Global fit for worm-gear TMD g_{1T}

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In Collaboration with:

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Precision QCD predictions
for ep Physics at the EIC

Center for Frontiers in Nuclear Science

Stony Brook University

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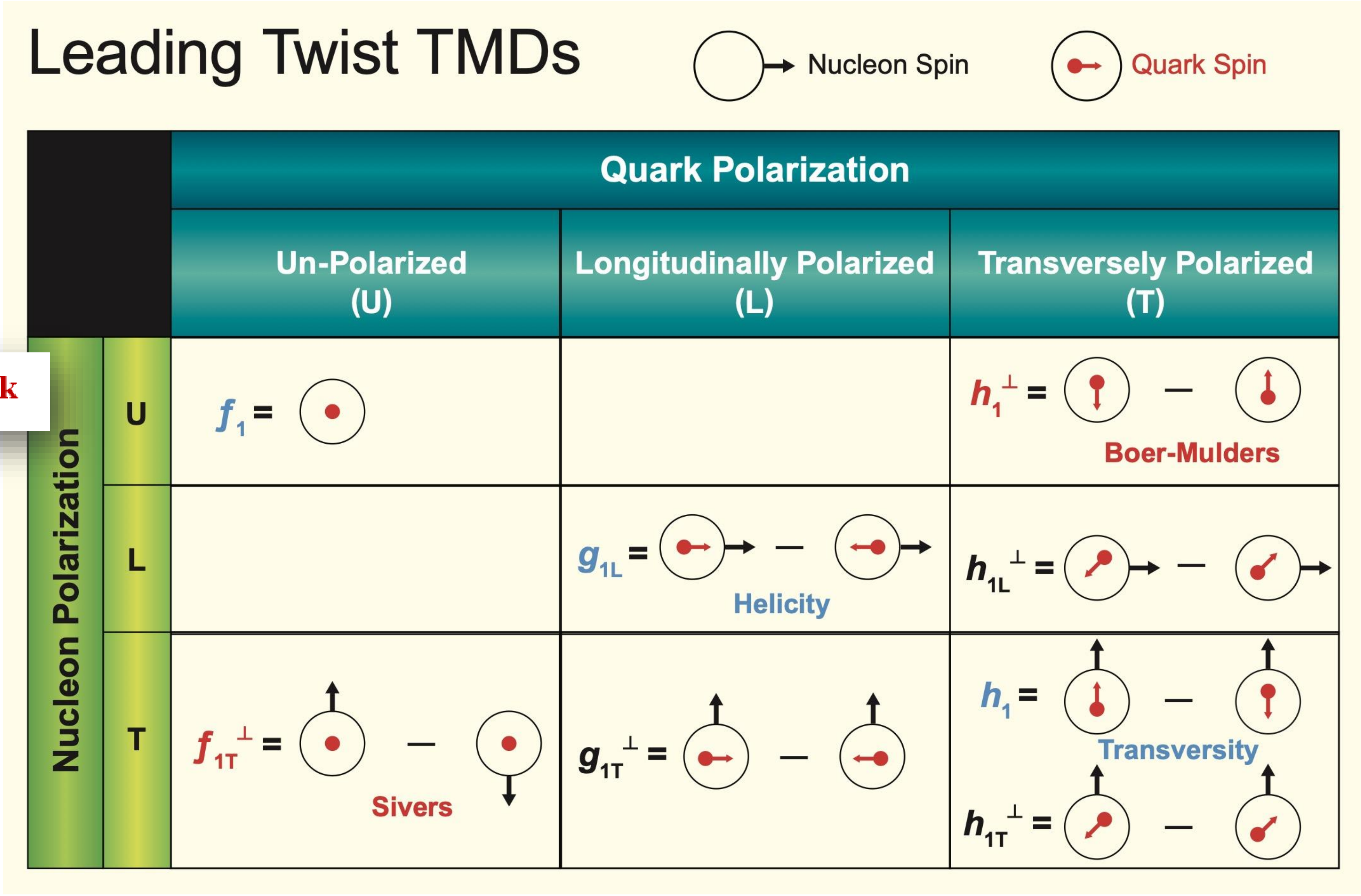
Outline

- **Introduction to TMDs**
- **Previous knowledge on g_{1T} : Model calculations, Lattice Calculations, Theoretical predictions**
- **Main extraction: Fit through Monte-Carlo technique**
- **Main fit results**
- **Comparison with theoretical predictions**
- **Comparison with Lattice QCD results**
- **Summary/Outlook**

Introduction

Mandatory Table

Fig. courtesy:
D. Pitonyak



Matteo Cerutti's talk

Introduction

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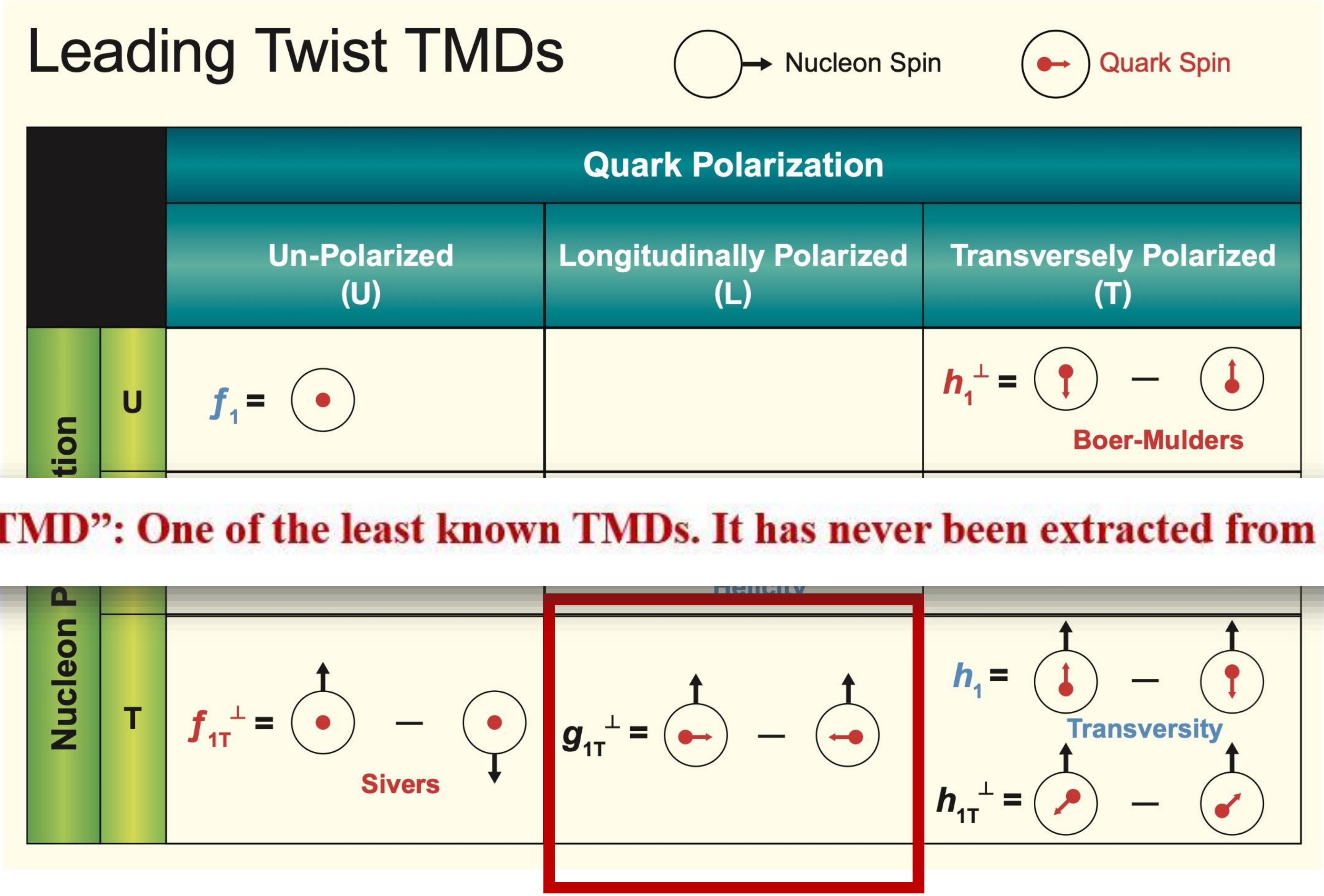
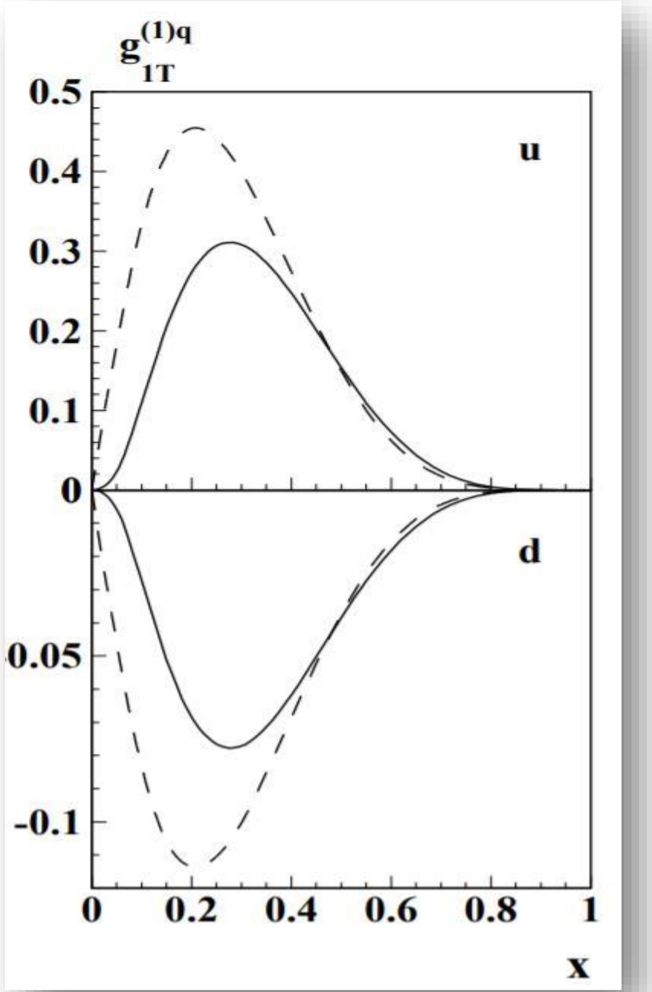


"Worm-gear TMD": One of the least known TMDs. It has never been extracted from experimental data.

Model calculations

Light-cone constituent quark model:

(Pasquini, Cazzaniga, Boffi, arXiv:0806.2298)



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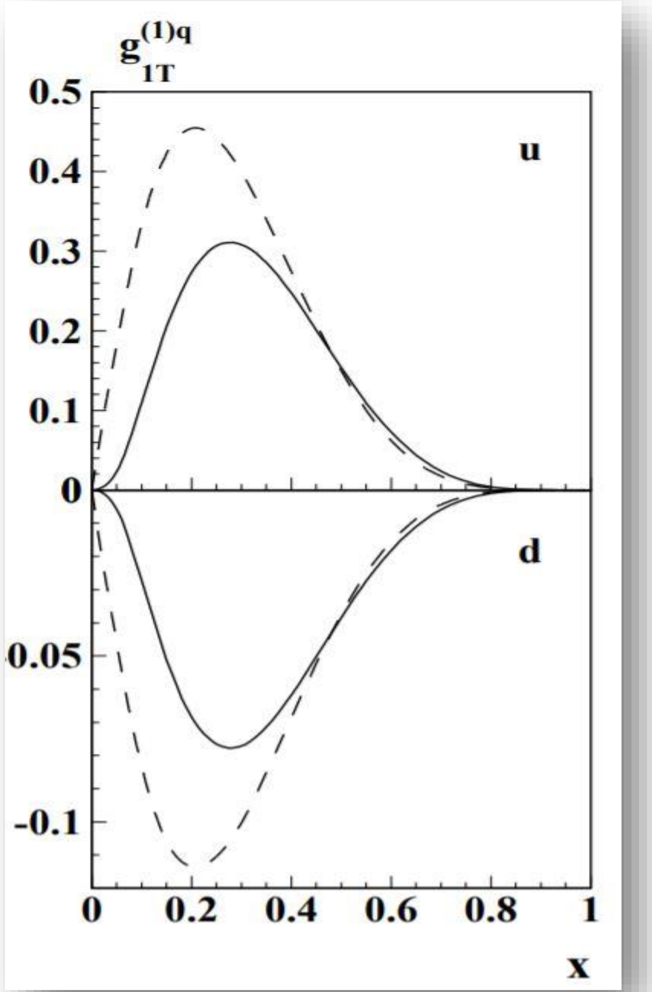
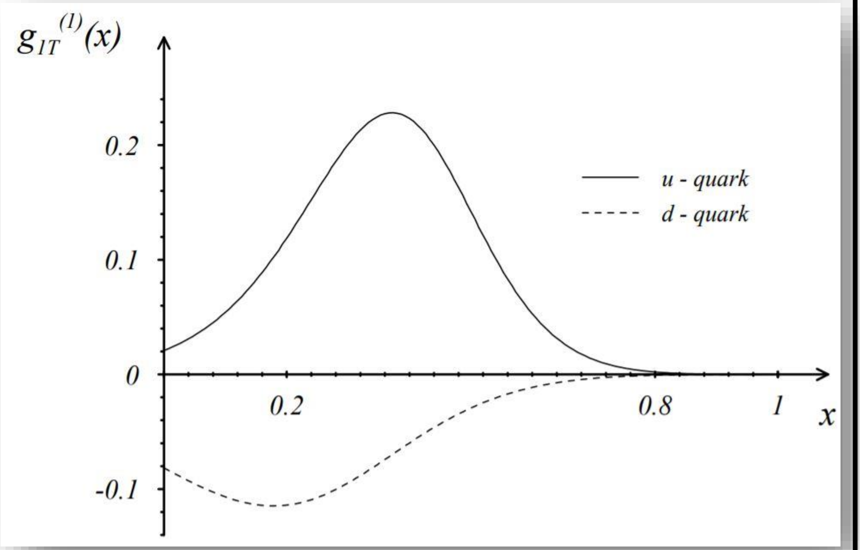


Spectator-diquark model:

(Jakob, Mulders, Rodrigues, hep-ph/9704335, Bacchetta, Conti, Radici, arXiv:0807.0323, ...)



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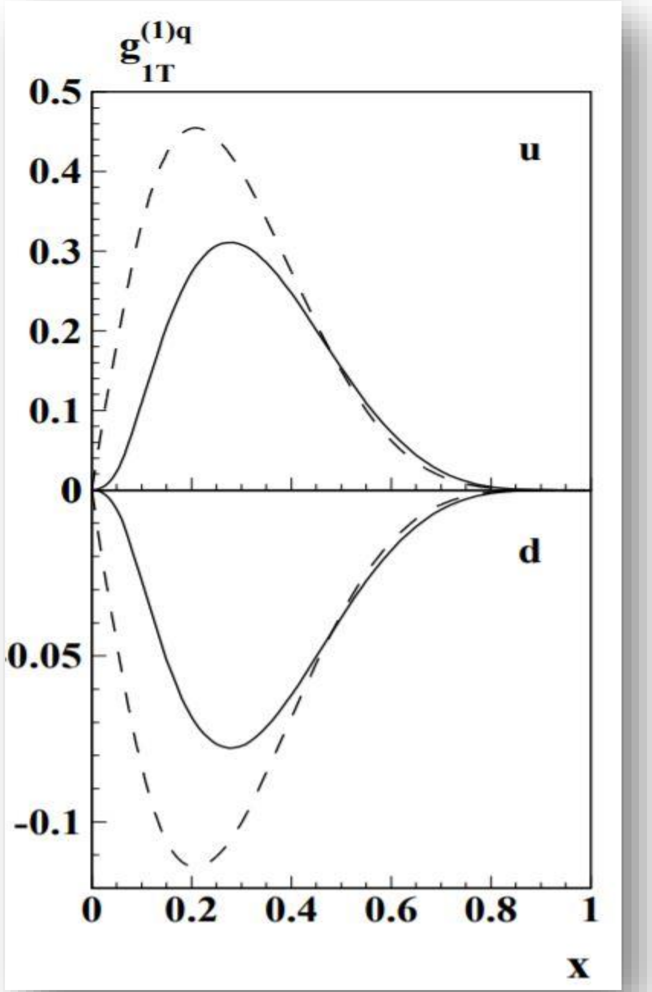

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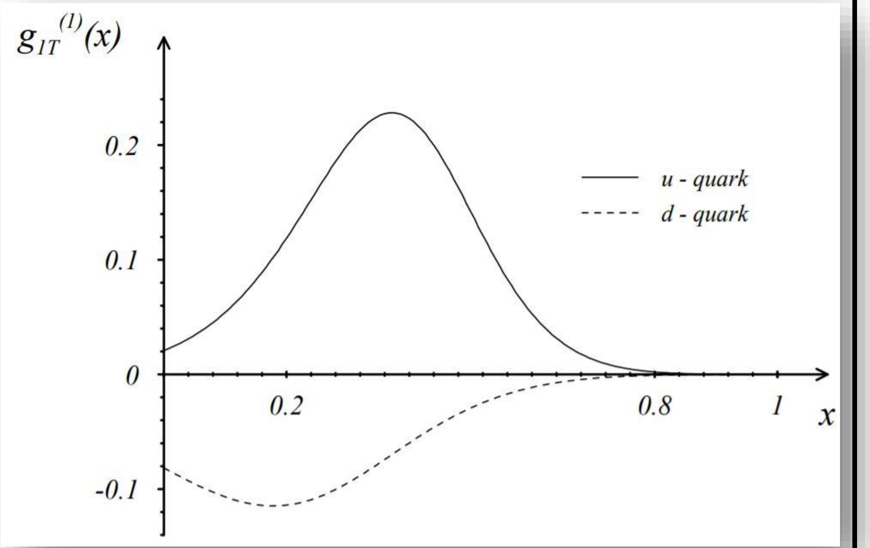
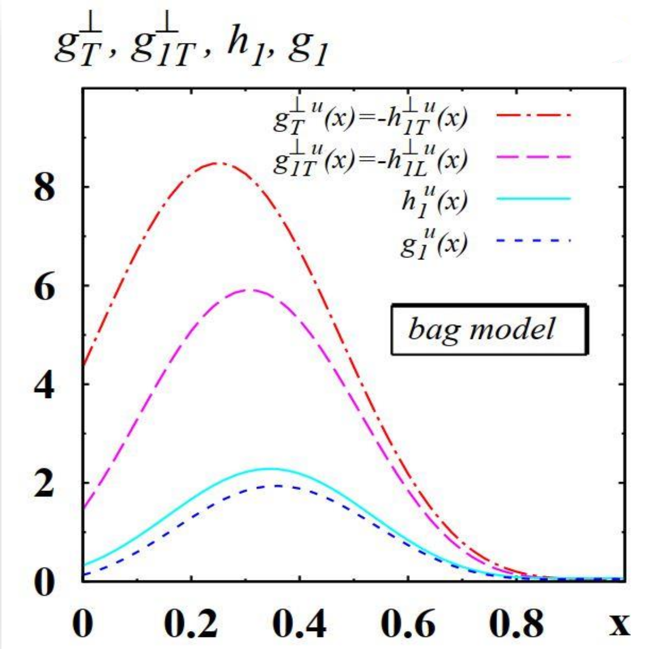


MIT bag model:

Avakian, Efremov, Schweitzer, Yuan, arXiv: 1001.5467



(The d-quark distributions have opposite signs and are smaller.)

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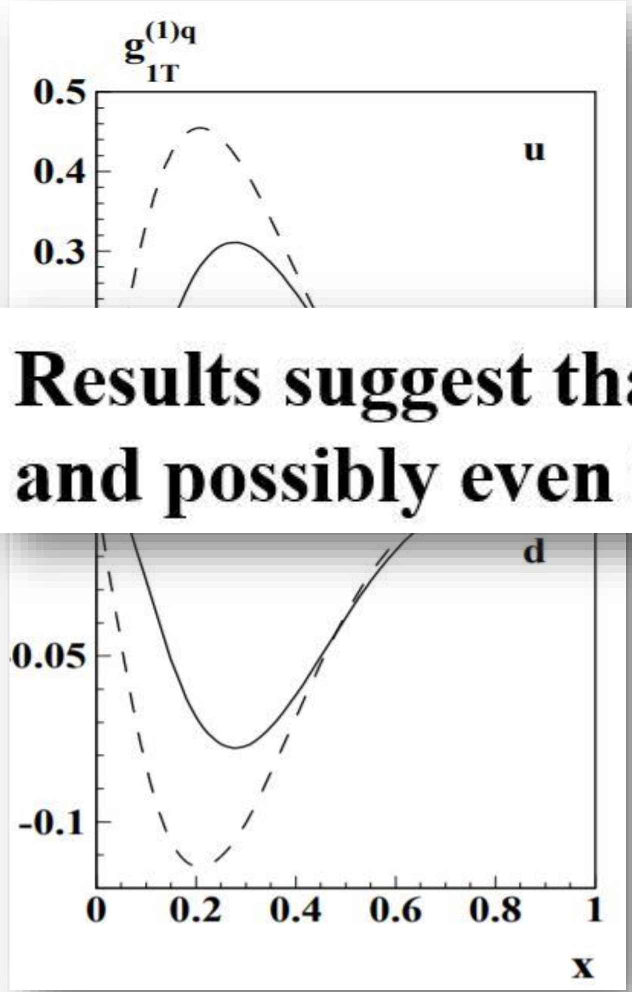

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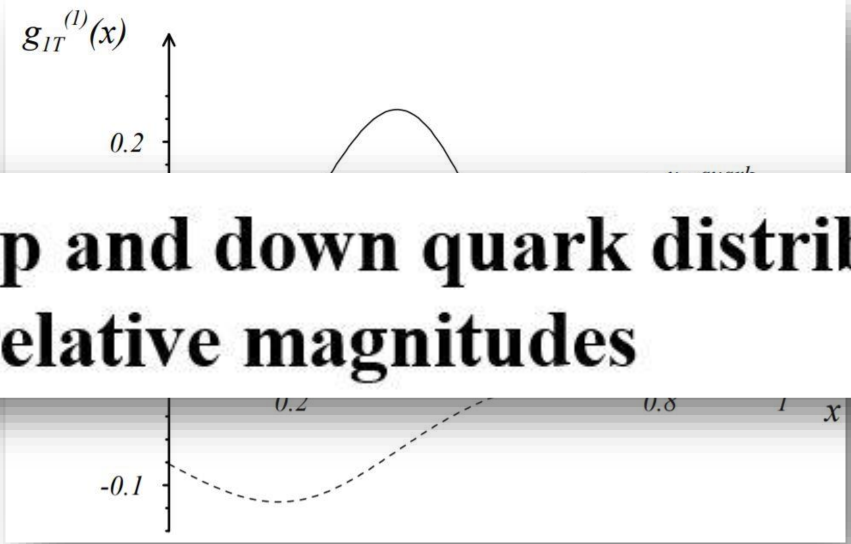

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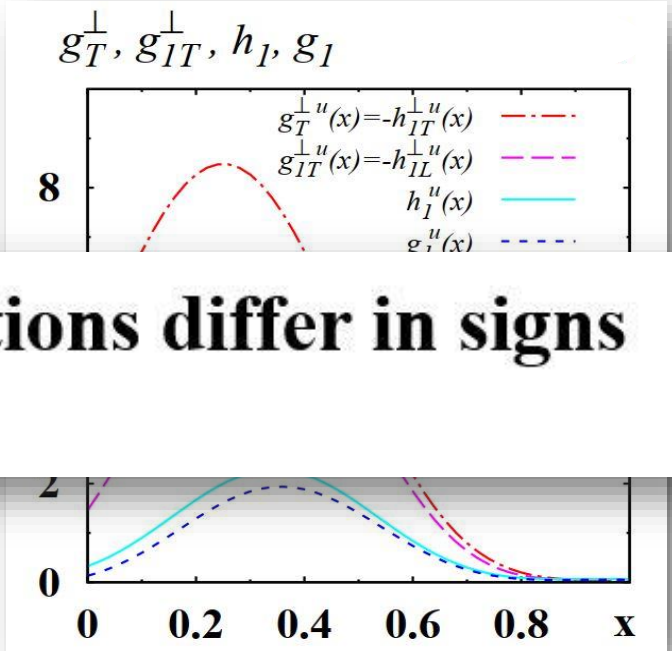

(The d-quark distributions have opposite signs and are smaller.)

Results suggest that up and down quark distributions differ in signs and possibly even in relative magnitudes

Lattice QCD calculations

Lattice QCD results (Musch et al., arXiv:0908.1283, arXiv:1011.1213)

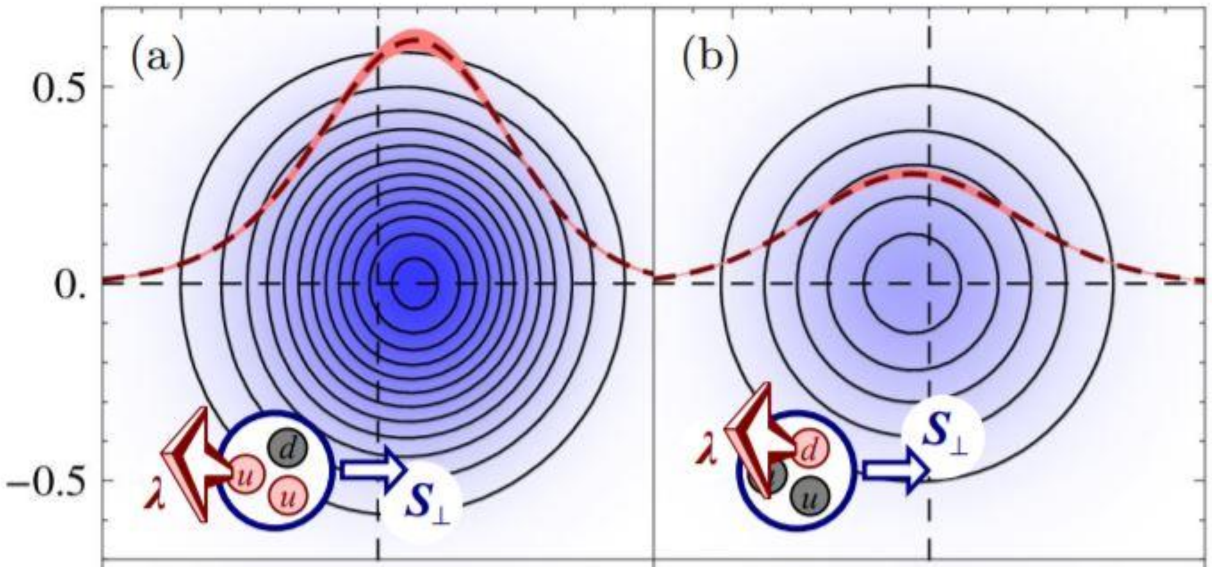

FIG. 3: Quark densities in the $\boldsymbol{k}_\perp$-plane, for $m_\pi \approx 500$ MeV. (a) ρ_L for u-quarks and $\lambda = 1$, $\boldsymbol{S}_\perp = (1, 0)$, (b) the same for d-quarks

- **Pioneering lattice QCD calculations done more than 10 years ago**

- **Lattice results support that up and down down quark distributions come in with different signs and different magnitudes**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theoretical predictions

1.

2.

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theoretical predictions

1. Large $\mathbf{N_c}$ analysis: (Pobylitsa, hep-ph/ 0301236)

$$g^u_{1T}(x,\vec{k}^2_\perp) \; = - \; g^d_{1T}(x,\vec{k}^2_\perp) + 1/N_c\text{-suppressed}$$

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2. Wandzura-Wilczek-type (WW-type) relation: (Avakian et. al., 0709.3253, Kanazawa et. al., 1512.07233, …)

$$g_{1T}^{(1)q}(x) \equiv \int d^2\vec{k}_\perp \left(\frac{k_\perp^2}{2M^2}\right) g_{1T}^{q}(x,\vec{k}_\perp^2) \overset{\text{EOM}}{=} x\int_x^1 \frac{dy}{y} g_1^q(y) + x\,\tilde{g}_T^q(x)$$

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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fundamentals

Semi-inclusive Deep Inelastic Scattering: $\ell(l) + N(P,S) \to \ell'(l') + h(P_h) + X$

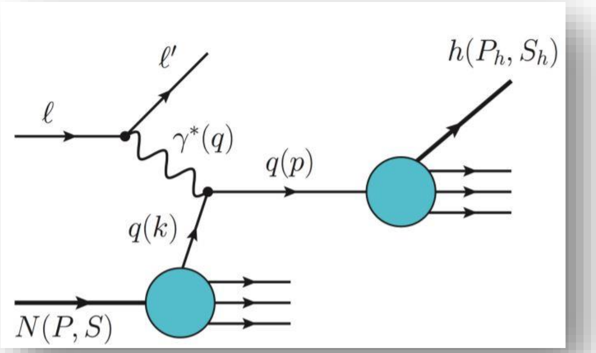

Fig. courtesy:
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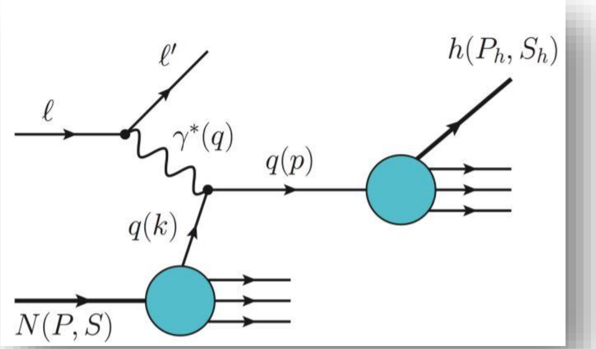

Fig. courtesy:
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Model-independent decomposition of cross-section: (Bacchetta et. al. 2007, …)

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz_h\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{\text{em}}^2}{x\,y\,Q^2}\left\{\left(1-y+\frac{1}{2}y^2\right)F_{UU}\right.$$

$$\left. + \lambda_l\,|\vec{S}_\perp|\,y\left(1-\frac{1}{2}y\right)\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} + \ldots\right\}$$

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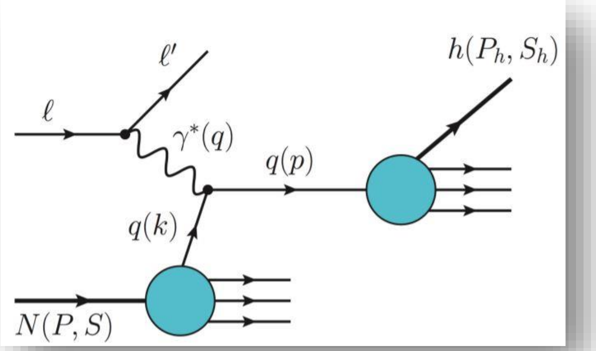
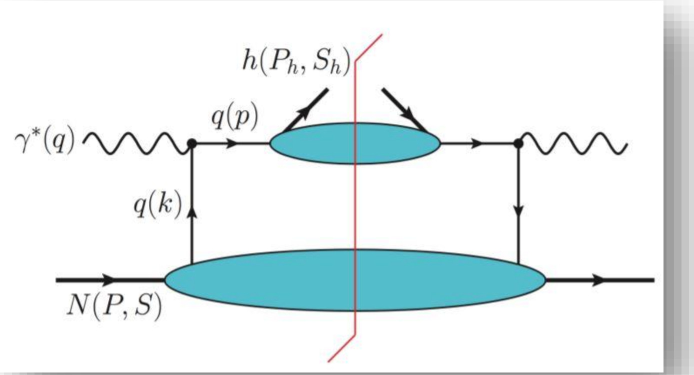


$q_T \ll Q$



Connection between structure functions and TMDs: (Bacchetta et. al. 2007, …)

$$F_{UU} = C\left[f_1(x,\vec{k}_\perp^2)\, D_1(z,\vec{P}_\perp^2)\right] \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = C\left[\frac{\vec{P}_{hT}\cdot\vec{k}_\perp}{|\vec{P}_{hT}|M}\, g_{1T}(x,\vec{k}_\perp^2)\, D_1(z,\vec{P}_\perp^2)\right]$$

$$C\left[w\, f\, D\right] = x\sum_q e_q^2 \int d^2\vec{k}_\perp \int d^2\vec{P}_\perp\, \delta^{(2)}\left(z\vec{k}_\perp + \vec{P}_\perp - \vec{P}_{hT}\right) w(\vec{k}_\perp, \vec{P}_\perp)\, f^q(x,\vec{k}_\perp^2)\, D^q(z,\vec{P}_\perp^2)$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Parameterization of g_{1T}

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Parameterization of g_{1T}

Gaussian ansatz:

$$g_{1T}^{q}(x,\vec{k}_{\perp}^{2},Q^2) = g_{1T}^{(1)q}(x,Q^2)\,\frac{2M_N^2\, e^{-\frac{\vec{k}_{\perp}^{2}}{\pi\langle k_{\perp}^{2}\rangle}}}{\pi\big(\langle k_{\perp}^{2}\rangle\big)^2}$$

$q=(u,d)$

where, $$g_{1T}^{(1)}(x,Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^{\beta} f_1(y,Q_0^2)}\, x^{\alpha}(1-x)^{\beta} f_1(x,Q^2)$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Presently available data insufficient to pin down the parameters:

$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz:

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- **Fix TMD width:**

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- **Fix TMD width:** $\frac{\langle k_\perp^2 \rangle\big|_{g_1}}{\langle k_\perp^2 \rangle\big|_{f_1}} \approx 0.76$

Lattice QCD Hagler et. al., hep-lat/ 0908.1283 (See also Bastami et. al., 1807.10606 that uses this idea to get $\langle k_\perp^2 \rangle\big|_{g_{1T}}$)

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$\langle k_\perp^2\rangle|_{f_1} = 0.53$ → $\therefore \langle k_\perp^2\rangle|_{g_{1T}} \approx 0.40$

Cammarota et. al., arXiv 2002.08384

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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- **Set alphas equal:** $\alpha^d = \alpha^u$

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- **Fix beta from WW approximation:**

$$g_{1T}^{(1)q}(x) \overset{x\to 1}{\approx} (1-x)\, g_1^q(x)$$

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Helicity & unpolarized PDFs have similar large-x behavior

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- **Fix TMD width:** $\langle k_\perp^2 \rangle\big|_{g_{1T}} \approx 0.40$
- **Set alphas equal:** $\alpha^d = \alpha^u$
- **Fix beta from WW approximation:**

$$g_{1T}^{(1)q}(x) \overset{x\to 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$

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$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz: $g_{1T}^q(x, \vec{k}_\perp^2, Q^2)$ … $q = (u, d)$

where, $g_{1T}^{(1)}(x, Q^2)$ … $(1-x)^\beta f_1(x, Q^2)$

3 free parameters:

$$n^u, \quad n^d, \quad \alpha$$

- **Fix TMD width:** $\langle k_\perp^2 \rangle\big|_{g_{1T}} \approx 0.40$
- **Set alphas equal:** $\alpha^d = \alpha^u$
- **Fix beta from WW approximation:**

$$g_{1T}^{(1)q}(x) \overset{x \to 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Presently available data insufficient to pin down the parameters:

$$\langle k_\perp^2 \rangle, \quad \alpha^d, \quad \beta^{u/d}$$

Gaussian ansatz: $g_{1T}^q(x, \vec{k}_\perp^2, Q^2)$ $q = (u, d)$

3 free parameters:

$$n^u, \quad n^d, \quad \alpha$$

where, $g_{1T}^{(1)}(x, Q^2)$ ... $(1-x)^\beta f_1(x, Q^2)$

We checked explicitly that using different values of TMD width and beta does not change the qualitative conclusions of our results

- Set alphas equal: $\alpha^d = \alpha^u$

$$g_{1T}^{(1)q}(x) \overset{x\to 1}{\approx} (1-x) f_1^q(x) \longrightarrow \therefore \beta^u = \beta^d = 1$$

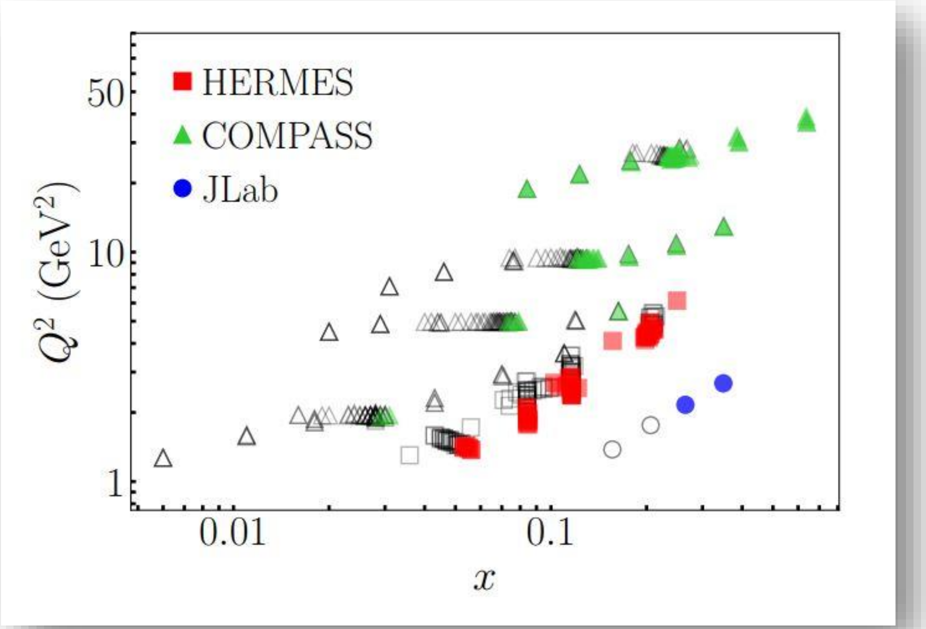
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

- **Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed**



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

- **Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed**
- **The quantity $g_{1T}^{(1)}(x)$ evolves according to $f_1(x)$:**

$$g_{1T}^{(1)}(x,Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^{\beta} f_1(y,Q_0^2)} x^{\alpha}(1-x)^{\beta} f_1(x,Q^2)$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

- Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed
- The quantity $g_{1T}^{(1)}(x)$ evolves according to $f_1(x)$
- Actual evolution of $g_{1T}^{(1)}(x)$ follows a more complicated pattern: (Zhou, Yuan, Liang (2008))

$$g_{1T}^{(1)}(x) \equiv \tilde{g}(x)$$

$$\begin{aligned}\frac{\partial \tilde{g}(x_B,\mu^2)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int \frac{dxdy}{x}\Bigg\{&\tilde{g}(x)\delta(y-x)\left[C_F\left(\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right)-\frac{C_A}{2}\frac{1+z^2}{1-z}\right]\\ &+\tilde{G}_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{2x_B^2}{xy}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{(x_B^2+xy)(2x_B-x-y)}{(x_B-y)(x-y)y}\right]\\ &+G_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xy}{(y-x_B)y}\right]\Bigg\}\end{aligned}$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Remarks about evolution

- **Given the precision and range in Q of the data, a rigorous implementation of TMD evolution not needed**
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DGLAP kernel

$$\frac{\partial \tilde{g}(x_B,\mu^2)}{\partial \ln\mu^2} = \frac{\alpha_s}{2\pi}\int\frac{dxdy}{x}\left\{\tilde{g}(x)\delta(y-x)\left[C_F\left(\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right)-\frac{C_A}{2}\frac{1+z^2}{1-z}\right]\right.$$
$$+\tilde{G}_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{2x_B^2}{xy}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{(x_B^2+xy)(2x_B-x-y)}{(x_B-y)(x-y)y}\right]$$
$$\left.+G_D(x,y)\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{y}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xy}{(y-x_B)y}\right]\right\}$$

$$g_{1T}^{(1)}(x)\equiv\tilde{g}(x)$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Experimental data

Dataset	Target	Identified hadron	No. of points
HERMES	p	π^+	26
Airapetian et. al., arXiv: 2007.07755		π^-	26
		π^0	8
COMPASS	p	$h^+ \approx (\pi^+, \mathrm{K}^+)$	33
Parsamyan, PoS: QCDEV2017		$h^- \approx (\pi^-, \mathrm{K}^-)$	31
JLab	n	π^+	2
Huang, arXiv: 1108.0489		π^-	2
Total			**128**

Cut:

$$\frac{q_T}{Q} < 0.50$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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Total			**128**

Cut:

$$\frac{q_T}{Q} < 0.50$$

Very less points!

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize:

$$\chi^2 = \sum_{\mathrm{H+C+J}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2}$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Minimize weighted chi-squared:

$$\chi^2 = \sum_{\mathrm{H+C}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2} + w \sum_{\mathrm{J}} \frac{(\mathrm{exp.\,data} - \mathrm{theory})^2}{(\mathrm{exp.\,error})^2}$$

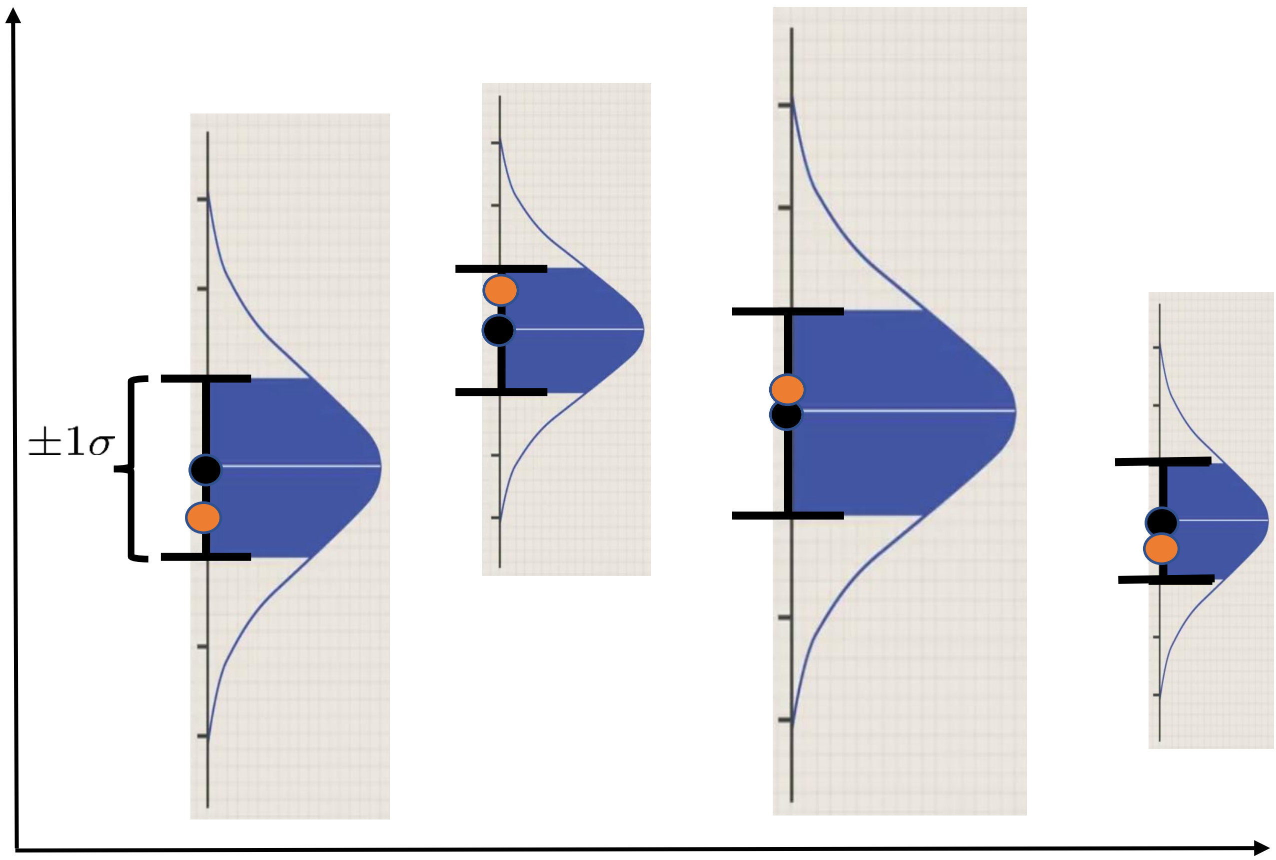
(Echevarria, Kang, Terry, arXiv: 2009.10710)

Give JLab data weight similar to HERMES & COMPASS data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

- Theory
- Fit to exp. data
- Generate pseudo-data



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Fitting procedure: Monte-Carlo technique

- Theory
- Fit to exp. data
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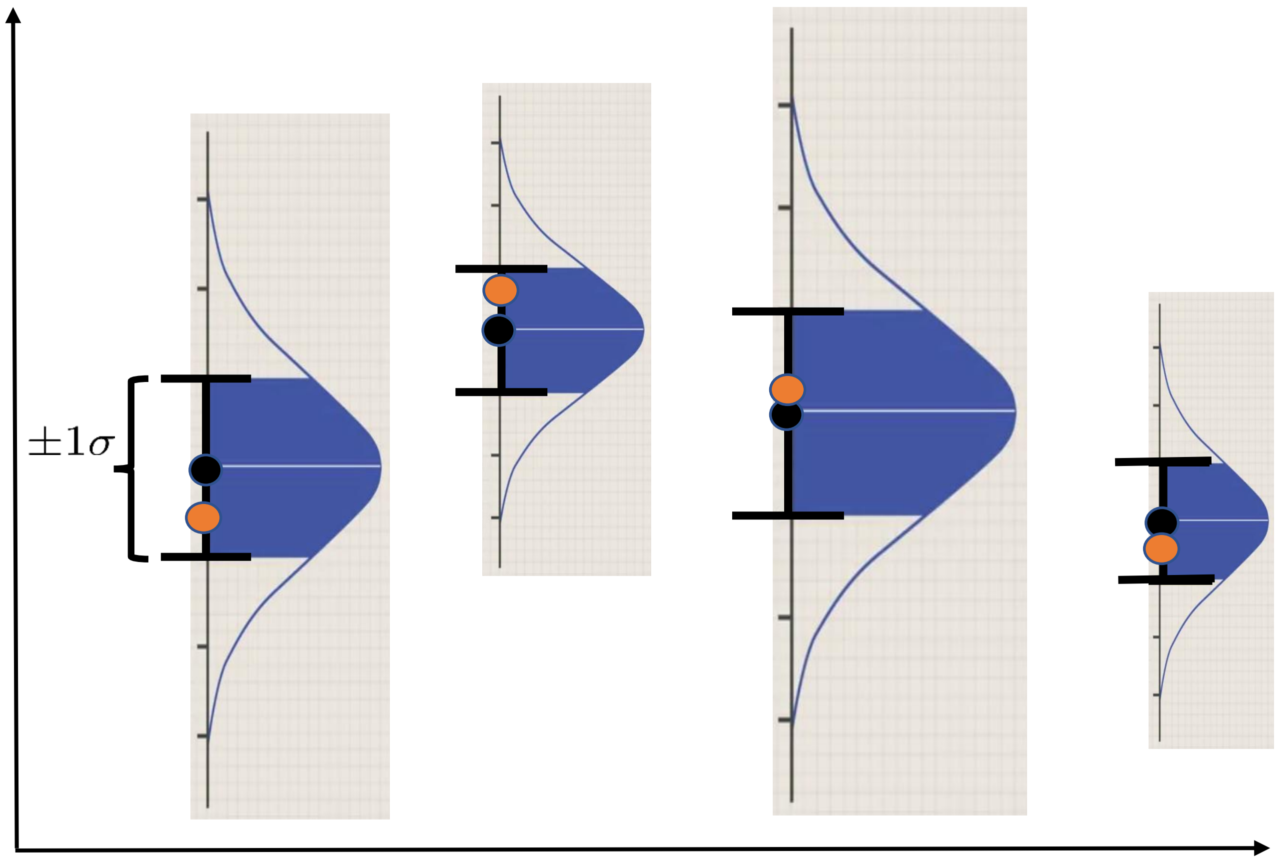
- Fit to pseudo-data



Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

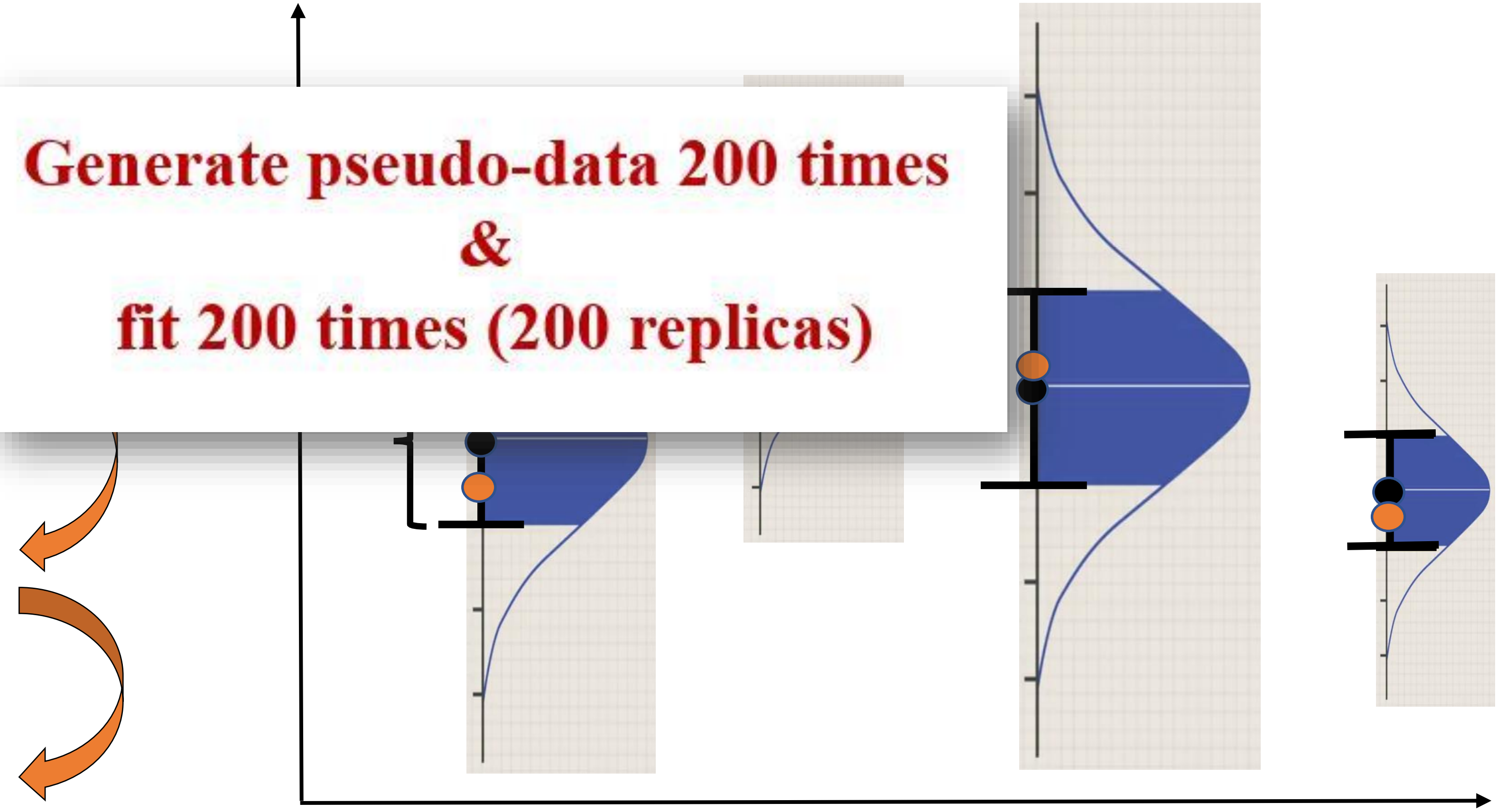
Fitting procedure: Monte-Carlo technique

Theory

Fit to exp. data

Generate pseudo-data

Fit to pseudo-data



Generate pseudo-data 200 times
&
fit 200 times (200 replicas)

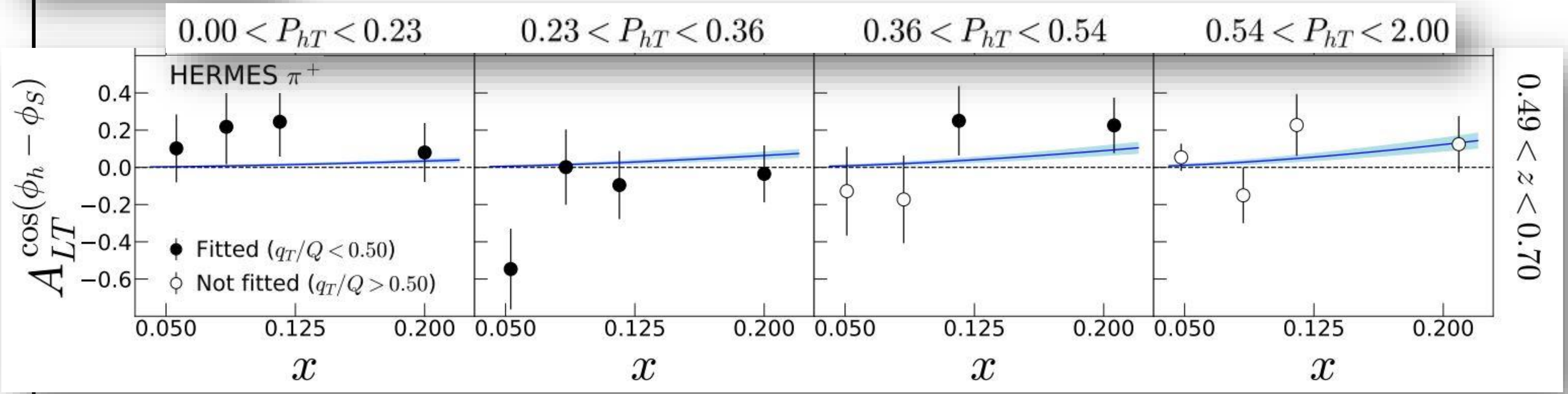
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

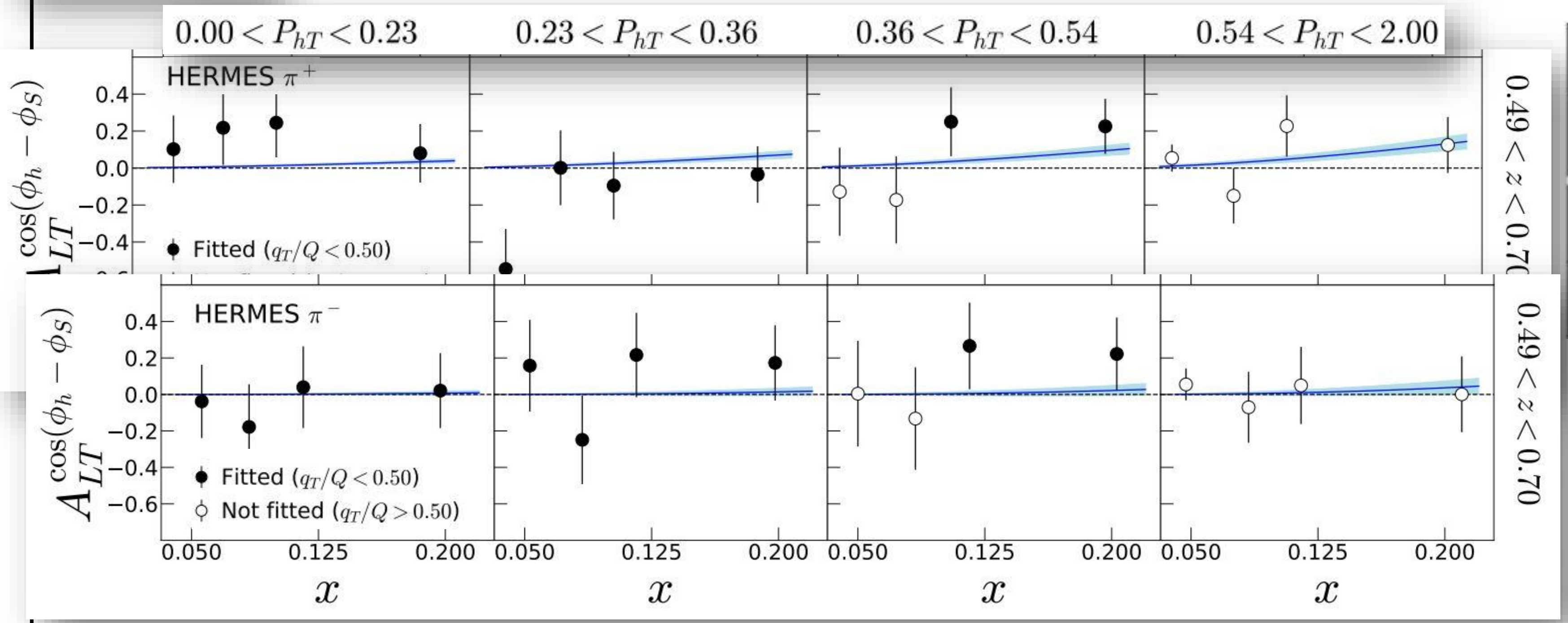


Data set	$\chi^2_w/N_{\text{pts.}}\|_{\text{Main}}$
HERMES π^+	1.20

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

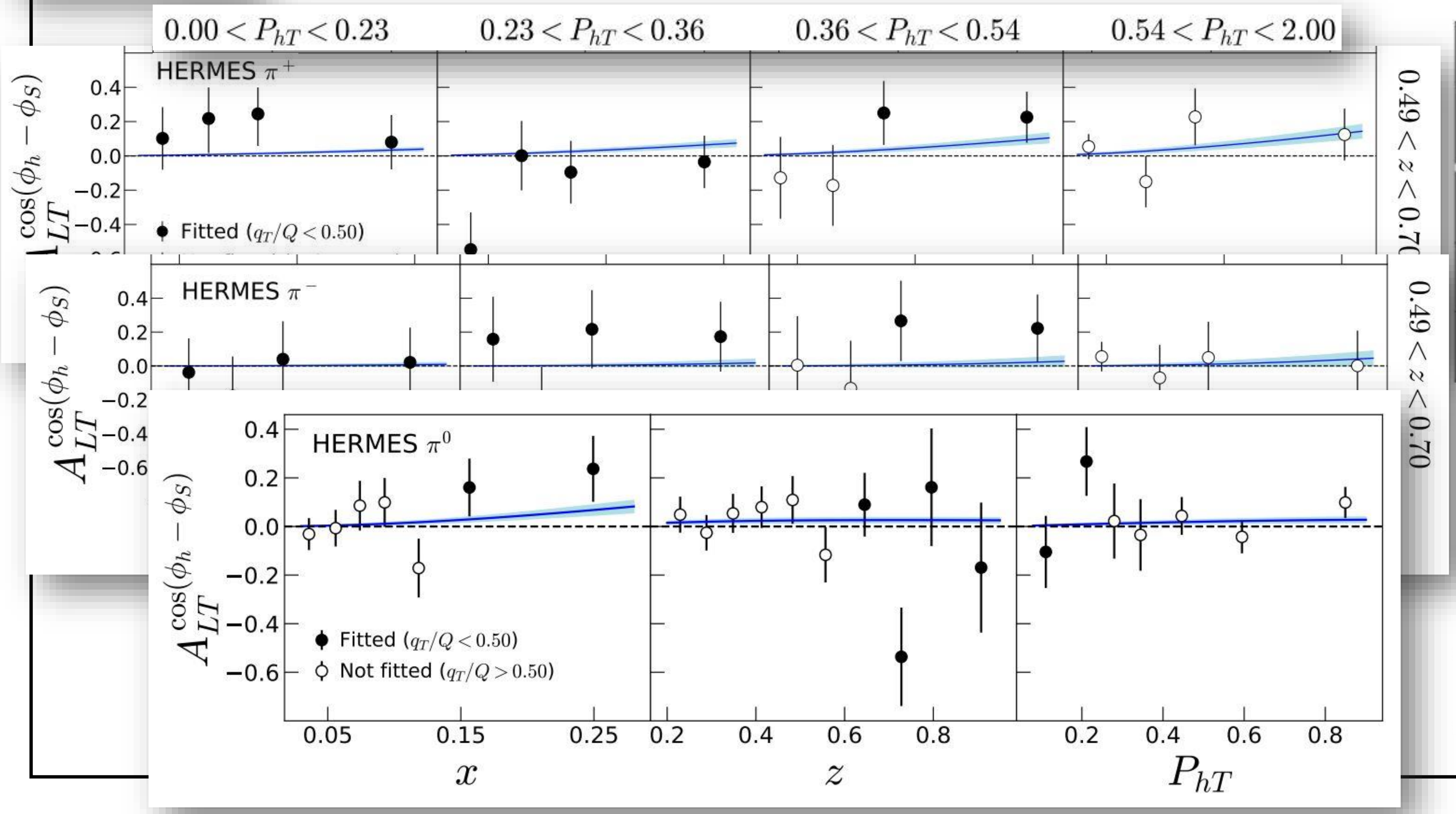


Data set	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Main}}$
HERMES π^+	1.20
HERMES π^-	0.88

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data

Sample results

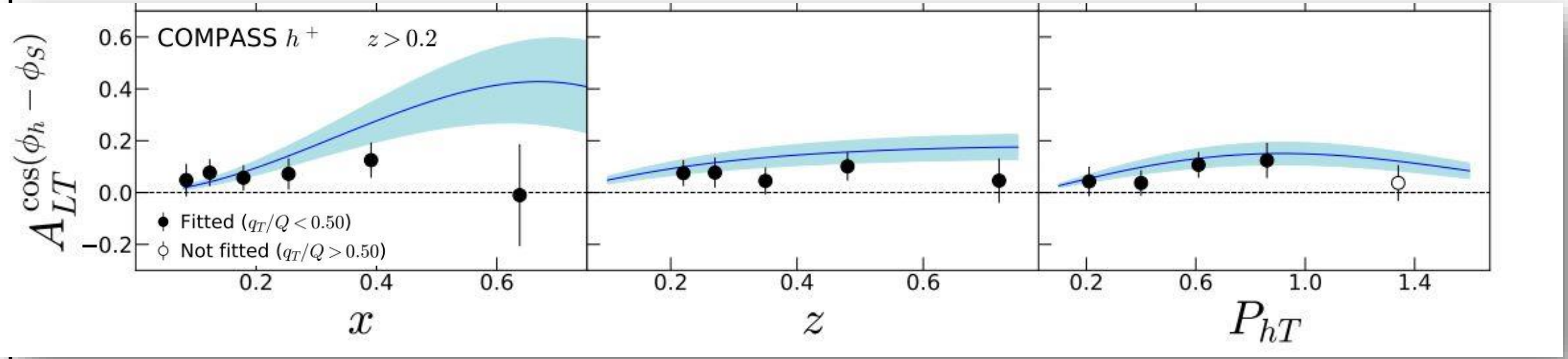


Data set	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Main}}$
HERMES π^+	1.20
HERMES π^-	0.88
HERMES π^0	1.94

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Sample results

Theory versus data

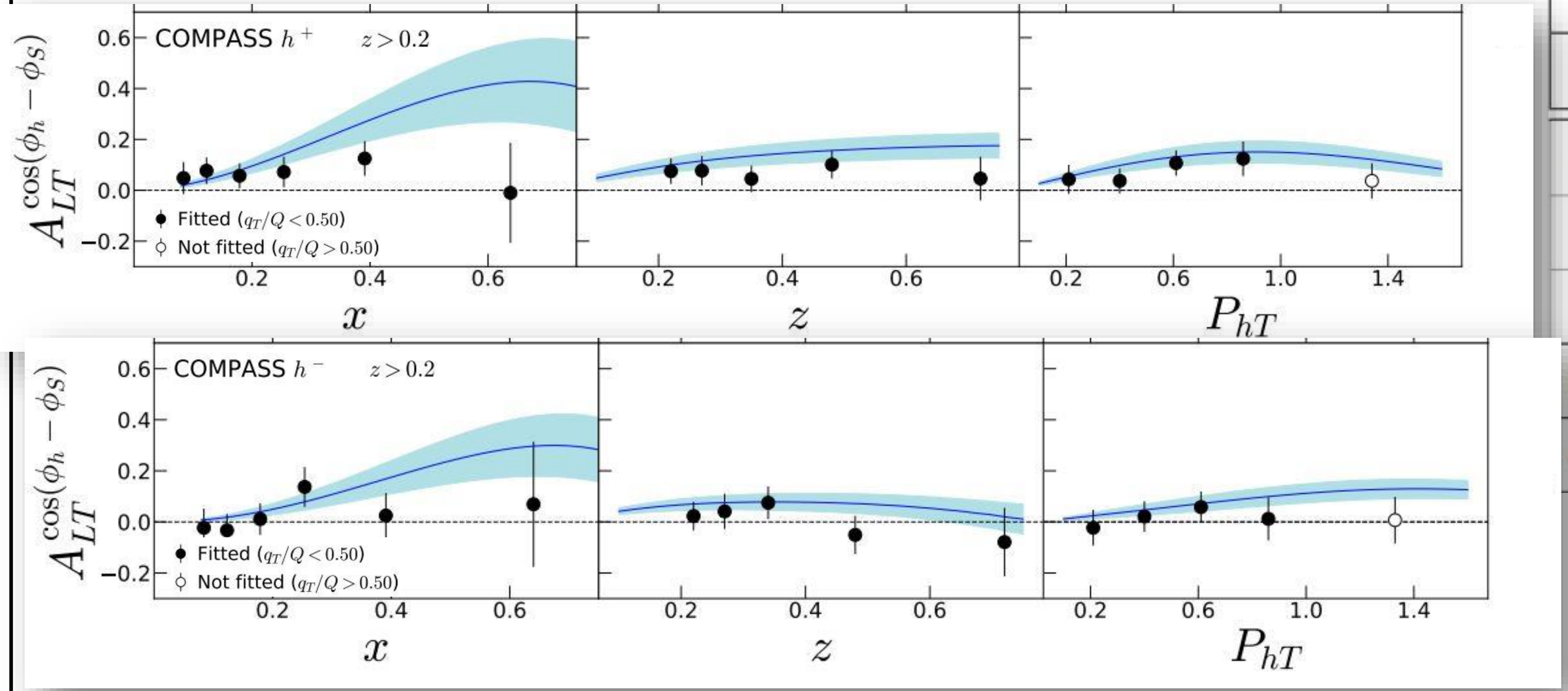


Data set	$\chi^2_w/N_{pts.}\|_{Main}$
HERMES π^+	1.20
HERMES π^-	0.88
HERMES π^0	1.94
COMPASS h^+	0.97

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Sample results

Theory versus data



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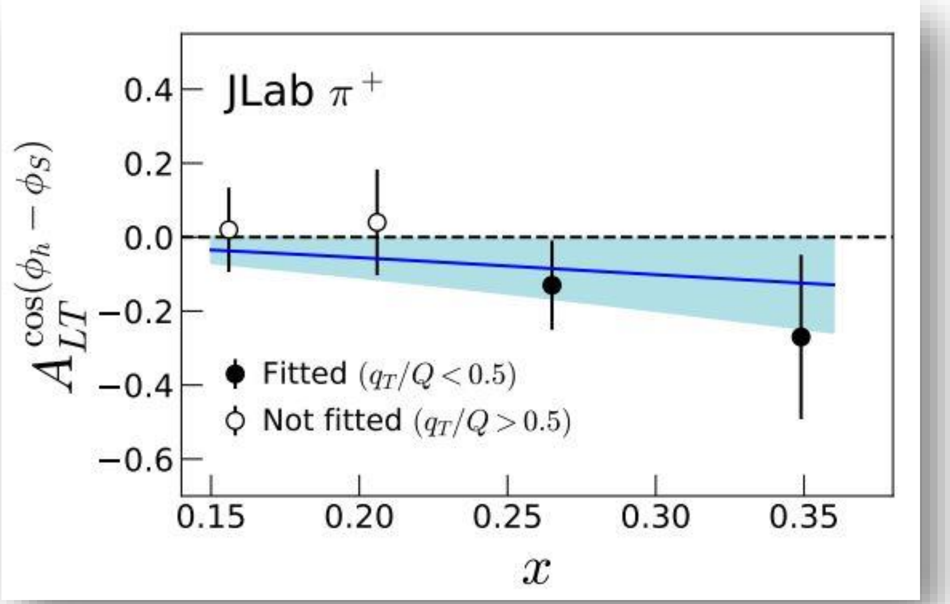
COMPASS h^+	0.97
COMPASS h^-	0.71

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data



Data set	$\chi^2_w/N_{pts.}\|_{Main}$
HERMES π^+	1.20
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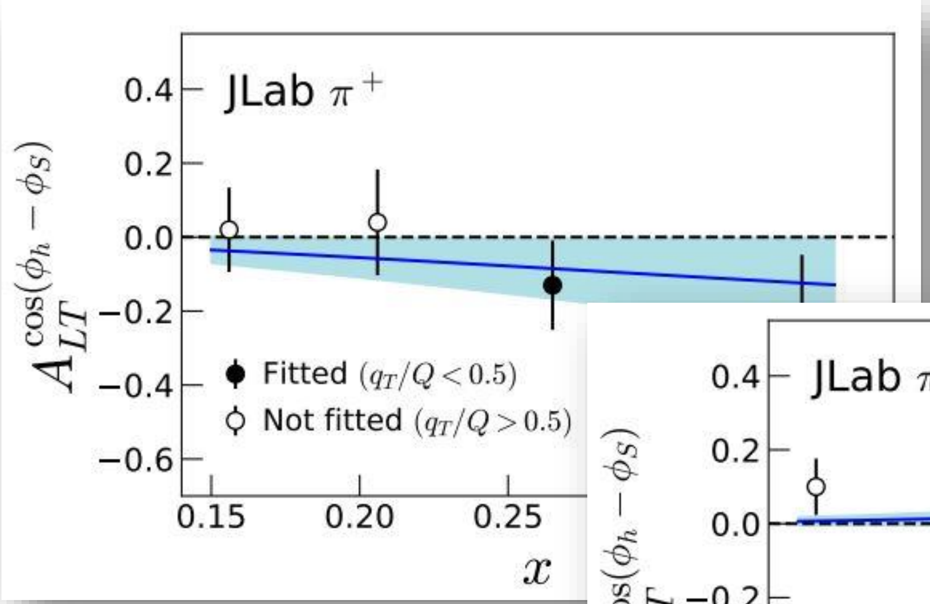
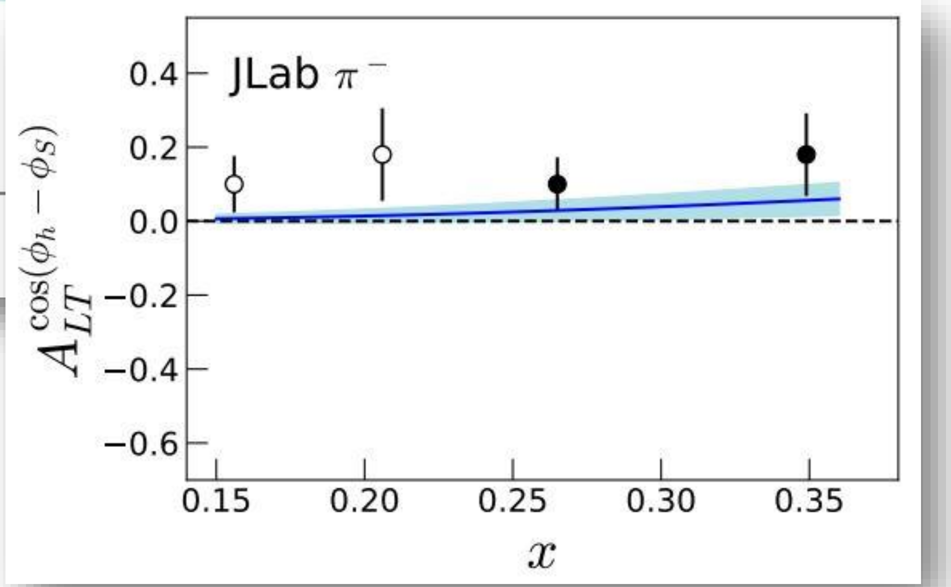
JLab π^+	0.31

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Theory versus data





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HERMES π^-	0.88
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COMPASS h^+	0.97
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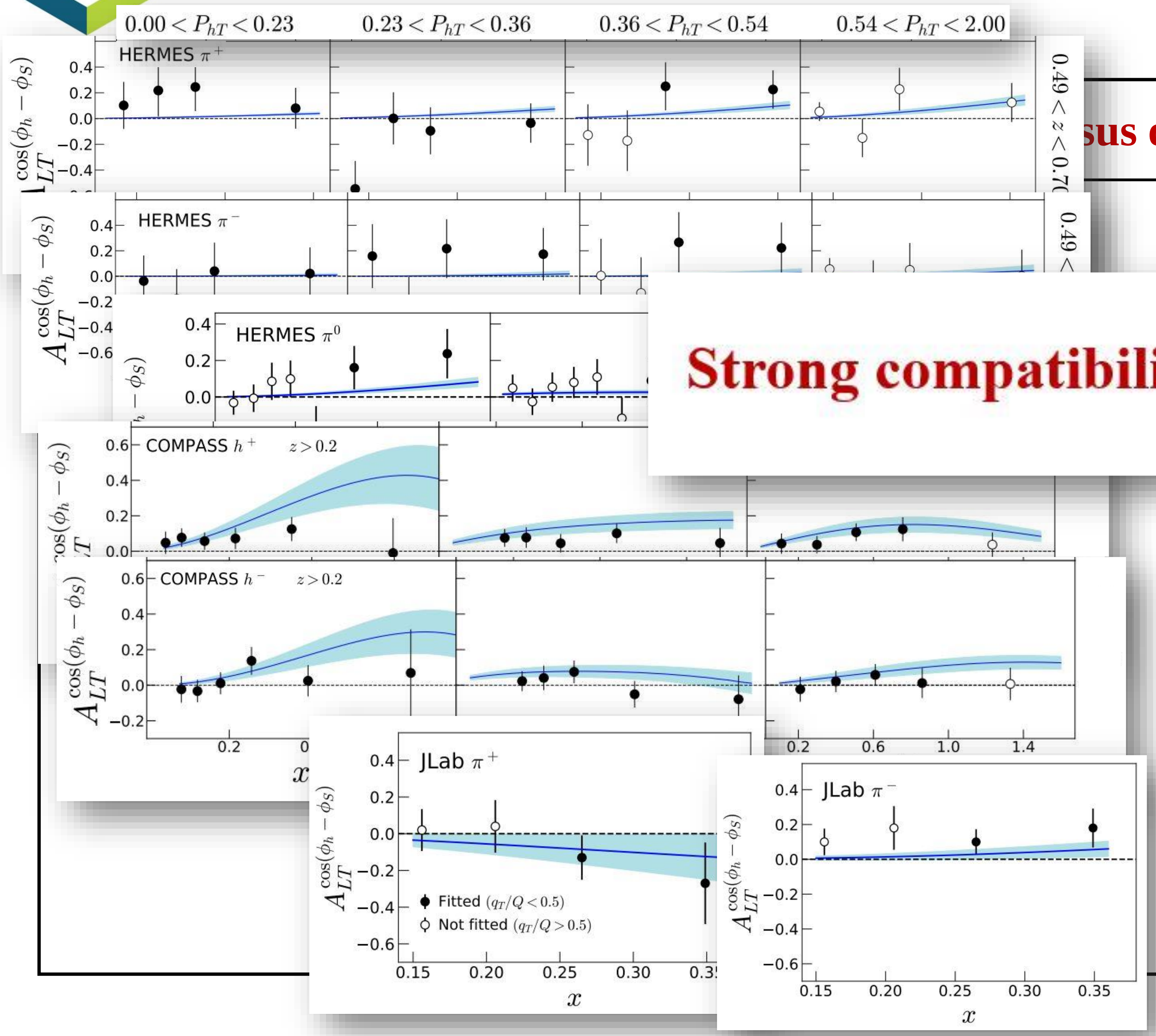
JLab π^-	1.13

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

…sus data

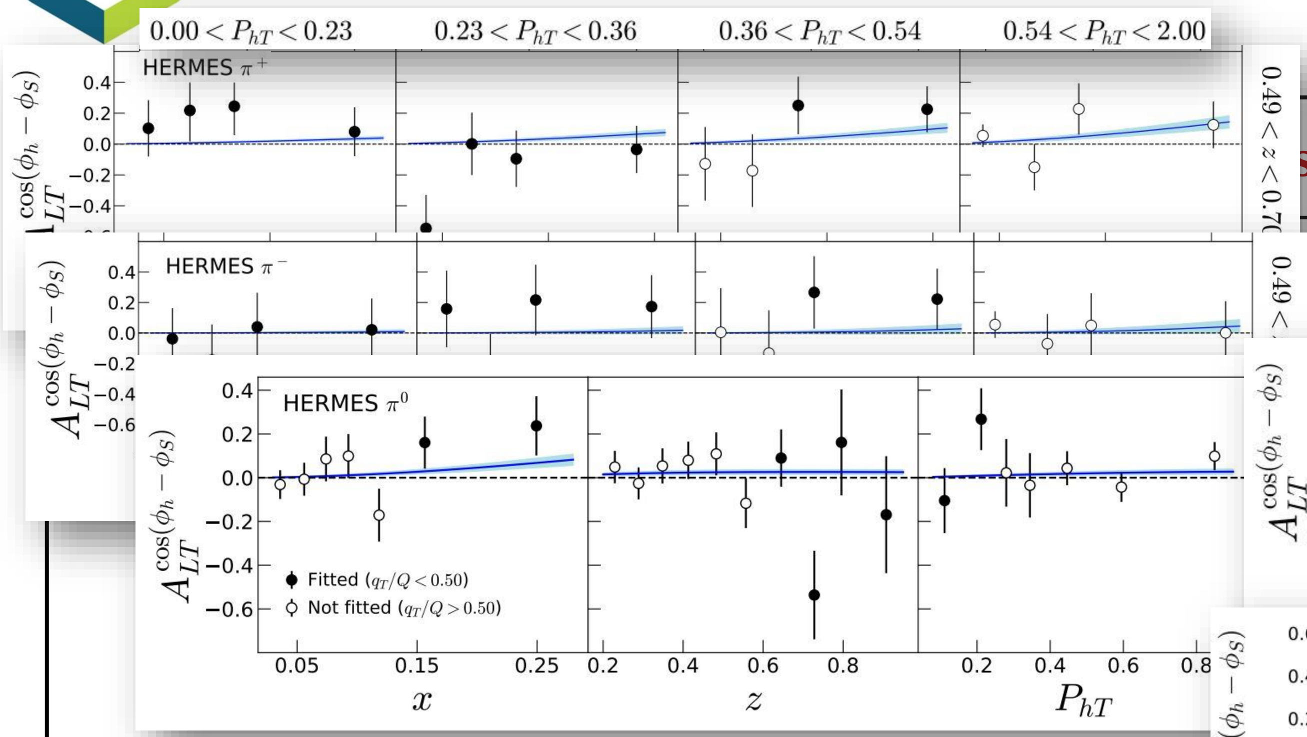


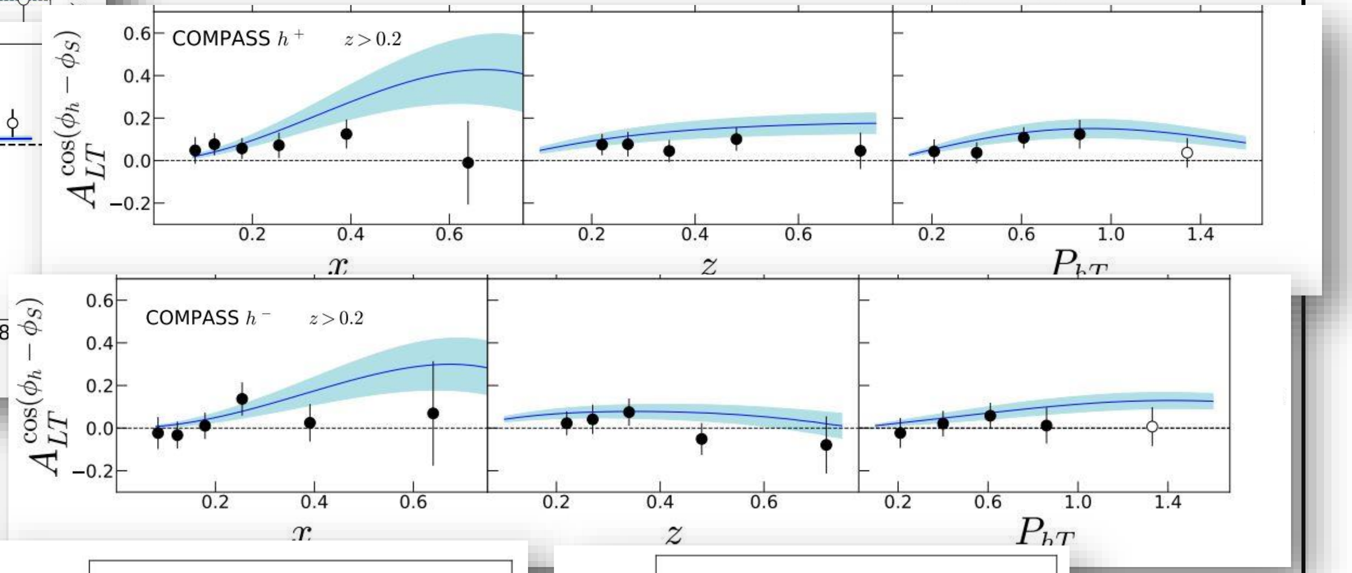
Strong compatibility between our theory and data

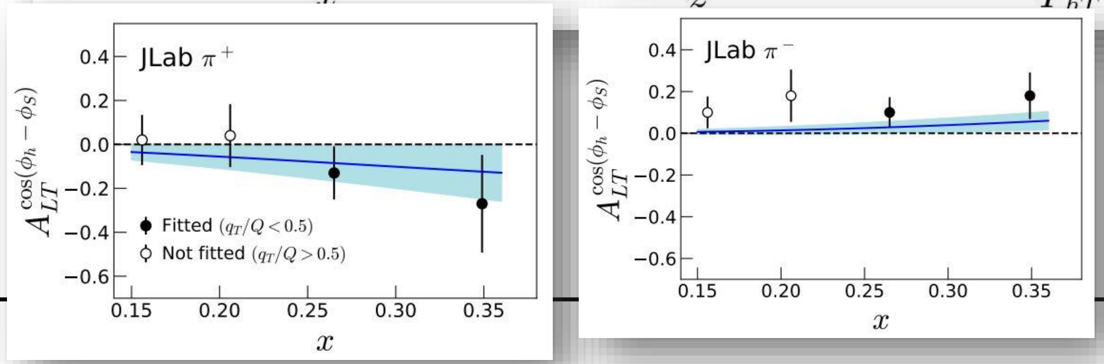
HERMES π^-	0.88
HERMES π^0	1.94
COMPASS h^+	0.97
COMPASS h^-	0.71
JLab π^+	0.31
JLab π^-	1.13
Global	**0.86**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

sus data

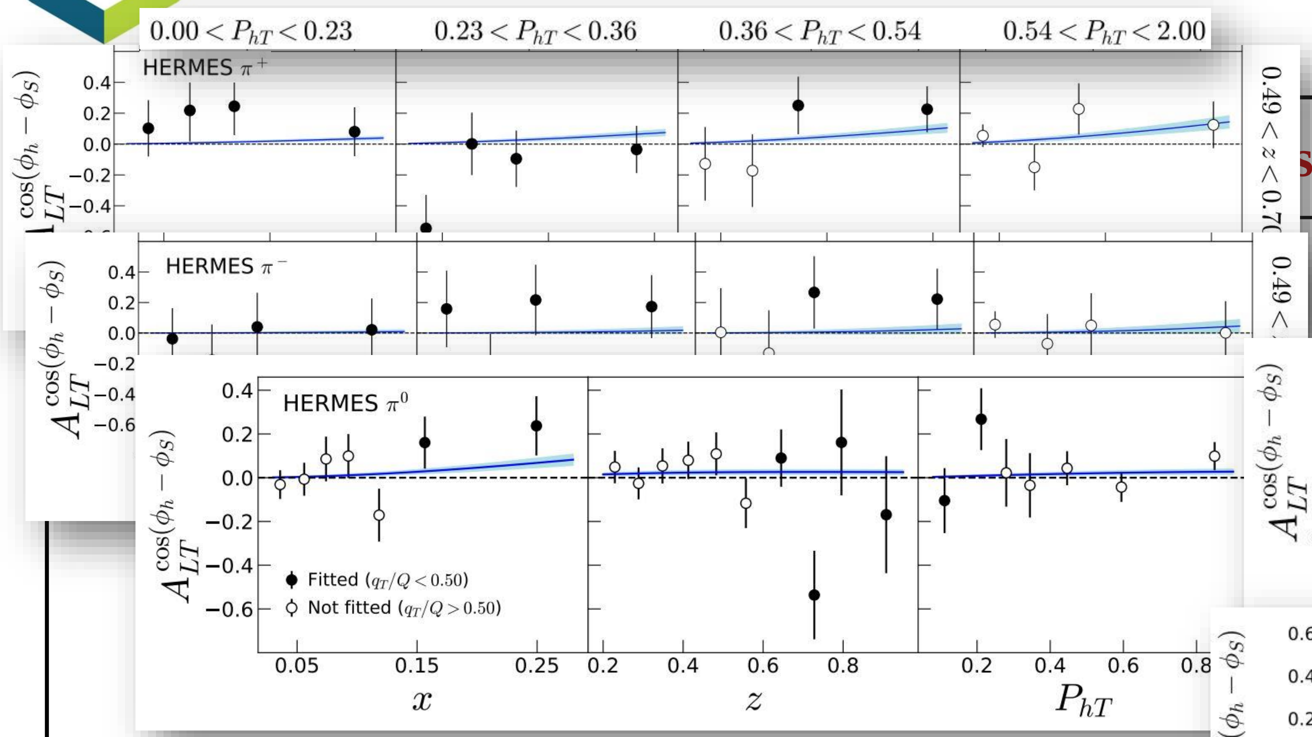


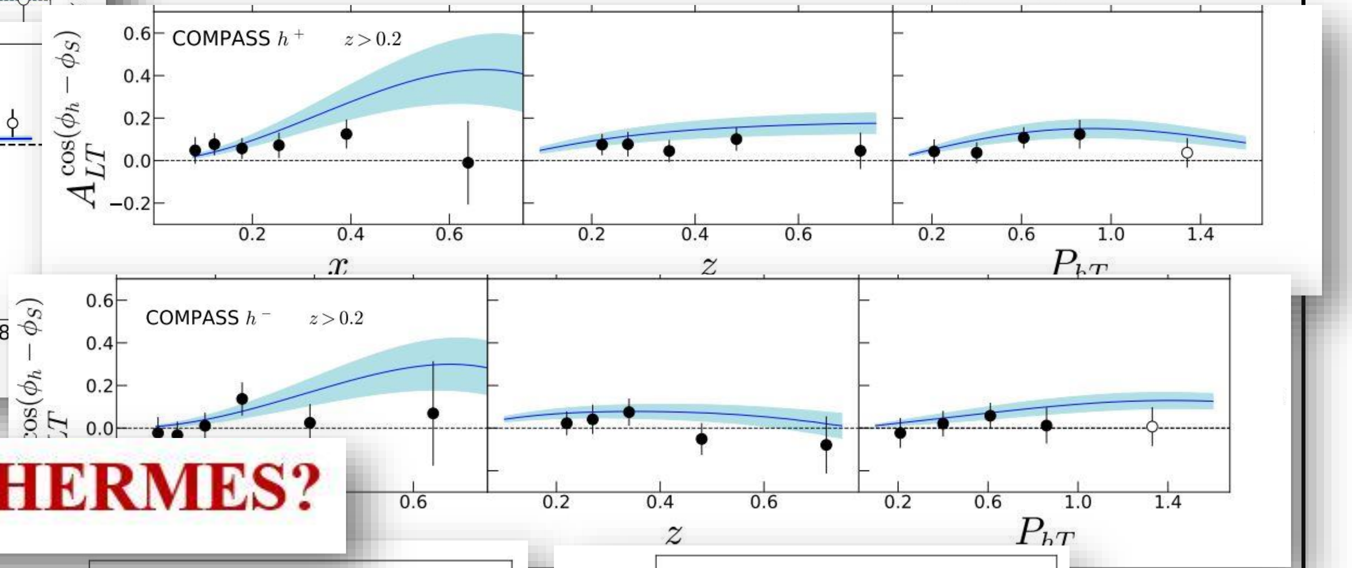




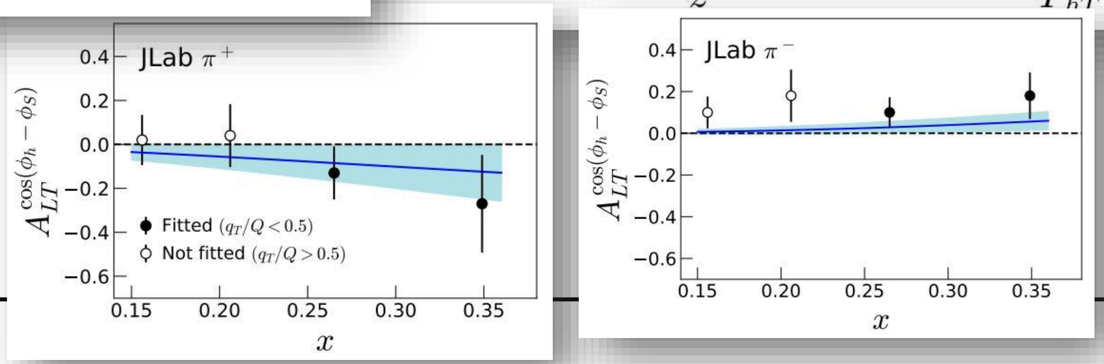
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

sus data





Smallness of the uncertainty bands for HERMES?

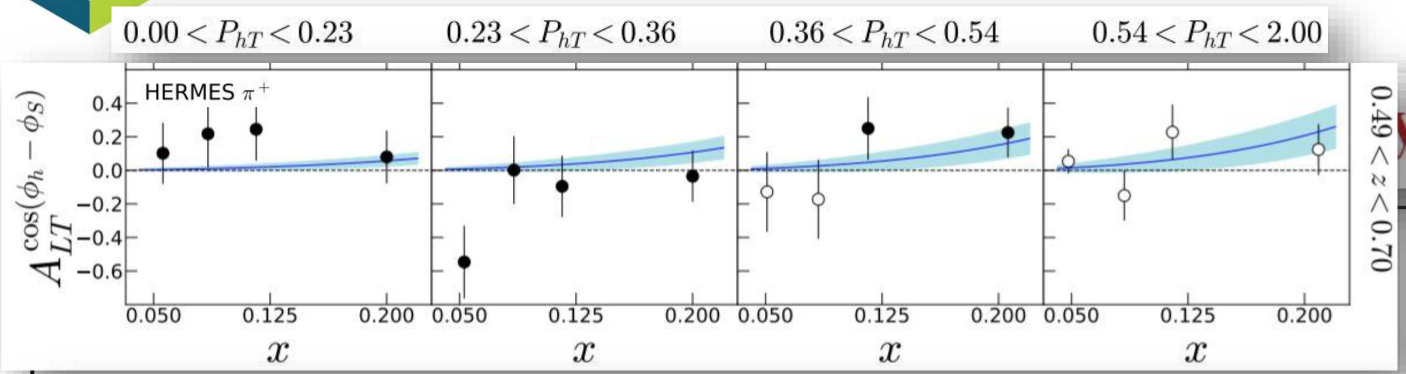


Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

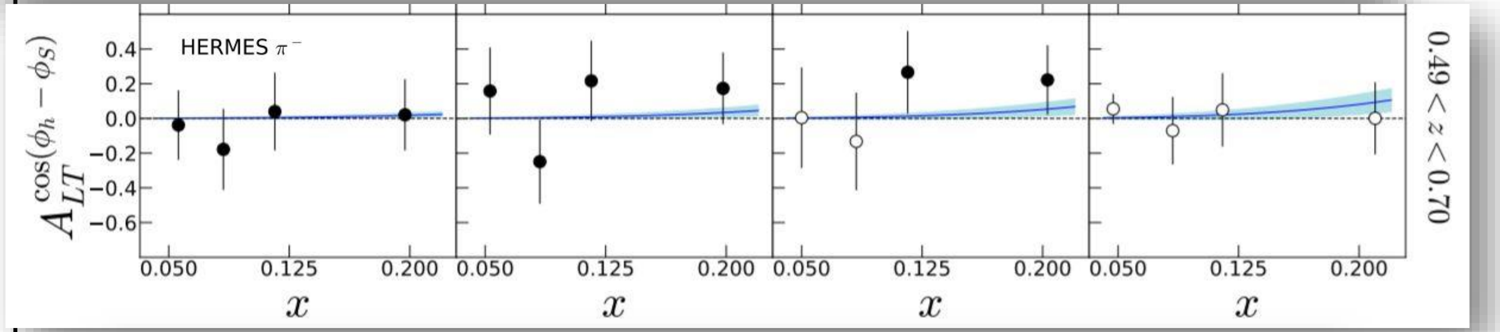
Smallness of the uncertainty bands for HERMES?

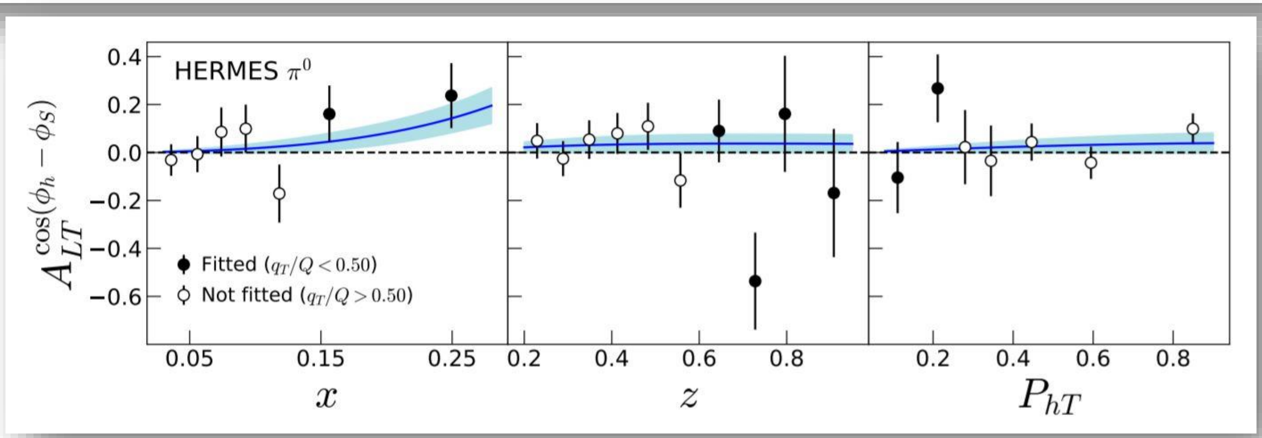
- **Performed a fit of HERMES + JLab**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

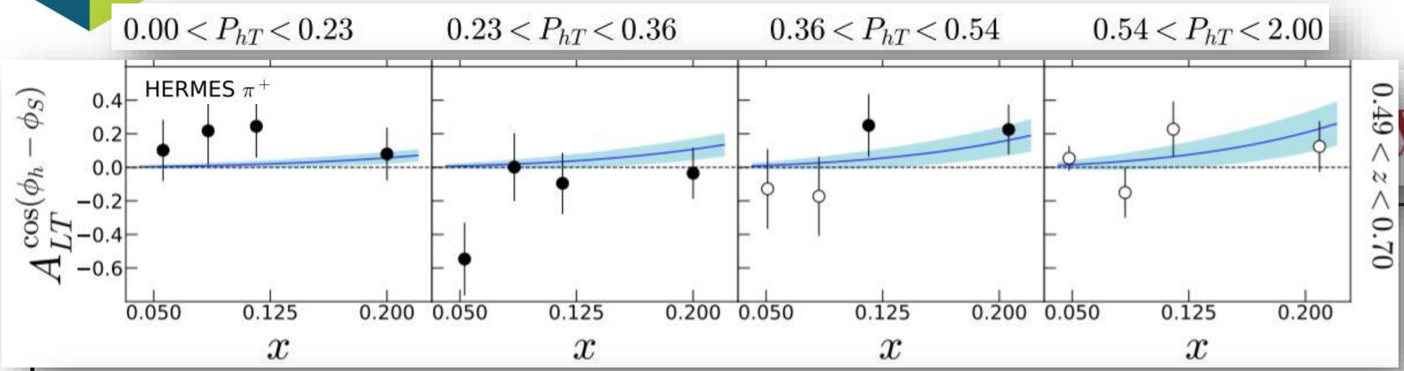


y bands for HERMES?





Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data



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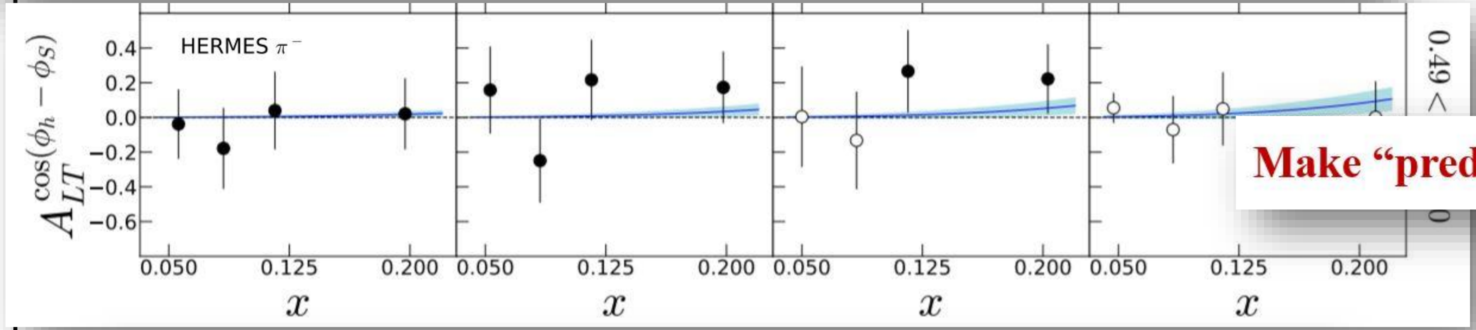
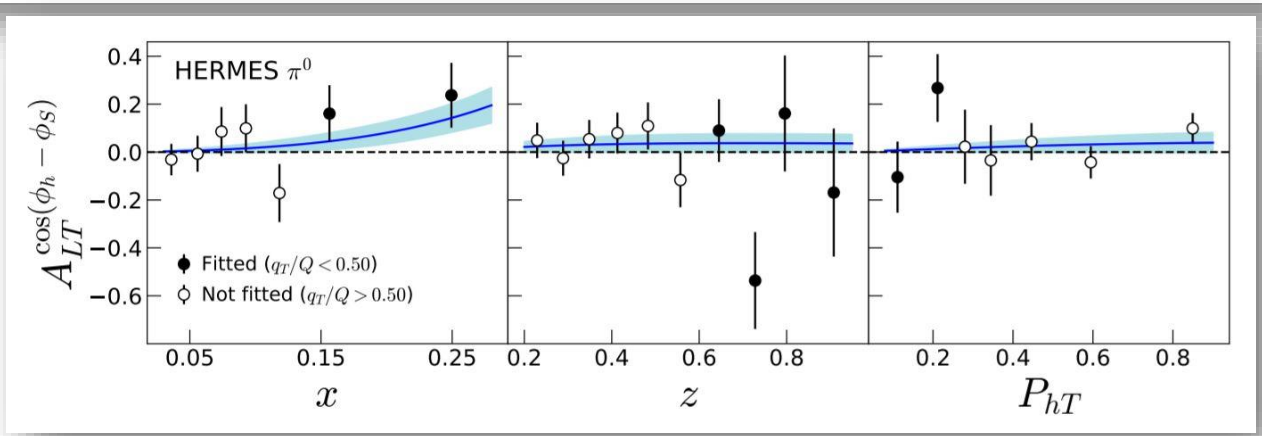
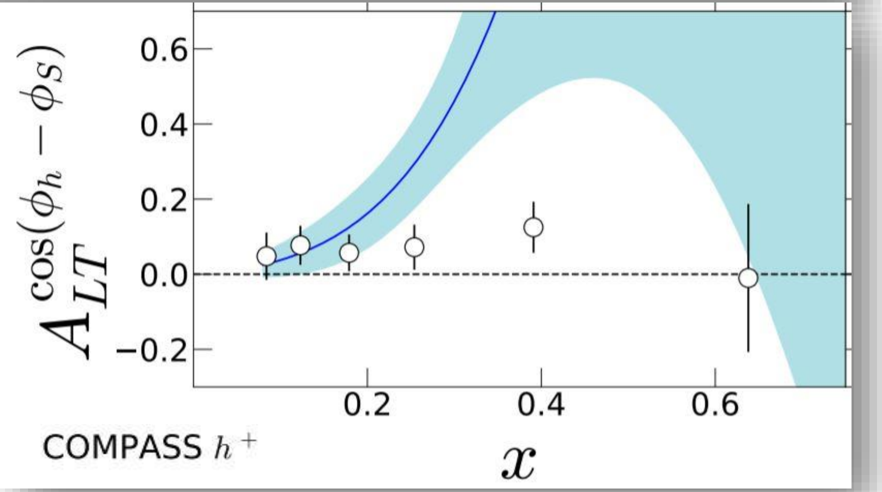
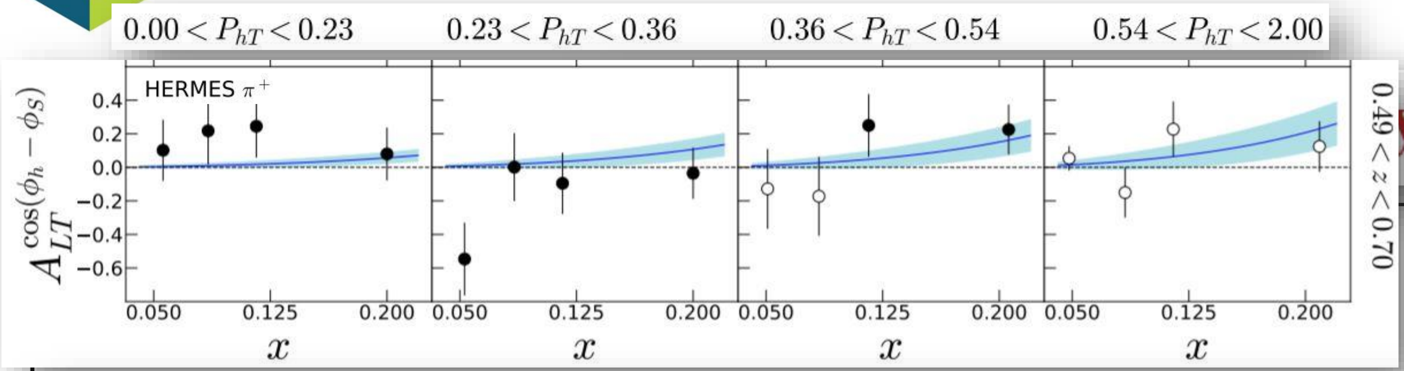


Make "predictions" for COMPASS using HERMES + JLab

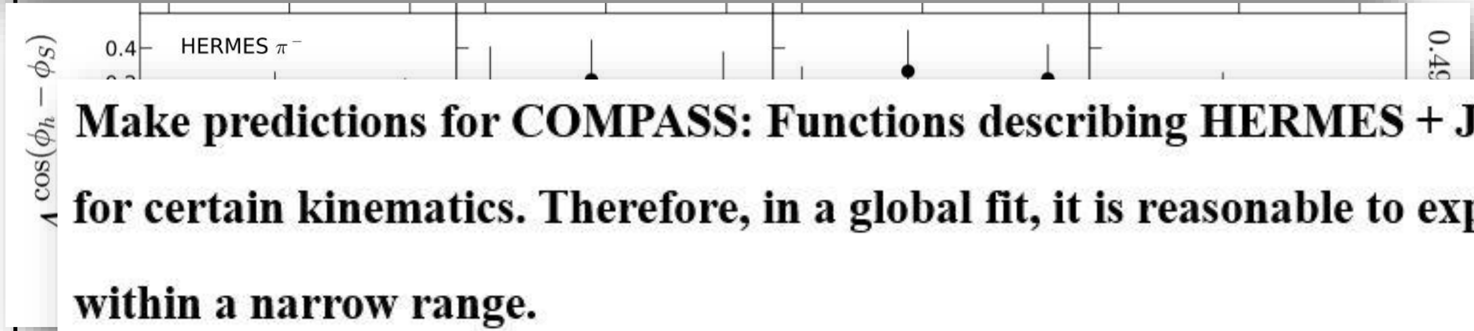




Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

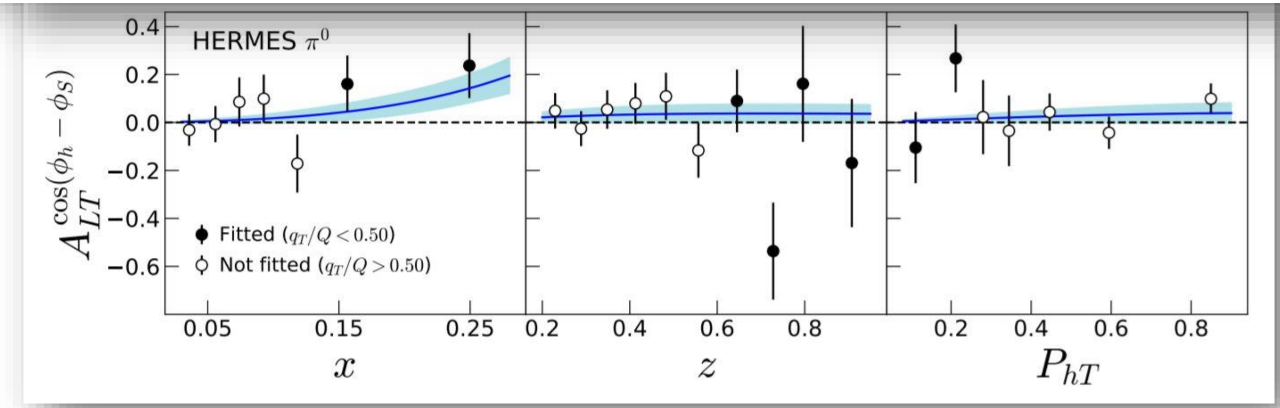


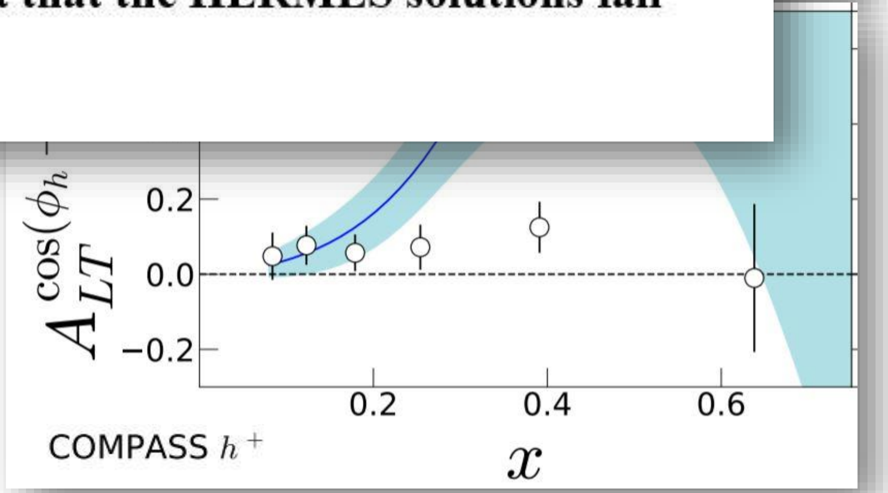
...y bands for HERMES?



Make predictions for COMPASS: Functions describing HERMES + JLab are too large to describe COMPASS for certain kinematics. Therefore, in a global fit, it is reasonable to expect that the HERMES solutions fall within a narrow range.

...Lab





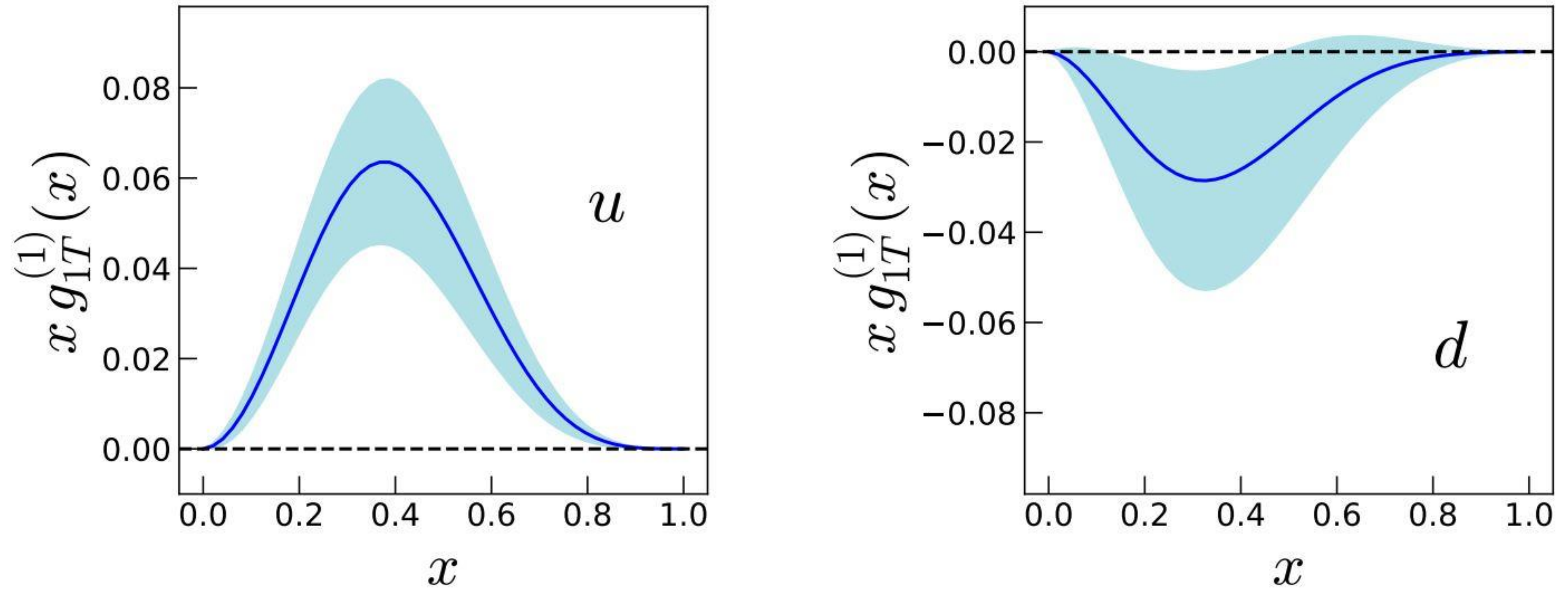
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Results for the x-dependence

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Average value of all replicas at a given *x* + 1-sigma error band

Results for the x-dependence

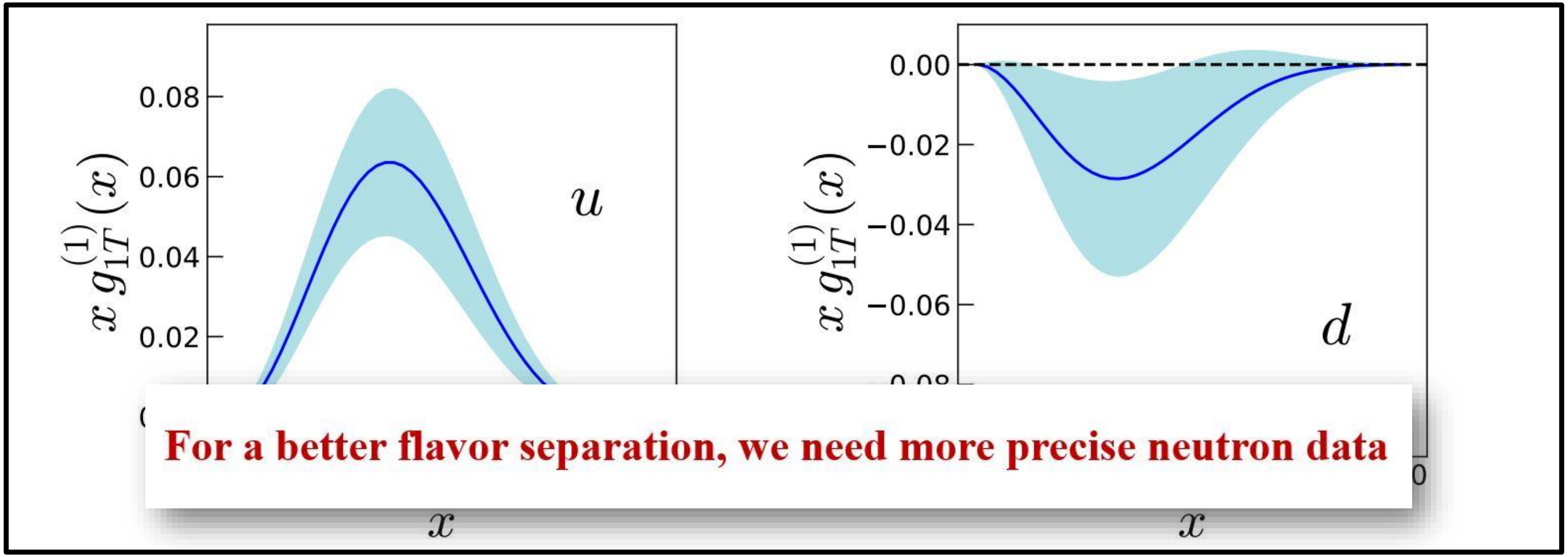


- **Up quark distribution is positive**
- **Down quark distribution is mostly negative**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Average value of all replicas at a given x + 1-sigma error band

Results for the x-dependence



For a better flavor separation, we need more precise neutron data

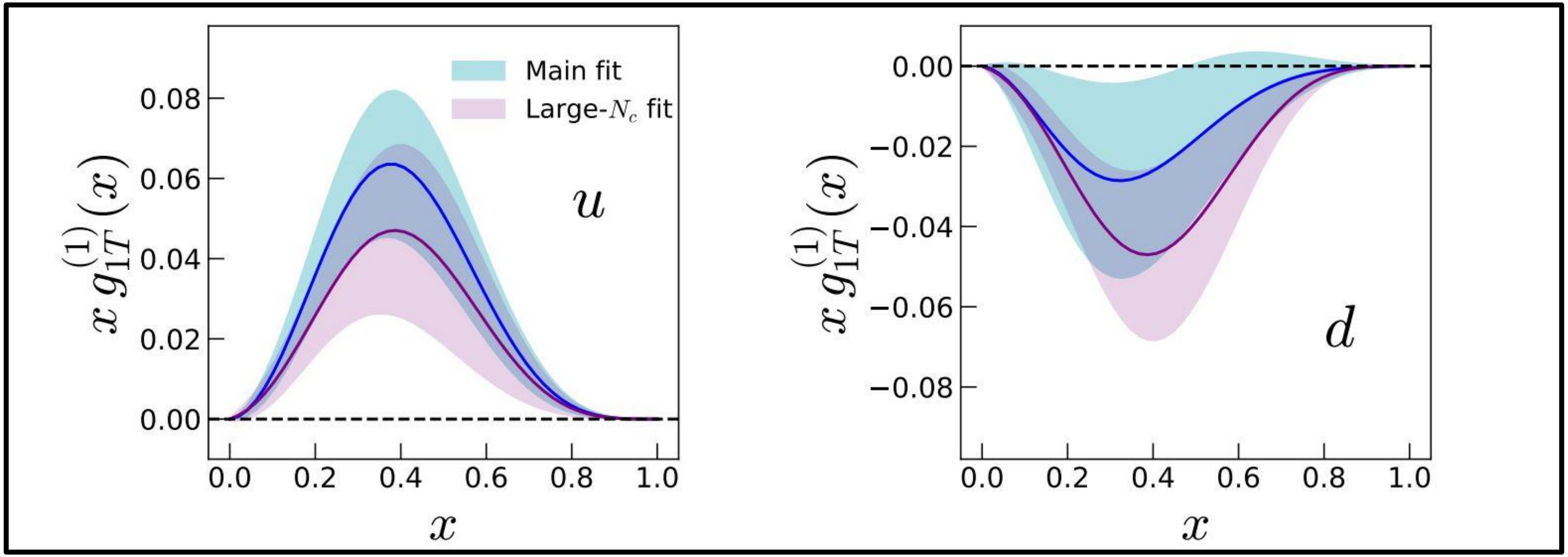
- **Up quark distribution is positive**
- **Down quark distribution is mostly negative**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions

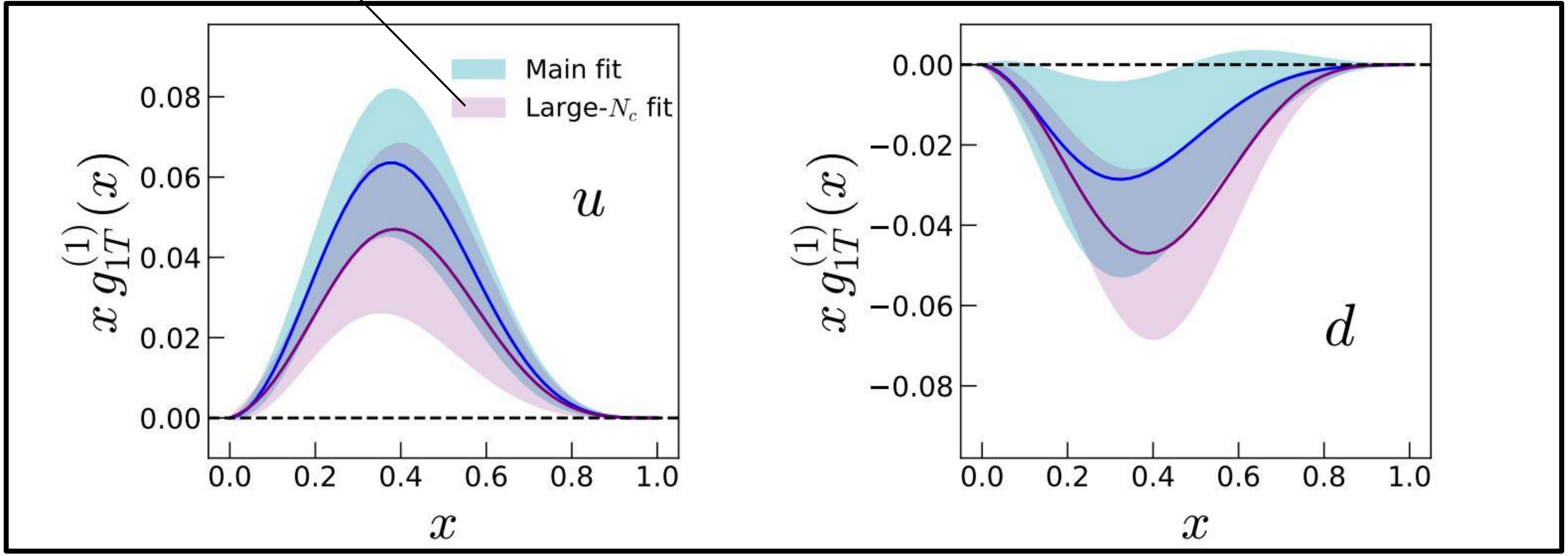


- **Qualitative agreement with large-N_c fit**
- **Slight preference to violate large-N_c approx.**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Set: $g_{1T}^{u}(x,\vec{k}_{\perp}^{2}) = -g_{1T}^{d}(x,\vec{k}_{\perp}^{2})$

Test of theoretical predictions



- **Qualitative agreement with large-N_c fit**
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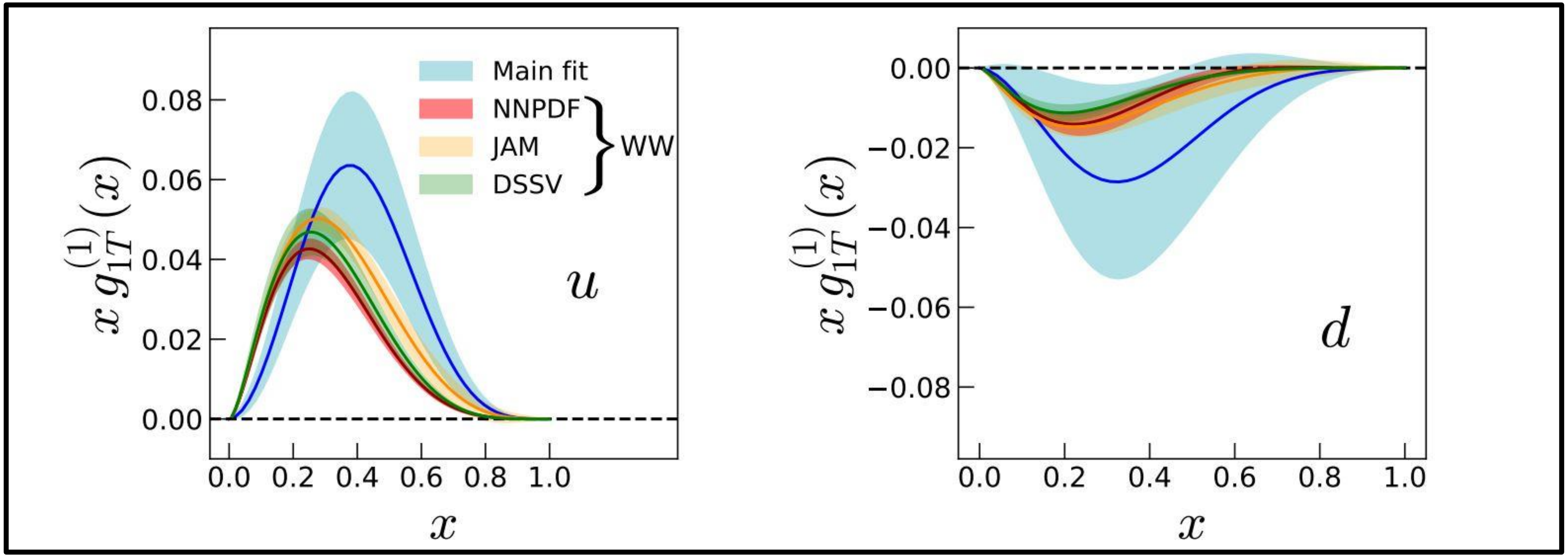
Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Test of theoretical predictions

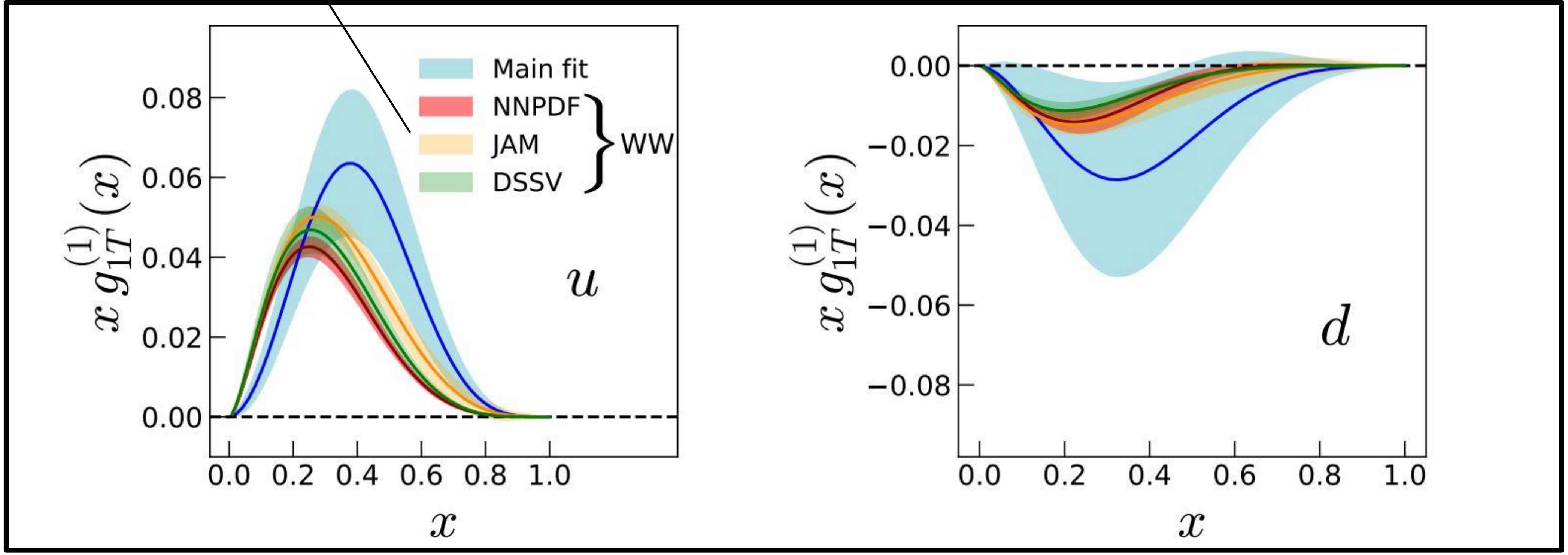


- **Qualitative agreement with WW-type approx.**
- **Hints of slight violation of WW-type approx.**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Set: $g_{1T}^{(1),q}(x) = x \int_x^1 dy\, \frac{g_1^q(y)}{y}$

Test of theoretical predictions



- **Qualitative agreement with WW-type approx.**
- **Hints of slight violation of WW-type approx.**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

Summary of $\chi^2_w/N_{\text{pts.}}$					
Data set	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Main}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{Large-}N_c}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{NNPDF}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{JAM}}$	$\chi^2_w/N_{\text{pts.}}\vert_{\text{DSSV}}$
HERMES π^+	1.20	1.23			
HERMES π^-	0.88	0.88			
HERMES π^0	1.94	2.01			
COMPASS h^+	0.97	0.51			
COMPASS h^-	0.71	0.53			
JLab π^+	0.31	0.06			
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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

cal predictions?

Similar or better chi-squared for some data sets

Summary of $\chi^2_w/N_{pts.}$					
Data set	$\chi^2_w/N_{pts.}\|_{Main}$	$\chi^2_w/N_{pts.}\|_{Large\text{-}N_c}$	$\chi^2_w/N_{pts.}\|_{NNPDF}$	$\chi^2_w/N_{pts.}\|_{JAM}$	$\chi^2_w/N_{pts.}\|_{DSSV}$
HERMES π^+	1.20	1.23			
HERMES π^-	0.88	0.88			
HERMES π^0	1.94	2.01			
COMPASS h^+	0.97	0.51			
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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Violation of existing theoretical predictions?

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HERMES π^+	1.20		1.19	1.19	1.19
HERMES π^-	0.88		0.85	0.85	0.85
HERMES π^0	1.94		1.98	1.95	1.96
COMPASS h^+	0.97		0.71	1.02	0.89
COMPASS h^-	0.71		0.71	0.81	0.80
JLab π^+	0.31		0.81	0.78	0.96
JLab π^-	1.13		1.15	0.93	0.93

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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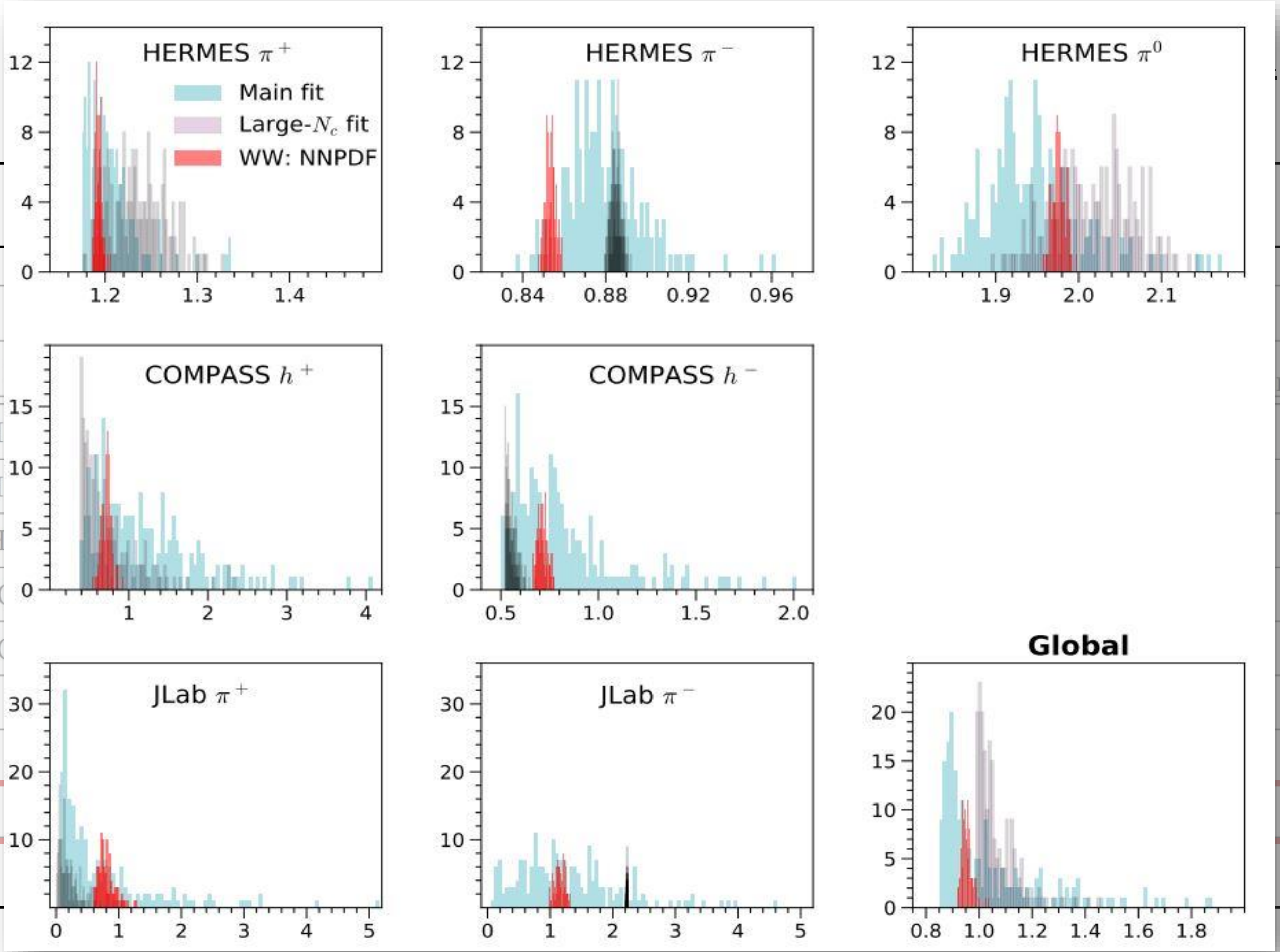
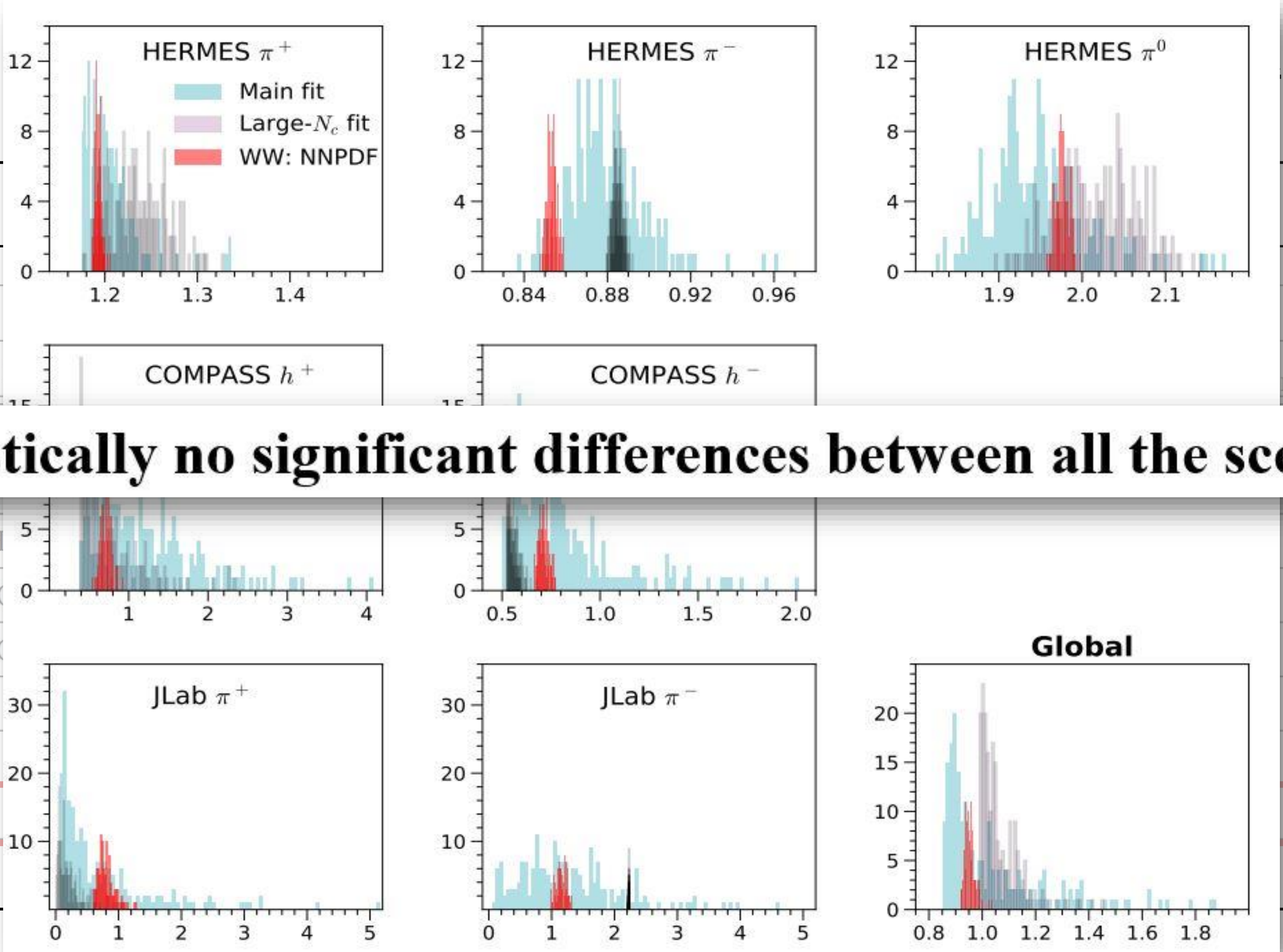
COMPASS h^-	[illegible]	[illegible]	[illegible]	[illegible]	0.80
JLab π^+	0.31	0.06	0.81	0.78	0.96
JLab π^-	1.13	2.23	1.15	0.93	0.93
Global	**0.86**	**0.99**	**0.95**	**0.94**	**0.97**

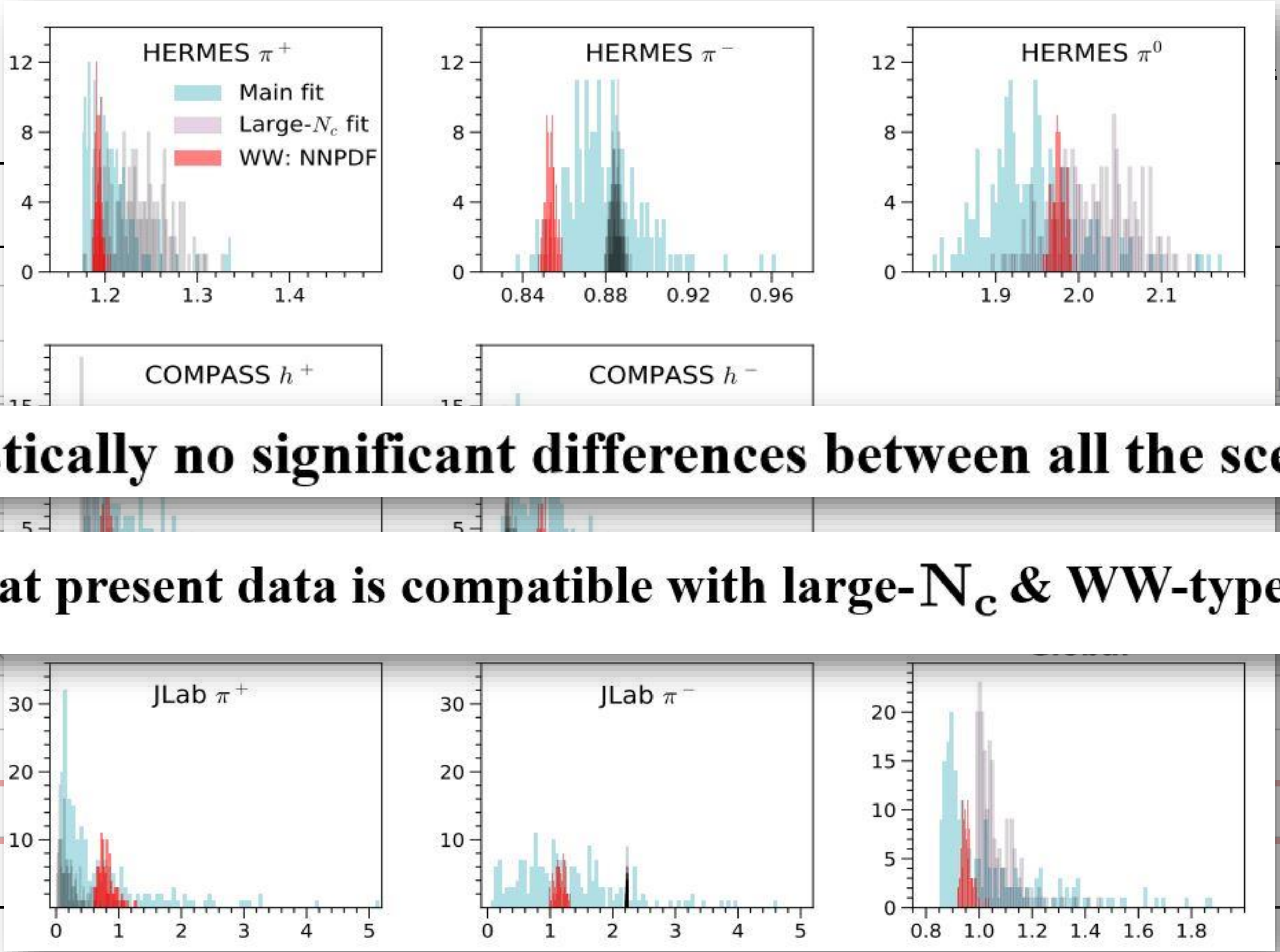
Our global chi-squared is consistently better



HERMES π+
Main fit
Large-Nc fit
WW: NNPDF
HERMES π−
HERMES π0
COMPASS h+
COMPASS h−
JLab π+
JLab π−
Global



Statistically no significant differences between all the scenarios



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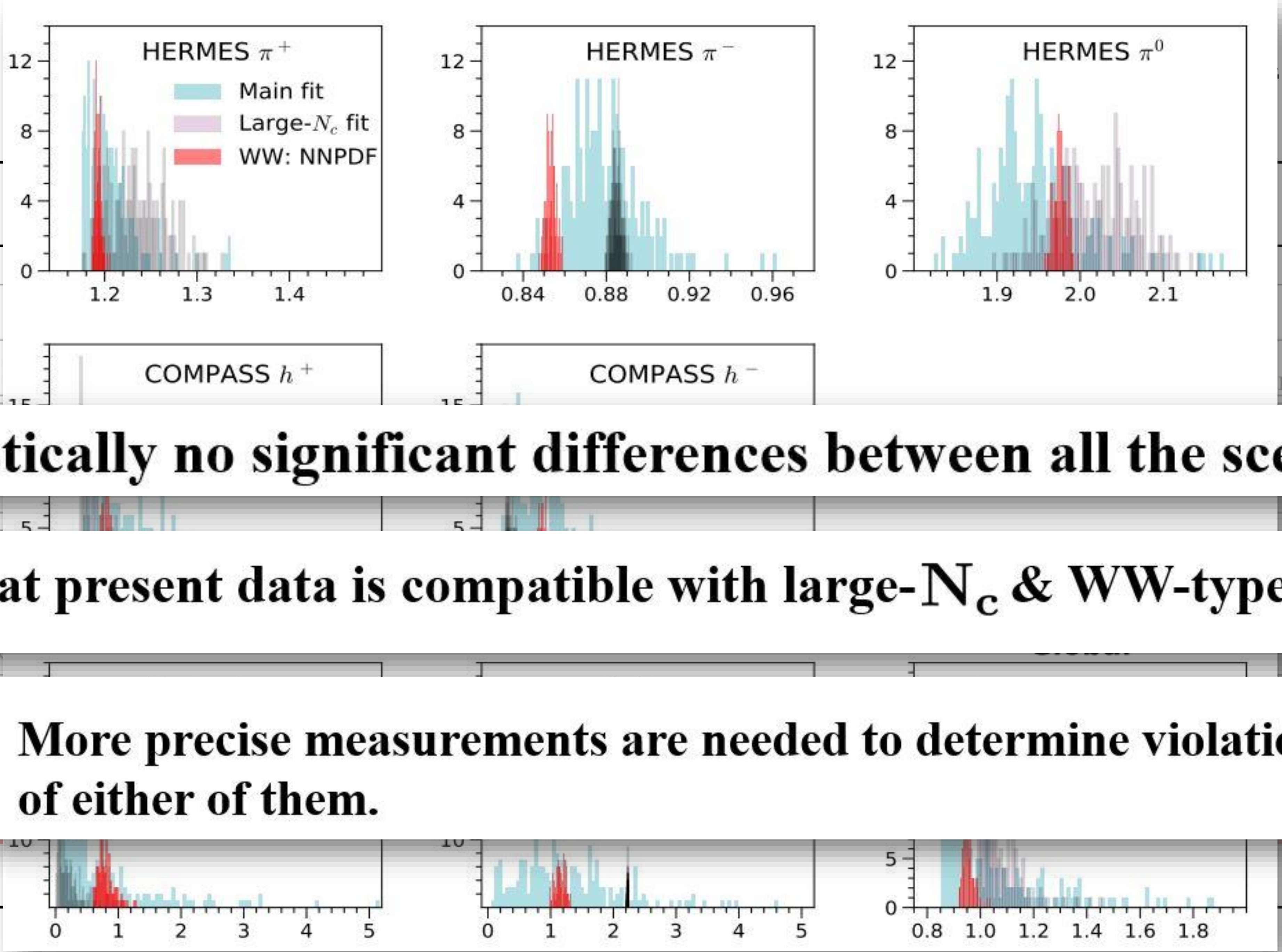
Hence, at present data is compatible with large-N_c & WW-type approx.



Statistically no significant differences between all the scenarios

Hence, at present data is compatible with large-N_c & WW-type approx.

More precise measurements are needed to determine violation of either of them.

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Why "weighted" technique after-all?

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Why "weighted" technique after-all?

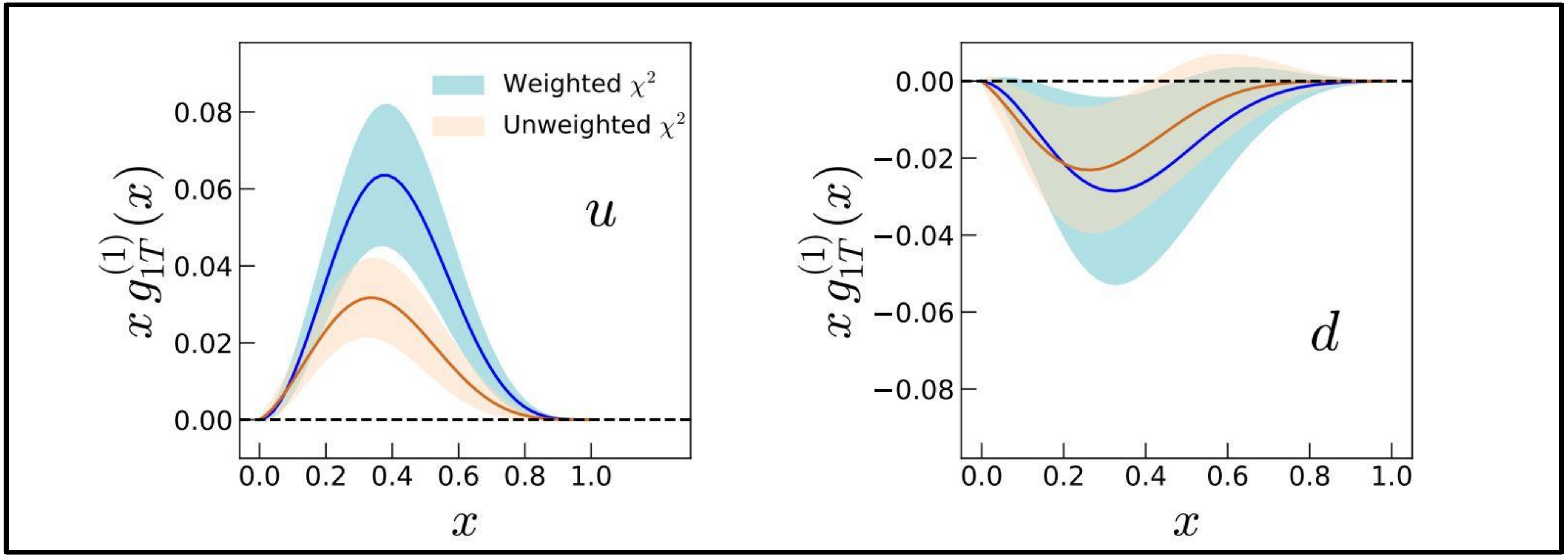
$$\chi^2 = \sum_{\text{H+C+J}} \frac{(\text{exp. data} - \text{theory})^2}{(\text{exp. error})^2}$$

Without weighting the chi-squared, we overfit COMPASS data, and we don't fit JLab π^- data particularly well.

HERMES π^-	0.89	0.88	0.85	0.85	0.85
HERMES π^0	2.03	2.03	1.98	1.95	1.96
COMPASS h^+	0.39	0.40	0.71	1.02	0.89
COMPASS h^-	0.54	0.53	0.71	0.81	0.80
JLab π^+	0.42	0.15	0.81	0.78	0.96
JLab π^-	1.88	2.23	1.15	0.93	0.93
Global	**0.83**	**0.83**	**0.93**	**1.02**	**0.99**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

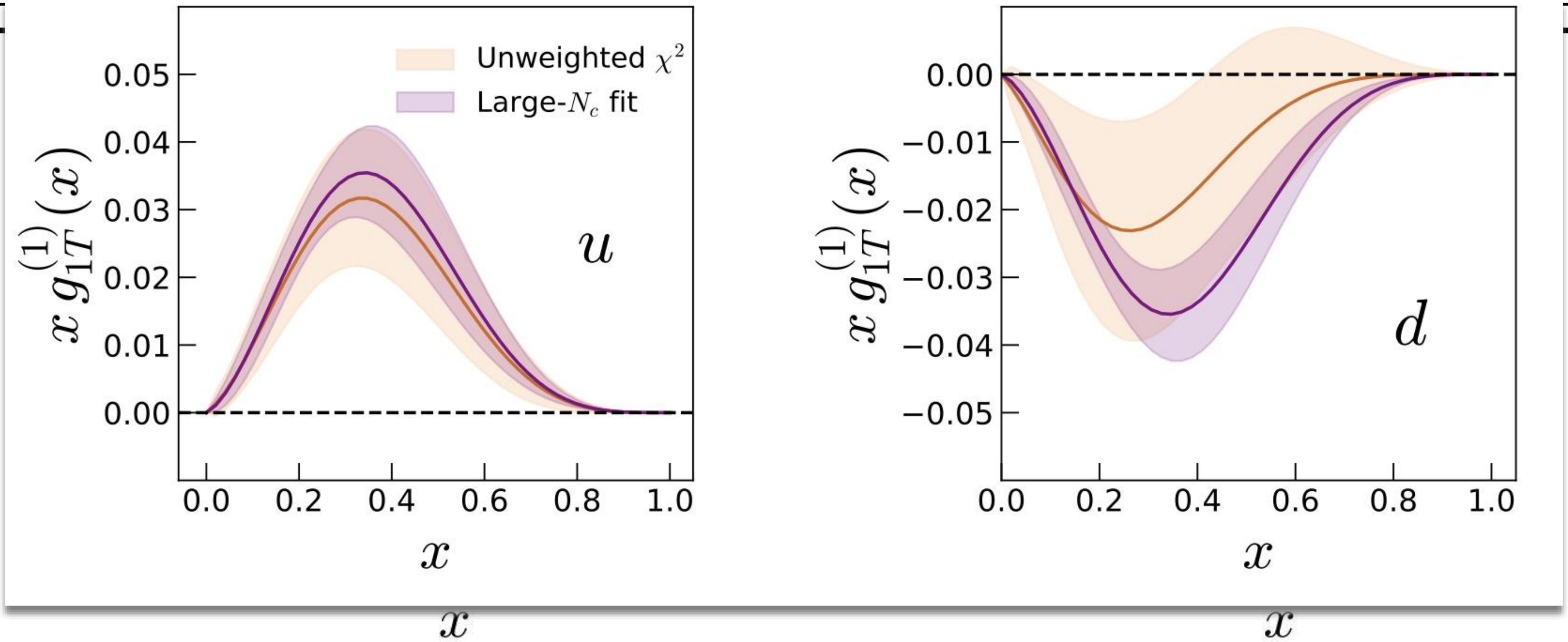
Weighted versus unweighted methods



- **Larger up quark distribution needed to describe JLab π^- data**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Weighted versus unweighted methods



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Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

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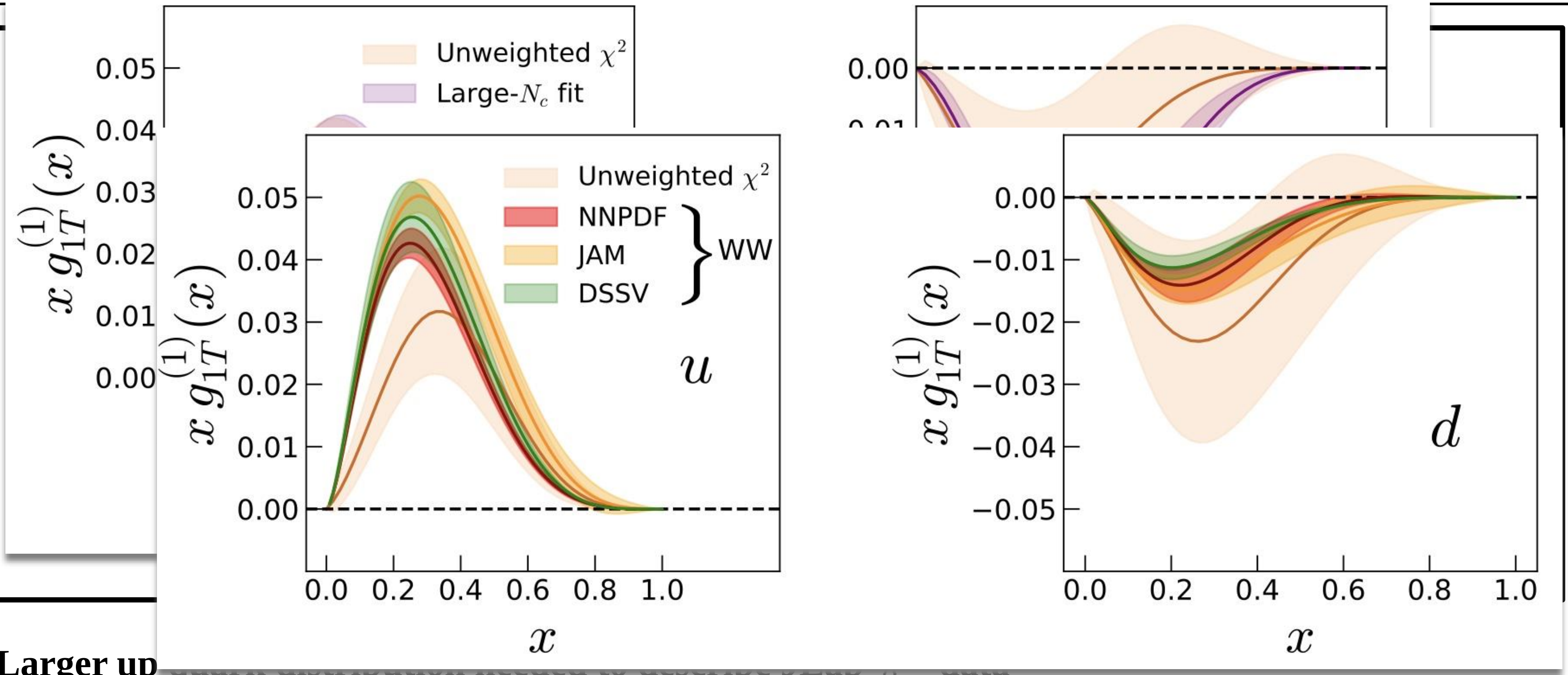


- Larger up

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

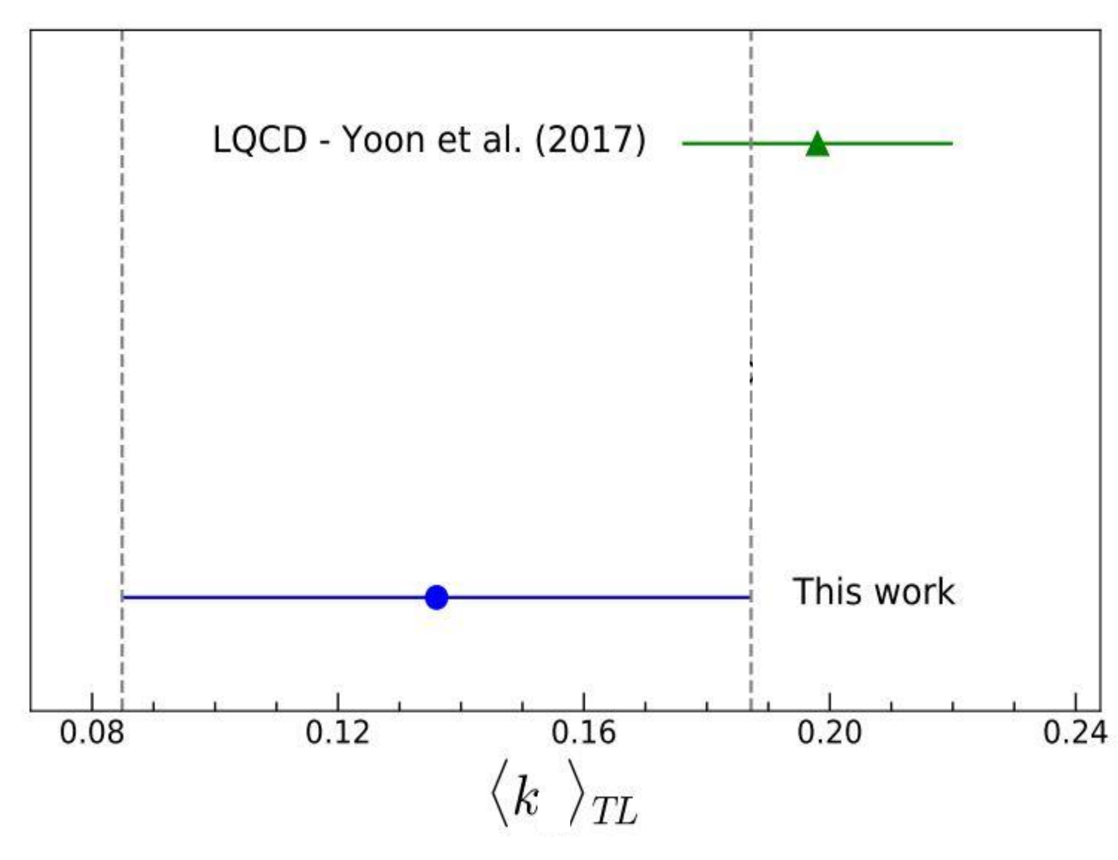
Comparison with lattice QCD results

Calculation of worm-gear shift:

$$[\langle k \ \rangle_{TL}](Q^2) \equiv M \frac{\int_0^1 dx \left[g_{1T}^{(1)u}(x,Q^2) - g_{1T}^{(1)d}(x,Q^2)\right]}{\int_0^1 dx \left[f_1^u(x,Q^2) - f_1^d(x,Q^2)\right]}$$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results



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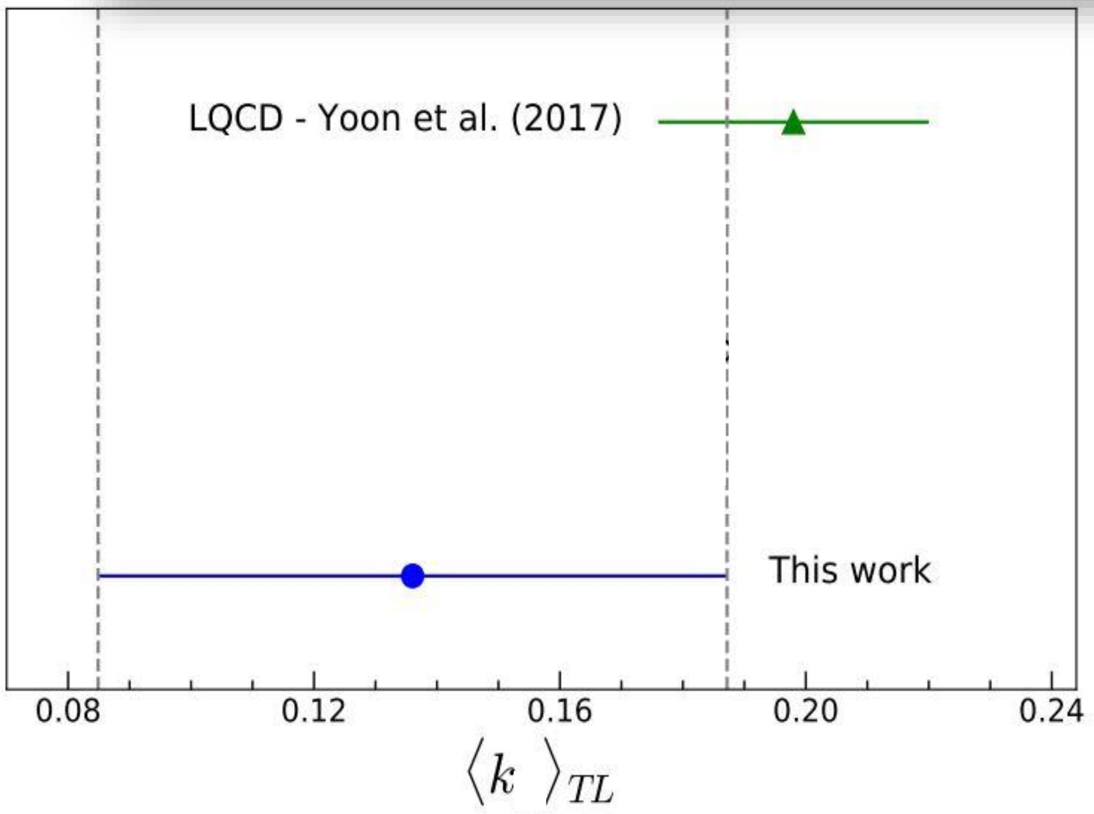
- **Consistency between lattice results & our main fit result**

Caveats in LQCD calculations:

- **No definite scale:** $Q \approx \frac{1}{a}$
- **Limits** $\hat{\zeta} \to \infty$ **&** $b_T \to 0$ **cannot be taken**

MES, COMPASS & JLab data

ce QCD results



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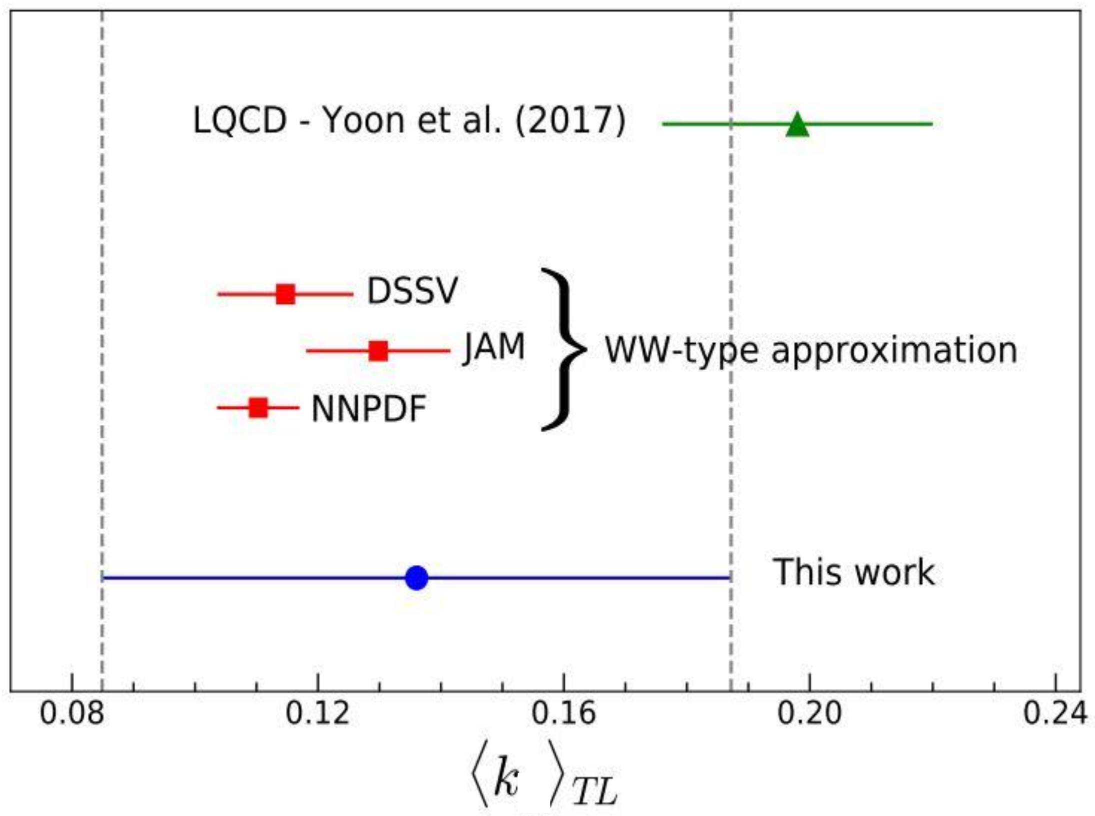
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Comparison with lattice QCD results



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- **Consistency between lattice results & our main fit result**
- **It is encouraging that lattice QCD & exp. data are in reasonable agreement**
- **Consistency between results from WW-type approx. & our main fit result**

Summary/Outlook

Summary

- We have performed the first extraction of g_{1T} from experimental data, obtaining a very good fit simultaneously of HERMES, COMPASS, JLab data on SIDIS
- Additional deuteron and/or neutron measurements are needed for a cleaner flavor separation
- Qualitative agreements with large-$\mathrm{N_c}$ & WW-type approximation
- Although there is an indication of a slight violation of both the theoretical predictions, the data is not precise enough to affirm the degree of violation (if any)
- Any clear signal of violation of WW-type approximation would be a probe of quark-gluon-quark correlations
- Encouraging agreement in the worm-gear shift with lattice QCD results

Summary/Outlook

Outlook

- **Impact of SoLID data on our current fit (In Collaboration with Vlad Khachatryan …)**

- **Extend analysis to extract** $h_{1L}^{\perp}$

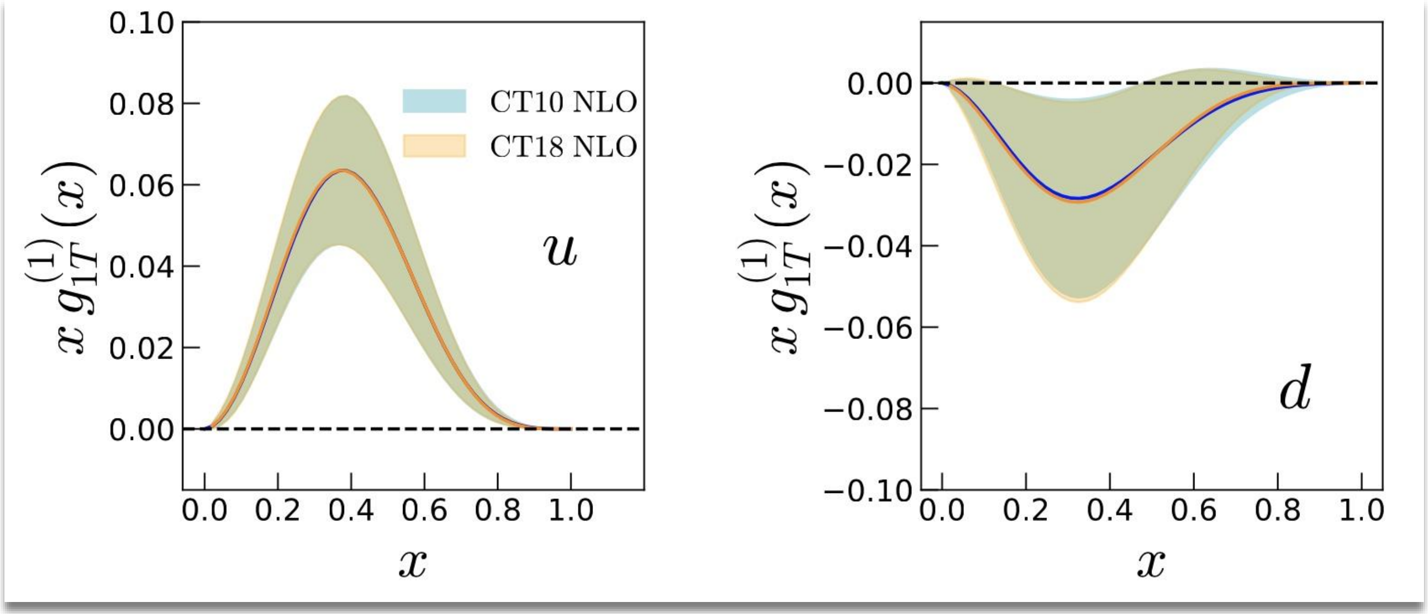
….

Backup slides

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpolarized PDF sets do not change our results**



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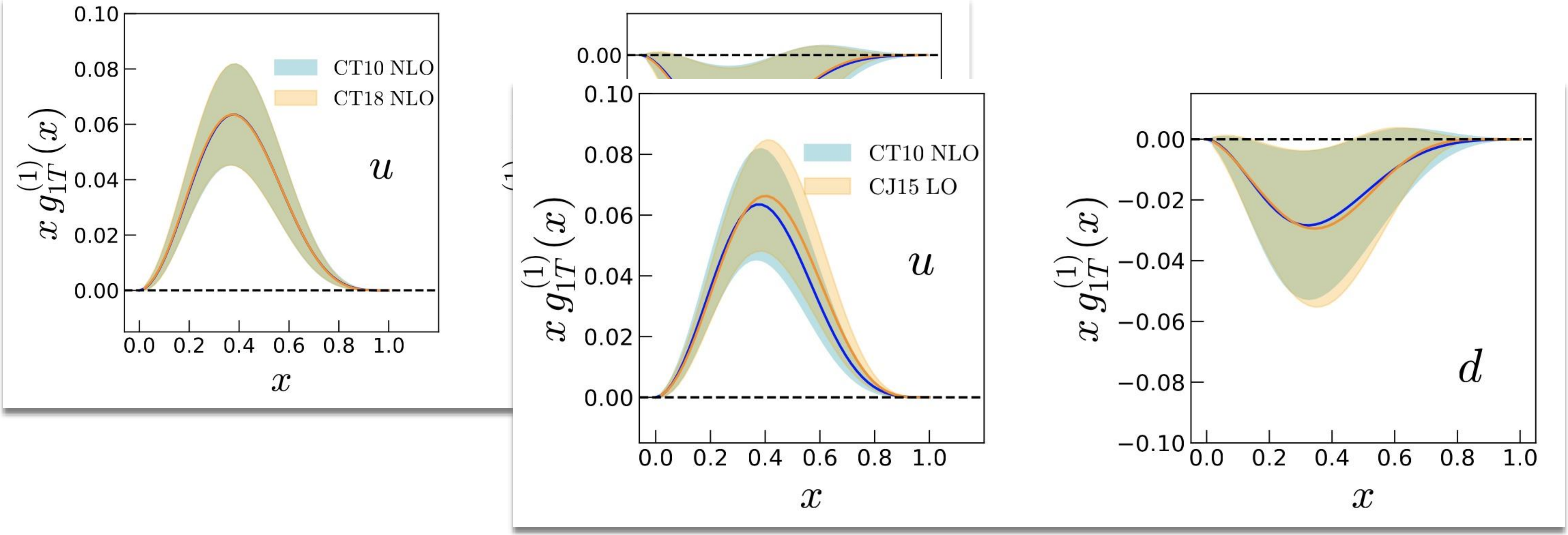


Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpolarized PDF sets do not change our results**

- **Parameterizing in terms of helicity PDF does not change our results**

  - **We were agnostic about any connection to helicity PDF through the WW-type relation**
  - **Try:**

$$g_{1T}^{(1)}(x,Q^2) = \frac{n}{\int_0^1 dy\, y^{\alpha+1}(1-y)^\beta f_1(y,Q_0^2)} x^\alpha (1-x)^\beta f_1(x,Q^2)$$

$f_1(x,Q^2) \rightarrow g_1(x,Q^2)$

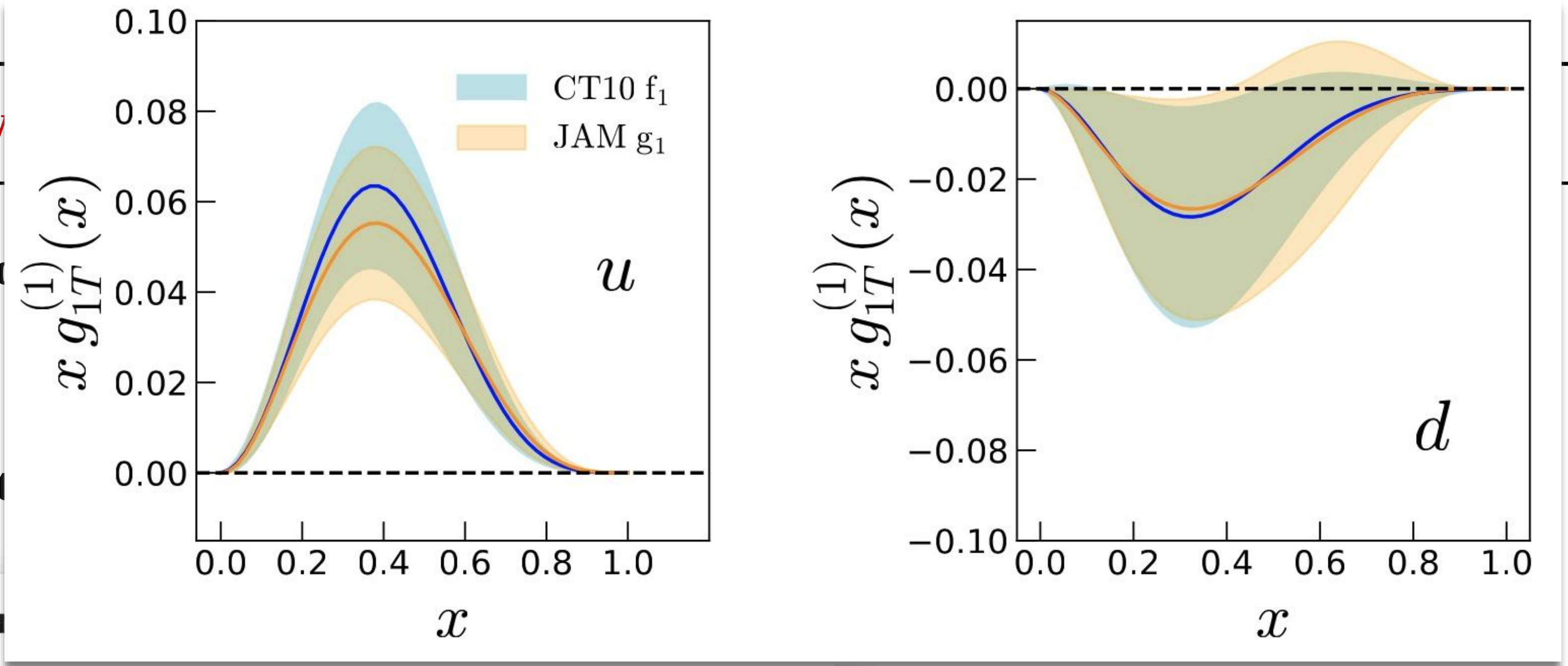
$f_1(y,Q_0^2) \rightarrow g_1(y,Q_0^2)$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests w

- **Different unpolarized PDF sets do**

- **Parameterizing in terms of helicit**



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$g_1(x,Q^2)$

$g_1(y,Q_0^2)$

Extraction of g_{1T} TMD from HERMES, COMPASS & JLab data

Tests with other PDF sets (not shown in paper)

- **Different unpolarized PDF sets do not change our results**
- **Parameterizing in terms of helicity PDF does not change our results**
- **One can also try parameterization in terms of integrals convolutions with helicity ("inspiration" from from WW-type relations):**
    - **Like above, we do not expect changes**