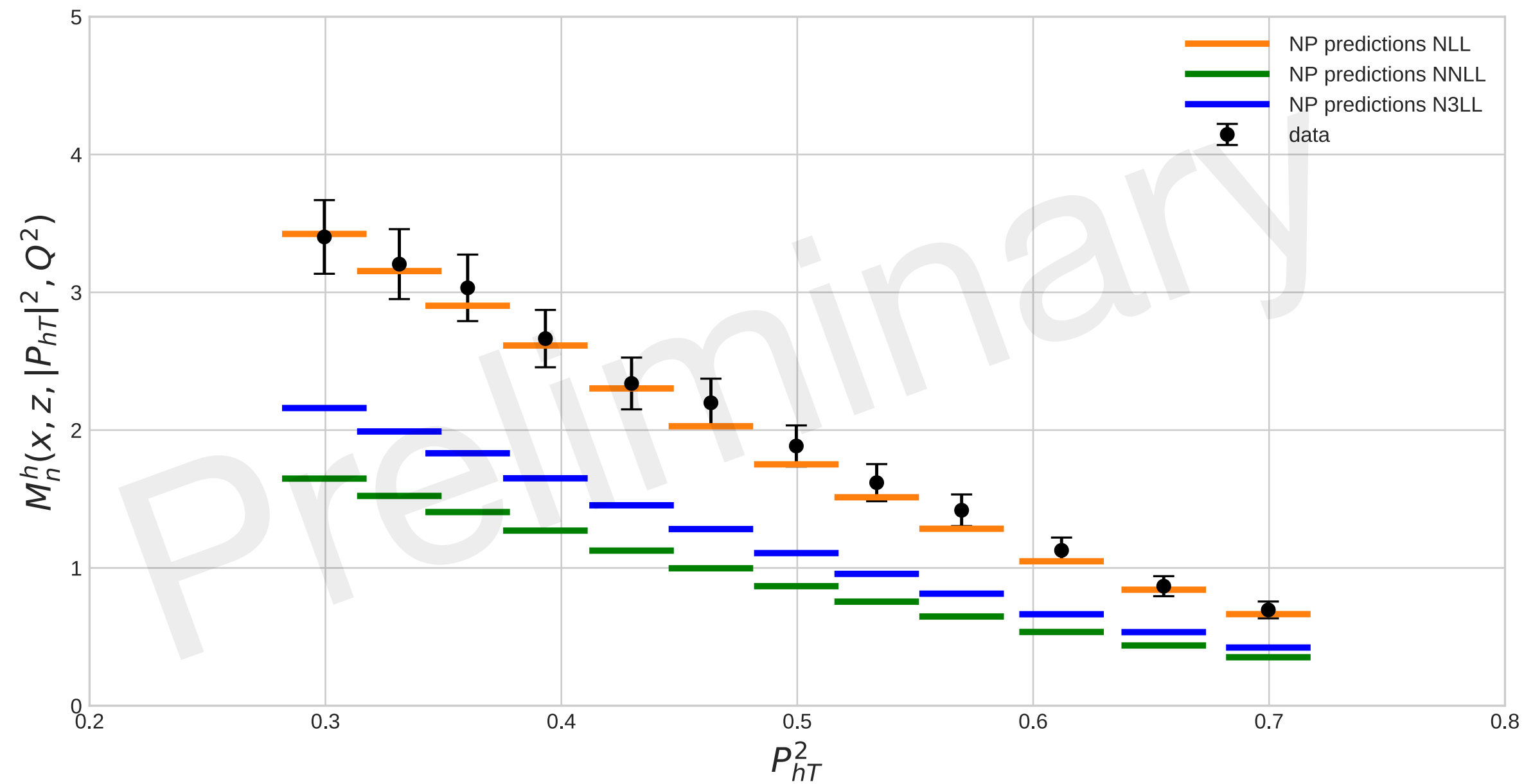
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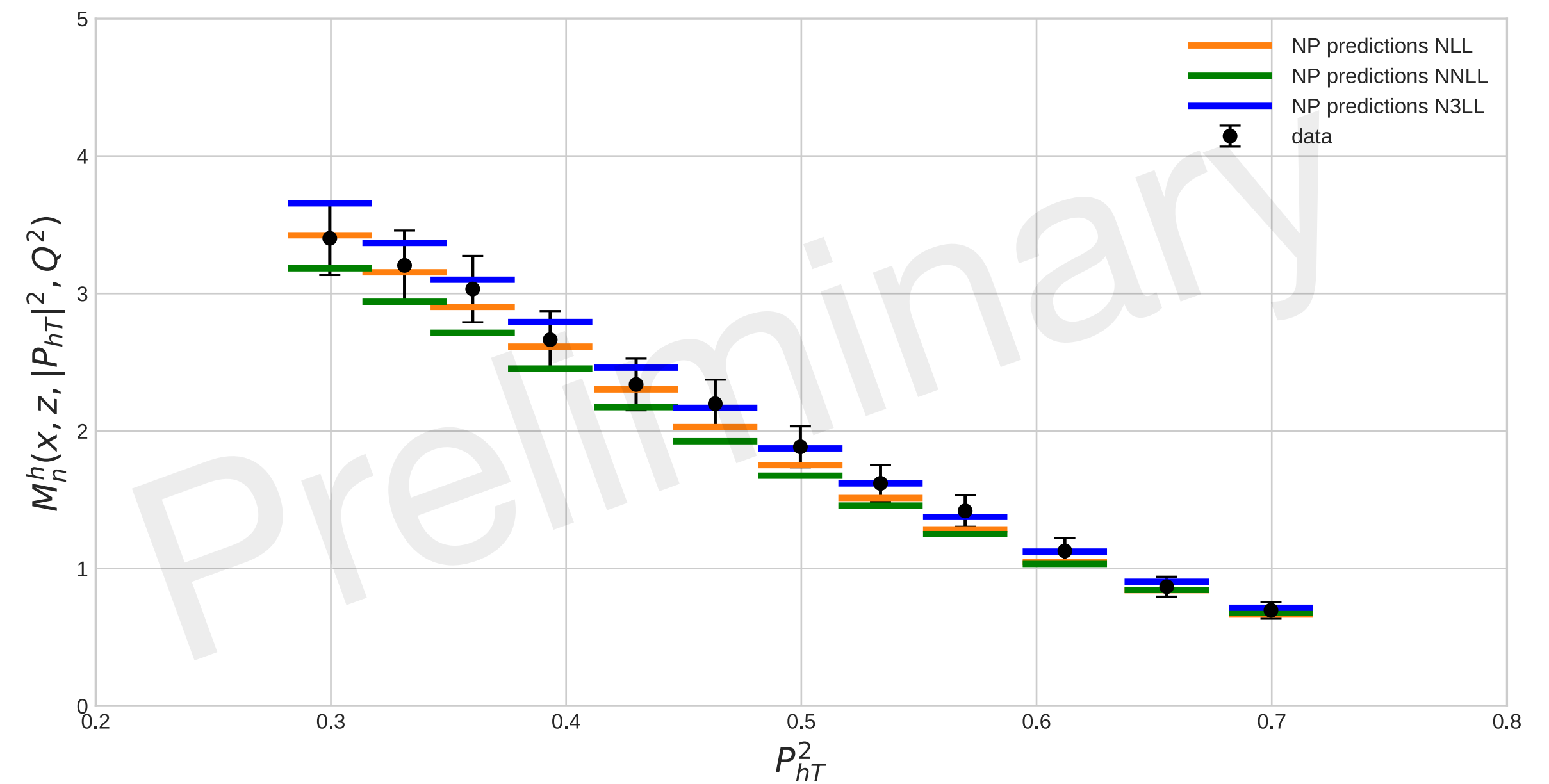
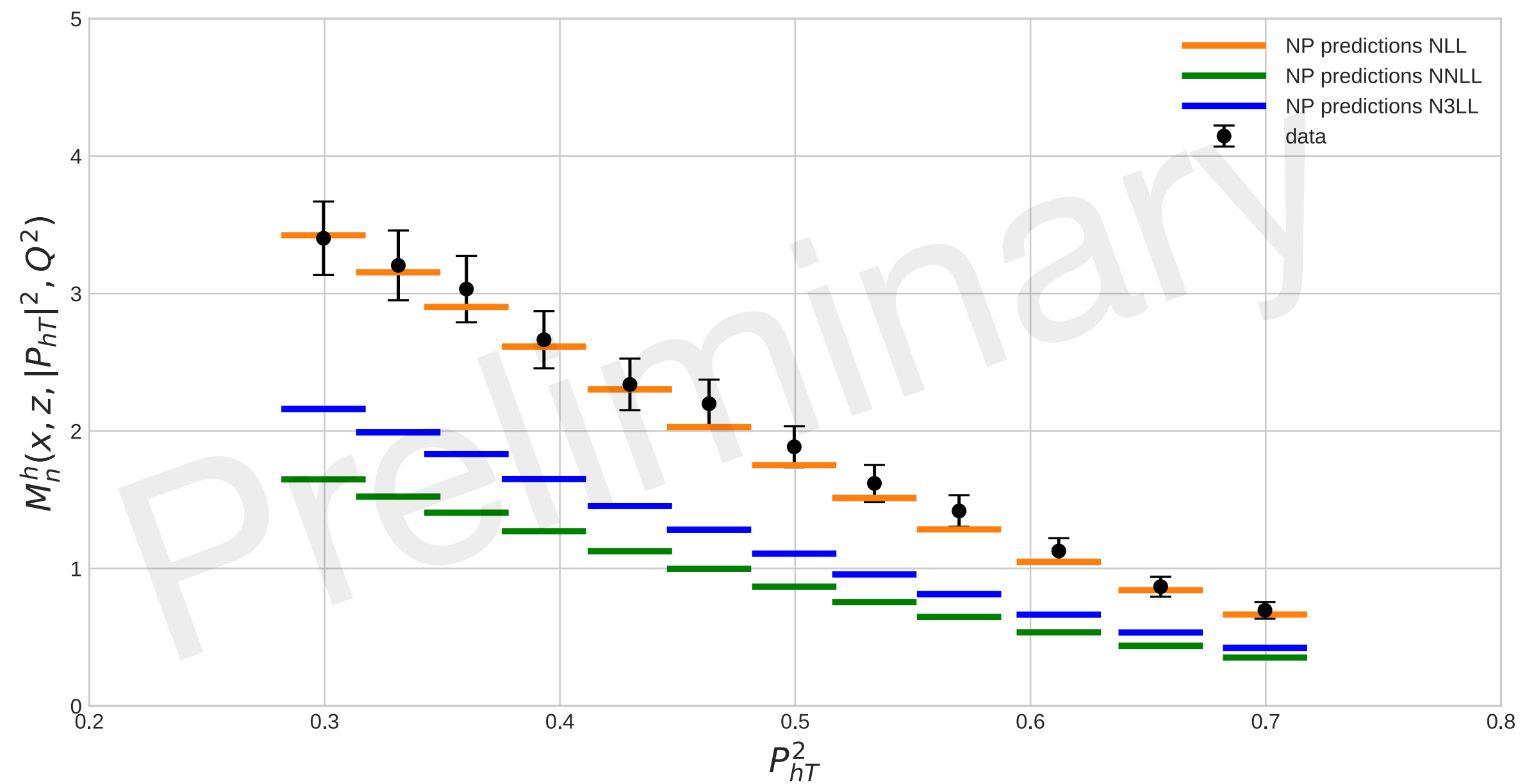
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Present situation at low Q

COMPASS multiplicity

Full Hard Factor

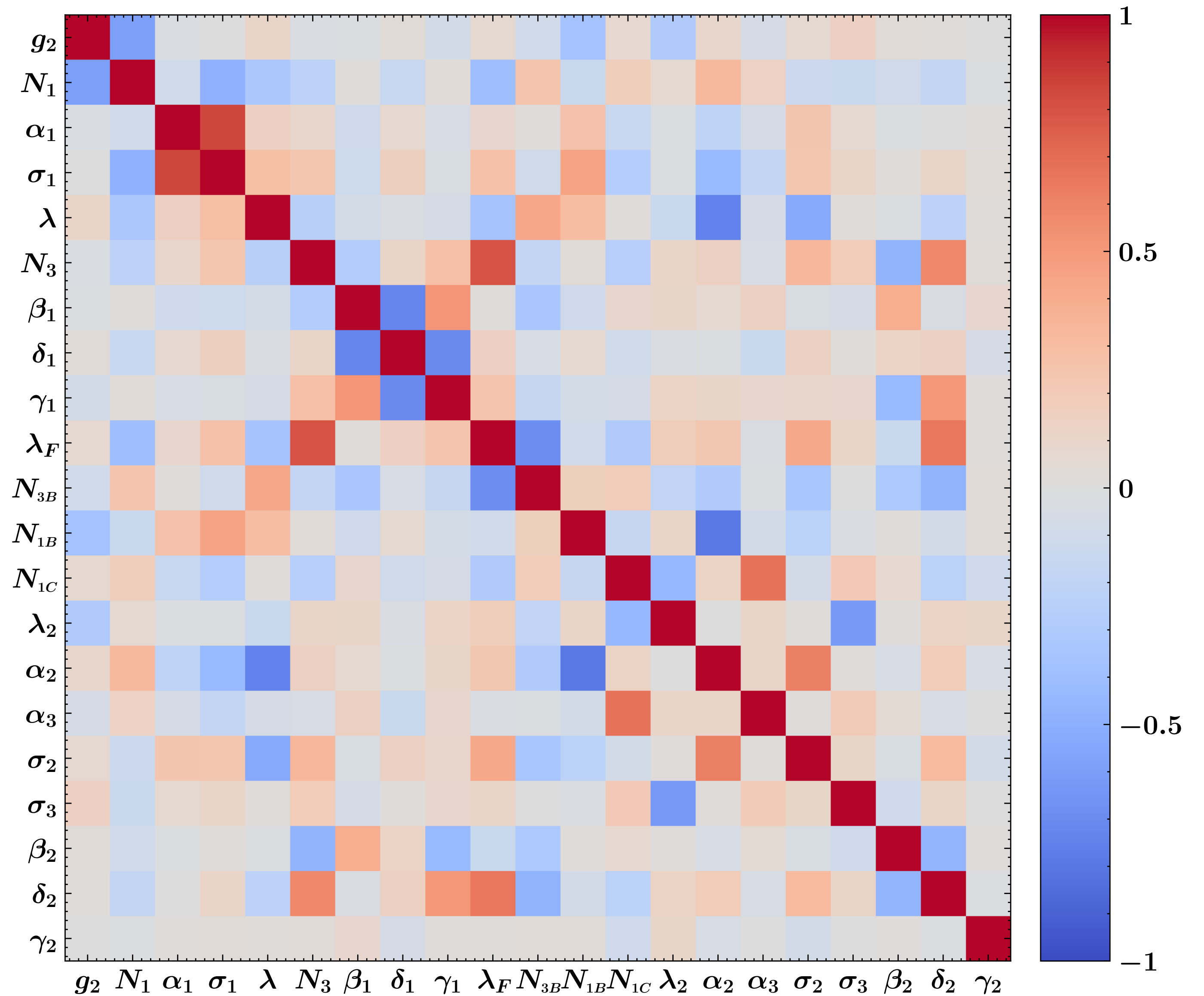
Hard Factor = 1



	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2013 arXiv:1309.3507	parton model	✓	✗	✗	✗	1538
Torino 2014 arXiv:1312.6261	parton model	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✗	✗	✓	✓	223
EIKV 2014 arXiv:1401.5078	NLL	1 (x,Q ²) bin	1 (x,Q ²) bin	✓	✓	500 (?)
SIYY 2014 arXiv:1406.3073	NLL'	✗	✓	✓	✓	200 (?)
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL'	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL'	✗	✗	✓	✓	457
SV 2019 arXiv:1912.06532	N ³ LL ⁻	✓	✓	✓	✓	1039
Pavia 2019 arXiv:1912.07550	N ³ LL	✗	✗	✓	✓	353
MAP22 arXiv:2206.07598	N ³ LL ⁻	✓	✓	✓	✓	2031

Results of the baseline fit

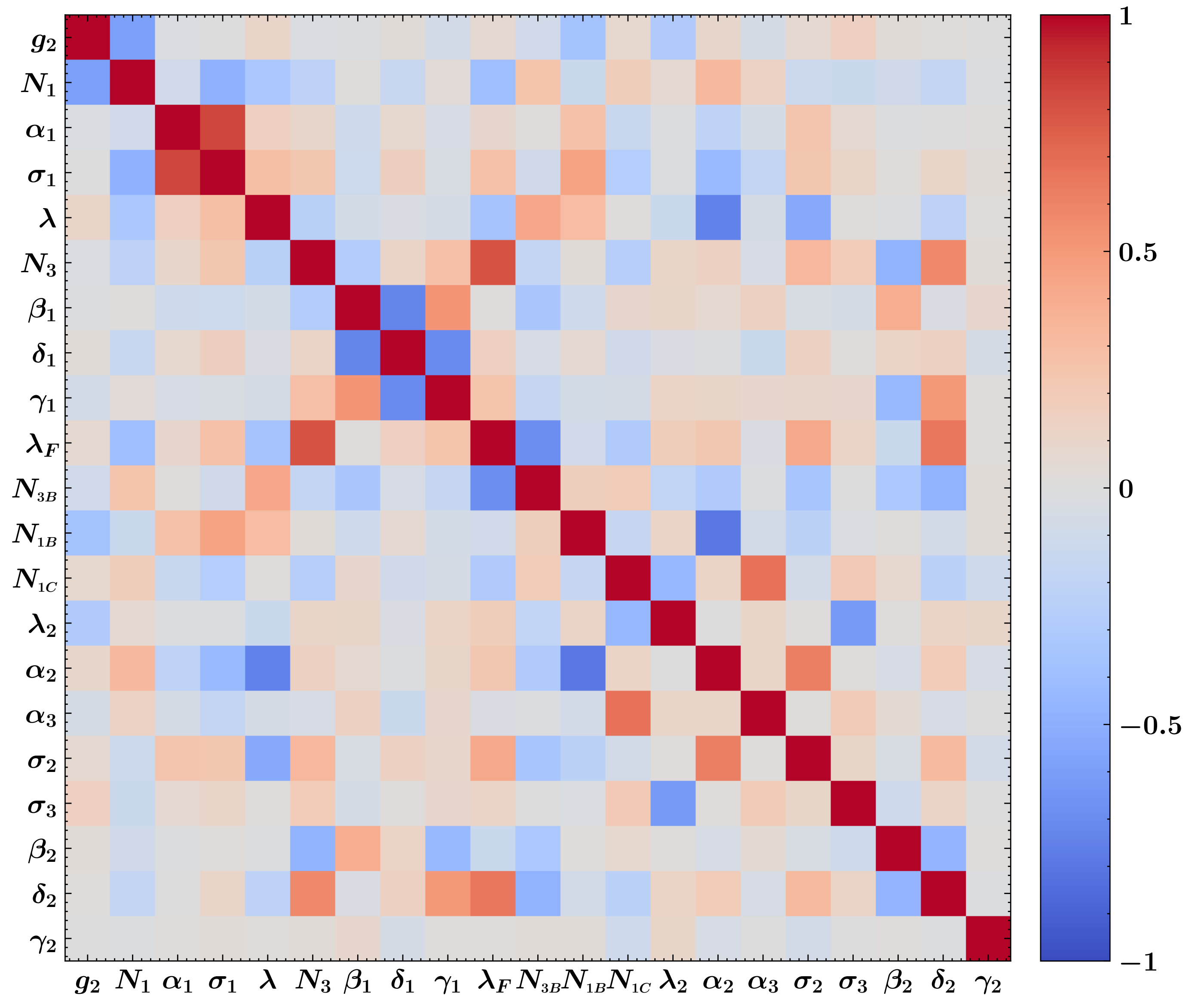
Error propagation
↓
250 Montecarlo replicas



Results of the baseline fit

Error propagation
↓
250 Montecarlo replicas

Correlation matrix
↓
Hints of the
appropriateness of the
chosen functional form



Cut q_T/Q for SIDIS dataset

$$P_{hT}|_{\max} = \min[\min[c_1 Q, c_2 z Q] + c_3 \text{ GeV}, c_4 z Q]$$

baseline
(c)

$$\begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 1 \end{cases}$$

$$\begin{array}{l} q_T/Q = 0.4 \\ \text{(a)} \end{array} \begin{cases} c_1 = 0.2 \\ c_2 = 0.5 \\ c_3 = 0.3 \\ c_4 = 0.4 \end{cases} \quad \begin{array}{l} < \text{baseline} \\ \text{(b)} \end{array} \begin{cases} c_1 = 0.15 \\ c_2 = 0.4 \\ c_3 = 0.2 \\ c_4 = 1 \end{cases}$$

$$\begin{array}{l} > \text{baseline} \\ \text{(d)} \end{array} \begin{cases} c_1 = 0.2 \\ c_2 = 0.6 \\ c_3 = 0.4 \\ c_4 = \infty \end{cases} \quad \begin{array}{l} \text{PV17} \\ \text{(e)} \end{array} \begin{cases} c_1 = 0.2 \\ c_2 = 0.7 \\ c_3 = 0.5 \\ c_4 = \infty \end{cases}$$

